3

CIE Standard Colorimetric System

As described in Chapter 2, there are two types of color specification system. One, exemplified by the Munsell system, is based on color appearance. The other, exemplified by the CIE system, is based on additive color mixture. The Munsell color system can be understood intuitively because actual color chips form the basis of the system. However, additional operations such as interpolation or extrapolation are necessary for the specification of an arbitrary color, and the results are thus of relatively low precision. On the other hand, high precision can be obtained in the CIE system by using spectrophotometric colorimetry, and color can be specified precisely for any arbitrary color stimulus. Accordingly, the CIE system is commonly used in industrial and other quantitative applications. In this chapter, the specification of the CIE standard colorimetric system is described in detail.

3.1 *RGB* COLOR SPECIFICATION SYSTEM

In a color mixture system, once the color matching functions have been determined, the tristimulus specification of any arbitrary color stimulus can be determined easily. However, standardization is

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necessary in order to be able to compare results, because the color matching functions depend on the basic and reference stimuli. Accordingly, the Commission Internationale de l'Eclairage (CIE) established standard color matching functions in 1931 based on the following principles (CIE 1986, 2004a).

1. The reference stimuli [R], [G] and [B] are monochromatic lights of wavelength λ_R=700.0 nm, λ_G=546.1 nm, and λ_B=435.8 nm, respectively.
2. The basic stimulus is the white color stimulus of the equi-energy spectrum. The amounts of the reference stimuli, [R], [G] and [B], required to match the basic stimulus are in the ratio 1.0000:4.5907:0.0601 when expressed in photometric units, and 72.0966:1.3791:1.0000 when expressed in radiometric units.

Thus an equi-energy white light of $1.0000 + 4.5907 + 0.0601 = 5.6508$ lm can be matched by additive color mixing of 1.0000, 4.5907 and 0.0601 lm of the reference stimuli [R], [G] and [B], respectively. By dividing each of these amounts by their respective luminous efficiencies, the ratio of the radiometric quantities of the three stimuli can be obtained, i.e., $243.783 : 4.66333 : 3.38134 = 72.096 : 1.3791 : 1.0000$. In establishing the color matching functions, the CIE adopted an average of the data reported by Guild (Figure 3.1) obtained from seven observers (Guild 1931) and those by Wright (Figure 3.2) from ten observers (Wright 1928–1929). The color matching functions thus obtained are assumed to be representative of people having normal color vision. The color matching functions are shown in Figure 3.3. Note 3.1 gives details of the procedure by which these functions were derived from Guild and Wright's results.

Figure 3.1 John Guild (1889–1979)

Figure 3.2 William David Wright (1908–1998)

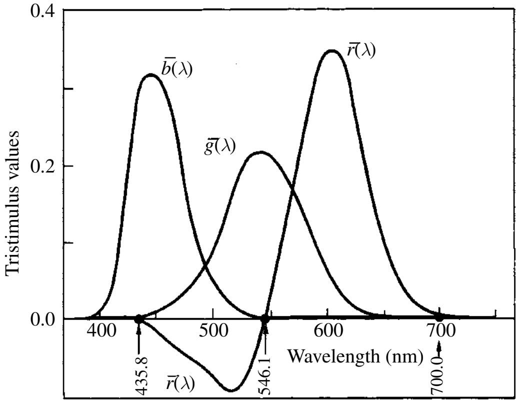

Figure 3.3 Color matching functions $\overline{r}(\lambda)$, $\overline{g}(\lambda)$ and $\overline{b}(\lambda)$ of the CIE 1931 *RGB* color specification system

As described before, the color matching functions are the amounts of the reference stimuli [R], [G] and [B] needed to match monochromatic stimuli of each wavelength. However, as shown in Figure 3.3, the color matching functions include a negative portion. At first sight, this seems strange because a negative value signifies that a negative amount of the reference stimulus is needed to make the match. This happens because of the following fact. Even though an observer may try various ways to mix the three reference stimuli, he or she will find it impossible to match any monochromatic light stimulus [F_λ], using monochromatic reference stimuli [R], [G] and [B], because the test light is always too saturated, except in the trivial case where the test stimulus is identical to one of the reference stimuli. Thus, in a practical color matching experiment, the saturation of [F_λ] is lowered by mixing it with a certain amount of

[R], for example, and then matching the mixture by a combination of [G] and [B]. The color equation in this case is expressed by

$$[F_\lambda] + R[R] = G[G] + B[B] \tag{3.1}$$

Assuming that Grassmann's Laws apply, the equation can be transformed into the following form

$$[\mathrm{F}_\lambda] = -R[\mathrm{R}] + G[\mathrm{G}] + B[\mathrm{B}] \tag{3.2}$$

Thus, the first term $-R$ is negative, and this is the reason why partly negative color matching functions are obtained.

This color equation can be regarded as a vector equation in three-dimensional space by taking [R], [G] and [B] as vector components. The three-dimensional space so constructed is used for the geometrical expression of colors and is called a *color space*. Thus, as shown in Figure 3.4, any color [F] can be located in the space at the point defined by the matching amounts of [R], [G] and [B], i.e., R, G and B. The intersection (r, g, b) of the vector [F] and the unit plane $R + G + B = 1$ is commonly used to express color [F] according to the following equations

$$\begin{aligned} r &= R/(R+G+B) \\ g &= G/(R+G+B) \\ b &= B/(R+G+B) \end{aligned} \tag{3.3}$$

It is apparent that $r + g + b = 1$. Thus, two coordinates out of the three, for example (r, g), are sufficient to locate the color [F] in the unit plane. The coordinates (r, g, b) determined in this manner are called chromaticity coordinates, and a diagram showing two

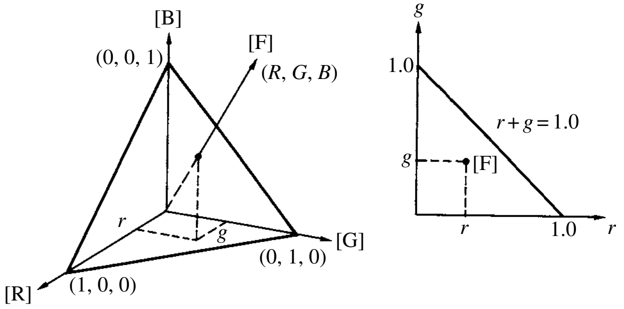

Figure 3.4 Three-dimensional expression of color [F] and chromaticity diagram

chromaticity coordinates in a plane is called a chromaticity diagram. The chromaticity coordinates of color [F] define a point in the chromaticity diagram known as a chromaticity point. The psychophysical property defined by the chromaticity coordinates is called the chromaticity of [F]. The chromaticity coordinates for monochromatic stimuli are called spectral chromaticity coordinates, and the curve obtained by connecting the chromaticity points of the monochromatic stimuli in wavelength order is known as the spectrum locus. The straight line connecting the two ends of the spectrum locus is called the purple boundary, and it represents the additive color mixture of the monochromatic stimuli (blue and red) located at the ends of the visible spectrum. Along the purple boundary, the color changes continuously from blue to red via various shades of purple.

The system for expressing color established by the definitions and normalization above is called the CIE 1931 *RGB* Color Specification System. Figure 3.5 shows the *rg* chromaticity diagram of this system and the chromaticity point W_E (1/3, 1/3) for the equi-energy white color. The chromaticity points for all real colors, i.e., those that can exist in practice, are located inside the region enclosed by the spectrum locus and the purple boundary. However, colors outside this region, for example a chromaticity point with $r = 1.0$ and $g = 1.0$, can be considered from a mathematical point of view. These colors cannot exist in practice and are therefore called imaginary colors. However, as is described in Section 3.2, imaginary colors are used for converting the *RGB* color specification system into a convenient *XYZ* color specification system.

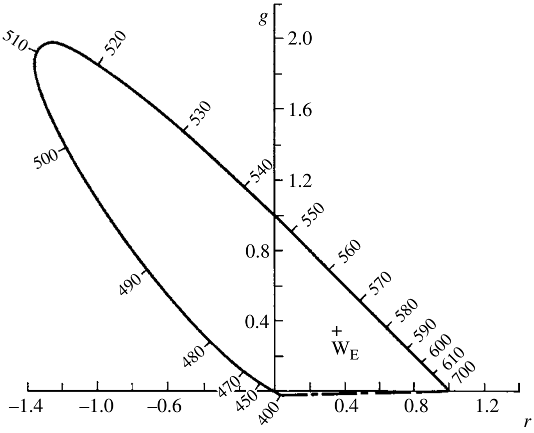

Figure 3.5 Chromaticity diagram for the CIE 1931 *RGB* color specification system and chromaticity point W_E for the equi-energy white color