# Predictive Modelling of the Warming up Times for Thermoelectric Converters

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**Abstract.** In the present article the question of simulation of nonstationary process of heat transport in a sensitive element of the thermoelectric transformer with an uninsulated seal is considered. The model allows defining and comparing the heating time of a thermocouple junction up to the given temperature for different types of thermocouples. In this article the values obtained by means of modelling the heat time of different thermocouples, as well as a temperature field in a sensitive element of the thermoelectric transformer are given.

## Introduction

Thermoelectric converters (TEC) are the most widespread temperature gages that applied in control and management systems for technological processes. Herewith an important role plays the accuracy of measurements which is defined by a large number of factors, including tests duration. [1].

## Problem Statement

Within limits of error it is possible to define the setting time of the converter for each type of thermo-converters and its classes of admission by a predictive model.

When modeling the heat conduction problem for the area representing significantly heterogeneous system, including a thermocouple junction, a protective cover and in between area filled by powder of an aluminum oxide is considered (Figure 1).
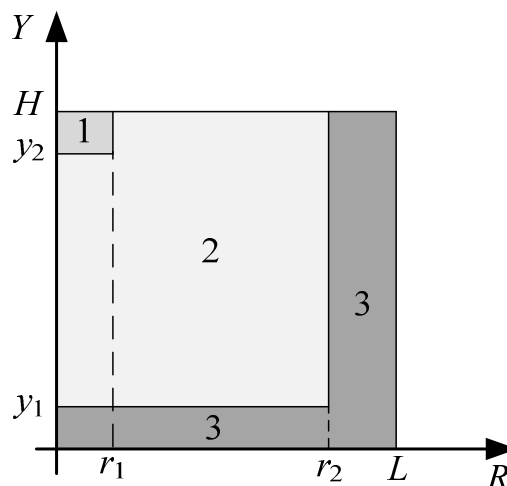


Fig. 1. Diagram of area of the solution of the task:
1) thermocouple junction; 2) powder of an oxide of aluminum; 3) protective cover; H – height of a section of the TEC sensitive element on an axis Y, L – TEC sensitive element radius on an axis R

Nonstationary process of heat transport due to heat conduction for a seal, the space filled with powder, and a protective cover is described by differential equations [2]:

$$C_1\rho_1 \frac{\partial T_1}{\partial t} = \lambda_1 \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial y} \right), \tag{1}$$

$t > 0, 0 < r < r_1, y_2 < y < H;$

$$C_2\rho_2 \frac{\partial T_2}{\partial t} = \lambda_2 \left( \frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial y} \right), \tag{2}$$

$t > 0, 0 < r < r_2, y_1 < y < y_2;$

$t > 0, r_1 < r < r_2, y_2 < y < H.$

$$C_3\rho_3 \frac{\partial T_3}{\partial t} = \lambda_3 \left( \frac{\partial^2 T_3}{\partial r^2} + \frac{1}{r}\frac{\partial T_3}{\partial r} + \frac{\partial^2 T_3}{\partial y} \right), \tag{3}$$

$t > 0, 0 < r < L, 0 < y < y_1;$

$t > 0, r_2 < r < L, y_1 < y < H.$

where $\rho_1$ - density of a seal material, $C_1$ - specific heat capacity of a seal material, $\lambda_1$ - coefficient of heat conduction of a seal material, $\rho_2$ - density of powder, $C_2$ – specific heat capacity of powder, $\lambda_2$ – coefficient of powder heat conduction, $\rho_3$ – density of a protective cover material, $C_3$ – specific heat capacity of a protective cover material, $\lambda_3$ – coefficient of heat conduction of a protective cover material.

The mathematical model includes the following edge conditions:

$$t = 0; T = T_0; \tag{4}$$

$$r = 0; \frac{\partial T}{\partial r} = 0; \tag{5}$$

$$r = L; T = T_r; \tag{6}$$

$$y = 0; T = T_r; \tag{7}$$

$$y = H; \frac{\partial T}{\partial t} = 0. \tag{8}$$

On boundaries "an Al2O3 thermocouple powder seal", "the powder Al2O3-protective a cover", "a protective cover air" (fig.1) conditions were accepted:

$$T_1(r_1,z) = T_2(r_1,z); \tag{9}$$

$$-\lambda_1 \frac{\partial T_1}{\partial r}\bigg|_{r=r_1} = -\lambda_1 \frac{\partial T_1}{\partial r}\bigg|_{r=r_1}; \tag{10}$$

$$T_2(r_2,z) = T_3(r_2,z); \tag{11}$$

$$-\lambda_2 \frac{\partial T_2}{\partial r}\bigg|_{r=r_2} = -\lambda_3 \frac{\partial T_3}{\partial r}\bigg|_{r=r_2}; \tag{12}$$

$$T_3(r_3,z) = T_4(r_3,z); \tag{13}$$

$$-\lambda_3 \frac{\partial T_3}{\partial r}\bigg|_{r=r_3} = -\lambda_4 \frac{\partial T_4}{\partial r}\bigg|_{r=r_3}; \tag{14}$$

$$T_1(r; z_3) = T_2(r; z_3); \tag{15}$$

$$-\lambda_1 \frac{\partial T_3}{\partial z}\bigg|_{z=z_3} = -\lambda_2 \frac{\partial T_2}{\partial z}\bigg|_{z=z_3}; \tag{16}$$

$$T_2(r; z_2) = T_3(r; z_2); \tag{17}$$

$$-\lambda_2 \frac{\partial T_2}{\partial z}\bigg|_{z=z_2} = -\lambda_3 \frac{\partial T_3}{\partial z}\bigg|_{z=z_2}; \tag{18}$$

$$T_3(r; z_1) = T_4(r; z_1); \tag{19}$$

$$-\lambda_3 \frac{\partial T_3}{\partial z}\bigg|_{z=z_1} = -\lambda_4 \frac{\partial T_4}{\partial z}\bigg|_{z=z_1}. \tag{20}$$

For the solution of a differential equation in private derivatives the method of finite differences on the basis of the implicit four-point difference diagram is used.

For this purpose the area of the solution of the task is broken into an uniform finite-difference grid of 200 to 200 nodes with a step on r coordinate: $h_r=2.51 \cdot 10^{-2}$ mm, on y coordinate: $h_y=4.02 \cdot 10^{-2}$ mm.

The temporal grid with a step on t time = 0,001 is used.

The numerical analysis of the temperature fields was made taking into account the heatphysical characteristics provided in table 1.

Table 1 Heatphysical characteristics of materials TEC [3-5]

| № material | Material name | Heat conduction coefficient λ, W / (m·J) | Specific heat capacity with, J / (kg·J) | Density ρ, kg/m³ |
|---|---|---|---|---|
| 1 | Thermocouple junction type S | 50.4 | 139 | 20710 |
| 1 | Thermocouple junction type *L* | 24.75 | 713 | 8920 |
| 1 | Thermocouple junction type *K* | 33.1 | 768 | 8825 |
| 2 | Powder Al₂O₃ | 6.57 | 850 | 1520 |
| 3 | Protective cover Steel | 15 | 462 | 7900 |

In case of numerical modelling, the values of geometrical parameters are accepted: H = 8 mm, L = 5 mm, the size of a thermocouple junction is 1.6 mm.

Comparing of warm-up period of a seal to the values within limits of error for different types of thermocouples was the purpose of these researches.

Allowed measurement errors of temperature measuring by thermoelectric transformers are given in table 2.

Table 2 Value of an allowed measurement error of TEC temperature [6]

| TEC Type | Permissible deviation limit from NSC, K |
|---|---|
| *S* (2 tolerance class) | ± 1,5 in the range of temperatures from 273 to 873 K inclusively |
| *K* (1 tolerance class) | ± 1,5 in the range of temperatures from 233 to 648 K inclusively ± 0,004·\| $T - 273$ \| in the range of temperatures over 648 to 1273 K inclusively |
| *L* (2 tolerance class) | ± 2,5 in the range of temperatures from 233 to 573 K inclusively; ± 0,0075·\| $T - 273$ \| in the range of temperatures over 648573 to 1073 K inclusively |

Operation modes of thermocouples in which temperature of the measured environment changed from 323 to 873 K were considered.
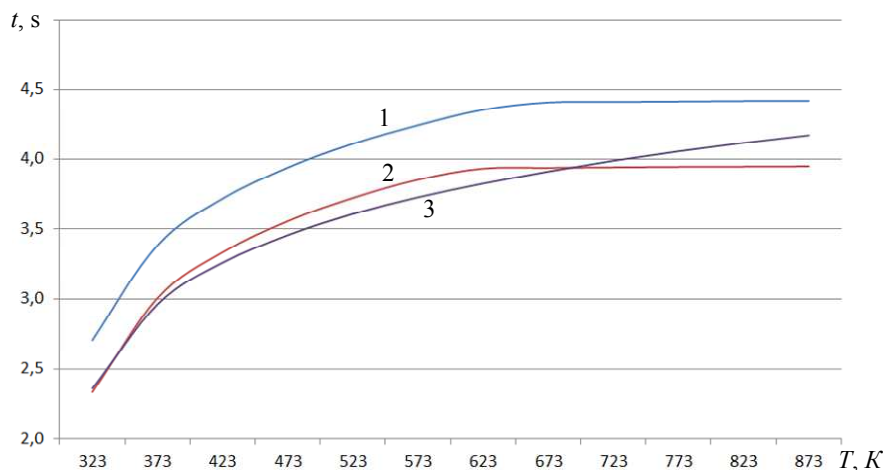


Fig. 2 Dependence of warm-up period of a thermocouple junction on the temperature of the measured environment:
1) thermocouple of K type; 2) thermocouple of L type; 3) thermocouple of S type

The analysis of figure 2 shows that the dependence of the warm-up period for researched thermotransformers has non-linear character and aims at constant value in the field of temperatures exceeding the range in which the permissible deviation from NSC has constant character.

## Results and Discussion

In figure 3 the temperature distribution in a sensitive element of the thermoelectric L type transformer  is shown when heating to a temperature of 873 K in 0,5 s from the warming up beginning (fig. 3, a) and after the end of the heating up process (fig. 3, b).

The analysis of fig. 3 shows that in an original timepoint there is an intensive heating up of a protective cover due to  the maintenance of constant temperature on y boundary = 0 and r = L (fig. 1). The heating speed of a thermocouple junction is defined by the geometrical sizes of the area filled with powder of an aluminum oxide, and also its heatphysical characteristics.



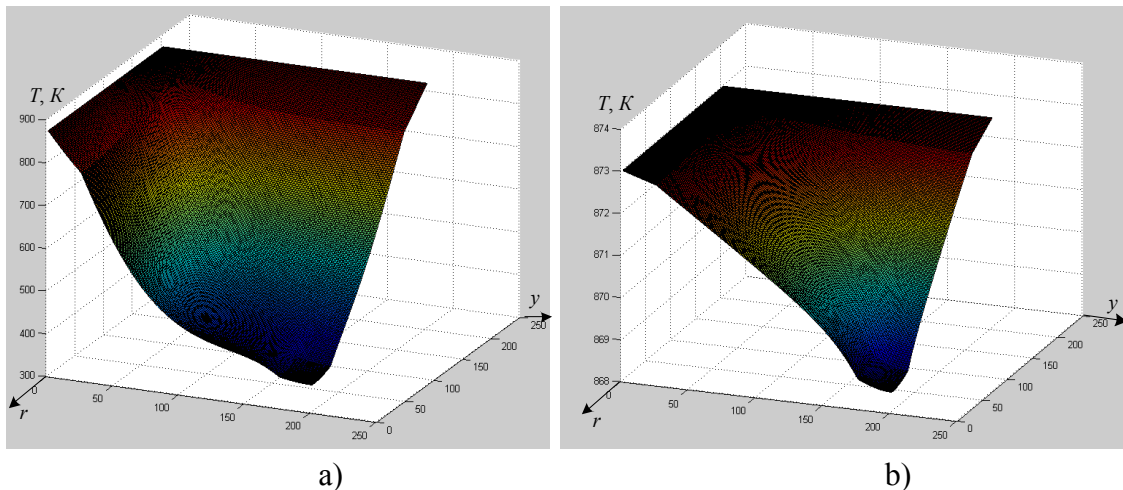a)                                                  b)

Fig. 3 Temperature distribution in the TEC sensitive element:
a – through 0,5 s from the warming up beginning; b – after the end  of the heating up process

## Conclusion

The described prognostic model is developed for the ordinary thermoelectric thermometers which thermocouple junction isn't isolated by a ceramic tip.

By means of model, the analysis of heating up duration of different types thermo-transformers and their accuracy classes is performed. It is set that among three researched types of thermocouples the greatest time for heating up to the temperature lying in the field of a permissible deviation from NSC is required for the K thermocouple. For S and L thermocouples the time of achievement of the provided temperature is lower, thus permissible deviations from NSC for all researched thermocouples are various.

## Acknowledgments

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