Detection accuracy of the temporary state of complex signals using phase-frequency tracking methods with equilibrium and non-equilibrium processing

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Abstract. The report proposes the phase-frequency tracking methods of complex signals based on optimal and suboptimal processing of PFC and group delay functions (FGZ). At the same time the cases of correlated and uncorrelated samples of PFC and FGZ are analyzed herein. It is demonstrated that the correlation in the samples does not change the processing structure, but only the weight factors are changed.

The phase-frequency algorithms with equilibrium and non-equilibrium processing are developed based on the proposed methods. It is shown that the transition to the equilibrium processing enables significant level down the requirements to apriori information on the properties of the useful signal, while the non-equilibrium processing increases the resolution of the signals significantly.

The conducted analytical argument and results of the simulation experiments on the produced simulated wave fields have testified that at propagation of the complex signals in dispersive media, these algorithms can assure rather high detection accuracy of the signal temporary state, even when the signal-to-noise ratio is close to 1. The results of the simulation experiments are justified by the real data obtained in processing of the seismic wave fields.

Keywords: phase-frequency characteristic, equilibrium and non-equilibrium processing, time position signal

1 INTRODUCTION

In the performance of a wide range of the tasks in such areas as radio detection, navigation and communication and geophysics, there arise the problems related to improper detection accuracy of the temporary state of complex signals, that is primarily conditioned by the presence of interference ranges of useful signals, dispersion nature of their propagation medium and the presence of intense incoherent noise [1]. Under such conditions the customary correlation methods for detection of the temporary state of the complex signals prove to be very inefficient [2-3] and require using the innovative approaches that make use of less apriori information on the properties of the useful signal while working sufficiently stable at signal propagation in the disper-

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sive and absorption media. In this respect, the most promising are the phase-frequency methods of the complex signals tracking, based on the effective processing of the information extracted from the phase-frequency characteristic (PFC) of the signal[4]. The prerequisite for their successful implementation is the fact that the phase of the signal, more precisely the complex law of variation of its PFC, provides for information that allows more efficient detection of the signal and measurement of its temporary state under high-amplitude noise [5].

2 METHODS DEVELOPMENT

The most common solution for determining signal's temporary state is to evaluate one of the non-energy parameters of the normal random process and not account specificity of the time parameter. At the same time the view of the signal`s temporary state in exponential basis is totally determined by its phase-frequency characteristic (PFC). Therefore, the optimal process of the PFC signal implements the optimal method for determining its temporary state [6].

Let it is given mixture of additive deterministic signal *s(t)* and Gaussian noise *n(t)*:

$$x(t,\tau) = s(t-\tau) + n(t) \, , \quad (1)$$

where τ - signal's temporary state.

It is required to analyze phase-frequency algorithms to investigate the accuracy of complex signals` temporary provisions with the use of equilibrium and non-equilibrium processes.

As it is known that the optimal method for determining the phase of signal's temporary state observed on the background of Gaussian noise is implemented as searching for maximum likelihood of the function [5]:

$$L(t) = \sum_{k=1}^{m} \delta(\omega_k) \cdot \cos\left(\Delta\phi(\omega_k) - \omega_k t\right) . \quad (2)$$

Where

$\Delta\phi(\omega_k) = \phi_x(\omega_k) - \phi_s(\omega_k)$ - the deviation of the signal's phase from the phase spectrum of the mixture of signal and noise;

$\delta(\omega_k) = \dfrac{A(\omega_k)}{\sigma(\omega_k)}$ - the peak signal to noise ratio at the frequency ω_k , *m* - number of analyzed frequency components.

It is not difficult to show that in case of a strong signal from the expression (1) the temporary state of the signal [7] can be directly accessed:

$$\tau_{opt} = \frac{\sum_{k=1}^{m} \delta^2(\omega_k) \cdot \omega_k \cdot \Delta\phi(\omega_k)}{\sum_{k=1}^{m} \delta^2(\omega_k) \cdot {\omega_k}^2} \quad (3)$$

In this case, the variance of (3) is:

$$D\left(\tau_{opt}\right)=\left[\sum_{k=1}^{m}\delta^2\left(\omega_k\right)\cdot\omega_k{}^2\right]^{-1}. \qquad (4)$$

If the values in the sample phase response are dependent among themselves, the optimal estimate of the temporary state of a strong signal in the integral form is as follow [7]:

$$\tau_{opt}=\frac{\int_{\Omega}V(\omega)\cdot\left[\phi_x(\omega_k)-\phi_s(\omega_k)\right]d\omega}{\int_{\Omega}V(\omega)d\omega}. \qquad (5)$$

Where

$$V(\omega)=\int_{\Omega}R^{-1}(\omega,\omega')\cdot\omega' d\omega'. \qquad (6)$$

$R(\omega,\omega')$ - positive definite matrix, consisting of the elements of the frequency correlation function PFC mixture; Ω - analyzed frequency band, and the variance of (5) is

$$D\left(\tau_{opt}\right)=\left[\int_{\Omega}\int_{\Omega}R^{-1}(\omega,\omega')\cdot\omega\cdot\omega' d\omega\, d\omega'\right]^{-1}. \qquad (7)$$

Comparison of (3) and (5) shows that when measuring the temporary state of a strong signal correlation values phase response in the sample mixture leads only to a change in the weighting procedures for handling phase response. Therefore, the expression (2) can be used to estimate the state of the signal, both uncorrelated and correlated sample PFC for any signal to noise ratio. Also it can be shown in (3) that in the case of a weak signal and uncorrelated variance of the sample PFC the time expression of the signal is:

$$D\left(\tau_{opt}\right)\approx\frac{4}{\pi}\cdot\left[\sum_{k=1}^{m}\delta^2\left(\omega_k\right)\cdot\omega_k{}^2\right]^{-1}. \qquad (8)$$

In practice, the optimal estimates of the signal's temporary state by maximizing the expression (2) is often not possible, since the distribution of the signal / noise ratio in the test band Ω, forming weights in (2) is usually unknown. It is therefore proposed to use the so-called phase frequency algorithms with equilibrium and non-equilibrium processing.These algorithms can be obtained from the optimal method by replacing in (2) the weight function $\delta\left(\omega_k\right)$ with other specially selected function. In general, the likelihood function (criterion time position signal) for these algorithms can be written as:

$$L(t)=\sum_{k=1}^{m}w\left(\omega_k\right)\cdot\cos\left(\phi(\omega_k,t)\right). \qquad (9)$$

Where $w\left(\omega_k\right)$ - frequency weighting function, the form of which depends on the implemented phase-frequency algorithm.

For the equilibrium algorithm weighting function $w\left(\omega_k\right)$ has the value of unity in the whole frequency band. For the algorithm of the non-equilibrium processing

$w(\omega_k)$ can have any form, but numerous experiments with different weight functions show [8], that the one with a triangular weight function is the best:

$$w(\omega)=\frac{4}{3\omega_m}\begin{cases} 0,\ \omega<\omega_L \\ \frac{2}{\omega_m}(\omega-\omega_L),\ \omega_L<\omega<\omega_m \\ -\frac{1}{\omega_m}(\omega-\omega_H),\ \omega_m<\omega<\omega_H \end{cases}, \qquad (10)$$

where ω_H and ω_L are, respectively, the higher and lower frequency bounds constraining $w(\omega_k)$, and $w(\omega_m)$ is the peak frequency of $w(\omega_k)$; $\omega_m=2\omega_L$; $\omega_H=2\omega_m$.

In the general case, Eq. (9) can be a picking result, because there is some analogy between estimating times in the conventional algorithms and the lowpass filtering. Indeed, the likelihood function equation (9) is an inverse discrete Fourier transform of the digitally filtered initial data, with the bandpass [7]:

$$H(\omega_k)=\frac{w(\omega_k)}{|X(k)|},\ k=\overline{1,m}, \qquad (11)$$

where $|X(k)|$ is the amplitude response.

As follows from (11), this bandpass filtering first straightens the amplitude spectrum and then weighs it with the specified weight coefficients, while the phase patterns in the initial record remain the same. Straightening the amplitude response in the case of a linear phase response is known [9] to compress the signal and, hence, to make it better resolved in the record. Furthermore, this kind of filter makes it possible to manage the bandpass frequency by specifying the weight coefficients $w(\omega_k)$ and thus either to strengthen or to weaken different frequency components of the signal.

It is not difficult to show that for a strong signal and uncorrelated variance of the sample PFC the time position for the equilibrium processing algorithms is:

$$D(\tau)=\frac{\sum_{k=1}^{m}\frac{\omega_k^{\,2}}{\delta^2(\omega_k)}}{\left(\sum_{k=1}^{m}\omega_k^{\,2}\right)^2} \qquad (12)$$

.

And for non-equilibrium processing algorithms

$$D(\tau)=\frac{\sum_{k=1}^{m}\frac{w^4(\omega_k)\cdot\omega_k^{\,2}}{\delta^2(\omega_k)}}{\left(\sum_{k=1}^{m}w^2(\omega_k)\cdot\omega_k^{\,2}\right)^2}. \qquad (13)$$

In comparison of (12) and (13) with (4) the transition to the equilibrium and non-equilibrium algorithms reduces the accuracy of the estimates, but this approach requires much less a priori information about the signal be recorded, namely only the information about the values of its PFC in analyzed frequency band.

At signals propagation in dispersive medium the temporal position of the signal is determined by its group delay [10]. In that event, the optimal method to determine the temporal position of the signals is realized in the form of the search procedure of the maximum of the plausibility function of the group delay[11]:

$$lnI(\tau) = \sum_{k=1}^{m} \gamma(\omega_k) \cos\left(\omega_k (\Delta t_{gr}(\omega_k) - \tau)\right) \tag{14}$$

where, $\Delta t_{gr}(\omega_k) = t_{gr}^{x}(\omega_k) - t_{gr}^{s}(\omega_k)$

$t_{gr}^{x}(\omega)$ - function of the group delay (FGD) of the noisy signal mixture;

$t_{gr}^{s}(\omega)$ - signal FGD;

$\gamma(\omega_k)$ - signal / noise ratio in the derivatives at the frequency ω_k .

By analogy with the above the algorithms with the FGD equilibrium and non-equilibrium processing may be formed based on formula (14). Here, $\gamma(\omega_k)$ is taken along the whole frequency band to be equal to unit for the equilibrium algorithms, and $\gamma(\omega_k)$ is described by formula (10) for non-equilibrium algorithms.

3 RESULTS

To investigate the effectiveness of the proposed algorithms for the case of a weak signal and the correlated sampling PFC numerical experiments were conducted. For this purpose a mathematical model of the form (1) was built, where the desired signal on the momentum from the bell tower of the envelope had duration of 60 ms, and the noise was generated with a random number and normal distribution. PFC mixture of signal and noise was calculated by DFT in the range of 20-60 Hz in steps of 1 Hz sampling. With implementation of algorithms with a non-equilibrium process as a function the weight function has the form of a triangle.

Figure 1 demonstrates plots of the displacement, and Figure 2 shows plots of the standard deviation estimates of the temporal state of the signals received by algorithms with equilibrium and non-equilibrium PFC processing depending on the signal-to-noise ratio.

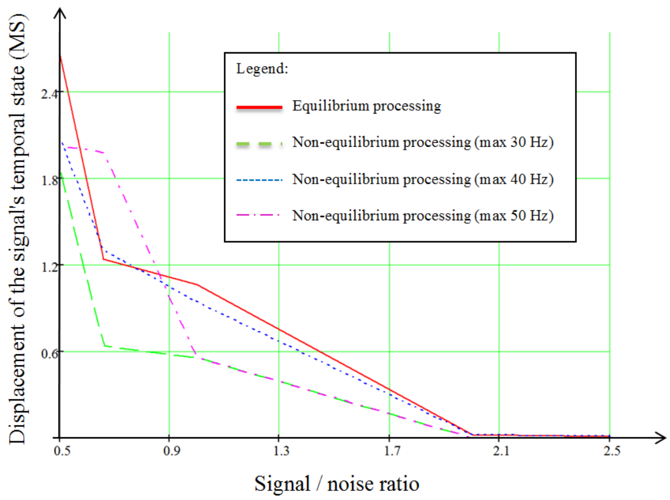


Fig. 1. The dependence of the bias in the temporal state of the signal from the signal-to-noise ratio

From the analysis results the accuracy of the estimates of phase response of the method is quite large, so with a signal-to-noise ratio of 0.9, the shift time position estimation is in the range of 1.2 to 2.2 ms (Fig. 1), and the standard deviation assess the temporary state of the signal with a signal-to-noise ratio of 1.3 is less than 17 ms (Fig. 2).In this case, the non-equilibrium processing methods generally provide more accurate estimates in the sense of reducing the standard deviation (Fig. 2).For the signal-to-noise ratio less than 1, the movement of the maximum of the weighting function in the higher frequencies leads to a reduction of the standard deviation (Fig. 2), but the minimum value of the bias is already provided with the equilibrium phase frequency methods of treatment (Fig. 1).

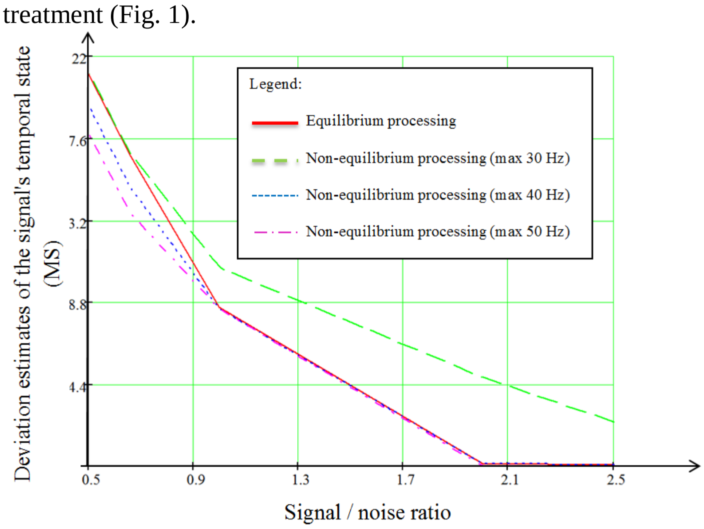
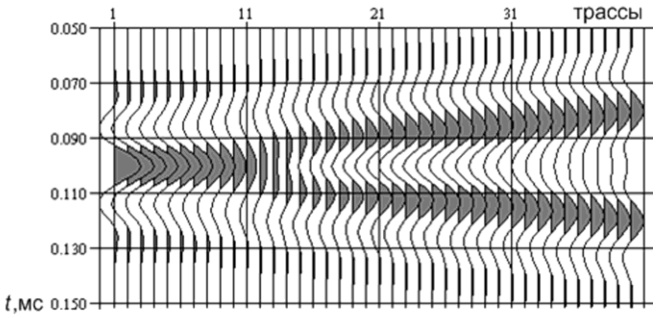
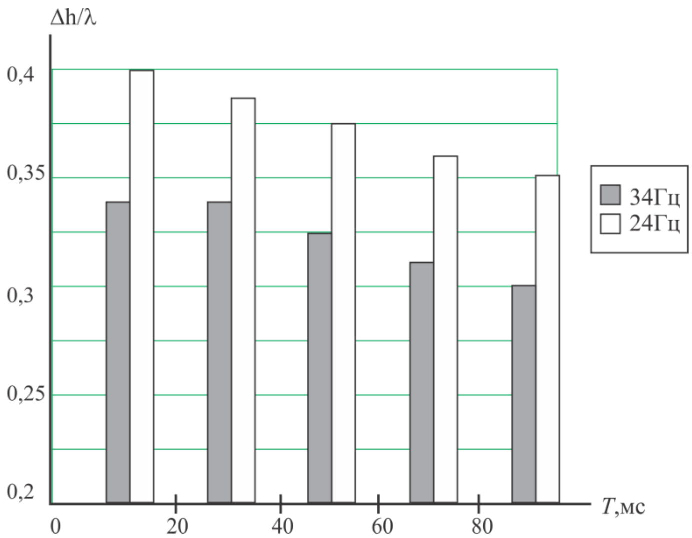


Fig. 2. The dependence of the standard deviation estimates the temporary position of the signal from the signal-to-noise ratio

A synthetic wavefield with two identical waveforms (Fig. 3, a) was used to study the resolution of the non-equilibrium picking algorithm. We used triangular function (10), as a weight, and a bell-shaped wavelet with the central frequencies 24 and 34 Hz.



a)



б)

Fig. 3. Resolution of the non equilibrium phase/frequency picking algorithm. a: synthetic seismogram; b: respective vertical resolution (Δh), λ is the wavelength.

The non-equilibrium algorithm shows quite a high resolution, of the order of ¼ wavelength in the vertical dimension (Fig. 3, b), and is thus applicable to data distorted by interference [12].

The developed algorithms have been used to process the wave seismic fields resulted in the oil and gas search in the Tomsk region. As an example, Figure 4 shows the fragments of the original wave field (Fig. 4,a) and the field after application of the phase-frequency algorithm with non-equilibrium processing (Fig. 4,b). The comparison of the findings shows, that the phase-frequency algorithms allowed significant increase of the recording resolution, and thereby identification of a number of reflection horizons, which are not visible in the original field. At the same time, the temporal position of the identified reflection horizons is well consistent with a priori geological information obtained from geophysical studies of the wells drilled earlier on the given area.

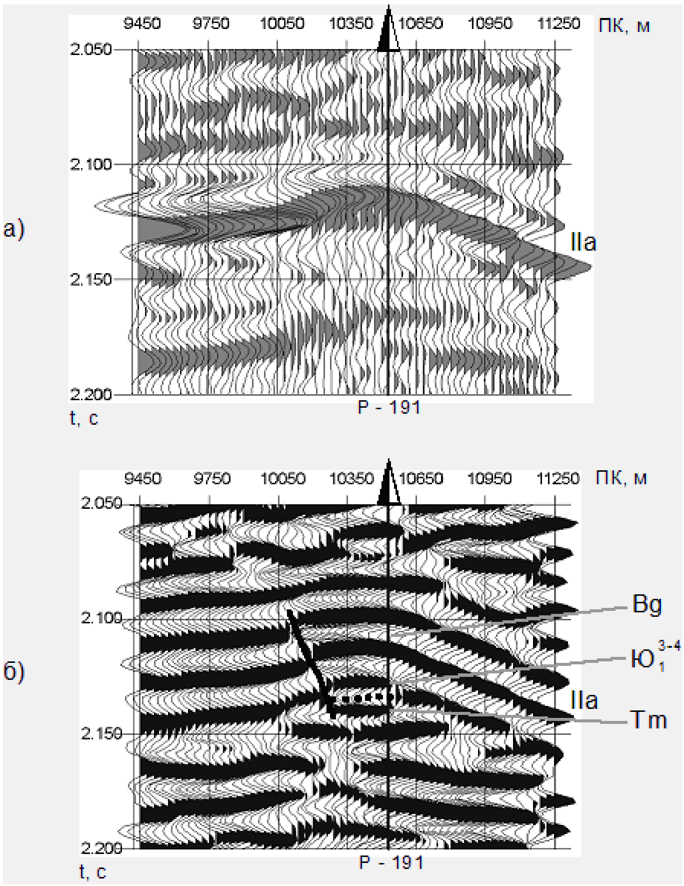


Fig. 4. Fragments of the wave seismic field. a: the original field; b - the field after the application of phase-frequency algorithms.

4 CONCLUSIONS

To conclude the research it is necessary to point out that the results of analysis show that the accuracy of the phase frequency method estimates is sufficiently high. At the same time, the non - equilibrium processing methods generally provide more accurate estimates in the sense of reducing the standardgdeviation.

Thus checking the accuracy of the study showed that the phase frequency methods with equilibrium and non-equilibrium processing provide sufficient accuracy to obtain time position signal estimates. In addition, their implementation requires just information of the PFC recorded signals. In the future we plan to study the accuracy of these methods in the case of calculating PFC with the derivatives of the coefficients of the Fourier series.

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