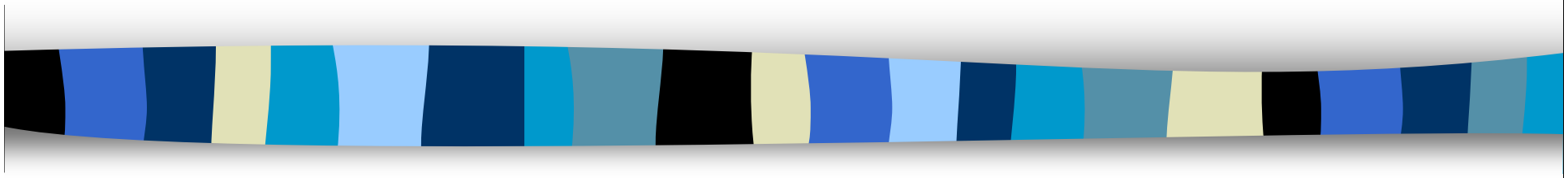
Трёхфазные цепи

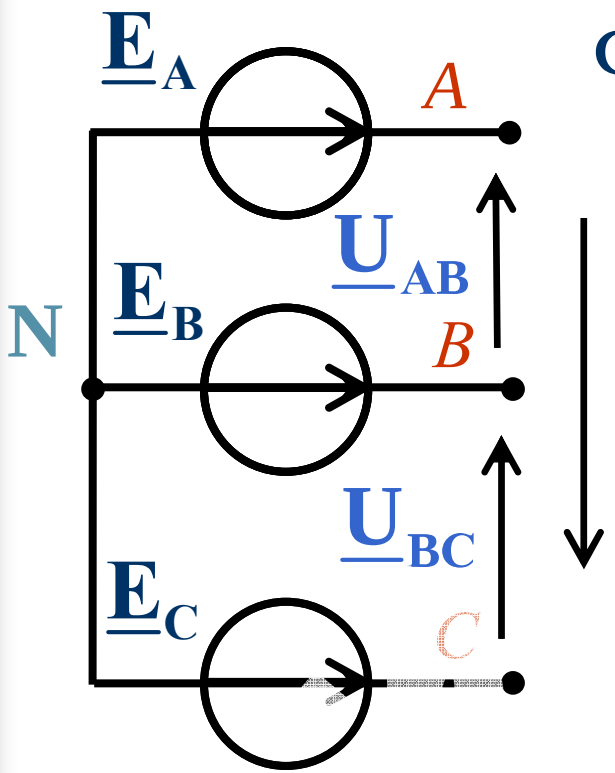


Фазовый оператор

$$a = 1e^{j120^\circ}$$

$$a^2 = 1e^{-j120^\circ}$$

Фазные ЭДС генератора



$$\underline{E}_A = Ee^{j0}$$

$$\underline{E}_B = E \cdot e^{-j120^\circ} = a^2 \underline{E}_A$$

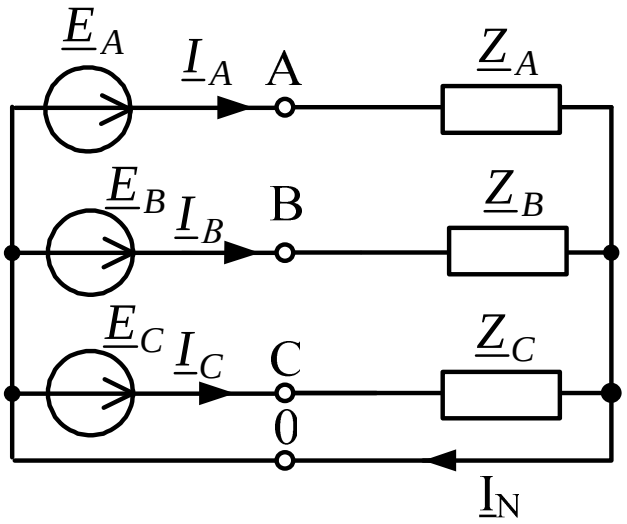
$$\underline{E}_C = E \cdot e^{j120^\circ} = a\underline{E}_A$$

Линейные напряжения :

$$\begin{cases} \underline{U}_{AB} = U_{Л} \cdot e^{j\lambda} = \sqrt{3}\underline{E}_{A} e^{j30} \\ \underline{U}_{BC} = a^{2} \cdot \underline{U}_{AB} \\ \underline{U}_{CA} = a \cdot \underline{U}_{AB} \end{cases}$$

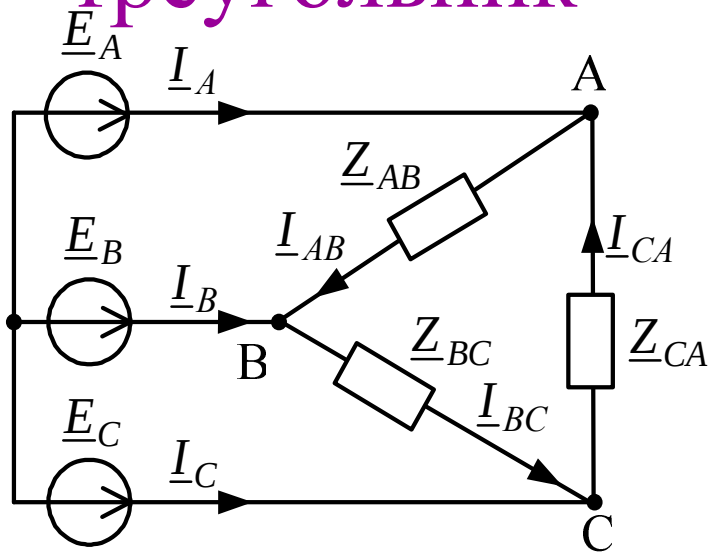
Схемы соединения нагрузки:

звезда



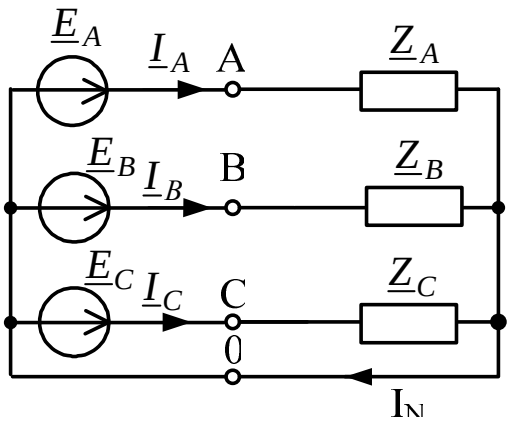
$\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$ – линейные токи, равные фазным

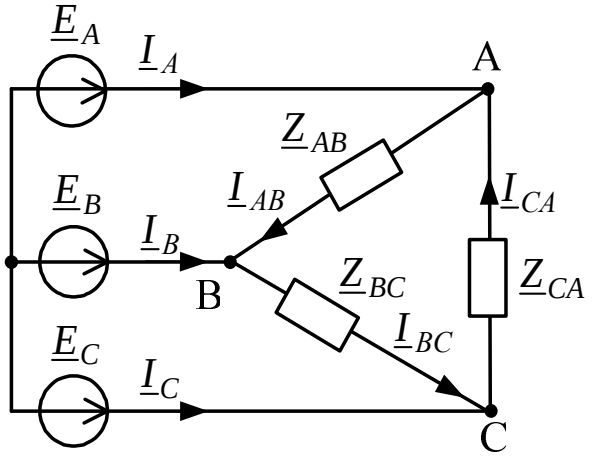
треугольник



$\underline{I}_{AB}$, $\underline{I}_{BC}$, $\underline{I}_{CA}$ – фазные токи

$\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$ – линейные токи





Симметричная нагрузка

$(\underline{Z}_A=\underline{Z}_B=\underline{Z}_C)$

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}$$

$$\underline{I}_B = a^2 \underline{I}_A$$

$$\underline{I}_C = a\ \underline{I}_A$$

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C$$

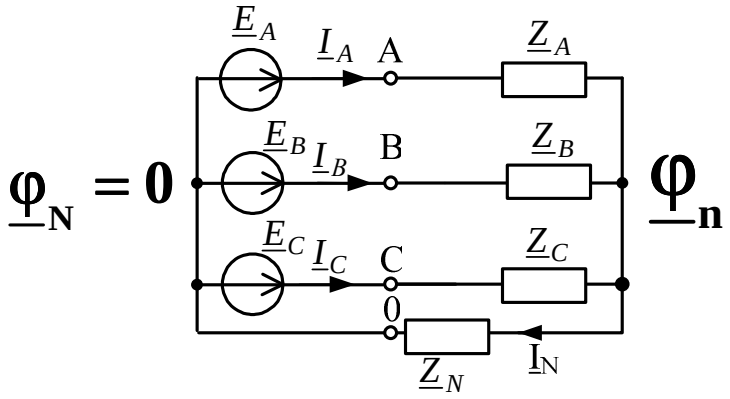
$(\underline{Z}_{AB}=\underline{Z}_{BC}=\underline{Z}_{CA})$

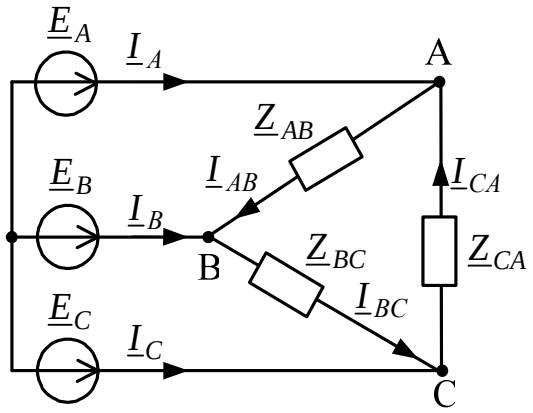
$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}},$$

$$\underline{I}_{BC} = a^2 \underline{I}_{AB}$$

$$\underline{I}_{CA} = a\ \underline{I}_{AB}$$

$$\underline{I}_A = \sqrt{3}\underline{I}_{AB} e^{-j30}$$





Несимметричная нагрузка

$(\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C)$

$$\underline{\varphi}_n \left(\frac{1}{\underline{Z}_A} + \frac{1}{\underline{Z}_B} + \frac{1}{\underline{Z}_C} + \frac{1}{\underline{Z}_N} \right) =$$

$$= \frac{\underline{E}_A}{\underline{Z}_A} + \frac{\underline{E}_B}{\underline{Z}_B} + \frac{\underline{E}_C}{\underline{Z}_C}$$

$$\underline{I}_A = \frac{-\underline{\varphi}_n + \underline{E}_A}{\underline{Z}_A}; \quad \underline{I}_B = \frac{-\underline{\varphi}_n + \underline{E}_B}{\underline{Z}_B};$$

$$\underline{I}_C = \frac{-\underline{\varphi}_n + \underline{E}_C}{\underline{Z}_C}$$

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C$$

$(\underline{Z}_{AB} \neq \underline{Z}_{BC} \neq \underline{Z}_{CA})$

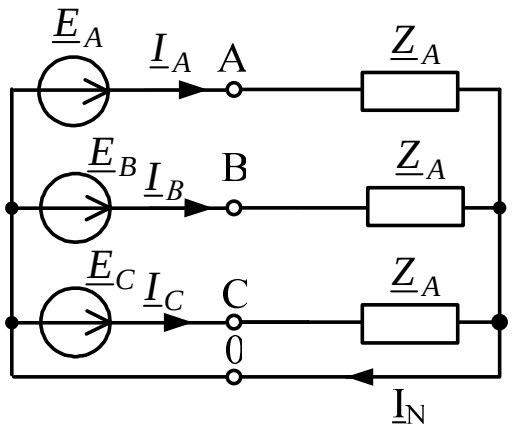
$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}}; \quad \underline{I}_{BC} = \frac{\underline{U}_{BC}}{\underline{Z}_{BC}};$$

$$\underline{I}_{CA} = \frac{\underline{U}_{CA}}{\underline{Z}_{CA}}$$

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB}$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}$$

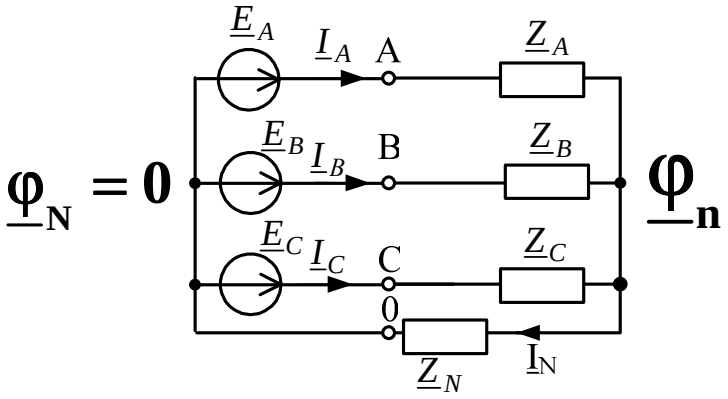


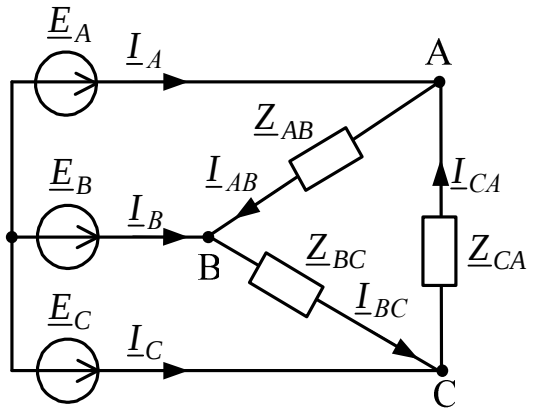
$$\underline{Z}_N = \underline{0}$$

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}; \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B};$$

$$\underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C}$$

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C$$





Несимметричная нагрузка

$(\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C)$

$$\underline{\varphi}_n \left(\frac{1}{\underline{Z}_A} + \frac{1}{\underline{Z}_B} + \frac{1}{\underline{Z}_C} + \frac{1}{\underline{Z}_N} \right) =$$

$$= \frac{\underline{E}_A}{\underline{Z}_A} + \frac{\underline{E}_B}{\underline{Z}_B} + \frac{\underline{E}_C}{\underline{Z}_C}$$

$$\underline{I}_A = \frac{-\underline{\varphi}_n + \underline{E}_A}{\underline{Z}_A}; \quad \underline{I}_B = \frac{-\underline{\varphi}_n + \underline{E}_B}{\underline{Z}_B};$$

$$\underline{I}_C = \frac{-\underline{\varphi}_n + \underline{E}_C}{\underline{Z}_C}$$

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C$$

$(\underline{Z}_{AB} \neq \underline{Z}_{BC} \neq \underline{Z}_{CA})$

$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}}; \quad \underline{I}_{BC} = \frac{\underline{U}_{BC}}{\underline{Z}_{BC}};$$

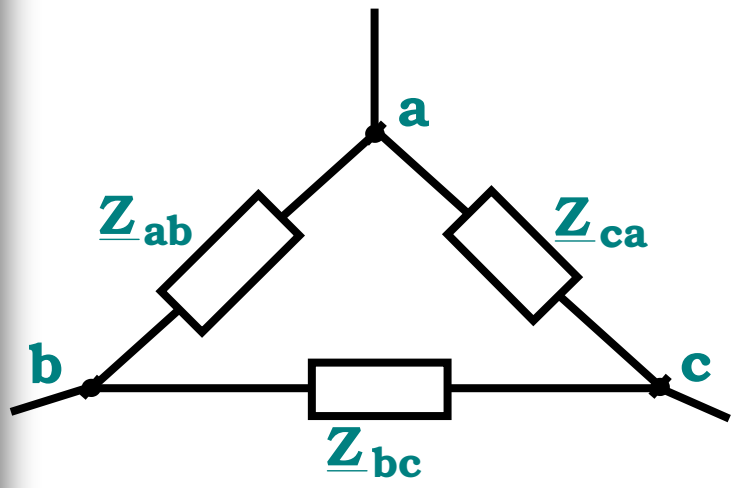
$$\underline{I}_{CA} = \frac{\underline{U}_{CA}}{\underline{Z}_{CA}}$$

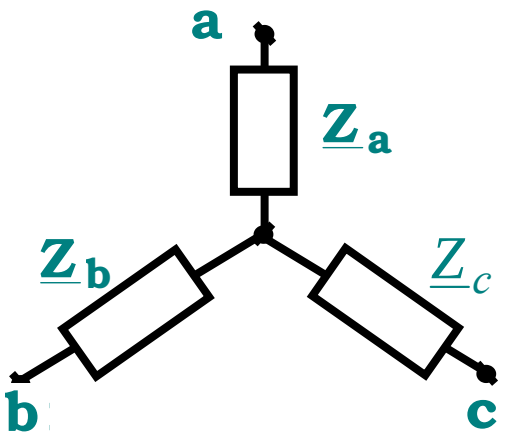
$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB}$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}$$

Преобразование треугольника в звезду и наоборот





$$\underline{Z}_{ab} = \underline{Z}_a + \underline{Z}_b + \frac{\underline{Z}_a \underline{Z}_b}{\underline{Z}_c}$$

$$\underline{Z}_{bc} = \underline{Z}_b + \underline{Z}_c + \frac{\underline{Z}_b \underline{Z}_c}{\underline{Z}_a}$$

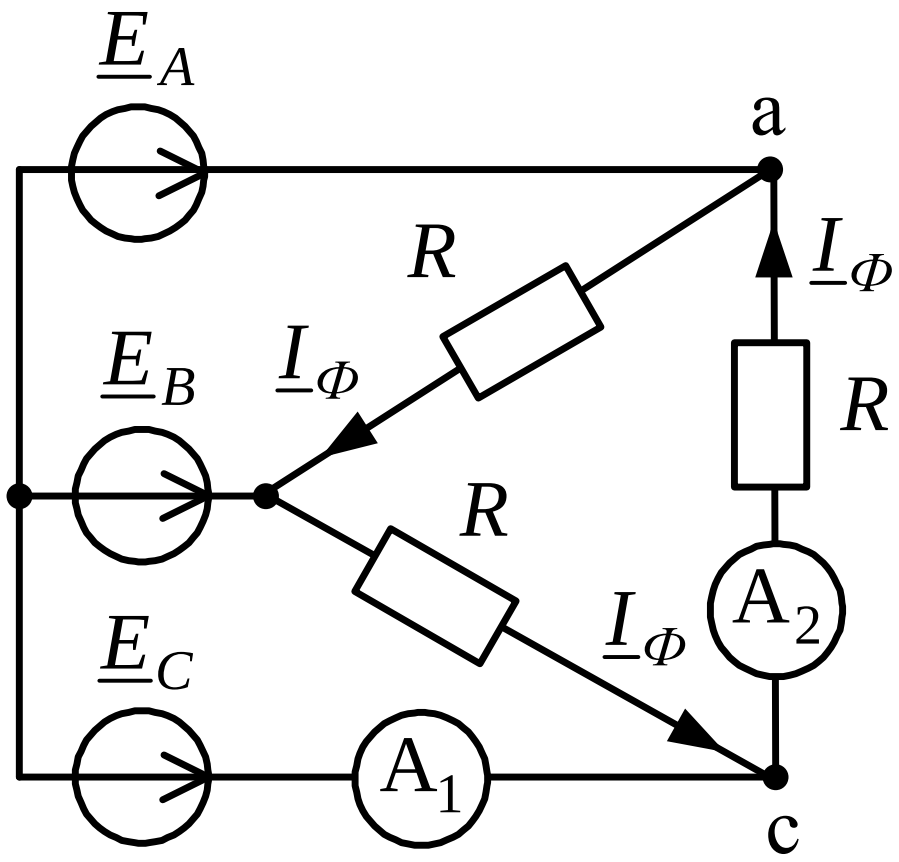
$$\underline{Z}_{ca} = \underline{Z}_c + \underline{Z}_a + \frac{\underline{Z}_c \underline{Z}_a}{\underline{Z}_b}$$

$$\underline{Z}_a = \frac{\underline{Z}_{ab} \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

$$\underline{Z}_b = \frac{\underline{Z}_{ab} \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

$$\underline{Z}_c = \frac{\underline{Z}_{ca} \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

Пример 1:



В симметричной трёхфазной цепи фазное напряжение генератора 127 В, сопротивление фаз нагрузки R=11 Ом.

Определить показания амперметров.

$$E = 127 \text{ В}$$

$$U_{Л} = \sqrt{3}E = \sqrt{3} \cdot 127 \text{ В} = U_{Ф}$$

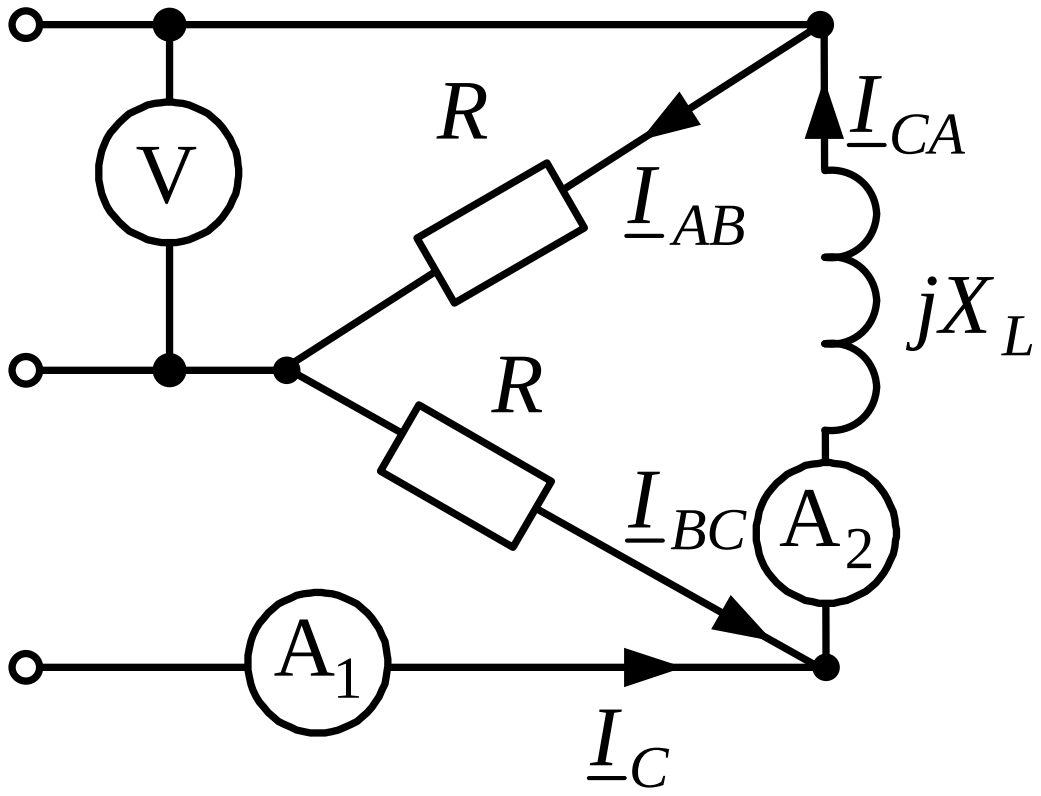
$$I_{ф} = \frac{U_{Ф}}{R} = \frac{\sqrt{3} \cdot 127}{11} = 20 \text{ А}$$

$$I_{A2} = 20 \text{ А}$$

$$\left|\underline{I}_{Л}\right| = \left|\underline{I}_{Ф}\right|\sqrt{3} = 34,64 \text{ А}$$

$$I_{A1} = \left|\underline{I}_{C}\right| = 34,64 \text{ А}$$

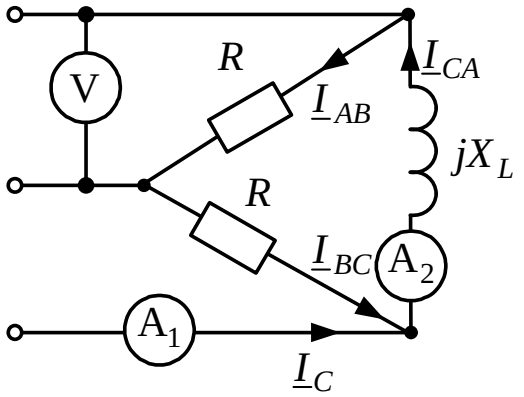
Пример 2:



В несимметричной трёхфазной цепи напряжение вольтметра 220 В, сопротивление фаз нагрузки $R=X_L=11$ Ом.

Определить показания амперметров.

$$U_Л = U_Ф = 220$$



$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}$$

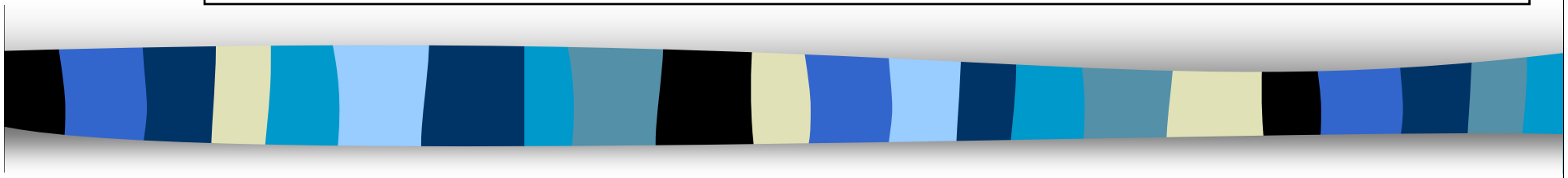
$$\underline{I}_{BC} = \frac{\underline{U}_{BC}}{R} = \frac{a^2 \underline{U}_{AB}}{R} = \frac{220e^{-j120}}{11} = 20e^{-j120} \text{ A}$$

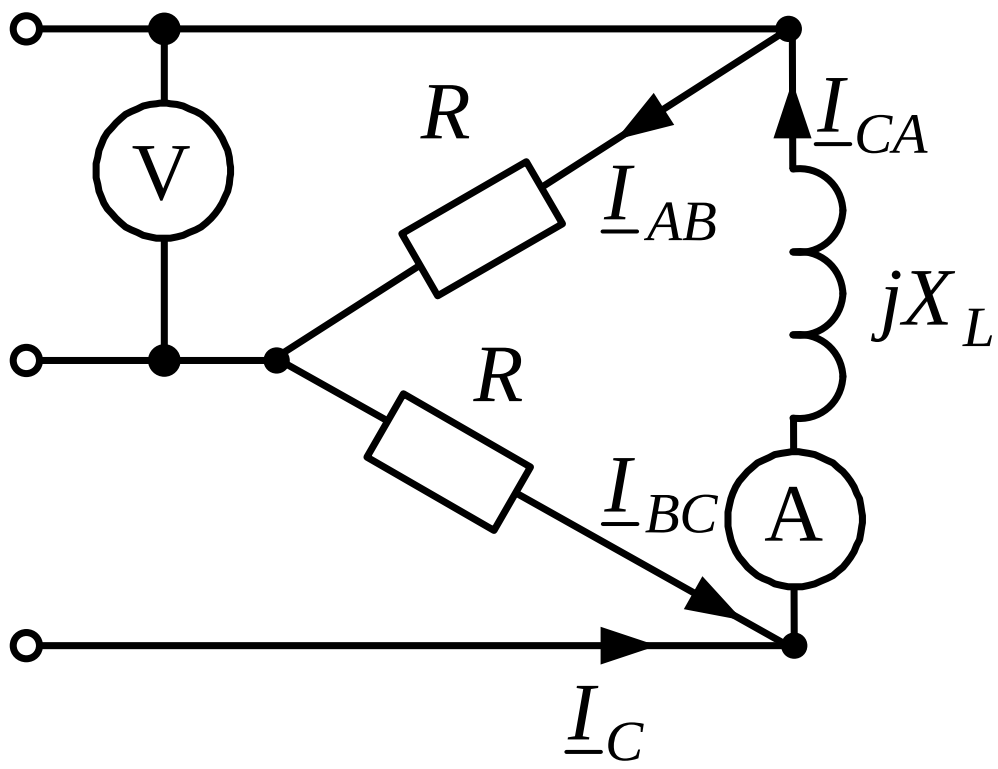
$$\underline{I}_{CA} = \frac{\underline{U}_{CA}}{jX_L} = \frac{a\ \underline{U}_{AB}}{jX_L} = \frac{220e^{j120}}{j11} = 20e^{j30} \text{ A}$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = 20e^{j30} - 20e^{-j120} = 38,62e^{j45} \text{ A}$$

$$I_{A1} = \left|\underline{I}_C\right| = 38,62 \text{ A} \qquad I_{A2} = \left|\underline{I}_{CA}\right| = 20 \text{ A}$$

Пример 3:





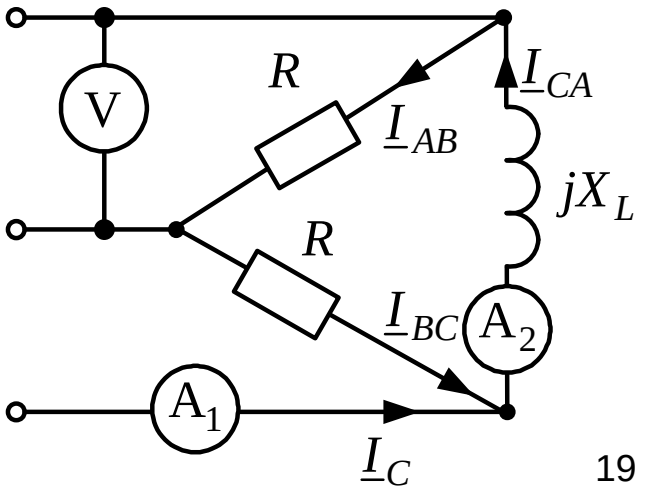
В несимметричной трёхфазной цепи, сопротивление фаз нагрузки $R=X_L=$ 11 Ом, Показания амперметра 38,62 А.

Определить показания вольтметра

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = \frac{a\ \underline{U}_{AB}}{jX_L} - \frac{a^2 \underline{U}_{AB}}{R} =$$

$$I_A = \frac{a\ U_V}{jX_L} - \frac{a^2 U_V}{R} = U_V \left(\frac{a}{jX_L} - \frac{a^2}{R} \right)$$

$$U_V = \frac{I_A}{\left(\frac{a}{jX_L} - \frac{a^2}{R} \right)} =$$



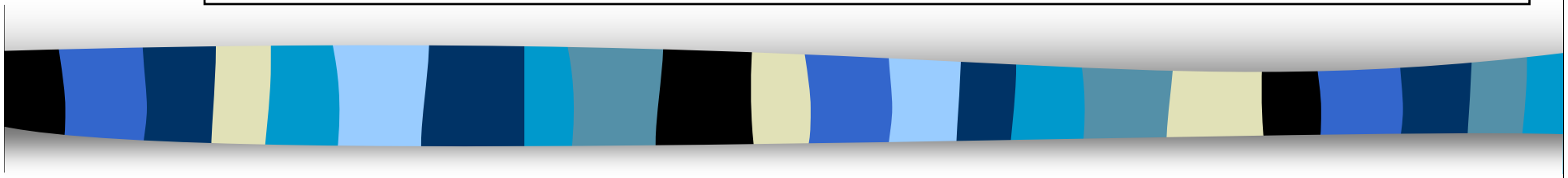
$$= \frac{38,62\ j11}{\left(\frac{e^{j120}j11}{j11} - \frac{e^{-j120}j11}{11}\right)} = \frac{38,62 \cdot j11}{e^{j120} - e^{-j30}} =$$

$$= \frac{j484,8}{(-0,5 + j0.867) - (0,86 - j0,5)} = \frac{484,8e^{j90}}{-1,366+1,366i} =$$

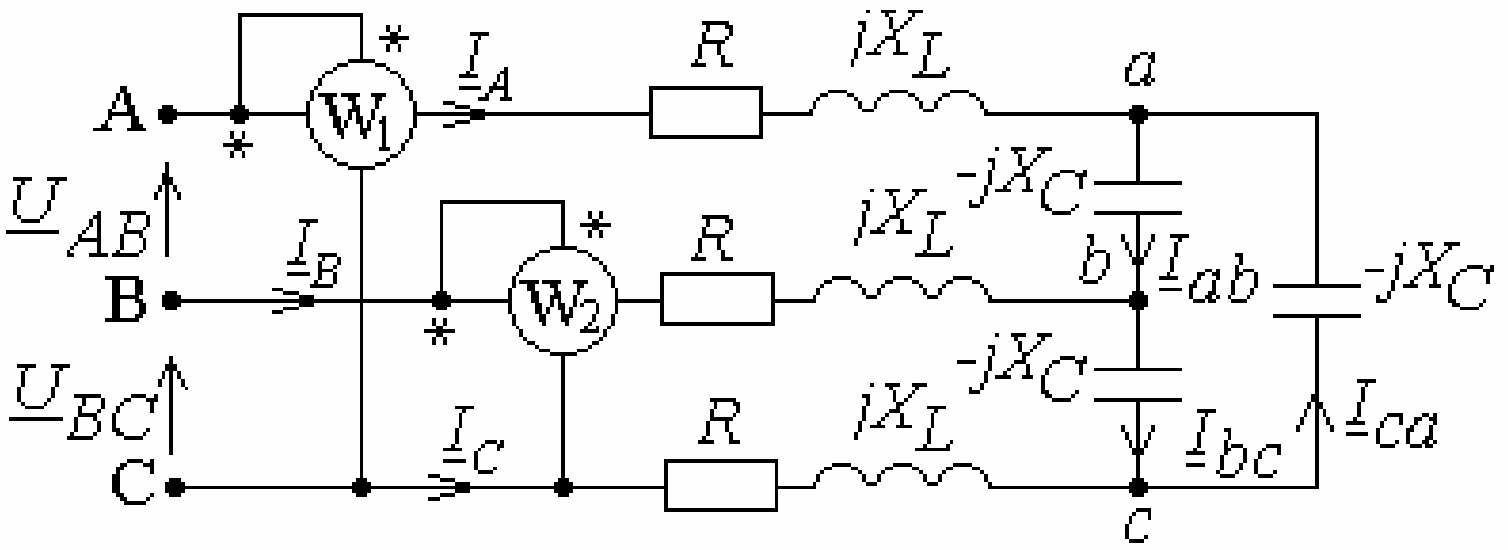
$$= \frac{424,8e^{j90}}{1,932e^{j135}} = 220e^{-j45}$$

$$U_V = 220\ В$$

Пример 4:



Определить показания ваттметров и активную мощность, потребляемую цепью.



$$\underline{U}_{AB} = 173e^{j60^\circ}$$

$$R = 25 \text{ Ом};$$

$$X_L = 24 \text{ Ом}; \; X_C = 72 \text{ Ом};$$

1. Преобразуем “треугольник” сопротивлений abc в эквивалентную звезду с сопротивлениями

$$X_C' = X_C / 3 = 24 \text{ Ом}$$

2. По заданному линейному напряжению найдем фазную ЭДС:

$$\underline{E}_A = \frac{\underline{U}_{AB}}{\sqrt{3}\cdot e^{j30^\circ}} = \frac{173e^{j60^\circ}}{\sqrt{3}\cdot e^{j30^\circ}} = 100e^{j30^\circ}\ \text{В}$$

3. Найдем линейные токи:

$$\underline{I}_A = \frac{\underline{E}_A}{R + jX_L - jX_C^{'}} = \frac{100e^{j30^\circ}}{25 + j24 - j24} = 4e^{j30^\circ} \quad A$$

$$\underline{I}_B = a^2 \underline{I}_A = e^{-j120^\circ} \cdot 4e^{j30^\circ} = 4e^{-j90^\circ}$$

4. Определим фазные токи “треугольника” *abc* :

$$\underline{\mathbf{I}}_{\mathbf{ab}} = \frac{\underline{\mathbf{I}}_{\mathbf{A}}}{\sqrt{3}} \cdot \mathbf{e}^{\mathbf{j30^\circ}} = \frac{\mathbf{4e^{j30^\circ}}}{\sqrt{3}} \cdot \mathbf{e}^{\mathbf{j30^\circ}} = \mathbf{2,31e^{j60^\circ}}$$

$$\underline{\mathbf{I}}_{\mathbf{bc}} = \mathbf{a}^2 \underline{\mathbf{I}}_{\mathbf{ab}} = \mathbf{2,31e^{-j60^\circ}}$$

$$\underline{\mathbf{I}}_{\mathbf{ca}} = \mathbf{a}\underline{\mathbf{I}}_{\mathbf{ab}} = \mathbf{2,31e^{j180^\circ}} = \mathbf{-2,31}$$

5. Найдем приложенные к ваттметрам напряжения:

а) к первому ваттметру

$$\underline{U}_{AC} = -\underline{U}_{CA} = -a\underline{U}_{AB} = -e^{j120^\circ} \cdot 173e^{j60^\circ} =$$

$$= -173e^{j180^\circ} = 173$$

б) ко второму ваттметру

$$\underline{U}_{BC} = a^2\underline{U}_{AB} =$$

$$= e^{-j120^\circ} \cdot 173e^{j60^\circ} = 173e^{-j60^\circ}$$

6. Определим показания ваттметров:

$$P_1 = U_{AC} \cdot I_A \cdot \cos(\underline{U}_{AC} \hat{} \underline{I}_A) =$$

$$= U_{AC} \cdot I_A \cdot \cos(\psi_{AC} - \psi_A) =$$

$$= 173 \cdot 4 \cdot \cos(0 - 30^\circ) = 600 \text{ Вт}$$

$$P_2 = U_{BC} \cdot I_B \cdot \cos(\underline{U}_{BC} \hat{} \underline{I}_B) =$$

$$= U_{BC} \cdot I_B \cdot \cos(\psi_{BC} - \psi_B) =$$

$$= 173 \cdot 4 \cdot \cos(-60 + 90^\circ) = 600\,\text{Вт}$$

тогда сумма показаний ваттметров равна

$$P = P_1 + P_2 = 600 + 600 = 1200\ \text{Вт}$$

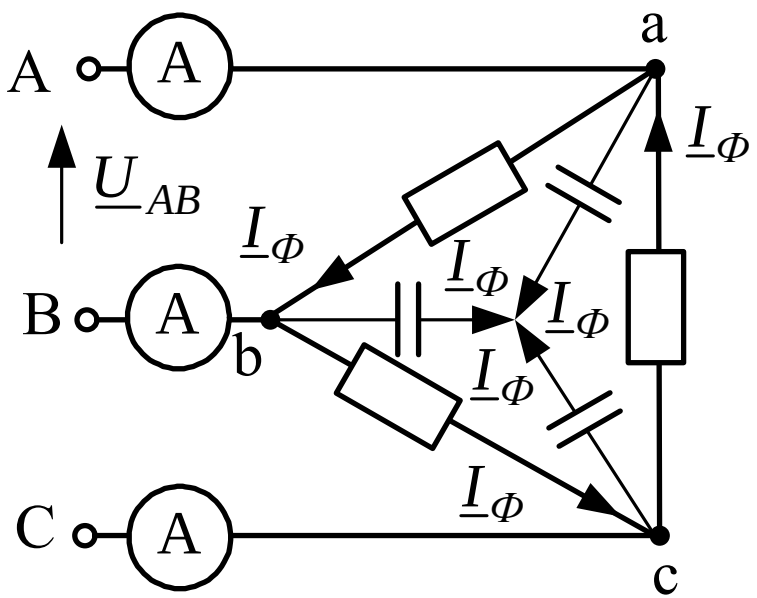
7. Найдем активную потребляемую цепью мощность:

$$P_П = 3(I_A)^2 R = 3 \cdot 4^2 \cdot 25 = 1200 \text{ Вт}$$

т.е. расчет проведен верно, т.к. $P = P_1 + P_2 = P_П$.

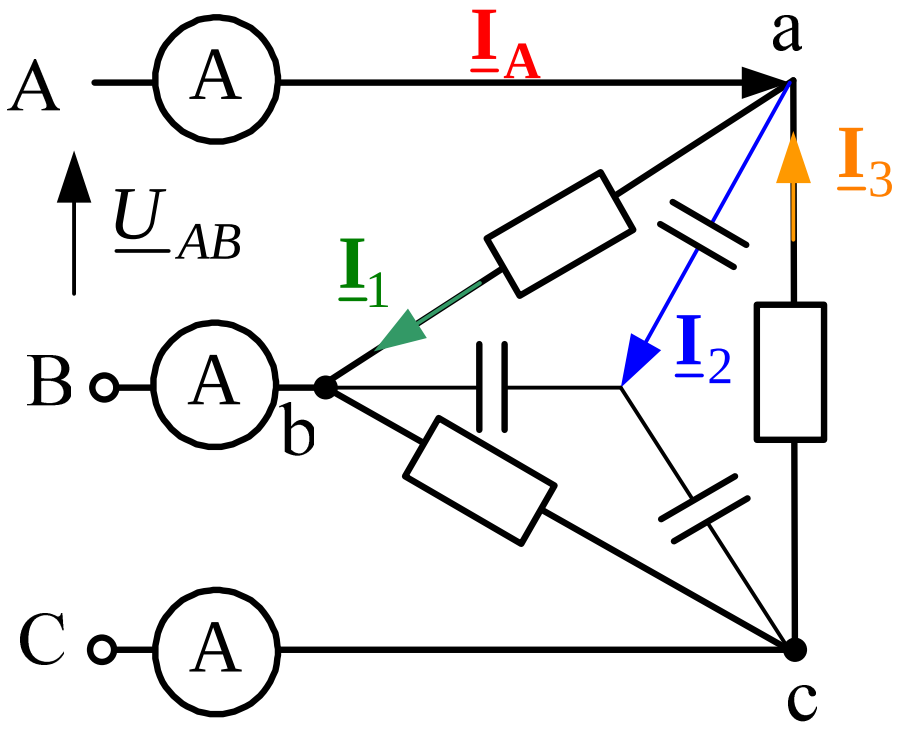
Действительно, для измерения активной мощности симметричной трехфазной цепи, достаточно включить один ваттметр и удвоить его показание.

Пример 5:



$$\left|\underline{\mathbf{I}}_{\mathbf{Ф}}\right| = \mathbf{4\ A}$$

Определить показания амперметров



$$\underline{\mathbf{I}}_{\mathbf{A}} = \underline{\mathbf{I}}_{\mathbf{1}} + \underline{\mathbf{I}}_{\mathbf{2}} - \underline{\mathbf{I}}_{\mathbf{3}} = \mathbf{4e^{jx1}} + \mathbf{4e^{jx2}} - \mathbf{4e^{jx3}} =$$

$$= \frac{\underline{\mathbf{U}}_{\mathbf{AB}}}{\mathbf{R}} + \frac{\underline{\mathbf{U}}_{\mathbf{A}}}{-\mathbf{jX_C}} - \frac{\underline{\mathbf{U}}_{\mathbf{CA}}}{\mathbf{R}} =$$

$$\underline{U}_{AB} = \underline{U}_A \sqrt{3} e^{j30}$$

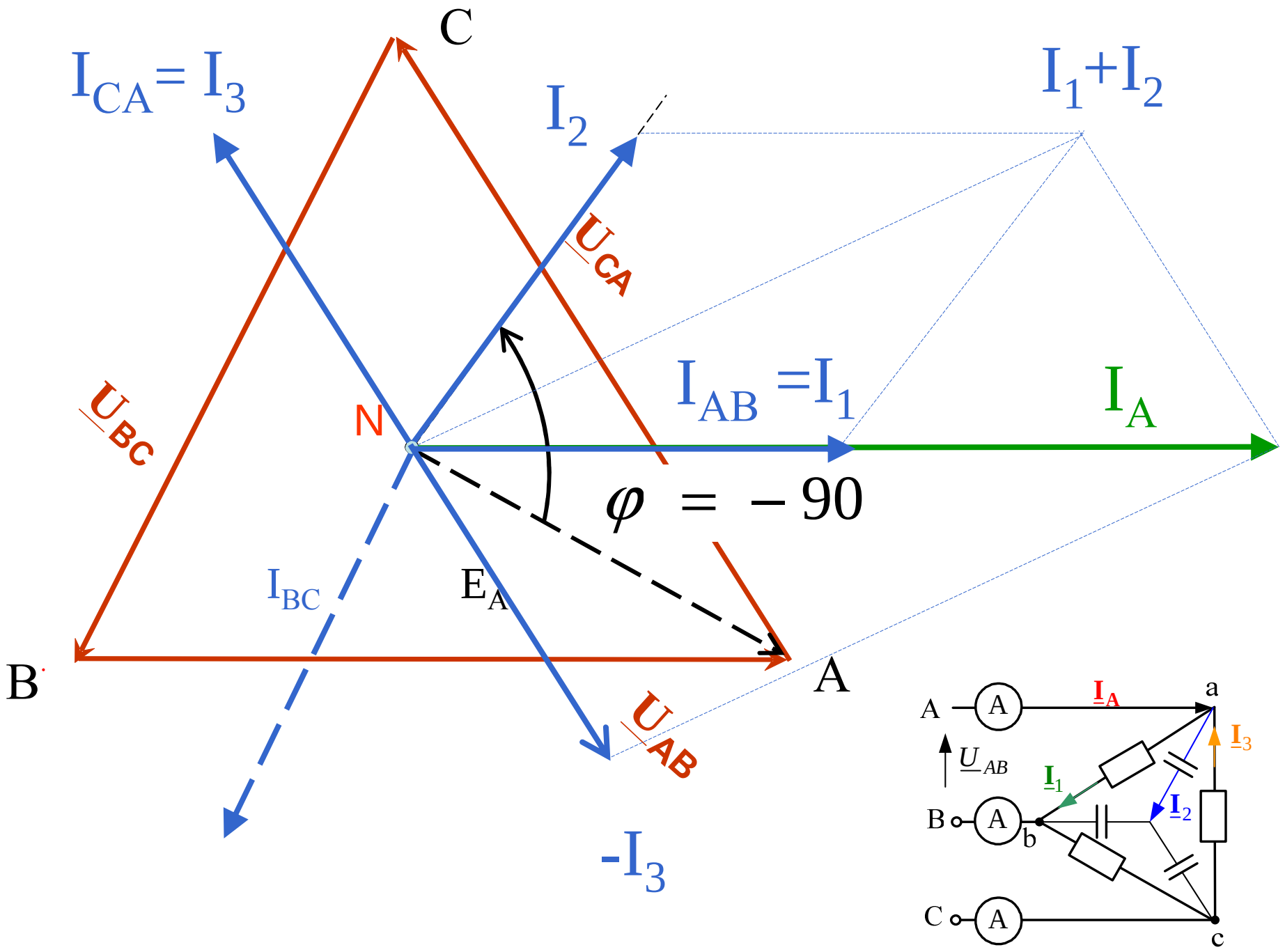
$$= \frac{\underline{U}_{AB}}{R} + \frac{\frac{\underline{U}_{AB}}{\sqrt{3}} e^{-j30}}{-jX_C} - \frac{\underline{U}_{AB} \cdot e^{j120}}{R} =$$

$$= \underbrace{\frac{\underline{U}_{AB}}{R}}_{=4} + \underbrace{\frac{\frac{\underline{U}_{AB}}{\sqrt{3}} e^{-j30}}{X_C} e^{-j90}}_{=4} - \underbrace{\frac{\underline{U}_{AB}}{R} e^{j120}}_{=4} =$$

$$= 4 + 4e^{+j60} - 4e^{j120} =$$

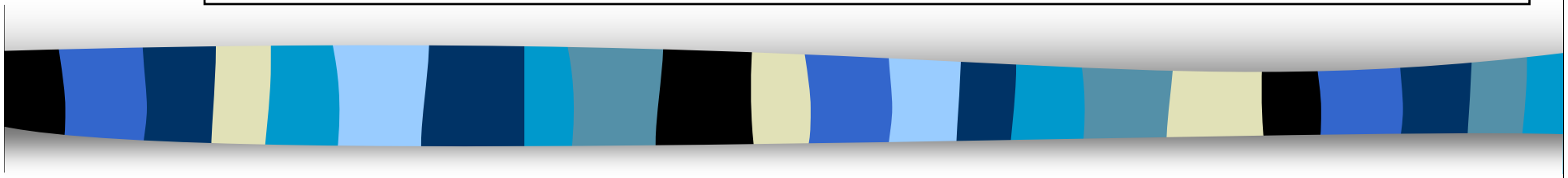
$$= 4 + (2 + j3,464) - (-2 + j3,464) = 8 \text{ A}$$

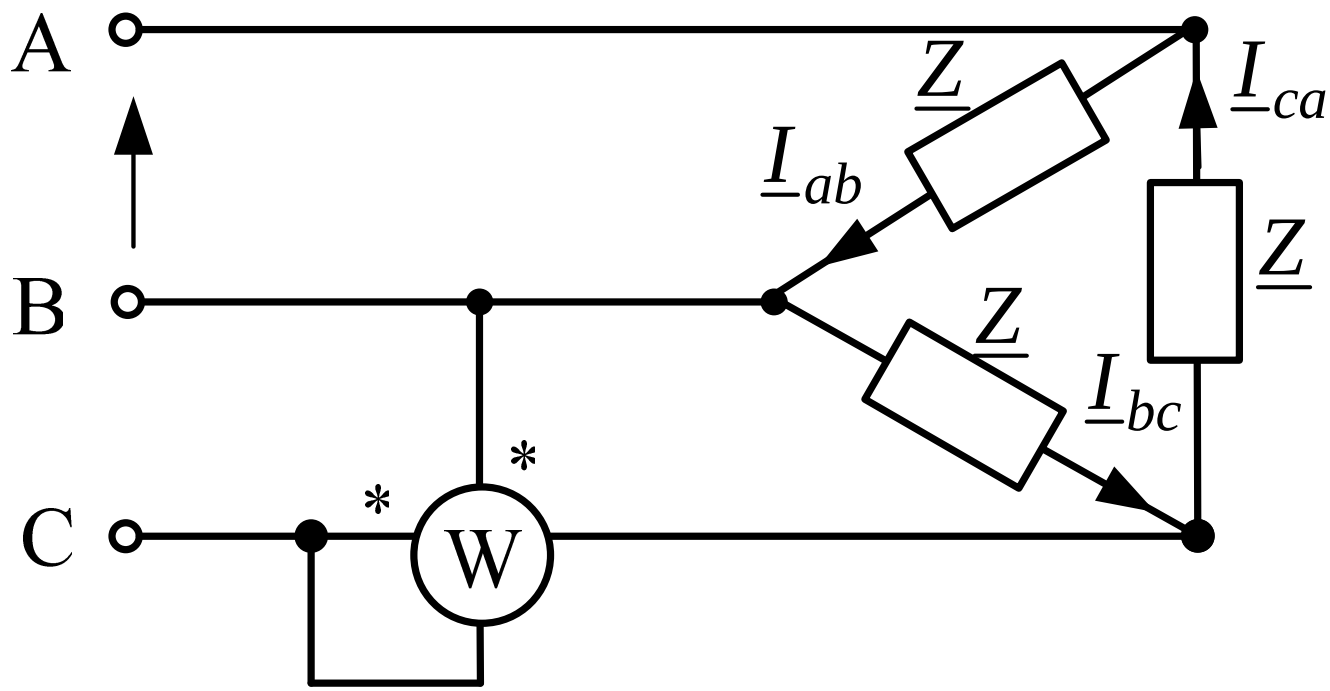
Решение с помощью векторной диаграммы


C
$I_{CA}= I_3$
I_2
I_1+I_2
$\underline{U}_{CA}$
$\underline{U}_{BC}$
N
$I_{AB} =I_1$
I_A
$\varphi = -90$
I_{BC}
E_A
B
A
$\underline{U}_{AB}$
$-I_3$

A
A
$\underline{I}_A$
a
$\underline{I}_3$
$\underline{U}_{AB}$
$\underline{I}_1$
$\underline{I}_2$
B
A
b
C
A
c

Пример 6:

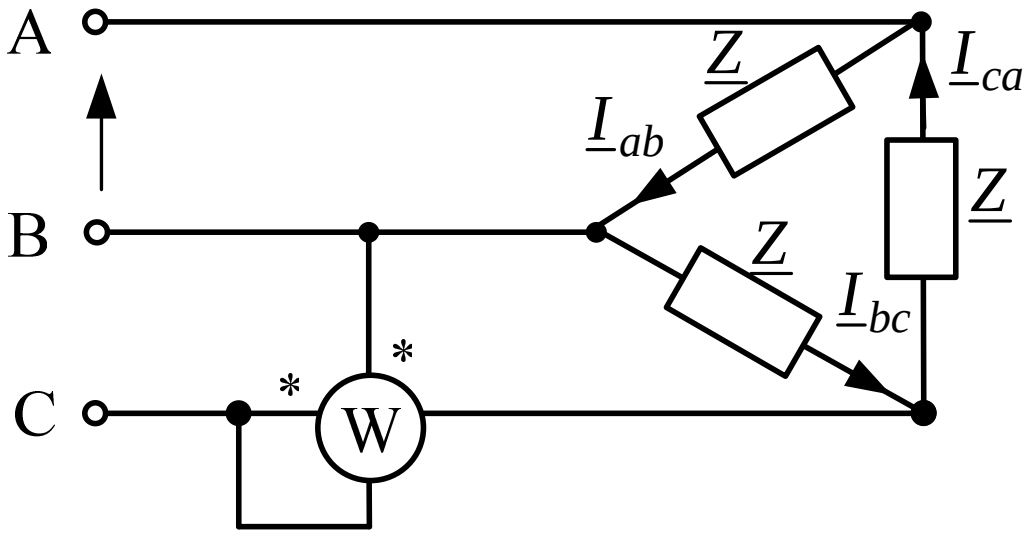




Найти показание ваттметра, если

$$\underline{E}_A = 500\,e^{j10^0} \quad B$$

$$\underline{Z}_\phi = \quad 120+\text{j}30 \text{ Ом.}$$



$$P_W = \left|U_{BC}\right| \cdot \left|I_C\right| \cdot \cos\varphi,\ \text{Вт}$$

$$\underline{U}_{BC} = a^2 \underline{U}_{AB} = a^2 \sqrt{3} \underline{E}_A e^{j30} = e^{-j120} \sqrt{3} 500 e^{j10} e^{j30} =$$

$$= 500\sqrt{3}\, e^{j(-120+10+30)^0} = 866{,}025\, e^{-j80^0}$$

$$\underline{I}_C = a\underline{I}_A = a\frac{\underline{E}_A}{\underline{Z}/3} = e^{j120}\frac{500e^{j10}}{\frac{120+j30}{3}} = \frac{500e^{j130}}{40+j10} =$$

$$= \frac{500e^{j130}}{41{,}2e^{j14}} = 12{,}127e^{j116}$$

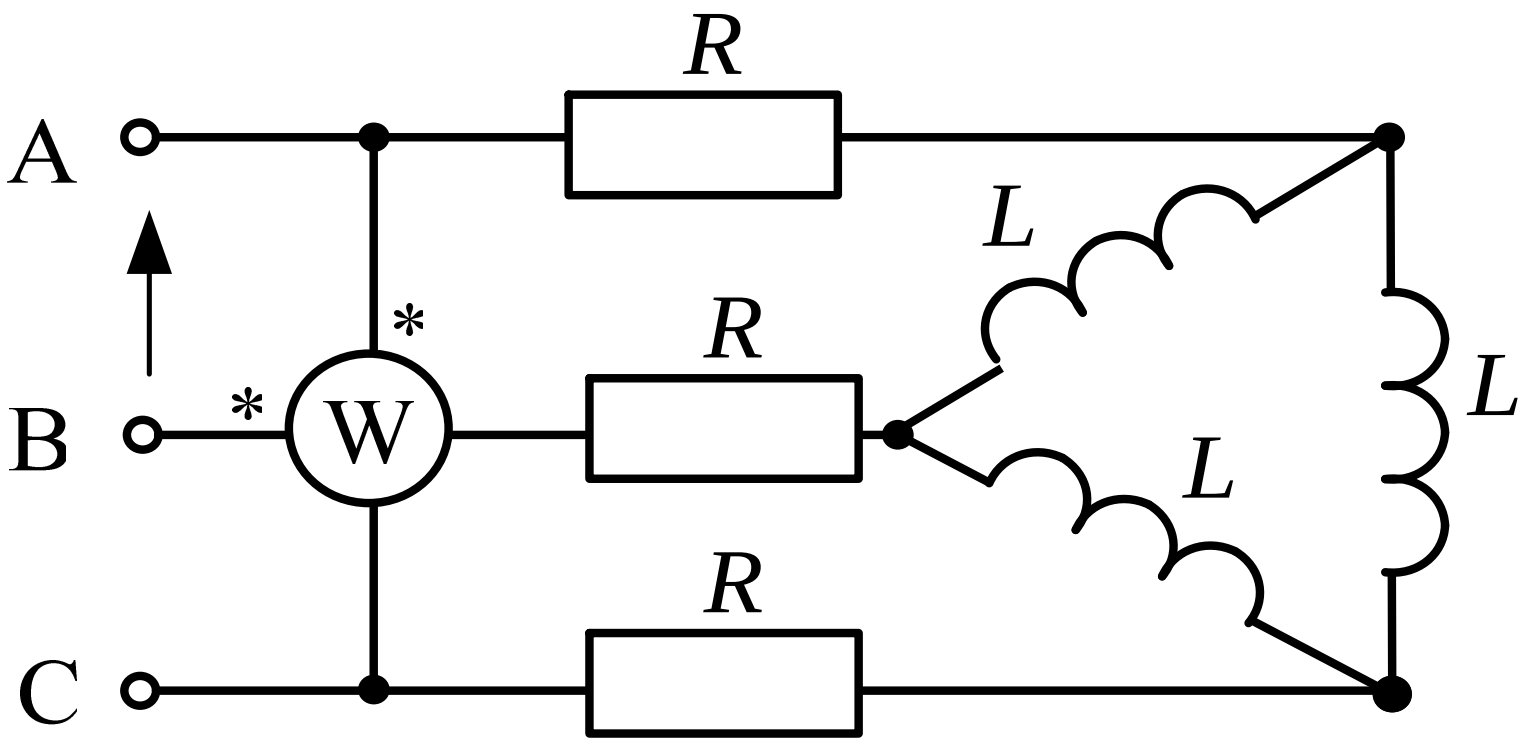
$$P_W = \left|U_{BC}\right| \cdot \left|I_C\right| \cdot \cos\varphi =$$

$$= 866{,}025 \cdot 12{,}127 \cos(-80-116) =$$

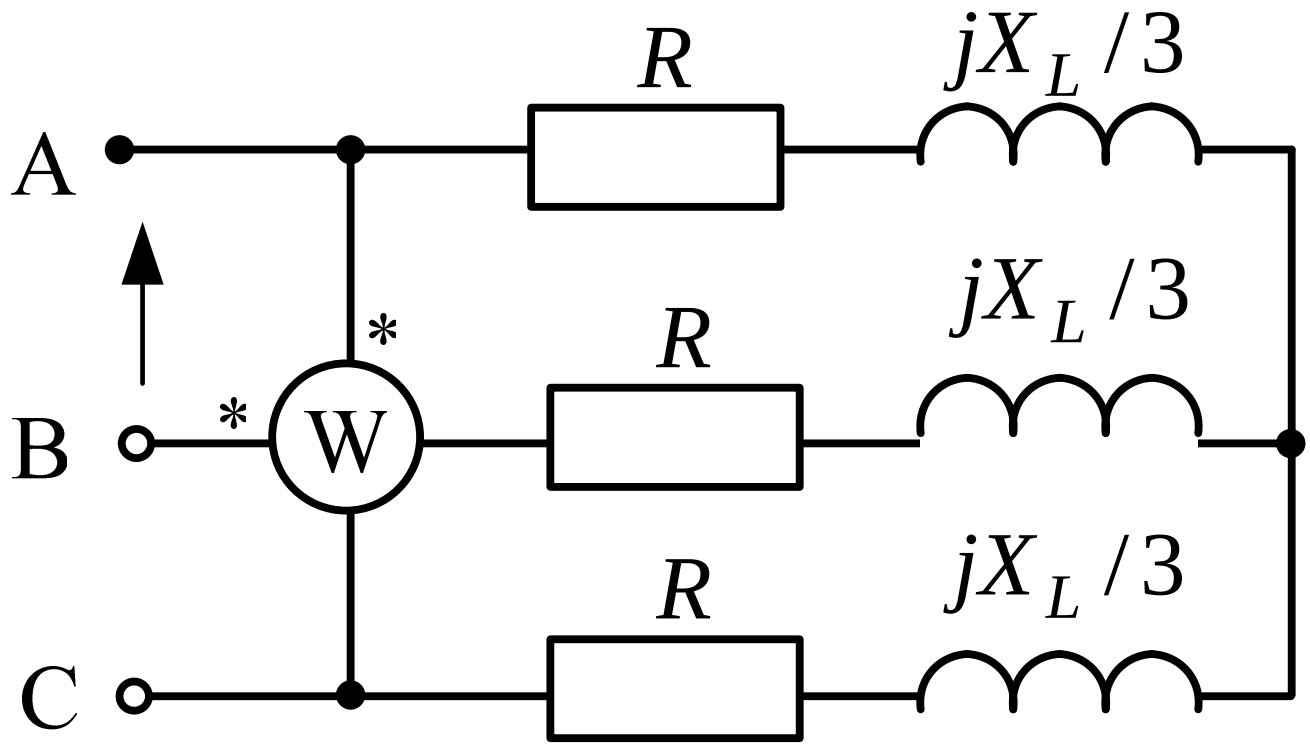
$$= -10097{,}096 \text{ Вт}$$

Пример 7:

Например: P_W=?



$$\mathbf{P_W = \left|U_{AC}\right| \cdot \left|I_B\right| \cdot \cos\varphi,\ Вт}$$



$$\underline{\mathbf{I}}_{\mathbf{A}} = \frac{\underline{\mathbf{E}}_{\mathbf{A}}}{\underline{\mathbf{Z}}} = \frac{\underline{\mathbf{E}}_{\mathbf{A}}}{\mathbf{R} + \frac{\mathbf{jX_L}}{\mathbf{3}}}$$

$$\underline{\mathbf{I}}_{\mathbf{B}} = \mathbf{a}^{\mathbf{2}}\underline{\mathbf{I}}_{\mathbf{A}}$$

$$P = \left|\underline{U}_{AC}\right|\left|\underline{I}_{B}\right| \cos\left(\hat{\underline{U}_{AC}\underline{I}_{B}}\right) = \mathrm{Re}(\underline{U}_{AC}, \underline{I}_{B}^{*})$$

$$\underline{U}_{AC} = -\underline{U}_{CA} = -a\ \underline{U}_{AB} =$$

$$= \underline{U}_{AB} e^{j(120-180)} = \underline{U}_{AB} e^{-j60} =$$

$$= \underline{E}_{A} \sqrt{3} e^{j(-60+30)} = \underline{E}_{A} \sqrt{3} e^{-j30}$$