Task 2 (Random Events)
(Kitayeva A.V.)

The lottery: a tax on people who flunked math.
Monique Lloyd

Coincidences, in general, are great stumbling blocks in the way of that class of thinkers who have been educated to know nothing of the theory of probabilities - that theory to which the most glorious objects of human research are indebted for the most glorious of illustrations.
Arsene Dupin, in *The Murders in the Rue Morgue*, Edgar Allen Poe

1. Construct following events (using the experiment of rolling two dice):
   (a) Let A be the event that the total number of dots equal 8.
   (b) B defined by requiring that neither of the two numbers be a six.
   (c) C: first number smaller than second.

2. Suppose that k distinct letters (to different friends) have been written, each with a corresponding (uniquely addressed) envelope. Then, for some strange reason, the letters are placed in the envelopes purely randomly (after a thorough shuffling).
   (a) What is the probability of all letters being placed correctly?
   (b) What is the probability that none of the k letters are placed correctly?
   (c) What is the probability of exactly one letter being placed correctly?

3. Four players are dealt 5 cards each. What is the probability that at least one player gets exactly 2 aces (a task ago, we could not solve this problem).

4. There are 100,000 lottery tickets marked 00000 to 99999. One of these is selected at random. What is the probability that the number on it contains 84 (consecutive, in that order) at least once.

5. Suppose the experiment consists of rolling two dice (red and green), the event A is: «the total number of dots equals 6», B is: «the red die shows an even number». Compute Pr(B/A).

6. Two players are dealt 5 cards each. What is the probability that they will have the same number of aces?

7. Let Pr(A) = 0.1, Pr(B) = 0.2, Pr(C) = 0.3 and Pr(D) = 0.4; A,B,C,D – independent events. Compute $\Pr[(A \cup B) \cap \overline{C \cup D}]$.

8. Let us return to Example 2 of the previous task (lottery with 100,000

tickets) and compute the probability that a randomly selected ticket has an 8 and a 4 on it (each at least once, in any order, and not necessarily consecutive).

9. The same question, but this time we want at least one 8 followed (sooner or later) by a 4 (at least once). What makes this different from the original question is that 8 and 4 now don't have to be consecutive.

10. Out of 10 dice, 9 of which are regular but one is 'crooked' (6 has a probability of 0.5), a die is selected at random (we cannot tell which one, they all look identical). Then, we roll it twice.
We will answer three question:

(a) Given that the first roll resulted in a six (Event S_1), what is the (conditional) probability of getting a six again in the second roll (Event S_2)?

(b) Are S_1 and S_2 independent?

(c) Given that both rolls resulted in a six, what is the (conditional) probability of having selected the crooked die?

11. Ten people have been arrested as suspects in a crime one of them must have committed. A lie detector will (incorrectly) incriminate an innocent person with a 5% probability, it can (correctly) detect a guilty person with a 90% probability.

(a) One person has been tested so far and the lie detector has its red light flashing (implying: 'that's him'). What is the probability that he is the criminal?

(b) All 10 people have been tested and exactly one incriminated. What is the probability of having the criminal now?

12. Two men take one shot each at a target. Mr. A can hit it with the probability of 1/4 , Mr. B's chances are 2/5 (he is a better shot). What is the probability that the target is hit (at least once)?
Here, we have to (on our own) assume independence of the two shots.

13. What is more likely, getting at least one 6 in four rolls of a die, or getting at least one double 6 in twenty four rolls of a pair of dice?

14. Four people are dealt 13 cards each. You (one of the players) got one ace. What is the probability that your partner has the other three aces? (Go back three questions to get a hint).

15. A, B, C are mutually independent, having (the individual) probabilities of 0.25, 0.35 and 0.45, respectively. Compute $\Pr[(A \cap \overline{B}) \cup C]$.

16. Two coins are flipped, followed by rolling a die as many times as the number of heads shown. What is the probability of getting fewer than 5 dots in total?

17. Consider the previous example. Given that there were exactly 3 dots in total, what is the conditional probability that the coins showed exactly one head?

18. Jim, Joe, Tom and six other boys are randomly seated in a row. What is the probability that at least two of the three friends will sit next to each other?

19. Shuffle a deck of 52 cards. What is the probability that the four aces will end up next to each other (as a group of four consecutive aces)?

20. Consider a 10 floor government building with all floors being equally likely to be visited. If six people enter the elevator (individually, i.e. independently) what is the probability that they are all going to (six) different floors?

21. (Extension of the previous example). What if the floors are not equally likely (they issue licenses on the 4th floor, which has therefore a higher probability of 1/2 to be visited by a 'random' arrival − the other floors remain equally likely with the probability of 1/16 each).

22. Within the next hour 4 people in a certain town will call for a cab. They will choose, randomly, out of 3 existing (equally popular) taxi companies. What is the probability that no company is left out (each gets at least one job)?

23. There are 10 people at a party (no twins). Assuming that all 365 days of a year are equally likely to be someone's birth date [not quite, say the statistics, but we will ignore that] and also ignoring leap years, what is the probability of:

(a) All these ten people having different birth dates?

(b) Exactly two people having the same birth date (and no other duplication).

24. A simple padlock is made with only ten distinct keys (all equally likely). A thief steals, independently, 5 of such keys, and tries these to open your lock. What is the probability that he will succeed?

Answers 2

1. (a) This of course consists of the subset: {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} (five simple events).

(b) This correspond to the subset:

1,1 1,2 1,3 1,4 1,5
2,1 2,2 2,3 2,4 2,5
3,1 3,1 3,3 3,4 3,5
4,1 4,2 4,3 4,4 4,5
5,1 5,2 5,3 5,4 5,5.

(c) Subset:

1,2 1,3 1,4 1,5 1,6
2,3 2,4 2,5 2,6

3,4 3,5 3,6
4,5 4,6
5,6.

2. The sample space of this experiment is thus a list of all permutations of k objects (123 132 213 231 312 321), when k = 3 (we will assume that 123 represents the correct placement of all three letters). In general, there are k! of these, all of them equally likely (due to symmetry, i.e. none of these arrangements should be more likely than any other).

(a) Solution (fairly trivial): Only one out of k! random arrangements meets the criterion, thus the answer is 1/k! (astronomically small for k beyond 10).

(b) Solution is this time a lot more difficult. First we have to realize that it is relatively easy to figure out the probability of any given letter being placed correctly, and also the probability of any combination (intersection) of these, i.e. two specific letters correctly placed, three letters correct..., etc. [this kind of approach often works in other problems as well; intersections are usually easy to deal with, unions are hard but can be converted to intersections]. Let us verify this claim. We use the following notation: A_1 means that the first letter is placed correctly (regardless of what happens to the rest of them), A_2 means the second letter is placed correctly, etc. Pr(A_1) is computed by counting the number of permutations which have 1 in the correct first position, and dividing this by k!. The number of permutations which have 1 fixed is obviously (k − 1)! [we are effectively permuting 2, 3, ... k, altogether k − 1 objects]. Pr(A_1) is thus equal to (k − 1)! / k! . The probability of A_2, A_3, etc. can be computed similarly, but it should be clear from the symmetry of the experiment that all these probabilities must be the same, and equal to Pr(A_1) = 1/k (why should any letter have a better chance of being placed correctly than any other?). Similarly, let us compute Pr($A_1 \cap A_2$), i.e. probability of the first and second letter being placed correctly (regardless of the rest). By again counting the corresponding number of permutations (with 1 and 2 fixed), we arrive at (k − 2)! / k!=1/k/(k−1) . This must be the same for any other pair of letters, e.g. Pr($A_3 \cap A_7$) = 1/k/(k−1) , etc. In this manner we also get

Pr($A_1 \cap A_2 \cap A_3$) =Pr($A_3 \cap A_7 \cap A_{11}$) = 1/k/(k−2)/(k−2), etc.

So now we know how to deal with any intersection. All we need to do is to express the event 'all letters misplaced' using intersections only, and evaluate the answer, thus:

Pr($\overline{A_1} \cap \overline{A_2} \cap ... \cap \overline{A_k}$) [all letters misplaced] = Pr($\overline{A_1 \cup A_2 \cup ... \cup A_k}$) =

= 1 − Pr($A_1 \cup A_2 \cup ... \cup A_k$) =

$$= 1 - \sum_{i=1}^{k} \Pr(A_i) + \sum_{i<j}^{k} \Pr(A_i \cap A_j) + ... + (-1)^k \Pr(A_1 \cap A_2 \cap ... \cap A_k) =$$

$$= 1 - k \cdot 1/k + \binom{k}{2}\frac{1}{k(k-1)} - \binom{k}{3}\frac{1}{k(k-1)(k-2)} + ... + (-1)^k \frac{1}{k!} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^k \frac{1}{k!}.$$

For k = 3 this implies 1 − 1 + 1/2−1/6=1/3 (check, only 231 and 312 out of six permutations). For k = 1, 2, 4, 5, 6, and 7 we get: 0 (check, one letter cannot be misplaced), 50% for two letters (check), 37.5% (four letters), 36.67% (five), 36.81% (six), 36.79% (seven), after which the probabilities do not change practically (i.e., surprisingly, we get the same answer for 100 letters, a million letters, etc.).

Can we identify the limit of the 1 −1+ 1/2! − 1/3! +... sequence? Yes, of course, this is the expansion of $e^{-1} \approx$ .36788.

c. Similarly, the probability of exactly one letter being placed correctly is

$$k\frac{1}{k} - 2\binom{k}{2}\frac{1}{k(k-1)} + 3\binom{k}{3}\frac{1}{k(k-1)(k-2)} + ... \mp k\frac{1}{k!} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + ... \mp \frac{1}{(k-1)!}$$

(the previous answer short of its last term!). This equals to 1, 0, 50%, 37.5%, ... for k = 1, 2, 3, 4, ... respectively, and has the same limit.

3. Solution: Let A_1 be the event that the first player gets exactly 2 aces, A_2 means that the second player has exactly 2 aces, etc. The question amounts to finding $\Pr(A_1 \cup A_2 \cup A_3 \cup A_4)$. By our formula, this equals $\sum_{i=1}^{4} \Pr(A_i) - \sum_{i<j}^{4} \Pr(A_i \cap A_j) + 0$ [the intersection of 3 or more of these events is empty − there are only 4 aces]. For $\Pr(A_1)$ we get $\binom{4}{2}\binom{48}{3}/\binom{52}{5} = 3.993\%$ (the denominator counts the total number of five-card hands, the numerator counts only those with exactly two aces) with the same answer for $\Pr(A_2)$, ... $\Pr(A_4)$ (the four players must have equal chances). Similarly $\Pr(A_1 \cap A_2) = \binom{4}{2,2,0}\binom{48}{3,3,42}/\binom{52}{5,5,42} = 0.037\%$ (the denominator represents the number of ways of dealing 5 cards each to two players, the numerator counts only those with 2 aces each − recall the 'partitioning' formula), and the same probability for any other pair of players.
Final answer: $4\Pr(A_1) - 6\Pr(A_1 \cap A_2) = 15.75\%$.

4. Solution: Let's introduce four events: A means that the first two digits of the ticket are 84 (regardless of what follows), B: 84 is found in the second and third position, C: 84 in position three and four, and D: 84 in the last two positions. Obviously we need $\Pr(A \cup B \cup C \cup D) = \Pr(A)+\Pr(B)+\Pr(C)+\Pr(D) - \Pr(A \cap C) - \Pr(A \cap D) - \Pr(B \cap D) + 0$ [the remaining possibilities are all null events − the corresponding conditions are incompatible, see the Venn diagram].
The answer is $4 \times 1000/100{,}000 - 3 \times 10/100{,}000 = 0.04 - 0.0003 = 3.97\%$ (the logic of each fraction should be obvious − there are 1000 tickets which belong to A, 10 tickets which meet conditions A and C, etc.).

5. Solution: Let us use the old sample space, indicating simple events of A by ○, of B by ×, and of the $A \cap B$ overlap by ⊗ :

○
× × × ⊗ × ×
○
× ⊗ × × × ×
○
× × × × × ×

Pr(B/A) is clearly the number of the overlap simple events ⊗ divided by the number of simple events in A (either ○ or ⊗), as these are all equally likely.
Answer: Pr(B/A) = 2/5.

6. Solution: We partition the sample space according to how many aces the first player gets, calling the events $A_0, A_1, \ldots, A_4$. Let B be the event of our question (both players having the same number of aces). Then, by the formula of total probability: $\Pr(B) = \Pr(A_0)\Pr(B/A_0) + \Pr(A_1)\Pr(B/A_1) + \Pr(A_2)\Pr(B/A_2) + \Pr(A_3)\Pr(B/A_3) + \Pr(A_4)\Pr(B/A_4) =$

$$\binom{4}{2}\binom{48}{3}/\binom{52}{5} \times \binom{4}{0}\binom{43}{5}/\binom{47}{5} + \binom{4}{1}\binom{48}{4}/\binom{52}{5} \times \binom{3}{1}\binom{44}{4}/\binom{47}{5} + \binom{4}{2}\binom{48}{3}/\binom{52}{5} \times \binom{2}{2}\binom{45}{3}/\binom{47}{5} + \binom{4}{3}\binom{48}{2}/\binom{52}{5} \times 0 + \binom{4}{4}\binom{48}{0}/\binom{52}{5} \times 0 = 49.33\%.$$

7. Solution: $= \Pr(A \cup B) \cdot [1 - \Pr(C \cup D)] =$
$= [0.1 + 0.2 - 0.02] \cdot [1 - 0.3 - 0.4 + 0.12] = 11.76\%$.

8. Solution: Define A: no 8 at any place, B: no 4. We need $\Pr(\overline{A} \cap \overline{B})$ (at least one 8 and at least one 4) = $\Pr(\overline{A \cup B})$ [De Morgan] = $1 - \Pr(A \cup B) = 1 - -\Pr(A) - \Pr(B) + \Pr(A \cap B)$. Clearly $A = A_1 \cap A_2 \cap ... \cap A_5$, where A_1: 'no 8 in the first place', A2: 'no 8 in the second place', etc. $A_1, A_2, ..., A_5$ are mutually independent (selecting a random 5 digit number is like rolling an 10-sided die five times), thus $\Pr(A) = \Pr(A_1) \cdot \Pr(A_2) \cdot ... \cdot \Pr(A_5) = (9/10)^5$.
Similarly, $\Pr(B) = (9/10)^5$. Now, $A \cap B \equiv C_1 \cap C_2 \cap ... \cap C_5$ where C_1: not 8 nor 4 in the first spot, C_2: not 8 nor 4 in the second, etc.; these of course are also independent, which implies $\Pr(A \cap B) = (8/10)^5$.
Answer: $1 - 2(9/10)^5 + (8/10)^5 = 14.67\%$.

9. Solution: We partition the sample space according to the position at which 8 appears for the first time: $B_1, B_2, ..., B_5$, plus B_0 (which means there is no 8). Verify that this is a partition. Now, if A is the event of our question (8 followed by a 4), we can apply the formula of total probability thus:
$\Pr(A) = \Pr(A/B_1)\cdot\Pr(B_1) + \Pr(A/B_2)\cdot\Pr(B_2) + \Pr(A/B_3)\cdot\Pr(B_3) + \Pr(A/B_4)\cdot \Pr(B_4) +$
$+ \Pr(A/B_5) \cdot \Pr(B_5) + \Pr(A/B_0) \cdot \Pr(B_0)$. Individually, we deal with these in the following manner (we use the third term as an example): $\Pr(B_3) = (9/10)^2 (1/10)$ (no 8 in the first slot, no 8 in the second, 8 in third, and anything after that; then multiply due to independence), $\Pr(A/B3) = 1-(9/10)^2$ (given the first 8 is in the third slot, get at least one 4 after; easier through complement: $1 - \Pr$(no 4 in the last two slots)).
Answer: $\Pr(A) = [1 - (9/10)^4](1/10) + [1 - (9/10)^3](9/10)(1/10) + [1 - (9/10)^2](9/10)^2 (1/10) + [1 - (9/10)](9/10)^3 (1/10) + 0 \cdot (9/10)^4 (1/10) = 8.146\%$.

10. The sample space of the complete experiment, including probabilities, is

r6 6	0.9(1/6)(1/6)	■○
r6 $\overline{6}$	0.9(1/6)(5/6)	■
r$\overline{6}$ 6	0.9(5/6)(1/6)	○
r$\overline{6}$ $\overline{6}$	0.9(5/6)(5/6)	
c6 6	0.1(1/2)(1/2)	■○
c6 $\overline{6}$	0.1(1/2)(1/2)	■
c$\overline{6}$ 6	0.1(1/2)(1/2)	○
c$\overline{6}$ $\overline{6}$	0.1(1/2)(1/2).	

(a) Solution: In our sample space we mark off the simple events contributing to S_1 (by ■) and to S_2 (by ○) and compute $\Pr(S_1 \cap S_2) / \Pr(S_1)$ (by adding the corresponding probabilities).
Answer: (9+9)/(9+9+45+9) (the common denominator of 360 cancels out) = 25%.

(b) Let us check it out, carefully! $\Pr(S_1 \cap S_2) ?= \Pr(S_1) \cdot \Pr(S_2)$.
Solution: $18/360 (=1/20) \neq (72/360)^2 (=1/25)$.
Answer: No.

(c) Answer: 9/(9+9) = 50%.

11. (a) Solution: Using c for 'criminal', i for 'innocent' r for 'red light flashing' and g for 'green', we have the following sample space:

cr	(1/10)(9/10)=0.090	■ ○
cg	(1/10)(1/10)=0.010	
ir	(9/10)(1/20)=0.045	■
ig	(9/10)(19/20)=0.855	

Answer: Pr(C/R) = 0.090/(0/090+0/045)=2/3 (far from certain!).

(b) A simple event consists now of a complete record of these tests (the sample space has of 2^{10} of these), e.g. rggrggrggg. Assuming that the first item represents the criminal (the sample space must 'know' who the criminal is), we can assign probabilities by simply multiplying since the tests are done independently of each other. Thus, the simple event above will have the probability of $0.9\times0.95^2\times0.05\times0.95^2\times0.05\times0.95^3$, etc. Since only one test resulted in r, the only simple events of relevance (the idea of a 'reduced' sample space) are:

rggggggggg	$0.9\cdot0.95^9$
grgggggggg	$0.9\cdot0.95^8\cdot0.05$
…………	
gggggggggr	$0.9\cdot0.95^8\cdot0.05$

Given that it was one of these outcomes, what is the probability it was actually the first one?
Answer: $(0.9 \times 0.95^9)/(0.9 \times 0.95^9 + 9 \times 0.1 \times 0.95^8 \times 0.05)$= 95% (now we are a lot more certain − still not 100% though!).

12. Solution (using an obvious notation):
$\Pr(A\cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = 1/4 + 2/5 - 1/10 = 55\%$.
Alternately: $\Pr(A\cup B) = 1 - \Pr(\overline{A\cup B}) = 1 - \Pr(\overline{A} \cap \overline{B}) = 1 - (3/4))3/5) = 55\%$
(replacing Pr(at least one hit) by 1 − Pr(all misses)).

13. Solution: Let's work it out. The first probability can be computed as 1−Pr(no sixes in 4 rolls) = $1 - (5/6)^4$ (due to independence of the individual rolls) = 51.77%. The second probability, similarly, as 1 − Pr(no double six in 24 rolls of a pair) = $1- (35/36)^{24}$ = 49.14% (only one outcome out of 36 results in a double six).
Answer: Getting at least one 6 in four rolls is more likely.

14. We can visualize the experiment done sequentially, with you being the first player and your partner the second one (even if the cards were actually dealt in a different order, that cannot change probabilities, right?). The answer is a natural conditional probability, i.e. the actual condition (event) is decided in the first stage (consider it completed accordingly). The second stage then consists of dealing 13 cards out of 39, with 3 aces remaining.
Answer: $\binom{3}{3}\binom{36}{10}/\binom{39}{13}$ = 3.129%.
The moral: conditional probability is, in some cases, the 'simple' probability.

15. Solution: = $\Pr(A\cap \overline{B}) + \Pr(C) - \Pr(A\cap \overline{B} \cap C) = 0.25\times0.65+0.45 - 0.25\times 0.65 \times 0.45$ =53.94%.

16. Solution: Introduce a partition A0, A1, A2 according to how many heads are obtained. If B stands for 'getting fewer than 5 dots', the total-probability formula gives: Pr(B) $=\Pr(A_0)\Pr(B/A_0)+\Pr(A_1)\Pr(B/A_1)+\Pr(A_2)\Pr(B/A_2) = (1/4)\cdot1+(2/4)\cdot(4/6)+(1/4)\cdot(6/36)$ = 62.5%.
The probabilities of A_0, A_1, and A_2 followed from the sample space of two flips: {hh, ht, th, tt}; the conditional probabilities are clear for $\Pr(B|A_0)$ and $\Pr(B|A_1)$, $\Pr(B|A_2)$ requires going back to 36 outcomes of two rolls of a die and counting those having a total less than 5: {11, 12, 13, 21, 22, 31}.

17. Solution: We are given the outcome of the second stage to guess at the outcome of the first stage. We need the Bayes rule, and the following (simplified) sample space:

03	$(1/4)\cdot0$ ■	
$0\overline{3}$	$(1/4)\cdot1$	
13	$(1/2)\cdot(1/6)$	■ ○
$1\overline{3}$	$(1/2)\cdot(5/6)$	

23	(1/4) ·(2/36) ■
$2\overline{3}$	(1/4) ·(34/36)

where the first entry is the number of heads, and the second one is the result of rolling the die, simplified to tell us only whether the total dots equaled 3, or did not (3). Pr(1|3) = (1/12)/(1/12+1/72) = 85.71%. Note that here, rather atypically, we used bold digits as names of events.

18. Solution: Let's introduce A: «Jim and Joe sit together», B: «Jim and Tom sit together», C: «Joe and Tom sit together». We need $\Pr(A \cup B \cup C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(A \cap B) - \Pr(A \cap C) - \Pr(B \cap C) + \Pr(A \cap B \cap C)$. There is 9! random arrangements of the boys, 2 × 8! will meet condition A (same with B and C), 2×7! will meet both A and B (same with A ∩C and B ∩C), none will meet all three.
Answer: 3 × (2×8!)/9!-3×(2×7!)/9! = 58.33%.

19. Answer: 4!49!/52! = 0.0181% (=1/5525) (for small probabilities, the last number telling us that this will happen, on the average, only in 1 out of 5525 attempts — conveys more information than the actual percentage).

20. Solution: The experiment is in principle identical to rolling a 9-sided die (there are nine floors to be chosen from, exclude the main floor!) six times (once for each person − this corresponds to selecting his/her floor). The sample space thus consists of 9^6 equally likely outcomes (each looking like this: 248694 − ordered selection, repetition allowed). Out of these, only 9 × 8 × 7 × 6 × 5 ×4 = P_9^6 consist of all distinct floors.

Answer: $P_9^6/9^6$ = 11.38%.

21. Solution: The sample space will be the same, but the individual probabilities will no longer be identical; they will now equal to $(1/2)^i(1/16)^{6-i}$ where i is how many times 4 appears in the selection (248694 will have the probability of $(1/2)^2(1/16)^4$, etc.). We have to single out the outcomes with all six floors different and add their probabilities. Luckily, there are only two types of these outcomes: (i) those without any 4: we have P_8^6 of these, each having the probability of $(1/16)^6$, and (ii) those with a single 4: there are 6 × P_5^8 of these, each having the probability of $(1/2)(1/16)^5$.
Answer: $P_8^6(1/16)^6 + 6P_8^5(1/2)(1/16)^5$ = 2.04% (the probability is a lot smaller now).

22. Solution: This is again a roll-of-a-die type of experiment (this time we roll 4 times − once for each customer − and the die is 3-sided − one side for each company). The sample space will thus consist of 3^4 equally likely possibilities, each looking like this: 1321. How many of these contain all three numbers? To achieve that, we obviously need one duplicate and two singles. There are 3 ways to decide which company gets two customers. Once this decision has been made (say 1223), we simply permute the symbols (getting $\binom{4}{2,1,1}$ distinct 'words').

Answer: $3\binom{4}{2,1,1}/3^4$ = 4/9 = 44.44%.

23. (a) Solution: This, in principle, is the same as choosing 6 different floors in an elevator (two examples ago).
Answer: $P_{365}^{10}/365^{10}$ = 88.31%.

(b) Solution: This is similar to the previous example where we needed exactly one duplicate. By a similar logic, there are 365 ways to choose the date of the duplication, $\binom{10}{2}$ ways of placing these into 2 of the 10 empty slots, and P_{364}^{8} of filling out the remaining 8 slot with distinct birth dates.

Answer: $365\binom{10}{2}P_{364}^{8}/365^{10}$ = 11.16% (seems reasonable).

These two answers account for 99.47% of the total probability. Two or three duplicates, and perhaps one triplicate would most likely take care of the rest; try it!

24. Solution: Again, a roll-of-a-die type of experiment (10 sides, 5 rolls). The question is in principle identical to rolling a die to get at least one six. This, as we already know, is easier through the corresponding complement.

Answer: $1 - (9/10)^5$ = 40.95%.