TOMSK POLYTECHNIC UNIVERSITY

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# ELECTRICAL ENGINEERING AND ELECTRONICS.
# GUIDE TO LABORATORY EXERCISES

*Recommended for publishing as a guide to laboratory exercises*
*by the Editorial Board of Tomsk Polytechnic University*

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The guide includes laboratory exercises on the main topics of electrical engineering such as Kirchhoff's laws, DC and AC circuits, three-phase circuits, series and parallel resonance, transient processes and nonlinear circuits. Special attention is devoted to application of electronics elements of circuits, transistors, diodes, and Zener diodes. The text is supplemented by examples in which application of the calculation methods is illustrated. The guide is intended for training students majoring in the specialty 140400 «Electro engineering and power engineering».

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# INTRODUCTION

This manual is your guide to the first electronics laboratory in the electrical engineering program. It is assumed that by completing the first electronics laboratory course you are familiar with basic electronic measurements and instrumentation, as well as with elements of data analysis, presentation of results, and reporting. Professional engineering practice requires using proper experimental methods and procedures. They include not only good measurement techniques, but also proper recording of all relevant information, preparing tables and graphs, etc. Almost as important as obtaining good data is their proper presentation which often determines success in this laboratory course as it does in engineering practice. Upon completion of the first laboratory course you should be very familiar with effective laboratory practices and professional style data presentation. They will be a great asset in your future.

The experiments in this lab manual are designed to give the student practical experience in working with DC and AC circuits operating in steady-state and transient modes, with three-phase circuits, and with nonlinear circuits. The laboratory will complement and support the theory taught in the lectures, and should help the student to apply his knowledge of electrical engineering.

Every week before lab, each student should read over the laboratory experiment and work out the various calculations, etc. that are outlined in the guide.

During experimental work in laboratory a student should observe the following rules:

1. Return parts and jumper wires to correct bins when you are finished with them.
2. Do not put suspected defective parts back in the bins. Give them to the Lab Technician for testing or disposal.
3. Report all equipment problems to Lab Instructor or Lab Technician.
4. Most experiments have several parts; students must alternate in doing these parts as they are expected to work in group.
5. Each student must have a laboratory notebook. The notebook should be a permanent document that is maintained and witnessed properly, and that contains accurate records of all lab sessions.
6. Laboratory and equipment maintenance is the responsibility of not only the Lab Technician, but also the students. A concerted effort to keep the equipment in excellent condition and the working environment well-organized will result in a productive and safe laboratory.

## Safety in the Laboratory

To minimize electric shock hazard, the experiments are designed for low-voltage; however one should never assume that electric circuits are safe. Few milliamps of current through the body can be lethal. For your safety you must follow safety rules particularly:

- Turn off power before working on circuits.
- Know the location of emergency power-off switch.
- Make sure that the transformers and equipments are plugged into utility lines, have no exposed wiring. Check with the instructor if you are not certain about the procedure.

## Laboratory Notebook

The laboratory notebook is a record of all work pertaining to the experiment. This record should be sufficiently complete so that you or anyone else of similar technical background can duplicate the experiment and data by simply following your laboratory notebook. Record everything directly into the notebook during the experiment. Do not use scratch paper for recording data. Do not trust your memory to fill in the details at a later time.

## Guidelines for Laboratory Notebook

- State the objective of the experiment.
- Draw the ciruit digram and mention the values of resistances etc. which are used.
- Make a note of all the measuring instruments you have used.
- Mention the formulas used.
- Create a table and write down the readings, including the units.
- Show all your calculation neatly and SYSTEMATICALLY. Do this is an organized manner.
- Attach graph if any.
- Be concise. Complete sentences are not necessary as long as the context is clear.
- If mistakes are made, they should not be erased. Just bracket them and make a short note explaining the problem.
- Make entries as the lab progresses; don't assume you can fill it in later. The instructor will ask to see it during the lab.
- Draw the figure using pencil before you come to the lab so that you can make corrections to it in case you need to do so by erasing and redrawing. This will ensure tidy and neat work.

- Prepare the READING TABLE using pencil and ruler and not just by sketching lines. Sketching gives rise to crooked lines and gives the lab notebook a haphazard look.

## General Lab Report Format

Following the completion of each laboratory exercise in Electrical Engineering courses, a report must be written and submitted for grading. The purpose of the report is to completely document the activities of the design and demonstration in the laboratory.

Reports should be complete in the sense that all information required to reproduce the experiment is contained within. Writing useful reports is a very essential part of becoming an engineer. In both academic and industrial environments, reports are the primary means of communication between engineers. There is no one best format for all technical reports but there are a few simple rules concerning technical presentations which should be followed. Adapted to this laboratory they may be summarized in the following recommended report format:

- Title page
- Introduction
- Experimental Procedure
- Experimental Data
- Conclusions

Detailed descriptions of these items are given below.

**Title Page:**

The title page should contain the following information

- Your name
- Course number (including section)
- Experiment number and title
- Date submitted
- Instructors Name

**Introduction:**

It should contain a brief statement in which you state the objectives, or goals of the experiment. It should also help guide the reader through the report by stating, for example, that experiments were done with three different circuits or consisted of two parts etc. or that additional calculations or data sheets can be found in the appendix,
or at the end of the report.

**The Procedure**

It describes the experimental setup and how the measurements were made. Include here circuit schematics with the values of components.

Mention instruments used and describe any special measurement procedure that was used.

**Results/Questions:**

This section of the report should be used to answer any questions presented in the lab handout. Any tables and/or circuit diagrams representing results of the experiment should be referred to and discussed/explained with detail. All questions should be answered very clearly in paragraph form. Any unanswered questions from the lab handout will result in loss of points on the report. The best form of presentation of some of the data is graphical. In engineering presentations a figure is often worth more than a thousand words. There are some simple rules concerning graphs and figures which should always be followed. If there is more than one figure in the report, the figures should be numbered. Each figure must have a caption following the number. For example, *"Figure 1.1: TTL Inverter"*

**Conclusion:**

A brief conclusion summarizing the work done, theory applied, and the results of the completed work should be included here. Data and analyses are not appropriate for the conclusion.

***Notes***

- Any drawings in the report done by hand must be done with neatness, using a straight edge and drawing guides wherever possible. Free hand drawings will not be accepted.
- It is your responsibility to obtain the instructor's signature and to include the signed sheet with your final experiment report.
- Each student must submit an individual report based on an individual effort.

## Laboratory exercise No. 1

## INVESTIGATION OF BRANCHED CIRCUIT WITH DIRECT CURRENTS

**The objective of the exercise:**
Experimental verification of Kirchhoff's and Ohm's laws, verification of superposition and reciprocity principles, and the linear relationship theorem.

### Theoretical Background

*Kirchhoff's current law.* The algebraic sum of currents in a node of electrical circuit is equal to zero. The currents flowing into the node are usually taken with negative sign, and the currents flowing out of the node are usually taken with positive sign:

$$\sum I = 0$$

*Kirchhoff's voltage law.* The algebraic sum of voltage in a contour is equal to the algebraic sum of electromotive forces (e.m.f) in this contour. In the left-hand part of the equation voltage drops across the elements should be taken with positive sign if the direction of the contour tracing coincides with the current direction in the element. In the right-hand side of equation the e.m.f are positive if arrows of e.m.f sources coincide with the direction of the contour tracing:

$$\sum IR = \sum E .$$

Equations for currents are independent for all nodes in a circuit except for one node. Equations for voltages are independent for all contours of plane (planar) circuit except for the external contour.

*Superposition principle.* The current in any branch of a linear electric circuit including several sources can be represented by the sum of currents generated by individual sources.

*Reciprocity principle.* Suppose that in a passive linear electric circuit two branches *ab* and *cd* are singled out, a source of e.m.f $E_{ab} = E$ is cut out of branch *ab* and current $I_{cd} = I$ is measured in branch *cd*. If one includes the same source of e.m.f. in branch *cd* ($E_{cd} = E$) and measures the current in branch *ab* ($I_{ab} = I$), both currents $I_{ab}$ and $I_{cd}$ will be equal.

*Theorem of linear relationships.* When one parameter of a linear electric circuit (e.m.f, resistance, or current) is changed, any other two parameters of this circuit (currents or voltages) is bound by a linear relation of the form $y = ax + b$.

*Compensation theorem.* Arbitrary tow-polar element can be substituted by a source with the e.m.f equal to the voltage drop across the terminals of the two-pole element and with the same polarity. In this case currents and voltages in the remaining part of the circuit do not change.

For any electrical circuit it is possible to obtain mutually independent equations describing voltages. To this purpose one should write the equation for any contour of this circuit, after that some branch in this contour should be removed and equations for each of the remaining unbroken contours be written.

For a linear electric circuit Kirchhoff's equations are linear. The consequences of this fact are the principle of influence independence of sources and the superposition principle. According to the last principle the current in any branch of circuit can be considered as a sum of current components, each component being generated by only one source of e.m.f. or by a current source which are switched in turn.

**Electrical Circuit**

To perform the experiments of exercise the electric circuit represented in Fig. 1.1 is used. Sources of electric power are accumulator batteries internal resistance of which is small in comparison with the resistances of resistors $R_1$, $R_2$, and $R_3$ so the batteries can be considered as sources of e.m.f. $E_1$ and $E_2$.

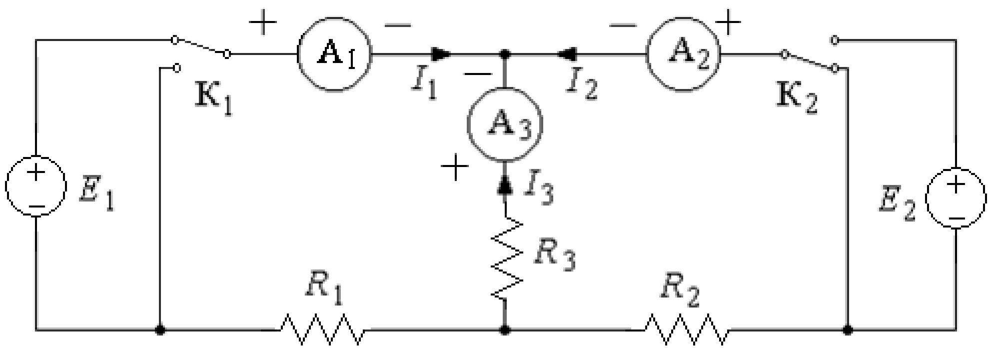



Fig. 1.1

As e.m.f. sources $E_1$ the battery should be taken with controlled output voltage. E.m.f. of source $E_2$ does not change in all experiments.

With the use of keys $K_1$ and $K_2$ these sources can be switched on or off the circuit or substituted by a conductor with zero resistance.

Before starting experiments we should measure voltages generated by the sources of e,m.f. by voltmeters. The voltage of $E_1$ must be set according the variant of the exercise, the value of $E_1$ is given in Table 1.1.

Table 1.1

| Variant | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $E_1$ | V | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $R_1$ | Ohm | 100 | 150 | 220 | 100 | 150 | 220 | 100 | 220 |
| $R_2$ | Ohm | 150 | 100 | 100 | 220 | 220 | 150 | 220 | 100 |
| $R_3$ | Ohm | 220 | 220 | 150 | 150 | 100 | 100 | 150 | 330 |

While assembling the circuit a special attention should be paid to the polarity of the sources of e.m.f and the instruments.

## Review Questions

Study theoretical material and answer the following questions:

1. In which way should the contours be chosen to make independent the Kirchhoff's equations of the circuit?
2. In Fig. 1.2 the structural scheme of some circuit is represented (branches are shown by lines, nodes by points). Determine the number of independent equations that should be written for this circuit according to the Kirchhoff's current and voltage laws.

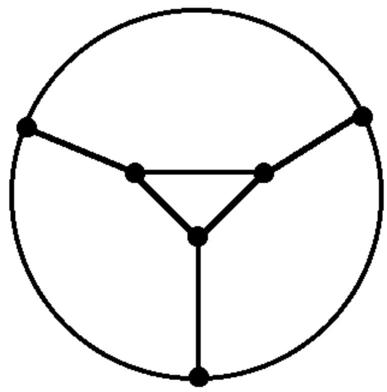


Fig. 1.2

3. In Fig. 1.3 the circuit under investigation is shown without meters. Write down for this circuit the equations according to Kirchhoff's current and voltage laws.
4. What are the rules of sign definition in Kirchhoff's current and voltage laws?
5. What is the essence of superposition principle? How can one verify this principle using the circuit drawn in Fig. 1.1? Can this principle be applied to calculation of powers?

6. Explain this principle by applying it to the circuit drawn in Fig. 1.3. Write the equations for analytical verification of the superposition principle (prove the equivalence of the equations for two currents).

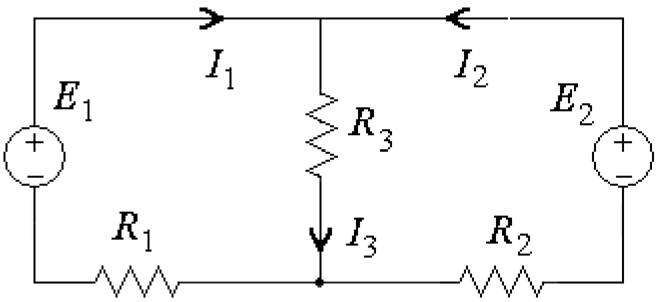



Fig. 1.3

**Experimental Investigations**

1. Measuring the circuit parameters.

Using the multimeter as a voltmeter measure the voltage of the source $E_2$ and set e.m.f. of $E_1$ according the variant of exercise (Table 1.1). In all experiments the value of this voltage should be fixed. Using the multimeter as an ohmmeter measure resistances $R_1$, $R_2$, and $R_3$. The obtained results write down in Table 1.2.

Table 1.2

| $E_1$ | $E_2$ | $R_1$ | $R_2$ | $R_3$ |
|---|---|---|---|---|
| V | V | Ω | Ω | Ω |
| | | | | |

2. Experimental verification of the first and second Kirchhoff's law and the superposition principle. The corresponding electrical circuit is presented in Fig. 1.1.

*Experiment 1.* E.m.f. $E_1$ is switched on, $E_2$ is switched off. Key $K_1$ is churned up, $K_2$ is churned down.

*Experiment 2.* EMF $E_2$ is switched on. $K_2$ is churned up.

*Experiment 3.* The both sources of EMF are switched on.

Write down the readings of the devices in Table 1.3. In the fourth row of the table write the sum of readings obtained in experiments 1 and 2. Compare this result with the readings of devices taken in experiment 3.

Sum up values of currents measuerd in experiments 1 - 3 according to the Kirchhoff's current law. Write down the result in the right column of the table. Requirements of the Kirchhoff's current law should be met in all experiments.

Calculate the sum of voltages across the passive elements in the left and right contours of the circuits for all three experiments. The results should be equal to the voltages of e.m.f. sources.

Table 1.3

| Experiments | Voltages of e.m.f. sources | | Reading of devices | | | Results of calculations | | |
|---|---|---|---|---|---|---|---|---|
| | $U_1$ | $U_2$ | $I_1$ | $I_2$ | $I_3$ | $\sum IR(1)$ | $\sum IR(2)$ | $\sum I$ |
| | V | V | mA | mA | mA | V | V | mA |
| 1 | $E_1$ | 0 | | | | | | |
| 2 | 0 | $E_2$ | | | | | | |
| 3 | $E_1$ | $E_2$ | | | | | | |
| Calculations | $E_1$ | $E_2$ | | | | Validation of superposition principle | | |

3. Verification of the linear relationship theorem for currents $I_2$ and $I_3$. EMF $E_1$ is varied, $E_2$ remains constant.

*Experiment 4.* Change the polarity of e.m.f. $E_1$ (the source $E_2$ is connected to the circuit) and write down the measured currents to Table 1.4.

Table 1.4.

| No. | $U_1$ | $U_1$ | $I_3$ | $I_2$ | Notes |
|---|---|---|---|---|---|
| | V | V | mA | mA | |
| 4 | | | | | Coefficient of the equation $I_3 = aI_2 + b$ $a = ..., b = ...$ |
| 3 | | | | | |
| 2 | | | | | |

To the same table write the results of experiments 2 and 3 (Table 1.3). Draw the plot of dependence $I_3$ upon $I_2$ by the data in Table 1.4. Note that this dependence is linear ($I_3 = aI_2 + b$) and calculate the coefficients *a* and *b*.

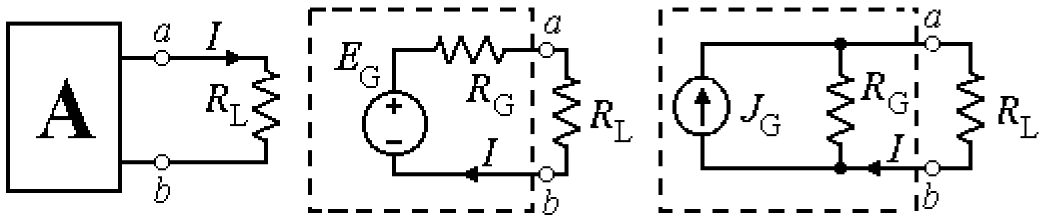
4. Draw general conclusions to the laboratory exercise. In the conclusions you should point out the fulfillment of the objectives. Explain the discrepancy between theoretical and experimental data if such discrepancy has been registered.

## Laboratory exercise No. 2

## ACTIVE TWO-POLAR CIRCUIT

**The objectives of the exercise:**
To prove possibility of representation of active two-polar element by an equivalent generator. To calculate the parameters of the equivalent generator.

### Theoretical Background




a b c

Fig. 2.1.

Frequently in problems of electroengineering calculation of currents is required only in one branch of a circuit. Relatively to the terminals of this branch the remaining part of the circuit can be viewed as an active two-polar element (Fig 2.1 *a*). This element is possible to represent in the form of equivalent generator including a source of e.m.f. (Fig 2.1 *b*) or a source of current (Fig 2.1 *c*) and active resistance $R_G$ ($R_G$ is the internal resistance of the generator).

E.m.f. of equivalent generator is equal to the voltage between points *ab* measured in the open-circuit regime (resistance $R_G$ equals infinity). Internal resistance of generator is equal to the resistance of circuit relatively points *ab* (it should be noted that resistance of e.m.f. sources equals zero and resistance of current sources equals infinity). Then the equivalent generator is represented according the scheme in Fig. 2.1 *c*, the current of the source $J_G$ can be found as a short-circuit current ($R_L = \infty$).

$$J_G = I_{SC} = E_G / R_G$$

Parameters of equivalent generator can be found experimentally by measuring the voltage across the load and current in the load for two different resistances of $R_L$. It is so called the method of two loads. The next possibility to find the parameters of the generator is measuring the short-circuit current ($R_L$ = 0) and open-circuit voltage ($R_L$ = ∞).

Calculation of generator parameters in the method of two loadings is performed according the following equations:

$$E_G = \frac{U_2 I_1 - U_1 I_2}{I_1 - I_2} \qquad R_G = \frac{U_2 - U_1}{I_1 - I_2} \tag{2.1}$$

where $U_1$ and $U_2$ are readings of the voltmeter corresponding to two loads, and $I_1$ and $I_2$ are reading of amperemeter corresponding to the same loads.

Parameters of the generator also can be calculated out of short-circuit and open-circuit regimes as

$$E_G = U_{ID}, \quad R_G = U_{OS} / I_{SC} \tag{2.2}$$

where $U_{OS}$ are readings of the voltmeter in the open-circuit regime ($I$ = 0) and $I_{SC}$ are reading of the voltmeter in the short-circuit regime ($U_{ab}$ = 0). Calculation of the load current can be performed according the equation (for circuit in Fig. 2.1 *b*):

$$I = \frac{E_G}{R_G + R_L} \tag{2.3}$$

or by

$$I = \frac{J_G}{1 + R_L / R_G} \tag{2.4}$$

(circuit in Fig. 2.1 *c*). Here $R_L = U_{ab} / I$ is the resistance of load (Fig. 2.2), $R_G$ is the resistance of generator, and $I_G = I_{SC}$.

Power dissipated in the load and the coefficient of efficiency are the following:

$$P_L = I^2 R_L \text{ and } \eta = P_L / (IE_G).$$

Their values depend on ratio of $R_G$ and $R_L$.

The external characteristic of the generator is the dependence of $U_{ab}$ on $I (U_{ab} = f(I))$. This characteristic allows one to find graphically the current and voltage of load. To do so we need to draw the volt-ampere characteristic of the load $R_L$ $(U_L = R_L I)$ in the same axes as the characteristic of the generator. The point of intersection gives us $U_L$ and $I_L$.

## Electrical Circuit

The schematic of electrical circuit is drawn in Fig. 2.2.

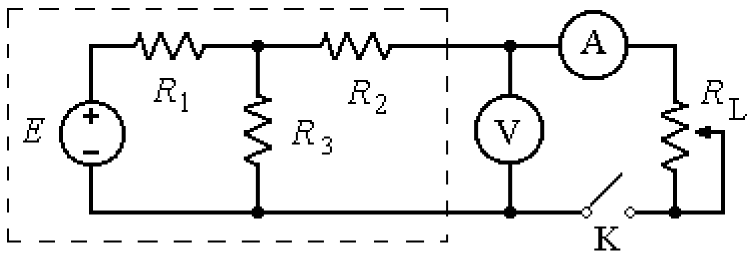



Fig. 2.2.

The part of the circuit marked by a dashed line represents an active two-polar element including source of e.m.f. $E$ = 15 V and resistors $R_1$, $R_2$, $R_3$. Parameters of these resistors should be taken from Table 1.1 in accordance to the variant of exercise.

As a load the variable resistor $R_L$ = 0 ÷ 1000 Ohms should be taken. DC current and voltage on the output of the two-polar elements are measured by voltmeter with permissible limit of 20 V and by ampermeter with limit of 200 mA.

When the switch $K$ is opened, the open-circuit regime is realized in the circuit ($I$ = 0). Closed switch and zero resistance of load give us the short-circuit regime ($U_{ab}$ = 0).

## Review Questions

1. Write down the equations which allow one to calculate the parameters of the equivalent generator out of open and short-circuit regimes.
2. Assuming that the parameters of the circuit in Fig. 2.2 ($E$, $R_1$, $R_2$, and $R_3$) are known obtain the formulae for calculation of $E_G$ and $R_G$.
3. Show analytically that the maximum power dissipated in the load is observed when $R_L = R_G$.

Note: For maximum power $\frac{\partial P_H}{\partial R_H} = 0.$

## Experimental Investigations

1. Measure actual values of resistors $R_1$, $R_2$, and $R_3$ and the voltage of e.m.f. source $E$. In Table 2.1 write down the obtained values of parameters.
2. Build the electrical circuit according to Fig. 2.2. Measure voltage $U_{ab}$ and current $I$ in short-circuit and open-circuit regimes.
3. Measure $U_{ab}$ and $I$ for six different resistances of load. Measurements should be done with equal intervals of load changes.
4. Calculate e.m.f. of the equivalent generator and resistance of the generator. Calculations should be done
   (a) using short-circuit and open-circuit regimes (Eq. 2.2):
   (b) according to the method of two loads (Eq. 2.1).

Results of calculation ($E_G$ и $R_G$) write down to Table 2.1

Table 2.1

| $E$ =… V | $R_1$ =… Ω | | | $R_2$ =… Ω | | $R_3$ = … Ω | | |
|---|---|---|---|---|---|---|---|---|
| Experiment | $I$ | $U_{ab}$ | $R_L$ | $E_G$ | $R_G$ | $\ln(R_L/R_G)$ | $P_L$ | η |
| | mA | V | Ω | V | Ω | – | W | % |
| Open-circuit | 0 | | ∞ | | | ∞ | 0 | 100 |
| 1<br>2<br>. . .<br>6 | | | | | | | | |
| Short-circuit | | 0 | 0 | | | ∞ | 0 | 0 |

5. In all experiments calculate values of loading according Ohm's law, power $P_L$ dissipated in the load, efficiency coefficient η, and function $\ln(R_H / R_Г)$. The results write down to the Table.
6. Draw external characteristic of the generator $U_{ab} = f(I)$ and for definite value of the load $R_L$ define graphically the value of current in the load $I$ and voltage $U_{ab}$ across the load. For the same load calculate current $I$ according Eq. 2.3. Compare the results.
7. Using the data from Table 2.1 draw the functions $P_H = f[\ln(R_H / R_Г)]$ and $\eta(P_H)$. Find out the resistance of load taking the maximum power and coefficient of efficiency corresponding to the maximum power.

8. Using the parameters of the circuit ($E$, $R_1$, $R_2$, and $R_3$) calculate analytically e.m.f. $E_G$ and the internal resistance of the generator $R_G$. Compare the data with results of experiments.
9. Draw conclusions to the laboratory exercise.

## Laboratory exercise No. 3

## CAPACITOR AND INDUCTOR IN A CIRCUIT WITH SINUSOIDAL CURRENTS

**Objectives of the exercise:**

To learn how to measure the parameters of a capacitor and inductor and to draw vector diagrams, to verify the Kirchhoff's laws for a circuit with sinusoidal currents.

### Theoretical Background

In contrast to an ideal capacitor a real one has some losses of energy in the form of heat due to isolation imperfections. In analysis of electric circuits such a capacitor is represented as a equivalent circuit with elements connected in parallel. Parameters $g$ and $C$ of the equivalent circuit can be defined.

In case of parallel connection of $R$, $L$, and $C$ elements according to Ohm's and Kirchhoff's laws one can write the following equation for the input current in complex representation:

$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = \dot{U}\dot{Y},$$

where $\dot{Y} = g - jb = y\exp(-j\varphi)$ is the complex conductivity of the circuit, $g$ is the active, $b = b_L - b_C$ is the reactive, and $y$ is the whole conductivity; $\varphi = \arctan(g/\omega C)$ is the angle of phase shift between voltage and current, $b_L = 1/(\omega L)$ is inductive conductivity and $b_C = \omega C$ is the capacitive one. Voltage across the capacitor lags in phase behind the current (the angle of phase shift is negative, **(**$-90° < \varphi < 0$**,** $b = -b_C$).

A real inductance coil also has thermal losses in contrast to the ideal one. The equivalent circuit for such an inductor is usually represented as $R$ and $L$ elements connected in series. These parameters can be determined experimentally using the readings of devices and a vector diagram corresponding to the circuit.

In case of series connection of $R$, $L$, and $C$ elements according to Obm's and Kirchhoff's laws one can write in complex representation the following equation for the input voltage:

$$\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C = \dot{I}\dot{Z},$$

where $\dot{Z} = R + jR = z\exp(+j\varphi)$ is the impedance of the circuit, $R$ is the active and $X = X_L - X_C$ is the reactive parts of impedance; $\varphi = \arctan(X/R)$ is the angle of phase shift between voltage and current, $X_L = \omega L$ is the inductive resistance and $X_C = 1/(\omega C)$ is the capacitive one.

Current in the inductor lags in phase behind the voltage (the angle of phase shift is positive, $90° > \varphi > 0$, $X = X_L$).

## Review Questions

1. Which physical phenomena are reflected in the equivalent circuit of the capacitor by *g* and *C* elements and in the equivalent circuit of the inductor by *R* and *L* elements?
2. Define the limits in which can be varied the phase shift between voltage and current at the input of the passive two-polar element?
3. Write down Kirchhoff's current law for the circuits represented in Fig. 3.1 and Kirchhoff's voltage law for the circuits represented in Fig. 3.2. The equations should be written both for instantaneous and complex values of voltages and currents.

## Electrical Circuit

Electrical circuits including an active-capacitive and active-inductive element which should be investigated in the laboratory exercise are drawn in Figs 3.1 and 3.2 correspondingly.

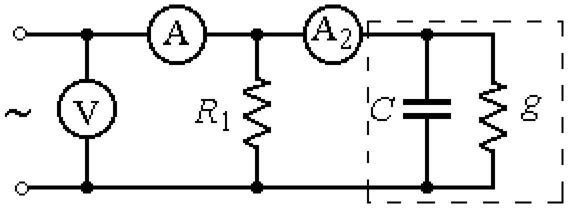



Fig. 3.1.

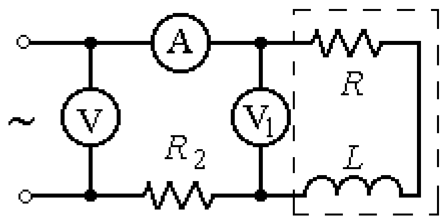



Fig. 3.2.

By dashed lines in these pictures a capacitor with parameters *g* and *C* and inductor with parameters *R* and *L* are marked. Both circuits are connected to a generator of sinusoidal voltage with frequency *f* and effective value *U* which can be changed.

Values of *U* and *f* as well as other parameters of circuits can be taken from Table 3.1 according to the variant of the exercise. Measurements of currents are done with an ampermeter with limits of measurements of 200 mA and measurements of voltage by a voltmeter with limits of 20 V.

Table 3.1

| **Variant** | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{U}$ | V | 4 | 4.5 | 5 | 4 | 4.5 | 5 | 4.5 | 4 | 4.5 | 5 |
| $\boldsymbol{f}$ | Hz | 500 | 1000 | 2000 | 1000 | 500 | 500 | 1000 | 2000 | 1000 | 500 |
| $\boldsymbol{R_1}$ | Ω | 330 | 150 | 100 | 220 | 470 | 680 | 470 | 220 | 330 | 470 |
| $\boldsymbol{C}$ | mkF | 1 | 1 | 1 | 1 | 1 | 0.47 | 0.47 | 0.47 | 0.47 | 0.47 |
| $\boldsymbol{R_2}$ | Ω | 680 | 680 | 1000 | 1000 | 470 | 220 | 220 | 470 | 470 | 150 |
| $\boldsymbol{L}$ | mH | 100 | 100 | 100 | 100 | 100 | 40 | 40 | 40 | 40 | 40 |

## Experimental Investigations

**A) Investigation of an active-capacitive circuit.**

1. Build the circuit shown in Fig. 3.2 with parameters chosen according the variant of exercises (Table 3.1).
2. Set the frequency and the voltage of the generator and write down the parameters of the circuit to Table 3.2.

Table 3.2

| $f =$ Hz, $\omega = 2\pi f =$ rad/s, $R_1 =$ Ohms, $C =$ mkF | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Experiment** | | | **Results of calculations** | | | | | | | |
| $U$ | $I$ | $I_2$ | $I_1$ | $\varphi_2$ | $g$ | $C$ | $\dot{I}$ | $\dot{I}_2$ | $\dot{I}_1$ | $\dot{I}_1 + \dot{I}_2$ |
| V | mA | mA | mA | grad | Sm | mkF | mA | mA | mA | mA |
| | | | | | | | | | | |

3. Calculate the current running in resistor $R_1$ assuming that the initial phase angle of the input voltage equals zero. Draw vectors of these current and voltage along the axis of real numbers in a vector diagram. Draw other two currents of the circuit in the diagram. Vectors $\underline{I}_1$ and $\underline{I}_2$ form a parallelogram with diagonal represented by current $\underline{I}$. Vertex of this parallelogram can be found using a pair of compasses. After drawing the diagram measure the phase angle of current $\underline{I}_2$ and write it down to table 3.2.
4. Write complex values of all current in Table 3.2 and calculate parameters $g$ and $C$ of the capacitor.

**B) Investigation of an active-inductive circuit.**

1. Assemble the circuit shown in Fig. 3.1 with parameters chosen according the variant of exercises (Table 3.2).
2. Set the frequency and the voltage of the generator and write down the parameters of the circuit to Table 3.3.

Table 3.3

| $f$ = Hz, $\omega = 2\pi f$ = rad/s, $R_2$ = Ohms, $L$ = mH | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Experiment** | | | **Results of calculations** | | | | | | | |
| $U$ | $I$ | $U_1$ | $U_2$ | $\varphi_1$ | $R$ | $L$ | $\dot{U}$ | $\dot{U}_1$ | $\dot{U}_2$ | $\dot{U}_1+\dot{U}_2$ |
| V | mA | V | V | grad | Ω | mH | V | V | V | V |
| | | | | | | | | | | |

3. Calculate the voltage across resistor $R_2$ assuming that the initial phase angle of the input current equals zero $\dot{U}_2 = U_2 = R_2 I$. Draw vectors of these current and voltage along the axis of real numbers in a vector diagram. Draw vectors $\dot{U}_1$ and $\dot{U}$ in the diagram. These vectors form a triangle one vertex of which can be found using a pair of compasses with the origin of coordinate system. The second vortex can be found using a pair of compasses
4. Measure the angle $\varphi_1$ between the current and voltage of the inductor in the diagram and calculate parameters of the inductor ($R$ and $L$). The obtained results write down tot Table 3.3.
5. Using the diagram measure the complex values of voltages $\dot{U}$, $\dot{U}_1$, and $\dot{U}_2$ and write the results to Table 3.3. Compare the value of voltage $\dot{U} = \dot{U}_1 + \dot{U}_2$ with the corresponding value measured experimentally.
6. Draw the conclusions to the exercise.

## Laboratory exercise No. 4

## THREE-PHASE CIRCUIT WITH THE LOAD CONNECTED AS A STAR

**The objective of the exercise:**

Investigation of three-phase circuit with symmetrical and asymmetrical loads and the influence of a zero wire on distribution of voltages in the circuit. Drawing of vector diagrams corresponding to experimental results.

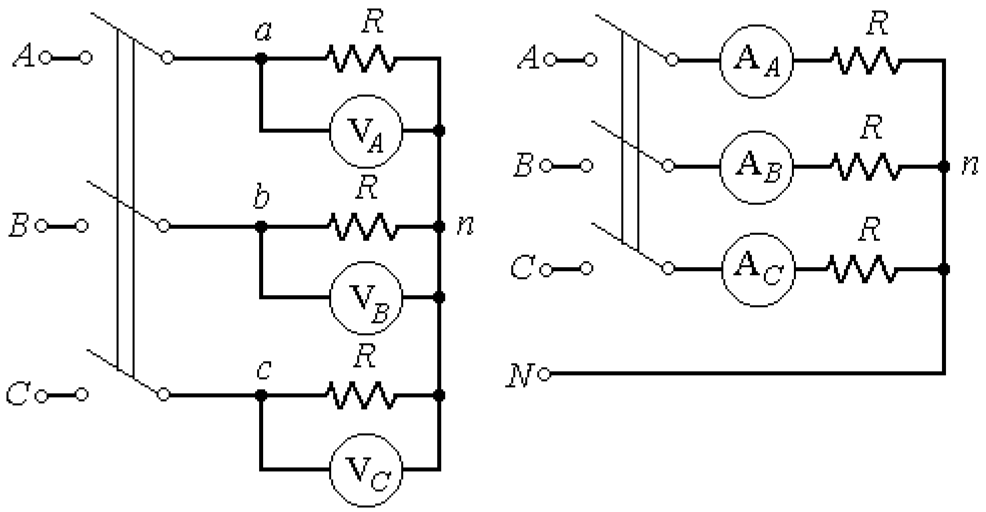
## Theoretical Background

In a three-phase circuit with the load connected as a star liner currents are equal to phase currents and linear voltages to the differences of phase voltages: $\dot{U}_{AB}=\dot{U}_A-\dot{U}_B$ . In symmetrical regime $U_L=\sqrt{3}U_{Ph}$ . A voltage between neutrals of a generator and load appears in an asymmetric three-phase circuit without a zero wire. In this regime phase voltages are different.

In a three-phase circuit with a neutral the current of this wire is not equal zero if the load is asymmetric, but all phase voltages are identical.

## Electrical Circuit

The electrical circuits investigated in this exercise are shown in Figs 4.1a (the circuit without neutral) and 4.1b (the circuit with neutral).




(a) (b)

Fig. 4.1

The circuits are supplied by a three-phase generator with symmetrical system of e.m.f. The effective value of phase voltage is 7 volts and frequency is 50 Hz. Parameters of the load correspond to the variant of the exercise, they are shown in Table 4.1. In symmetrical regime resistors of 1kΩ should be included in all branches of the circuit. The load is changed in a phase known as a “special phase”. In this phase we include resistors corresponding to the variant (Table 4.1), a capacitor and inductor. Also we investigate the open-circuit regime (the load in the special phase equals infinity), and the short-circuit regime (**only in the circuit without a neutral**).

In the circuit without a neutral we measure phase voltages with a voltmeter with limits of measurements of 20 V. Phase currents in this circuit are calculated according Ohm's law. The voltage between neutral points should be defined with the use of vector diagram. Also with the diagram we define the current of the special phase when a capacitor or inductor is included in the circuit. The phase angle of this current should be used in drawing of the diagram corresponding to circuits with the same loading and zero wire.

In the circuit including the neutral we measure phase currents with a amperemeter with limits of measurements of 200 mA. The current of the zero wire should be defined out of the vector diagram.

## Review Questions

Study theoretical material and answer the following questions:

1. A loading of what kind is called a symmetrical load?
2. Write equations relating linear and phase voltages for a circuit with symmetric and asymmetric load connected as a star.
3. Let's assume that we have a symmetric three-phase circuit loaded by three resistors. Draw the vector diagram corresponding to this circuit. Show the shift of point *n* (the neutral of the load) in the diagram if the load in phase *A* is increased (decreased).
4. Draw the vector diagram corresponding to asymmetrical circuit with a neutral loaded by three unequal resistors.

## Experimental Investigations

1. Measure the linear and phase voltages of the given three-phase generator. Write the values of these voltages to column "Notes" in tables 4.2 and 4.3.

Table 4.1

| **Variant** | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{R_1}$ | kΩ | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 |
| $\boldsymbol{R_2}$ | Ω | 680 | 680 | 680 | 680 | 680 | 470 | 470 | 470 | 470 | 470 |
| ***L*** | mH | 200 | 140 | 100 | 100 | 200 | 140 | 100 | 100 | 200 | 140 |
| ***C*** | mkF | 1 | 1.22 | 1.47 | 1 | 1.22 | 1.47 | 1 | 1.22 | 1.47 | 1 |
| **Special phase** | | *A* | *A* | *B* | *C* | *B* | *C* | *A* | *B* | *C* | *A* |

2. Assemble the circuit shown in Fig. 9.1 *a* and measure the voltages in a symmetric regime. Write the reading of voltmeters in the upper row of Table 4.1. Calculate the ratio $U_L/U_{Ph}$.

Table 4.2

| **Experimental data** | | | **Results of calculation** | | | | **The load of special phase** | **Notes** |
|---|---|---|---|---|---|---|---|---|
| $U_A$ | $U_B$ | $U_C$ | $I_A$ | $I_B$ | $I_C$ | $U_{nN}$ | | $U_{Ph}$ = V |
| V | V | V | mA | mA | mA | V | | $U_L$ = V |
| | | | | | | | 1kΩ | Symm. regime |
| | | | | | | | $R_1$ | A circuit without neutral |
| | | | | | | | $R_2$ | |
| | | | | | | | $L$ | |
| | | | | | | | $C$ | |
| | | | | | | | ∞ | |
| | | | | | | | 0 | **Short circuit** |

3. Measure the voltages in the three-phase circuit without neutral with the following loads in the special phase:

a. Decrease of the active load. Resistor *R* in the special phase should be changed for resistor $R_1 > R$;
b. Increase of the active load. Resistor *R* in the special phase should be changed for resistor $R_2 < R$;
c. Inductive load *L* in the special phase;
d. Capacitive load *C* in the special phase;
e. Infinite load in the special phase.
f. Zero load in the special phase. Remove the resistor out of the special phase and include in it a wire.

**The results obtained in experiments 3 and 4 write down to Table 4.2.**

4. Calculate the currents in all phases. Write down the results to Table 4.2.

5. Draw vector diagrams for all regimes. Here 7 vector diagrams should be drawn. Begin the drawings with a triangle of linear voltages. In this case the position of neutral point *n* can be found using a pair of compasses.

6. Define the voltage $U_{nN}$ between neutral points of the generator and load out of vector diagrams. The results write down to Table 4.2.
7. Assemble the circuit shown in Fig. 4.1 *b* and measure the current in symmetric regime. All currents should be identical and the current of neutral is equal to zero.
8. Measure all currents in the same regimes as in experiments 4 (**except of the short-circuit regime**). The results write down to Table 4.3.

**Table 4.3**

| Experiments and calculations | | | | The load of special phase | Notes |
|---|---|---|---|---|---|
| $I_A$ | $I_B$ | $I_C$ | $I_N$ | | $U_{Ph}$ = V |
| mA | mA | mA | mA | | $U_L$ = V |
| | | | | $R_1$ | A circuit with a zero wire |
| | | | | $R_2$ | |
| | | | | $L$ | |
| | | | | $C$ | |
| | | | | ∞ | |

9. Draw vector diagrams for all regimes. Here 5 vector diagrams should be draw
10. Analyze the diagrams and draw conclusions to this exercise.

## Laboratory exercise No. 5

## THREE-PHASE CIRCUIT WITH THE LOAD CONNECTED AS A TRIANGLE

**The objective of the exercise:**
Investigation of three-phase circuit with symmetrical and asymmetrical loads connected as a rectangle. Drawing of vector diagrams corresponding to experimental results.

### Theoretical Background

In a three-phase circuit with a load connected as a star linear voltage is equal to phase voltage, and linear current is equal to the difference of phase current, for example, $\dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA}$. If a load is a symmetrical one, $I_L = \sqrt{3} I_{Ph}$.

## Electrical Circuit

In the laboratory exercise the circuit presented in Fig. 5.1 is analyzed experimentally. The circuit is supplied by symmetric three-phase generator with phase voltage of 7 V and frequency of 50 Hz. Linear currents are measured by amperemeters with limit of measurements of 200 mA, phase currents should be calculated according Ohm’s law or obtained with the use of vector diagrams.

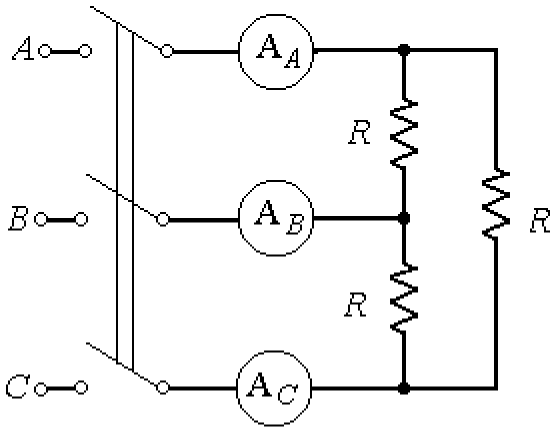
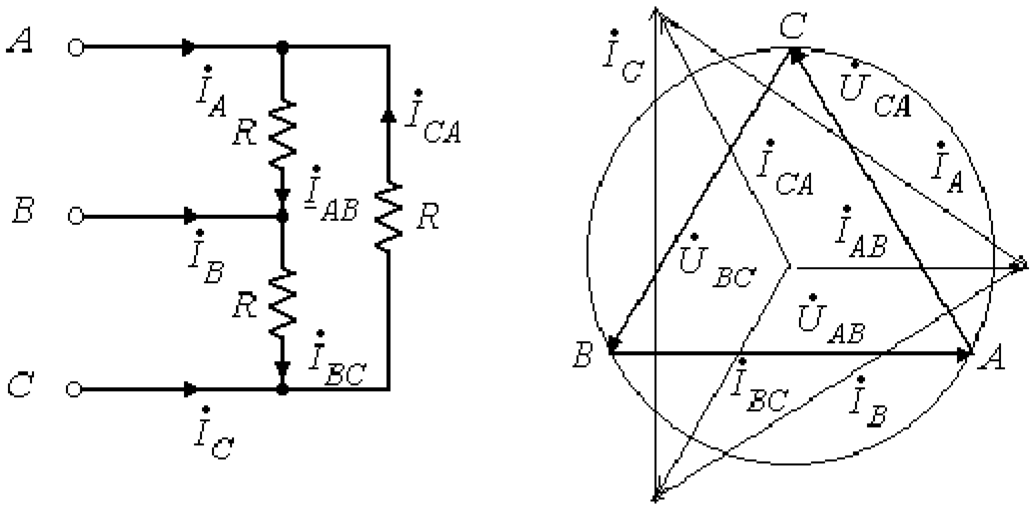



Fig. 5.1.

In symmetric regime 1 kΩ resistors are included into all phases of load. In asymmetric regimes we include several different loads into the special phase. These loads are resistors with resistance lesser $R_1 < R$ or greater $R_2 > R$ than 1 kΩ, an inductor, and capacitor. Parameters of the devices listed above are given in Table 5.1. Above that, we should break a wire in the special phase and break a line

Table 5.1

| **Variant** | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{R_1}$ | kΩ | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 | 4.7 | 2.2 |
| $\boldsymbol{R_2}$ | Ω | 680 | 680 | 680 | 680 | 680 | 470 | 470 | 470 | 470 | 470 |
| ***L*** | mH | 200 | 140 | 100 | 100 | 200 | 140 | 100 | 100 | 200 | 140 |
| ***C*** | mkF | 1 | 1.22 | 1.47 | 1 | 1.22 | 1.47 | 1 | 1.22 | 1.47 | 1 |
| **Special phase** | | *AB* | *AB* | *BC* | *CA* | *BC* | *CA* | *AB* | *BC* | *CA* | *AB* |




a b

Fig.5.2

## Review Questions

Study theoretical material and answer the following questions:

1. Show linear and phase currents in electrical circuit presented in Fig. 5.1.
2. Write equations relating linear and phase currents in symmetric and asymmetric regimes.
3. A vector diagram corresponding to a three-phase circuit with a load connected as a triangle is shown in Fig. 5.2.
4. Explain how we should change the diagram if phase *AB* is broken;
5. Explain how we should change the diagram if a linear wire *A* is broken;

## Experimental Investigations

1. Measure all linear voltages. The mean value of these voltages write down to Table 5.2.
2. Assemble electrical circuit according to Fig. 5.1 with parameters corresponding to the given variant and measure all currents in symmetric regime. Reading of the amperemeters write down to Table 5.2.
   a) Measure currents in asymmetric regimes:
   b) Reduce active load in the special phase ;
   c) Increase active load in the special phase ;
   d) Remove the load out of the special phase (break the special phase);
   e) Include an inductor into the special phase ;
   f) Include a capacitor into the special phase ;
   g) Break the line that is not connected with the special phase.

Results of all experiments wrote down to Table 5.2. Also write in this table the values of currents calculated according Ohm's law.

Table 5.2

| $I_A$ | $I_B$ | $I_C$ | $I_{AB}$ | $I_{BC}$ | $I_{CA}$ | $U_L =$ (V) |
|---|---|---|---|---|---|---|
| mA | mA | mA | mA | mA | mA | The load in the special phase |
| | | | | | | Active symmetric |
| | | | | | | Active, $R_1 < R$ |
| | | | | | | Active, $R_2 > R$ |
| | | | | | | The load is excluded ($R = \infty$) |
| | | | | | | An inductor |
| | | | | | | A capacitor |
| | | | | | | The line is broken |

3. Draw vector diagrams corresponding to all experiments.
4. Analyze the obtained results and formulate conclusions to the laboratory exercise.

## Laboratory exercise No. 6

## ELECTRICAL CIRCUITS WITH INDUCTION COUPLED ELEMENTS

**The objective of the exercise:**
Experimental investigation of a circuit with inductors connected in series aiding and opposing manners and in parallel aiding and opposing manner. Calculation of mutual induction of inductors and drawing of vector diagrams corresponding to experimental results.

### Theoretical Background

A real inductor without a core can be represented as a combination of active resistor and an ideal inductor connected in series. If we know the resistance of the active element (it can be measured by a voltmeter), other parameters of the circuit can be calculated using the formulas:

$$Z = \frac{U}{I},\ X = \sqrt{Z^2 - R^2},\ L = \frac{X}{\omega}$$

The total resistance of elements (the modulus of impedance) connected in series can be found as $Z = \sqrt{R_{Eq}^2 + X_{Eq}^2}$, where $R_{Eq} = R_1 + R_2$ and $X_{Eq} = X_1 + X_2$.

If current $i_1$ generates in the first inductor some magnetic flux and part $\Phi_{21}$ of this flux goes through the coils of the second inductor, and again, the part of magnetic flux of the second inductor $\Phi_{12}$ generated by current $i_2$ goes through the coils of the first inductor, it is possible to say that these two inductors are coupled magnetically. Magnetic fluxes $\Phi_{21}$ and $\Phi_{12}$ are called by mutual induction fluxes.

But some part of the total magnetic flux is associated only with inductor that generates this flux. This part is called the flux of self induction ($\Phi_{11}$ and $\Phi_{22}$ in an example with two inductors). For a tightly-wound coil of wire, composed of $w$ identical loops magnetic fluxes are:

$$\psi_{11} = w_1\,\Phi_{11},\quad \psi_{22} = w_2\,\Phi_{22},\quad \psi_{12} = w_1\,\Phi_{12},\quad \psi_{21} = w_2\,\Phi_{21},$$

mutual and self inductions:

$$L_1 = \left.\frac{\psi_{11}}{i_1}\right|_{i_2=0},\quad M_{21} = \left.\frac{\psi_{21}}{i_1}\right|_{i_2=0},\quad L_2 = \left.\frac{\psi_{22}}{i_2}\right|_{i_1=0},\quad M_{12} = \left.\frac{\psi_{12}}{i_2}\right|_{i_1=0}.$$

For linear magnetic circuits according the principle of reciprocity: $M_{12} = M_{21} = M$.

To account for orientations of magnetic fluxes in induction coupled elements the “rule of dots” was introduced. Terminals of different inductors are marked by dots (or by asterisks) if currents with same directions relatively to these terminals generate magnetic fluxes of mutual inductance with same orientations.

Voltages across terminals of inductors can be calculated according Faraday’s law:

$$u_1 = u_{1L} + u_{1M} = \frac{d\psi_{11}}{dt} \pm \frac{d\psi_{12}}{dt} = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt},$$
$$u_2 = u_{2L} + u_{2M} = \frac{d\psi_{22}}{dt} \pm \frac{d\psi_{21}}{dt} = L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt}.$$

Positive sign in these equations corresponds to identical orientation of currents relatively marked terminals (it is so called aiding connection of inductors). Negative sing tells us that inductors connected in opposing manner.

In circuits with harmonic currents and voltages:

$$U_{1L} = X_{L_1} I_1, \quad U_{1M} = X_M I_2, \quad U_{2L} = X_{L_2} I_2, \quad U_{2M} = X_M I_1,$$

where $X_M = \omega M$ is impedance of mutual induction.

Induction coupling of elements influences the magnitude of the circuit impedance. For example, if inductors are connected in series and in aiding or opposing manner we obtain:

$$X_{Eq}^{\text{Aiding}} = X_1 + X_2 + 2X_M = \omega\,(L_1 + L_2 + 2M) = \omega\, L_{Eq}^{\text{Aiding}},$$

$$X_{Eq}^{\text{Opposing}} = X_1 + X_2 - 2X_M = \omega\,(L_1 + L_2 - 2M) = \omega\, L_{Eq}^{\text{Opposing}}.$$

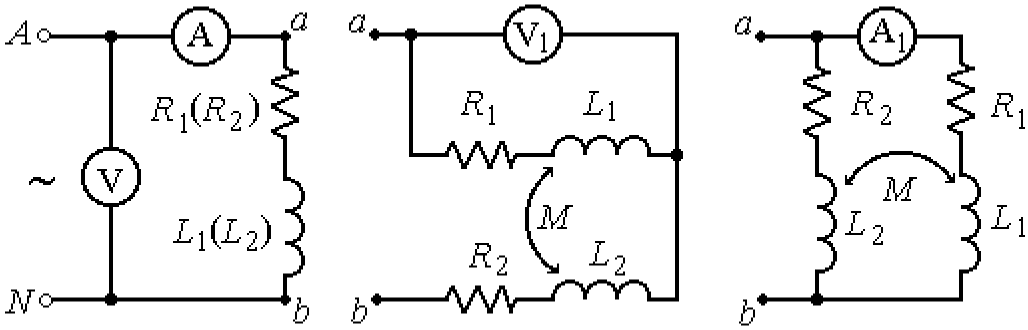
The impedance of mutual induction $X_M$ can be found experimentally with the use of these equations along with mutual induction $M$:

$$X_M = \frac{X_{Eq}^{\text{Aiding}} - X_{Eq}^{\text{Opposing}}}{4}, \quad M = \frac{X_M}{\omega}.$$

Dependence of impedance on the manner of connection can be used to define experimentally orientation of mutual fluxes. If the input current of the circuit is measured with different manners, the largest current corresponds to the opposing manner of element connection.

**Review Questions**

1. Write down in your text-book the Faraday’ law.
2. How it is possible to measure such parameters of inductor as *Z*, *R*, *X*, *L* with the use of ohmmeter, amperemeter, and voltmeter?
3. Explain the meaning of terms “connection in aiding manner” and “connection in opposing manner”
4. How we can find experimentally the manner in which inductors are connected?




*a* *b* *c*

Fig. 6.1.

The electrical circuit investigated in the exercise is shown in Fig. 6.1. The right-hand part of the circuit (i.e., the part of the circuit between points *a* and *b*) is changed in experiments. The left-hand part which includes amperemeter and voltmeter remains unchanged. The circuits are supplied by a three-phase source of e.m.f. Phase voltage of the source is applied to points *A* and *N*.

As induction coupled elements two identical inductors are used with 900 loops in each. To change the manner of connection we need to change terminals with which inductors are joined with the outer circuit. In this case the value of input current in the circuit depends on the manner of connection, so changing terminals we can decide if inductors are connected in aiding or in opposing manner.

In series with the first inductor we should connect a resistor $R_{Add}$ the value of which is given in Table 6.1. Sum of this resistor along with active resistance of the inductor forms active resistance $R_1$ included into electrical circuit (Fig. 6.1).

Table 6.1

| **Variant** | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{R_{Add}}$ | Ω | 10 | 22 | 47 | 100 | 150 | 220 | 150 | 100 | 47 | 22 |

To measure currents and voltages in the experiments we use amperemeters and voltmeters with limits of 200 mA and 20 V correspondingly.

## Experimental Investigations

1. Measure the active resistance ($R_2$) of the inductors with the ohmmeter. Connect in series with the first inductor active resistor $R_{Add}$, values of $R_2$ and $R_1 = R_2 + R_{Add}$ write to table 6.1.
2. Assemble the circuit shown in Fig. 6.1*a*. Measure currents and voltages in the circuit including into it the first inductor and then the second. Calculate parameters of inductors ($z$, $X$ and $L$) according the formulas given in paragraph “Theoretical Background”. Write the obtained results to table 6.2.
3. Assemble the circuit with inductors connected in series (Fig. 6.1*b*). Measure currents and voltages with inductor connected in aiding and opposing manners. Write the obtained results to Table 6.3.
4. Calculate the equivalent parameters of two inductors connected in series. Active equivalent resistance should be found as $R_{Eq} = R_1 + R_2$, other parameters according equations written given in paragraph *Theoretical Background*.
5. Calculate the impedance $X_M$ of induction coupling and mutual inductance of elements $M$.

Table 6.2

| **Number of inductor** | **Readings of devices** | | | **Results of calculations** | | |
|---|---|---|---|---|---|---|
| | $R$ | $U$ | $I$ | $z$ | $X$ | $L$ |
| | Ω | V | mA | Ω | Ω | Hn |
| 1 | | | | | | |
| 2 | | | | | | |

Table 6.3

| **Connection of inductors** | **Readings of devices** | | | | **Results of calculations** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | U | $I$ | $U_1$ | $R_{Eq}$ | $z_{Eq}$ | $X_{Eq}$ | $L_{Eq}$ | $X_M$ | $M$ |
| | V | mA | V | Ω | Ω | Ω | Hn | Ω | Hn |
| In aiding manner | | | | | | | | | |
| In opposing manner | | | | | | | | | |

6. Calculate all parameters of the circuits according to table 6.4. Draw vector diagrams for inductors connected in aiding and opposing manners using parameters from Table 6.4.

*Drawing of the diagrams should be done as following:*

a) Values of $R$ and $X$ for each inductor should be taken from Table 6.2, currents and $X_m$ from Table 6.3;
b) The first vector in a diagram is a vector of current with zero phase angle. After that is possible to draw vectors of voltages.
c) After drawing all vectors in a diagram draw in it the vector of input voltage by connecting corresponding points by a line. Draw also vectors of voltages across each inductor. Measure obtained voltages, write them to Table 6.4, and compare obtained data with results written to Table 6.3.

7. Assemble the circuit with inductors connected in parallel (Fig. 4.1*c*). Measure currents and voltages with inductor connected in aiding and opposing manners. Write the obtained results to Table 6.5.

8. Calculate the phase angle α between currents in inductors and current $I_2$ in the second inductor as:

$$\alpha = \operatorname{arctg}\frac{X_1 \pm X_M}{R_1} - \operatorname{arctg}\frac{X_2 \pm X_M}{R_2}, \quad I_2 = -I_1\cos\alpha + \sqrt{I^2 - I_1^2\sin^2\alpha},$$

here minus sign before term $X_M$ corresponds to aiding manner of connection and plus to opposing manner. Write the results of calculations to Table 6.5.

Table 6.4

| **Connection of inductors** | **Results of calculations** | | | | | | | **Results obtained from diagrams** | |
|---|---|---|---|---|---|---|---|---|---|
| | $I$ | $IR_1$ | $IX_1$ | $IX_M$ | $IR_2$ | $IX_2$ | $IX_M$ | $U$ | $U_1$ |
| | mA | V | V | V | V | V | V | V | V |
| In aiding manner | | | | | | | | | |
| In opposing manner | | | | | | | | | |

9. Calculate all parameters of the circuits according to table 6.6. Draw vector diagrams for inductors connected in aiding and opposing manners using parameters from Table 6.6.

*Drawing of the diagrams should be done as the following:*

a) Values of currents should be taken from Table 6.5, $R$ and $X$ for each inductor from Table 6.2, and $X_M$ from Table 6.3;

b) Begin drawing of the diagram with vectors of current. Along real axis draw vector $\underline{I}_1$ corresponding to a branch with the largest active resistance. Vector of current $\underline{I}_2$ goes after $\underline{I}_1$ with phase angle α. The sum of this current is equal to the input current *I*.
c) After currents draw all voltages in the circuit.
d) After drawing all vectors in a diagram draw in it the vector of input voltage by connecting corresponding points by a line. Measure this voltage in the diagram, write it down to Table 6.6, and compare obtained data with results written to Table 6.5.

Table 6.5

| **Connection of inductors** | **Readings of devices** | | | **Results of calculations** | |
|---|---|---|---|---|---|
| | $U$ | $I$ | $I_1$ | $\alpha$ | $I_2$ |
| | V | mA | mA | grad | mA |
| In aiding manner | | | | | |
| In opposing manner | | | | | |

Table 6.6

| **Connection of inductors** | $I_1R_1$ | $I_1X_1$ | $I_2X_M$ | $I_2R_2$ | $I_2X_2$ | $I_1 X_M$ | $U$ |
|---|---|---|---|---|---|---|---|
| | V | V | V | V | V | V | V |
| In aiding manner | | | | | | | |
| In opposing manner | | | | | | | |

10. Analyze the obtained results and formulate conclusions to the laboratory exercise.

## Laboratory exercise No. 7

## INVESTIGATION OF SERIES RESONANCE

**The objective of the exercise:**
Investigation of resonance in a circuit including inductor and capacitor connected in series. Registration of resonance obtained with variation of the generator frequency.

## Theoretical Background

Resonance is such a regime of electrical circuit including storages of energy (inductors and capacitors) in which phase shift between the input current and voltage equals zero.

If a circuit includes inductors and capacitors connected in series at some frequency reactive components of their impedances compensate each other. In this case we obtain series resonance in the circuit. Let's consider the simplest circuit with only one inductor characterized by parameters $R$ and $L$ and a single capacitor $C$ (Fig. 7.1). If the circuit is supplied by a harmonic source with voltage $u = U\sqrt{2}\sin(\omega t)$ and has the parameters

$$z = \sqrt{R^2 + X^2}, \quad X = X_L - X_C,$$

$$X_L = \omega L, \quad X_C = \frac{1}{\omega C}, \quad \phi = \arctan\frac{X}{R}.$$

a harmonic current $i = I\sqrt{2}\sin(\omega t - \varphi)$ with modulus $I = U / z$ flows in this circuit

It is clear that the phase difference is equal zero only if $X = 0$ or $X_L = X_C$, $\omega^2 LC = 1$. So to obtain resonance in the circuit we need to change frequency $\omega$, inductance $L$, or capacitance $C$. For given inductance and capacitance the resonance develops at frequency $\omega_{RES} = \frac{1}{\sqrt{LC}}$.

Effective values of voltages across the inductor $U_L = X_L I$ and across the capacitor $U_C = X_C I$ are equal at resonance, and for large quality factors $Q = \frac{\rho}{R} > 1$ their magnitudes are much larger than the magnitude of the input voltage $U$.

## Electrical Circuit

The electrical circuit of the exercise is shown in Fig. 7.1. The circuit is supplied by a generator with alternating frequency $f = \frac{\omega}{2\pi}$. Changes of frequency may result in changes of the generator voltage, in this case the effective value of voltage should be tuned according the requirements of the experiment.

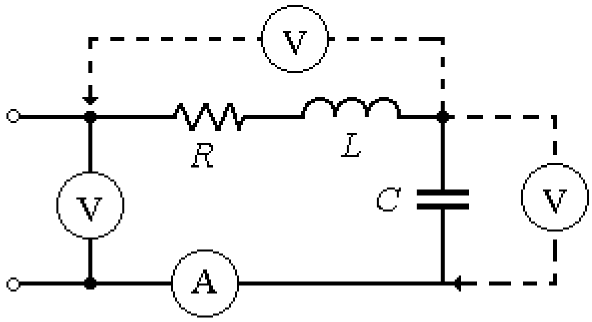



Fig. 7.1

The inductor is represented in the circuit as an active resistor and an ideal inductor connected in series. This active resistance *R* can be measured by the ohmmeter while parameters of circuit elements should be chosen according the variant of the exercise (Table 7.1).

Table 7.1

| **Variant** | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| ***U*** | V | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 1.5 |
| ***L*** | mH | 100 | 100 | 100 | 100 | 40 | 40 | 40 | 40 | 10 |
| ***C*** | mkF | 1 | 0.47 | 0.22 | 0.1 | 1 | 0.47 | 0.22 | 0.1 | 1 |

Measurements of all voltages and currents are performed by voltemeter with 20 volt limit of measurements and by amperemeter with 200 mA limit, correspondingly.

## Review Questions

Answer the following questions:

1. What regime of electrical circuit is called resonance?
2. Which parameters we can change in the circuit shown in Fig. 7.1 to obtain resonance?
3. How we can calculate resonance frequency for the circuit shown in Fig. 7.1? Calculate the resonance frequency for your variant.
4. How changing frequency and measuring the input current in the circuit we can tell that resonance is achieved?
5. Draw qualitatively vector diagrams for the circuit in Fig. 7.1 corresponding to frequencies $f = f_{RES}$, $f < f_{RES}$, and $f > f_{RES}$.

## Experimental Investigations

1. Measure the active resistance of the inductors with the ohmmeter. Calculate resonance frequency $f_{RES}$ and quality factor $Q$ corresponding to

the variant of exercise. Write parameters of the circuit and results of calculations to Table 7.2

2. Assemble the circuit shown in Fig. 7.1 and register the readings of devices with frequency $f = f_{RES}$, with two values of frequency greater than $f_{RES}$, and with two values lesser than $f_{RES}$. With the second voltmeter measure the voltages across the inductor and capacitor. Write down the obtained results to Table 7.3.
3. For all frequencies calculate voltages $U_R = RI$ and $U_L = 6.28fLI$ and phase angle $\varphi = \pm\arccos(U_R / U)$. Positive sign in this equation corresponds to frequency $f > f_{RES}$, negative to $f < f_{RES}$. The results of calculations write down to Table 7.3.
4. Using the data of Table 7.3 draw dependences $I(f)$, $\varphi(f)$, $U_L(f)$, and $U_C(f)$. The last two graphs draw in the same coordinate system.
5. Calculate currents and voltages in the circuit corresponding to three frequencies taken from Table 7.3 ($f < f_{RES}$, $f = f_{RES}$, and $f > f_{RES}$). The results of calculations write down to Table 7.4.
6. Draw three vector diagrams corresponding to data of Table 7.4.
7. Compare theoretical and experimental results and draw general conclusion to the exercise.

Table 7.2

| $R$ | $L$ | $C$ | $f_{RES}$ | $Q$ |
|---|---|---|---|---|
| Ohm | mH | mkF | Hz | – |
| | | | | |

Table 7.3

| $U$ | $f$ | $I$ | $U_{IND}$ | $U_R$ | $U_L$ | $U_C$ | φ |
|---|---|---|---|---|---|---|---|
| V | Hz | mA | V | V | V | V | grad |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

Table 7.4

| $U$ | $f$ | $X_C$ | $X$ | z | $I$ | $U_C$ | $U_L$ | $U_R$ | $U_{KIN}$ | φ |
|---|---|---|---|---|---|---|---|---|---|---|
| V | Hz | Ω | Ω | Ω | mA | V | V | V | V | grad |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

## Laboratory exercise No. 8

## INVESTIGATION OF PARALLEL RESONANCE

**The objective of the exercise:**

Investigation of resonance in a circuit with inductor and capacitor connected in parallel. Registration of resonance obtained with variation of the generator frequency.

### Theoretical Background

In the regime of resonance phase difference between the input current of the circuit and input voltage is equal to zero. Resonance in a circuit with reactive elements connected in parallel is called *parallel resonance*. The simplest circuit where resonance is possible to observe is a circuit including an inductor (parameters *R* and *L*) and capacitor (*C*) connected in parallel (Fig 8.1).

If harmonic voltage $u = U\sqrt{2}\sin(\omega t)$ is applied to this circuit, the current $i = I\sqrt{2}\sin(\omega t - \phi)$ in it is also a harmonic function. Here

$$I = yU, \quad y = \sqrt{g^2 + b^2}, \quad g = R / z_{\text{IND}}^2, \quad b = b_K - b_C,$$

$$b_C = \omega C, \quad b_{\text{IND}} = \omega L / z_{\text{IND}}^2, \quad z_{\text{IND}} = \sqrt{R^2 + (\omega L)^2}, \quad \phi = \arctan(b / g).$$

These equations illustrate the main property of the regime: the input current of the circuit has the same phase angle as the input voltage (φ = 0) and the imagine part of the input conductivity equals zero (*b* = 0). Resonance in such a circuit is possible to obtain by varying frequency of the input voltage or by varying capacitance or inductions of elements. For a circuit with given parameters *R*, *L*, and *C* the resonance frequency is defined by the equation

$$\omega_{\text{RES}} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}, \quad f_{\text{RES}} = \frac{\omega_{\text{RES}}}{2\pi}.$$

In the regime of resonance effective values of currents running in the inductor $I_{\text{IND}} = U / z_{\text{IND}}$ and in the capacitor $I_C = \omega CU$ can be much bigger than the effective value of the input current $I = Uy$.

## Electrical Circuit

The circuit investigated in the current laboratory exercise is shown in Fig. 8.1. It's supplied by the source of sinusoidal voltage the frequency of which is possible to change by a switch labeled *Frequency*. Effective value of the voltage can be changed with a switch *Amplitude*. In all experiments the magnitude of voltage should be kept at 6 V.

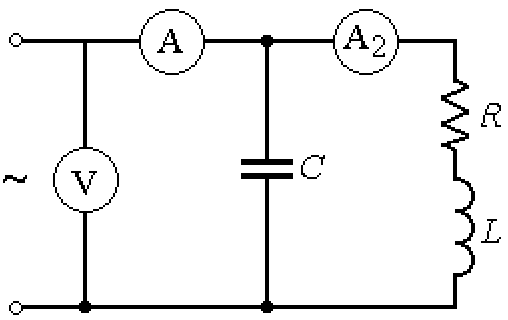



Fig. 8.1.

The inductor in the circuit is represented by active resistor and ideal inductance coil characterized by parameters $R$ and $L$ connected in series. Parameters $C$ and $L$ are given in Table 8.1, while active resistance $R$ of the inductor should be measured by the ohmmeter.

Table 8.1

| **Variant** | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|---|
| ***L*** | mH | 40 | 40 | 40 | 40 | 40 | 100 | 100 | 100 | 100 |
| ***C*** | mkF | 1 | 0.47 | 0.57 | 0.22 | 1.22 | 0.47 | 1 | 1.1 | 0.57 |

The effective value of input voltage is measured by voltmeter with 20 V limit of measurements, currents in inductor and capacitor are measured by amperemeters with limits of 200 mA. In this circuit the current in the capacitor can be calculated according Ohm's law.

## Review Questions

Study theoretical material and answer the questions:

1. What is the difference between parallel and series resonance?

2. Write the equation describing the regime of resonance in circuits drawn in Figs 8.1 and 8.2. Which parameters we can change to obtain resonance in these circuits?
3. How we can tell that the regime of resonance is achieved in the circuit if we can measure only the input current?
4. Draw three vector diagrams corresponding to the circuit in Fig. 8.1 if $f = f_{\text{RES}}$, $f < f_{\text{RES}}$, and $f > f_{\text{RES}}$

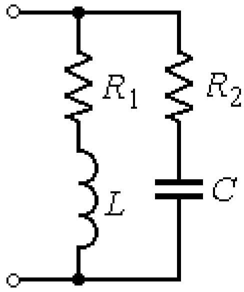



Fig. 8.2

## Experimental Investigations

1. Measure the active resistance $R$ of the inductor by the ohmmeter. Calculate resonance frequency and write this frequency and other parameters of the circuit to Table 8.2.

Table 8.2

| $R$ | $L$ | $C$ | $f_{\text{RES}}$ |
|---|---|---|---|
| Ω | mH | mkF | kHz |
| | | | |

2. Assemble the circuit shown in Fig. 8.1 and measure currents and voltages corresponding to five frequencies (for $f = f_{\text{RES}}$, for two values of frequency $f < f_{\text{RES}}$, and two values $f > f_{\text{RES}}$). Current in the capacitor calculate according Ohm's law. Phase shift between the input current and voltage calculate as

$$\phi = \arcsin \frac{I^2 + I_C^2 - I_{\text{IND}}^2}{2 I_C I}.$$

The results of calculation write to Table 8.3

Table 8.3

| $U$ | $f$ | $I$ | $I_{IND}$ | $I_C$ | φ |
|---|---|---|---|---|---|
| V | kHz | A | A | A | grad |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

3. Calculate currents in the circuit for three values of frequency ($f = f_{\text{RES}}$, $f < f_{\text{RES}}$, and $f > f_{\text{RES}}$) using equations given in paragraph *Theoretical background*. Phase shift between the input current and input voltage calculate as $\varphi = \text{arctg}(b/g)$. In calculations assume that exact values of $U$, $R$, $L$, and $C$ are given. The obtained results write to table 8.4 and compare with experimental data.

Table 8.4

| $U$ | $f$ | $b_C$ | $z_K$ | $y$ | $I$ | $I_{IND}$ | $I_C$ | φ |
|---|---|---|---|---|---|---|---|---|
| V | kHz | $\text{Ohm}^{-1}$ | $\text{Ohm}^{-1}$ | $\text{Ohm}^{-1}$ | A | A | A | grad |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

4. Draw vector diagrams according to Table 8.4.
5. Draw the functions $\varphi(f)$, $I(f)$, $I_{\text{IND}}(f)$, and $I_C(f)$ using the data from Table 8.4. All functions of current draw in the same coordinate system.
6. Compare theoretical and experimental results and draw conclusions to the laboratory exercise.

## Laboratory exercise No. 9

## INVESTIGATION OF TRANSIENTS IN A FIRST-ORDER CIRCUIT

**The objective of the exercise:**

Investigation of transients in a circuit including a capacitor (a first-order circuit). Comparison of experimental and analytical results.

### Theoretical Background

Transients are changes of currents or voltages in a circuit from one steady-state to other. Usually transients develop at switching. Power of real sources is always finite, so variables associated with changes of energy (current of the inductor or voltage across terminals of a capacitor) cannot be changed infinitely fast. Conservation of these variables is defined by laws of switching.

Solution to the system of differential equation describing the transient regime of a circuit consists of two components. It includes free component and induced component. For example, a current can be written as a sum $i(t) = i_{Ind}(t) + i_{Free}(t)$. The induced component is calculated as steady-state current or voltage corresponding to the regime after switching ($t = \infty$). The free component is a sum of exponential functions

$$i_{Free}(t) = \sum A_k e^{p_k t} \ ,$$

obtained with different roots $p_k$ of characteristic equation. Here coefficients $A_k$ are constant of integration calculated with the use of independent and dependent initial conditions.

The equation describing the free component in a circuit with only one inductor or capacitor includes only one exponential function and characteristic equation has one root. For example, a free component for the circuit drawn in Fig. 9.1*a* is the following:

$$i_{Free}(t) = Ae^{pt} .$$

The induced component for the circuit supplied by DC source is equal zero ($i_{Ind}(t) = 0$) because DC current does not flow through a capacitor. Before switching the capacitor has zero charge, and according to the second rule of switching

$$U_C(0_+) = U_C(0_-) = 0 .$$

So

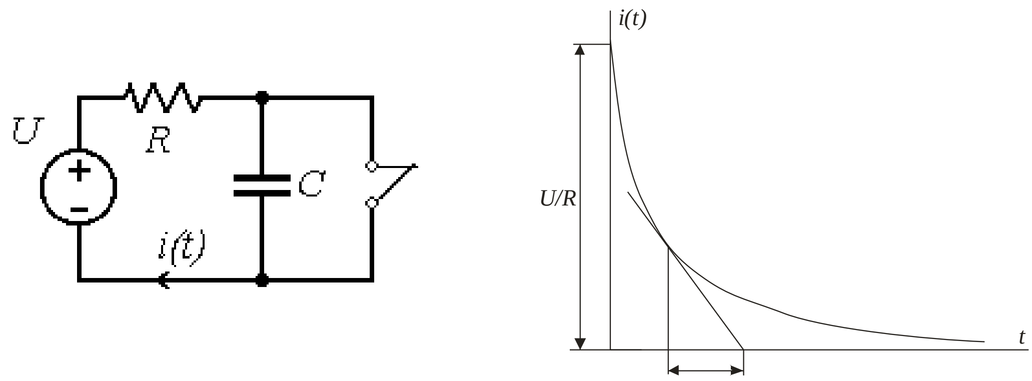
$$A = i_{Free}(0) = \left[ U - U_C(0) \right] / R .$$

In this case changes of current and voltage of the capacitor are described by equations

$$i(t) = \frac{U}{R} \exp(-t/RC) \qquad (9.1)$$

$$U_C(t) = U - i(t)R = U \left[ 1 - \exp(-t/RC) \right] .$$

Here $p = -t/RC$ is the root of characteristic equation and $\tau = 1/|p| = RC$ is the time constant. This constant can be found with the use of a tangent line to the graph obtained experimentally (Fig. 9.1*b*).




*a* *b*

Fig. 9.1

The equations describing current and voltage of the discharge can be obtained in the same way. They are the following:

$$i(t) = -\frac{U}{R}\exp(-t/RC), \quad U_C(t) = U\exp(-t/RC). \tag{9.2}$$

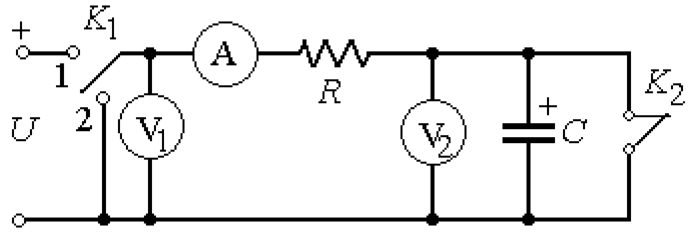



Fig. 9.2

Charge and discharge of capacitor are investigated with the use of the circuit shown in Fig. 9.2. The circuit is supplied from a DC source the voltage of which is given in Table 9.1. Also in this table values of *R* and *C* are written. Special attention should be paid to the polarity of capacitor.

Table 9.1

| Variant | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| *U* | V | 10 | 11 | 12 | 13 | 14 | 3 | 4 | 4 |
| *R* | kΩ | 100 | 122 | 133 | 147 | 200 | 22 | 22 | 33 |
| *C* | mkF | 100 | 100 | 100 | 100 | 100 | 470 | 470 | 470 |

Two voltmeters with 20 V limit of measurement are included into the circuit along with an amperemeter (200 mkA). The first voltmeter allows us to control the voltage of the source, the second voltmeter shows changes of voltage across terminals of capacitor. Current of charge and discharge is measured by the amperemeter.

Before development of transients key $K_1$ is connected with point 1 (the source is included into the circuit), key $K_2$ is closed. In this geometry of the circuit voltage across the capacitor equals zero ($U_C(0)=0$) and the current is $i_C(t)=U/R$. At the moment of switching key $K_2$ opens, voltage preserves its magnitude, and the current is still equal to the ratio of voltage and resistance ($i_C(t)=U/R$). So this formula gives us the current at the first moment after switching ($t=0_+$). After that the current and voltage should be registered simultaneously with equal intervals of time (usually 5 seconds).

Then the capacitor is charged, key $K_1$ is switched to position 2. Now we register discharge of capacitor in the same manner as it was done earlier.

## Review Questions

Study theoretical material and answer the questions:

1. How induced component, independent, and dependent initial conditions can be calculated?
2. How the time constant can be found theoretically and experimentally?
3. Calculate the root of characteristic equation, time constant, and the total time of transients for the circuit used in your experiments.

## Experimental Investigations

1. Assemble the circuit shown in Fig 9.2 and measure the value of current and voltage corresponding to $t=0_+$. In this experiment key $K_2$ is closed, key $K_1$ is in position 1. Write the current and voltage to Table 9.2.
2. Open $K_2$ and measure currents and voltages in the circuit with equal temporal intervals. Write the obtained results to Table 9.2.
3. Switch key $K_2$ to position 2 and measure currents and voltages of discharge. Write the results to Table 9.2.
4. Draw graphs of current and voltage (four graphs $i_C(t)$ and $U_C(t)$), define graphically the time constant for each graph, and calculate the mean value of the constant.
5. Calculate analytically the time constant as $\tau = RC$. Compare analytical and experimental results.

6. Calculate values of functions $i_C(t)$ and $U_C(t)$ for moments of time $t = \tau, 2\tau, 3\tau, 4\tau, 5\tau$. Draw the obtained points in the same coordinate system as experimental graphs.
7. Analyze the results and write conclusions to the exercise.

Table 9.2

| Experimental data | | | | | | Results of calculations | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Charge | | | Discharge | | | Charge | | | Discharge | | |
| *t* | *i*(t) | $u_C$(t) | *t* | *i*(t) | $u_C$(t) | *t* | *i*(t) | $u_C$(t) | *t* | *i*(t) | $u_C$(t) |
| s | mkA | V | s | mkA | V | s | mkA | V | s | mkA | V |
| 0 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |
| 15 | | | | | | | | | | | |
| 20 | | | | | | | | | | | |
| 25 | | | | | | | | | | | |
| 30 | | | | | | | | | | | |
| 35 | | | | | | | | | | | |
| 40 | | | | | | | | | | | |
| 45 | | | | | | | | | | | |
| … | | | | | | | | | | | |

## Laboratory exercise No. 10

## INVESTIGATION OF TRANSIENTS IN THE SECOND-ORDER CIRCUIT

**The objective of the exercise:**

Investigation of transients in a circuit including capacitor, inductor and resistor connected in series (a second-order circuit). Comparison of experimental and analytical results.

## Theoretical Background

An example of a first-order circuit is given in Fig. 10.1. After switching this circuit is described by the following equation written according the Kirchhoff's voltage law:

$$U_R + U_L + U_C = iR + L\frac{di}{dt} + \frac{1}{C}\int idt = E \qquad (10.1)$$

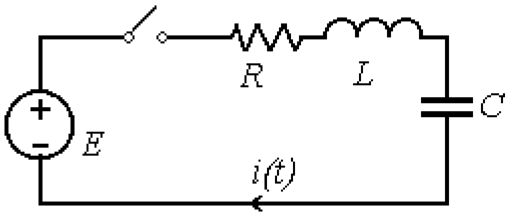



Fig. 10.1

After calculation of derivatives in the left and right-hand parts of this formula we obtain differential equation of the second order:

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0. \qquad (10.2)$$

In solution to this equation which is possible to obtain with classical approach the induced component is absent and form of the natural response depends on the roots $p_1$ and $p_2$ of characteristic equation:

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}. \qquad (10.3)$$

Here three variants are possible:

1. The roots are real, negative, and different ($p_1 \neq p_2 < 0$). In this case we observe overdumped response of the circuit.
2. Real, negative, and identical roots ($p_1 = p_2 = -\delta = -R/2L$) correspond to critical dumped transient process.
3. Complex conjugative roots $p_{1,2} = -\delta \pm j\omega_{Free}$ give us underdumped response. Here $\delta = \frac{R}{2L}$ is the coefficient of attenuation and $\omega_{Free} = \sqrt{LC^{-1} - \delta^2}$ is the cyclic frequency of oscillations. In this case changes of current are described by equation:

$$i(t) = A\exp(-\delta t)\cos(\omega_{Free}t + \alpha) = \frac{E}{\omega_{Free}L}\exp(-\delta t)\sin(\omega_{Free}t). \qquad (10.4)$$

In Eq. 10.4 the amplitude of oscillations is the following:

$$A = \frac{E}{\omega_{Free} L}$$

The current corresponding to underdumped response is presented in Fig. 10.2.

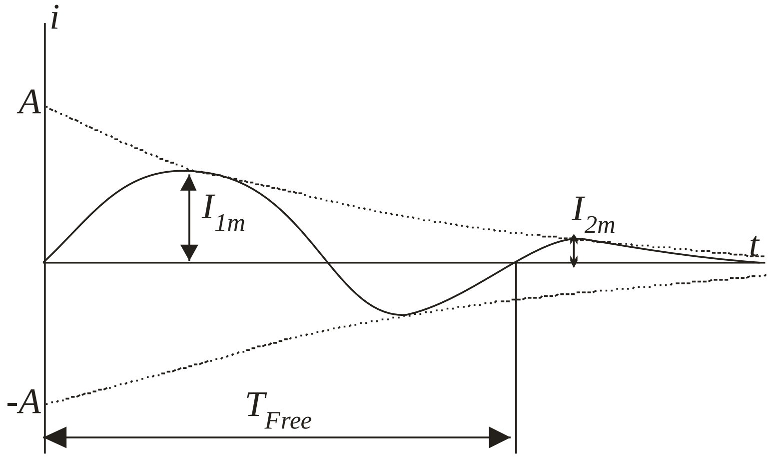



Fig. 10.2

To evaluate the rate of the current changes the decrement of oscillations Δ is usually used. This parameter is equal to the ratio of current magnitudes at two moments of time taken with the temporal interval between them of one period ($T_{\mathrm{Free}}$):

$$\Delta = \frac{i(t)}{i(t + T_{Free})} .$$

The logarithmic decrement is also possible to use for this purpose:

$$\Theta = \ln \Delta = \delta T_{Free} = T_{Free} / \tau_{Free} .$$

Here $\tau_{Free}$ is the time constant of the oscillation envelope (Fig. 10.2). To find the decrement experimentally we can use amplitudes of the first and second periods:

$$\Delta = \frac{I_{1m}}{I_{2m}} .$$

## Review Questions

Study theoretical material and answer the questions:

1. How is possible to write characteristic equation corresponding to differential equation of the second order?
2. What types of transients are possible in a second-order circuit including an inductor, resistor, and capacitor?

3. How is possible to calculate independent and dependent initial conditions in a second-order circuit?
4. Give the definition of the oscillation decrement and explain how this parameter is possible to find experimentally.

## Electrical Circuit

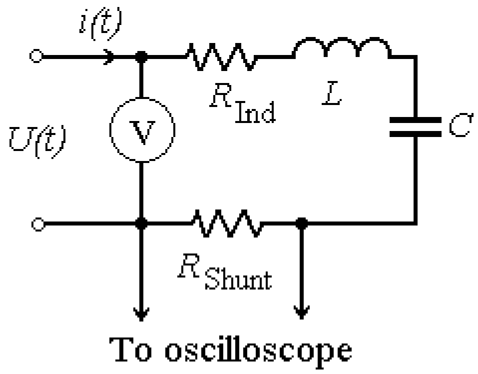
Transients in circuits including capacitors and inductors are so fast that they usually cannot be registered with amperemeters and voltmeters. But repetition of the process allows one to obtain representation of voltages in a screen of oscilloscope.

Table 10.1

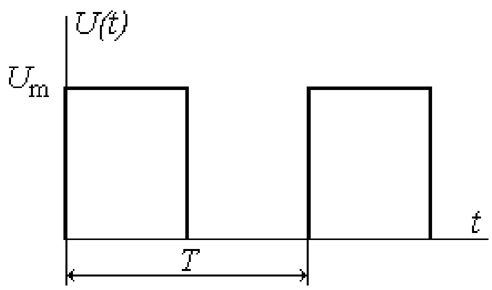
| Variant | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $L$ | mH | 10 | 10 | 40 | 40 | 40 | 40 | 100 | 100 |
| $C$ | mkF | 0.47 | 1 | 0.1 | 0.22 | 0.47 | 1 | 0.1 | 0.22 |
| $R_{shunt}$ | Ω | 22 | 22 | 100 | 100 | 100 | 100 | 220 | 220 |

Electrical circuit investigated in this exercise is shown in Fig. 10.3. The circuit is supplied from a source generating rectangular pulses of voltage with frequency of 200 Hz and amplitude of 8 V (Fig. 10.3*b*). This voltage is measured by a DC voltmeter with 20 V limits of measurements. Note that amplitude of 8 V corresponds to the effective value of 4 V registered by the voltmeter.

Parameters of the circuit should be taken from Table 10.1 according the variant of the exercise. $R_{Ind}$ is the internal resistance of the inductor. Voltage proportional to the current of transient process is registered by the oscilloscope connected in parallel with resistor $R_{Shunt}$.




*a*




*b*

Fig. 10.3

## Experimental Investigations

1. Assemble the circuit shown in Fig 10.3*a*. Values of $R_{shunt}$, $L$, and $C$ write down to Table 10.2. Set the frequency of pulses equal to 200 Hz and amplitude equal to 8 V.
2. Write the setting of oscilloscope (the time scale $m_t$ and scale of voltage $m_u$) to Table 10.2. Calculate the scale of current as $m_i=10^3 m_u/R_{shunt}$ and write it to the same table. Copy the graph of current from the screen of oscilloscope.
3. Using the graph of the current find the period of oscillation $T_{Free}$ and the time constant $\tau_{Free}$ (the time constant can be found with the use of a tangent line to oscillation envelope).
4. Measure amplitudes of oscillations corresponding to the first $I_{1m}$ and second $I_{2m}$ periods. Calculate decrement of oscillations Δ and logarithmic decrement of oscillations Θ.
5. Calculate the coefficient of attenuation $\delta = \tau_{Free}^{-1}$, cyclic frequency of oscillations $\omega_{Free} = 2\pi/T_{Free}$ and the integration constant $A = E/(\omega_{Free}L)$. Results of calculations write to Table 10.2.
6. According to Eq. 10.4 calculate values of the current at moments of time: t = 0, $T_{Free}$ /4, $T_{Free}$ /2, 3$T_{Free}$ /2, and $T_{Free}$, and draw corresponding points in the graph of current.
7. Analyze the results and write conclusions to the exercise.

Table 10.2

| Parameters | $E$ = 8 V | $L$ = | $C$ = | $R_{shunt}$ = | |
|---|---|---|---|---|---|
| Scales | $m_t$= ms/div | $m_u$ = ms/div | $m_i$= mA/div | | |
| Results from the graph | $T_{Free}$ = | $\tau_{Free}$ = | $I_{1m}$= | $I_{12m}$= | |
| Results of calculations | Δ = | Θ = | δ = | ω = | $A$ = |

## Laboratory exercise No. 11

## INVESTIGATION OF NONLINEAR DC CIRCUIR

**The objective of the exercise:**

Experimental investigation of nonlinear elements. Application of Zener diode to stabilize voltage. Calculation of equivalent characteristics of series-parallel connection of nonlinear elements.

## Theoretical Background

To analyze properties of nonlinear DC circuit we need to know volt-ampere characteristic (dependence of voltage on current or current on voltage) of all elements included into this circuit. Such a characteristic obtained with DC currents is called a static characteristic. If a circuit consists of several nonlinear elements with different types of connection, we can simplify it by representing groups of elements by equivalent volt-ampere characteristic.

For example, instead of several elements connected in series we can introduce an equivalent element with characteristic obtained on the base of Kirchhoff's voltage law:

$$U(I) = \sum_{k=1}^{n} U_k(I).$$

Here summation of voltages (points or characteristics) is done for several chosen values of currents.

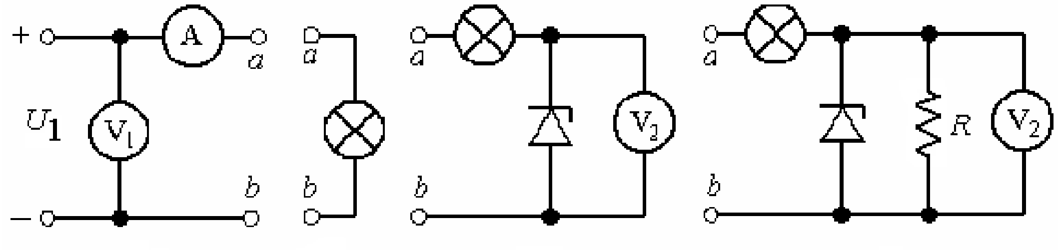
If elements are connected in parallel, according to Kirchhoff's current law we need to sum up currents with fixed values of voltages:

$$I(U) = \sum_{k=1}^{n} I_k(U).$$

Some nonlinear elements is possible to connect in such a way that the registered changes of voltage in the circuit would be much smaller comparing with changes of the input parameters. Such a circuit is called a voltage stabilizer. On the other hand, we can build a nonlinear circuit in which current in some branch does not depend on resistance of this branch. The devise is called a current stabilize.

## Electrical Circuit

Electrical circuit parts of which are shown in Fig. 11.1 *a, b, c,* and *d* is used in the current exercise. The voltage of the supply is varied in the interval $U_1 = 0 \div 15$ V. The part in Fig. 11.1 *a* is common for all experiments.




*a* *b* *c* *d*

Fig. 11.1

With contact *ab* of this part we connect a lamp (Fig. 11.1 *b*), Zener diode and a lamp (*c*), or the combination of these elements and a linear resistor (*d*). In corresponding experiments we can obtain volt-ampere characteristic of the lamp $U_L(I_L)$, of Zener diode $U_{ZD}(I_{ZD})$, and equivalent characteristic of the circuit $U_{Eq}(I_{Eq})$. In the last two experiments we can also register the dependence of voltage across the diode on the input voltage $U_2(U_1)$ in the regime of idling (Fig. 11.1 *c*) and with some load (Fig. 11.1 *d*). The resistance of the load and the voltage of operation point are given in Table 10.1. Currents and voltages should be measured with DC amperemeters and voltmeters with limits of 20 mA and 20V.

Table 11.1

| Variant | | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|---|
| *R* | kΩ | 1 | 2.2 | 1.68 | 2.53 | 3.2 | 3.67 | 1.22 |
| $U_{1P}$ | V | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

## Review Questions

Read the theory to the laboratory exercise and answer the following questions:

1. What element can be called *a nonlinear resistor*? Describe the properties of this element.
2. What laws are valid for nonlinear circuits?
3. How we can obtain equivalent volt-ampere characteristic of several nonlinear elements connected in series? In parallel?
4. Which element is possible to use for stabilization of voltage?
5. Which element is possible to use for stabilization of current?

## Experimental Investigations

1. Experimental registration of lamp characteristic. The circuit is shown in Fig. 11.1 *a* and *b*. The points of characteristic should be registered with equal intervals of current variations. The largest possible current is 35 mA. The results must be written to Table 11.2.
2. Experimental registration of Zener diode characteristic. The circuit is shown in Fig. 11.1 *a* and c. The points of characteristic should be registered with equal intervals of voltage variations. The largest possible voltage is 15 V. The results must be written to Table 11.2.
3. Experimental registration of equivalent characteristic for series-parallel connection of elements. The circuit is shown in Fig. 11.1 *a* and *d*. The points of characteristic should be registered with equal intervals of vol-

tage variations. The largest possible voltage is 15 V. The results must be written to Table 11.2.

4. In the same system of coordinates draw the graphs $U_L(I_L)$, $U_{ZD}(I_{ZD})$, and $U_{Eq}(I_{Eq})$ according to data written to table 11.2. In the same system draw the characteristic of linear resistor $U_R=RI_R$.
5. In the same system of coordinates draw the equivalent characteristic of a circuit obtained as a result of summation. Compare it with experimental characteristic (voltage $U_{1P}$).
6. Find in the graphs the currents $I_L = I_{Eq}$, $I_{ZD}$, $I_R$ and voltages $U_{JI}$, $U_{ZD}$, $U_R$ corresponding to the point of operation.
7. Calculate static resistance of the circuit in the operation point.
8. Draw two curves $U_2(U_1)$ obtained in the regime of idling and with the load $R$. For each curve find the intervals of stabilization and calculate the coefficient of stabilization

$$k_{CT} = \frac{\Delta U_1 / U_{1M}}{\Delta U_2 / U_{2M}},$$

here $\Delta U$ is variation of voltage in interval of stabilization, $U_{CP}$ is the mean voltage corresponding to this interval.

9. Write conclusions to the exercise.

Table 11.2

| *A lamp* | | *A Zener diode* | | | *Series-parallel connection* | | |
|---|---|---|---|---|---|---|---|
| $U_1 = U_L$ | $I = I_L$ | $U_1$ | $I = I_{ZD}$ | $U_2 = U_{ZD}$ | $U_1 = U_{Eq}$ | $I = I_{Eq}$ | $U_2$ |
| V | mA | V | mA | ***V*** | V | mA | ***V*** |
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## Laboratory exercise No. 12

## INVESTIGATION OF A SEMICONDUCTOR RECTIFIER

**The objective of the exercise:**
Experimental investigation of current rectification. Comparison of different rectifiers.

### Theoretical Background

Analysis of nonlinear AC circuits is carried out usually with the use of dynamic characteristics of elements included into the circuit. If the objective of investigation is calculation of the form of current or voltage functions, we need to use these characteristics to obtain the values of functions at definite moments of time. In circuits with low frequency currents static characteristics of elements are practically the same as their dynamic characteristics.

To rectify AC current the diodes are possible to use with typical dependence of current on voltage $i(u)$ shown in Fig. 12.1. Let's suppose that AC voltage is applied to such an element. To obtain corresponding dependence of the current on time we need to fix some moment of time $\omega t$ , find corresponding voltage $u(\omega t)$, then the current on characteristic of the diode, and obtain the point $i(\omega t)$. Repetition of this operation (we need ten or more points) allows us to obtain the current as a function of time. This function has asymmetric form, the spectrum of it includes the constant component, the first and high harmonics.

The amplitude of positive half-wave of the current is much larger than the amplitude of negative half-wave. This difference increases with increase of the voltage amplitude and ordinary it is so large that the negative half-wave of the current can be omitted. In this case the characteristic of an ideal diode can be use shown by dashed line in Fig. 12.1. The resistance of such a diode equals zero for a positive half-wave and infinity for a negative one. In the presents exercise we use semiconductor diodes with characteristics close to characteristics of an ideal devise.

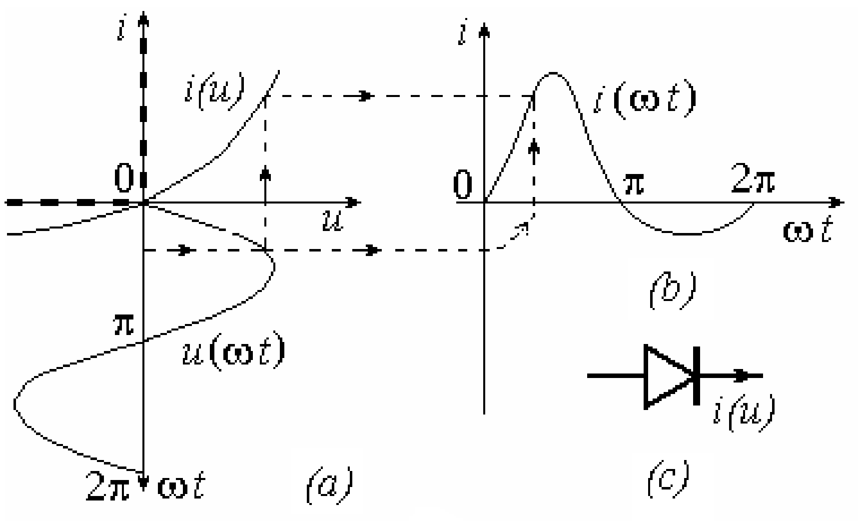


Fig. 12.1

Nonlinear circuits including diodes can be used to rectify AC currents. Quality of rectification is characterized by the coefficient of pulsation $k_P$ defined by the ratio of the AC component of the voltage ($U_2^{\sim}$) to its DC component $U_0$:

$$k_P = \frac{U_2^{\sim}}{U_0} = \frac{\sqrt{U_2^2 - U_0^2}}{U_0}. \qquad (19.1)$$

With decrease of the coefficient magnitude quality of rectification increases.

## Electrical Circuit

Electrical rectifies compared in the exercise are shown in Figs 12.2, 12.3, and 12.4. All circuits are supplied by a three-phase generator, but for the first two circuits only linear voltage is used. The effective value of this voltage is measured by voltmeter $V_1$. The rectifiers are loaded by resistor $R$ = 10 kΩ. DC voltmeter $V_0$ and AC voltmeter $V_2$ measure direct voltage $U_0$ and effective value of alternating voltage $U_2^{\sim}$. Limits of measurements for both voltmeters should be 20 V. The dependence of output voltage $u_2(\omega t)$ on time can be seen in the screen of oscilloscope. Zero level of voltage in screen is possible to register with opened switch K.

**1. Half-wave rectifier (Fig. 12.2)**

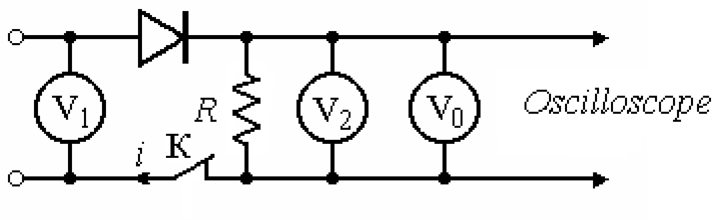


Fig. 12.2

During the positive half-period of the input voltage $u_1(\omega t)$ the diode is opened and the current flows through it. The diode is closed when the input voltage is negative, at what time the current is absent. So the pulsed current flows through the load, the value of this current changes but the direction remains the same. Picture in the screen of oscilloscope reproduces the form of current, and expansion of this function into Fourier series is described by the equation:

$$u_2(\omega t) = \frac{U_m}{\pi}\left[1 + \frac{\pi}{2}\sin(\omega t) - 2\left(\frac{1}{1\cdot 3}\cos(2\omega t) + \frac{1}{3\cdot 5}\cos(4\omega t) + ...\right)\right]. \quad (12.2)$$

The frequency of the first harmonic is the same as the frequency of input voltage ($f$ = 50 Hz), and the same is true for amplitudes $U_m$ of these two voltages. So $U_0 = U_m / \pi = 0.45\cdot U_1$, $U_2 = 0.5\cdot U_m = 0.707\cdot U_1$.

**2. Full-wave rectifier (Fig. 12.3)**

During the positive half-wave of the input voltage diodes $D_1$ and $D_2$ are opened, diodes $D_3$ and $D_4$ are closed. The reversed situation is observed when the input voltage is negative, here $D_1$ and $D_2$ are closed, $D_3$ and $D_4$ are opened. So the current through the load always has the same direction. Expansion of the corresponding voltage into Fourier series has the form:

$$u_2(\omega t) = \frac{2\cdot U_m}{\pi}\left[1 - 2\left(\frac{1}{1\cdot 3}\cos(2\omega t) + \frac{1}{3\cdot 5}\cos(4\omega t) + ...\right)\right]. \quad (12.3)$$

The frequency of the first harmonic of this signal is the same as the frequency of input voltage. The direct component of the output voltage and its effective value are the following:

$$U_0 = 2\cdot U_m / \pi = 0.9\cdot U_1, \quad U_2 = 0.707\cdot U_m = U_1.$$

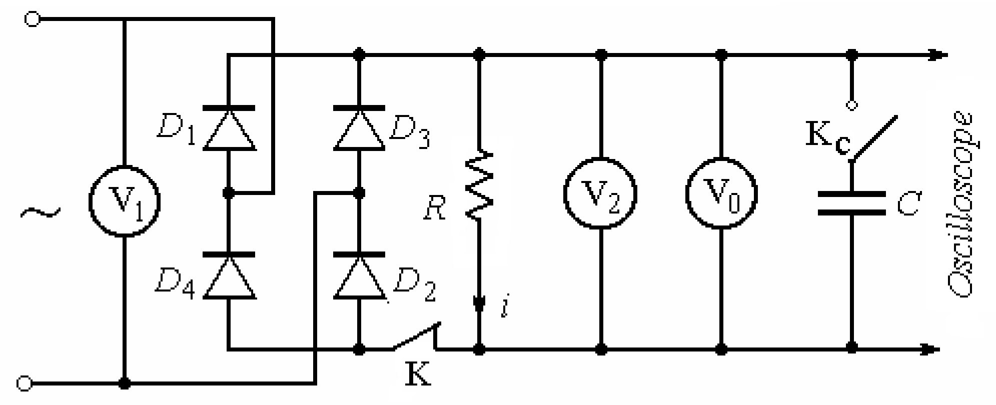


Fig. 12.3

To improve the quality of output current different filters can be used. The simplest of them is an ordinary capacitor through which DC current does not flow, while AC component becomes smoother due to cutting off the highest harmonics.

**3. Three-phase rectifier with a load connected to neutral (Fig. 12.4)**

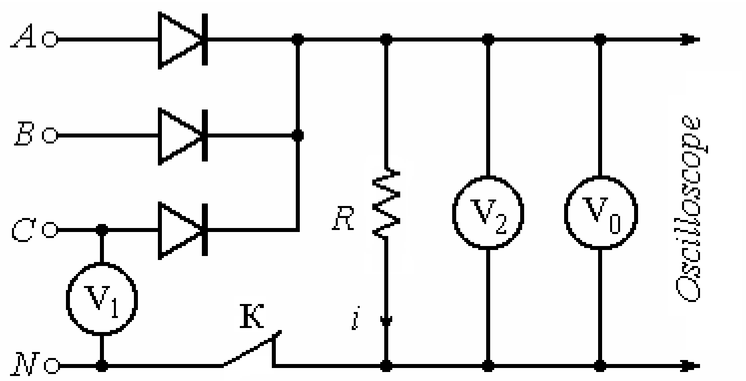


Fig. 12.4

Increase of rectified current quality is also possible by increasing the number of phases in a circuit. The diodes work in turn. The diode in a phase with large positive voltage is opened, while other diodes are closed. If we have, for example, the following voltage in phase A

$$\underline{U}_A = U_m / \sqrt{2} = a\underline{U}_B = a^2\underline{U}_C, \qquad (19.4)$$

when in the interval from $t$ = 0 to $t$ = $T$/12 the diode in phase $C$ is opened because in this interval $u_C > u_A > 0 > u_B$. During the next third part of the period the diode in phase $A$ is opened, then the diode in $C$ and so on. As a result we always have the current of the definite direction in the load. The maximum value of output voltage is $U_{\max} = U_m$, and the minimum $U_{\min} = 0{,}5 \cdot U_m$.

## Review Questions

Read the theory to the laboratory exercise and answer the following questions:

1. What is the difference between static and dynamic characteristics of nonlinear elements?
2. What kind of nonlinear element should be used to rectify AC current?
3. Draw the curve of the current in a load in the circuit shown in Fig. 12.2 if the circuit is supplied by sinusoidal source of voltage.

## Experimental Investigations

1. Assemble the circuit shown in Fig. 12.2. Obtain the curve of current in the screen. Open key *K* and draw the zero level in a semitransparent paper. Open the key and draw the form of the current. The readings of devices write to Table 12.1.

Table 12.1

| Rectifiers | | Experimental results | | | Results of calculations | |
|---|---|---|---|---|---|---|
| | | $U_1$ | $U_0$ | $U_2^{\sim}$ | $U_2$ | $k_P$ |
| | | V | V | V | V | - |
| Half-wave rectifier | | | | | | |
| Full-wave rectifier | Without filtration | | | | | |
| | $C$ = 1 mkF | | | | | |
| | $C$ = 1.47 mkF | | | | | |
| | $C$ = 1.69 mkF | | | | | |
| Three-phase rectifier | | | | | | |

2. Assemble the circuit shown in Fig. 12.3. Obtain the curve of current in the screen with opened key $K_C$. Draw the curve on semitransparent paper and write the readings of devices to Table 12.1. Repeat all operations with closed key $K_C$ and with different capacitors in the circuit.
3. Assemble the circuit shown in Fig. 12.4. Obtain the curve of current in the screen, draw it on semitransparent paper, and write the readings of devices to Table 12.1.
4. Calculate the effective value of input voltage according the equation $U_2 = \sqrt{U_0^2 + (U_2^{\sim})^2}$, calculate the coefficient of pulsations according Eq. 12.1 and write the results to Table 12.1.
5. Analyze the results of experiments and draw conclusions to the laboratory exercise.

### Laboratory exercise No. 13

## AN INDUCTOR WITH FERROMAGNETIC CORE IN A CIRCUIT OF AC CURRENT

**The objective of the exercise:**
Experimental investigation of nonlinear inductor.

### Theoretical Background

Inductors with ferromagnetic cores are often used in modern devices to amplify magnetic flux. Magnetic reluctance of such cores depends on the strength of magnetic field *H*. So an inductor with a core is a nonlinear element characterized by Weber-Ampere characteristic $\psi$(i).

If we assume that $\psi = BSw$ and $wi = Hl$ (*B* is the inductance of magnetic field, *S* is the square of core cross-section, *l* is the length of core), we obtain the curve $\psi(i)$ similar to the function *B*(*H*)., called the magnetic curve of material.

For ferromagnetic material these curves are possible to obtain experimentally. An example of the curve is shown in Fig. 13.1. The inductor is inertialess element, and if we apply sinusoidal voltage to it, we obtain aharmonic current in its wires. We assume that the voltage is described by the equation

$$u(t) = U_m \cos(\omega t).$$

In this case the magnetic flux can be found as

$$\psi(\omega t) = \int u dt = \Psi_m \sin(\omega t),$$

where $\Psi_m = U_m/\omega = U\sqrt{2}/\omega$ is the amplitude of magnetic flux.

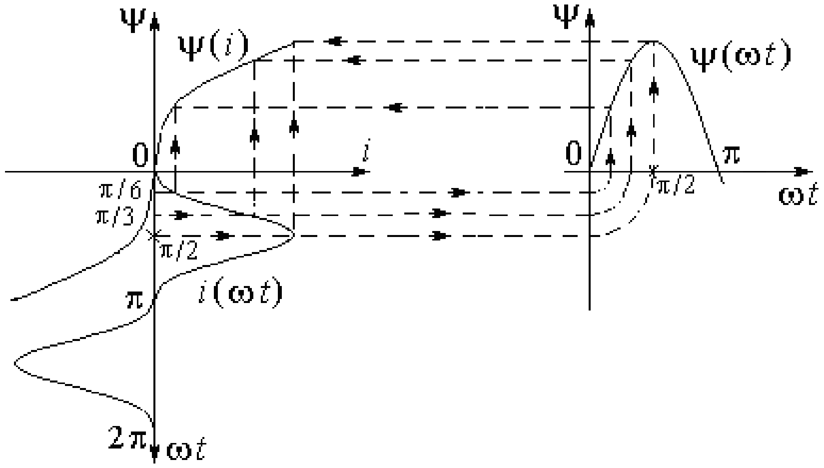

Fig.13.1

**Electrical Circuit**

To obtain the dependence of current on time $i(\omega t)$ the circuit is used shown in Fig. 13.2.

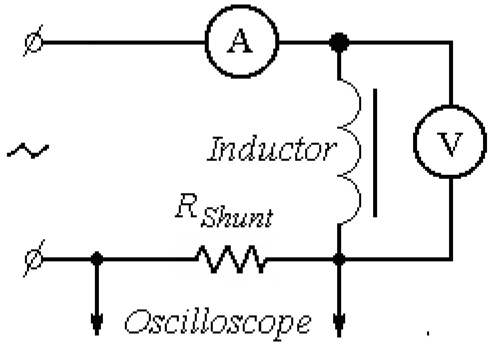


Fig. 13.2

The circuit is supplied by a source of sinusoidal voltage with frequency of 50 Hz and effective value of 24 V. As a nonlinear inductor the primary coil of a transformer is used (number of turns $w = 300$). The shunt is a linear resistor so the dependence of current through it on time $i(\omega t)$ is the same as dependence of voltage $u(\omega t)$.

Effective values of current and voltage is measured by AC amperemeter and voltmeter with limits of measurements of 200 mA and 20 V correspondingly.

The oscilloscope scales of voltage and time should be chosen in such a way that the curve fills the whole screen. The scale of current $m_i$ in mA/cm can be calculated as $m_i = 10^3 m_u /(10R_{\text{Shunt}})$, where $m_u$ is the scale of voltage.

**Review Questions**

Read the theory to the laboratory exercise and answer the following questions:

1. Why the dependence of magnetic flux on current is nonlinear for inductors with ferromagnetic core?
2. What material should be used to minimize losses in the core of inductor?

**Experimental Investigations**

1. Assemble the circuit presented in Fig. 13.2. Resistor $R_{\text{Shunt}}$ should be taken from Table 13.1 according the variant of the exercise.

Table 13.1

| Variant | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_{Shunt}$ | Ω | 47 | 57 | 69 | 80 | 55 | 65 | 90 | 100 | 110 | 122 |

2. Switch on the oscilloscope. Draw half-wave of the voltage on paper. Calculate the scale of current. This scale and reading of voltmeter and amperemeter write to Table 13.2.

Table 13.2

| $U$ | $I$ | $m_i$ |
|---|---|---|
| V | mA | mA/mm |
| | | |

3. Calculate the amplitude of magnetic flux $\Psi_m$ and draw the sinusoidal function $\psi(\omega t) = \Psi_m \sin(\omega t)$ with the same temporal scale as the scale of curve obtained with oscilloscope. In the same picture draw the curve of current $i(\omega t)$ in a way shown in Fig. 13.1.
4. Draw Weber-Ampere characteristic of the inductor $\psi(i)$ (the example is given in Fig. 13.1).
5. Write conclusions to the exercise.

**CONTENTS**

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