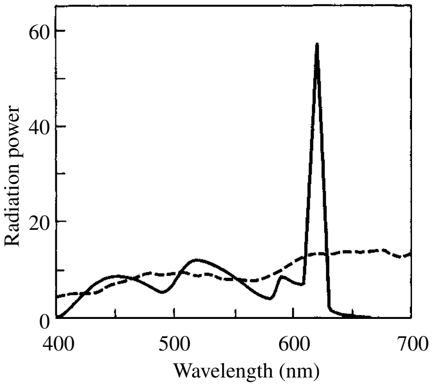

Figure 3.14 Actual skin color (broken lines) and skin color reproduced in color television (solid line) (Wyszecki and Stiles 1982). Reproduced by permission of Wiley

spectral radiant power of an actual skin color under daylight and that of the same skin color reproduced by color television (Wyszecki and Stiles 1982). Although these metamers give the same tristimulus values, there is a large difference in the spectral characteristics. The CIE has recommended evaluation methods for the degree of metamerism exhibited when there is a change in the spectral distribution of the illuminant or in the color matching functions of the observer (see Sections 7.4 and 7.5)

3.6 DOMINANT WAVELENGTH AND PURITY

As shown in Figure 3.15, the distance and direction from a specified chromaticity point W in the *xy* chromaticity diagram are sometimes used to specify chromaticity instead of x and y. The point W is known as the *white point* and represents an achromatic stimulus [W] (a color stimulus seen as achromatic under normal observation conditions). If the intersection of the spectrum locus with the straight line through the white point W and the test point F_1 is D, the color $[F_1]$ represented by F_1 can be obtained by properly mixing the white color stimulus [W] and the monochromatic light stimulus [D] represented by point D. The ratio of distances WF_1/WD is a scale that indicates how near $[F_1]$ is to the monochromatic stimulus [D], and is called the excitation purity, p_e of $[F_1]$. The wavelength of the monochromatic stimulus at the intersection D is called the dominant wavelength of $[F_1]$, and is denoted by the symbol λ_d. The excitation purity p_e can be expressed in terms of the chromaticity coordinates x_w and y_w of the white point

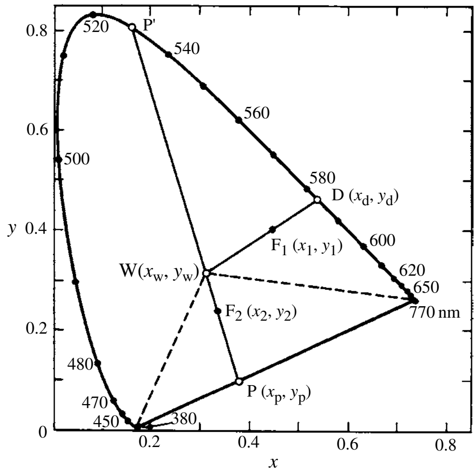

Figure 3.15 Dominant wavelength and excitation purity

W, the chromaticity coordinates x_1 and y_1 of the test point F_1, and the chromaticity coordinates x_d and y_d of the intersection point D

$$\begin{aligned} p_e &= WF_1/WD \\ &= (x_1 - x_w)/(x_d - x_w) \\ &= (y_1 - y_w)/(y_d - y_w) \qquad (3.17) \end{aligned}$$

The equations involving x and y are equivalent in Equations 3.17. The one having the larger divisor is recommended to obtain a result with higher precision.

When the color is in the purple region defined by broken lines in Figure 3.15, as is the case with color [F_2], the cross point P is not on the spectrum locus, but on the purple boundary. In such a case, the excitation purity p_e can be obtained as follows

$$\begin{aligned} p_e &= WF_2/WP \\ &= (x_2 - x_w)/(x_p - x_w) \\ &= (y_2 - y_w)/(y_p - y_w) \qquad (3.18) \end{aligned}$$

where x_p and y_p are the chromaticity coordinates of point P. The wavelength of the monochromatic stimulus [P′], where P′ is the point that the straight line extended in the direction of W crosses the spectrum locus, is used instead of the dominant wavelength. This wavelength is called the complementary wavelength, and is denoted by the symbol λ_c.

For light sources, the white point is usually set at $x_w = y_w = 1/3$, whereas for object colors it is usually set at the illuminant point. Then, instead of using chromaticity coordinates, the dominant wavelength or complementary wavelength can be used in combination with the excitation purity to define the color stimulus. Because, roughly speaking, the dominant wavelength expresses hue and the excitation purity expresses chroma, this method of specification is helpful in directly understanding the appearance of a color stimulus.

Let the X tristimulus values of the stimuli [W], [F$_1$] and [D] be X_w, X_1 and X_d, respectively, let the sums (stimulus sums) of the three tristimulus values be S_w, S_1 and S_d, respectively, and the x chromaticity coordinates be x_w, x_1, and x_d respectively. Then, the following relations can be obtained

$$\begin{aligned} &x_w = X_w/S_w \qquad x_1 = X_1/S_1 \qquad x_d = X_d/S_d \\ &X_1 = X_w + X_d \qquad S_1 = S_w + S_d \end{aligned} \tag{3.19}$$

By substituting these relations into Equations 3.17, the following can be obtained

$$\begin{aligned} p_e &= (x_1 - x_w)/(x_d - x_w) \\ &= (X_1/S_1 - X_w/S_w)/(X_d/S_d - X_w/S_w) \\ &= \{(X_w + X_d)/(S_w + S_d) - X_w/S_w\}/(X_d/S_d - X_w/S_w) \\ &= S_d/(S_w + S_d) \\ &= S_d/S_1 \end{aligned} \tag{3.20}$$

Thus, the excitation purity p_e is the ratio of the stimulus sum of the monochromatic stimulus [D] and the stimulus sum of the test stimulus [F$_1$]

In general, when an additive mixture of an achromatic stimulus and a monochromatic stimulus match a test stimulus, the mixing ratio is called the purity of the color stimulus. Excitation purity p_e above is one type of purity, but purity can be expressed differently by employing another definition using the ratio of the luminances of the stimuli [F$_1$] and [D]. This is called the colorimetric purity p_c, and is defined by the following equation

$$p_c = Y_d/Y_1 \tag{3.21}$$

where Y_1 and Y_d are the values of Y for the stimuli [F$_1$] and [D] respectively. Because the following relations hold,

$$Y_1 = y_1 S_1 \qquad Y_d = y_d S_d \tag{3.22}$$

Equation 3.22 can be substituted into Equation 3.21, and from Equation 3.20, the following relation can be obtained

$$\begin{aligned} p_c &= Y_d/Y_1 \\ &= y_d S_d/(y_1 S_1) \\ &= y_d p_e/y_1 \end{aligned} \tag{3.23}$$

As an example showing the use of purity, the just noticeable difference (*jnd*) in colorimetric purity Δp_c has been measured as shown in Figure 3.16 (Wyszecki and Stiles 1982). This result was obtained for monochromatic light with a wavelength of 650 nm, but a similar result can be obtained for other wavelengths. Figure 3.17

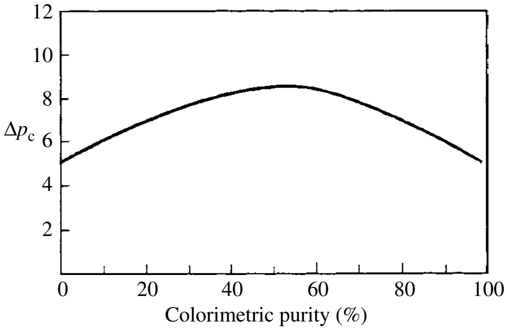

Figure 3.16 Just-noticeable difference Δp_c of colorimetric purity (at $\lambda = 650$ nm and white point of 4800 K) (Wyszecki and Stiles 1982). Reproduced by permission of Wiley

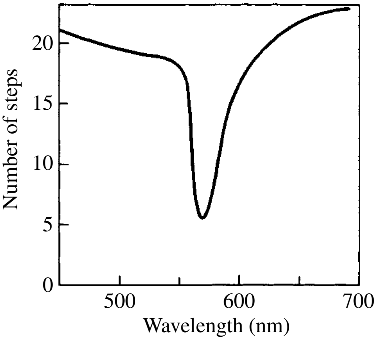

Figure 3.17 Number of just-noticeable steps between white light (4800 K) and monochromatic light (Wyszecki and Stiles 1982). Reproduced by permission of Wiley

shows the distinguishable number of colored lights between white light and monochromatic light derived from the result illustrated in Figure 3.16.

Color specification by means of dominant wavelength and purity has been used frequently because it can be readily understood, but recently chromaticity coordinates have been preferred. In particular, colorimetric purity has a discontinuity in the purple region, and is therefore used less because of this inconvenience.

3.7 COLOR TEMPERATURE AND CORRELATED COLOR TEMPERATURE

As described above, a color stimulus can be specified in three dimensions by tristimulus values, and in two dimensions by chromaticity coordinates or by dominant wavelength and excitation purity. The variables for the methods are (X, Y, Z), (x, y) and (λ_d, p_e), respectively. Of course, the two-dimensional specifications can only distinguish among a restricted set of colors that all have the same luminance.

To go one step further, for stimuli with the spectral power distribution of an ideal black body (see Section 3.8), the absolute temperature and the spectral power distribution of the radiation (black-body radiation) are in 1 to 1 correspondence. (Absolute temperature is a temperature scale first introduced by Lord Kelvin in 1848 and thus is sometimes called 'Kelvin temperature'. Originally, the unit for this scale was the 'degree Kelvin' (abbreviated °K), but it is now known simply as the kelvin (abbreviated K). The lowest theoretically possible temperature (absolute zero) is set at 0 K, and the temperature (0.01°C) of the triple point of water (a state in which ice, water, and water vapor co-exist) is defined as 273.16 K. Thus the absolute temperature can be obtained by adding 273.15 to the Celsius temperature (°C) used in everyday life). Thus, the colors of these stimuli can be specified by only one variable (absolute temperature of the black body). The color specification based on this concept is called color temperature or correlated color temperature. Color temperature (usually denoted by T_c) expresses the chromaticity of a given radiation by the temperature of the black body having the same chromaticity as that of the radiation. For radiation whose chromaticity is not exactly equal to that of a black body, correlated color temperature T_{cp} is defined as the temperature of the black body whose chromaticity is nearest to that of the radiation. The absolute temperature scale (in kelvin) is used for expressing these temperatures.