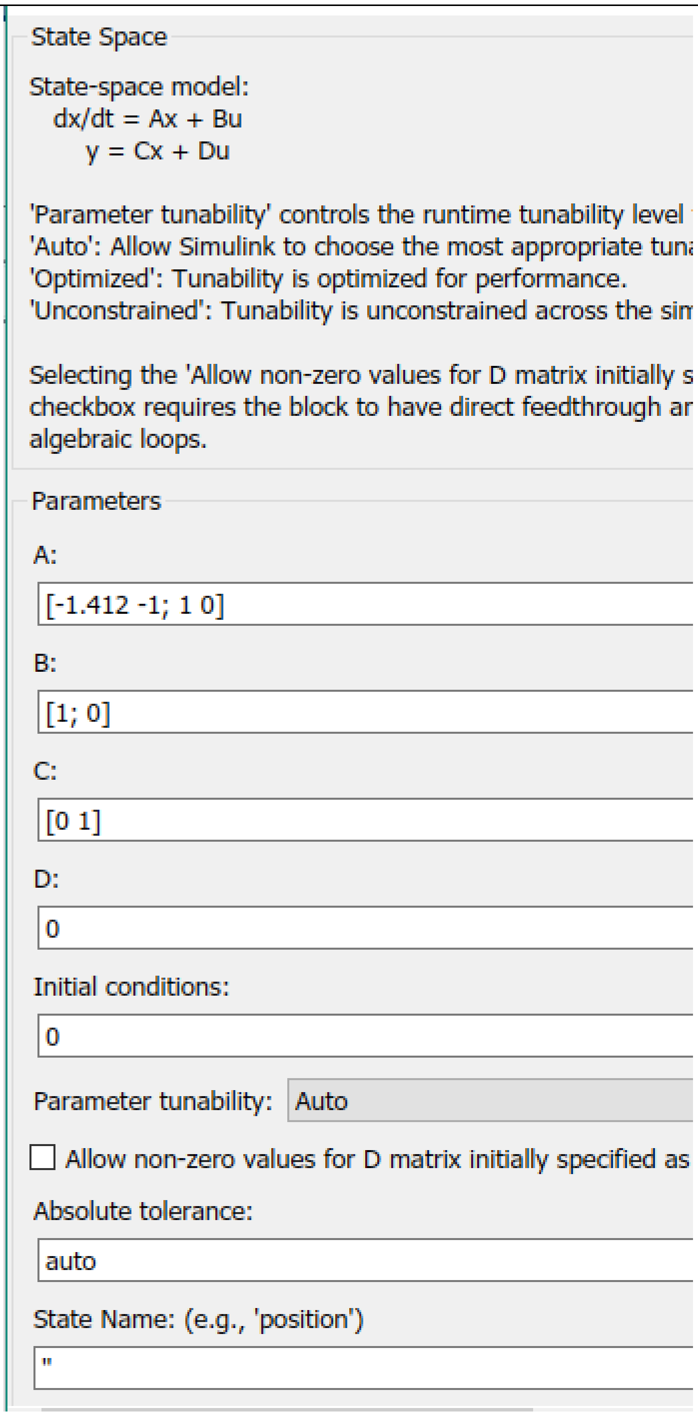
Формирование фильтра Баттерворта (Butterworth)

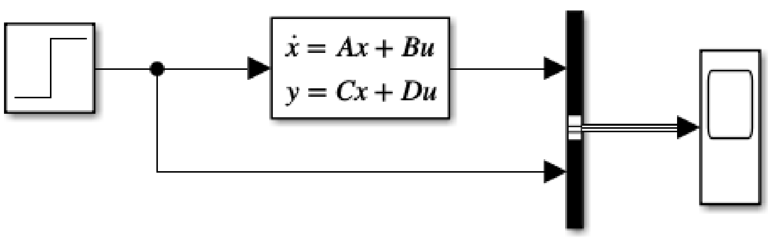
Сведем полученные полиномы в таблицу

n	***Полином Баттерворта*** $B(n,p) = a_0 p^n + a_1 p^{n-1} + a_2 p^{n-1} + \ldots . a_n, \quad a_0 = 1, \quad a_n = 1,$
2	$B(2,p) = p^2 + \sqrt{2} p + 1, \qquad a_0 = 1,\ a_1 = \sqrt{2},\ a_2 = 1$
3	$B(3,p) = p^3 + 2p^2 + 2p + 1, \qquad a_0 = 1,\ a_1 = 2,\ a_2 = 2,\ a_3 = 1$
4	$B(4,p) = p^4 + 2.613p^3 + 3.414p^2 + 2.6131p + 1, \qquad a_1 = 2.613,\ a_2 = 3.414,\ a_3 = 2.613$
5	$B(5,p) = p^5 + 3.2361p^4 + 5.2361p^3 + 5.2361p^2 + 3.2361p + 1,$ $a_1 = 3.2361,\ a_2 = 5.2361,\ a_3 = 5.2361,\ a_4 = 3.2361$
6	$B(6,p) = p^6 + 3.8637p^5 + 7.4641p^4 + 9.1416p^3 + 7.4641p^2 + 3.8637p + 1,$ $a_1 = 3.8637,\ a_2 = 7.4641,\ a_3 = 9.1416,\ a_4 = 7.4641,\ a_5 = 3.8637$
7	$B(7,p) = p^7 + 4.49396p^6 + 10.0978p^5 + 14.5918p^4 + 14.5918p^3 + 10.0978p^2 + 4.49396p + 1,$ $a_1 = 4.49396,\ a_2 = 10.0978,\ a_3 = 14.5918,\ a_4 = 14.5918,\ a_5 = 10.0978,\ a_6 = 4.49396$
8	$B(8,p) = p^8 + 5.1258p^7 + 13.137p^6 + 21.846p^5 + 25.688p^4 + 21.846p^3 + 13.137p^2 + 5.1258p + 1,$ $a_1 = 5.1258,\ a_2 = 13.137,\ a_3 = 21.846\ a_4 = 25.688,\ a_5 = 21.846,\ a_6 = 13.137\ a_7 = 5.1258$

Переходный процесс для фильтров Баттерворта второго порядка

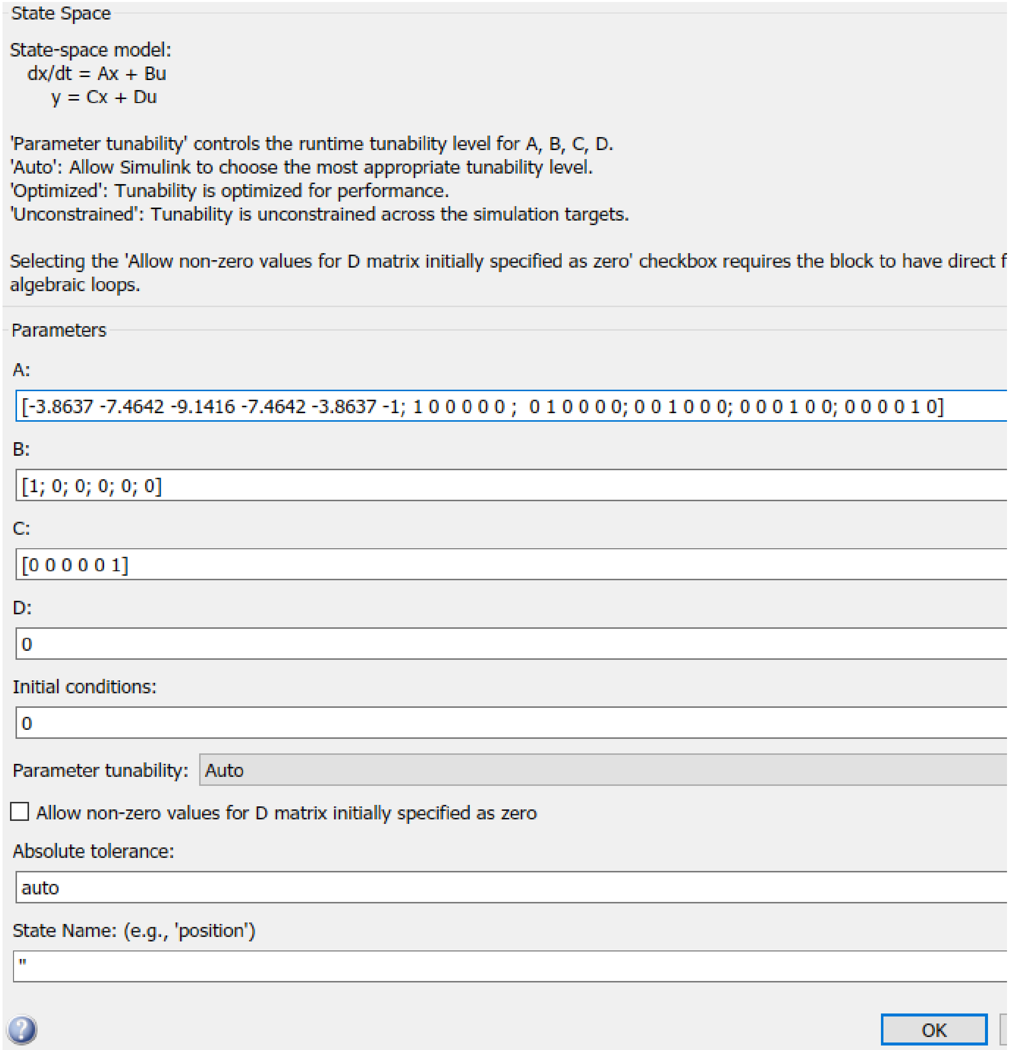
```
>> num=1;
>> den=[1 sqrt(2) 1]

>> [A,B,C,D]=tf2ss(num,den)
A =

  -1.4142   -1.0000
   1.0000         0

B =
    1
    0

C =
    0     1

D =
    0
```

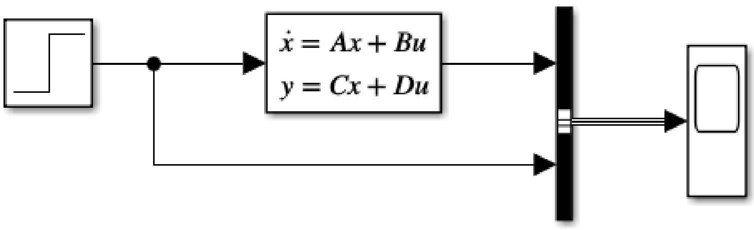

State Space
State-space model:
dx/dt = Ax + Bu
y = Cx + Du
'Parameter tunability' controls the runtime tunability level
'Auto': Allow Simulink to choose the most appropriate tun
'Optimized': Tunability is optimized for performance.
'Unconstrained': Tunability is unconstrained across the sin
Selecting the 'Allow non-zero values for D matrix initially s
checkbox requires the block to have direct feedthrough ar
algebraic loops.
Parameters
A:
[-1.412 -1; 1 0]
B:
[1; 0]
C:
[0 1]
D:
0
Initial conditions:
0
Parameter tunability:
Auto
Allow non-zero values for D matrix initially specified as
Absolute tolerance:
auto
State Name: (e.g., 'position')
''

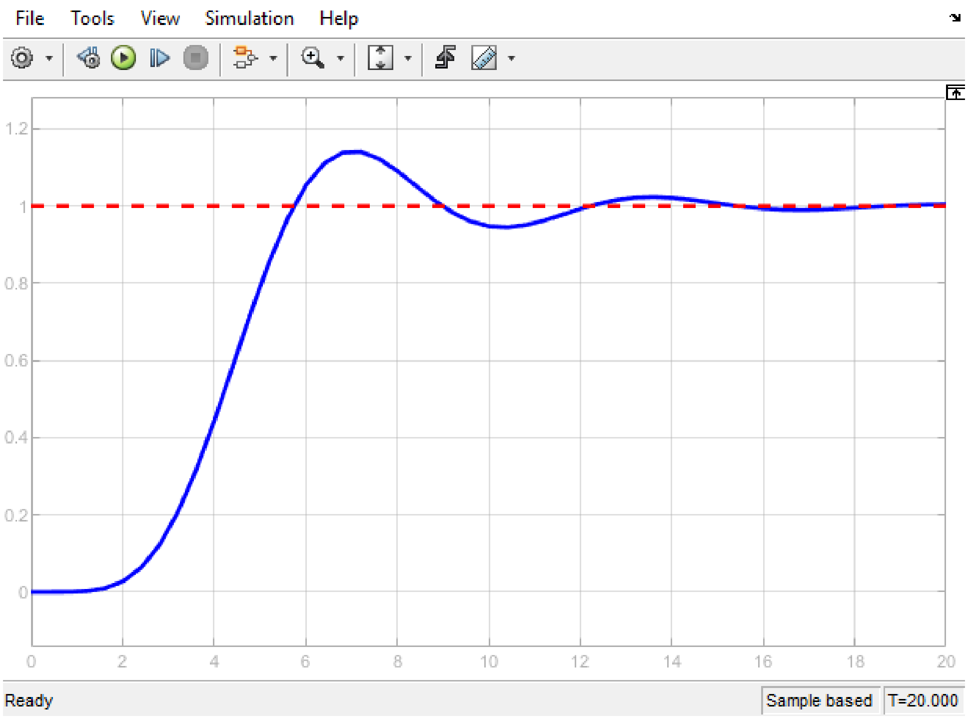

$\dot{x} = Ax + Bu$
$y = Cx + Du$

Переходный процесс для фильтров Баттерворта шестого порядка

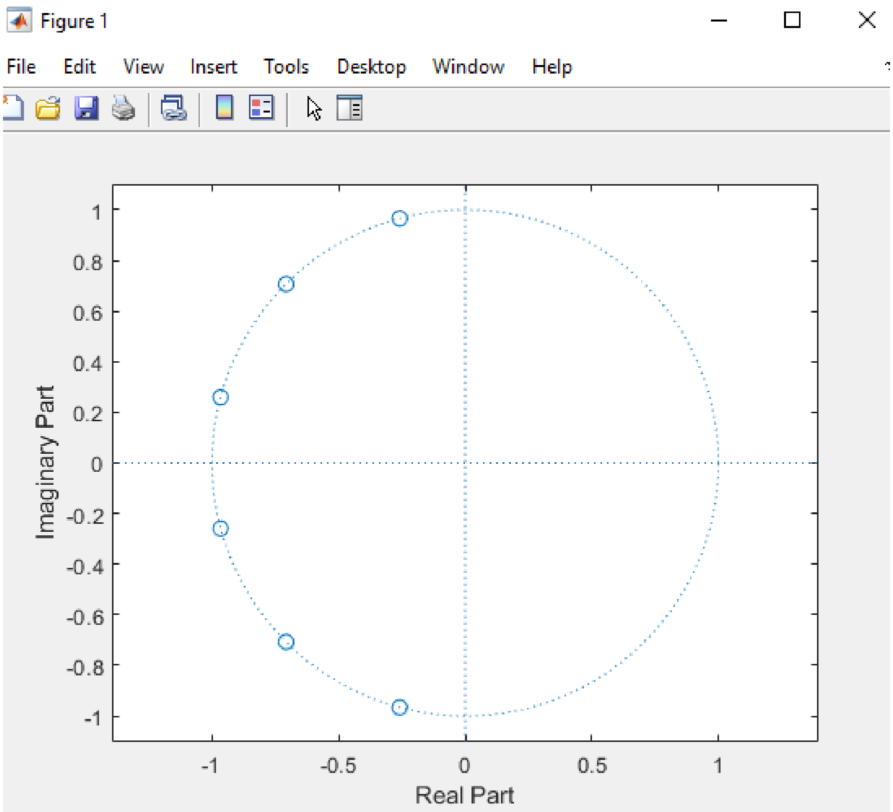
```
>> num=1;
>> den=[1 3.8637 7.4642 9.1416 7.4642 3.8637 1];
>> [A,B,C,D]=tf2ss(num,den) % преобразование передат. функции в метод ПС

A =

-3.8637   -7.4642   -9.1416   -7.4642   -3.8637   -1.0000

   1.0000         0         0         0         0         0

        0    1.0000         0         0         0         0

        0         0    1.0000         0         0         0

        0         0         0    1.0000         0         0

        0         0         0         0    1.0000         0

B =
     1
     0
     0
     0
     0
     0

C =

     0     0     0     0     0     1

D =

0
```


State Space
State-space model:
dx/dt = Ax + Bu
y = Cx + Du
'Parameter tunability' controls the runtime tunability level for A, B, C, D.
'Auto': Allow Simulink to choose the most appropriate tunability level.
'Optimized': Tunability is optimized for performance.
'Unconstrained': Tunability is unconstrained across the simulation targets.
Selecting the 'Allow non-zero values for D matrix initially specified as zero' checkbox requires the block to have direct f
algebraic loops.
Parameters
A:
[-3.8637 -7.4642 -9.1416 -7.4642 -3.8637 -1; 1 0 0 0 0 0 ; 0 1 0 0 0 0; 0 0 1 0 0 0; 0 0 0 1 0 0; 0 0 0 0 1 0]
B:
[1; 0; 0; 0; 0; 0]
C:
[0 0 0 0 0 1]
D:
0
Initial conditions:
0
Parameter tunability: Auto
Allow non-zero values for D matrix initially specified as zero
Absolute tolerance:
auto
State Name: (e.g., 'position')
''
OK


ẋ = Ax + Bu
y = Cx + Du




```
>> p=roots(den) % вычисление корней знаменателя

p =

  -0.2588 + 0.9659i
  -0.2588 - 0.9659i
  -0.9658 + 0.2594i
  -0.9658 - 0.2594i
  -0.7073 + 0.7069i
  -0.7073 - 0.7069i
>> zplane(p) % рисование карты полюсов
```



Представим знаменатель передаточной функции 6-го порядка как произведения передаточных функций второго порядка.

```
>> Sos=tf2sos(num,den)
```

Коэффициенты числителя Коэффициенты знаменателя

Sos =

1.0000	0	0	1.0000	1.9316	1.0000	для первой дроби ($w_1(p)$)
1.0000	0	0	1.0000	1.4146	1.0000	для второй дроби ($w_2(p)$)
1.0000	0	0	1.0000	0.5176	1.0000	для третьей дроби ($w_3(p)$)

```
p
>> a1=Sos(1,(4:6))
a1 =
   1.0000    1.9316    1.0000

>> a2=Sos(2,(4:6))
a2 =
   1.0000    1.4146    1.0000

>> a3=Sos(3,(4:6))
a3 =
   1.0000    0.5176    1.0000
```

Рисование частотных характеристик

```
>> w=[0:0.01:4];
>> z=1i*w;
>> H1=abs(1./(a1(1)+a1(2)*z+a1(3)*z.^2));
>> H2=abs(1./(a2(1)+a2(2)*z+a2(3)*z.^2));
>> H3=abs(1./(a3(1)+a3(2)*z+a3(3)*z.^2));
>> subplot(2,2,1);
>> plot(w,H1, 'g', 'LineWidth',2); grid on;
>> hold on
>> subplot(2,2,2);
>> plot(w,H2, 'r', 'LineWidth',2); grid on;
>> hold on
>> subplot(2,2,3);
>> plot(w,H3, 'b', 'LineWidth',2); grid on;
>> hold on
>> subplot(2,2,4);
>> H= H3.*H2.*H1;
>> plot(w,H, 'r', 'LineWidth',2); grid on;
```