which became detached was considerably less than those which became attached. At low atomic fluxes there was an increase in the number of detachment events and the growth rate began to depend on the roughness.

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ELECTRON SCATTERING IN PHOTOEMISSION

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Kane's one-dimensional model of the scattering of photoelectrons as they propagate to the emitting surface is generalized to the three-dimensional case. It is shown that this problem reduces to the Milne problem. An analytic expression is obtained for the probability of an electron's reaching the surface.

Photoelectron emission can be regarded as a three-stage process: excitation of the electrons by photons of the incident wave, propagation of the electrons to the emitting surface, and their transition through the solid–vacuum interface. As the electrons travel to the surface, they can undergo both elastic and inelastic scattering. The problem of electron transport has been studied from various points of view by different authors [1-3]. In a one-dimensional model [1], Kane obtained an exact analytic expression for the probability of escape of electrons with allowance for the possibility of their scattering with the production of electron–hole pairs and multiple scatterings on phonons. In [2], Beckmann first solved the problem of electron transport for an infinite space, in which the transition of the electrons through an imaginary surface was then simulated. Duckett [3] studied this problem in detail in the random-walk model. In the present paper it is shown that the problem of propagation of photoelectrons to the surface in the three-dimensional generalization of Kane's model [1] can be reduced to the Milne problem. An analytic expression is obtained

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for the probability of an electron's reaching the surface.

We consider the following problem, an electron propagates from depth x to the surface. We take into account electron scattering processes in which electron–hole pairs are formed (ee scattering) and interaction with phonons (ep scattering). As in [1], we assume that if an electron undergoes ee scattering, then neither the scattered initial electron nor the secondary electron can escape into the vacuum. We shall also ignore energy losses in ep interactions.

The probability that an electron undergoes ee scattering in a short interval Δr is $\Delta r/l_e$. The analogous probability for ep scattering is $\Delta r/l_p$ (l_e and l_p are the mean free paths between ee and ep interactions, respectively). An arbitrary distance r can be divided into N small intervals Δr: $r = N\cdot\Delta r$. The probability p(r) of an electron's traversing the distance r without scattering is $(1-\Delta r/l_e)^N \cdot (1-\Delta r/l_p)^N$ and for large N takes the form $p(r) = \exp(-cr)$, where $c=a+b$, $a=l_p^{-1}$, $b=l^{e-1}$.

Let $p_n(x)$ be the probability of the process in which the electron reaches the surface from depth x without ee scattering but undergoing ep scattering n times. This probability can be represented as the product of the following probabilities integrated over the intermediate states: a) the probability of the electron's traversing the distance r in the solid angle $\sin\theta d\theta d\varphi$ without scattering; b) the probability of a subsequent ep interaction; c) the probability of reaching the surface after a further n − 1 ep interactions. Assuming that the distribution over the directions of motion of the excited and scattered electrons is isotropic, we can write down the recursion relation

$$p_n(x) = \frac{a}{2}\left\{\int_0^{\pi/2} \sin\theta d\theta \int_0^{x/\cos\theta} dre^{-cr}p_{n-1}(x - r\cos\theta) + \int_{\pi/2}^{\pi} \sin\theta d\theta \int_0^{\infty} dre^{-cr}p_{n-1}(x - r\cos\theta)\right\},$$

which is readily transformed to

$$p_n(x) = \frac{a}{2}\int_0^{\infty} dyE_1(c\,|\,x-y\,|)\,p_{n-1}(y), \tag{1}$$

where $E_1(x)$ is the exponential integral. The function $E_n(x)$ is defined by

$$E_n(x) = \int_1^{\infty} \exp(-xt)\cdot t^{-n}dt,\ (n \geqslant 0).$$

The total probability of the electron's reaching the surface is

$$g(x) = \sum_{n=0}^{\infty} p_n(x).$$

Summing (1) from 1 to ∞, we obtain an equation for g(x):

$$g(x) = p_0(x) + (a/2)\int_0^{\infty} dyE_1(c\,|\,x-y\,|)\,g(y), \tag{2}$$

where $p_0(x) = (1/2)E_2(cx)$ is the probability of the electron's reaching the surface from depth x without scattering. We introduce the integral operator Λ,

$$\Lambda\Psi(x) = (a/2)\int_0^{\infty} dyE_1(c\,|\,x-y\,|)\,\Psi(y), \tag{3}$$

and define the function $\varphi(x)$ by $\varphi(x) = c/a - g(x)$.

In the notation of (3), the equation for this function is

$$(1-\Lambda)\,\varphi(x) = b/a. \tag{4}$$

The solution $\varphi_0(x)$ of the homogeneous equation

$$(1-\Lambda)\ \varphi_0(x) = 0 \tag{5}$$

describes the density distribution of neutrons in an infinite half-space without sources, and Eq. (5) itself is called the Milne equation [4]. In what follows, we require the commutation relations

$$\frac{d}{dx}[\Lambda\Psi(x)] - \Lambda\frac{d\Psi(x)}{dx} = \frac{a}{2}E_1(cx)\Psi(0), \tag{6}$$

$$\Lambda\int_0^x dy\Psi(y) - \int_0^x dy[\Lambda\Psi(y)] = (a/2c)\int_0^\infty dyE_2(cy)\Psi(y). \tag{7}$$

Differentiating (4) and (5) and using (6), we find that the functions $d\tilde{\varphi}(x)/dx$ and $d\tilde{\varphi}_0(x)/dx$ are solutions of the same equation

$$(1-\Lambda)\Psi(x) = a/2E_1(cx),$$

i.e., they differ only by a solution of the homogeneous equation (5):

$$\frac{d\tilde{\varphi}(x)}{dx} = \frac{d\tilde{\varphi}_0(x)}{dx} + \lambda\tilde{\varphi}_0(x) \tag{8}$$

(here $\tilde{\varphi}(x) = \varphi^{-1}(0)\varphi(x)$, $\tilde{\varphi}_0(x) = \varphi_0^{-1}(0)\varphi_0(x)$, λ is a constant). Integrating (8) from 0 to x, we obtain an expression for $\tilde{\varphi}(x)$:

$$\tilde{\varphi}(x) = \tilde{\varphi}_0(x) + \lambda\int_0^x dy\tilde{\varphi}_0(y). \tag{9}$$

Thus, the problem of the electron's reaching the surface has been reduced to the solution of the homogeneous equation (5), i.e., to the Milne problem. We introduce the function $\varphi_0^+(x)$, which is equal to $\varphi_0(x)$ for $x > 0$ and vanished for $x < 0$. Using the Wiener–Hopf method, one can show [4] that the Fourier transform of this function, continued analytically with respect to the transformation variable to the complex plane, has the form

$$\hat{\varphi}_0^+(p) = i\varphi_0(0)\cdot(p+ic)(p^2+1/L^2)^{-1}\exp[-F(p)], \tag{10}$$

where L is determined by the equation $\frac{aL}{2}\ln\frac{Lc+1}{Lc-1} = 1$, and

$$F(p) = \frac{1}{2\pi i}\int_{|t|=\infty,\,\mathrm{Re}\,t<0}^{|t|=\infty,\,\mathrm{Re}\,t>0}\frac{dt}{t-p}\ln\left[\frac{t^2+c^2}{t^2+1/L^2}\left(1-\frac{a}{t}\operatorname{arctg}\frac{t}{c}\right)\right]$$

(the contour of integration passes lower than t = p and does not intersect the cut along the imaginary axis from $-i\infty$ to $-ic$ and from ic to $i\infty$). The function $\varphi_0(x)$ can be found in accordance with the inversion formula and can be conveniently expressed in the form

$$\varphi_0(x) = \varphi_0(0)\{(1/2)(Lc+1)\exp[x/L - F(i/L)] - (1/2)(Lc-1)\exp[-x/L - F(-i/L)] - f(x)\},$$

$$f(x) = (a/2c)\int_1^\infty dy\left\{y(y+1)\left[\left(1-(a/2cy)\ln\frac{y+1}{y-1}\right)^2 + a^2\pi^2/(4c^2y^2)\right]\right\}^{-1}\exp[-cxy + F(iy)].$$

To determine the constant λ, we apply the operator $1-\Lambda$ to both sides of Eq. (9). Using (4) and (7), we find that $\lambda = -2bc/[a^2\mu\varphi(0)]$, where

$$\mu = \int_0^\infty dyE_2(cy)\tilde{\varphi}_0(y).$$

We introduce the function

$$\Phi(x) = \frac{d\tilde{\varphi}_0(x)}{dx} - \frac{1}{L^2}\int_0^x dy\tilde{\varphi}_0(y) - a\mu/(2bL^2).$$

It satisfies the equation

$$(1-\Lambda)\Phi(x) = (a/2)E_1(cx) - a^2\mu/(4bcL^2)E_2(cx)$$

and decreases exponentially at infinity. We multiply this equation by $\tilde{\varphi}_0(x)$ and integrate from 0 to ∞. Noting that

$$\int_0^\infty dx\varphi_0(x)(1-\Lambda)\Phi(x) = \int_0^\infty dx\Phi(x)(1-\Lambda)\varphi_0(x) = 0,$$

we determine μ: $\mu = (2L/\beta)\sqrt{1-\beta}$, $\beta = a/c$.

Thus, the probability of the electron's reaching the surface is

$$g(x) = \beta^{-1} + \left(g(0) - \frac{1}{\beta}\right)\tilde{\varphi}_0(x) + (\beta L)^{-1}\sqrt{1-\beta}\int_0^x dy\tilde{\varphi}_0(y).$$

Since g(x) cannot increase as $x \to \infty$, we conclude from the last equation that $g(0) = \beta^{-1}(1-\sqrt{1-\beta})$. It is easy to show that for all physically meaningful β $(0 \leq \beta \leq 1)$ the necessary condition $1/2 \leq g(0) \leq 1$ is satisfied. Thus, the solution of Eq. (2) with boundary condition specified at infinity has the form

$$g(x) = \beta^{-1}[1 - \sqrt{1-\beta}\tilde{\varphi}_0(x) + L^{-1}\sqrt{1-\beta}\int_0^x dy\tilde{\varphi}_0(y)]. \tag{11}$$

We assume further that the exciting light is damped exponentially in the matter. Then the mean probability of an electron's reaching the surface is

$$G = \alpha\int_0^\infty dxe^{-\alpha x}g(x) = \beta^{-1}[1 - \sqrt{1-\beta}(\alpha - 1/L)\varphi_0^{-1}(0)\hat{\varphi}^{+}(i\alpha)].$$

Substituting $\hat{\varphi}_0^{+}(i\alpha)$ from (10) in this equation, we finally obtain

$$G = (l_p/l)\{1 - \sqrt{1 - l/l_p}[(\alpha L + L/l)/(\alpha L + 1)]\exp[-F(i\alpha)]\}, \tag{12}$$

where we have introduced the effective mean free path $l \equiv c^{-1}$. One can show that when the electron–phonon interaction is switched off (which is achieved by going to the limit $l_p \to \infty$) Eq. (12) goes over into the well-known expression obtained in [5]: $G = (1/2)[1 - (1/\alpha l)\ln(1 + \alpha l)]$. For this it is necessary to note that in the limit $l_p \to \infty$

$$F(i\alpha) \Rightarrow (1/\alpha l_p)\ln(1+\alpha l), \text{ and } L \Rightarrow l[1+2\exp(-2l_p/l)].$$

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