LECTURE 5

MathCAD

With over a million users worldwide, Mathcad is today's standard for technical calculation. In science, engineering, design and economics; in industry, research and education; wherever figures need to be computed, you'll find Mathcad. That's because Mathcad is the complete, integrated environment for performing, documenting and communicating technical calculations. It gives you the tools you need sample data and build models, to document and communicate your analysis. Mathcad is ideal for formulating ideas, exploring problems, evolving solutions and sharing results. Within a Mathcad work-sheet you can perform 'live' numeric or symbolic calculations, add graphics and animations, annotate and format text. When you need to change a variable, Mathcad updates your results, formulae and graphs - instantly. You get instant feedback as you try different approaches, so you can experiment with what-if scenarios, establish limits, test results and make accurate decisions.

Sinusoidal steady-state analysis is greatly facilitated if the currents and voltages are represented as vectors in the complex-number plane known as phasors. Let's us consider phasor in complex plane. First we need to write a complex number. Then, form its presentation as a phasor in MathCAD and draw it using MathCAD plot interface, beside we will draw wave diagram of the steady-state process. Here is an example:

1. Rectangular form - real part *x* and imaginary part *y* $z = x + jy$

$z := 2 + j \cdot 5$

2. Exponential form $z = r\ exp(\theta)$

$|z| = 5.385$ <= Magnitude

$\frac{arg(z)}{deg} = 68.199$ <= Angle (argument)

$\bar{z} = 2 - 5i$ <= Complex conjugate

It is suitable to use Mathcad to get all presentation of complex numbers altogether (all at once - magnitude and phase, imaginary and real pats of complex number)

$$Z(z) := \left| \begin{array}{l} r \leftarrow |z| \\ \theta \leftarrow \dfrac{\arg(z)}{deg} \\ x \leftarrow \mathrm{Re}(z) \\ y \leftarrow \mathrm{Im}(z) \\ c \leftarrow \begin{pmatrix} r & \theta \\ x & y \end{pmatrix} \end{array} \right.$$

The same program can be used for addition, subtraction, division and multiplication of complex numbers

$$Z(2 + i\cdot 3) = \begin{pmatrix} 3.606 & 56.31 \\ 2 & 3 \end{pmatrix}$$

$z1 := 2 + i\cdot 3 \quad z2 := 5 - i\cdot 4 \quad z3 := -1 + i\cdot 2 \quad Z(z1 + z2 + z3) = \begin{pmatrix} 6.083 & 9.462 \\ 6 & 1 \end{pmatrix}$

$Z(z1 - z2) = \begin{pmatrix} 7.616 & 113.199 \\ -3 & 7 \end{pmatrix} \quad Z(z1\cdot z2) = \begin{pmatrix} 23.087 & 17.65 \\ 22 & 7 \end{pmatrix} \quad Z\left(\dfrac{z1}{z2}\right) = \begin{pmatrix} 0.563 & 94.97 \\ -0.049 & 0.561 \end{pmatrix}$

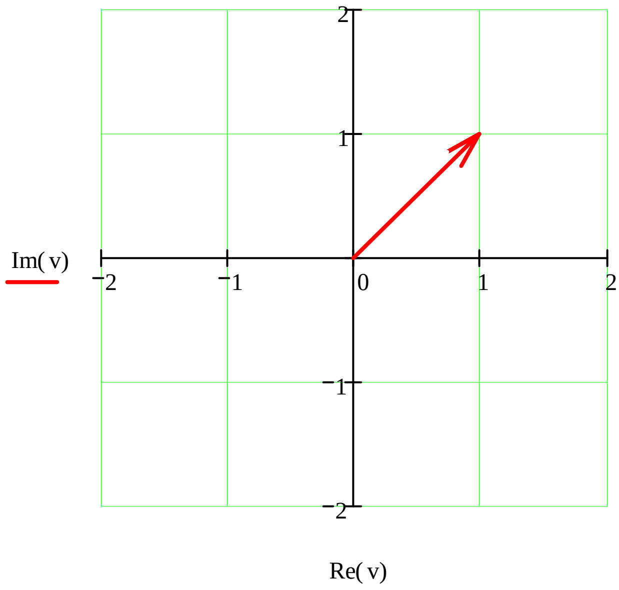
Complex plane representation of Phasor -vector

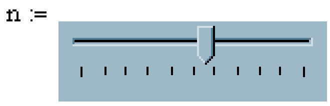
$a := 0.3$ <= reduce or enlarge arrow's wings

$z := 1. + i\cdot 1.$

$$V(z,a) := \begin{bmatrix} 0 \\ z \\ z + a\cdot e^{i\cdot(\arg(z)+165\cdot deg)} \\ z \\ z + a\cdot e^{i\cdot(\arg(z)-165\cdot deg)} \end{bmatrix}$$

$v := V(z,a)$



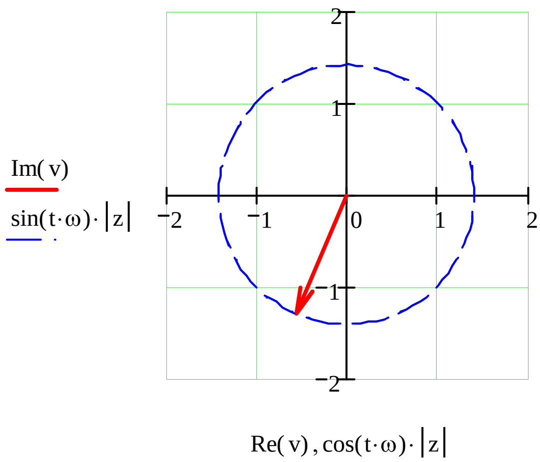


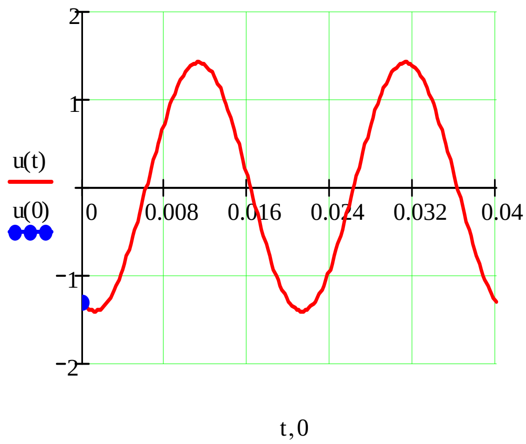
$n := FRAME$ $n = 56$ $N := 100$ $i := 0..N$ $\phi_i := \frac{2 \cdot \pi}{N} \cdot i$ $\omega := 314$ $T := \frac{2 \cdot \pi}{\omega}$

Counterclockwise Vector Rotation

$z := z \cdot e^{i \cdot \phi_n}$ $v := V(z, a)$ $u(t) := |z| \cdot \sin(\omega \cdot t + \arg(z))$ $t := 0, .01 \cdot T .. T \cdot 2$

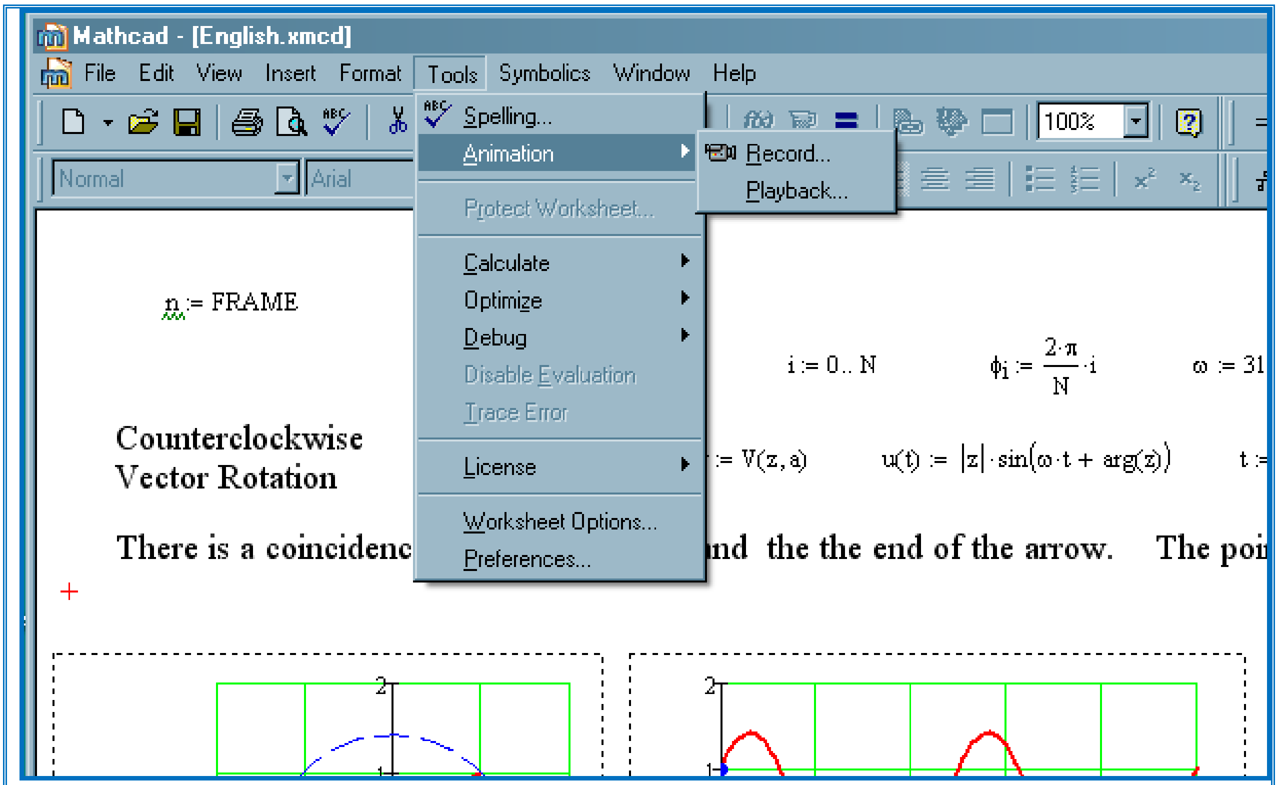
There is a coincidence between the point and the end of the arrow. The point is oscillating



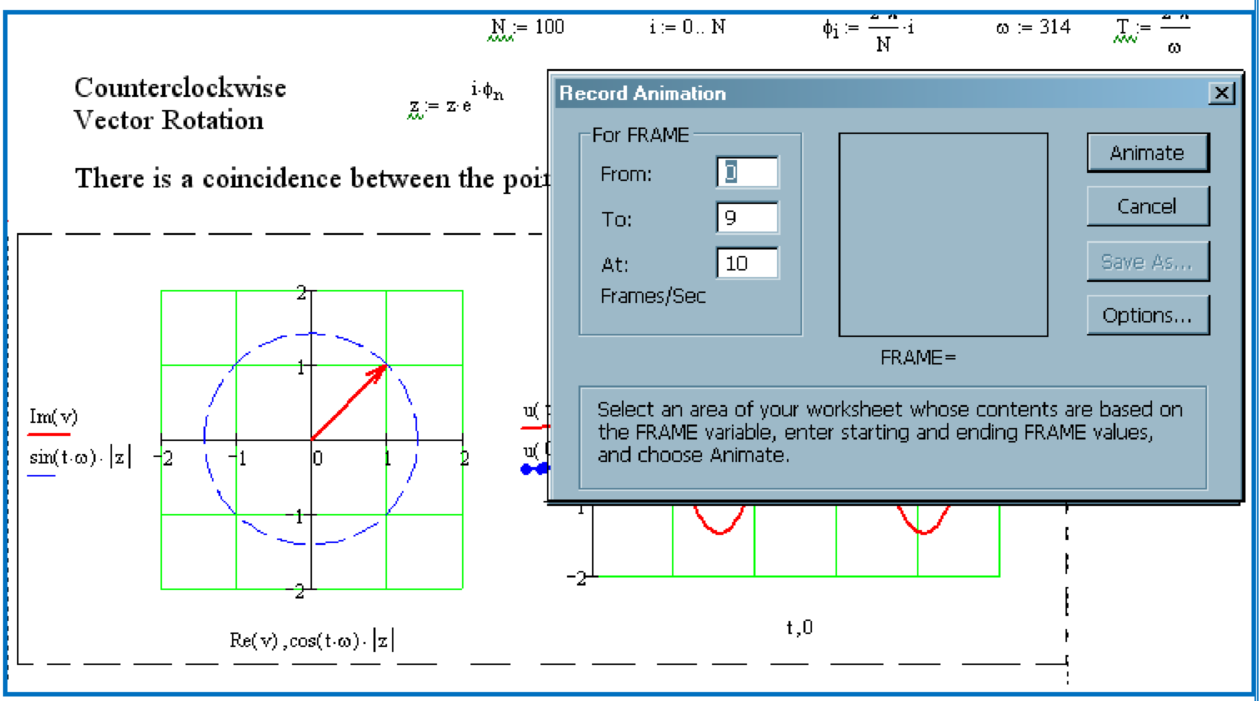


In order to get animation of your process you need to do the following sequence:

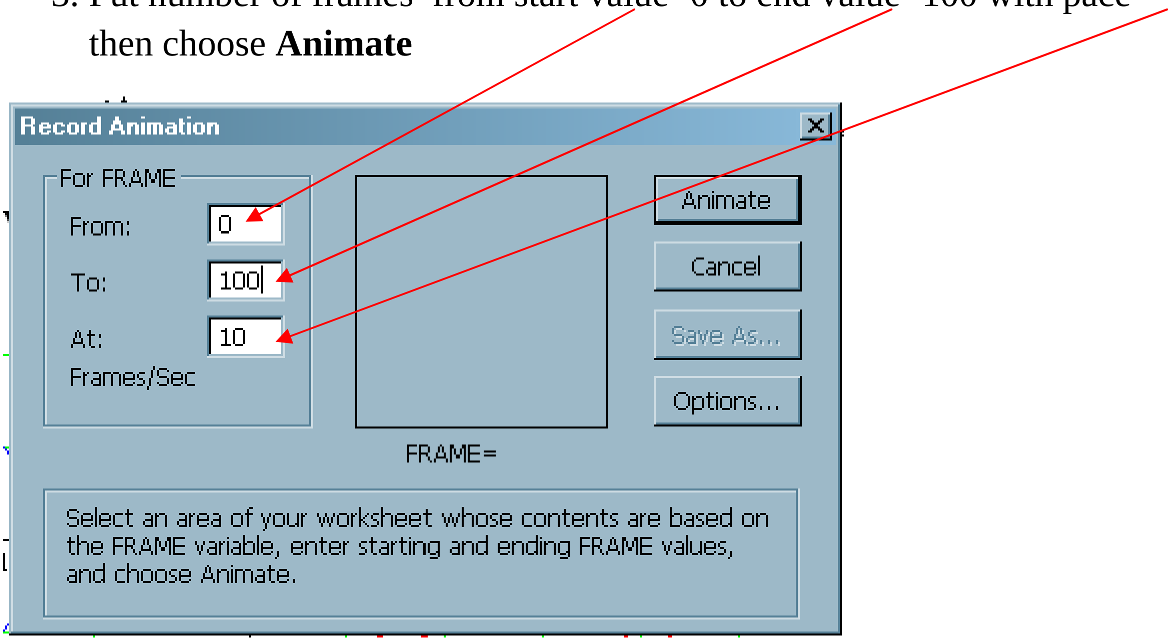
1. First step can be obtained by selecting **Tools/Animation/Record (see picture)**



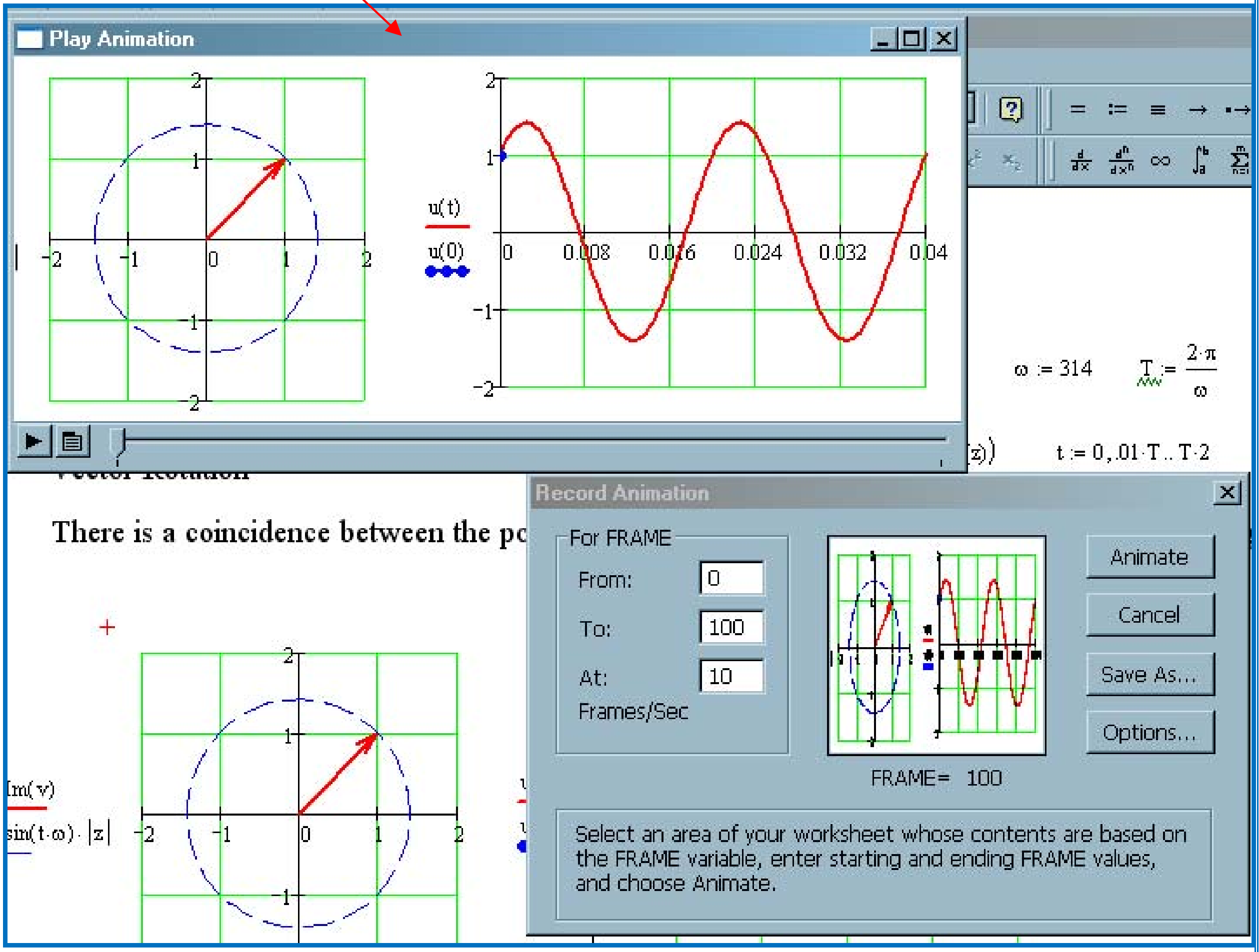
2. First step can be obtained by selecting region of animation (dashed line)



3. Put number of frames from start value=0 to end value=100 with pace = 10 then choose **Animate**



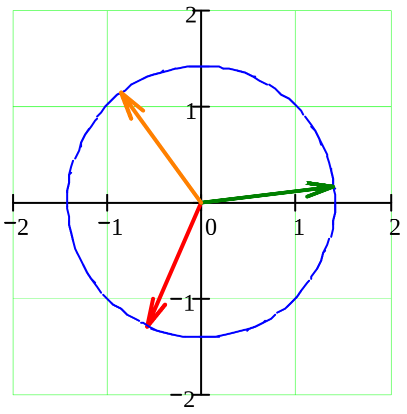
Then picture below appears

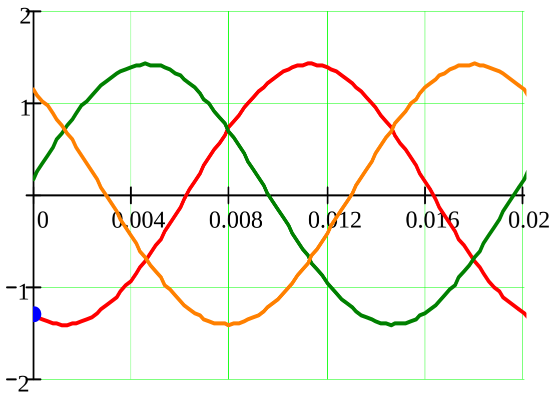


4. Finally choose **Save As** and save your file by giving it the name

Let's us consider 3-phase voltage $z1 := z \cdot e^{\frac{\pi}{3} \cdot 2j}$ $z2 := z \cdot e^{-\frac{\pi}{3} \cdot 2j}$ $v1 := V(z1, a)$

$u1(t) := |z1| \cdot \sin(\omega \cdot t + \arg(z1))$ $v2 := V(z2, a)$ $u2(t) := |z2| \cdot \sin(\omega \cdot t + \arg(z2))$



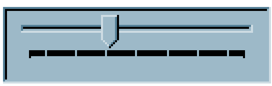


Plot tracing program

1. need to activate Control panel
2. need to activate TextBox
3. use script language

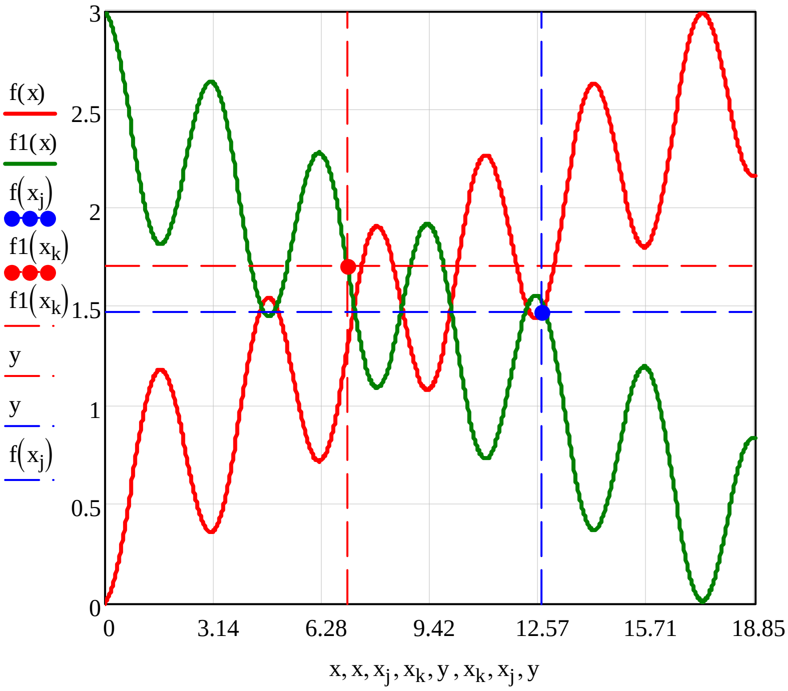
j :=

k :=



$N := 1000$ $i := 0..N$ $x_i := \frac{2 \cdot \pi}{N} \cdot i \cdot 3$ $y := 0, 0.01..6 \cdot \pi$

$f(x) := \sin(x)^2 + x \cdot 0.115$ $f1(x) := -f(x) + 3$

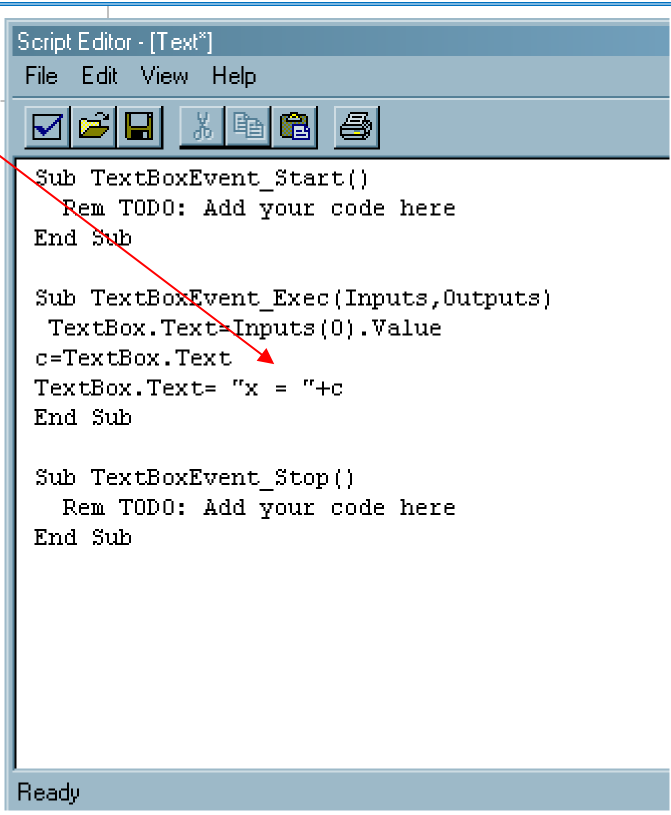


x = 12,6857511351956

f(x) = 1,47304551409016

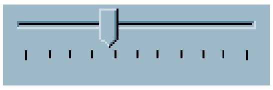
x1 = 7,0497339146555

f1(x) = 1,70812569114959



Representation of electric field in a complex plane

k :=

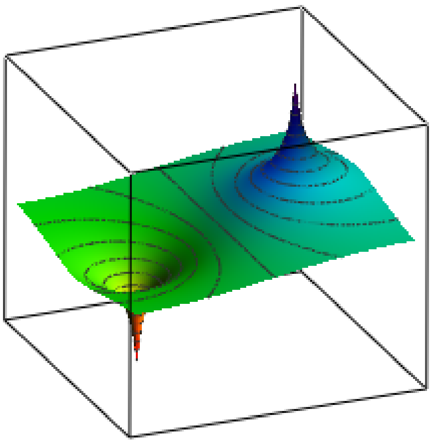


$N := 101 \quad i := 0..N \quad x_i := \frac{2}{N}\cdot i - 1$

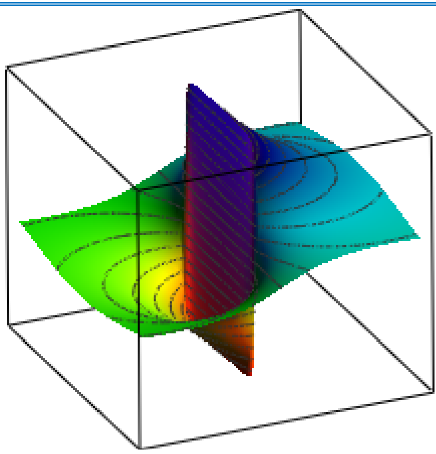
$z(x,y) := x + i\cdot y \quad x_0 := \frac{1}{N}\cdot(k+20) \quad i := i \quad w(x,y) := \ln\left(\frac{1}{z(x,y)}\right)$

Potential function of complex variables

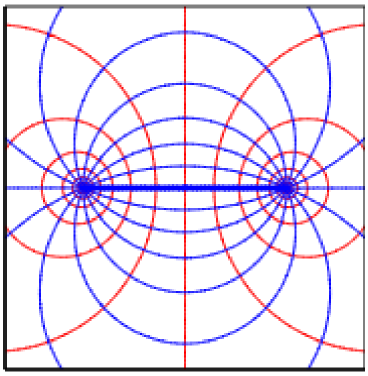
$W_{i,j} := w(x_i + x_0, x_j) - w(x_i - x_0, x_j)$



Real part of complex potential



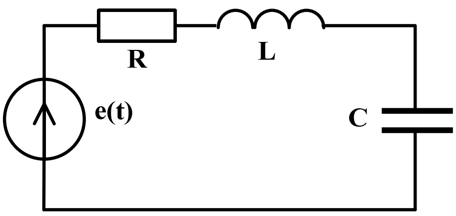
Imaginary part of complex potential



Contour plot of complex potential

MathCAD is a convenient program for solving transient processes in electric power engineering. A transient process is performed by ordinary differential equations. Let us consider example of using MathCAD for solving differential equation in state-space form

State-space matrix of electrical circuit differential equation



Using Kirchhoff's second law we can write

Given

$$i'_L \cdot L + U_C + i_L \cdot R = E$$

$$i_L = C \cdot U'_C$$

$$A(i_L, U_C, E) := \mathrm{Find}(i'_L, U'_C) \rightarrow \begin{bmatrix} \frac{-(U_C + i_L \cdot R - E)}{L} \\ \frac{i_L}{C} \end{bmatrix}$$

$$A(1,0,0) \rightarrow \begin{pmatrix} \frac{-R}{L} \\ \frac{1}{C} \end{pmatrix} \quad A(0,1,0) \rightarrow \begin{pmatrix} \frac{-1}{L} \\ 0 \end{pmatrix} \quad A(0,0,E) \rightarrow \begin{pmatrix} \frac{E}{L} \\ 0 \end{pmatrix} \quad A := \mathrm{augment}(A(1,0,0), A(0,1,0))$$

$$A \rightarrow \begin{pmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{pmatrix}$$

Form State -space Matrix

$$R := 10 \quad C := 150 \cdot 10^{-6} \quad L := 0.05$$

$$A := \begin{pmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \quad p := \mathrm{eigenvals}(A) \quad \omega := \left|\mathrm{Im}(p_0)\right| \quad T := \frac{2 \cdot \pi}{\omega}$$

$$t := 0, 0.01 \cdot T .. T \cdot 10 \quad N := 500 \quad E(t) := \sin(100 \cdot t) + 0.4 \cdot \sin(1000 \cdot t)$$

$$B(t) := \begin{pmatrix} \frac{E(t)}{L} \\ 0 \end{pmatrix} \quad D(t, x) := A \cdot x + B(t)$$

$i := 0 .. N \quad x_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ initial conditions of differential equation

$$x_1 := 0 \quad x_2 := 10 \cdot T$$

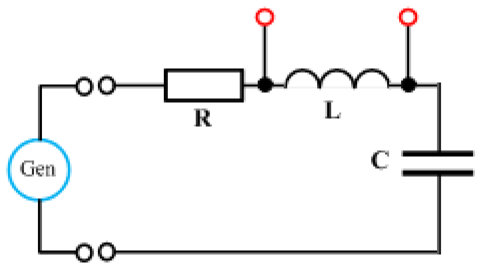
$$x := \mathrm{rkfixed}(x_0, x_1, x_2, N, D)$$

Returns a matrix of solution values for the differential equation specified by the derivatives in D and having initial conditions x_0 on the interval $[x_1, x_2]$ using a fixed step Runge-Kutta method. Parameter N controls the number of rows in the matrix output.

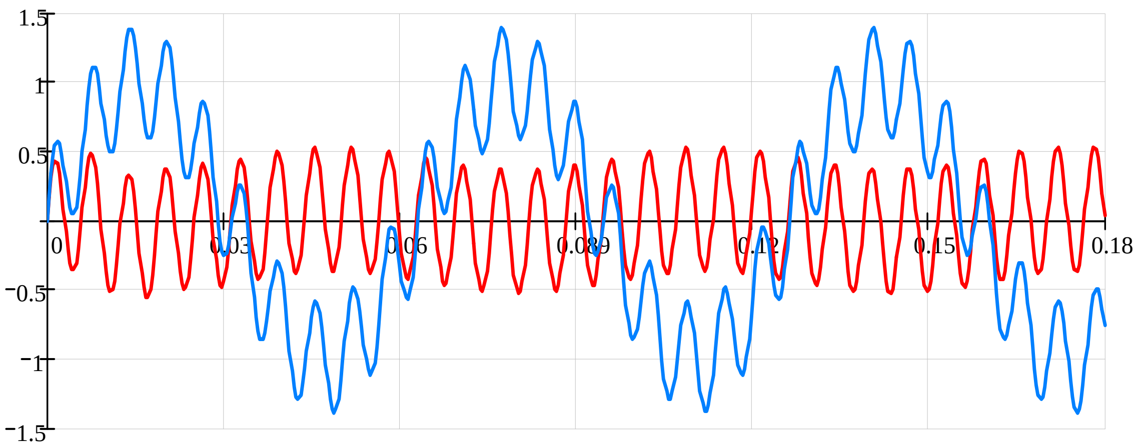
$$t := x^{\langle 0 \rangle} \qquad \begin{pmatrix} U_{L_i} \\ i_{C_i} \end{pmatrix} := \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix} \cdot D\left[\left(x^{\langle 0 \rangle}\right)_i, \begin{bmatrix} \left(x^{\langle 1 \rangle}\right)_i \\ \left(x^{\langle 2 \rangle}\right)_i \end{bmatrix}\right]$$

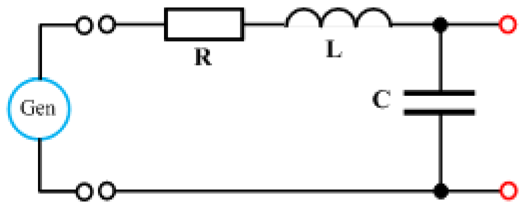
Generator

B := READRGB("C:/2.bmp")
A := READRGB("C:/1.bmp")
A := if(s = 1, A, B)
$F := \mathrm{if}\left(s = 1, x^{\langle 2 \rangle}, U_L\right)$



i := 0.. FRAME <= you can activate animation enabling evaluation





i := 0.. FRAME <= you can activate animation enabling evaluation