APPROVED
«___»_____________2020 г.

CALCULUS MATHEMATICS
COURSE SYLLABUS ()

STUDY MAJOR 09.03.01 Computer science and technology

QUALIFICATION (DEGREE) Bachelor

ADMISSION 2020
YEAR 2 Term 4

CREDITS 6

PREREQUISITES B2.B1, B2.B3, B1.1
COREQUISITES B3.B2, B3.V4.2

TIME DISTRIBUTION:

LECTURES	32	hrs.
LABORATORY WORKS	32	hrs.
PRACTICE	-	hrs.
CLASS HOURS	62	hrs.
SELF-STUDY	62	hrs.
TOTAL	124	hrs.

MODE OF STUDY Full-time

FINAL CERTIFICATION Test Final Test

DEPARTMENT Interdisciplinary

HEAD OF DEPARTMENT ____________________

HEAD OF DIRECTION ____________________

LECTURER ____________________ E.A.Kochegurova

2020

1. The purposes of the educational course

The main purposes of the course are:

- to provide students with theoretical foundations and methods of computational mathematics;
- to provide students with knowledge of typical mathematical problems statement and numerical methods of solving them;
- to develop practical skills of the numerical algorithms construction and error estimates for the numerical solution.

B2.B1, B2.B3, B1.1

B3.B2, B3.V4.2

2. The rank of discipline in educational program structure

The course refers to different parts of mathematics and natural science in the B2.V1.2 module.

Prerequisites are: Computer Science, Mathematical Analysis, Linear Algebra and Analytic Geometry. For successful completion of the course "Calculus Mathematics" students should learn "Integration", "Linear and non-linear equations", "Differential Equations", "Matrix calculus”. Students are also required to have experience in and knowledge of the algorithms development.

Knowledge and skills obtained in the course can be used in disciplines (co-requisites): "Programming", "Physics", "Computer Science".

3. Results of discipline studying

The main planned result is:

application of basic and special natural-science and mathematical knowledge in computer science and computer engineering, sufficient for complex engineering (R1).

As a result of studying the course, a student will have:

KNOWLEDGE (K1.5) OF:

- theoretical bases of studied numerical methods;
- design principles and limits on the use of computational methods;
- methods for monitoring of computation and error estimates;

SKILLS (S1.5) IN*:*

- applying of numerical methods for solving engineering problems;
- estimating calculation errors;
- selection numerical method for solving a mathematical problem according to its features and limitations to implement;
- development of algorithms for the numerical method.

COMPETENCES (C1.5)

- the ability to apply numerical method in practical problems;
- the ability to solve engineering tasks using modern mathematical software;
- the ability to make program implementations of numerical methods algorithms.

4. Course structure and content

4.1 Course content

Material is composed of the following sections:

1. Introduction and subject

The history of computational methods development. Principles and creations of computing methods . Principles of computing methods creation. Creation of algorithms for computing tasks. Computational experiment. Stability, complexity, convergence, correctness of the algorithm. Requirements for computing methods. Features of engineering mathematics solution. Mathcad, Matlab software.

2. Computational errors

Sources and classification of errors. Direct and inverse problems of the theory of errors. Full (total) error. Fatal and removable error. Absolute and relative errors. Fixed and floating- point representation. Computer digit grid. Significant and correct figures. Errors of elementary computing operations: sum, subtraction, multiplication, division. The general approach of computing algorithms errors.

3. Numerical Integration

Newton-Cotes Closed Quadrature Formulas. Left point, right point and midpoint quadrature formulas. Trapezoidal and Simpson's rules. Geometric illustration of quadrature formulas. Estimates of quadrature formulas: remainder term formula, by Runge principle. Simple Monte Carlo integration methods.

4. Solution Methods of Non-Linear Equations and Set Equations

Classification of equations and systems. Algebraic and transcendent equations. Direct and iterative numerical methods.

Features of iterative procedures. Scheme of solving nonlinear equations: separation and correction roots. Intervals theorem. Methods based on the theorem of intervals: bisection, secant and false position. Newton-Raphson method using derivative. Method of simple iterations Geometric illustration, formulas, convergence, initial approximation methods.

Newton-Raphson and simple iterations methods for solving systems of nonlinear equations. Convergence conditions and calculation scheme methods (algorithms).

5. Numerical Linear Algebra

Record form of linear algebraic sets. Existence and uniqueness of solution. Conditionality and stability of linear algebraic sets. Main problems of linear algebra. Direct and iterative numerical methods. Gauss elimination method - the main idea and implementation scheme (single-division and selection of main elements). Tasks associated with gauss method: triangular factorization of matrix, calculation of the determinant and inverse matrix.

Iterative method for solving linear algebraic sets: Jacobi iteration rule, Gauss-seidal iteration method. Concept of a matrix norm of and conditions of methods convergence.

6. Numerical solution of ordinary differential equations

Classification of differential equations. Cauchy problem and its solution methods. Instability of the Cauchy problem. Runge-Kutta methods – main idea. Order and accuracy of methods. Stability areas. Euler method, Higher-order schemes Runge-Kutta methods of

2-d, 4-th orders. Geometric illustration. Method errors. Runge-Kutta method with the adaptive step size control.

Higher-order differential equations and Systems of ordinary differential equations. Runge-Kutta formulas for higher-order differential equations.

7. Approximation of Data and Functions

Classification of approximation tasks: interpolation, smoothing, extrapolation (prediction). Criterion of closeness of functions. Interpolation of data: spline-extrapolation. Choice of nodes of interpolation. Spline functions: linear splines, quadratic splines and cubic splines. Data smoothing and fitting. Regression function. Geometric illustration of Regression. Criteria for a “best” fit. Least-squares regression method. The regression equations, linear and polynomial regression. Prediction of data. Short-term forecast. Efficiency of approximation.

4.2. Course structure

Table 1

Section	**Classroom activities (hrs)**			Self study	Midtests	**Total hours**
	Lectures	Practice	Labs			
1. Introduction and subject	2		4	6		12
2. Computational errors	2		2	4	2	10
3. Numerical Integration	4		4	8	2	6
4. Solution Methods of Non-Linear Equations and Set Equations	8		8	10	4	30
5. Numerical Linear Algebra	8		8	12	2	24
6. Numerical solution of ordinary differential equations	6		4	8	2	18
7. Approximation of Data and Functions	6		6	10	2	20
Total	36		36	58	14	124

4.3 Distribution of competences according to discipline sections

Table 2

	Competences	**Course sections**						
		1	2	3	4	5	6	7
1.	K.1.5*	+	+	+	-	-	+	+
2.	S. 1.5		+	+	-	-	+	+
3.	C.1.5		+	+	-	-	+	

* K-KNOWLEDGE
S-SKILLS
C- COMPETENCES

5. Educational technologies

The combination of educational technologies, given in Table 3, provides achievement of the expected results.

Table 3.

Teaching methods and forms of studying

Forms / Methods	Lectures	Labs.	Self-study
IT-methods	+	+	+
Teamwork		+	+
Case-study			
Role play			
Learning based on experience		+	
Advanced self-study			+
Projecting			
Searching	+		
Investigating			+

6. Self-study program

6.1 Current self-study consists of studying lecture materials, training for laboratory works and midtests. Time budget is 72 hours and includes:

1) studying of lecture materials and training for laboratory works (58 hrs.)
2) midtests preparations (14 hrs.)

6.2 Creative problem-oriented self-study involves conducting research and analysis of numerical algorithms, participation in conferences; students' review of scientific publications on a predetermined theme.

6.3 Self-study content

Report topics

1. Elementary maths error.
2. Computational errors of linear algebra equation solution.
3. Gauss elimination method: development of algorithmic structure, program implementation, evaluation of a system conditionality.

4. Jacobi iteration rule: evaluation of convergence conditions, get a form suitable for iterations, evaluation of convergence speed, method algorithmization.

5. Methods bisection, secant and Newton-Raphson for solving nonlinear equations: analytical and graphic separation of roots, algorithmization of methods, check of convergence conditions, evaluation of errors and iterations number.

6. Runge-Kutta methods 1, 2, 4th of orders for solving of differential equations and systems of differential equations: algorithm, error estimation by Runge rule.

7. Interpolation of the experimental data by Lagrange and Newton polynoms: algorithms, error estimation of interpolation by remainder term formulas and closeness criteria.

8. Iterative methods for solving systems of nonlinear equations

9. Multistep methods of the solution of differential equations of n-go of an order and systems of differential equations.

9. Multistep methods of the solution of differential equations of n-th order and systems of differential equations.

10. Data approximation on the basis of the least-squares method.

6.4 Self-study assessment

The assessment of self-study results is organized in two forms: self-checking and monitoring by teachers. Teacher’s evaluation is reflected in the rating plan.

6.5 Educational and methodical support for the students' self-study

- http://www.aics.ru/books.shtml?action=showbookcont&id=132
- www.wikibooks.org.
- www.intuit.ru.
- www.exponenta.ru.
- Кочегурова Е.А. Вычислительная математика. Учебное пособие. - Томск, изд. ТПУ, 2008. - 112 с.

7. Current and final assessment

Current and interim assessment of students’ achievements is carried out on a basis of the Rating – plan , which includes the results of laboratory works, self-study and midterm tests.

In the process of studying, students do two midterm tests in the following sections of the course:

1. Theory of errors.
2. Numerical integration.
3. Solution of linear algebra tasks.
4. Solution of the non-linear equations.
5. Solution of differential equations.
6. Tasks of functions approximation.

Sample test 1 and 2 are given in the Appendix A. Test 1 includes theoretical and practical tasks from next course sections: Computational errors, Numerical Integration, Numerical Linear Algebra. Test 2 includes - Solution Methods of Non-Linear and ordinary differential equations, Approximation of Data and Functions.

Final test (Appendix B) includes 2 practical and 2 theoretical tasks and combines all parts of the course.

The final academic ranking is calculated in accordance with the results of laboratory works, midterm and final tests.

8. Rating plan of the course

Course	**CALCULATING MATHEMATICS**	Weeks	16
		Credits	6
		Lectures, hrs	32
Term	**3**	Practice, hrs	
Year	**2**	Labs, hrs.	32
Lecturer	**Elena Kochegurova, assosiated professor**	Class work in total, hrs	64
Course	**CALCULATING MATHEMATICS**	Self-study, hrs	64
		TOTAL, hrs.	124
		Weeks	16
Term	**3**	Credits	6
Year	**2**	Lectures, hrs	32
Lecturer	**Elena Kochegurova, assosiated professor**	Practice, hrs	
		Labs, hrs.	32
		Class work in total, hrs	64

8. References

Main

1. Akai, T.J., Applied Numerical Methods for Engineers, Wiley, New York, NY, 1993.
2. Atkinson, K.E. and Han, W., Elementary Numerical Analysis, 3rd ed. Wiley, New York, 2004.
3. Balagurusamy, E., Numerical Methods, Tata McGraw-Hill, New Delhi, India, 2002.
4. Dukkipati, R.V., Numerical Methods through Solved Problems, New Age International Publishers (P) Ltd., New Delhi, India, 200
5. Ortega, J.M., Numerical Analysis—A Second Course, Academic Press, New York, NY, 1972.
6. Powell, M., Approximation Theory and Methods, Cambridge University Press, Cambridge, UK, 1981.
7. Rao, S.S., Applied Numerical Methods for Engineers and Scientists, Prentice Hall, Upper Saddle River, New Jersey, NJ, 2002.
8. Tyrtyshnikov, E.E., A Brief Introduction to Numerical Analysis, Birkhauser, Boston, 1997.

Additional

1. Chapra, S.C., Numerical Methods for Engineers with Software and Programming Applications, 4th ed., McGraw-Hill, New York, NY, 2002.
2. Steven C. Charpa, Raymond P.Canale, Numerical Methods for Engineers: With Programming and Software Applications, 3d ed., McGraw – Hill, New York, 1998.

Software and Web sources

1. Software MathCad, MathSoft, Inc.
2. http://www.aics.ru/books.shtml?action=showbookcont&id=132
3. www.wikibooks.org.
4. www.intuit.ru
5.www.exponenta.ru

9. Computer facilities used in a process of teaching

Personal computers with the operating system Windows-*.

This program is made in accordance with TPU Standards and Federal State Educational Standards (FSES) requirements for the study major of 09.03.01 «Computer science and technology»

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Author ____________________ Elena Kochegurova, assosiate professor

Reviewer ____________________

APPENDIX A

Samples of midterm tasks

Test № 1

Variant 1

Main objective of calculus mathematics.

Error of mathematical model.

How many significant figures in number 1223,0034.

Geometric interpretation of a method of rectangles.

Task correctness.

Main methods classes of set equation solving.

Main idea of the Seidel method .

Absolute error.

To calculate integral of function y(x)=x+1 on the interval [0, 3] by the trapezoidal rule with step h=1.

How to organize the monitoring of Gauss method reverse run.

Test № 2

Variant 1

Concept of the regression equation.

Euler's method for the solution of the Couchy problem.

Procedures of the software MathCad for solution of the nonlinear equations and systems of the nonlinear equations.

What is the basis function? Example of the basic functions.

Methods for solving of the nonlinear equations systems.

Solution of the differential equation of n-th order.

Concept of an iterative process.

Efficiency indicators of approximate data.

To get the regression equation by the least-squares method on basis {1, exp(x)}

Stages of an iterative task solution.

APPENDIX B

Final Test

Variant № 1

<table>
<tr><td>1</td><td>Give a geometrical illustration of the trapezoidal rule for solving the problem of numerical integration.</td></tr>
<tr><td>2</td><td>It is required to round number <i>X*=0,50156±0,00231</i>. To reject doubtful figures and to leave only sure signs. To delete dubious digits and keep the valid (correct) digits in the record</td></tr>
<tr><td>3</td><td>What is the difference between global and local interpolation? Explain graphically.</td></tr>
<tr><td>4</td><td>a. To get the regression equation by the least-squares method based on the given basic functions { 1, sin (x) } for the following experimental data
<table>
<tr><td><i>x</i></td><td>0</td><td>π/2</td><td>π</td><td>3π/2</td><td>2π</td></tr>
<tr><td><i>y</i></td><td>-0.5</td><td>1.75</td><td>0</td><td>-2.2</td><td>0.5</td></tr>
</table>
b. To give a graphic illustration of approximation.</td></tr>
</table>