

Mathematics in Engineering Education

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ABSTRACT: Mathematics occupies a particular place among the exact sciences. The language, theoretical methods, and logic of physical and engineering sciences are based on mathematics since it concentrates the most important results of exact thinking of scientists and engineers. Mathematics deals with the general structure of argumentation, no matter the physical meaning of the used concepts. Then other sciences apply the reasons and solutions to create and develop theoretical models of the phenomenon. Mathematics possesses fundamental and constantly growing importance for the technical progress. The modern teaching of mathematics has changed radically in the last decades. The Internet tools, computer software, and audio-visual aids revolutionized teaching. The access to the Internet and computer tools changed also the traditional education concept and resulted in new forms of education, for instance, the web-based distance learning. Taking into account the tendencies of rapid change in educational activities and the level of development of engineering education, international cooperation of universities is becoming an insistent need to provide students new educational opportunities.

INTRODUCTION

Mathematics is gaining in greater importance for technical progress, economy, medicine, *etc*. It is difficult to find a branch of human activity that can do without mathematical methods. Experimentalists, engineers and constructors apply mathematical approaches for statistic data processing of experiments; social scientists are based on mathematical methods to extrapolate the results of public-opinion poll obtained by sampling observation of a finite group; political scientists use mathematical models to predict the results of elections; business managers practice mathematics for creation and development of economical strategy of trading companies; students calculate their chances to pass an examination, *etc*.

The special role of mathematics in natural sciences is generally agreed. Everything seems to be all right, and one can be pleased saying about the place of mathematics in the system of engineering education. In reality, however, it is a wrong impression, since the level of math education and math culture is steadily decreasing during the last decades. It is urgent to express concern over math training in engineering education. Let us try to make clear the actual state of things.

MATHEMATICS IN THE SYSTEM OF ENGINEERING EDUCATING: TO PROVE OR NOT TO PROVE

Everybody got used to the wording "Mathematics is very important" which became a stock phrase to such a degree that the many forgot about the true meaning of mathematics. Proofs of theorems become a thing of the past in mathematical courses, even intended for engineering specialties. Mathematics is considered in practice no more than an useful auxiliary in the study of engineering sciences. A similar interpretation of mathematics has an influence like the narcotic effect: the illusion of the well-being and the paralysis of creative thinking in the future. The result will not keep waiting itself.

One has not to treat mathematics as only a means allowing to carry out transformations and calculations of expressions or to write down solutions of equations. The ability to reproduce some manipulations with expressions or functions has relation only to the simplest mathematics, since such actions require only trivial logic. Let us do not forget that a rigorous proof of a statement is one of the main goals of mathematics.

At first sight, engineers do not need rigorous proofs of mathematical statements, solving an engineering problem. However, they must have a high level of mathematical culture in order to solve serious engineering problems. Itself procedure of proving propositions forms the level of thinking, that is, rigorous mathematical proofs have to consider as an end in itself. It is not surprising that many philosophers receive mathematics training of exact reasoning to develop their own thinking.

Mathematics possess by a particular attraction due its logic. It should not wonder if many students hate mathematics. It is very difficult to study a subject, the matter of which consists in definitions, rules and formulations of theorems. Only a small part of students is able to orient in this information stream consisting of definitions and naked facts and to pick out the key points of the subject. The many do not understand the intrinsic logic of mathematics, and so they do not like mathematics.

Everybody likes mathematics who understands it.
Is it necessary to study "a cut-down version" of mathematics, if any process can be simulated by computer aids? The question seems to be quite reasonable.

Really, some professions require only the simplest mathematical knowledge due to possibilities of modern computer software which are practically unbounded. However, the ability to know and reproduce mathematical definitions or rules is necessary but not sufficient for a future engineer. Hard work is required of students in mathematical course in order to master clearly the intrinsic logic of mathematics; to get hands-on training in the proving of propositions, and then to apply a similar logical schema to special engineering courses.

ESTIMATION OF KNOWLEDGE

If a system of estimation of knowledge is based on a list of questions which students should know to pass an examination successfully, then the system stimulates memorizing the answers to the most asked questions. In order to avoid similar "stimulus", the list of questions should be replaced by the list of problems. The set of well thought-out problems allows the examiner to get a more objective judgment about a skill level of a student but not his (her) ability to reproduce some parts of a manual of mathematics. The matter of the problems has naturally to comply with requirements of education programs. Systems of estimation based on similar principles, stimulate students to work constructively, to study fundamental mathematical problems, and to apply other tools, for example, Internet-based tools or computer software.

MATHEMATICS GIVES A WAY OF CONVERSION OF ONE SET OF STATEMENTS TO ANOTHER

Mathematics combines a language and logic together. Many different (from the physical point of view) processes are described by an identical equation, the solution of which does not depend on the physical meaning of the process. Since math operates with the general structure of reasoning, other sciences can apply similar ideas for creation of the corresponding theoretical model. Engineers and physicists know from experience that it is very important to have an alternative formulation available to solve a difficult problem. By making use of mathematical and physical methods together one can express the laws of nature in different forms. For example, the principle of least action represents the generalization of some mechanical laws. However, its domain of applicability is not restricted by mechanical problems only, and it extends also over electrodynamic phenomena and quantum processes.

Mathematics helps us to understand deeper the mutual correspondence of different approaches to the same problem.

MATH PROVIDES THE WAY OF DESCRIPTION OF NATURE

Considering some problem, engineers and physicists express the suitable law of nature in a form of equation, then solve the equation, interpret its solution, and compare the result with data of experiments. However surprisingly it may seem that simplified mathematical model describes the real physical process! French physicist Dirac guesses right the corresponding mathematical equation and he discovered a law of relativistic quantum mechanics. Maxwell's electrodynamics was based on an erroneous conception of space. However, the physical theory expressed in a mathematical form, is true.

Any fundamental physical law can be formulated by means of a few mathematical models. Just that hypothesis stands a better chance to be valid, whose mathematical representation possesses the property of simplicity.

It seems that Nature knows mathematics and manages processes by means of math.

MATH REASONING

The structure of mathematical reasoning is suitable perfectly for any engineering science.
First, the primary concepts are introduced, the list of which should be as small as possible. Some concepts are considered to be undefined, that is, they are introduced by means of description of their properties, basing on intuitive notions. Other propositions assumed to be true statements, that is, they are postulated without proving.

Second, a proof of any statement takes its origin from a hypothesis, that is, the statement of the given fact. Then it is necessary to construct a logic chain consisting of true statements or demonstrations that results in a valid conclusion. It would be a good thing to accentuate specially that the matter of any proposition (e.g., a theorem) includes two parts: initial conditions and a conclusion. So the validity of any statement is restricted by requirements, assumptions, qualifications, or initial data. Some modification of the initial conditions implies a new conclusion.

NEW POSSIBILITIES OF MODERN INFORMATIONAL TECHNOLOGIES

Conditions of the incessant process of accumulation of knowledge require a reformation of engineering education. The Internet provided the access to new sources of information and changed the traditional education concept that resulted in an appearance of new forms of education.

Mathematical education of future engineers has to include the modern Internet-technologies and computer tools for training. However, taking into account the modern tendencies of teaching, we have not to run to extremes. Any computer systems cannot replace basic mathematical knowledge as tools for engineers to master technological needs currently and in the future. So it is important to find a balanced combination of tools based on new information technologies and the classic approach to math education. The fundamental component of mathematics keeps its importance, since it develops the ability to reason logically and to apply knowledge to special engineering courses. It is necessary to analyze the existing educational programs, approaches, and methodical materials in view of their correspondence to the modern tendencies of engineering education and to pick out the conceptions of central importance.

WEB-BASE COURSES

Originally, web-based systems have been created for organization of the online distance learning. However, the area of their application is not restricted by this form of education, since web-based systems possess many advantages over traditional forms of education, having great potential for the further development. For instance, web course tools (WebCT) includes some special means which allow tutors and students to discuss projects together in real time and to get the access to a different kind of information. In particular, the glossary page provides the access to information in a short form. A student gets the listing of all terms that start with the letter by clicking on the starting letter for the term. Therefore, it is not necessary to learn in advance all terms without distinction being waiting for if some problem refer to a term.

To discuss some additional advantages of web-based courses, let us consider a few fragments from our course that are represented in the figures below.

Assume that a student opens the page "The Most Important Limits" (Fig. 1) to obtain the general presentation about the section "Limits". Scanning the contents he/she discovers that:

- each of the most important limits is represented by the corresponding theorem;
- the theorems look quite inoffensively;
- each theorem gives the results of dividing zero by zero.

The button GRAPHIC ILLUSTRATION provides the access to the window with demonstrations in graphic form, while the button PROOF allows to obtain the access to the proof of Theorem 1 (Fig. 2-3).

Now the statement seems to be quite evident.

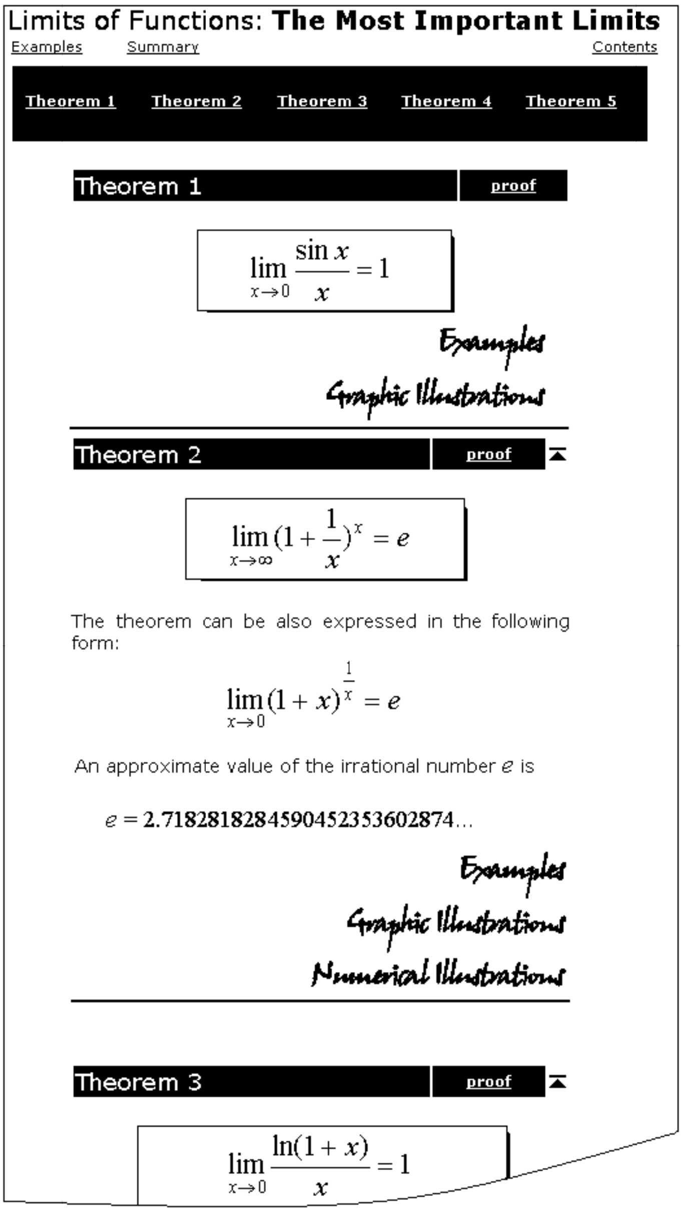


Fig. 1

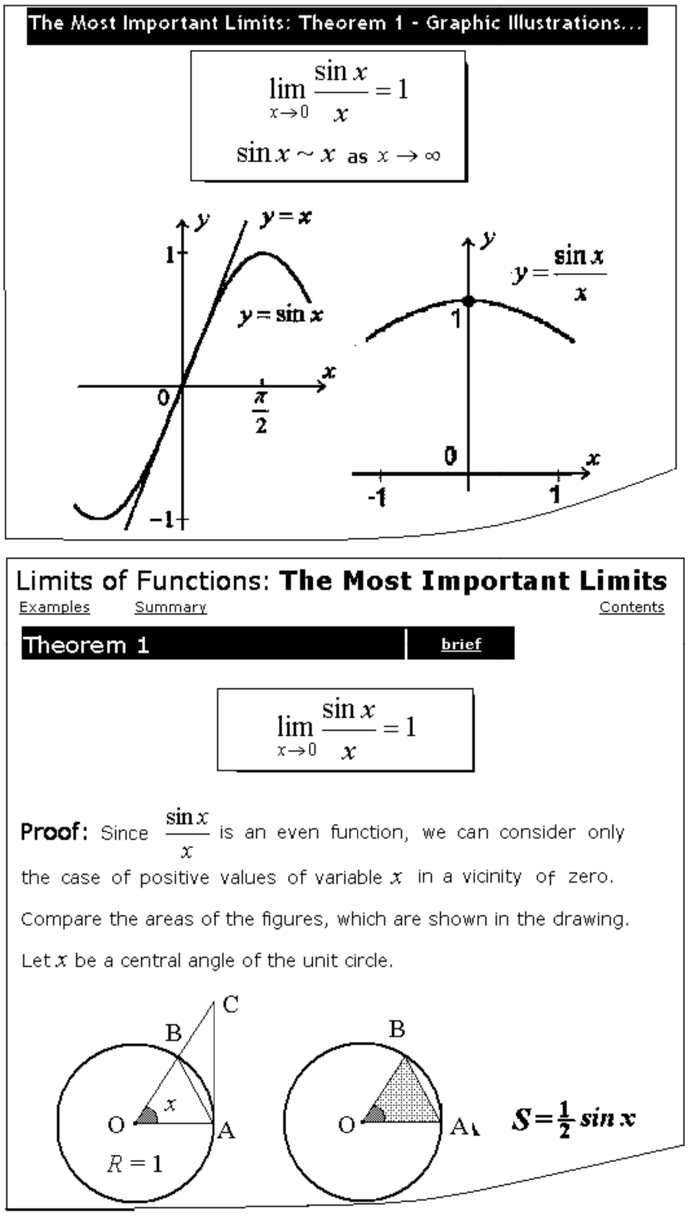


Fig. 2-3

Each web-page has an additional menu which provides the ability to navigate, for example, from the given page to the content of the page or the content of the course as well as to the next (or previous) topic.
The vocabulary and glossary allow students to get the access to the definition desired just at that moment as it is necessary.

The definition of the limit is represented by means of a few forms:

- Informal (Intuitive) Definition (Fig. 4);
- Rigorous Definition (Fig. 5);
- Helpful Rule (Fig. 6);
- Graphic Illustrations of Definition (Fig. 7).

The

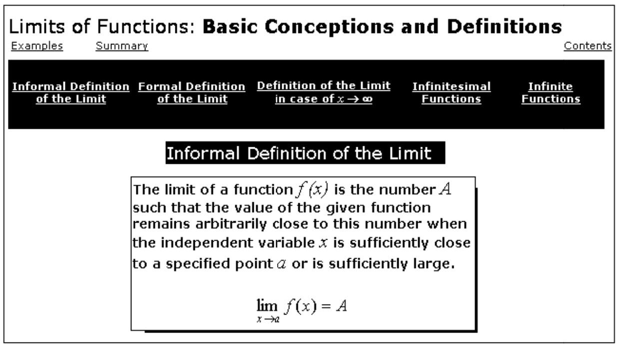


Fig. 4

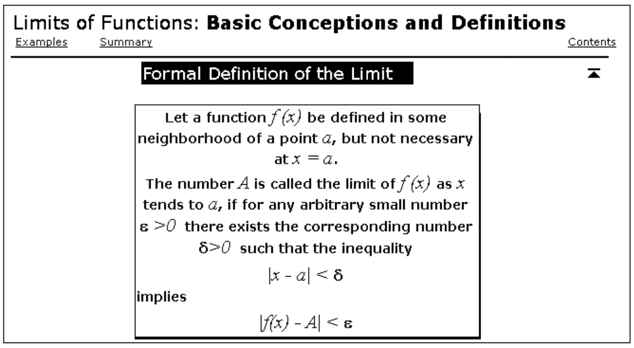


Fig. 5

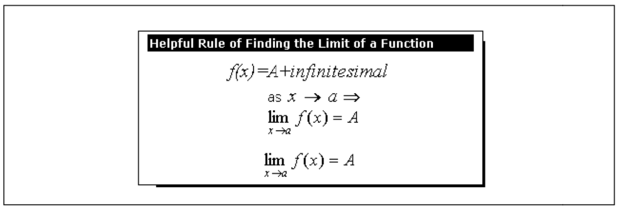


Fig. 6

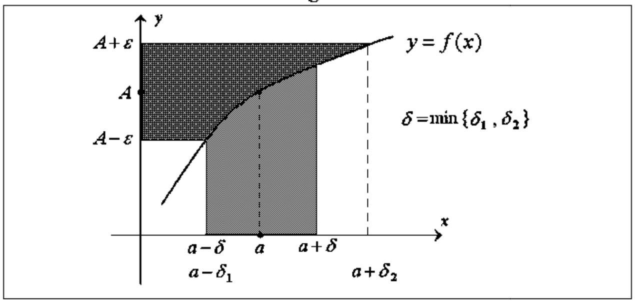
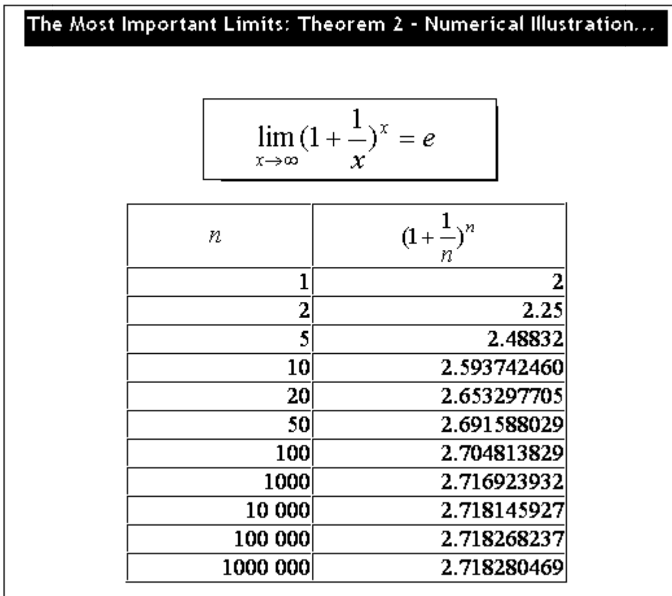


Fig. 7

Theorem 2 is also states “an impossible” proposition: 1 rased to a power (no matter the value of the buttons power?) is not 1!

buttons NUMERICAL ILLUSTRATION (see Fig. 8) and GRAPHIC ILLUSTRATIONS provide the access to the corresponding windows (see Fig. 8 and 9).


The Most Important Limits: Theorem 2 - Numerical Illustration...

$$\lim_{x\to\infty}(1+\frac{1}{x})^x = e$$

n	$(1+\frac{1}{n})^n$
1	2
2	2.25
5	2.48832
10	2.593742460
20	2.653297705
50	2.691588029
100	2.704813829
1000	2.716923932
10 000	2.718145927
100 000	2.718268237
1000 000	2.718280469

Fig. 8

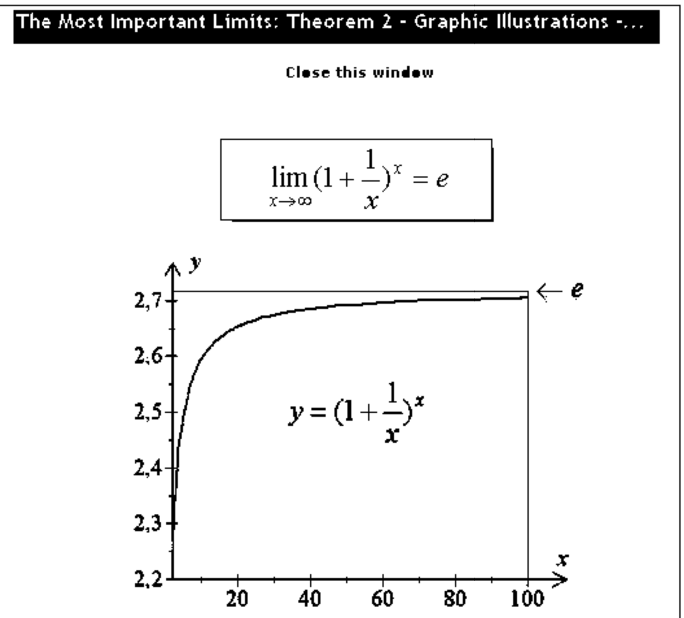


Fig. 9

CONCLUSION

Mathematics in the modern system of engineering education is unthinkable without using scientific computing environments.
A laborious calculating work (in numerical form as well as in symbolical one) can be readdressed to computer programs.

However, any computer systems cannot replace basic mathematical knowledge.