THERMODYNAMIC CYCLES OF IDEAL GAS ENGINES

THERMODYNAMICS

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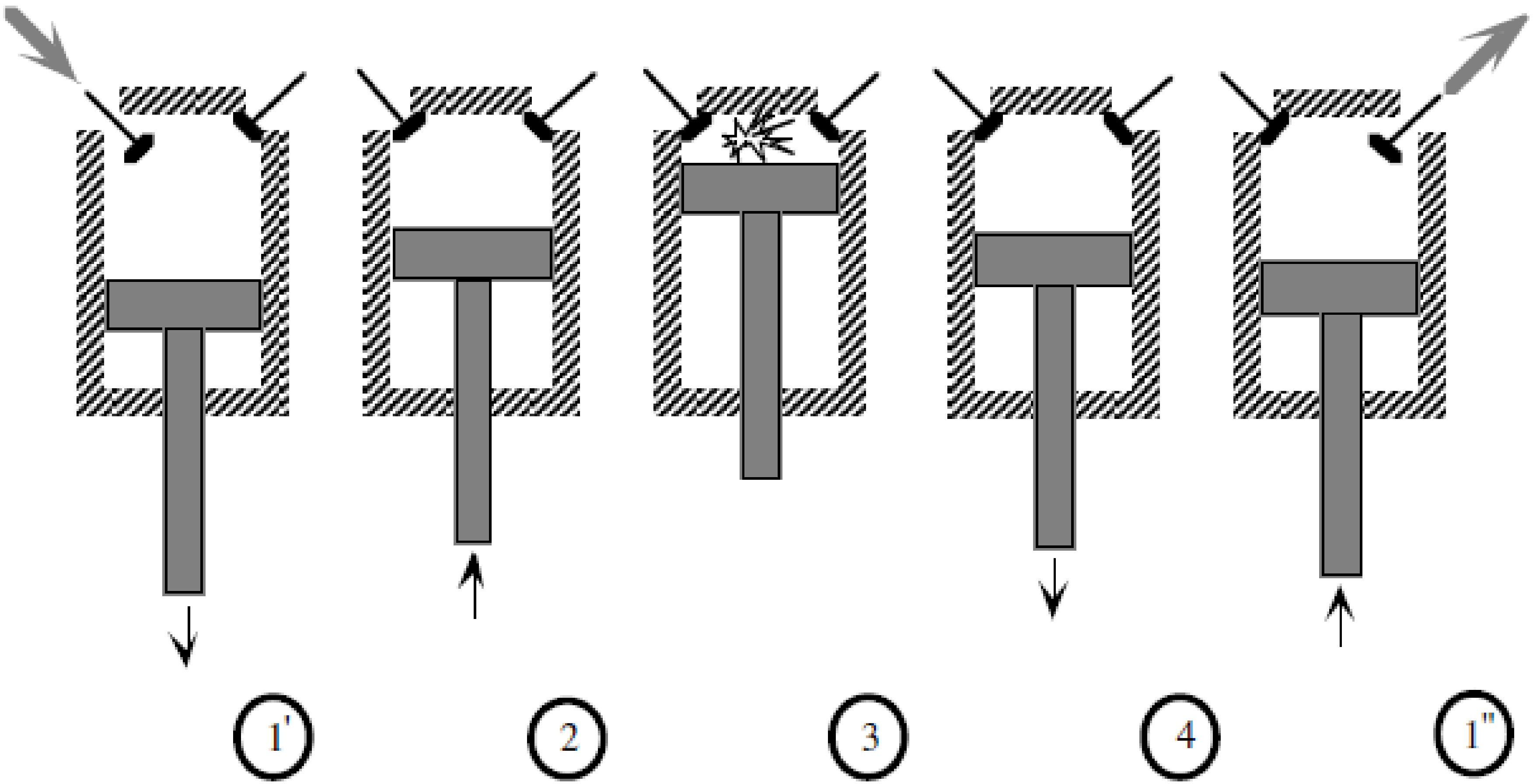
Thermodynamic cycles and heat engines

- This lecture is devoted to main points of heat engine work. Today we will concentrate on thermal engines with ideal gas as working medium.
- **Heat engines** are power plants designed to convert heat into mechanical work.
- Our focus today will be on defining general characteristics of thermal power plant cycles (like work done, heat supplied, heat discharge or cycle efficiency) using thermal parameters in key points.

The Otto Cycle

- The Otto cycle is an idealization of a set of processes used by spark ignition internal combustion engines (2-stroke or 4-stroke cycles). These engines a) inject a mixture of fuel and air, b) compress it, c) cause it to react, thus effectively adding heat through converting chemical energy into thermal energy, d) expand the combustion products, and then e) eject the combustion products and replace them with a new charge of fuel and air.

Otto cycle illustration



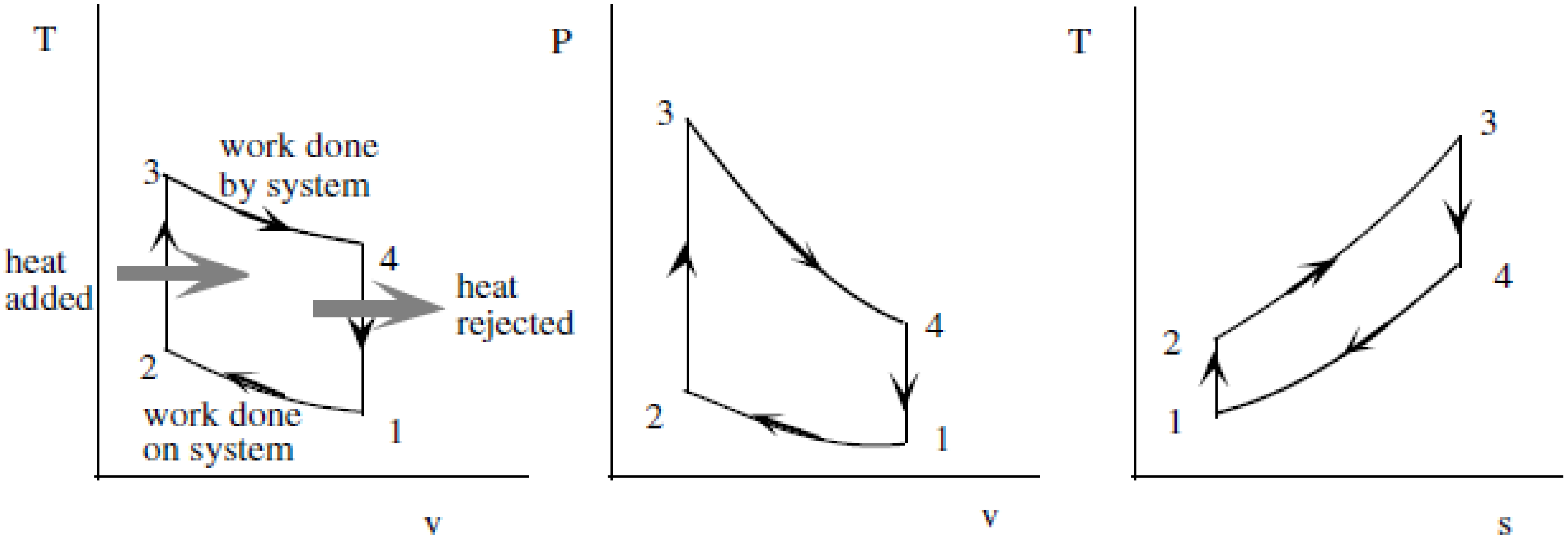
Otto cycle idealization

Representation of the heat engine as a thermodynamic cycle.

- Ingest mixture of fuel and air – not considered as a process
- 1' - 2 Compress mixture adiabatically
- 2 - 3 Ignite and burn mixture at constant volume (heat is added)
- 3 - 4 Expand mixture adiabatically
- 4 - 1" Cool mixture at constant volume

In order to get closer to real heat engines the irreversibility of compression and expansion process could be taken into account.

Otto cycle in diagrams



Otto cycle efficiency

- Cycle efficiency= $\dfrac{\textbf{Net work done by system}}{\textbf{Heat supplied to system}}$

- Net work done by system = (Work of expansion) – (Work of compression)

- Assuming both processes to be adiabatic and perfect:

$$\Delta w=(u_3-u_4)-(u_2-u_1)$$

- Assuming heat capacity of gas to be constant we get:

$$\Delta w=c_v[(T_3-T_4)-(T_2-T_1)]$$

Otto cycle efficiency

- Assuming compression ratio (and expansion ratio as well for perfect process) to be equal $v_1/v_2=v_4/v_3=r$ and process to be quasi-static, adiabatic and reversible we get:

$$T_2/T_1=(v_1/v_2)^{k-1}=T_3/T_4$$

- In that case net work would be equal to:

$$\Delta w=c_v T_1(T_4/T_1-1)(r^{k-1}-1)$$

- Taking into account heat of combustion ($q_{comb}=c_v(T_3-T_2)$) we get:

$$T_4/T_1-1=((T_3-T_2)/T_1)(1/r^{k-1}) \text{ and}$$

$$\Delta w=q_{comb}(r^{k-1}-1)/r^{k-1}$$

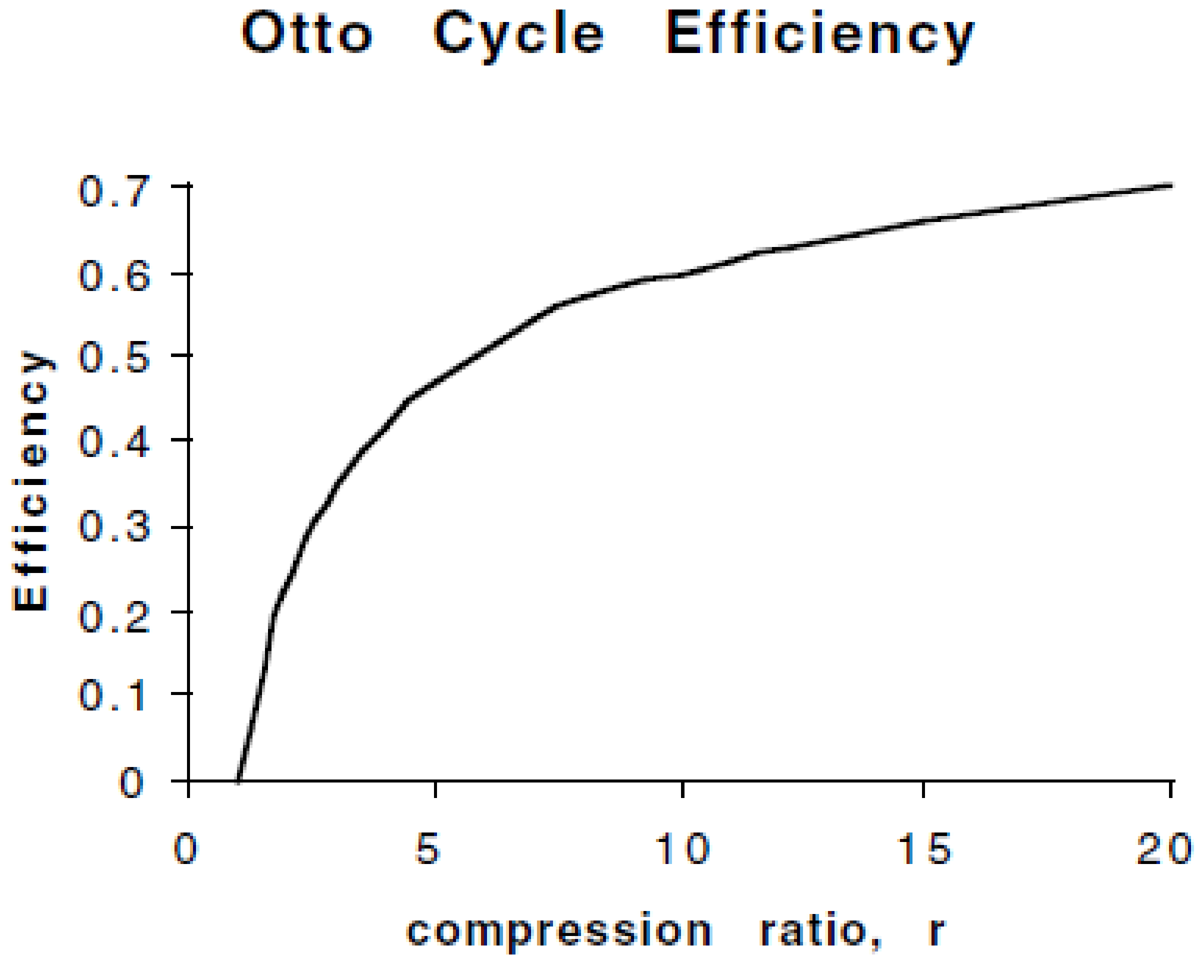
Otto cycle efficiency

- In the end, Otto cycle efficiency could be obtained as:

$$\eta=(\text{net work})/(\text{heat supplied})=(r^{k-1}-1)/r^{k-1}$$

$$\eta=1-(\text{heat discharged})/(\text{heat supplied})=1-q+q/r^{k-1}$$

Otto cycle efficiency dependence on compression ratio



Irreversible Otto cycle

- Taking into account efficiency of compression and expansion processes as:

$$\eta_{comp}=(T_2-T_1)/(T_{2r}-T_1) \text{ and } \eta_{exp}=(T_3-T_{4r})/(T_3-T_4)$$

- We get temperature in real points 2r and 4r as follows:

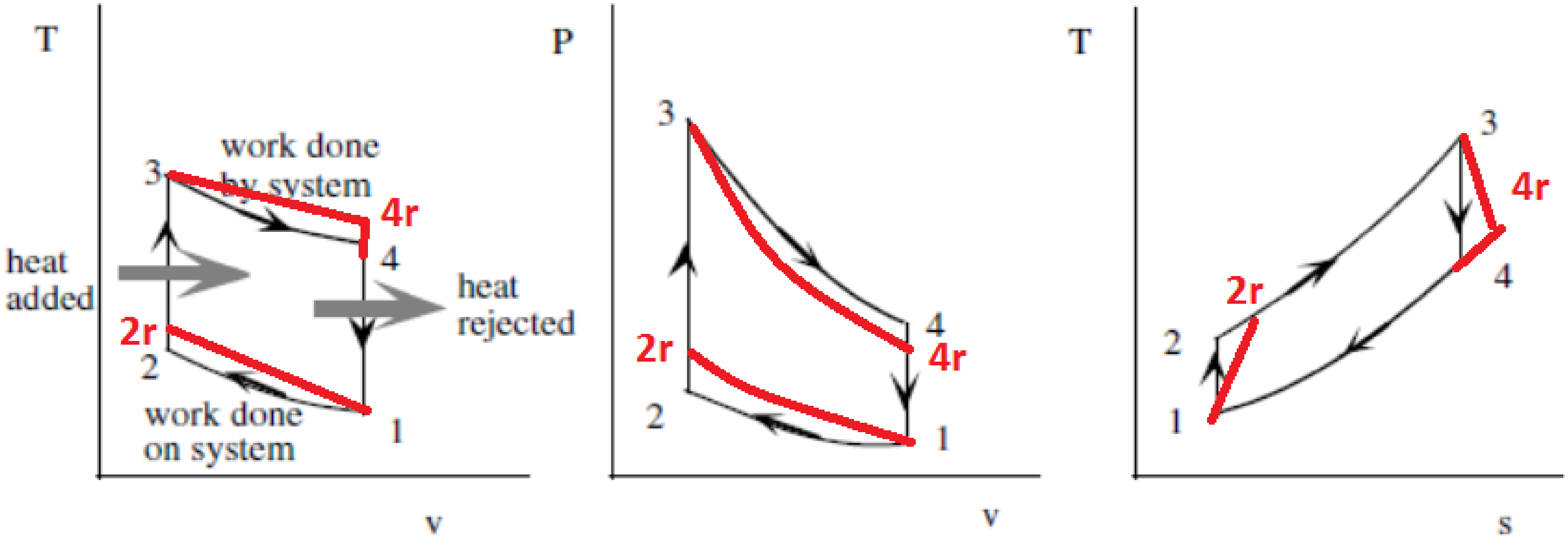
$$T_{2r}=T_1+(T_2-T_1)/\eta_{comp}=T_1(1+(r^{k-1}-1)/\eta_{comp})$$

$$T_{4r}=T_3-(T_3-T_4)\eta_{exp}=T_4(1-(1-r^{k-1})\eta_{exp})$$

- In that case net work could be written as (in case $T_3/T_1=\tau$):

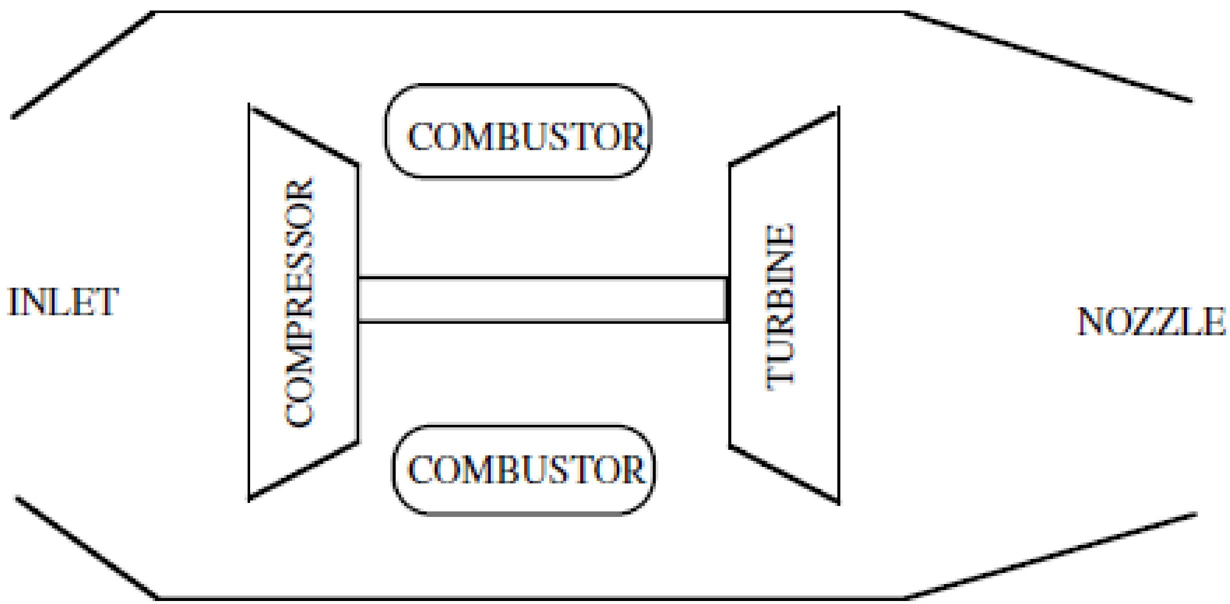
$$\eta=\frac{((1-r^{k-1})(T_3\eta_{exp}-T_1/\eta_{comp})}{T_3-T_1(1+(r^{k-1}-1)/\eta_{comp})}=\frac{((1-r^{k-1})(\tau\eta_{exp}-1/\eta_{comp})}{\tau-(1+(r^{k-1}-1)/\eta_{comp})}$$

Irreversible Otto cycle in diagrams

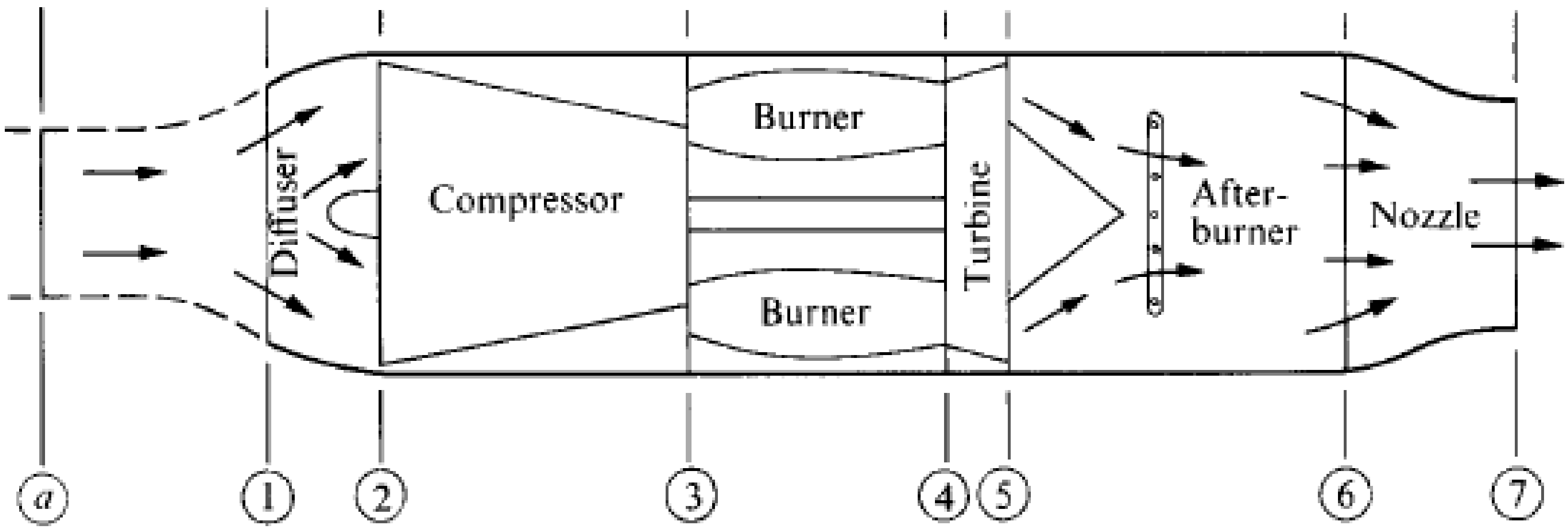


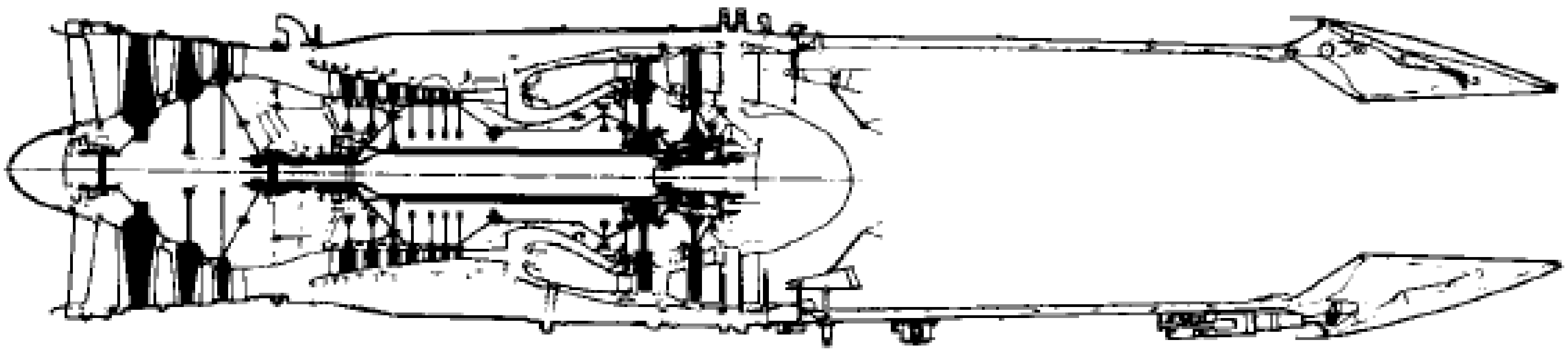
Brayton Cycle

- The Brayton cycle is an idealization of a set of thermodynamic processes used in gas turbine engines, whether for jet propulsion or for generation of electrical power.



Schematics of typical gas turbine engines

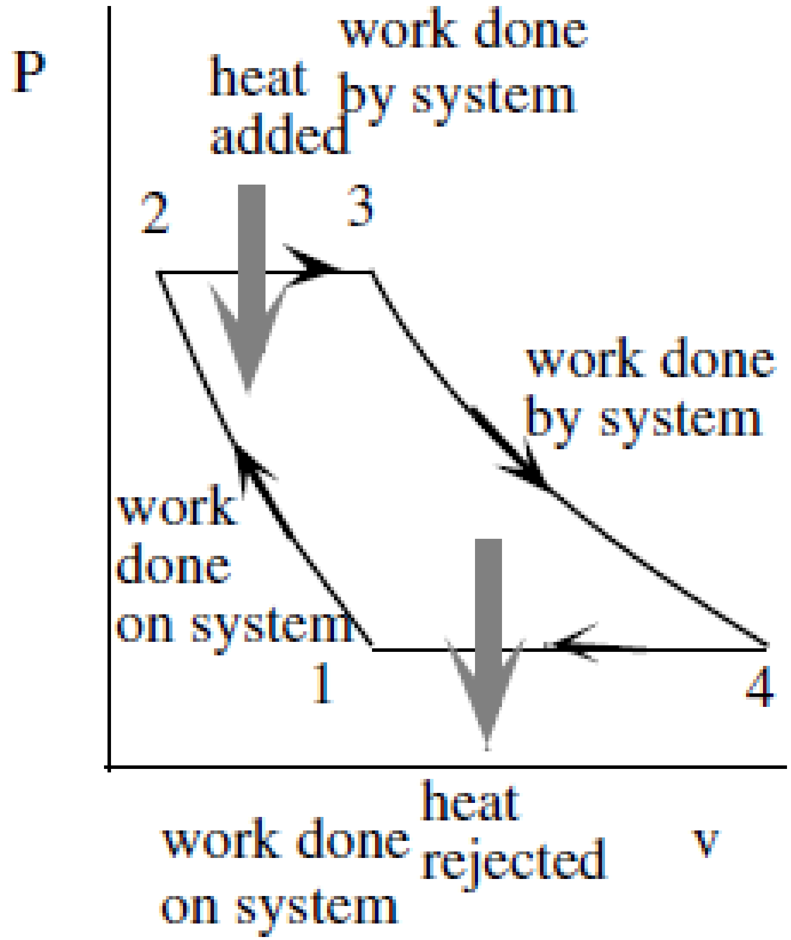


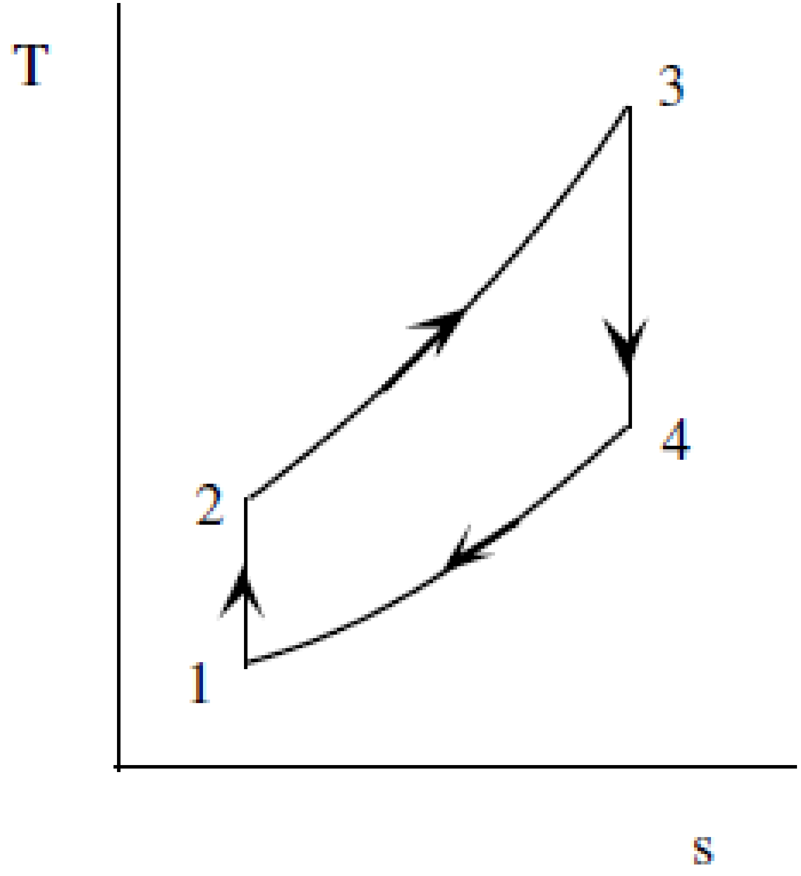


Brayton cycle

- The cycle consists of four processes: a) quasi-static adiabatic compression in the inlet (optional) and compressor, b) constant pressure heat addition in the combustor, c)quasi-static adiabatic expansion in the turbine and exhaust nozzle (optional), and finally d) constant pressure cooling to get the working fluid back to the initial condition.

Brayton cycle in diagrams





Brayton cycle in diagrams

- 1 - 2 Adiabatic, quasi-static compression in inlet and compressor
- 2 - 3 Combust fuel at constant pressure (i.e. add heat)
- 3 - 4 Adiabatic, quasi-static expansion in turbine
  - a. take work out and use it to drive the compressor
  - b. take remaining work out and use it to accelerate fluid for jet propulsion, or to turn a generator for electrical power generation.
- 4 - 1 Cool the air at constant pressure

Brayton cycle performance

- To derive expressions for the net work and the thermal efficiency of the cycle, and then to manipulate these expressions to put them in terms of typical design parameters so that they will be more useful.
- For any cyclic process heat and work transfers are numerically equal:

$$\Delta u = q - w$$

$$u_{final} = u_{initial}$$

$$\Delta u = 0 \text{ and } q = w$$

- We could just as well consider the difference between the heat added to the cycle in process 2-3, and the heat rejected by the cycle in process 4-1.

Brayton cycle parameters

- Heat added between 2-3 (combustor):
  - $\delta q = dh - v\,dp = c_p dT - v\,dp$
  - $q_{added} = c_p \Delta T = cp(T_3 - T_2)$
- Heat rejected between 4-1:
  - $\delta q = dh - v\,dp = c_p dT - v\,dp$
  - $q_{rejeced} = c_p \Delta T = cp(T_4 - T_1)$
- Work done:
  - $\Delta w = \Delta q = q_{added} - q_{rejected} = c_p[(T_3 - T_2) - (T_4 - T_1)]$

Brayton cycle efficiency

- To rewrite this equations in more useful form we will use assumptions about ideality of thermodynamic processes and working body to follow perfect gas laws.
- For gas turbine engines the most useful design parameters to use for these equations are often the inlet temperature (T_1), the compressor pressure ratio (p_2/p_1), and the maximum cycle temperature, the turbine inlet temperature (T_3).

Brayton cycle efficiency

- Rewriting equations in terms of design parameters we get:

$$\Delta w=c_pT_1(T_3/T_1-T_2/T_1-T_4/T_1+1)$$

$$p_2/p_1=p_3/p_4=(T_2/T_1)^{k/(k-1)}=(T_3/T_4)^{k/(k-1)}$$

- In this case we get:

$$\Delta w=c_pT_1[T_3/T_1-(p_2/p_1)^{(k-1)/k}-T_3/T_2+1]$$

$$\Delta w=c_pT_1[T_3/T_1-(p_2/p_1)^{(k-1)/k}-T_3/T_1(p_2/p_1)^{(1-k)/k}+1]$$

Brayton cycle efficiency

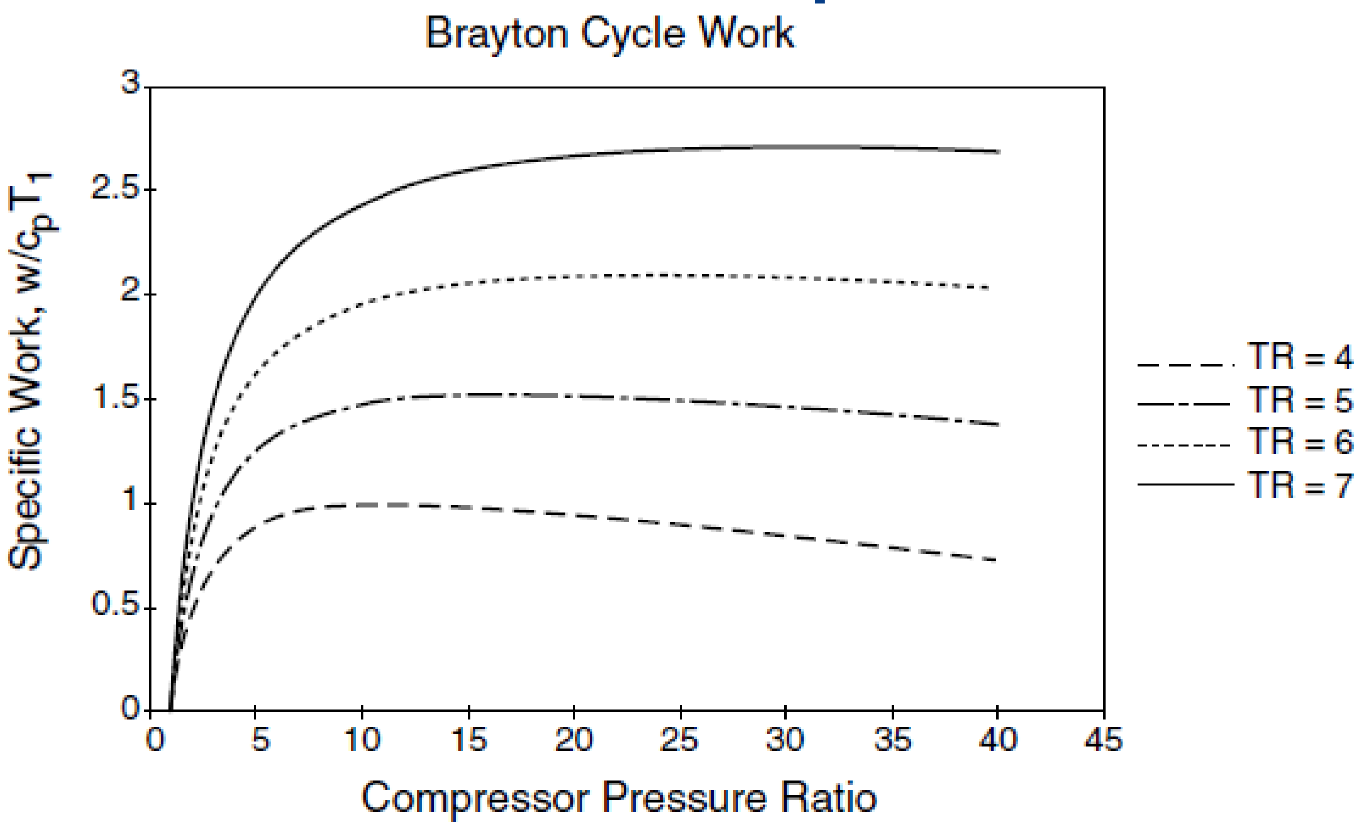
- As long as:

$$\eta=1-(T_4-T_1)/(T_3-T_2)=1-T_1/T_2[(T_4/T_1-1)/(T_3/T_2-1)]$$

- So we will have:

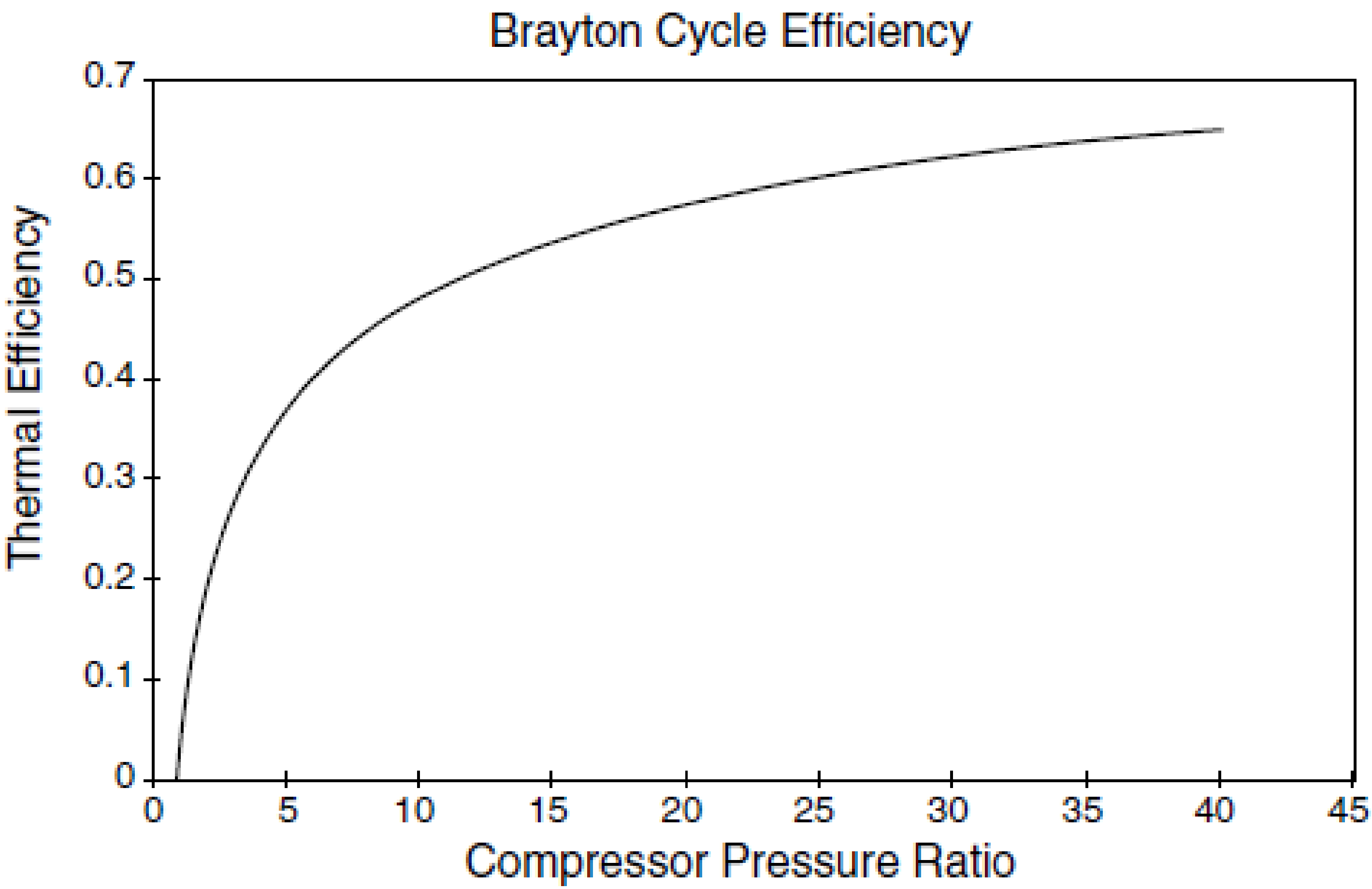
$$\eta_{Brayton}=1-T_1/T_2=1-1/(p_2/p_1)^{(k-1)/k}$$

Brayton cycle specific work dependence on compression ratio

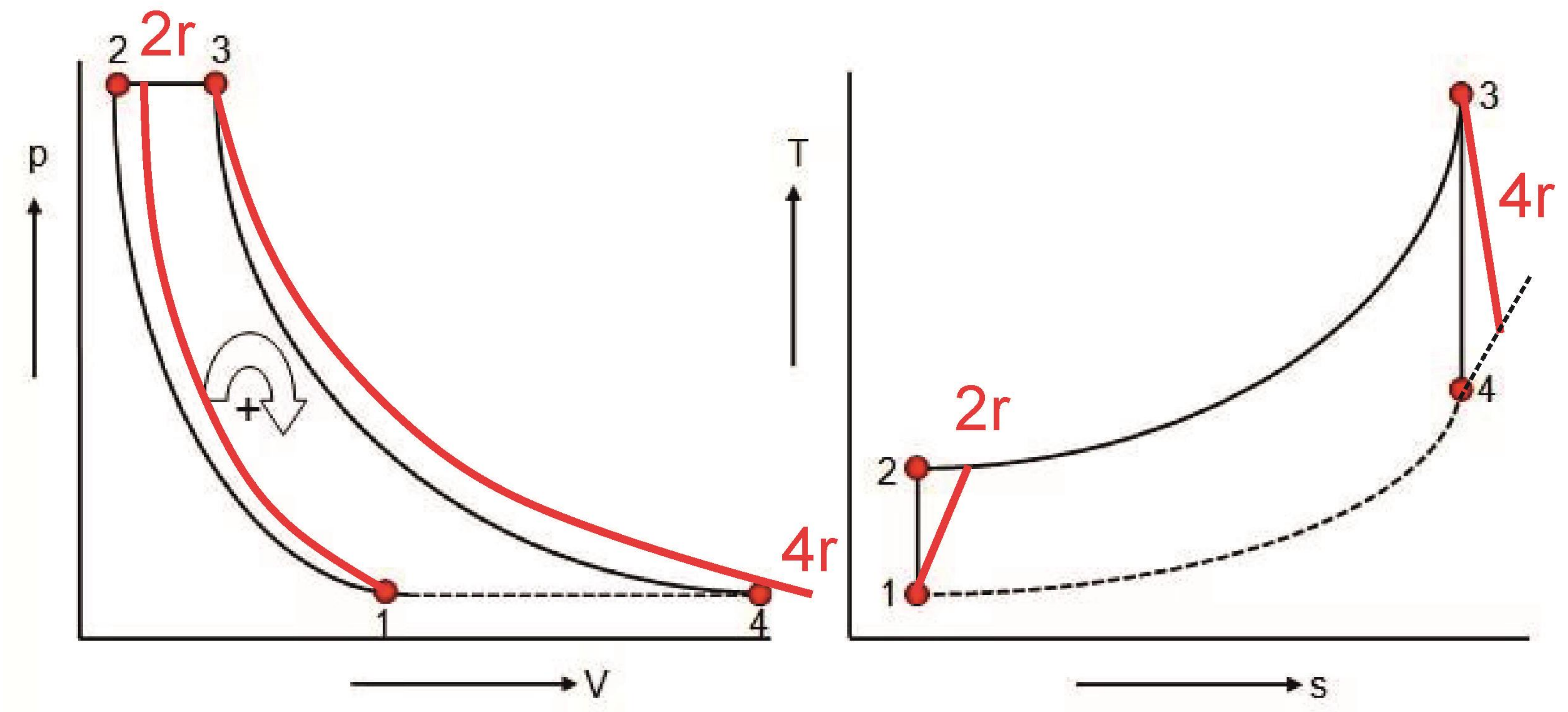


- In the plot above, TR = T_3/T_1. Note that for a given turbine inlet temperature, T_3, (which is set by material limits) there is a compressor pressure ratio that maximizes the work.

Brayton cycle efficiency dependence on compression ratio



Irreversible Brayton cycle



Irreversible Brayton cycle

- Taking into account efficiency of compression and expansion processes as:

$$\eta_{comp}=(T_2-T_1)/(T_{2r}-T_1) \text{ and } \eta_{exp}=(T_3-T_{4r})/(T_3-T_4)$$

- We get temperature in real points 2r and 4r as follows:

$$T_{2r}=T_1+(T_2-T_1)/\eta_{comp}=T_1(1+(\delta^{(k-1)/k}-1)/\eta_{comp})$$

$$T_{4r}=T_3-(T_3-T_4)\eta_{exp}=T_4(1-(1-\delta^{(1-k)/k})\eta_{exp})$$

- Work of turbine and work of compressor in this case:

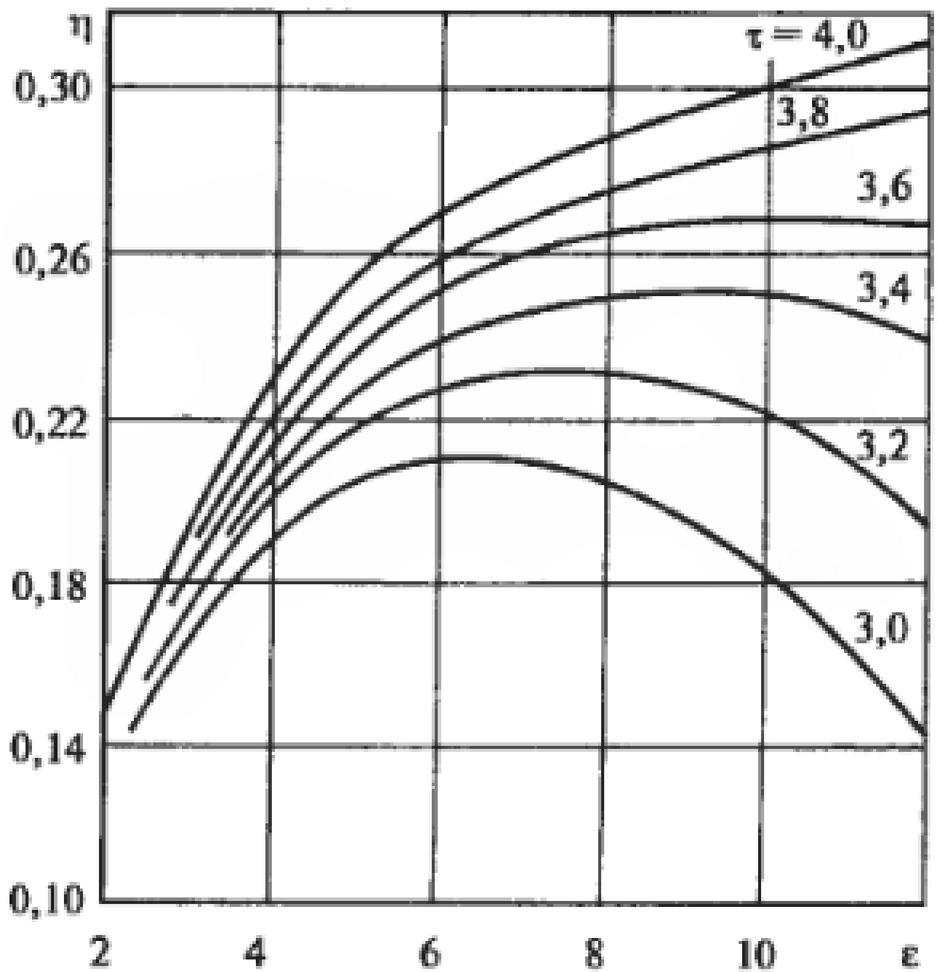
$$H_t=\eta_t c_p T_3(1-\delta^{(1-k)/k})$$

$$H_k=(1/\eta_k)c_p T_1(\delta^{(k-1)/k}-1)$$

Irreversible Brayton cycle efficiency

- Assuming $p_2/p_1=\delta$ and $T_3/T_1=\tau$ we will get following equation for efficiency dependence on this parameters:
- $$\eta = \frac{\tau\eta_t(1-\delta^{(1-k)/k}) - (\delta^{(k-1)/k}-1)/\eta_k}{\tau - 1 - (\delta^{(k-1)/k}-1)/\eta_k}$$

Irreversible Brayton cycle efficiency on compression ratio



Net work coefficient

- Net work coefficient is one more important feature of Brayton cycle:

$$\varphi=(H_t-H_k)/H_t=1-1/\eta_k\eta_t(\delta^{(k-1)/k}-1)/(1-\delta^{(1-k)/k})$$

- Or in more simple form:

$$\varphi=1-\delta^{(k-1)/k}/(\eta_k\eta_t\tau)$$

THANK YOU FOR ATTENTION