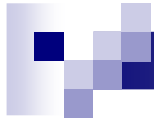



Операторный метод расчета переходных процессов

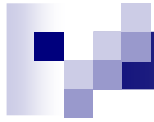


Порядок расчета переходных процессов операторным методом



1. Определяются независимые
начальные условия

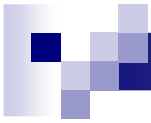
$$\mathbf{i}_L(\mathbf{0}_-) = \mathbf{i}_L(\mathbf{0}) \quad \mathbf{u}_C(\mathbf{0}_-) = \mathbf{u}_C(\mathbf{0})$$

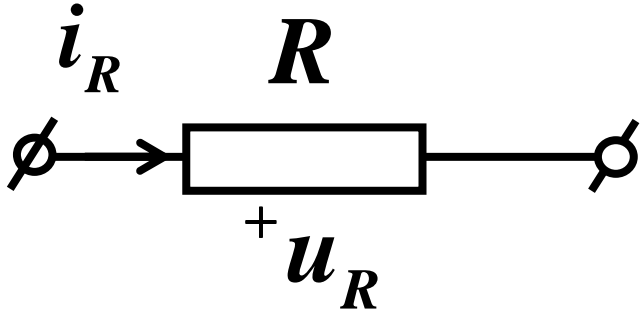
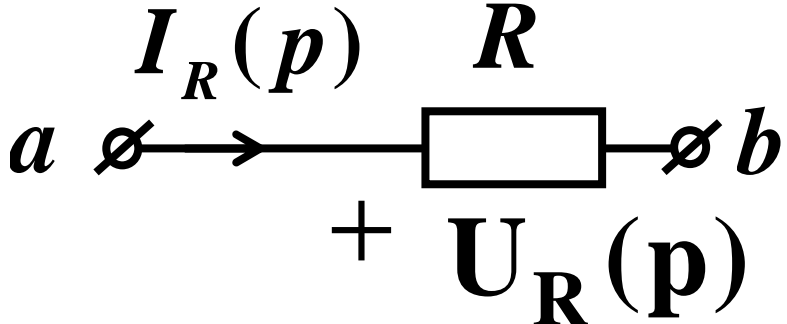


2. Для схемы после коммутации изображается операторная схема, которая рассчитывается любым методом

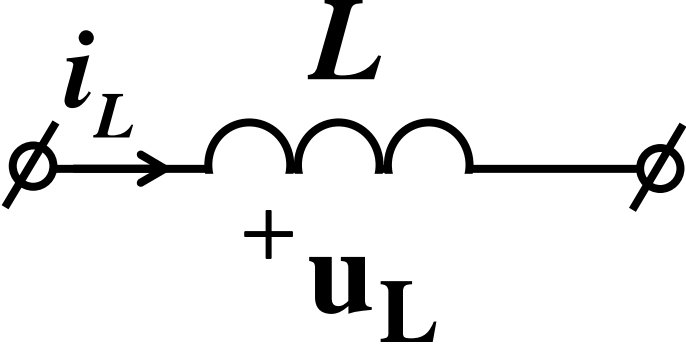
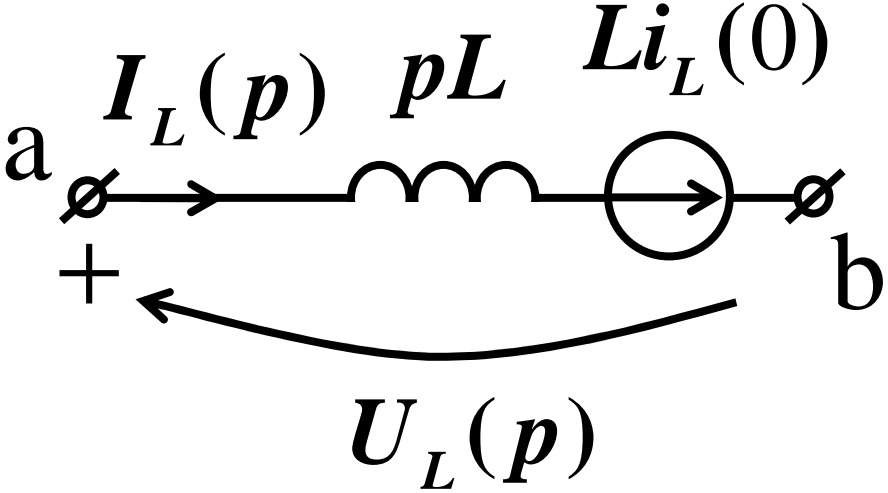


$f(t)$ - -оригинал	$F(p)$ - -изображение
b	b/p
$b e^{-\alpha t}$	$b/(p + \alpha)$
$b \cdot t e^{-\alpha t}$	$\frac{b}{(p + \alpha)^2}$
$\sin(\omega t)$	$\frac{\omega}{p^2 - \omega^2}$
$\cos(\omega t)$	$\frac{p}{p^2 + \omega^2}$

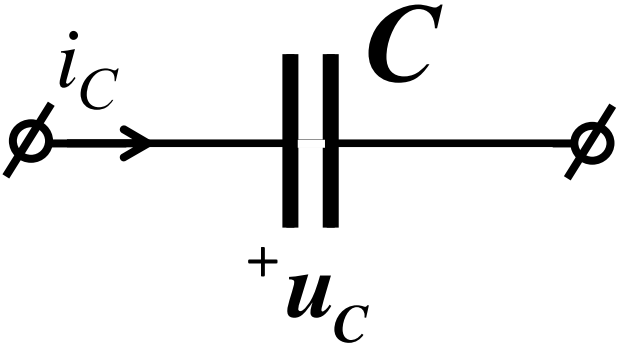
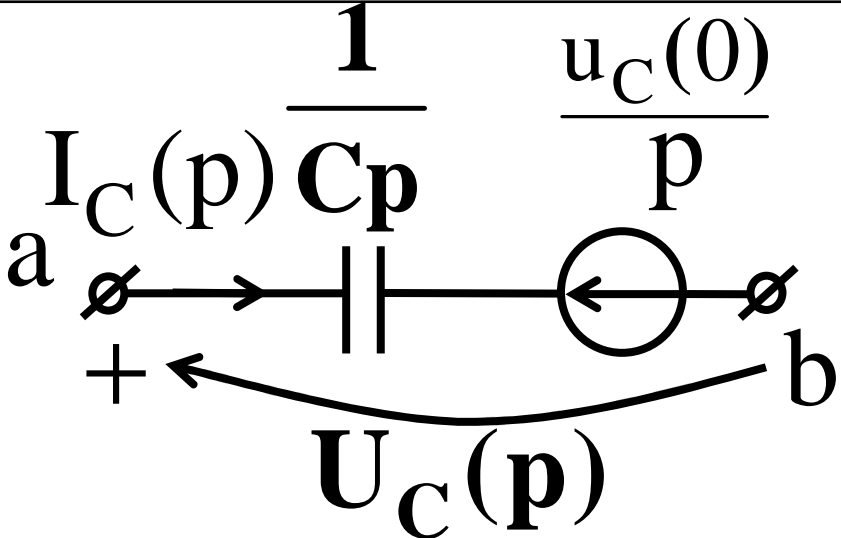


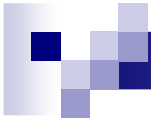
<p><i>Резистивный элемент</i></p>	
<p><i>Операторная схема</i></p>	
<p><i>Закон Ома</i></p>	$U_R(p) = R \cdot I_R(p)$

2.

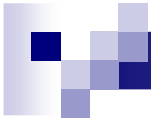
<p>Индуктивный элемент</p>	 <p>A circuit diagram of an inductor with inductance L. The current i_L flows from left to right through the inductor. The voltage u_L is indicated with a '+' sign on the left terminal and a '-' sign on the right terminal.</p>
<p>Операторная схема</p>	 <p>An operator circuit diagram of an inductor. The current is $I_L(p)$ and the voltage is $U_L(p)$. The circuit consists of an inductor with impedance pL and a current source $Li_L(0)$ in parallel. The terminals are labeled 'a' and 'b'. The voltage $U_L(p)$ is indicated with a '+' sign at terminal 'a' and a '-' sign at terminal 'b'.</p>
<p>Закон Ома</p>	$U_L(p) = Lp \cdot I_L(p) - L \cdot i_L(0_+)$

3.

	
<p>Операторная схема</p>	
<p>Закон Ома</p>	$U_C(p) = \frac{1}{Cp} \cdot I_C(p) + \frac{u_C(0_+)}{p}$



$$F(p) = \frac{D(p)}{B(p)}$$




3. По теореме разложения
определяются напряжения и
токи переходного процесса в
функции времени




Тогда

$$i(t) = \sum_{k=0}^n \frac{D(p_k)}{B'(p_k)} e^{p_k t};$$



где p – корни
характеристического
уравнения определяются из
уравнения:

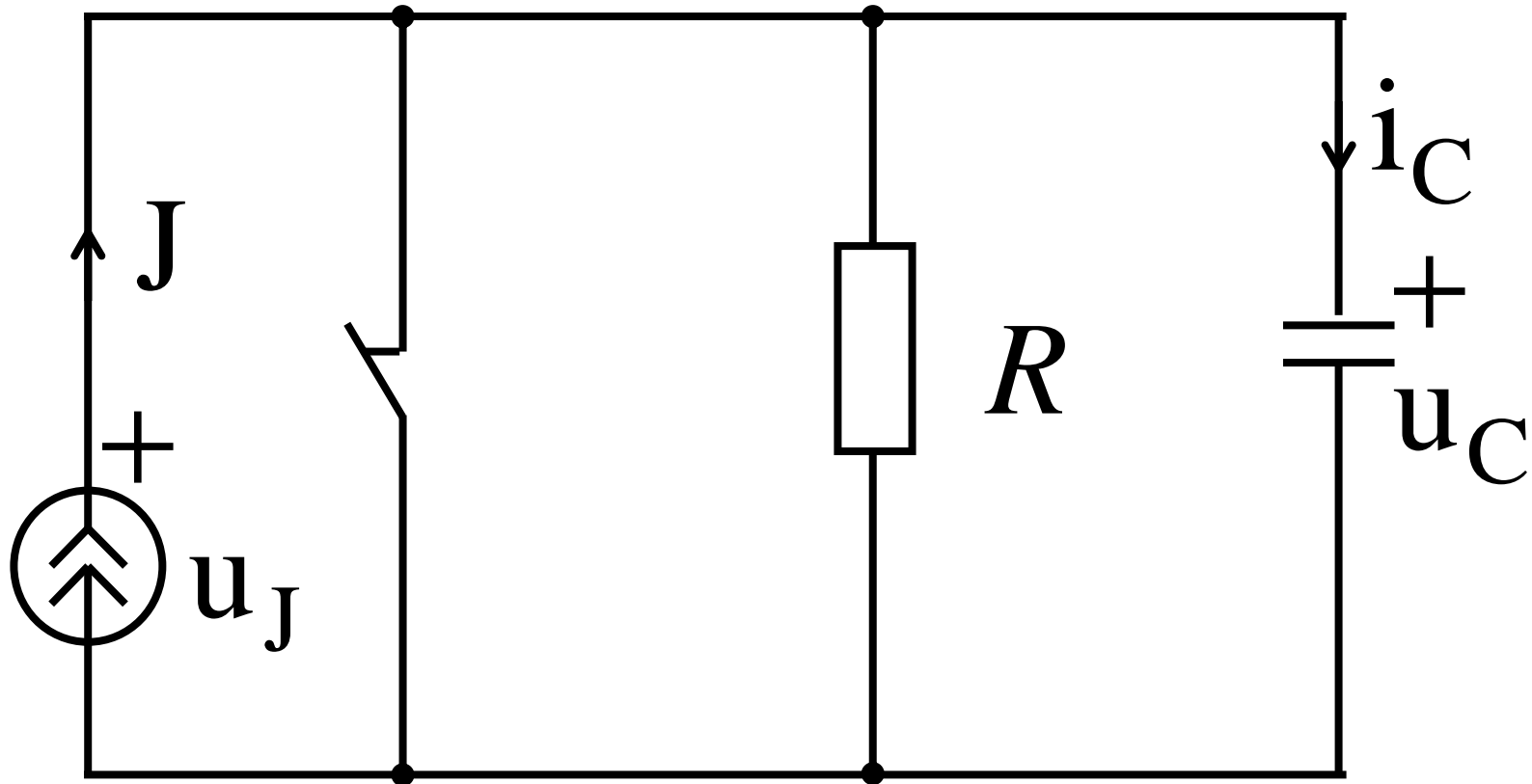
$$B(p) = 0$$




При комплексно- сопряженных
корнях

$$f(t) = 2 \operatorname{Re} \left[\frac{D(p_1)}{B'(p_1)} \cdot e^{p_1 t} \right]$$

Пример 1:






Дано: $\mathbf{J(t) = 10e^{-50t} \text{ A}}$

$$\mathbf{R = 50 \text{ Ом}}$$


$$\mathbf{C = 200 \text{ мкФ}}$$

Определить: $\mathbf{i_C(t) = ?}$



1. Определяем независимые
начальные условия ($t = 0_-$):

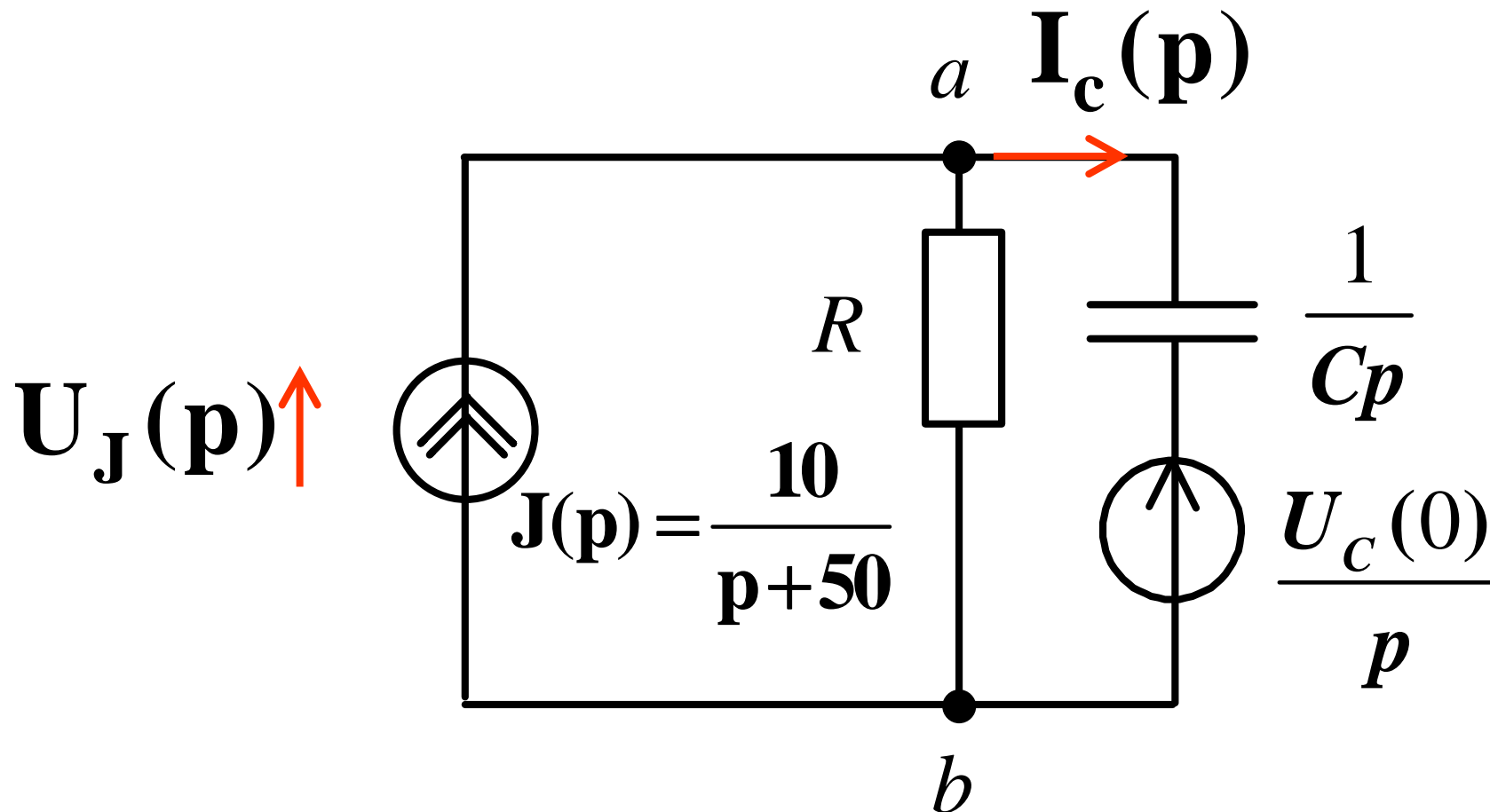
$$\mathbf{u}_C(\mathbf{0}_-) = \mathbf{0}$$



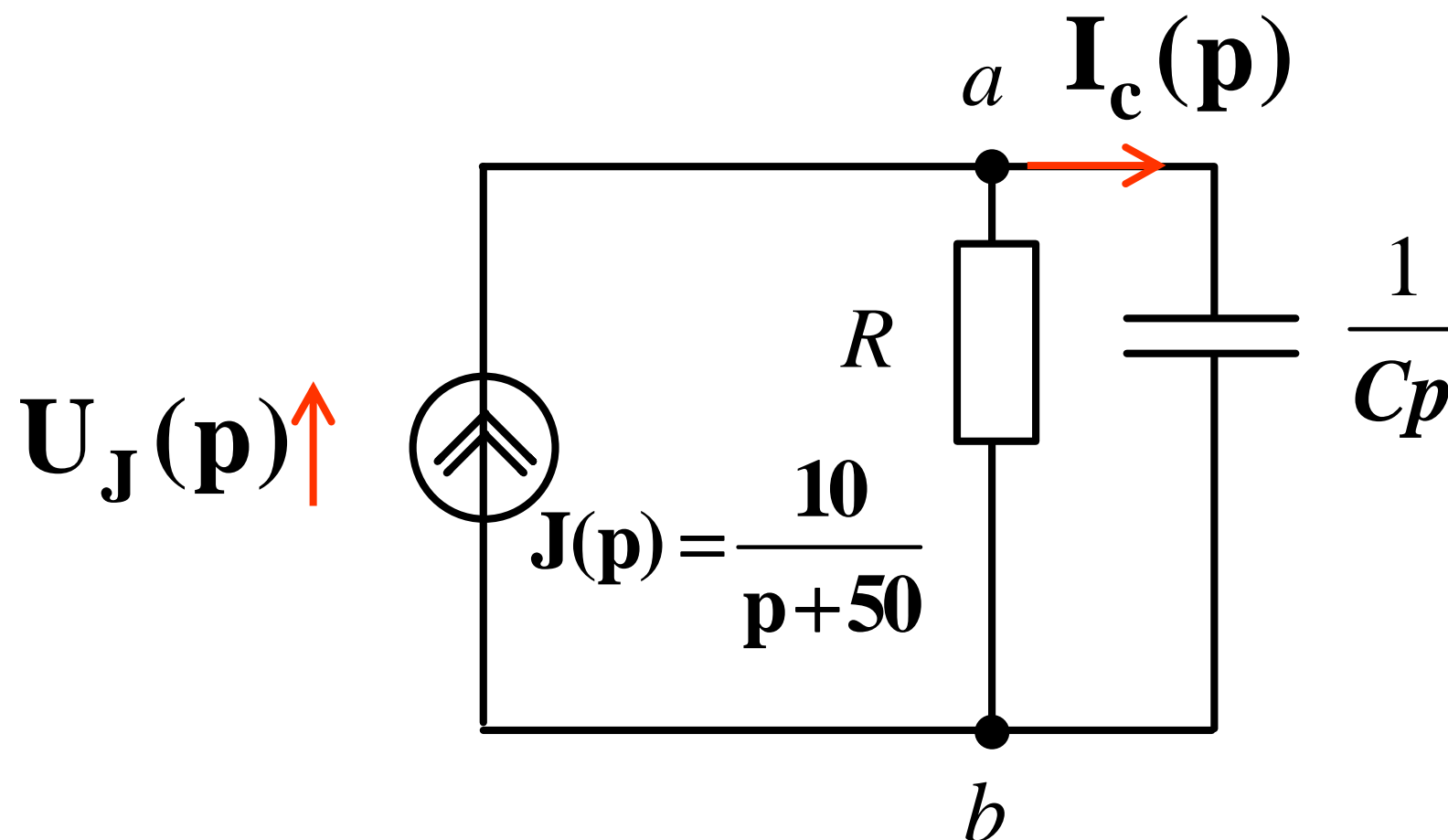
2. Операторная схема после КОММУТАЦИИ


$$\mathbf{u}_C(\mathbf{0}_+) = \mathbf{u}_C(\mathbf{0}_-) = \mathbf{0} \text{ В}$$

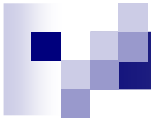
Операторная схема



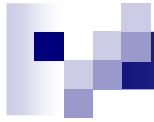
Операторная схема




$$\begin{aligned} I_C(p) &= \frac{J(p)R}{R + \frac{1}{C_p}} = \\ &= \frac{\left(\frac{10}{p + 50} R \right) C_p}{RC_p + 1} = \end{aligned}$$



$$= \frac{10CRp}{(p + 50)(RCp + 1)} = \frac{D(p)}{B(p)}$$



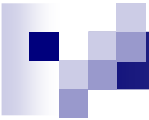
3. По теореме разложения
определяем $\mathbf{i}_C(t)$

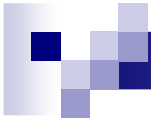
$$\mathbf{D(p)} = \mathbf{10p};$$

$$\mathbf{B(p)} = (\mathbf{p + 50})(\mathbf{RCp + 1}) = \mathbf{0} \Rightarrow$$

$$\mathbf{p_1 = -50; p_2 = -\frac{1}{RC}}$$

$$\begin{aligned} \mathbf{B'(p)} &= \left((\mathbf{p + 50})(\mathbf{RCp + 1}) \right)' = \\ &= \left(\mathbf{p^2 RC + p + 50RCp + 50} \right)' = \\ &= \mathbf{2pRC + 1 + 50RC} \end{aligned}$$


$$\begin{aligned}i_C(t) &= \sum_{k=1}^{n=2} \frac{D(p_k)}{B'(p_k)} e^{p_k t} = \\&= \frac{10(-50)}{2pRC + 1 + 50RC} e^{-50t} + \\&+ \frac{10(-100)}{2\left(-\frac{1}{RC}\right)RC + 1 + 50RC} e^{-\frac{1}{RC}t} =\end{aligned}$$



$$\begin{aligned} &= \frac{10(-50)}{2(-50) + 150} e^{-50t} + \frac{10(-100)}{2(-100) + 150} e^{-100t} = \\ &= -10e^{-50t} + 20e^{-100t} \end{aligned}$$



Документ Mathcad

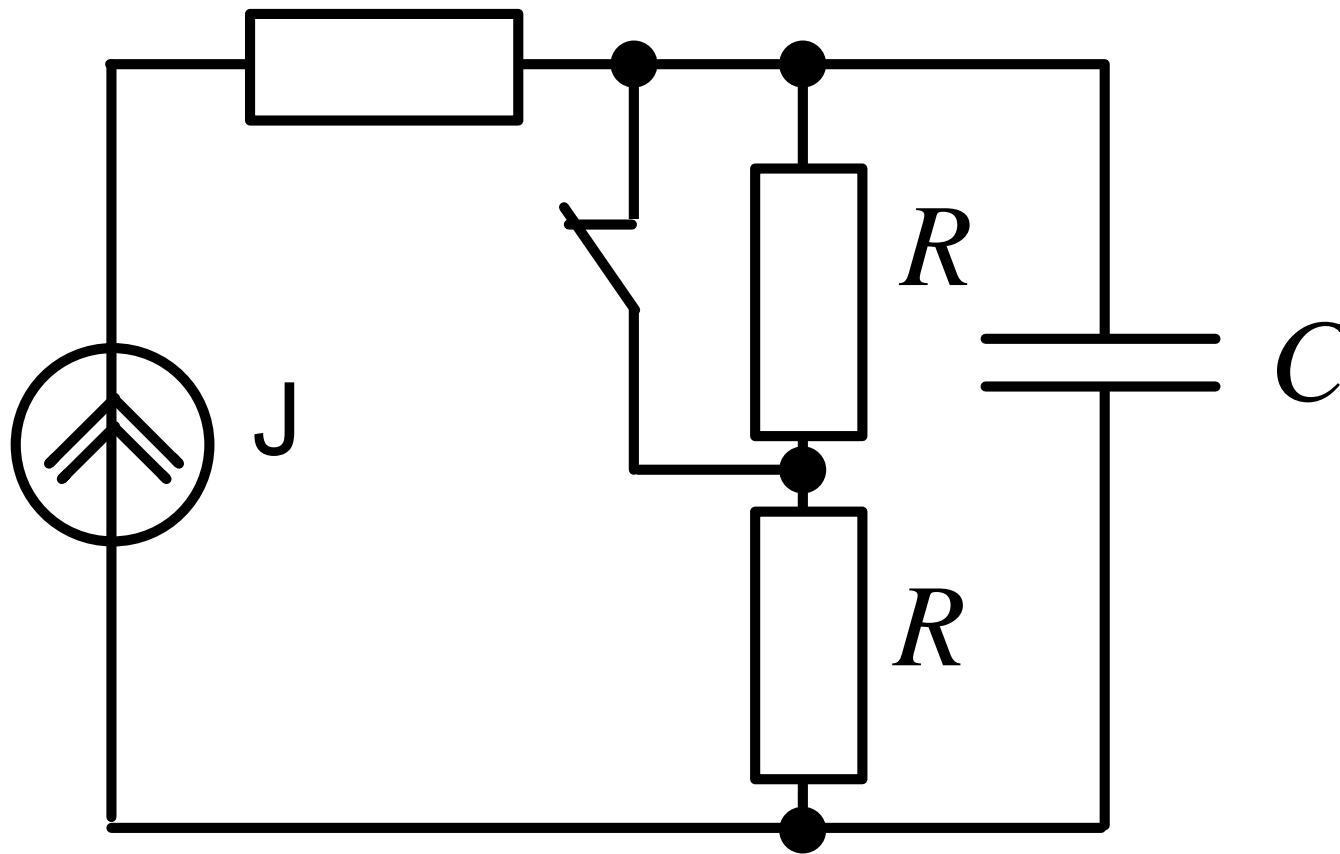
$$R := 50 \quad c := 2 \cdot 10^{-4}$$

$$I(p) := \frac{\frac{10}{p + 50} \cdot R}{\frac{1}{c \cdot p} + R} \text{ simplify} \rightarrow 10 \cdot \frac{p}{(p + 50) \cdot (100 + p)}$$

$$I(t) := I(p) \text{ invlaplace, } p \rightarrow -10 \cdot \exp(-50 \cdot t) + 20 \cdot \exp(-100 \cdot t)$$

Пример 2:

R



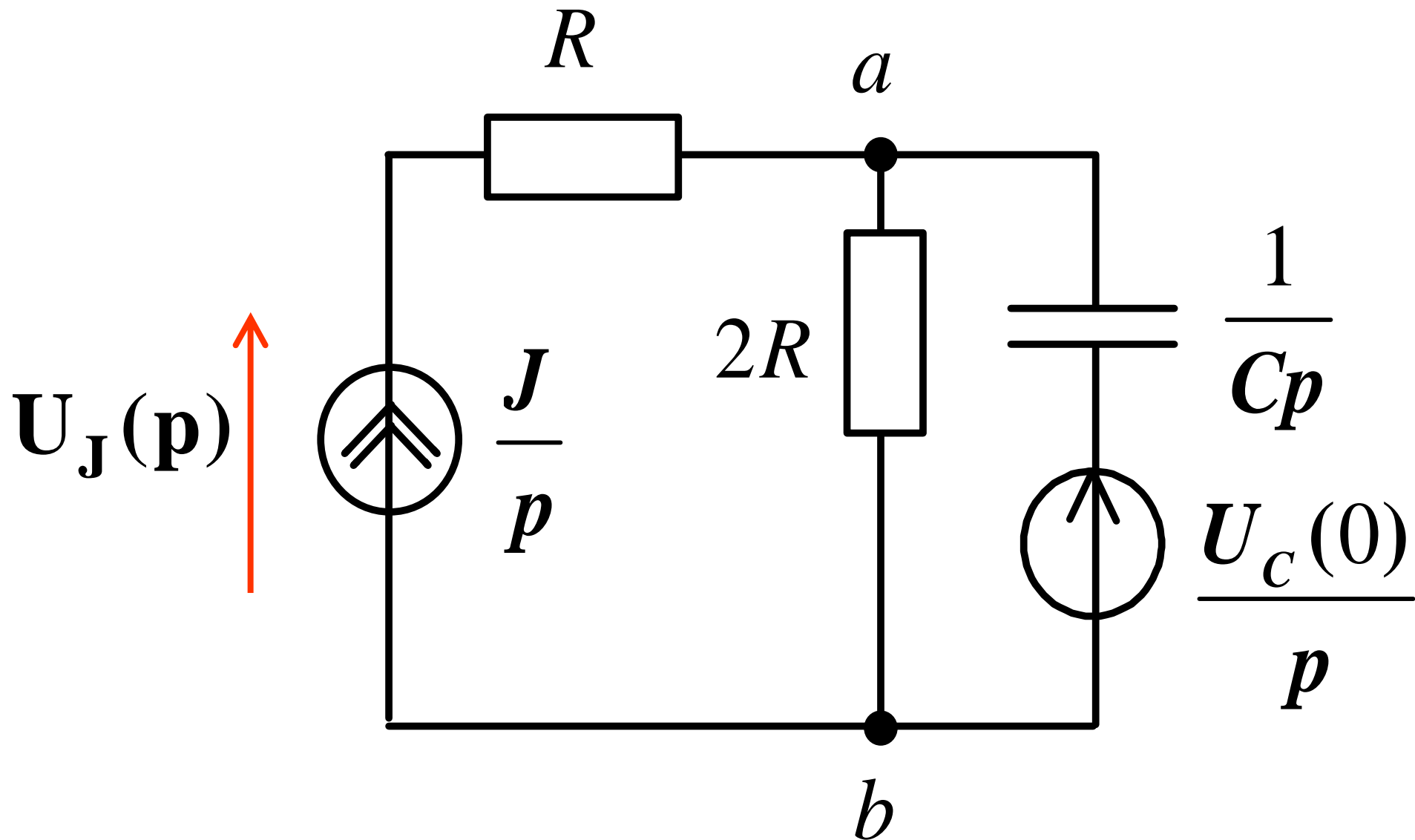


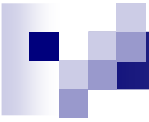
1. ННУ $(t = 0_-)$

⋮

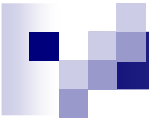
$$u_C(0_-) = JR$$

1. Операторная схема

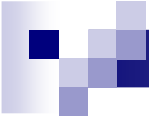



$$U_J(p) = \frac{J}{p} R + \varphi_a$$

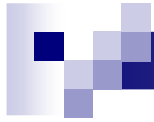
$$\varphi_a(p) \left(\frac{1}{2R} + \frac{1}{Cp} + \frac{1}{R+\infty} \right) = \frac{J}{p} + \frac{U_C(0)}{p \frac{1}{Cp}}$$




$$\begin{aligned}
 \varphi_a(\mathbf{p}) &= \frac{\mathbf{J} + U_C(\mathbf{0})\mathbf{C}}{\frac{1}{2R} + C_p} = \\
 &= \frac{(\mathbf{J} + U_C(\mathbf{0})c_p)2R}{p(1 + C_p2R)} = \\
 &= \frac{\mathbf{J}2R + U_C(\mathbf{0})c_p2R}{p(1 + C_p2R)} = \frac{\mathbf{D}(\mathbf{p})}{\mathbf{B}(\mathbf{p})}
 \end{aligned}$$


$$U_J(p) = \frac{J}{p}R + \varphi_a(p) - \varphi_b(p)$$

$$U_J(p) = \frac{JR}{p} + \frac{J2R + U_c(0)Cp2R}{p(1 + Cp2R)}$$



3. По теореме разложения
определяем $\mathbf{u}_J(t)$

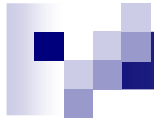

$$\mathbf{D}(\mathbf{p}) = \mathbf{J}^2\mathbf{R} + \mathbf{U}_c(\mathbf{0})\mathbf{C}_p^2\mathbf{R};$$

$$\mathbf{B}(\mathbf{p}) = \mathbf{p} \left(\mathbf{1} + \mathbf{C}_p^2\mathbf{R} \right) =$$

$$= \mathbf{p}^2 \mathbf{2RC} + \mathbf{p}$$

$$\mathbf{B}'(\mathbf{p}) = \left(\mathbf{p}^2 \mathbf{2RC} + \mathbf{p} \right)' =$$

$$= \mathbf{4RCp} + \mathbf{1};$$




$$B(p) = p(1 + Cp2R) = 0 \Rightarrow$$


$$p_1 = 0; \quad p_2 = -\frac{1}{2RC}$$

ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ

$$u_J(t) = \sum_{k=1}^{n=2} \frac{D(p_k)}{B'(p_k)} e^{p_k t} + JR = \quad =$$

$$= \frac{D(0)}{B'(0)} e^{0t} + \frac{D\left(-\frac{1}{2RC}\right)}{B'\left(-\frac{1}{2RC}\right)} e^{-\frac{1}{2RC}t} + JR =$$


$$u_J(t) = JR + \frac{J2R}{1} + \frac{J2R + JR2RC \left(-\frac{1}{2RC} \right)}{4RC \left(-\frac{1}{2RC} \right) + 1} e^{-\frac{1}{2RC}t}$$


$$= 3JR + \frac{2JR + (-JR)}{-1} e^{-\frac{1}{2RC}t} =$$

$$u_J(t) = 3JR - JR e^{-\frac{1}{2RC}t}$$



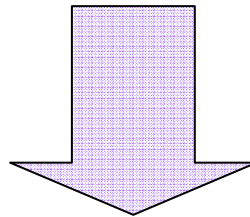
Используя теорему
разложения, определить
оригинал



Пример 3


$$\begin{aligned} F(p) = U(p) &= \frac{2 \cdot 10^4 p + 2 \cdot 10^6}{p(p^2 + 200p + 2 \cdot 10^4)} = \\ &= \frac{D(p)}{B(p)}, \quad (\text{Bc}) \end{aligned}$$


$$\mathbf{B(p) = p(p^2 + 200p + 2 \cdot 10^4) = 0}$$



$$\mathbf{p_1 = 0}$$

$$\mathbf{p_{2,3} = -100 \pm j100 \left(\frac{1}{c} \right)}$$


$$\mathbf{B}(p) = p(p^2 + 200p + 2 \cdot 10^4)$$

$$\begin{aligned}\mathbf{B}'(p) &= (p^3 + 200p^2 + 2 \cdot 10^4 p)' = \\ &= 3p^2 + 400p + 2 \cdot 10^4,\end{aligned}$$

ТОГДА

$$\mathbf{u}(t) = \sum_{\kappa=1}^{n=3} \frac{\mathbf{D}(p_{\kappa})}{\mathbf{B}'(p_{\kappa})} \cdot e^{p_{\kappa}t}$$



Или

$$u(t) =$$

$$= \frac{2 \cdot 10^4 \cdot 0 + 2 \cdot 10^6}{0^2 + 400 \cdot 0 + 2 \cdot 10^4} e^{0 \cdot t} +$$

$$+ 2 \operatorname{Re} \left[\frac{2 \cdot 10^4 \cdot p_2 + 2 \cdot 10^6}{3p_2^2 + 400p_2 + 2 \cdot 10^4} e^{p_2 t} \right] =$$



T.e.

$$u(t) = 100 + 2 \operatorname{Re} \left[70,5 e^{-j135^\circ} e^{(-100+j100)t} \right] =$$


$$= 100 + 2 \operatorname{Re} \left[70,5 e^{j(-135^\circ+100t)} e^{-100t} \right] =$$

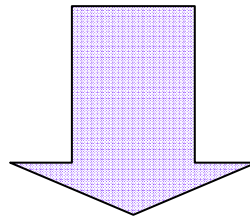
$$= 100 + 141 e^{-100t} \cos(100t - 135^\circ), \quad B$$



Пример 4

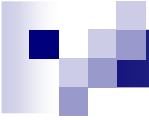
$$\begin{aligned} I(p) &= \frac{p^2 + p + 0,5}{p(p^2 + 2p + 1)} = \\ &= \frac{D(p)}{B(p)}, \quad (Ac) \end{aligned}$$


$$\mathbf{B(p) = p(p^2 + 2p + 1) = 0}$$




$$\mathbf{p_1 = 0}$$

$$\mathbf{p_2 = p_3 = -1 \left(\frac{1}{c} \right)}$$



Используем метод
неопределённых
коэффициентов

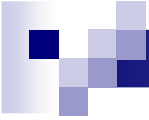
$$\frac{a}{p} + \frac{b}{p+1} + \frac{c}{(p+1)^2} =$$
$$= \frac{(a+b)p^2 + (2a+b+c)p + a}{p(p+1)^2} =$$



Сравнивая коэффициенты
числителей, находим

$$\begin{cases} (a + b) = 1 \\ (2a + b + c) = 1 \\ a = 0,5 \end{cases}$$

$$\begin{cases} a = 0,5 \\ b = 0,5 \\ c = -0,5 \end{cases}$$



Оригиналы каждой из простых дробей определим по таблице

$$\mathbf{i(t) = 0,5 + 0,5e^{-t} - 0,5te^{-t} \quad (A)}$$