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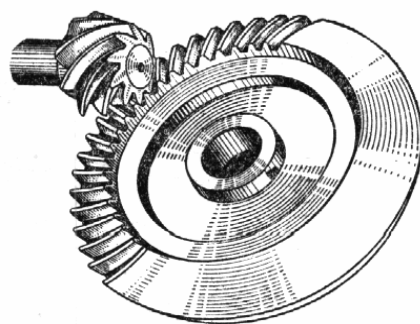
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**М.В.Горбенко, Н.Б.Кривцова**

**АНГЛИЙСКИЙ ЯЗЫК**

**ТЕОРИЯ МЕХАНИЗМОВ И МАШИН**

*ПРОФЕССИОНАЛЬНАЯ СОСТАВЛЯЮЩАЯ  
ЯЗЫКОВОЙ ПОДГОТОВКИ*



Учебно-методическое пособие для студентов  
машиностроительных специальностей

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Г 681 Теория механизмов и машин. Профессиональная составляющая языковой подготовки: Учебно-методическое пособие. Изд. 2-е исправленное и дополненное. – Томск: Изд-во ТПУ, 2006. – 110 с.

Пособие предназначено для выработки и совершенствования навыков профессионального общения инженеров-механиков на английском языке.

В пособии приведены аутентичные тексты, соответствующие дисциплине «Теория механизмов и машин», различного уровня сложности, отражающие современные направления в данной области как отечественных, так и зарубежных ученых, упражнения к текстам и словарная база.

Методика изложения, структура оформления оригинальных англоязычных источников сохранена.

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# I. STRUCTURAL AND KINEMATIC ANALYSIS OF LEVER MECHANISMS

## Terminology study of component parts of mechanisms

Classification of mechanisms and machines. Links. Kinematic pairs.  
Structural groups (Assur's groups).

Methodical materials:

- text;
- models of mechanisms;
- pictures with kinematic pairs, mechanisms, schemes of mechanisms;
- tasks on the theme.

## INTRODUCTION

### 1. Mechanics of Machines and their Basic Sections

Mechanisms are widely used in all branches of modern technology. The same types of mechanisms having the same computation and design methods are used in machines of various branches. The Theory of Machines and Mechanisms (TMM) develops general kinematic and dynamic methods of design applied to mechanisms of various branches of engineering. The special disciplines studying the theory and methods of mechanisms and machines' design, used in separate branches of engineering, widely use general methods developed in TMM, therefore TMM is a general-engineering discipline.

Base for TMM is physics, mathematics and theoretical mechanics. The TMM jointly with sciences "Strength of Materials and Theory of Elasticity", "Machine Elements" and "Production Engineering" is theoretical basis for current engineering industry.

The Theory of Machines and Mechanisms considers the scientific basis of machine design and their investigation methods. It is a science studying machines both as a complex unit and as separate links having the purpose of their analysis and providing people with knowledge for creation of new ones.

The purpose of creating of machines is to increase the productivity and to make worker's manual labour easier or to extend his or hers physical and intellectual opportunities. The term "machine" covers a great number and variety of objects used by a man for labour and physiological functions.

The following definition is common:

The **machine** is a device intended for transformation of energy,

materials and information.

The machines are means of production that use the forces nature of to facilitate work.

From the point of view of functions, carried out by machines, it is possible to classify all machines into the following groups:

- a) Power machines;
- b) Material-working machines;
- c) Information (supervisory control) machines;
- d) Cybernetic.

The **power machine** is a machine intended for conversion of energy from one form to another (transforming any kind of energy into mechanical one – the machine-engine, and vice versa – machine-generator).

The **material-working machine** is a machine for conversion of materials. The working machines are subdivided into transport and technological ones.

The transport machine is a working machine, in which conversion of materials consists only in changes of their position in space.

The **technological machine** is a working machine, in which the conversion of a material consists in changes of the form, location or properties of a material or machined object.

The **information machine** is a machine for conversion of information. These machines are subdivided into **supervisory control machines** and **computing** ones.

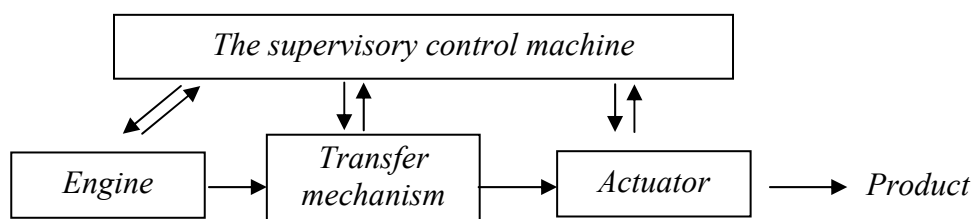
The **cybernetic machine** is a machine replacing or simulating various mechanical, physiological or biological processes, inherent to a man and animate nature.

If the processes of energy or materials conversion are carried out without direct participation of man, the machines performing such operations are named **automatic-machine devices**.

The set of the automatic-machine devices connected with each other and intended for performance of certain technological processes, is called an **automatic transfer line**.

The set of the machine-engine, transmission, working machine and supervisory control machine is called a **machine unit**.

It can be represented by the diagram:



The **actuator** consists of one or several mechanisms.

**Mechanism** – is a system of bodies intended for conversion of motion of one or several bodies into the required motion of other ones.

By the functional purpose all mechanisms are subdivided into the following kinds:

1. Mechanisms of engines and converters.
2. Transmissions.
3. Executive mechanisms (actuators).
4. Mechanisms of control and regulation.
5. Feeders, transporters, sorters of machined objects.
6. Mechanisms of the automatic count, weighing and packing.

The mechanisms which are included in the structure of a machine can incorporate firm, liquid, gaseous bodies, electrical and magnetic devices, which work according to the basic principles of mechanics, thermal physics, electrical engineering and electronics.

Therefore, the Theory of Machines and Mechanisms is the science studying the structure, kinematics and dynamics of mechanisms in connection with their analysis and synthesis.

## **2. Basic concepts and definitions**

All problems of the theory of mechanisms can be divided into two groups. The first group of problems is connected with the research of structural, kinematic and dynamic properties of mechanisms, i.e. with their analysis. The second group of problems deals with the design of mechanisms with the predetermined structural, kinematic and dynamic properties for realization of required movements, i.e. with the synthesis of mechanisms.

The mechanism's movement depends on their structure and applied forces. Therefore, in the study of the theory of mechanisms, it seems to be convenient to divide all problems of mechanisms' analysis into two parts:

1. Structural and kinematic analysis.
2. Dynamic analysis of mechanisms.

The *purpose of structural and kinematic analysis* of mechanisms is to study the mechanism's structural theory and research the motion of mechanisms from the geometrical point of view, without the consideration of the forces causing this motion.

The *purpose of dynamic analysis* is to study the methods of force definition, acting on the elements, forming a mechanism during their motion, and to study the interrelation between the motion of the elements, their masses and acting forces.

## EXERCISES TO THE CHAPTER “MECHANICS OF MACHINES AND THEIR BASIC SECTIONS”

### *1. Learn these words and phrases*

1. branch	- отрасль
2. mechanism	- механизм
3. kinematic	- кинематический
4. dynamic method	- динамический метод
5. apply	- применять
6. computation	- применение , вычисление
7. therefore	- поэтому; следовательно
8. jointly	- совместно
9. strength of materials	- сопротивление материалов
10. discipline	- дисциплина
11. theory of Elasticity	- теория упругости
12. producing Engineering	- технология производства
13. consider	- рассматривать
14. productivity	- производительность
15. manual labour	- физический труд
16. device	- механизм, прибор
17. carry out	- выполнять
18. power machine	- энергетическая машина
19. material working machine	- технологическая машина
20. supervisory control machine	- контрольно-управляющая машина
21. information machine	- информационные машины
22. cybernetic machine	- кибернетическая машина
23. conversion	- превращение
24. property	- свойство
25. location	- размещение
26. inherent	- присущий
27. animate nature	- живая природа
28. automatic transfer line	- автоматическая линия
29. automatic –machine device	- машина-автомат
30. transmission	- передача, коробка передач
31. working machine	- станок
32. machine unit	- машинный агрегат
32. actuator	- исполнительный механизм
33. converter	- преобразователь
34. executive mechanism	- исполнительный механизм
35. feeder	- питатель, подающий механизм

36. gaseous body	- газообразное тело
37. thermal physics	- тепловая физика
38. liquid body	- жидкое тело
39. deal with	- иметь дело с чем-либо
40. geometrical	- геометрический
41. interrelation	- взаимоотношения
42. acting force	- действующая сила
43. structural analyses	- структурный анализ
44. dynamic analyses	- динамический анализ
45. intend for	- предназначаться
46. investigation method	- метод исследования

**2. Read and translate the words having the same root:**

Act-action-active-activity-interaction  
 Different-difference-differentiate-indifference  
 Locate-located-location  
 Apply-application  
 Engine-engineering-engineer  
 Create-creation-creative-creativity  
 Performance-performing-perform  
 Similar-similarity-similarly  
 Technology-technological-technician-technique  
 Produce-production-productivity-productive  
 Conversion-convert

**3. Form words by means of the following prefixes or suffixes and translate them into Russian:**

-action: consider  
 -ly: general, final  
 -ion: rotate, act, direct  
 -al: rotation, dimension  
 -mis: understand  
 -ex: change  
 -ant: distance

**4. Give Russian equivalents to the following**

1. strength of materials	12. subdivide
2. facilitate	13. production engineering
3. actuator	14. apply
4. participation	15. material-working machine



5. location	16. transmission
6. deal with	17. simulate
7. acting forces	18. feeder
8. executive forces	19. gaseous body
9. liquid	20. therefore
10. inherent	21. consider
11. extend	22. intend for

**5. Give English equivalents to the following**

1. превращение	13. машина-двигатель (электрогенератор)
2. совместно с	14. выполнять
3. развивать	15. контрольно-управляющая машина
4. механизм, прибор	16. автоматическая линия
5. кибернетический	17. тепловая физика
6. основа	18. свойство
7. питатель	19. физический труд
8. геометрический	20. соединять
9. взаимоотношение	21. обеспечивать
10. замещать	22. методы проектирования
11. производительность	
12. дисциплина	

**6. Match the words and phrases in column A with those in column B**

A	B
1. machine elements	A. замещать
2. branches of engineering	B. средства
3. automatic counts	C. взаимоотношение
4. converter	D. автоматический приборный механизм
5. transmission	E. сортировочное устройство
6. replace	F. детали машин
7. automatic - machine- device	G. отрасли инженерии
8. sorter of machined objects	H. коробка передач
9. means	I. автоматический подсчет
10. interrelation	J. преобразователь

**7. Give definitions to the following words and phrases**

1. Machine unit
2. Mechanism

3. Cybernetic machine
4. Machine
5. Power machine
6. Automatic- machine device
7. Technological machine
8. Information machine
9. Automatic transfer line
10. Material-working machine

**8. Match the phrases in column A with those in column B**

<b>A</b>	<b>B</b>
1. From the point of view of functions	...to make worker's manual labour easier.
2. The working machines	...studying machines.
3. The purpose of creating of machines is	...we can classify all mechanisms into 6 groups.
4. The mechanism movement	...is to study the methods of force definition, interrelation between elements' motion.
5. The purpose of dynamic analysis	...are subdivided into 2 groups.
6. It is the science	...depends on their structure and applied forces.

**9. Fill in the gaps.**

The **machine** is a ..... intended for transformation of energy, ..... and information.

The **power machine** is a machine ..... for conversion of energy from one form to ..... (transforming any kind of energy into mechanical – the machine-engine, and vice versa – machine-generator).

The **material-working machine** is a machine for conversion of ..... . The working machines are ..... transport and ..... ones.

The transport machine is a ..... machine, in which conversion of materials consists only in ..... of its position in space.

The **technological machine** is a working machine, in which the conversion of a material consists in changes of the ..... , location or ..... of a material or machined object.

The **information machine** is a machine for conversion of ..... . These machines are subdivided into ..... and **computing** ones.

The **cybernetic machine** is the machine replacing or ..... various mechanical, physiological or ..... processes, inherent to the man and ..... nature.

If the processes of energy or materials conversion are carried out without ..... participation of ....., the machines performing such operations are named .....- **machine devices**.

The set of the automatic-machine devices connected with each other and intended for ..... of certain technological process, is called an ..... **transfer line**.

The set of the machine-engine, transmission, ..... machine and supervisory control machine is called a **machine** .....

**10. Translate the following sentences paying attention to the infinitive forms of the verbs:**

1. The purpose of machine creation is *to increase* the productivity.
2. It is possible *to classify* all machines into several groups.
3. The machines are means of production which use the forces of nature *to facilitate* work.
4. It seems to be convenient *to divide* all problems of mechanisms' analysis into two parts.

**11. Translate the sentences paying attention to the Participle II.**

1. Mechanisms are widely used in all branches of modern technology.
2. The machine is a device intended for transmission of energy, materials and information.
3. The working machines are subdivided into transport and technological ones.
4. The mechanisms which are included in the structure of a machine can incorporate firm, liquid, gaseous bodies.
5. All problems of the theory of mechanisms can be divided into two groups.

**12. Work in pairs.**

1. Imagine that you are at the exam in TMM. One of you is a student, the other one is a teacher. The teacher asks you about the main concepts of TMM.
2. You are a group of students visiting the plant. The mechanical engineer shows you different types of machines. You are interested in them and ask him detailed questions about each machine.
3. Create your own machine. Your task is to make up an advertisement for this machine.

## II. THE STRUCTURAL AND KINEMATIC ANALYSIS OF MECHANISMS

### 1. Structure of Mechanisms

Any mechanism consists of separate details. Each mobile detail or a group of details forming one rigid mobile system of bodies is called a **mobile link** of the mechanism.

All motionless details forming one motionless rigid system of bodies are called a motionless link or a rack.

The connection of two adjoining links that admits their relative movement, is called a kinematic pair.

Surfaces, lines and points of a link, on which it can adjoin another link and form a kinematic pair, are referred to as elements of a link.

The connected system of links that forms kinematic pairs is called a kinematic chain.

Motionless link of mechanisms is a **frame** or a **rack**. Now, let's consider mobile links and give the name to each link according to its movement:

1. **Crank** is a link, which performs complete circular movement around some centre (Fig. II.1).

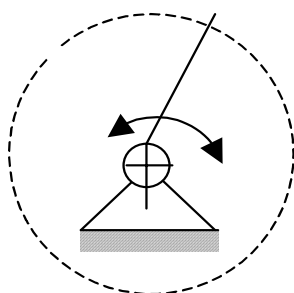


Fig. II.1.

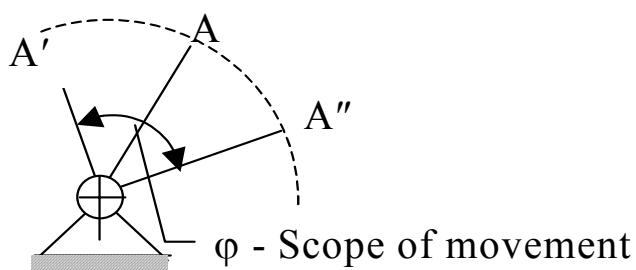
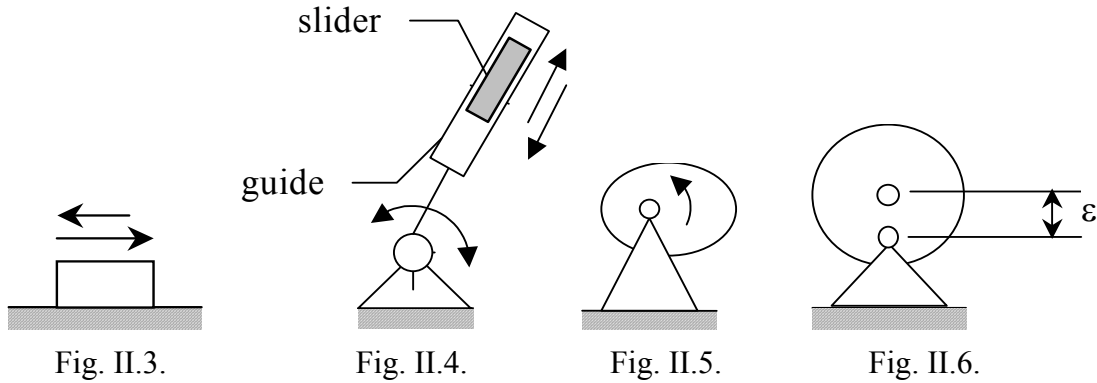


Fig. II.2.

2. **Rocker** is a link that performs incomplete rotary movement. (Fig. II.2).
3. **Connecting rod** is a link that performs complex movement (translational and rotary).
4. **Slider** is a link that performs only translational movement (Fig. II.3).
5. **Guide** is a mobile guide link (Fig. II.4).
6. **Cam** is a detail as a plate, disk, cylinder that has a shaped working surface used for realization of the movement of a follower of the executive mechanism according a given law (Fig. II.5).

7. **Eccentric** (special case of a cam) is disk or cylinder that fits eccentrically on the shaft, i.e. with the displacement of a geometrical axis of a disk concerning an axis of the shaft. Eccentric is a link that performs rotational movement but not around its geometrical axis (Fig. II.6).



The mobile and motionless links can be incorporated differently, but this connection always provides relative movement, forming kinematic pairs.

If an element of a kinematic pair (KP) is a surface (flat, cylindrical, spherical...), such KP is called a pair of **lower degree**. If an element of KP is a line or a point, such KP is called a **pair of higher degree**.

## 2. Classification of Kinematic Pairs

The possible connections of links in kinematic pairs are rather various. They can impose restrictions on relative movements of links and determine both the character and the quantity of movements.

The following kinds of classification of kinematic pairs are used:

- 1) by the relative character of movement - rotary "R" and translational "T" kinematic pairs;
- 2) by the character of kinematic pair's element (lower KP and a higher degree one);
- 3) by the number of motion freedom.

It is known, that any object in space has six degrees of freedom.

The movement of any object can be imagined as sliding lengthways and rotational around three arbitrarily chosen and mutually perpendicular axes  $X$ ,  $Y$ ,  $Z$ .

Thus, generally, each object has six kinds of possible independent motions in space – three rotational, around the axes  $X$ ,  $Y$ ,  $Z$ , and three translational movements, along the same axes (Fig. II.7). Therefore, if no conditions of constraint were not imposed on the movement of the first link of a kinematic pair which is accepted for an absolutely rigid body, the movement of such a link could be imagined as the movement consisting of

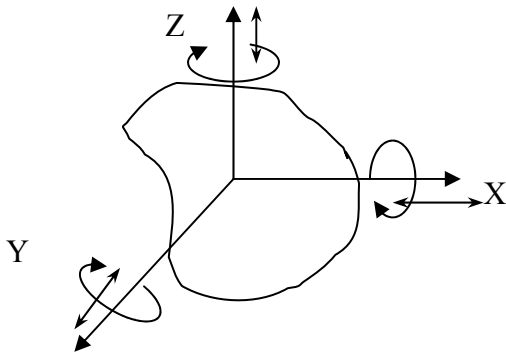


Fig. II.7.

six above-mentioned movements concerning the chosen system of coordinates connected to the second link. However, the link joined into a kinematic pair with another link, imposes conditions of constraint on relative movement of links. It is obvious, that the number of these conditions of constraint can be only an integer and should be less

than six. If the constraint number is equal to six, the links lose relative mobility and kinematic pair becomes a rigid connection of parts. At the same time the number of conditions of constraint cannot be less than one, in this case links do not contact and, therefore, kinematic pair ceases to exist. It will be two bodies in space moving independently one from each other.

“S” is used to designate the number of conditions of constraint.

$$1 \leq S \leq 5 \quad (I.1)$$

Hence, the number of degrees of freedom (commonly designated by H) of a kinematic pair link in relative movement can be expressed by the following relation:

$$H = 6 - S \quad (I.2)$$

It is obvious that

$$1 \leq H \leq 5 \quad (I.3)$$

"H" determines the quantity of possible movements, which can either be independent from each other, or connected by some additional geometrical dependencies imposing functional constraint between movements (for example as in kinematic pair a screw-lead nut).

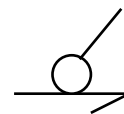
Let's consider for example, some kinematic pairs, for which the separate elementary movements are not functionally connected with each other.

**Sphere - plane** (Fig. II.8). As one can see links may perform five relative movements - three rotational and two translational.

$$RRRTT \quad H = 5 \quad S = 6 - H = 1.$$

Such kinematic pairs, having one constraint (S = 1), are called first class pairs and are designated  $P_1$  (five-mobile pair of higher degree).

The legend for such pairs in accordance with GOST 2770-68 is:



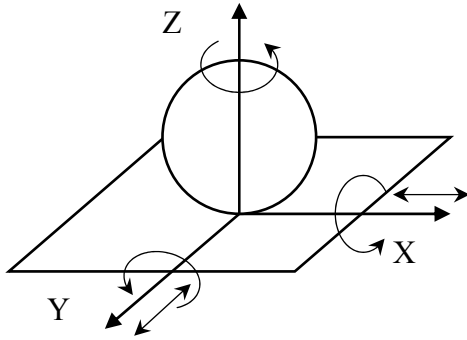


Fig. II.8.

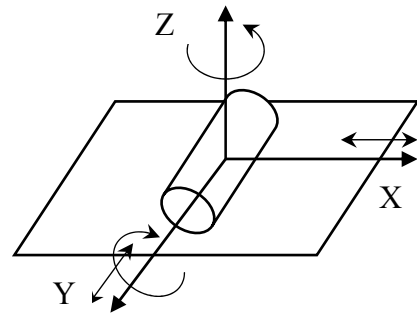


Fig. II.9.

All kinematic pairs are divided into classes (5 classes) depending on the number of constraint conditions.

**Cylinder - plane** (Fig. II.9).

$$RRTT \quad H = 4 \quad S = 6 - H = 2.$$

This is a second class kinematic pair of higher degree  $P_2$ .

The legend is

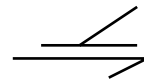


**Prism - plane** (Fig. II.10).

$$RTT \quad H = 3 \quad S = 6 - H = 3.$$

This is a third class kinematic pair  $P_3$  (lower).

The legend is



**Cylindrical KP** (Fig. II.11).

$$RT \quad H = 2 \quad S = 4.$$

This is the fourth class kinematic pair  $P_4$ .

The legends are

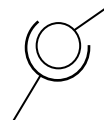


**Spherical KP** (Fig. II.12).

$$RRR \quad H = 3 \quad S = 6 - H = 3.$$

This is the third class kinematic pair  $P_3$ .

The legend is



**Spherical with a finger KP** (Fig. II.13).

$$RR \quad H = 2 \quad S = 4.$$

This is the fourth class kinematic pair  $P_4$ .

The legend is



**Cylindrical with collars** (Fig. II.14).

$$R \quad H = 1 \quad S = 5.$$

This is the fifth class kinematic pair  $P_5$ .

The legends are



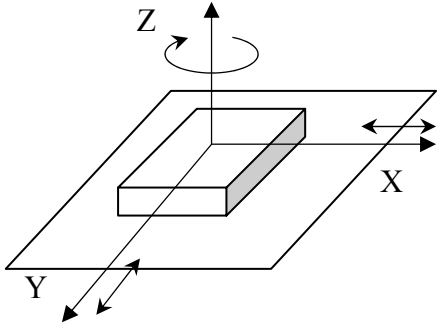


Fig. II.10.

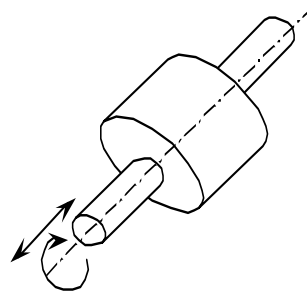


Fig. II.11.

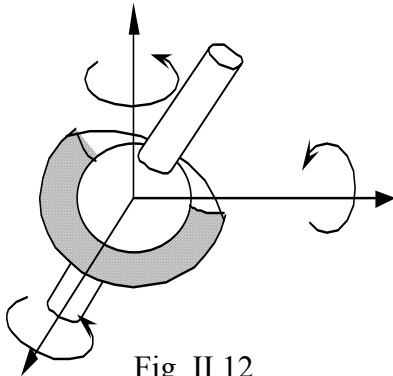


Fig. II.12.

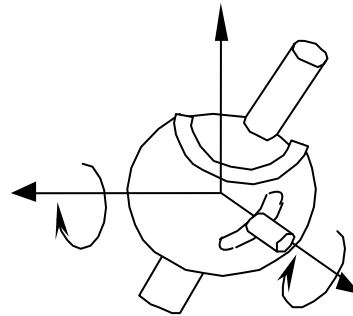


Fig. II.13.

**Rectilinear KP** (Fig. II.15).  $T = H = 1$   $S = 5$ . This is the fifth class kinematic pair  $P_5$ . The legends are

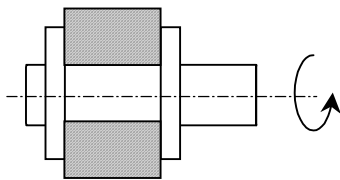
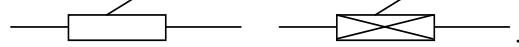


Fig. II.14.

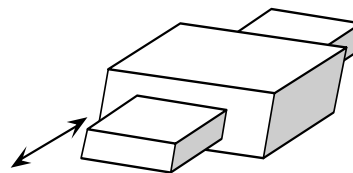


Fig. II.15.

**Screw KP** (Fig. II.16). This pair has only one independent motion. The angle of rotation  $\varphi$  responds to translation shift  $l$ , defined as

$$l = \frac{h}{2\pi} \varphi = C\varphi.$$

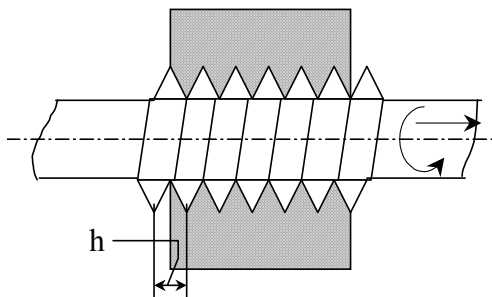
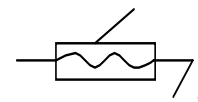


Fig. II.16.

Both movements are connected by single-valued dependence, therefore

$$H = 1, S = 6 - 1 = 5.$$

The legend is





**EXERCISES TO THE CHAPTER  
“THE STRUCTURAL AND KINEMATIC ANALYSIS OF  
MECHANISMS”**

***1. Learn these words and phrases***

1. mobile detail	- подвижная деталь
2. rigid	- жесткий
3. mobile link	- подвижное звено
4. motionless link	- неподвижное звено
5. frame	- стойка, рама, станина
6. adjoin	- примыкать
7. admit	- допускать
8. relative movement	- относительное движение
9. kinematic chain	- кинематическая цепь
10. according to	- согласно чему-либо
11. crank	- кривошип
12. circular movement	- круговое движение
13. scope of movement	- размах движения
14. rotary movement	- вращательное движение
15. rocker	- коромысло
16. connecting rod	- шатун
17. complex movement	- сложное движение
18. slider	- ползун
19. translational movement	- поступательное движение
20. bar link	- ограничивающее звено
21. reciprocating link	- возвратно-поступательное звено
22. cam	- кулачок
23. guide	- направляющая
24. cylinder	- цилиндр
25. eccentric	- эксцентрик
26. shaft	- вал
27. axis	- ось
28. pair of lower degree	- низшая кинематическая пара
29. pair of higher degree	- высшая кинематическая пара
30. impose restrictions	- вводить ограничение
31. determine	- определять
32. number of motion freedom	- степень подвижности
33. arbitrarily	- произвольно
34. mutually perpendicular	- взаимно перпендикулярный
35. thus	- так; таким образом

36. constraining condition	- ограничивающие условия
37. obvious	- очевидный
38. integer	- целое число
39. hence	- следовательно; отсюда
40. cease	- прекратить
41. screw – lead nut	- винт подачи
42. sphere – plane	- шар – плоскость
43. legend	- обозначение
44. cylinder – plane	- цилиндр – плоскость
45. prism – plane	- призма – плоскость
46. cylindrical KP	- цилиндрическая кинематическая пара
47. spherical KP	- сферическая кинематическая пара
48. spherical with a finger	- сферическая с пальцем
49. cylindrical with collars	- цилиндрическая с буртиками
50. rectilinear KP	- прямолинейная кинематическая пара
51. screw KP	- винтовая кинематическая пара

**2. Give the English equivalents of:**

Применение - применять

Действовать - деятельность- действие

Произвольный - произвольно

Вращаться – вращение - вращающийся

Перпендикуляр – перпендикулярно - перпендикулярный

Движение – подвижный - двигаться

Направление - прямой- направлять

Иметь отношение- связанный- отношение- связанный

**3. Give Russian equivalents to the following words and phrases:**

1. rack	10. prism plane
2. motionless link	11. guide
3. displacement	12. spherical with a finger
4. pusher movement	13. connecting rod
5. pair of higher degree	14. arbitrarily chosen
6. slider	15. cease to exist
7. screw	16. screw-lead nut
8. cylindrical with collars	17. impose restrictions
9. hence	18. scope of movement

**4. Give English equivalents to the following**

1. поступательная кинематическая пара	10. согласно чему-либо
2. эксцентрик	11. примыкать
3. шатун	12. неподвижное звено
4. низшая кинематическая пара	13. кулачок
5. сложное движение	14. кривошип
6. вал	15. ось
7. ограничивающие условия	16. размах движения
8. отсюда	17. таким образом
9. взаимно-перпендикулярный	18. число степеней свободы
	19. определять

**5. Match the words and phrases in column A with those in column B**

A	B
1) single-valued dependence	a) относительное движение
2) conditions of constraint	b) кулачок
3) cylindrical kinematic pair	c) однозначно-определенная зависимость
4) slider	d) винт подачи
5) displacement	e) допускать
6) cam	f) произвольно выбранный
7) screw-lead nut	g) вводить ограничения
8) spherical with a finger	h) подвижное звено
9) admit	i) стойка
10) relative movement	j) сферическая с пальцем
11) frame	k) шатун
12) connecting rod	l) кривошип
13) crank	m) размещение
14) arbitrarily chosen	n) ползун
15) impose restrictions	o) сдерживающие условия
16) mobile link	p) цилиндрическая кинематическая пара

**6. Give definitions to the following words and phrases**

1. pair of higher degree	8. kinematic chain
2. slider	9. eccentric
3. crank	10. motionless link
4. mobile link	11. rocker
5. rack	12. shaft

6. cam	13. complex movement
7. pair of lower degree	14. kinematic pair

**7. Answer the following questions**

1. Can a crank perform complete circular movement?
2. How many classifications of kinematic pairs do you know?
3. What does a mobile link mean?
4. How many degrees of freedom does any freely moving body have?
5. Can a mobile link perform translational movement?
6. Which KP is called a pair of a higher degree?
7. How many classes of kinematic pairs do you know (according to the number of constraint conditions)?

**8. Make up sentences using the following words and phrases.**

1. Are, following, of, kinds, the, pairs, used, kinematic
2. Translational, slider, a, performs, is, link, movement, only, that
3. Independent, kinds, body, in, six, each, has, space, a, possible, motions
4. Equal, six, lose, mobility, if, number, links, relative, the, is, constraint, to, the
5. Rack, mechanism, designated, link, motionless, of, was, as
6. A, complex, that, connecting, is, movement, rod, link, performs
7. Incorporated, mobile, links, be, the, motionless, and, can, differently
8. May, relative, links, perform, five, movements
9. Is, incomplete, link, a, performs, rotary, called, that, rocker, movement

**9. Write suitable words in each gap.**

1. **Crank** is a link, which performs complete ..... movement about some centre.
2. The link that performs ..... rotary movement is called .....
3. **Connecting rod** is a link that performs ..... movement (..... and rotary).
4. .... is a link that performs only translational movement.
5. Guide is a mobile guide -.....
6. Cam is a detail as a plate, ....., cylinder that has a shaped working surface used for realization of a .....movement of the ..... mechanism according a given law.
7. .... (cam special case) is disk or cylinder that fits eccentrically on the ....., i.e. with the displacement of a geometrical ..... of a disk concerning an axis.

8. Eccentric is a link that performs ..... movement but not around its .....axis.

**10. Match the phrases in column A with those in column B.**

A	B
1. If the constraint number is equal to six...	A. ...has only one independent motion.
2. Any freely moving object in space...	B. ...three rotational and two translation
3. All kinematic pairs are divided into 5 classes...	C. ...links lose relative mobility
4. Links can perform five relative movements...	D. ...has six degrees of freedom
5. At the same time...	E. ...the number of conditions of constraint cannot be less than one
6. Screw KP...	F. ...depending on the number of conditions of constraint.

**11. Use the words in brackets in the correct form: active or passive.**

1. Crank (to perform) complete circular movement.
2. The movement of any body (can, imagine) as sliding lengthways and rotational around axes.
3. If the constraint number is equal to six, links (lose) relative mobility.
4. The link (to perform) incomplete rotary movement (to call) rocker.
5. Motionless links of mechanisms (to design) as a frame or rack.

**3. Kinematic chains**

Any mechanism consists of some number of details connected with each other. The connection of links by kinematic pairs is called a kinematic chain.

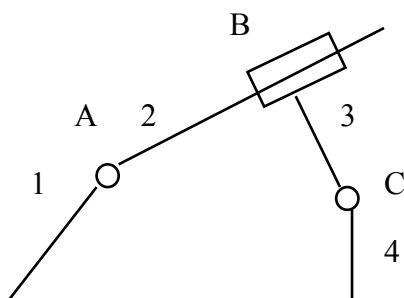


Fig. II.17.

Example of a kinematic chain consisting of four links is given in Fig. II.17.

The links 1 and 2 form rotary kinematic pair of 5-th class, 2 and 3 form rectilinear KP of 5-th class, 3 and 4 - rotary KP of 5-th class.

All kinematic chains (KC) are subdivided into single-stranded KC (simple) (Fig. II.18. b, c) and complex KC (Fig. II.18, a).

If each link of kinematic chain enters no more than into two kinematic pairs, such kinematic chain is called a single-stranded (simple) KC.

If at least one link enters into more than two KP, such kinematic chain is called complex KC.

All kinematic chains are divided into closed and open-ended KP.

If each link of a kinematic chain enters at least into two KP, such kinematic chain is called a closed KC (Fig. II.18c).

If at least one link in a kinematic chain enters only into one KP, such chain is called an open-ended KC (or open) (Fig. II.18a,b).

Kinematic chains can be either flat or spatial (three-dimensional).

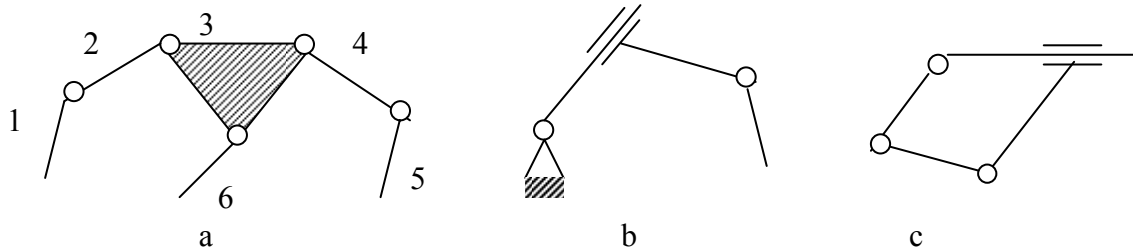


Fig. II.18. Examples of kinematic chains:  
a - complex KC; b - open KC; c - closed KC.

Besides that kinematic chains can be definite and indefinite.

If in a kinematic chain, at a given movement one or several links are named driver(s), other links (except for the rack) are named driven link(s), and they receive quite certain movement, such a kinematic chain is called a definite KC.

Now it is possible to give another definition to the notion "mechanism".

Mechanism is a kinematic chain, in which at the given movement of one or several links all the others perform the single-valued movements.

To study the movement of any mechanism, it is not enough to know its structure, i.e. number of links, number and classes of kinematic pairs, it is also necessary to know the sizes of separate links influencing movement, their mutual disposition etc (Fig. II.19.).

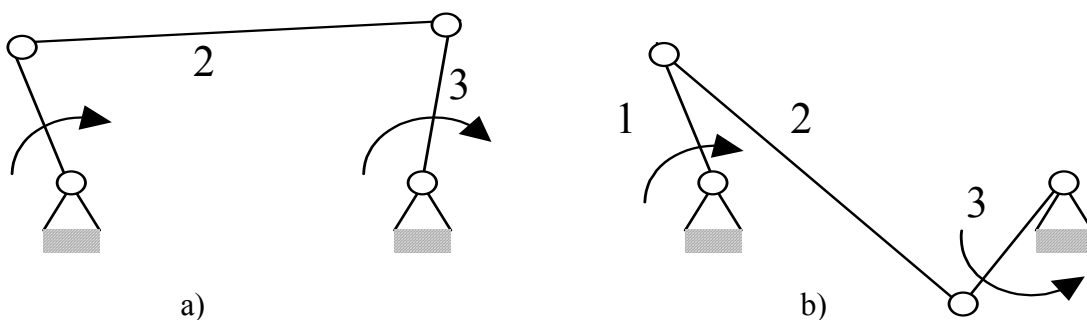


Fig. II.19. Different configurations of mechanisms having equal links

Therefore, studying the movement of a mechanism' links, one usually makes a kinematic scheme, which represents its kinematic model. Kinematic

model is diagrammed in a chosen scale with exact observance of all sizes and forms influencing the disposition, speed and acceleration of mechanism points.

All superfluties, not principal for movement, should be excluded in order not to complicate the drawing (Fig. II.20).

The kinematic scheme allows investigation of all basic kinematic parameters of the work of a mechanism. The number of drivers determinates the number of motion freedom, or number of its degrees of freedom concerning the rack.

We have defined that kinematic chains can incorporate a KP of 1 - 5th classes:  $P_1, P_2, P_3, P_4, P_5$ . Number of degrees of freedom of a separate link, which has not been connected with others, is equal to six. If number of links of a kinematic chain is  $k$ , then degrees of freedom of a kinematic chain is equal to  $6k$ . From this number, it is necessary to subtract the number of degrees of freedom, which are subtracted by link entrance into kinematic pairs. Therefore, the number of degrees of freedom  $H$  of kinematic chain is:

$$H = 6k - 5P_5 - 4P_4 - 3P_3 - 2P_2 - P_1. \quad (I.4)$$

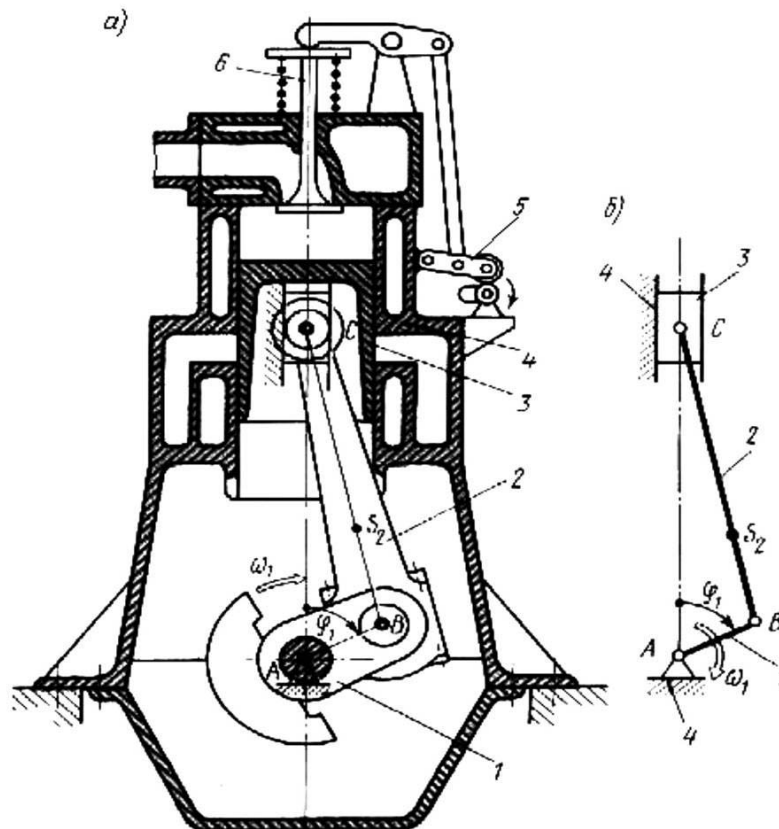


Fig. II.20. Layout (a) and kinematic scheme (b) of an internal combustion engine

Number of degrees of freedom of a mechanism concerning a motionless link (rack) is usually considered and designated as *W – motion freedoms*:

$$W=H-6=6(k-1)-5P_5-4P_4-3P_3-2P_2-P_1, \quad (I.5)$$

or

$$W=6n-5P_5-4P_4-3P_3-2P_2-P_1, \quad (I.6)$$

where *n* - is a mobile link number,

$P_5...P_1$  - are kinematic pair numbers according to its class.

This formula carries the name Somov's-Malyshev's formula (for the first time it was deduced by P.I.Somov in 1887y., but in a slightly different form and was advanced by A.P.Malyshev in 1923y.). This is a **formula of mobility** or a **structural formula of a general view kinematic chain**.

- In a specific case of a flat mechanism (when all links move parallel to one general planes) for the movement of all links of the mechanism 3 common restrictions are imposed. The structural formula will be of the next form:

$$W=(6-3)n-(5-3)P_5-(4-3)P_4-(3-3)P_3, \\ W=3n-2P_5-P_4. \quad (I.7)$$

P.L.Chebyshev deduced this structural formula for a general view of flat mechanisms in 1869.

- In a specific case of open-ended kinematic chains, the number of mobile links is equal to the number of kinematic pairs:

$$n=P_1+P_2+P_3+P_4+P_5. \quad (I.8)$$

Substituting this dependence in the Somov's-Malyshev's formula, one can obtain

$$W=P_5+2P_4+3P_3+4P_2+5P_1. \quad (I.9)$$

Let's consider some examples:

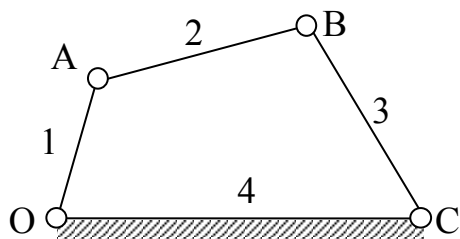


Fig. II.21.

1. Four-bar linkage (Fig. II.21.)

$$n=3 \quad P_4=0 \quad P_5=4 \\ W=3 \cdot 3-2 \cdot 4-0=1.$$

Hence, it is enough for such mechanism to initialize motion of one link and all others will have quite certain movements.

2. Five-link chain (Fig. II.22.).  $n=4$   
 $P_4=0 \quad P_5=6$

$$W=3 \cdot 4-2 \cdot 6-0=0.$$

It means that it is not the mechanism, it is the rigid construction - **girder**.



3. Five-link chain (Fig. II.23.).  $n = 4$   $P_4 = 0$   $P_5 = 5$

$$W = 3 \cdot 4 - 2 \cdot 5 - 0 = 2.$$

In order for such kinematic chain to become a mechanism, it is necessary to initialize motion to its two links and only then all others receive the certain movement.

Therefore, the number of motion freedom is a number of independent parameters, for which it is necessary to preset values in order for a given KC to be a mechanism.

One can see that Chebyshev's formula incorporates only  $P_4$  and  $P_5$

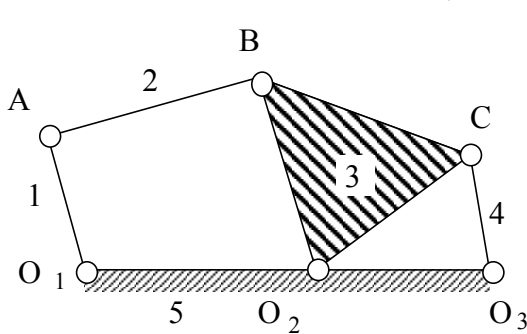


Fig. II.22.

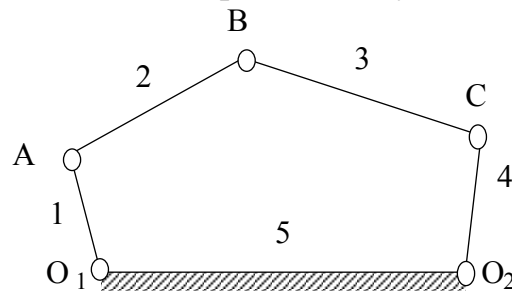


Fig. II.23.

kinematic pairs and it means that any flat mechanism may be created by only 4th and 5th classes kinematic pairs.

Let's consider two more examples:

1. Articulated parallelogram (Fig. II.24.).

$$n = 4 \quad P_4 = 0 \quad P_5 = 6$$

$$W = 3 \cdot 4 - 2 \cdot 6 - 0 = 0.$$

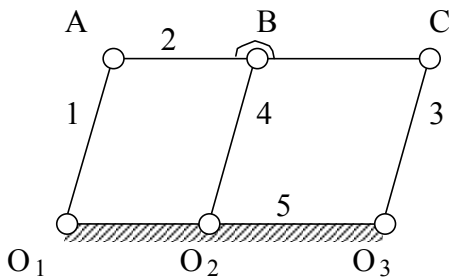


Fig. II.24.

According to Chebyshev's formula the given construction is motionless, but this fact does not correspond to the reality, as at equality of links 1, 3, 4 and pairwise equality of  $O_1O_2$  and  $AB$ ,  $O_2O_3$  and  $BC$  this is an articulated parallelogram. Having removed link 4 with its kinematic pairs  $B$  and  $O_2$ , movements of links 2 and 3 with driver link 1 does not change at all.

$$n' = 3 \quad P_4 = 0 \quad P_5 = 4. \quad W = 3 \cdot 3 - 2 \cdot 4 = 1.$$

Links which are not influenced by the driven links movement laws are called **passive**. Link 4 is passive.

2. Cam mechanism with translating pusher 3 and intermediate roller 2 (Fig. I.25.).

$$n = 3 \quad P_5 = 3 (R_{1,4}, R_{2,3}, T_{3,4}) \quad P_4 = 1 (RT_{2,1})$$

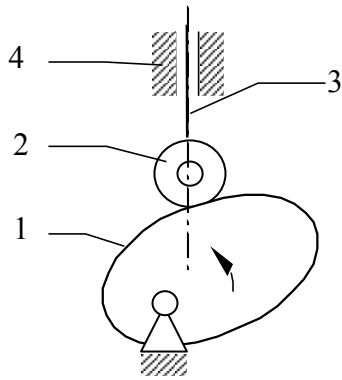


Fig. II.25.

$$W = 3 \cdot 3 - 2 \cdot 3 - 1 = 2.$$

Having removed a roller 2 (or having fixed it motionlessly to the pusher 3) the movement of a driven link - follower 3 will not change.

$$n' = 2; P_5 = 2 (R_{1,4}, T_{3,4}); P_4 = 1 (RT_{2,1})$$

$$W = 2 \cdot 3 - 2 \cdot 3 - 1 = 1.$$

Similar passive links are applied to reduce the loading of mechanism links, help the mechanism of an uncertain position, or (as in

the last example) to decrease the friction force (by replacement of sliding friction to rolling friction), which results in improvement of mechanism performance.

#### 4. Classification of mechanisms

There are exceedingly a lot of types of mechanisms which are used in the modern technology. The study of their various properties with reference to each separate mechanism is practically impossible and inexpediently by virtue of variety of existing kinds of mechanisms, that is why, not only existing kinds of mechanisms are used in engineering but they are constantly being improved and created. Therefore, for systematic study of mechanisms it is convenient to adhere to some certain classification and to study general properties intrinsically to the given class of mechanisms.

Some general properties of mechanisms inherent to separate classes are usually put as a basis of one or another classification of mechanisms. The following classification types are mostly applied:

1. By constructive design:
  - a). lever mechanisms;
  - b). connecting rod-crank mechanisms;
  - c). crank-and-slider mechanisms;
  - d). rocker mechanisms;
  - e). gears units;
  - f). cam mechanisms;
  - g). frictional mechanisms etc.
2. By functional purpose (pumps, conveyors, internal combustion engines etc.).
3. By structural attributes.

The Russian scientist professor Leonid Vladimirovich Assur offered the last classification in 1914 – 1917.

He thought the formation of a mechanism as a consecutive attachment to a driver link and a rack of kinematic chains, with a zero degree of mobility, or as it is now common to name, the attachment to the initial mechanism of Assur's groups (groups of links with a zero degree of mobility).

**Initial mechanism** is a mechanism consisting of one link for which the law of motion is given, and connected by kinematic pair with a frame (Fig. II.26).

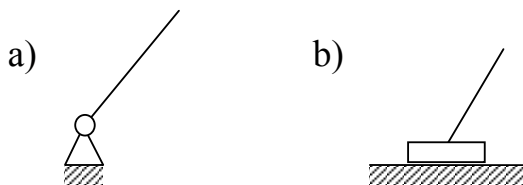


Fig. II.26. Initial mechanisms

For both mechanisms represented in Fig. II.26

$$n = 1, \quad P_5 = 1.$$

The **structural group (Assur's group)** is the kinematic chain having a zero motion freedom.

$$W=0.$$

For planar mechanisms consisting only from kinematic pairs of 4th and 5th classes the equation can be written as:

$$W=3n-2p_5-p_4=0.$$

If the mechanism is formed only by pairs of 5th classes ( $P_4=0$ ):

$$W_{gr.}=3n-2p_5=0.$$

Therefore

$$P_5 = \frac{3}{2}n, \quad (I.10)$$

but  $n$  and  $P_5$  can take only integer values, hence the last condition is carried out only with certain parities of  $n$  and  $P_5$ .

$n$	2	4	6	8	...
$P_5$	3	6	9	12	...

### Questions for discussion.

What purposes are machines intended for?

List main groups of machines from the point of view of their functions.

Give examples of material-working machines.

What does "material-working machine" mean?

What distinctions between transport machines and technological ones exist?

What does "power machine" mean?

What types of transmissions do you know?

When can we say "It is automatic transfer line"? Explain please.

Give the definition of the term “mechanism”.

What types of bodies can incorporate mechanism?

Let's imagine: we have an electric drive before us.

How can you subdivide it?

What types of mechanisms can you find in its structure?

We can see a lot of constructions around of us. Can you give the distinctive feature of mechanism?

Is “The theory of machines and mechanisms” a science? Is it difficult?

Why is it necessary for mechanical engineers?

What are the purposes of studying this discipline?

What can you say about structural and kinematic analysis of mechanisms?

List the types of links you know.

Give concepts: kinematic pair, link. Tell about principles of kinematic pair classification.

Give the definition of a kinematic chain. Tell about the classification of kinematic chains.

What is a motion freedom of the mechanism?

#### **Tasks for seminar:**

1. Classify the links of the mechanism. Give classificational characteristic of the mechanism itself **according to models**, analysing the character of links movements.
2. Classify kinematic pairs according to character of relative movements and according to the classes. Describe movements. (**Cards in Appendix II.1**).
3. Give the classification of mechanism and their links using the picture or scheme of mechanism. Explain. (**Cards in Appendix II.2**).

## EXERCISES TO THE CHAPTER “KINEMATIC CHAINS”

### 1. Learn these words and phrases

1. single-stranded	- цепь с одной ветвью
2. complex chain	- сложная цепь
3. spatial	- пространственный
4. three-dimensional	- трехмерный – (пространственный)
5. driver link	- ведущее звено
6. driven link	- ведомое звено
7. kinematic scheme	- кинематическая схема
8. observance	- соблюдение
9. acceleration	- ускорение
10. exclude	- исключать
11. complicate	- запутать, усложнить
12. layout	- схема; план
13. internal combustion	- двигатель внутреннего сгорания
14. subtract	- вычитать
15. deduce a formula	- выводить формулу
16. substitute	- заменить
17. initialize	- привести в первоначальное значение
18. girder	- ферма
19. preset value	- предопределить значение
20. articulated parallelogram	- шарнирный параллелограмм
21. correspond	- соответствовать
22. validity	- значимость
23. pair wise equality	- попарное равенство
24. roller	- ролик
25. pusher	- толкатель
26. friction force	- сила трения
27. exceedingly	- чрезвычайно
28. by virtue	- благодаря
29. inexpediently	- нецелесообразно
30. adhere	- придерживаться чего-либо
31. intrinsically	- существенно
32. lever mechanism	- рычажный механизм
33. gear	- зубчатая передача
34. pump	- насос
35. conveyer	- конвейер
36. attributive	- свойство
37. consecutive	- последовательный
38. initial mechanism	- начальный механизм

**2. Find the words in the text which mean the opposite of these:**

Open-ended; simple; definite; not obligatory; soft; possible; less; simple; mobile; driver link

**3. Arrange the words into synonyms.**

1. many	a) utilize
2. important	b) perform
3. kind	c) get
4. receive	d) type
5. little	e) significant
6. use	f) several
7. carry out	g) small

**4. Give Russian equivalents to the following words**

1. spatial	11. pairwise equality
2. mutual disposition	12. to preset values
3. internal combustion engine	13. complicate
4. observance	14. single-stranded
5. superfluity	15. kinematic scheme
6. initial mechanism	16. subtract
7. articulated parallelogram	17. initialize
8. driven link	18. girder
9. follower	19. roller
10. crank-and-slider	20. acceleration

**5. Give English equivalents to the following words**

1. исключить	11. благодаря
2. влияющее движение	12. чрезвычайно
3. шарнирный параллелограмм	13. кривошипно-ползунный механизм
4. зубчатый механизмы	14. ролик
5. рычажный механизм	15. кинематическая схема
6. заменять	16. выводить формулу
7. существующие виды	17. привести в первоначальное положение
8. ведущее звено	18. значимость
9. двигатель внутреннего сгорания	19. соответствовать
10. кулачковый механизм	20. пространственный

**6. Match the words and phrases in column A with those in column B**

A	B
1. pairwise equality	a. ведущее звено
2. superfluity	b. ферма
3. subtract	c. попарное равенство
4. gear mechanisms	d. шарнирный параллелограмм
5. follower	e. исключать
6. girder	f. пространственный
7. substitute	g. нецелесообразно
8. exclude	h. излишество
9. spatial	i. толкатель
10. single-stranded	j. насос
11. driver link	k. кривошипно-ползунный механизм
12. articulated parallelogram	l. значимость
13. roller	m. кинематическая схема
14. adhere	n. вычитать
15. inexpediently	o. придерживаться
16. pump	p. заменять
17. frictional mechanism	q. зубчатые механизмы
18. validity	r. цепь с одной ветвью
19. crank-and-slider	s. ролик
20. kinematic scheme	t. фрикционный механизм

**7. Answer the following questions**

1. What is a kinematic chain?
2. What is a complex KP?
3. How many kinds of kinematic chains do you know?
4. What does a driver mean?
5. Is it enough to know the structure of mechanism?
6. What does a kinematic scheme represent?
7. Who deduced a formula of mobility or structural formula of a general view kinematic chain?
8. Which link is called a passive link?
9. What types of mechanisms do you know (according to constructive design)?
10. What types of mechanisms do you know (according to the functional purpose)?
11. What is an initial mechanism?

**8. Match the phrases in column A with those in column B**

A	B
1. The connection of links by kinematic pairs is ...	A. ...is diagrammed in a chosen scale with exact observance of all sizes.
2. All kinematic chains are...	B. ...a number of independent parameters, for which it is necessary to preset values in order for a given KP.
3. To study the movement of mechanism ...	C. ...called a kinematic chain.
4. Kinematic model ...	D. ... is not enough to know its structure.
5. Initial mechanism ...	E. ... are called passive.
6. The number of degrees of freedom is ...	F. ...subdivided into single-stranded KC and complex KC.
7. The kinematic chains can be ...	G. ...either spatial or flat.
8. Links which are not influenced by the driven links movement laws...	H. ...consists of one link for which the law of motion is given, connected by kinematic pair with a rack.

**9. Agree or disagree to these statements.**

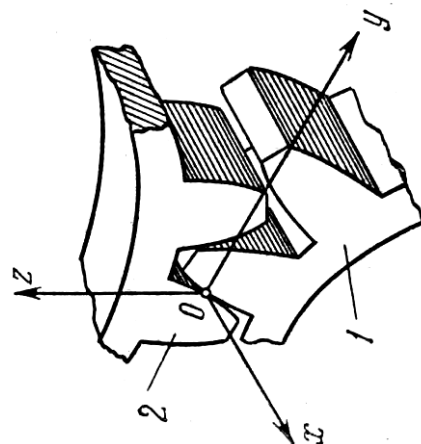
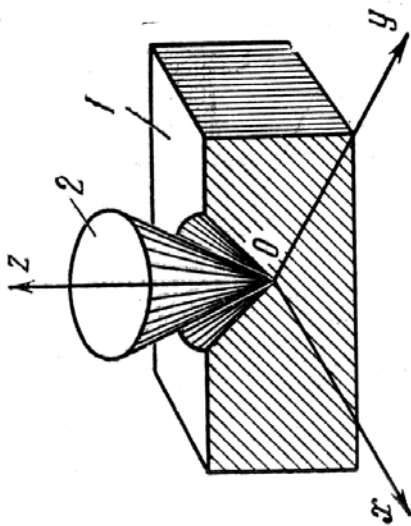
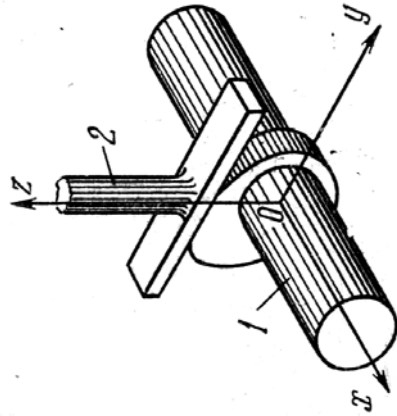
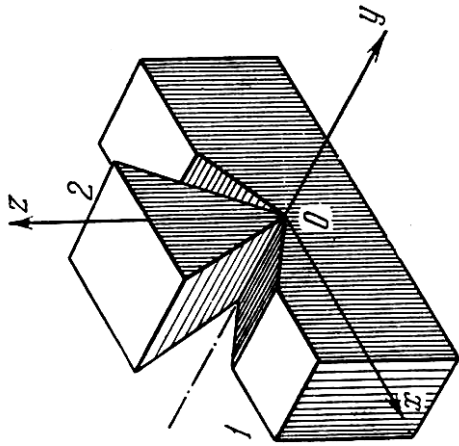
1. Initial mechanism is a mechanism that consists of two links.
2. Girder is a firm construction.
3. Rectilinear KP is the fourth class of kinematic pair.
4. Each body has six independent motions.
5. Screw KP has two independent motions.
6. Mechanisms are constantly improved.
7. Passive links act on the movement laws of driven links.
- 8 Kinematic chains may be of several types.

**10. Work in pairs.**

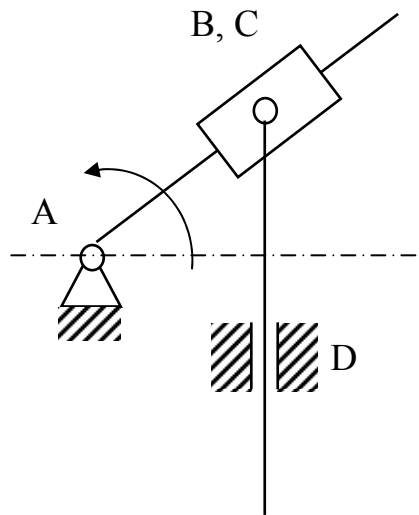
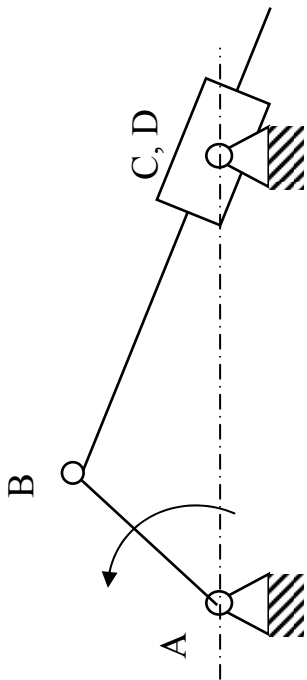
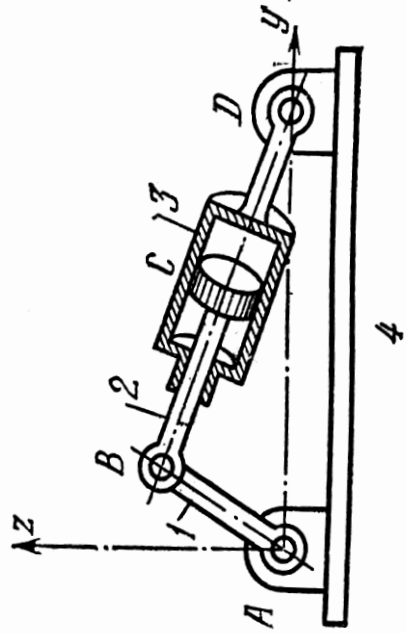
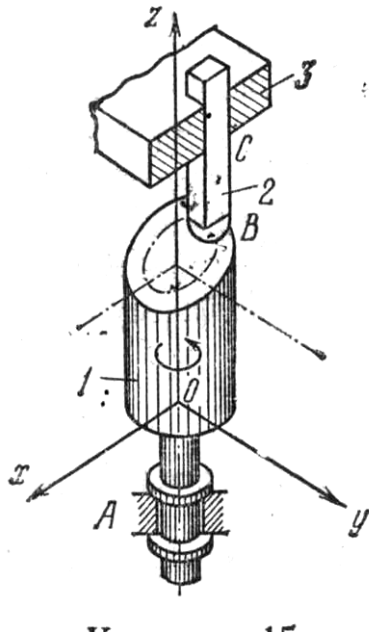
1. Speak about different kinds of kinematic chains. Draw schemes and give the detailed characteristic of each chain.
2. Tell about the classification of mechanisms.



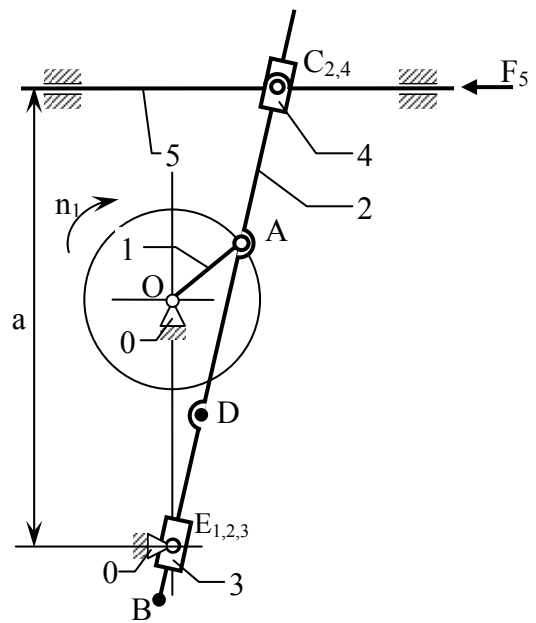
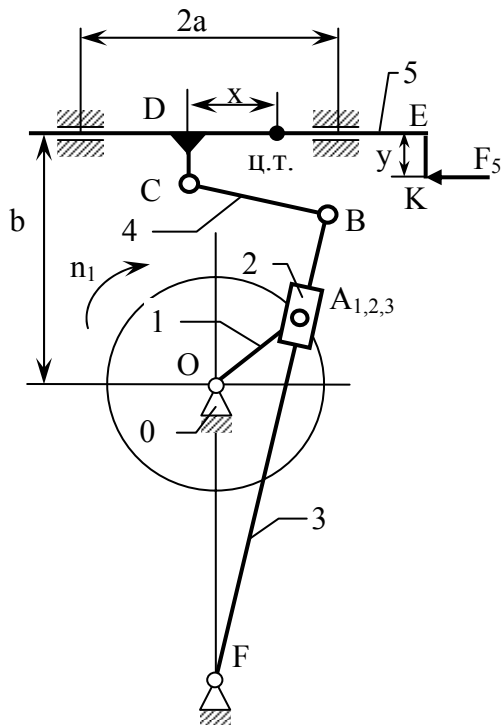
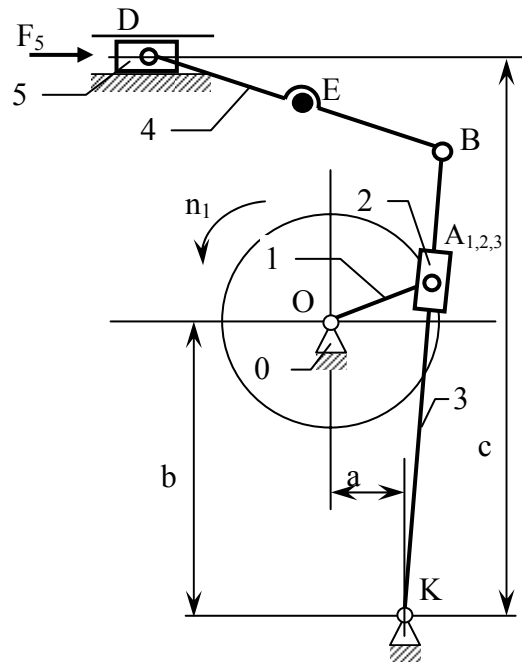
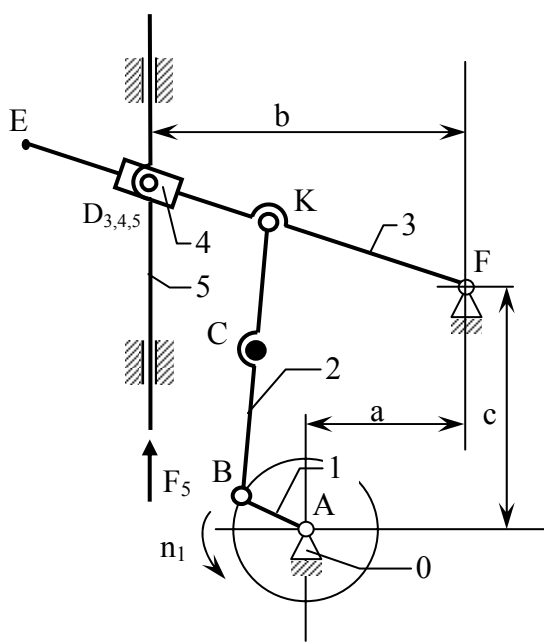
Appendix II.1



Appendix II.2



### Appendix II.3



## III. TYPES OF GEARS

### Study of gear terminology

Types of gears. Classification. Basic elements of gears.

Methodical materials:

- non-adapted texts:
  - a) textbooks,
  - b) references,
  - c) periodical scientific and popular-science editions;
- models of mechanisms;
- pictures of gear elements, gear mechanisms, schemes of mechanisms;
- tasks and exercises on theme.

This chapter will address gear geometry, the kinematic relations, and the consequential forces and moments associated with the four principal types of gears. The forces and moments are transmitted to meshing gears and to the shafts and bearings to which they are attached. The torsion moment allows motion and power transmission. Other moments and forces affect the shaft and its bearings. The next two chapters will address stress, strength, safety, and reliability.

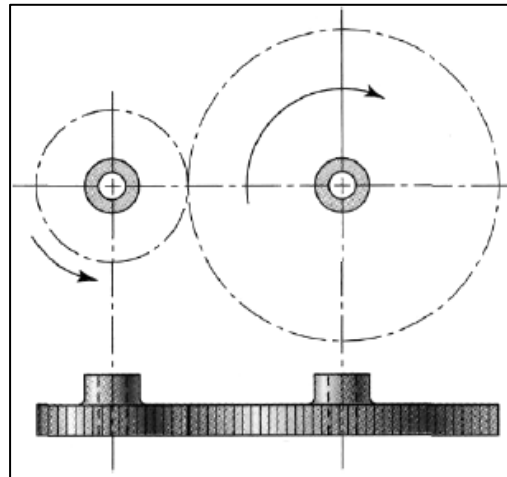
### Types of Gears

*Spur gears*, illustrated in Fig. III.1, have teeth parallel to the axis of rotation and are used to transmit motion from one shaft to another, parallel, shaft. Of all types, the spur gear is the simplest and, for this reason, will be used to develop the primary kinematic relationships of the tooth form.

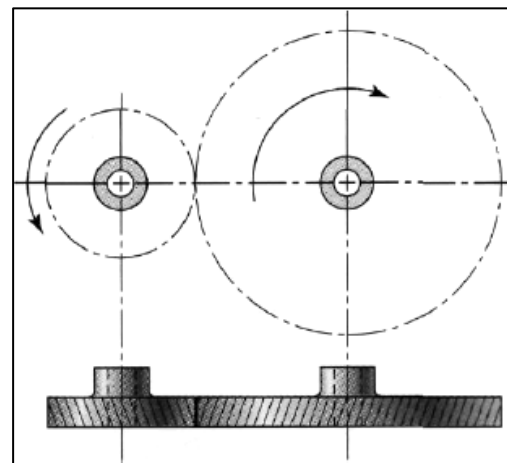
*Helical gears*, shown in Fig. III.2, have teeth inclined to the axis of rotation. Helical gears can be used for the same applications as spur gears and, when so used, are not as noisy, because of the more gradual engagement of the teeth during meshing. The inclined tooth also develops thrust loads and bending couples, which are not present with spur gearing. Sometimes helical gears are used to transmit motion between nonparallel shafts.

*Bevel gears*, shown in Fig. III.3, have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts. The figure actually illustrates *straight-tooth bevel gears*. *Spiral bevel gears* are cut so the tooth is no longer straight but forms a circular arc. *Hypoid gears* are quite similar to spiral bevel gears except that the shafts are offset and nonintersecting.

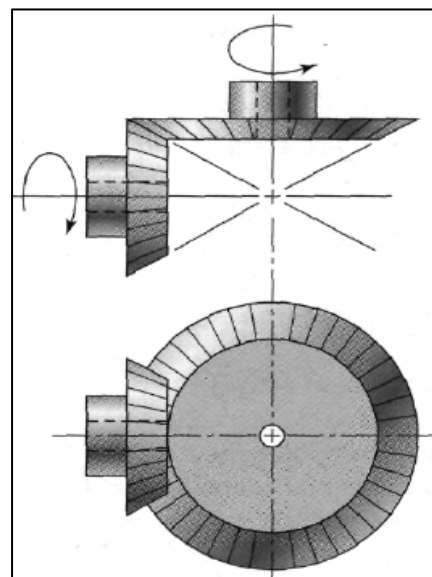
**Figure III.1**  
Spur gears are used to transmit rotary motion between parallel shafts.



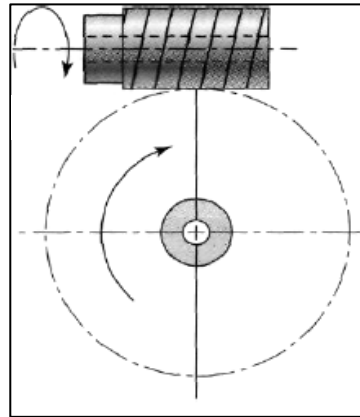
**Figure III.2**  
Helical gears are used to transmit motion between parallel or nonparallel shafts.



**Figure III.3**  
Bevel gears are used to transmit rotary motion between intersecting shafts.



**Figure III.4**  
Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.



Shown in Fig. III.4 is the fourth basic gear type, the worm and worm gear. As shown, the worm resembles a screw. The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand. Worm-gear sets are also made so that the teeth of one or both wrap partly around the other. Such sets are called single-enveloping and double-enveloping worm-gear sets. Worm-gear sets are mostly used when the speed ratios of the two shafts are quite high, say, 3 or more.

## 2. Nomenclature

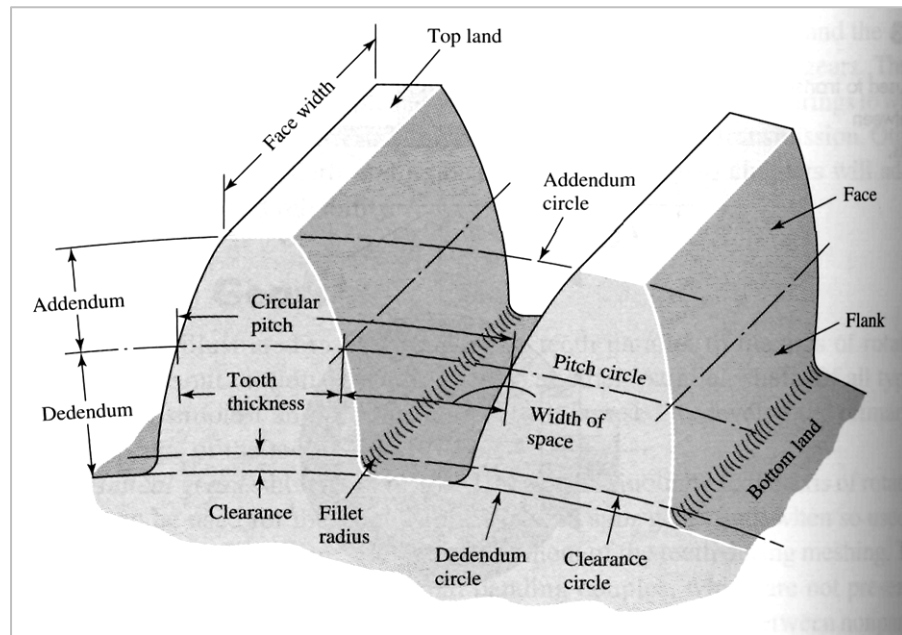
The terminology of spur-gear teeth is illustrated in Fig. III.5. The *pitch circle* is a theoretical circle upon which all calculations are usually based; its diameter is the *pitch diameter*. The pitch circles of a pair of mating gears are tangent to each other. A *pinion* is the smaller of two mating gears. The larger is often called the *gear*.

The *circular pitch*  $p$  is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. Thus the circular pitch is equal to the sum of the *tooth thickness* and the *width of space*.

The *module*  $m$  is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.

The *diametral pitch*  $P$  is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module. Since diametral pitch is used only with U.S. units, it is expressed as teeth per inch.

**Figure III.5**  
Nomenclature of spur-gear teeth.



The *addendum*  $a$  is the radial distance between the *top land* and the pitch circle. The *dedendum*  $b$  is the radial distance from the *bottom land* to the pitch circle. The whole *depth*  $h_t$  is the sum of the addendum and the dedendum.

The *clearance circle* is a circle that is tangent to the addendum circle of the mating gear. The *clearance*  $n$  is the amount by which the dedendum in a given gear exceeds addendum of its mating gear. The *backlash* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.

You should prove for yourself the validity of the following useful relations:

$$P = \frac{N}{d}$$

where  $P$  = diametral pitch, teeth per inch

$N$  = number of teeth

$d$  = pitch diameter, in

$$m = \frac{d}{N}$$

where  $m$  = module, mm;  $d$  = pitch diameter, mm

$$p = \frac{\pi d}{N}$$

where  $p$  = circular pitch

$$pP = \pi$$

### 3. Tooth Systems<sup>1</sup>

A *tooth system* is a standard which specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle. The standards were originally planned to attain interchangeability of gears of all tooth numbers, but of the same pressure angle and pitch.

Table III.1 contains the standards most used for spur gears. A 14.5° pressure angle was once used for these but is now obsolete; the resulting gears had to be comparatively larger to avoid interference problems.

Table III.2 is particularly useful in selecting the pitch or module of a gear. Cutters are generally available for the sizes shown in this table.

Table III.3 lists the standard tooth proportions for straight bevel gears. These sizes apply to the large end of the teeth. The nomenclature is defined in Fig. III.2.

Standard tooth proportions for helical gears are listed in Table III.4. Tooth proportions are based on the normal pressure angle; these angles are standardized the same as for spur gears. Though there will be exceptions, the face width of helical gears should be at least 2 times the axial pitch to obtain good helical-gear action.

Tooth forms for worm gearing have not been highly standardized, perhaps because there has been less need for it. The pressure angles used depend upon the lead angles and must be large enough to avoid undercutting of the worm-gear tooth on the side at which contact ends. A satisfactory tooth depth, which remains in about the right proportion to the lead angle, may be obtained by making the depth a proportion of the axial circular pitch. Table III.5 summarizes what may be regarded as good practice for pressure angle and tooth depth.

**Table III.1**  
Standard and Commonly used  
Tooth Systems for Spur Gears

Tooth System	Pressure Angle $F$ , deg	Addendum $a$	Dedendum $b$
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	22.5	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

<sup>1</sup> Standardized by the American Gear Manufacturers Association (AGMA). Write AGMA for a complete list of standards, because changes are made from time to time. The address is: 1500 King Street, Suite 201, Alexandria, VA 22314.



**Table III.2**  
Tooth Sizes in General Uses

<b>Diametral Pitch</b>	
Coarse	2, 2.25, 2.5, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200
<b>Modules</b>	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

**Table III.3**  
Tooth Proportions for  
20° Straight Bevel-  
Gear Teeth

<b>Item</b>	<b>Formula</b>										
Working depth	$h_k = 2.0 / P$										
Clearance	$c = (0.188 / P) + 0.002$ in										
Addendum of gear	$a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{90})^2}$										
Gear ratio	$m_G = N_G / N_P$										
Equivalent 90° ratio	$m_{90} = m_G$ when $\Sigma = 90^\circ$ $m_{90} = \sqrt{m_G \frac{\cos \gamma}{\cos \Gamma}}$ when $\Sigma \neq 90^\circ$										
Face width	$F = \frac{A_o}{3}$ or $F = \frac{10}{P}$ , whichever is smaller										
Minimum number of teeth	<table border="1"> <tr> <td>Pinion</td> <td>16</td> <td>15</td> <td>14</td> <td>13</td> </tr> <tr> <td>Gear</td> <td>16</td> <td>17</td> <td>20</td> <td>30</td> </tr> </table>	Pinion	16	15	14	13	Gear	16	17	20	30
Pinion	16	15	14	13							
Gear	16	17	20	30							

**Table III.4**  
Standard  
Tooth  
Proportions  
for Helical  
Gears

<b>Quantity*</b>	<b>Formula</b>	<b>Quantity*</b>	<b>Formula</b>
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D + d}{2}$

	Pinion pitch diameter	$\frac{N_P}{P_n \cos \psi}$	Gear outside diameter	$D + 2a$
	Gear pitch diameter	$\frac{N_G}{P_n \cos \psi}$	Pinion outside diameter	$d + 2a$
	Normal arc tooth thickness	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D - 2b$
	Pinion base diameter	$d \cos \phi_t$	Pinion root diameter	$d - 2b$
			Internal gears:	
	Gear base diameter	$D \cos \phi_t$	Center distance	$\frac{D - d}{2}$
	Base helix angle	$\tan^{-1}(\tan \psi \cos \phi_t)$	Inside diameter	$D - 2a$
			Root diameter	$D + 2b$

\*All dimensions are in inches, and angles are in degrees

**Table III.5**  
Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle $\lambda$ , deg	Pressure Angle $\Phi_t$ , deg	Addendum $a$	Dedendum $b_G$
0-15	14½	$0.3683p_x$	$0.3683p_x$
15-30	20	$0.3683p_x$	$0.3683p_x$
30-35	25	$0.2865p_x$	$0.3314p_x$
35-40	25	$0.2546p_x$	$0.2947p_x$
40-45	30	$0.2228p_x$	$0.2578p_x$

The *face width*  $FG$  of the worm gear should be made equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle as shown in Fig. III.6

**EXERCISES TO THE CHAPTER “TYPES OF GEARS”  
LEARN THESE WORDS AND PHRASES**

- |     |                             |  |
|-----|-----------------------------|--|
| 1.  | addendum                    | - головка зуба, высота головки<br>зуба                                     |
| 2.  | addendum circle, top circle | - окружность вершин зубьев   |
| 3.  | adjacent tooth              | - примыкающий зуб  |
| 4.  | attain interchangeability   | - достичь взаимозаменяемости   |
| 5.  | backlash                    | - люфт   |
| 6.  | bevel gear                  | - коническая зубчатая передача   |
| 7.  | bottom land                 | - поверхность впадин   |
| 8.  | circular pitch              | - окружной шаг   |
| 9.  | clearance                   | - зазор  |
| 10. | cutter                      | - резец  |
| 11. | dedendum                    | - ножка зуба, высота ножки зуба  |
| 12. | dedendum circle             | - окружность впадин зубьев   |
| 13. | diametral pitch             | - питч   |
| 14. | double-enveloping worm-gear | - двухзаходная червячная<br>передача                                       |
| 15. | engagement                  | - зацепление   |
| 16. | (tooth) face                | - рабочая поверхность зуба от<br>вершины зуба до делительной<br>окружности |
| 17. | fillet radius               | - радиус переходной кривой   |
| 18. | flank                       | - поверхность зуба ниже<br>делительной окружности                          |
| 19. | helical gear                | - косозубое зубчатое колесо  |
| 20. | hypoid gear                 | - гипоидное колесо   |
| 21. | incline                     | - наклонять  |
| 22. | interchangeability          | - взаимозаменяемость   |
| 23. | interference problems       | - проблемы столкновения  |
| 24. | intersecting shafts         | - пересекающиеся валы  |
| 25. | mating gear                 | - сопряженное зубчатое колесо  |
| 26. | meshing                     | - зацепление   |
| 27. | module                      | - модуль, коэффициент  |
| 28. | nomenclature                | - перечень, номенклатура   |
| 29. | nonintersecting shaft       | - непересекающиеся валы  |
| 30. | obsolete                    | - устарелый, вышедший из<br>употребления                                   |
| 31. | offset                      | - смещать  |
| 32. | pinion                      | - шестерня   |

33. pitch circle	- делительная окружность
34. pitch diameter	- диаметр делительной окружности
35. pressure angle	- угол давления
36. ratio	- отношение, пропорция
37. single-enveloping worm-gear	- однозаходная червячная передача
38. spiral bevel gear	- спиральнозубая коническая передача
39. spur gear	- прямозубое зубчатое колесо
40. straight-tooth bevel gear	- прямозубая коническая передача
41. tangent	- касательный
42. thrust load	- осевая нагрузка
43. tooth thickness	- толщина зуба
44. top land	- поверхность вершин зубьев
45. transmit motion	- передавать движение
46. undercutting	- подрез
47. width of space	- ширина впадины
48. working depth	- глубина захода
49. worm gear	- червячная передача, червячное колесо
50. wrap	- заходить друг на друга

**2. Find words in the text which mean the opposite of these:**

Addendum; parallel; intersecting; calm; complex; right-hand; smaller; useless; new; different.

**3. Fill in the following table.**

Noun	Verb

Usage, rotation, measure, inclination, mesh, development, transmission, calculation, correspondence, perform.

**4. Give Russian equivalents to the following words and phrases**

1. backlash	11. fillet radius
2. bevel gear	12. flank
3. engagement	13. diametral pitch

4. working depth	14. transmit motion
5. attain	15. nonintersecting shafts
6. helical gear	16. dedendum
7. application	17. tooth thickness
8. spur gear	18. spiral bevel gear
9. top land	19. thrust load
10. clearance	20. pinion

**5. Give English equivalents to the following words and phrases**

1. терминология	11. ширина рабочей поверхности зуба
2. пересекающиеся валы	12. устарелый
3. однозаходная червячная передача	13. взаимозаменяемость
4. отношение, пропорция	14. гипоидное колесо
5. рабочая поверхность	15. зацепление
6. проблемы столкновения	16. осевая или аксиальная нагрузка
7. поверхность впадин	17. окружный шаг
8. высота головки зуба	18. резец
9. диаметральный шаг зубчатого зацепления	19. высота ножки зуба
10. сопряженные зубчатые колеса	20. коническое колесо со спиральным зубом

**6. Match the notions and their definitions**

1. Gear	A. Any of the narrow pointed parts that stands out from a gear.
2. Axis	B. The usually imaginary line around which a body moves.
3. Nomenclature	C. The distance between the objects and another passing beneath or beside it.
4. Diameter	D. A system of naming things, especially in science.
5. Cutter	E. Something, especially one consisting of a set of toothed wheels.
6. Clearance	F. A type of fastener that is like a nail but has a raised edge winding round it.
7. Tooth	G. An instrument used for cutting.
8. Screw	H. A straight line going from one side of a circle to other side, passing through the center of the circle.

**7. Find the endings for the given beginnings:**

A	B
1. The module is ...	A. ...the amount by which the width of a space exceeds the thickness of the engaging tooth.
2. A pinion is ...	B. ...when the speed ratios of the two shafts are high say, 3 or more.
3. Helical gears are used ...	C. ...the distance, measured on the pitch circle, from a point on the tooth to a corresponding point on an adjacent tooth.
4. Worm gears are used ...	D. ...to transmit motion between nonparallel shafts.
5. The circular pitch is ...	E. ...the smaller of two mating gears.
6. The backlash is...	F. ...the ratio of the pitch diameter to the number of teeth.

**8. Say whether these statements true or false**

1. Helical gears have teeth parallel to the axis of rotation.
2. Hypoid gears are similar to straight-tooth bevel gears.
3. The worm gear resembles a screw.
4. Addendum is the radial distance between the top land and clearance.
5. The pitch circles of a pair of mating gears are not tangent to each other.
6. Spur gear is the most complex gear.
7. Bevel gears are used to transmit motion between nonintersecting shafts.

**9. Make up sentences using the following words and phrases**

1. Teeth, the, axis, rotation, spur, have, of, gears, parallel, to.
2. Are, similar, bevel, gears, hypoid, to, spiral, gears.
3. Used, motion, nonparallel, helical, are, to, between, gears, transmit, shafts.
4. Between, land, circle, addendum, the, radial, is, top, pitch, distance, the, and.
5. Is, larger, gear, pinion, than.

**10. Choose the right variant: a, b, c.**

1. *Helical gears have ..... teeth.*
  - a) .parallel
  - b). inclined
  - c). straight
2. *Hypoid gears are similar to ..... ones.*
  - a). worm
  - b). spur

- c). spiral bevel
- 3. *The addendum is the radial distance between the.....*
  - a). bottom land and pitch circle
  - b). top land and the pitch circle
  - c). flanks of adjacent teeth
- 4. *..... is one of the simplest gears*
  - a). Helical
  - b). Spur
  - c). Worm

**11. Translate the following sentences paying attention to the Participle II of the verbs.**

- 1. Helical gears *can be used* for the same application.
- 2. Bevel gears have teeth *formed* on the conical surfaces.
- 3. The standards *were originally planned* to attain interchangeability of gears.
- 4. The *inclined* tooth also develops thrust loads.
- 5. The forces and moments *are transmitted* to meshing gears.
- 6. The circular pitch is the distance, *measured* on the pitch circle.

**12. Work in pairs. Make up a dialogue about different types of gears.**

## IV. GEARS AND GEARING

### (PART I)

#### 1. Types of gears

*External spur gears* are cylindrical gears with straight teeth cut parallel to the axes. Gears transmit drive between parallel shafts. Tooth loads produce no axial thrust. Excellent at moderate speeds but tend to be noisy at high speeds. Shafts rotate in opposite directions.

*Internal spur gears* provide compact drive arrangements for transmitting motion between parallel shafts rotating in the same direction.

*Helical gears* are cylindrical gears with teeth cut at an angle to the axes. Provide drive between shafts rotating in opposite directions, with superior load carrying capacity and quietness than spur gears. Tooth loads produce axial thrust.

*Crossed helical gears* are helical gears that mesh together on non-parallel axes.

*Straight bevel gears* have teeth that are radial toward the apex and are of conical form. Designed to operate on intersecting axes, bevel gears are used to connect two shafts on intersecting axes. The angle between the shafts equals the angle between the two axes of the meshing teeth. End thrust developed under load tends to separate the gears.

*Spiral bevel gears* have curved oblique teeth that contact each other smoothly and gradually from one end of a tooth to the other. Meshing is similar to that of straight bevel gears but is smoother and quieter in use. Left hand spiral teeth incline away from the axis in an anti-clockwise direction looking on small end of pinion or face of gear, right-hand teeth incline away from axis in clockwise direction. The hand of spiral of the pinion is always opposite to that of the gear and is used to identify the hand of the gear pair. Used to connect two shafts on intersecting axes as with straight bevel gears. The spiral angle does not affect the smoothness and quietness of operation or the efficiency but does affect the direction of the thrust loads created. A left-hand spiral pinion driving clockwise when viewed from the large end of the pinion creates an axial thrust that tends to move the pinion out of mesh.

*Zerol bevel gears* have curved teeth lying in the same general direction as straight bevel teeth but should be considered to be spiral bevel gears with zero spiral angle.

*Hypoid bevel gears* are a cross between spiral bevel gears and worm gears. The axes of hypoid bevel gears are non-intersecting and non-parallel. The distance between the axes is called the offset. The offset permits higher ratios of reduction than is practicable with other bevel gears. Hypoid bevel gears have curved oblique teeth on which contact begins gradually and continues smoothly from one end of the tooth to the other.

*Worm gears* are used to transmit motion between shafts at right angles, that do not lie in a common plane and sometimes to connect shafts at other angles. Worm gears have line tooth contact and are used for power transmission, but the higher the ratio the lower the efficiency.

## **2. Definitions of Gear Terms**

The following terms are commonly applied to the various classes of gears:

*Active face width* is the dimension of the tooth face width that makes contact with a mating gear.

*Addendum* is the radial or perpendicular distance between the pitch circle and the top of the tooth.

*Arc of action* is the arc of the pitch circle through which a tooth travels from the first point of contact with the mating tooth to the point where contact ceases.



*Arc of approach* is the arc of the pitch circle through which a tooth travels from the first point of contact with the mating tooth to the pitch point.

*Arc of recession* is the arc of the pitch circle through which a tooth travels from its contact with a mating tooth at the pitch point until contact ceases.

*Axial pitch* is the distance parallel to the axis between corresponding sides of adjacent teeth.

*Axial plane* is the plane that contains the two axes in a pair of gears. In a single gear the axial plane is any plane containing the axis and any given point.

*Axial thickness* is the distance parallel to the axis between two pitch line elements of the same tooth.

*Backlash* is the shortest distance between the non-driving surfaces of adjacent teeth when the working flanks are in contact.

*Base circle* is the circle from which the involute tooth curve is generated or developed.

*Base helix angle* is the angle at the base cylinder of an involute gear that the tooth makes with the gear axis.

*Base pitch* is the circular pitch taken on the circumference of the base circles, or the distance along the line of action between two successive and corresponding involute tooth profiles. The *normal base pitch* is the base pitch in the normal plane and the *axial base pitch* is the base pitch in the axial plane.

*Base tooth thickness* is the distance on the base circle in the plane of rotation between involutes of the same pitch.

*Bottom land* is the surface of the gear between the flanks of adjacent teeth.

*Center distance* is the shortest distance between the non-intersecting axes of mating gears, or between the parallel axes of spur gears and parallel helical gears, or the crossed axes of crossed helical gears or worm gears.

*Central plane* is the plane perpendicular to the gear axis in a worm gear, which contains the common perpendicular of the gear and the worm axes. In the usual arrangement with the axes at right angles, it contains the worm axis.

*Chordal addendum* is the radial distance from the circular thickness chord to the top of the tooth, or the height from the top of the tooth to the chord subtending the circular thickness arc.

*Chordal thickness* is the length of the chord subtended by the circular thickness arc. The dimension obtained when a gear tooth caliper is used to measure the tooth thickness at the pitch circle.

*Circular pitch* is the distance on the circumference of the pitch circle, in the plane of rotation, between corresponding points of adjacent teeth. The length of the arc of the pitch circle between the centers or other corresponding points of adjacent teeth.

*Circular thickness* is the thickness of the tooth on the pitch circle in the plane of rotation, or the length of arc between the two sides of a gear tooth measured on the pitch circle.

*Clearance* is the radial distance between the top of a tooth and the bottom of a mating tooth space, or the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

*Contact diameter* is the smallest diameter on a gear tooth with which the mating gear makes contact.

*Contact ratio* is the ratio of the arc of action in the plane of rotation to the circular pitch, and is sometimes thought of as the average number of teeth in contact. This ratio is obtained most directly as the ratio of the length of action to the base pitch.

*Contact ratio – face* is the ratio of the face advance to the circular pitch in helical gears.

*Contact ratio – total* is the ratio of the sum of the arc of action and the face advance to the circular pitch.

*Contact stress* is the maximum compressive stress within the contact area between mating gear tooth profiles. Also called the Hertz stress.

*Cycloid* is the curve formed by the path of a point on a circle as it rolls along a straight line. When such a circle rolls along the outside of another circle the curve is called an *epicycloid*, and when it rolls along the inside of another circle it is called a *hypocycloid*. These curves are used in defining the former American Standard composite Tooth Form.

*Dedendum* is the radial or perpendicular distance between the pitch circle and the bottom of the tooth space.

*Diametral pitch* is the ratio of the number of teeth to the number of inches in the pitch diameter in the plane of rotation, or the number of gear teeth to each inch of pitch diameter.

Normal diametral pitch is the diametral pitch as calculated in the normal plane, or the diametral pitch divided by the cosine of the helix angle.

*Efficiency* is the torque ratio of a gear set divided by its gear ratio.

*Equivalent pitch radius* is the radius of curvature of the pitch surface at the pitch point in a plane normal to the pitch line element.

## EXERCISES TO THE CHAPTER “GEAR AND GEARING”

### LEARN THESE WORDS AND PHRASES (PART 1)

- |     |                          |   |
|-----|--------------------------|---|
| 1.  | active face width        | - рабочая ширина зуба   |
| 2.  | anti-clockwise direction | - направление против часовой стрелки  |
| 3.  | arc of action            | - дуга зацепления   |
| 4.  | arc of approach          | - дуга делительной окружности, проходимая зубом от начальной точки контакта до полюсной точки |
| 5.  | arc of recession         | - дуга делительной окружности, проходимая зубом от полюсной точки до выхода из зацепления     |
| 6.  | axial pitch              | - осевой шаг  |
| 7.  | axial plane              | - осевая плоскость  |
| 8.  | axial thickness          | - толщина в осевом направлении  |
| 9.  | base circle              | - основная окружность   |
| 10. | base helix angle         | - угол наклона зуба по основной окружности  |
| 11. | base pitch               | - шаг по основной окружности  |
| 12. | base tooth thickness     | - толщина зуба по основной окружности   |
| 13. | center distance          | - межосевое расстояние  |
| 14. | central plane            | - центральная плоскость   |
| 15. | chordal addendum         | - высота головки зуба, измеренная от хорды  |
| 16. | chordal thickness        | - толщина зуба по хорде   |
| 17. | circular thickness       | - толщина зуба по начальной окружности  |
| 18. | circumference            | - окружность  |
| 19. | clearance                | - зазор   |
| 20. | contact diameter         | - диаметр контакта  |
| 21. | contact ratio            | - коэффициент перекрытия  |
| 22. | contact ratio-face       | - окружной коэффициент перекрытия   |
| 23. | contact stress           | - контактное напряжение   |
| 24. | contact-ratio-total      | - общий коэффициент перекрытия  |
| 25. | cosine                   | - косинус   |
| 26. | crossed helical gear     | - винтовая зубчатая передача  |
| 27. | cycloid                  | - циклоида  |
| 28. | epicycloid               | - эпициклоида   |
| 29. | external spur gear       | - прямозубое колесо внешнего  |

	зацепления
30. helical gear	- косозубое зацепление
31. hypocycloid	- гипоциклоида
32. hypoid bevel gear	- гипоидная коническая передача
33. internal spur gear	- прямозубое колесо внутреннего зацепления
34. involute	- эвольвента
35. normal base pitch	- шаг по основной окружности нормально в плоскости, перпендикулярной поверхности зуба
36. oblique	- наклонный
37. reduction	- уменьшение, сокращение
38. subtend	- стягивать
39. zerol bevel gear	- нулевая коническая передача

**1. Fill in the following table.**

Noun	Adjective	Verb
Rotation		
		intersect
	permissible	
Inclination		
		circle
	compressive	
Travel		
		affect
	directive	
Correspondence		

**2. Give Russian equivalents to the following words**

1. spiral bevel gears	11. circular pitch
2. active face width	12. contact ratio
3. axial plane	13. diametral pitch
4. zerol bevel gears	14. cycloid
5. crossed helical gears	15. bottom land
6. external spur gear	16. base circle
7. act of action	17. hypocycloid
8. arc of approach	18. axial pitch
9. chordal addendum	19. internal spur gear
10. central plane	

**3. Give English equivalents to the following words**

1. гипоидная коническая передача	11. контактное напряжение
2. рабочая ширина зуба	12. шаг по нормальной окружности
3. дуга поворота зуб. колеса от полюса до выхода из зацепления	13. гипоциклоида
4. осевой шаг	14. винтовая зубчатая передача
5. шаг по основной окружности	15. дуга зацепления
6. циклоида	16. полюс зацепления
7. толщина по хорде	17. зубчатые колеса внешнего зацепления
8. межосевое расстояние	18. толщина в осевом направлении
9. основная окружность	19. косинус
10. коэффициент перекрытия	20. измерение высоты головки зуба по хорде

**4. Find the endings for the given beginnings**

<b>A</b>	<b>B</b>
1. Zerol bevel gears ...	A. ... is the radial distance from the circular thickness to the top of the tooth.
2. Axial pitch ...	B. ... the surface of the gear between the flanks of adjacent teeth.
3. Circular pitch ...	C. ...is the distance on the base circle in the plane of rotation between involutes of the same pitch.
4. Chordal addendum ...	D. ...is the ratio of the face advance to the circular pitch in helical gears.
5. Bottom land ...	E. ... is the distance parallel to the axis between corresponding sides of adjacent teeth
6. Base tooth thickness...	F. ...have curved teeth lying in the same direction as straight bevel teeth
7. Contact ratio-face...	G. ...the distance on the circumference of the pitch circle

**5. Can you say what comes next? Try to give as many answers as possible.**

1. distance between .....
2. mating .....
3. the ratio of.....

4. have teeth .....
5. the diameter of.....
6. straight teeth.....
7. shafts rotate .....
8. the angle between .....
9. to drive.....

**6. Answer the following questions.**

1. What is the difference between external spur gear and internal spur gear?
2. What is the crossed helical gear?
3. What kind of teeth do spiral bevel gears have?
4. What do we call zero bevel gears?
5. What is the difference between arc of action and arc of recession?
6. What is the axial plane?
7. Is central plane perpendicular to the gear axis?
8. What is the difference between the contact –ratio face and contact ratio-total?

**7. Agree or disagree to these statements.**

1. Axial thickness is the distance perpendicular to the axis.
2. Bottom land is the surface of the gear between the flanks of the tooth.
3. Internal spur gears transmit motion between nonparallel shafts.
4. Crossed helical gears mesh together on non-parallel axes.
5. Dedendum is the radial distance between the pitch circle and bottom land.
6. External spur gears are noisy at high speed.
7. Central plane is the plane parallel to the gear axis in a worm gear.

**9. Fill in the gaps inserting the verbs in brackets in the correct form.**

1. Chordal thickness is the length of the chord (subtend) by the circular thickness arc.
2. Gears (transmit) drive between parallel shafts.
3. The spiral angle (not, effect) the smoothness of operation.
4. Worm gears (use) to transmit motion between shafts at right angles.
5. Spiral bevel gears (have) oblique teeth.
6. The distance between the axes (call) the offset.
7. The axes of hypoid bevel gears (to be) non-intersecting and non-parallel.

## IV. GEARS AND GEARING

### (PART II)

*Face advance* is the distance on the pitch circle that a gear tooth travels from the time pitch point contact is made at one end of the tooth until pitch point contact is made at the other end.

*Fillet radius* is the radius of the concave portion of the tooth profile where it joins the bottom of the tooth space.

*Fillet stress* is the maximum tensile stress in the gear tooth fillet.

*Flank of tooth* is the surface between the pitch circle and the bottom land, including the gear tooth fillet.

*Gear ratio* is the ratio between the numbers of teeth in mating gears.

*Helical overlap* is the effective face width of a helical gear divided by the gear axial pitch.

*Helix angle* is the angle that a helical gear tooth makes with the gear axis at the pitch circle, unless specified otherwise.

*Hertz stress*, see *Contact stress*.

*Highest point of single tooth contact (HPSTC)* is the largest diameter on a spur gear at which a single tooth is in contact with the mating gear.

*Interference* is the contact between mating teeth at some point other than along the line of action.

*Internal diameter* is the diameter of a circle that coincides with the tops of the teeth of an internal gear.

*Internal gear* is a gear with teeth on the inner cylindrical surface.

*Involute* is the curve generally used as the profile of gear teeth. The curve is the path of a point on a straight line as it rolls along a convex base curve, usually a circle.

*Land*: The top land is the top surface of a gear tooth and the *bottom land* is the surface of the gear between the fillets of adjacent teeth.

*Lead* is the axial advance of the helix in one complete turn, or the distance along its own axis on one revolution if the gear were free to move axially.

*Length of action* is the distance on an involute line of action through which the point of contact moves during the action of the tooth profile.

*Line of action* is the portion of the common tangent to the base cylinders along which contact between mating involute teeth occurs.

*Lowest point of single tooth contact (LPSTC)* is the smallest diameter on a spur gear at which a single tooth is in contact with its mating gear. Gear set contact stress is determined with a load placed on the pinion at this point.

*Module* is the ratio of the pitch diameter to the number of teeth, normally the ratio of pitch diameter in mm to the number of teeth. Module in the inch system is the ratio of the pitch diameter in inches to the number of teeth.

*Normal plane* is a plane normal to the tooth surfaces at a point of contact and perpendicular to the pitch plane.

*Number of teeth* is the number of teeth contained in a gear.

*Outside diameter* is the diameter of the circle that contains the tops of the teeth of external gears.

*Pitch* is the distance between similar, equally-spaced tooth surfaces in a given direction along a given curve or line.

*Pitch circle* is the circle through the pitch point having its center at the gear axis.

*Pitch diameter* is the diameter of the pitch circle. The operating pitch diameter is the pitch diameter at which the gear operates.

*Pitch plane* is the plane parallel to the axial plane and tangent to the pitch surfaces in any pair of gears. In a single gear, the pitch plane may be any plane tangent to the pitch surfaces.

*Pitch point* is the intersection between the axes of the line of centers and the line of action.

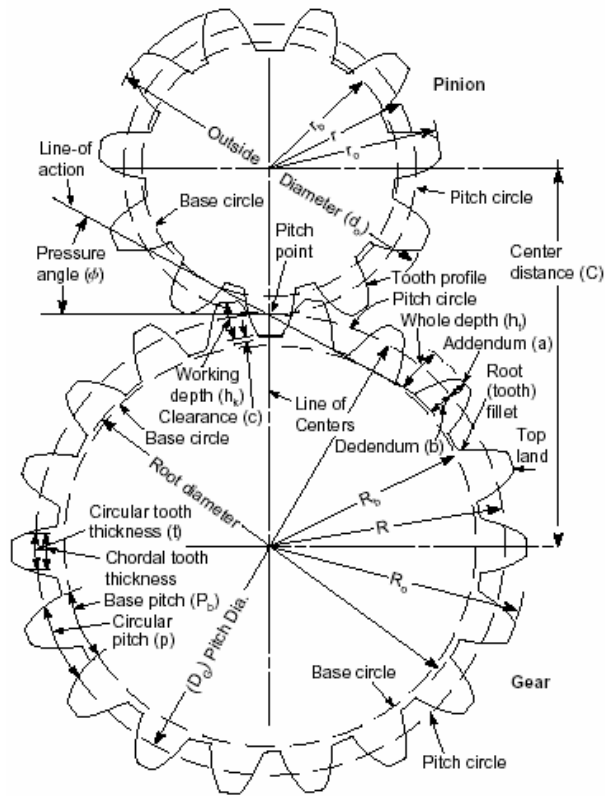
*Plane of rotation* is any plane perpendicular to a gear axis.

*Pressure angle* is the angle between a tooth profile and a radial line at its pitch point. In involute teeth, the pressure angle is often described as the angle between the line of action and the line tangent to the pitch circle. *Standard pressure angles* are established in connection with standard tooth proportions. A given pair of involute profiles will transmit smooth motion at the same velocity ratio when the center distance is changed. Changes in center distance in gear design and gear manufacturing operations may cause changes in pitch diameter, pitch and pressure angle in the same gears under different conditions. Unless otherwise specified, the pressure angle is the *standard pressure angle at the standard pitch diameter*. The *operating pressure angle* is determined by the center distance at which a pair of gears operate. In oblique teeth such as helical and spiral designs, the pressure angle is specified in the transverse, normal or axial planes.

*Principle reference planes* are pitch plane, axial plane and transverse plane, all intersecting at a point and mutually perpendicular.

*Rack*: A rack is a gear with teeth spaced along a straight line, suitable for straight line motion. A basic rack is a rack that is adopted as the basis of a system of interchangeable gears. Standard gear tooth dimensions are often illustrated on an outline of a basic rack.

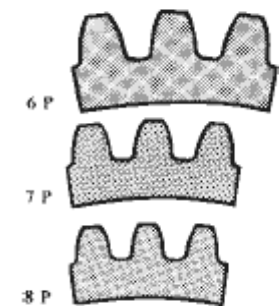




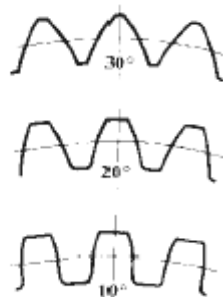
**Nomenclature:**

- |                              |                                 |                                   |
|------------------------------|---------------------------------|-----------------------------------|
| $\phi$ = Pressure Angle      | $a_G$ = Addendum of Gear        | $a_P$ = Addendum of Pinion        |
| $A$ = Addendum               | $c$ = Clearance                 | $C$ = Center Distance             |
| $b$ = Dedendum               | $D_G$ = Pitch Diameter of Gear  | $D_P$ = Pitch Diameter of Pinion  |
| $D$ = Pitch Diameter         | $D_O$ = Outside Diameter        | $D_R$ = Root Diameter             |
| $D_B$ = Base Circle Diameter | $h_k$ = Working Depth of Tooth  | $h_t$ = Whole Depth of Tooth      |
| $F$ = Face Width             | $p$ = Circular Pitch            | $P$ = Diametral Pitch             |
| $m_G$ = Gear Ratio           | $N_G$ = Number of Teeth in Gear | $N_P$ = Number of Teeth in Pinion |
| $N$ = Number of Teeth        |                                 |                                   |

**Comparative Sizes and Shape of Gear Teeth**



Gear Teeth of Different Diametral Pitch



Shapes of Gear Teeth of Different Pressure Angles

*Roll angle* is the angle subtended at the center of a base circle from the origin of an involute to the point of tangency of a point on a straight line from any point on the same involute. The radian measure of this angle is the tangent of the pressure angle of the point on the involute.

*Root diameter* is the diameter of the circle that contains the roots or bottoms of the tooth spaces.

*Tangent plane* is a plane tangent to the tooth surfaces at a point or line of contact.

*Tip relief* is an arbitrary modification of a tooth profile where a small amount of material is removed from the involute face of the tooth surface near the tip of the gear tooth.

*Tooth face* is the surface between the pitch line element and the tooth tip.

*Tooth surface* is the total tooth area including the flank of the tooth and the tooth face.

*Total face width* is the dimensional width of a gear blank and may exceed the effective face width as with a double-helical gear where the total face width includes any distance separating the right-hand and left-hand helical gear teeth.

*Transverse plane* is a plane that is perpendicular to the axial plane and to the pitch plane.

In gears with parallel axes, the transverse plane and the plane of rotation coincide.

*Trochoid* is the curve formed by the path of a point on the extension of a radius of a circle as it rolls along a curve or line. A trochoid is also the curve formed by the path of a point on a perpendicular to a straight line as the straight line rolls along the convex side of a base curve. By the first definition, a trochoid is derived from the *cycloid*, by the second definition it is derived from the *involute*.

*True involute form diameter* is the smallest diameter on the tooth at which the point of tangency of the involute tooth profile exists. Usually this position is the point of tangency of the involute tooth profile and the fillet curve, and is often referred to as the TIF diameter.

*Undercut* is a condition in generated gear teeth when any part of the fillet curve lies inside a line drawn at a tangent to the working profile at its lowest point. Undercut may be introduced deliberately to facilitate shaving operations, as in pre-shaving.

*Whole depth* is the total depth of a tooth space, equal to the addendum plus the dedendum and equal to the working depth plus clearance.

*Working depth* is the depth of engagement of two gears, or the sum of their addendums. The standard working distance is the depth to which a

tooth extends into the tooth space of a mating gear when the center distance is standard.

Definitions of gear terms are given in AGMA Standards 112.05, 115.01, and 116.01 entitled “Terms, Definitions, Symbols and Abbreviations,” “Reference Information—Basic Gear Geometry,” and “Glossary—Terms Used in Gearing,” respectively; obtainable from American Gear Manufacturers Assn., 1500 King. St., Alexandria, VA 22314.

## EXERCISES TO THE CHAPTER “GEAR AND GEARING”

### LEARN THESE WORDS AND PHRASES (PART II)

- |   |  |
|---|--|
| 1. efficiency                               | - производительность; мощность;<br>кпд               |
| 2. convex side                              | - выпуклая сторона                                   |
| 3. curvature                                | - изгиб, кривизна                                    |
| 4. equivalent pitch radius                  | - радиус кривизны в точке контакта<br>(профиля зуба) |
| 5. fillet stress                            | - напряжение изгиба у ножки зуба                     |
| 6. gear ratio                               | - отношение угловых скоростей;<br>передаточное число |
| 7. helical overleap                         | - перекрытие винтового зубчатого<br>колеса           |
| 8. helix angle                              | - угол наклона зуба                                  |
| 9. highest point of single tooth<br>contact | - наивысшая точка контакта<br>однопарного зацепления |
| 10. interference                            | - интерференция                                      |
| 11. internal diameter                       | - внутренний диаметр                                 |
| 12. involute                                | - эвольвента   |
| 13. lead                                    | - шаг винта; винтовые линии                          |
| 14. length of action                        | - длина зацепления                                   |
| 15. line of action                          | - линия зацепления                                   |
| 16. lowest point of single tooth<br>contact | - наинизшая точка контакта<br>однопарного зацепления |
| 17. number of teeth                         | - число зубьев                                       |
| 18. operating pressure angle                | - действительный угол зацепления                     |
| 19. outside diameter                        | - внешний диаметр                                    |
| 20. pitch point                             | - полюс зацепления                                   |
| 21. plane of rotation                       | - плоскость вращения                                 |
| 22. pressure angle                          | - угол давления                                      |
| 23. roll angle                              | - угол обката  |
| 24. root diameter                           | - диаметр окружности впадин                          |

	зубьев
25. shaving operation	- операция шевингования
26. standard pressure angle	- стандартный угол давления
27. standard pressure angle at the standard pitch	- стандартный угол зацепления на начальной окружности
28. tangent plane	- касательная плоскость
29. tooth face	- верхняя часть зуба
30. tooth profile	- профиль зуба
31. tooth surface	- поверхность зуба
32. top land	- поверхность вершин зубьев
33. torque ratio	- отношение крутящих моментов
34. total face width	- общая ширина поверхности зуба
35. transverse plane	- плоскость поперечного сечения
36. true involute form diameter	- наименьшая точка эвольвентной части зуба
37. whole depth	- общая глубина захода

**2. Read and translate the words having the same root:**

Tense – tensile – tension  
 Coincide – coincidence  
 Complicate – complicity – complicated  
 Section – intersection – sectional – sector  
 Adoptive – adopt  
 Include – inclusion – inclusive  
 Moderate – moderation  
 Length – lengthen – long  
 Move – motionless – movement – movable  
 Work – worker – worked

**3. Give Russian equivalents to the following words and phrases.**

1. arc of action	11. interference
2. contact ratio	12. fillet stress
3. land	13. efficiency
4. outside diameter	14. lead
5. line of action	15. undercut
6. total face width	16. root diameter
7. true involute form diameter	17. transverse plane
8. tip relief	18. rack
9. number of teeth	19. pressure angle
10. pitch point	20. tangent plane

**4. Give English equivalent to the following words and phrases.**

1. угол давления	11. дуга зацепления
2. отклонение профиля зуба	12. мощность
3. плоскость вращения	13. перекрытие винтового зубчатого колеса
4. напряжение у ножки зуба	14. длина зацепления
5. отношение угловых скоростей	15. осевой шаг
6. высота головки зуба, измеренная по хорде	16. циклоида
7. контактное напряжение	17. толщина зуба по начальной окружности
8. толщина в осевом направлении	18. производительность
9. винтовые зубчатые передачи	19. количество зубьев
10. рабочая ширина зуба	

**5. Match the notions and their definitions.**

<b>A</b>	<b>B</b>
1. Internal gear	A. is the number of teeth contained in a gear
2. Efficiency	B. is the surface of the gear between the flanks of the adjacent teeth
3. Plane of rotation	C. is the total depth of a tooth space
4. Whole depth	D. is the arc of the pitch circle through which a tooth travels from the first point of contact with mating tooth to the point where contact ceases
5. Number of teeth	E. is a gear with teeth on inner cylindrical surface
6. Flank of tooth	F. is the torque ratio of a gear set divided by its gear ratio
7. Bottom land	G. is any plane perpendicular to a gear axis
8. Arc of action	H. is the surface between the pitch circle and the bottom land, including the gear tooth fillet

**6. Say whether these statements true or false.**

1. The operating pressure angle is determined by the distance at which a single tooth operates.
2. A rack is a gear with teeth spaced along a curve line.
3. Working depth is the sum of gears' addendums.

4. Plane of rotation is not perpendicular to a gear axis.
5. The surface between the pitch line elements and the tooth tip is called tooth surface.
6. True involute form diameter is the largest diameter on the tooth.

**7. Complete the sentences using the words below:**

**Words:** coincides, teeth, tensile, torque, point, plane, tip, inner



1. Number of teeth is the number of ..... contained in a gear.
2. Internal gear is a gear with teeth on the ..... cylindrical surface.
3. Fillet stress is the maximum ..... stress in the gear tooth fillet.
4. Tangent plane is the plane tangent to the tooth surfaces at a ..... or line of contact.
5. Plane of rotation is any ..... perpendicular to a gear axis.
6. Tooth face is the surface between the pitch line element and the tooth .....
7. Efficiency is the ..... ratio of a gear set divided by its gear ratio.
8. Internal diameter is the diameter of a circle that ..... with the tops of the teeth of an internal gear.



**8. Make up a story “My first acquaintance with gears”. Express your impression.**

**9. Classify gears and their elements given on pictures.**

**10. Classify gears and their elements given on cards. Name all required elements.**

**11. Solve problems. Use formulas that you know and given in tables IV.3 and IV.4.**

PROBLEMS		
 ANALYSIS	1.1	A 17-tooth spur pinion has a diametral pitch of 8, runs at 1120 r/min, and drives a gear at a speed of 544 r/min. Find the number of teeth on the gear and the theoretical center-to-center distance.
 ANALYSIS	1.2	A 15-tooth spur pinion has a module of 3 mm and runs at a speed of 1600 r/min. The driven gear has 60 teeth. Find the speed of the driven gear, the circular pitch, and the theoretical center-to-center distance.

 ANALYSIS	1.3	A spur gearset has a module of 4 mm and a velocity ratio of 2.80. The pinion has 20 teeth. Find the number of teeth on the driven gear, the pitch diameters, and the theoretical center-to-center distance.
 ANALYSIS	1.4	A 21 -tooth spur pinion mates with a 28-tooth gear. The diametral pitch is 3 teeth/in and the pressure angle is 20°. Make a drawing of the gears showing one tooth on each gear. Find and tabulate the following results: the addendum, dedendum, clearance, circular pitch, tooth thickness, and base circle diameters; the lengths of the arc of approach, recess, and action; and the base pitch and contact ratio.

<b>Table IV.3</b>		<b>Metric Spur Gear Design Formulas</b>
<b>To obtain:</b>	<b>From known</b>	<b>Use this formula*</b>
Pitch diameter $D$	Module; diametral pitch	$D = mN$
Circular pitch $p_c$	Module; diametral pitch	$p_c = m\pi = \frac{D}{N}\pi = \frac{\pi}{P}$
Module $m$	Diametral pitch	$m = \frac{25.4}{P}$
No. of teeth $N$	Module and pitch diameter	$N = \frac{D}{m}$
Addendum $a$	Module	$a = m$
Dedendum $b$	Module	$b = 1.25m$
Outside diameter $D_o$	Module and pitch diameter or number of teeth	$D_o = D + 2m = m(N + 2)$
Root diameter $D_r$	Pitch diameter and module	$D_r = D - 2.5m$

Base circle diameter $D_b$	Pitch diameter and pressure angle $f$	$D_b = D \cos \phi$
Base pitch $p_b$	Module and pressure angle	$p_b = m\pi \cos \phi$
Tooth thickness at standard pitch diameter $T_{std}$	Module	$T_{std} = \frac{\pi}{2} m$
Center distance $C$	Module and number of teeth	$C = \frac{N_1 + N_2}{2} m$
Contact ratio $m_p$	Outside radii, base-circle radii, center distance, pressure angle	$m_p = \frac{\sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C \sin \phi}{m\pi \cos \phi}$
Backlash (linear) $B$ (along pitch circle)	Change in center distance	$B = 2(\Delta C) \tan \phi$
Backlash (linear) $B$ (along pitch circle)	Change in tooth thickness, $T$	$B = \Delta T$
Backlash (linear) (along line of action) $B_{LA}$	Linear backlash (along pitch circle)	$B_{LA} = B \cos \phi$
Backlash (angular) $B_a$	Linear backlash (along pitch circle)	$B_a = 6.880 \frac{B}{D}$ (arc minutes)
Min. number teeth for no undercutting, $N_c$	Pressure angle	$N_c = \frac{2}{\sin^2 \phi}$



<b>Table IV.4</b>	<b>Helical Gears on Parallel Shafts</b>
To find:	Formula
Center distance $C$	$C = \frac{N_p + N_G}{2P_{dn} \cos \psi}$
Pitch diameter $D$	$D = \frac{N}{P_d} = \frac{N}{P_{dn} \cos \psi}$
Normal diametral pitch $P_{dn}$	$\frac{P_d}{\cos \psi}$
Normal circular pitch $p_n$	$p \cos \psi$
Pressure angle $\phi$	$\tan^{-1} \frac{\tan \phi_n}{\cos \psi}$
Contact ratio $m_p$	$\frac{\sqrt{D_G^2 - D_b^2} + \sqrt{D_p^2 - D_b^2} + 2C \sin \phi}{2p \cos \phi} + \frac{F \sin \psi}{p_n}$
Velocity ratio $m_G$	$\frac{N_G}{N_p} = \frac{D_G}{D_p}$

## BEVEL GEARING - GENERAL

Bevel gears may be classified as follows:

- Straight bevel gears
- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears

A straight bevel gear was illustrated in Fig. III-3. These gears are usually used for pitch-line velocities up to 1000 ft/min (5 m/s) when the noise level is not an important consideration. They are available in many stock sizes and are less expensive to produce, than other bevel gears, especially in small quantities.

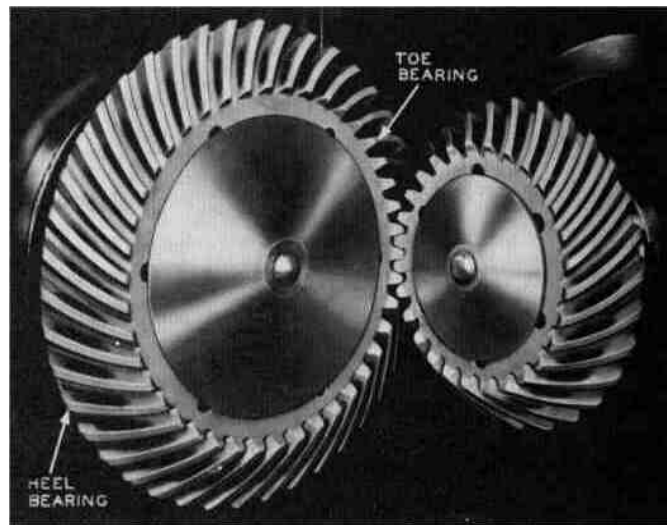
A *spiral bevel* gear is shown in Fig. IV-1; the definition of the *spiral angle* is illustrated in Fig. IV-2. These gears are recommended for higher speeds and where the noise level is an important consideration. Spiral bevel gears are the bevel counterpart of the helical gear; it can be seen in Fig. IV-1 that the pitch surfaces and the nature of contact are the same as for straight

bevel gears except for the differences brought about by the spiral-shaped teeth.

The *Zerol bevel gear* is a patented gear having curved teeth but with a zero spiral angle. The axial thrust loads permissible for Zerol bevel gears are not as large as those for the spiral bevel gear, and so they are often used instead of straight bevel gears. The Zerol bevel gear is generated using the same tool as for regular spiral bevel gears. For design purposes, use the same procedure as for straight bevel gears and then simply substitute a Zerol bevel gear.

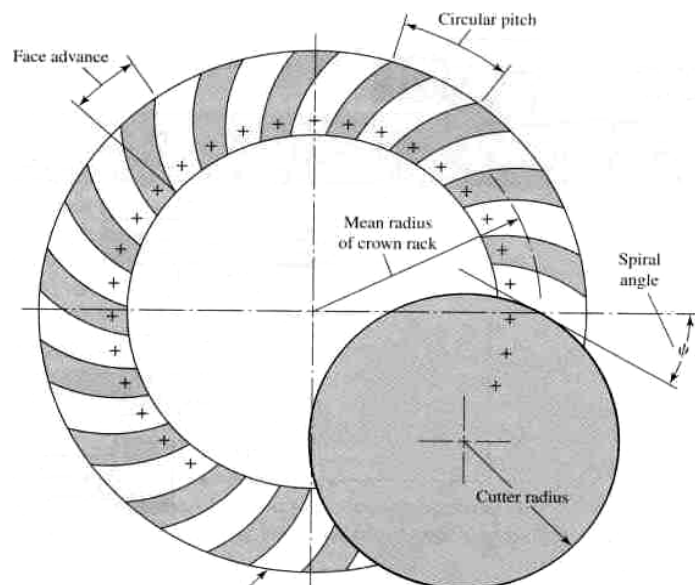
**Figure IV-1**

Spiral bevel gears.  
(Courtesy of Gleason Works, Rochester, N.Y.)



**Figure IV-2**

Cutting spiral-gear teeth on the basic crown rack.



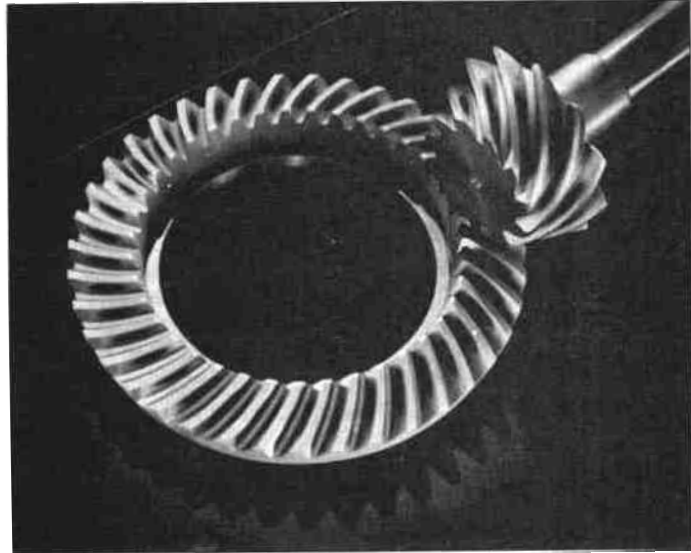
It is frequently desirable, as in the case of automotive differential applications, to have gearing similar to bevel gears but with the shafts offset. Such gears are called *hypoid gears*, because their pitch surfaces are hyperboloids of revolution. The tooth action between such gears is a combination of rolling and sliding along a straight line and has much in

common with that of worm gears. Figure IV-3 shows a pair of hypoid gears in mesh.

Figure IV-4 is included to assist in the classification of spiral bevel gearing. It is seen that the hypoid gear has a relatively small shaft offset. For larger offsets, the pinion begins to resemble a tapered worm and the set is then called *spiroid gearing*.

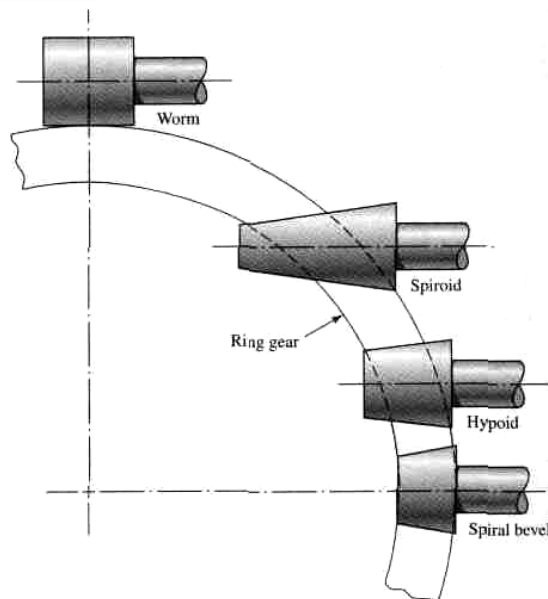
**Figure IV-3**

Spiroid bevel gears.  
(Courtesy of Gleason Works, Rochester, N.Y.)



**Figure IV-4**

Comparison of intersecting- and offset-shaft bevel-type gearings. (By permission, from Gear Handbook, McGraw-Hill, New York, 1962, p. 2-24.)



### **Straight Bevel Gears**

When gears are used to transmit motion between intersecting shafts, some form of bevel gear is required. A bevel gearset is shown in Fig. IV-5. Although bevel gears are usually made for a shaft angle of  $90^\circ$ , they may be produced for almost any angle. The teeth may be cast, milled, or generated. Only the generated teeth may be classed as accurate.

The terminology of bevel gears is illustrated in Fig. IV-5. The pitch of bevel gears is measured at the large end of the tooth, and both the circular pitch and the pitch diameter are calculated in the same manner as for spur gears. It should be noted that the clearance is uniform. The pitch angles are defined by the pitch cones meeting at the apex, as shown in the figure. They are related to the tooth numbers as follows:

$$\tan \gamma = \frac{N_p}{N_G} \quad \tan \Gamma = \frac{N_G}{N_p} \quad (IV.1)$$

where the subscripts  $P$  and  $G$  refer to the pinion and gear, respectively, and where  $\gamma$  and  $\Gamma$  are, respectively, the pitch angles of the pinion and gear.

**Figure IV-5**  
Terminology of bevel gears.

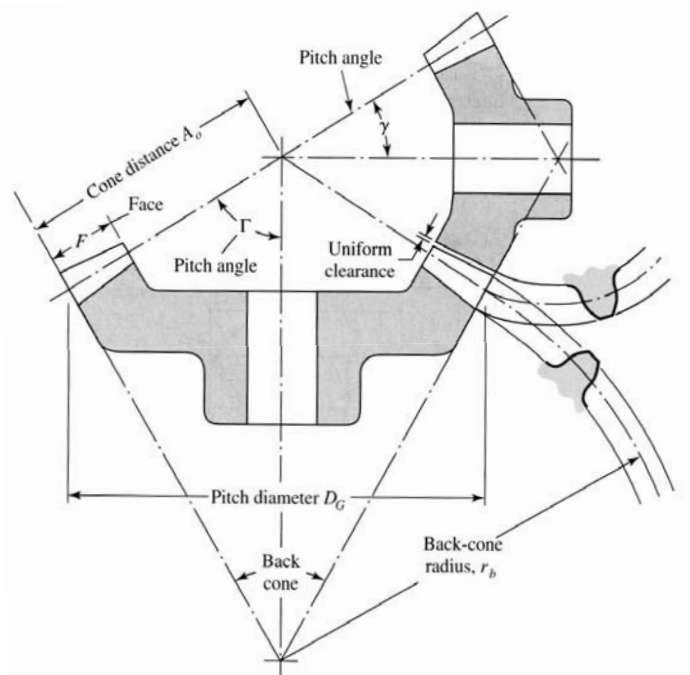


Figure IV-5 shows the shape of the teeth, when projected on the back cone, is the same as in a spur gear having a radius equal to the back-cone distance  $r_b$ . This is called Tredgold's approximation. The number of teeth in this imaginary gear is

$$N' = \frac{2\pi r_b}{P} \quad (IV.2)$$

where  $N'$  is the *virtual number of teeth* and  $c$  is the circular pitch measured at the large end of the teeth. Standard straight-tooth bevel gears are cut by using a  $20^\circ$  pressure angle, unequal addenda and dedenda, and full-depth teeth. This increases the contact ratio, avoids undercut, and increases the strength of the pinion.

## EXERCISES TO THE CHAPTER “BEVEL GEARING – GENERAL”

### 1. Words for understanding the text.

1. straight bevel gear	прямозубая коническая передача
2. spiral bevel gear	спиральнозубая коническая передача
3. zerol bevel gear	нулевая коническая передача
4. hypoid bevel gear	гипоидная зубчатая передача
5. spiroid bevel gear	спироидная зубчатая передача
6. stock (size)	ассортимент по размеру
7. permissible	допустимый, позволительный
8. hyperboloid of revolution	гиперболоид вращения
9. back-cone distance	длина образующей дополнительного конуса (равна радиусу делительной окружности эквивалентного колеса)
10. to occur	случаться, происходить
11. resultant	получающийся в результате
12. mounting distance	монтажное расстояние
13. crown rack	зубчатый венец
14. cutter radius	радиус резца

### 2. Read and translate the text.

### 3. Find the endings for the given beginnings:

A	B
1. Spiral bevel gears	a). have curved teeth but with zero spiral angle
2. Spiroid gears	b). are usually used for pitch-line velocities up to 1.000 ft/min
3. Hypoid gears	c). are recommended for higher speeds
4. A straight bevel gear	d). are similar to bevel gears but with shafts offset
5. Zerol bevel gears	e). resemble tapered worm

### 4. Match the notions and their definitions.

1). available	a). the force pushing an object
2). thrust	b). circular movement round a fixed point
3). revolution	c). able to be had, obtained, used, seen
4). permissible	d). to help or support
5). assist	e). allowed; that is permitted

**5. Think whether the given statements are true or false.**

1. Bevel gears are usually made for a shaft angle of 80.
2. The axial thrust loads for zero bevel gears are larger than for the spiral bevel gears.
3. Noise level is very important for spiral bevel gears.
4. Hypoid gears have large shaft offset.
5. Straight bevel gears resemble worm gears.

**6. Enlarge your vocabulary. Find word combination in the text which means the following:**

уровень шума; неважное обстоятельство; использовать ту же самую методику; пересекающиеся валы; зубцы могут быть отлиты; допустимый; зазор; вершина; иметь много общего; относительно маленькое смещение вала; подстрочный индекс; обычно; в небольших количествах; за исключением; передавать движение.

**7. Work in pairs. Make up a dialogue about different kinds of bevel gears.**

**8. Check your grammar. Insert prepositions given below. Some prepositions may be used twice.**

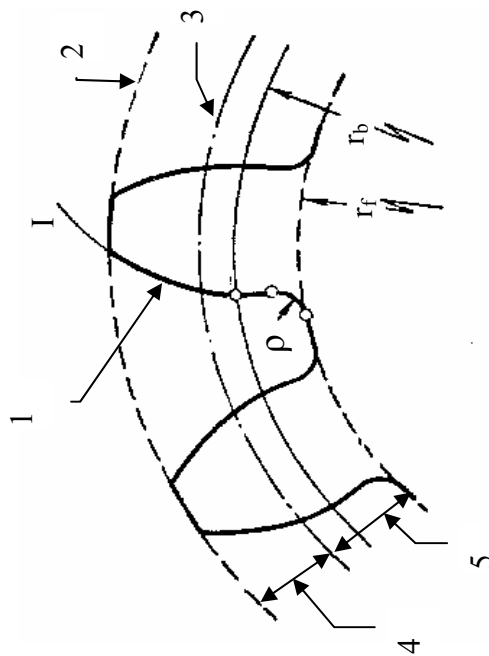
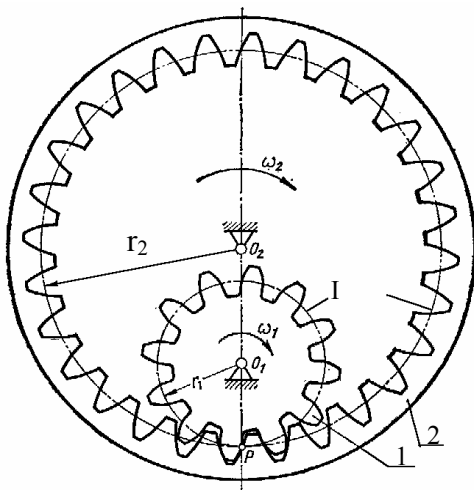
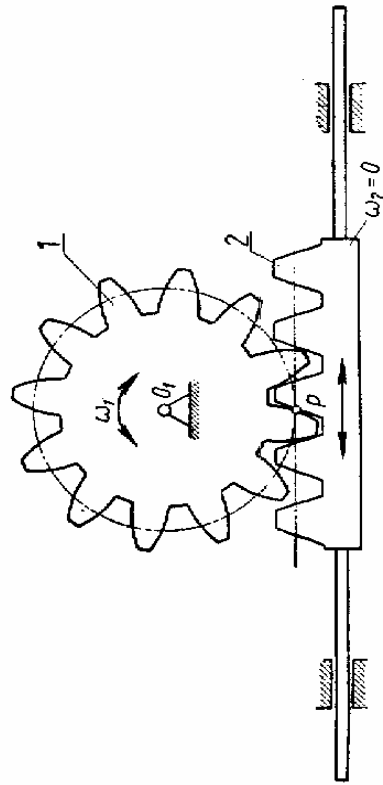
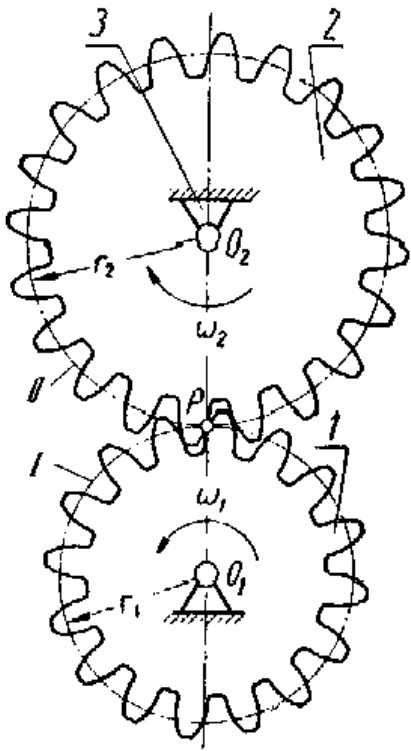
Spiral bevel gears are recommended \_\_\_\_\_ higher speeds and where the noise level is an important consideration. Spiral bevel gears are the bevel counterpart \_\_\_\_\_ the helical gear; it can be seen \_\_\_\_\_ Fig. IV-1 that the pitch surfaces and the nature \_\_\_\_\_ contact are the same as \_\_\_\_\_ straight bevel gears except the differences brought \_\_\_\_\_ by the spiral-shaped teeth.

**(for, about, of, in)**

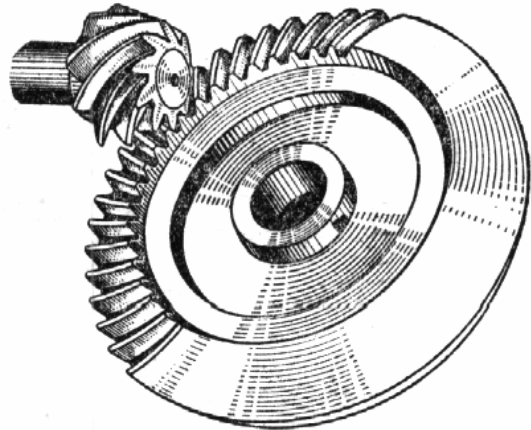
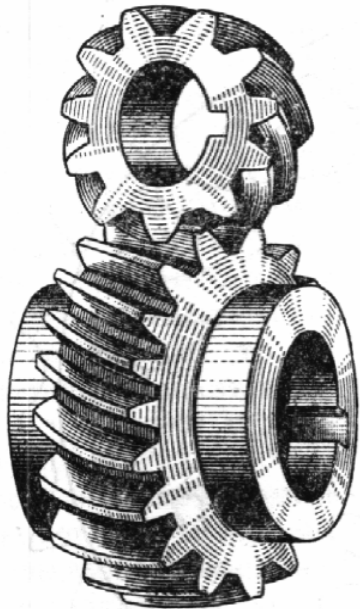
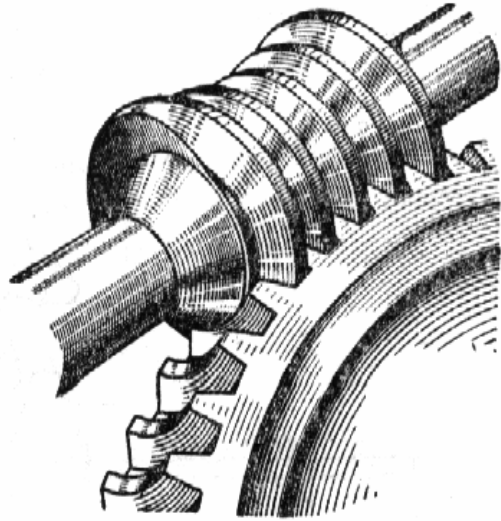
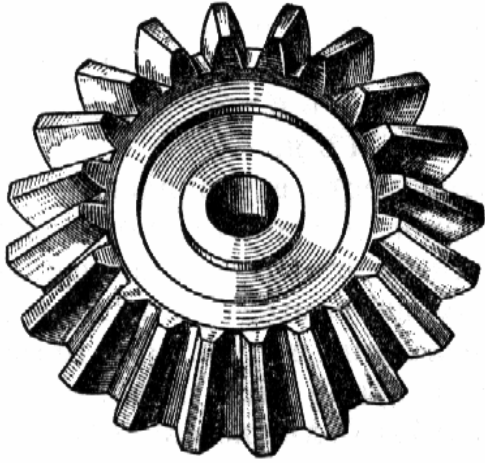
**9. Complete the sentences using correct Passive Voice of the verbs in brackets.**

1. A straight bevel gear (use) for pitch-line velocities up to 1.000 ft/min.
2. Standard straight-tooth bevel gears (cut) by using 20 pressure angle.
3. Such gears (call) hypoid gears.
4. It (see) that the hypoid gear has a relatively small shaft offset.
5. The definition of the spiral angle (illustrate) here.
6. Bevel gears usually (make) for a shaft angle of 90. They (may, produce) for almost any angle.

Appendix IV.1



Appendix IV.2





1

Draw the scheme of meshing and show the following elements:

1. *Tip relief*
2. *Tooth face*
3. *Whole depth*
4. *bottom land*

3

Draw the scheme of meshing and show the following elements:

1. *Roll angle*
2. *Base circle*
3. *Outside diameter*
4. *Pitch diameter*

2

Draw the scheme of meshing and show the following elements:

1. *Line of action*
2. *Pitch*
3. *Tooth surface*
4. *Addendum*

4

Draw the scheme of meshing and show the following elements:

1. *Pitch point*
2. *Pitch circle*
3. *Pressure angle*
4. *Root diameter*

## V. PLANETARY GEAR DRIVES

Read, translate the following text. Compare the style of the following text with the style of the text above. What is the difference between scientific, colloquial and belles styles? Give your conclusions.

### 1. A SIMPLIFIED APPROACH FOR DETERMINING POWER LOSSES AND EFFICIENCY OF PLANETARY GEAR DRIVES

By Eugene I. Radzimovsky

Associate Professor of Mechanical Engineering University of Illinois Urbana, III.

In a planetary gear drive, the power circulating within the system elements may differ appreciably from that being transmitted. This unique characteristic, which is a result of the relative motion of the gear members, is present in some degree in all planetary gear trains and has a significant effect on the performance and efficiency of systems transmitting power continuously. Power losses become a function of the power developed in the gearing rather than of the actual power transmitted through the train.

Several methods for determination of power losses and efficiencies of planetary gearing have been proposed, and expressions for a number of specific gear arrangements have been developed. Most of these analyses are based on the principle of equilibrium of moments first suggested by Massot.<sup>1</sup>

This article presents a new approach to the problem and offers a general method of solution, utilizing the concept of an "equivalent" conventional gear train in conjunction with the particular kinematic characteristics of planetary systems. Several representative planetary gear arrangements are analyzed to illustrate application of the principles discussed.

#### Nomenclature

$d$  = Pitch diameter of gear, in.  
 $e$  = Efficiency of planetary gear pair

$m_p$  = Overall speed ratio of planetary gear train  
 $n$  = Number of meshing pairs in a gear

$e_t$ = Overall efficiency of planetary gear train	train
$F$ = Tangential force acting at the pitch circle of the driving gear, lb	$P_i, P_o$ = Input and output power, respectively, ft-lb per sec
$L$ = Tooth mesh power loss in a planetary gear pair, ft-lb per sec	$R$ = Planetary velocity ratio defined by Equation 6
$L_t$ = Total tooth mesh power loss in a planetary gear train, ft-lb per sec	$v$ = Pitch-line velocity of gear pair, fps
$l$ = Tooth mesh loss in a conventional gear pair, ft-lb per sec	$v_e$ = Velocity of engagement of a gear pair, fps
$l_t$ = Total tooth mesh loss in a conventional gear train, ft-lb per sec	$\Delta$ = Ratio of tooth mesh loss to input power in a gear pair
$m$ = Speed ratio of gear pair	$\eta$ = Efficiency of conventional gear pair
	$\eta_t$ = Overall efficiency of conventional train
	$\omega$ = Angular velocity, radian·per sec

**Basic Considerations:** Certain general concepts of gear performance will be useful in analysis of planetary gear systems and perhaps should be reviewed.

The tooth mesh loss in a pair of gears with fixed centers is

$$l = \Delta Fv = \Delta P_i \quad (1)$$

where symbols are defined in Nomenclature. The efficiency of a drive system consisting of one such pair of gears is

$$\eta_t = \eta = \frac{P_o}{P_i} = \frac{P_i - l}{P_i} = 1 - \Delta. \quad (2)$$

For a train with more than one pair of gears,

$$\eta_t = \eta_1 \eta_2 \dots = (1 - \Delta_1)(1 - \Delta_2), \quad (3)$$

where subscripts 1 and 2 identify individual meshing pairs in the train. If  $\Delta_1 = \Delta_2 = \dots = \Delta_n = \Delta$ , then

$$\eta_t = (1 - \Delta)^n = \eta^n \quad (4)$$

Principal contributing sources of power losses in gear trains are as follows:

1. Sliding friction between meshing tooth surfaces.
2. Oil churning.
3. Friction in shaft support bearings.

Tooth mesh loss, as the first type will be called here, depends upon the geometry of tooth profiles and upon a "mean coefficient of friction"

between the mating surfaces. This mean coefficient of friction is a function of the acting tooth forces, gear material, contact surface condition, and the velocity of relative motion of these surfaces in the different phases of engagement. The last factor is also a function of geometry of the gearing.

Although analytical expressions have been developed for determination of tooth mesh loss, the relationships involved are rather complex and not always reliable. They appear to be best suited for comparative evaluation of different gear-tooth forms rather than for calculation of absolute values of gear losses. Available experimental data offer a more reliable source of information on tooth mesh power losses in gears than do the theoretical expressions. The following conclusions can be made:

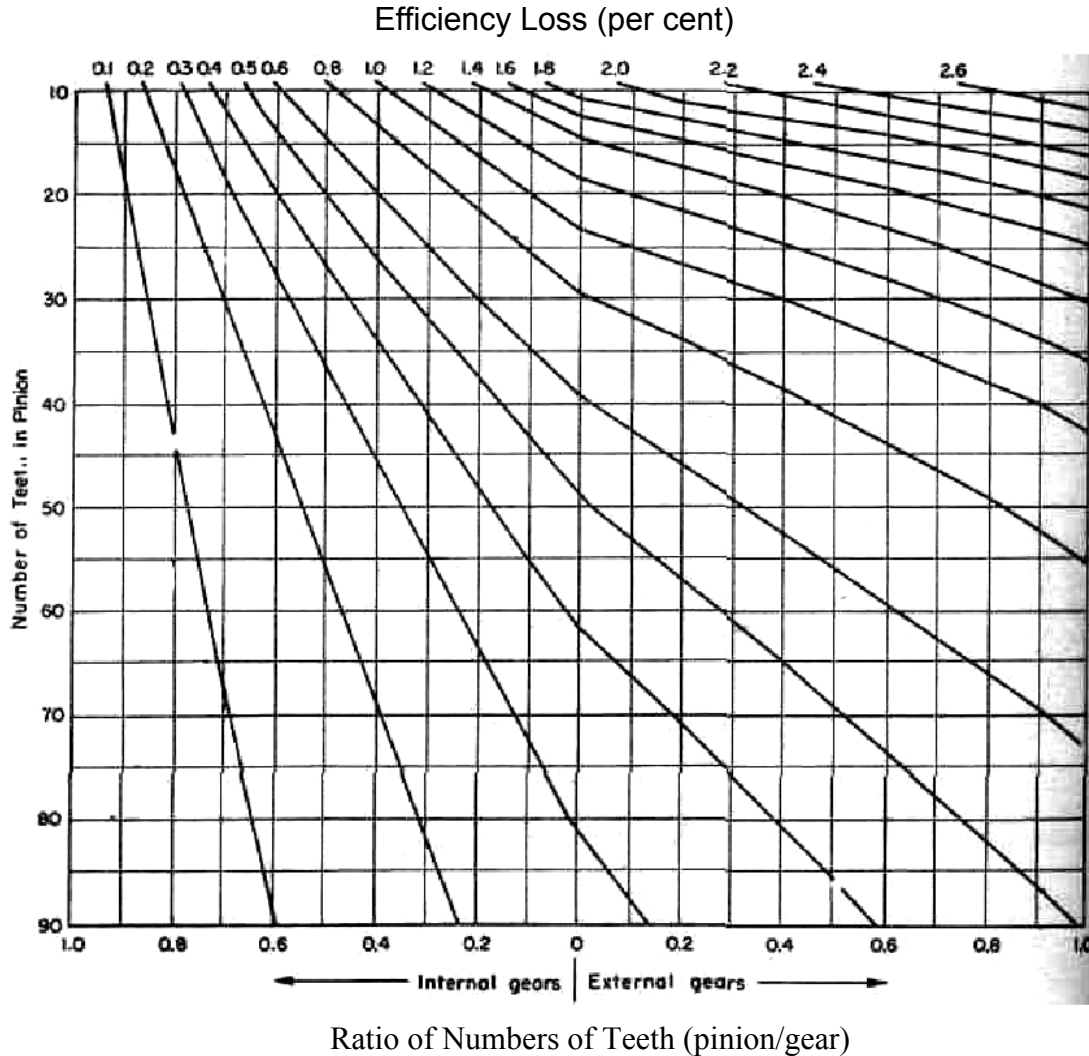
1. Tooth mesh losses in internal gears are somewhat less than those in external gears.
2. Pitch-line velocity has small influence on gear efficiency.
3. With the exception of light load conditions, it may be assumed that the ratio,  $\Delta$ , of tooth mesh loss to input power does not depend upon this power.
4. The difference in losses for several standard tooth forms in common use is so small that the same values of efficiency may be assumed for various tooth forms.
5. Losses from oil churning are independent of the load transmitted, but depend upon gear velocities, gear-drive design, and viscosity of lubricants. It seems that these losses can be neglected for gear trains which transmit relatively large power, but must be taken into account in the total efficiency of high-velocity, low-power gear systems.
6. For practical purposes, the loss in a pair of spur gears may be assumed to be proportional to the product of the pitch-line velocity and the tangential force acting at the pitch circle of the driving gear.

Merritt<sup>2</sup> has developed a chart, Fig. 1, which gives the value of  $\Delta$  for external and internal gear pairs as a function of the speed ratio of the two gears. In this chart, the abscissas represent the ratios between the numbers of teeth on pairs of gears; the numerator of the ratio is always the number of teeth on the smaller gear. Range of chart values includes pinions varying in size from 10 to 90 teeth. Plots are based on the sliding friction between a pair of spur gears of British Standard form having 20-degree pressure angle and full depth. A mean coefficient of friction equal to 0.08 is assumed. Fig. 1 may be used for a first

approximation in calculation of efficiency of gear trains, if more pertinent data are not available.

The chart values include loss for ball or roller bearings.

Fig. 1 — Efficiency losses from sliding friction between a pair of full-depth 20-degree spur gears of British standard form. Losses vary negligibly for different tooth forms and pressure angles



## 2. Equivalent System Concept

Tooth mesh loss of a pair of gears in a planetary train depends upon the same factors as for pair of gears with fixed centers. Here, also, it can be assumed that the loss in engagement is proportional to the product of the tangential force at the pitch circle of the driving gear and the velocity of engagement:

$$L = \Delta Fv \tag{5}$$

The essential difference between Equations 1 and 5 is in the magnitude of the velocity of engagement. In a conventional gear train with fixed axes of gear rotation,  $v_e$  is equal to pitch-line velocity  $v$  of the gear, and the maximum limit of the product  $Fv$  is equal to the input power of the train. In a planetary gear train, however, the velocity of engagement is affected by the relative motion of the planet cage. Thus, velocity of engagement  $v_e$  is not equal to pitch line velocity  $v$ ; and product  $Fv_e$  is not equal to the input power at the driving gear of the pair.

For comparison, two gear pairs will be considered, Fig. 2: one in a planetary gear train, the other in a conventional train. Assume that the geometry, size, and other characteristics of the corresponding gears are the same. Tangential forces  $F$  acting on both gear pairs are equal, and the pitch-line, or absolute, velocity of the sun gear in the planetary train is the same as that for the corresponding gear in the conventional train.

The ratio of tooth mesh losses in these "equivalent" gear pairs is

$$\frac{L}{l} = \frac{\Delta F v_e}{\Delta F v} = \frac{v_e}{v} = R \quad (6)$$

where  $R$  is defined here as the planetary velocity ratio. Therefore, the tooth mesh loss in a pair of gears in a planetary train is

$$L = lR = \Delta F v R = P_i \Delta R = P_i R(1 - \eta) \quad (7)$$

where  $l$  is the loss in an equivalent gear pair with fixed centers and the same input power as the planetary pair. Fig. 2.

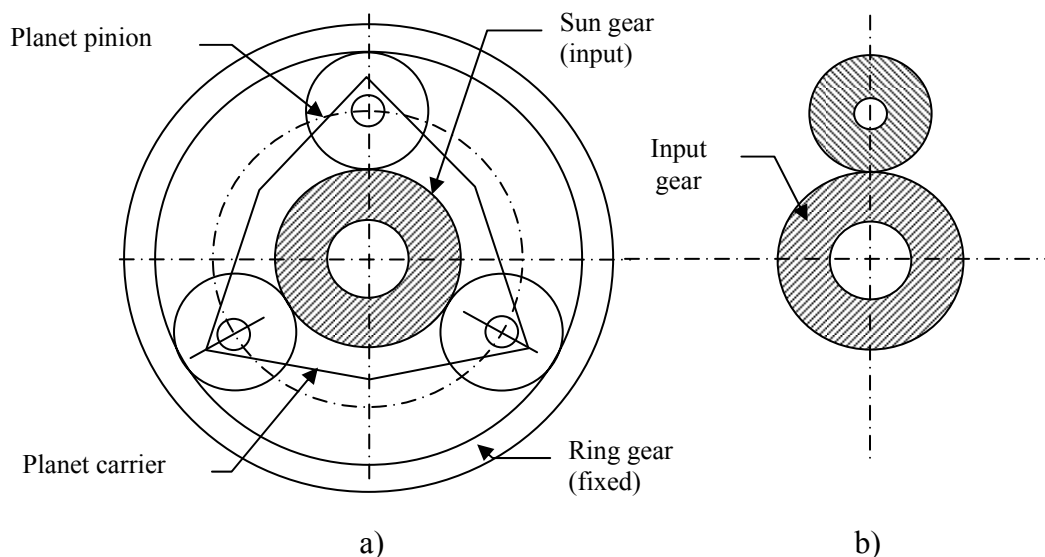


Fig. 2—Comparison of equivalent gear pairs in, *a*, simple planetary system and *b* conventional gear train

The efficiency of this planetary gear pair is given by

$$e = \frac{P_0}{P_i} = \frac{P_i - L}{P_i} = 1 - \Delta R = 1 - R(1 - \eta) \quad (8)$$

This same method may be applied for determining losses in any planetary gear pair and, consequently, for determining the efficiency of the entire planetary train.

### 3. Simple Planetary Train

This method may be best illustrated by an example. Consider the simple planetary gear train, Fig. 2a, which is represented schematically in Fig. 3a. This system is one of the most compact planetary gear arrangements having a wide range of application possibilities in design. Design of an actual planetary train of this type is shown in Fig. 4. In this train, Fig. 3a, planet gear *B* engages simultaneously with sun gear *A*, which is keyed to input shaft *D*, and with fixed internal (ring) gear *C*. Planet gear *A* rotates freely around its own axis on a shaft mounted to the planet cage, *G*, which is keyed to and drives output shaft *H*.

The speed ratio of this train is

$$m_p = \frac{\omega_D}{\omega_H} = \frac{\omega_A}{\omega_G} = 1 + \frac{d_C}{d_A} \quad (9)$$

where subscripts identify the corresponding system elements. The value of  $m_p$  is always positive in this arrangement and input and output shafts always rotate in the same direction.

Assume now that the planet cage is "stopped" by adding to the entire system an additional angular velocity,  $-\omega_H$ , equal in absolute value to the angular velocity of the output shaft, but opposite in direction. The relative motions of all members of the planetary train are unchanged. However, planet gear *A* now becomes an idler, and the entire train may be considered as a conventional gear train with fixed axes of rotation. Since the external torque applied to input shaft does not change, the tangential forces acting on the gears are the same as those in the actual planetary train. The only change is in the pitch-line velocities of the gears; they are now equal to the velocities of engagement of the gears in the planetary train. This train with "stopped" planet cage is depicted in Fig. 3b.

In Fig. 3b, the absolute angular velocity,  $\omega'_A$ , of gear A' is given by

$$\omega'_A = \omega_A + (-\omega_H) = \omega_A - \frac{\omega_A}{m_p} = \omega_A \left( 1 - \frac{1}{m_p} \right) \quad (10)$$

Velocity  $\omega'_A$ , which is the pitch-line velocity of gear  $A'$ , is thus also the velocity of engagement of gear  $A$  in Fig. 3a. Since the value of  $m_p$  for this planetary train is always greater than one, the value of  $\omega'_A$  is always positive if  $\omega_A$  is positive. The planetary velocity ratio,  $R_{AB}$ , for gears  $A$  and  $B$  is

$$R_{AB} = \frac{\omega'_A}{\omega_A} = \frac{\omega_A \left(1 - \frac{1}{m_p}\right)}{\omega_A} = 1 - \frac{1}{m_p} \quad (11)$$

This result means that the velocity of engagement of gear  $A$  in the planetary train, Fig. 3a, is  $R_{AB}$  times the pitch-line velocity of this gear, and the power developed by gear  $A$  is  $R_{AB}$  times the power at input shaft  $D$ .

With the planet cage stopped and planet gear  $B$  serving as an idler, Fig. 3b, all gears in the train have the same pitch-line velocity. Therefore,  $R_{BC}$ , the planetary velocity ratio for gears  $B$  and  $C$ , is equal to  $R_{AB}$ , and  $R_{AB} = R_{BC} = R$  is a constant value for a given planetary train.

To derive an expression for the efficiency of the planetary train, it is important to know which gear in the train with the stopped planet cage is the driver. The fact that gear  $A$  is the driver in the planetary train, Fig. 3a, does not necessarily mean that gear  $A'$  has the same function in the stopped system, Fig. 3b.

In any gear train, an external torque applied to the input shaft acts in the direction of rotation of this shaft. Therefore, the product of torque and angular velocity at the input shaft is always positive. The external resisting torque acting on the output shaft, however, always acts in a direction opposite to that of shaft rotation. Therefore, the product of external torque and angular velocity for the output shaft is always negative.

For the present case, Fig. 3, let it be assumed that  $\omega_D$ ,  $\omega_{D'}$ , and the external torque acting on shaft  $D'$  all have positive values. Thus, the product of torque and angular velocity at shaft  $D'$  is positive, and gear  $A'$  is the driver. In this instance, the position of the driving gear is the same in both trains, Figs. 3a and b.

The power,  $P_{AB}'$ , developed between gears  $A'$  and  $B'$  in the equivalent train, Fig. 3b, is:  $P_{AB}' = P_i R$ . Equal power is developed by gears  $A$  and  $B$ , Fig. 3a. If the tooth mesh loss ratio for gears  $A'$  and  $B'$  is  $\Delta_1$ , and the for gears  $B'$  and  $C'$  is  $\Delta_2$ , the power delivered at output shaft  $H'$  is

$$P_H' = P_i R (1 - \Delta_1)(1 - \Delta_2) = P_i R \eta_t \quad (12)$$



where  $\eta_t$  = the overall efficiency of a conventional gear train consisting of the same gears as the planetary train. Therefore, in the planetary train and its equivalent, which have equal losses,

$$L_t = P_i R - P_i R \eta_t = P_i R (1 - \eta_t) \quad (13)$$

The efficiency of a planetary train of this type is

$$e_t = \frac{P_i - L_t}{P_i} = \frac{P_i - P_i R (1 - \eta_t)}{P_i} = 1 - R (1 - \eta_t) \quad (14)$$

From Equations 11 and 14

$$e_t = 1 - (1 - \eta_t) \left( 1 - \frac{1}{m_p} \right) \quad (15)$$

Ring gear  $C$  is always larger than sun gear  $A$  so that the value of  $R$  for this planetary train is always smaller than one but larger than 0.5. The tooth mesh losses in this train, then, are smaller than, but not less than half, the tooth losses in a conventional train consisting of the same gears. Consequently, the efficiency of this type of planetary train is always higher than the efficiency of a corresponding conventional train but lower than the efficiency of one pair of gears with fixed centers and the same unit tooth mesh loss ratio,  $\Delta$ .

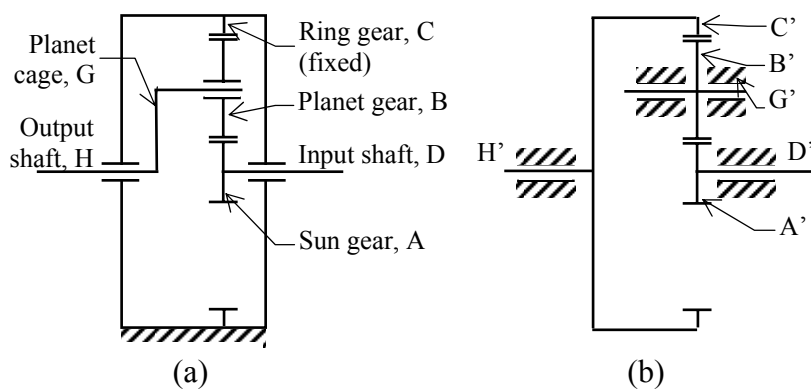


Fig. 3—Simple planetary gear system showing, *a*, schematic arrangement of system elements and, *b*, equivalent system obtained by "stopping" the planet cage. Prime notations identify corresponding members

Generally speaking, this efficiency may be considered as highly satisfactory for planetary gear systems. However, this type of train can be de-designed only for relatively small speed ratios.

**Sample Calculations:** A planetary gear train of the type shown in Fig. 3 has the following design data:  $d_C = 12$  in.,  $d_A = 2$  in.,  $d_B = 5$  in., and  $m_p = 7$ . For gears  $A$  and  $B$ , efficiency  $\eta_1 = 0.98$ , and for gears  $B$  and  $C$ ,  $\eta_2 = 0.99$ .

The efficiency of a conventional train with the same gears is  $\eta_t = \eta_1 \eta_2 = 0.9702$ . For the planetary train, Fig. 3a, using Equation 11,  $R_{AB} = R_{BC} = R = 1 - (1/7) = 6/7$ . From Equation 14,  $e_t = 1 - (6/7)(1 - 0.9702) =$

0.97446. In this case, the velocity of engagement in the planetary train is somewhat less than that in a corresponding conventional train consisting of the same gears and having the same angular velocity and power at the input shaft. Since the tangential forces acting on the gears are the same, the power developed by the planetary gears is lower than in the conventional train. Dynamic tooth loads and wear, which depend upon the tangential force and the velocity of engagement, are also lower in the planetary train.

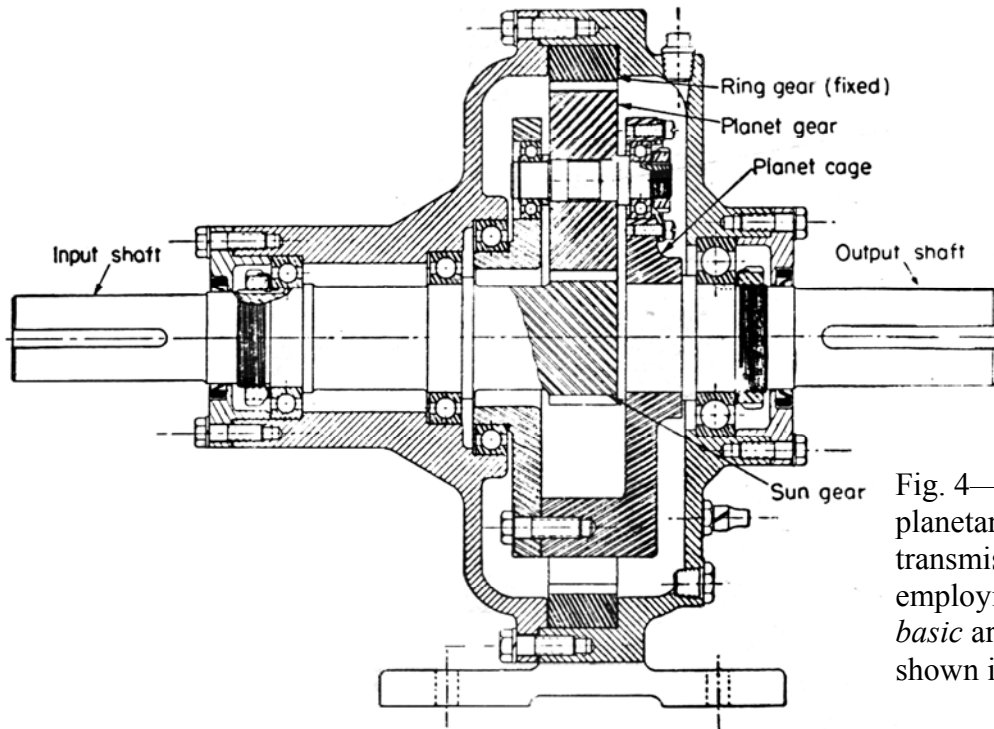


Fig. 4—Typical planetary gear transmission design employing the *basic* arrangement shown in Fig. 3a

**Modified Train Arrangement:** A planetary gear train similar to the one just considered, but with gear *A* as the fixed member, is shown schematically in Fig. 5. Internal gear *C* becomes the driver, and output shaft *C* is connected to planet cage *G*. The speed ratio of this train can be determined from the following relationship:

$$m_p = \frac{\omega_D}{\omega_H} = \frac{\omega_C}{\omega_G} = 1 + \frac{d_A}{d_C} \quad (16)$$

After the angular velocity  $-\omega_H$  is added to the entire system, as in the previous analysis, the angular velocity of gear *C* becomes

$$\omega'_C = \omega_C + (-\omega_H) = \omega_C \left( 1 - \frac{1}{m_p} \right) \quad (17)$$

The value of  $m_p$  is always positive and greater than one; therefore, the direction of  $\omega'_c$  is positive if  $\omega_C$  is positive. The torque acting on

input shafts  $D$ , and  $D'$ , is also positive. Thus, the product of torque and angular velocity for gear  $C'$  will also be positive, making this gear the driver in the train with the stopped planet cage.

Planetary velocity ratio, output power, tooth mesh loss and efficiency for this train can be determined from Equations 11 to 15. There is a difference, however, in the value of  $R$ . In this train,  $R$  is always smaller than 0.5. Therefore, the efficiency of this train is considerably higher than that of a conventional train with comparable gear arrangement. Here again, however, the speed ratios possible with trains of this type are relatively small.

Other combinations of fixed, input and output members are also possible with the basic system shown in Fig. 2a. Each arrangement will constitute a different mechanism having different speed ratio and efficiency. These efficiencies may be easily determined by the foregoing method.

#### 4. Compound Systems

The speed ratio of a planetary train can be increased by using the so-called compound arrangement shown schematically in Fig. 6. In this train, sun gear  $A$  is connected to input shaft  $D$ . Output shaft  $C$  is connected to planet cage  $G$  which carries planet gears  $B$  and  $J$ . Planet gear  $J$  engages the fixed ring or internal gear,  $C$ . Speed ratio of this train is

$$m_p = \frac{\omega_D}{\omega_H} = 1 + \frac{d_C d_B}{d_A d_J} \quad (18)$$

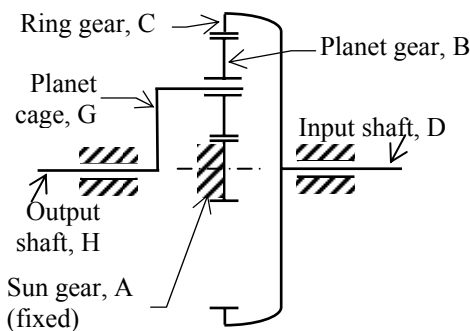


Fig. 5—Simple planetary gear train in which the sun gear is the fixed member. The train is a modification of the arrangement shown in Fig. 3a

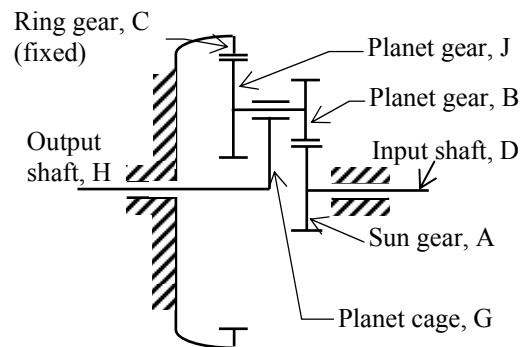


Fig. 6—Compound planetary gear train for large speed ratios

This expression shows that the speed ratio which can be obtained through this train is approximately of the same magnitude as can be obtained by means of a conventional gear train having two pairs of gears.

Assume that input shaft  $D$  revolves clockwise. The speed ratio  $m_p$  is always positive; therefore, the angular velocity of output shaft  $H$ ,  $\omega_H = \omega_A/m_p < \omega_A$ , is also positive.

When angular velocity  $-\omega_H = -\omega_A/m_p$  is added to the system to stop the planet cage, the value of  $\omega_A'$  which is the new absolute angular velocity of gear  $A$  is given by Equation 10. Because the value of  $m_p$  for this train is greater than one,  $\omega_A'$  is positive if  $\omega_A$  is positive. Therefore, the product of torque and angular velocity for gear  $A'$  will be positive, making this gear the driver in the train with the stopped cage.

Following the same reasoning, it can be shown, that the expression for efficiency for this planetary train is the same as for the previous two types (Equation 14). The difference lies in the value of  $R$ . For this train,

$$R = \frac{\omega'_A}{\omega_A} = 1 - \frac{1}{m_p} = \frac{d_C d_B}{d_A d_J + d_C d_B} \quad (19)$$

It can be seen that  $R$  for this train is always smaller than one. The efficiency of this train must therefore be somewhat higher than that of a comparable conventional gear train consisting of two gear pairs with the same tooth mesh loss ratios.

**Planetary Differential:** A typical differential planetary gear train Fig. 7a, has the planet cage connected to the input shaft, and sun gear  $K$  is driven member. The speed ratio of this train is

$$m_p = \frac{\omega_D}{\omega_H} = \frac{1}{1 - \frac{d_A d_J}{d_B d_K}} = \frac{d_B d_K}{d_B d_K - d_A d_J} \quad (20)$$

If the value of  $m_p$  is positive, the output and input shafts rotate in the same direction. If  $m_p$  is negative, they revolve in opposite directions. The most interesting peculiarity of this train is that a speed ratio of any magnitude, even infinity ( $\omega_H = 0$ ), can theoretically be obtained.

The efficiency of this train may be determined by the methods discussed previously. Assume that input shaft  $C$  revolves in a clockwise (positive) direction and  $d_A d_J / d_G d_K < 1$  so that  $m_p > 1$  and is positive. To stop planet cage  $G$ , let the entire system be revolved at an angular velocity  $-\omega_D$ . With the planet cage stopped, the absolute angular velocity of gear  $K$  becomes

$$\omega'_K = \omega_K + (-\omega_D) = \frac{\omega_D}{m_p} - \omega_D = -\omega_D \left( 1 - \frac{1}{m_p} \right) \quad (21)$$

For the assumed conditions, the value of  $\omega_K'$  is negative. Actually, in the planetary train, Fig.7a, gear K has a positive direction of rotation and, being connected to the output shaft, is loaded by an external torque acting in a direction opposite to its motion. Thus, the external torque acting on shaft C and gears K and K' is negative. The product of negative angular velocity and negative torque is positive. Therefore, gear K' is the driver in the equivalent train obtained by stopping the planet cage.

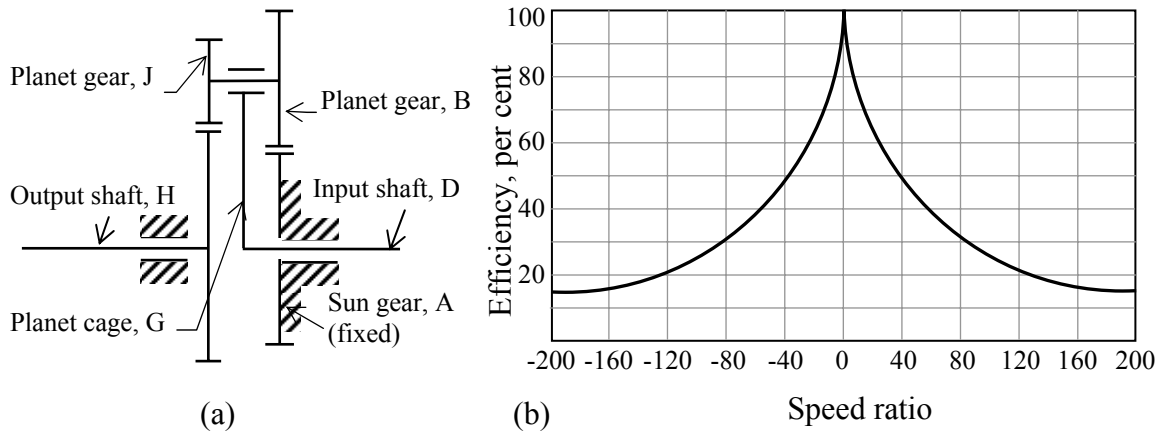


Fig.7–Basic differential planetary gear system showing, a, arrangement of system elements and, b, relationship between efficiency and speed ratio

This case is an example of an interesting peculiarity in analysis. The gear that must be considered as the driver for analysis of train performance is actually connected to the driven shaft of planetary train and appears, at first, to be a driven member. Consequently, with the planet cage fixed, gear A', becomes the driven member. Now the expression for efficiency may be derived. If the power at output shaft C of the planetary train is  $P_0$ , the power developed in the engagement of gears J and K is  $P_0R$ . Since gear K' is considered as the driver, the tooth mesh loss in this train is

$$L_t = P_0R(1 - \eta_t) \quad (22)$$

Efficiency of this train is

$$e_t = \frac{P_0}{P_0 + P_0R(1 - \eta_t)} = \frac{1}{1 + R(1 - \eta_t)} \quad (23)$$

Planetary velocity ratio  $R$  is

$$R = \frac{\omega'_K}{\omega_K} = \frac{-\omega_D \left(1 - \frac{1}{m_p}\right)}{\frac{\omega_D}{m_p}} = -(m_p - 1) \quad (24)$$

The value of  $R$  is negative. This result means that the motion of gear  $K$  relative to gear  $J$  in this planetary train is opposite in direction to the absolute velocity of gear  $K$ . However, the direction of relative rotation does not influence the magnitude of tooth mesh loss in the gear pair. Therefore, the absolute value of  $R$  must be used in the calculation of train losses and efficiency. Introducing this value into Equation 23 gives

$$e_t = \frac{1}{1 + (1 - \eta_t)(m_p - 1)} = \frac{1}{1 + (1 - \eta_t) \left( \frac{d_A d_J}{d_B d_K - d_A d_J} \right)} \quad (25)$$

Equation 25 shows that as the speed ratio of this train increases, the absolute value of  $R$  and, consequently, the tooth mesh losses in the train increase, while efficiency decreases. For trains of this type, having a large speed ratio, the velocities of gear engagement are relatively high and the efficiency relatively low. The power developed at the gears in such trains is  $R$  times higher than in a conventional train having the same size gears and the same speed and power at the *output* shaft.

For the train shown in Fig. 7a, assume that  $1 < d_A d_J / d_B d_K < 2$ . Such a train will reduce the speed and reverse the direction of rotation of the output shaft, so that  $m_p$  is now negative with an absolute value greater than one (Equation 20). In the planetary train, Fig. 7a, the angular velocity of the gear  $K$  is now negative, and the external resisting torque acting on output shaft  $H$  is positive.

When the planet cage is stopped, the angular velocity of gear  $K$  becomes

$$\omega'_K = -\omega_K + (-\omega_D) = -(\omega_K + \omega_D) \quad (26)$$

which is a negative value. The product of external torque acting on this gear and angular velocity is negative. Therefore, gear  $K'$  is a driven member in the equivalent train with a stopped planet cage. Gear  $A'$  is the driver in this case.

The power developed by gear  $K$  is  $P_0 R$ . The power of engagement of the driving gear  $A'$  is  $P'_{AB} = P_0 R / \eta_t$ . Tooth mesh loss in this train is

$$L_t = \frac{1}{\eta_t} P_0 R (1 - \eta_t) \quad (27)$$

Therefore, the efficiency of this train, considering tooth mesh loss only, is

$$e_t = \frac{P_0}{P_0 + \frac{1}{\eta_t} P_0 R (1 - \eta_t)} = \frac{\eta_t}{\eta_t + (1 - \eta_t) R} \quad (28)$$

The value of  $R$  for this train is given by

$$R = \frac{\omega'_K}{\omega_K} = \frac{-(\omega_K + \omega_D)}{\omega_K} = -(1 - m_p) \quad (29)$$

The absolute value of  $R$  must be used in Equation 28 for efficiency calculations.

**Sample Calculations:** A planetary differential train. Fig. 7a, has the following design data:

$d_A d_J / d_B d_K = 1.025$  and  $\eta_t = 0.97$ . From Equation 20,  $m_p = -40$ ; from Equation 29,  $R = -41$ ; and from Equation 28, using the absolute value of  $R=41$ ,  $e_t=0.4409$ . In a planetary gear train of this type with  $P_o = 1$  hp, the power developed by gears  $J$  and  $K$  is more than 41 hp while that developed by gears  $A$  and  $B$  is  $41/0.97 = 42.26$  hp. These conditions must be taken into account when designing gears for the planetary train.

The relationship between efficiency and speed ratio for, the differential planetary gear train, Fig. 7a, is shown in Fig. 7b. The left-hand portion of the curve represents the cases where speed ratio is negative, and the right-hand portion those with positive speed ratios. When  $m_p = +1$ , the entire system is revolving as one unit, without relative motion of its parts, and  $e_t = 1$  or 100 per cent. Actually, however, such a ratio is not possible in practice because it requires that  $d_A d_J / d_B d_K = 0$ . When,  $m_p = -1$ , the absolute value of  $R = 2$ , and the losses in this train are about twice as large as in a conventional train consisting of the same gears.

The chart indicates that efficiency of this type of planetary train decreases rapidly with the increasing speed ratio, and that gears in trains having large speed ratios operate under extremely severe service conditions. Apparently, trains of this type are most suitable for applications where speed ratio or power transmitted are relatively low.

**Reversed Power Flow:** If, in the train depicted in Fig. 7a, the sun gear  $K$  is made the input member and the shaft connected to the planet cage becomes the output member, a different mechanism will be obtained, Fig. 8a.

The speed ratio of this modified train is

$$m_p = 1 - \frac{d_A d_J}{d_B d_K} \quad (30)$$

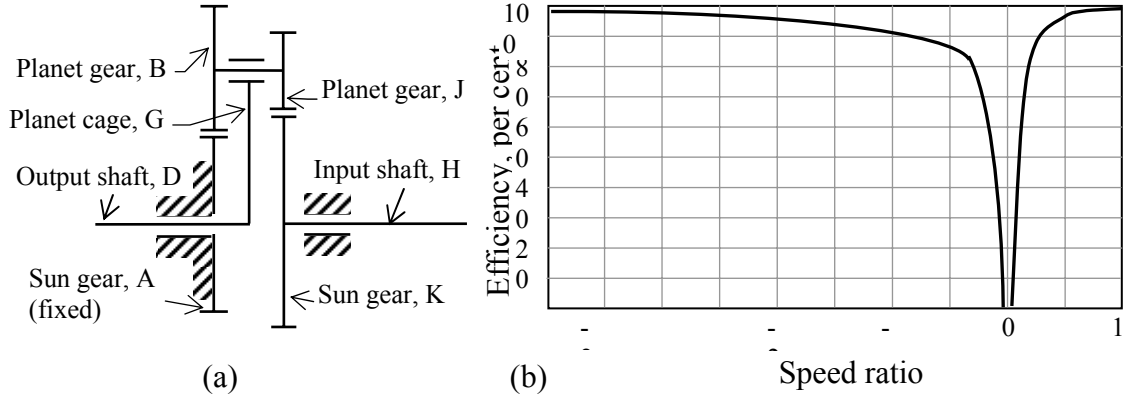


Fig. 8—Differential planetary gear system showing, *a*, arrangement of system elements and, *b*, relationship between efficiency and speed ratio. This train is basically the same as the one shown in Fig. 7*a* except that direction of power flow has been reversed

The value of  $m_p$  is positive or negative depending upon the value of  $d_A d_J / d_B d_K$ . Assume input shaft *H*, connected to gear *K*, revolves clockwise. The external torque applied to this shaft acts in a positive direction. Assume also, that  $d_A d_J / d_B d_K < 1$  so that  $1 > m_p > 0$ . This arrangement will provide a speed increase without reversing the direction of rotation. A positive speed ratio greater than one is theoretically impossible for this train, because it requires the value of  $d_A d_J / d_B d_K$  to be less than zero.

To "stop" planet cage *G*, it will be necessary to revolve the entire system at an angular velocity,  $-\omega_G$ . Then, the angular velocity of gear *K* becomes

$$\omega'_K = \omega_K + (-\omega_G) = \omega_K - \frac{\omega_K}{m_p} = -\omega_K \left( \frac{1}{m_p} - 1 \right) \quad (31)$$

For the assumed conditions, the value of  $\omega'_K$  is negative. Therefore, the product of positive torque and negative angular velocity is negative, and gear *K'* will be a driven member in the equivalent train with a stopped planet cage. Gear *A'* will thus be the driver.

If gear *A'* is the driver,  $P'_{AB} = P_i R / \eta_t$  and the tooth mesh loss in the planetary train is

$$L_t = \frac{1}{\eta_t} P_i R (1 - \eta_t) \quad (32)$$



Efficiency of the train may be expressed by

$$e_t = \frac{P_i - L}{P_i} = \frac{\eta_t - (1 - \eta_t)R}{\eta_t} \quad (33)$$

where

$$R = \frac{\omega'_K}{\omega_K} = -\left(\frac{1}{m_p} - 1\right) \quad (34)$$

Introducing the absolute value of  $R$  into Equation 33 gives

$$e_t = \frac{\eta_t - (1 - \eta_t)\left(\frac{1}{m_p} - 1\right)}{\eta_t} \quad (35)$$

It is interesting to note that  $e_t=0$  when

$$\eta_t = 1 - \eta_t\left(\frac{1}{m_p} - 1\right)$$

and this mechanism becomes self-locking.

Consider now a train of this type which has ratio  $d_A d_J / d_B d_K > 1$ . Such a mechanism will increase speed and reverse the direction of rotation when  $1 < d_A d_J / d_B d_K < 2$ . It will reduce speed and reverse the direction of rotation if  $d_A d_J / d_B d_K > 2$ . This latter case will be assumed for illustration. In this reduction drive,  $m_p$  is negative. Angular velocity of the planet cage is  $\omega_G = \omega_K / m_p$  and the angular velocity of gear  $K$  with the planet cage stopped becomes

$$\omega'_K = \omega_K + \left(-\frac{\omega_K}{m_p}\right) = \omega_K \left(1 - \frac{1}{m_p}\right) \quad (36)$$

Because  $m_p$  in this case has a negative value,  $\omega'_K$  is positive. The product of torque and velocity at gear  $K'$  is positive, making this gear the driver in the equivalent train. Therefore, the tooth mesh loss in this train is given by Equation 13, and the efficiency of planetary trains of this type, in which the speed ratio is negative ( $d_A d_J / d_B d_K > 1$ ), can be determined from Equation 14.

When a  $R(1 - \eta_t) = 1$ ,  $e_t = 0$ . The value of  $R$  for this train is given by Equation 11, where  $m_p$  is a negative value. The relationship between efficiency and speed ratio for this planetary train arrangement, Fig. 8a, is shown in Fig. 8b. This chart is based on the assumption that  $\eta_t = 0.97$ . When  $m_p = +1$ , the entire system revolves as a unit and  $\eta_t = 1$ , or 100 per

cent. Actually, such a speed ratio is impossible to realize in practice because it requires that  $d_A d_J / d_B d_K = 0$ . Also, since the value of this diameter ratio cannot be negative,  $m_p = +1$  is the theoretical maximum limit for positive speed ratios in this train.

When  $\eta_t = R(1 - \eta_t)$ ,  $\eta_t = 0$ . For  $\eta_t = 0.97$  this condition corresponds to  $m_p = +0.03$ . When the speed ratio is negative,  $\eta_t = 0$  corresponds approximately to  $m_p = -0.031$ . Efficiency increases as the absolute value of speed ratio increases, approaching the value of  $\eta_t$  asymptotically as a higher limit. For  $m_p = -25$ , the efficiency of the train is 0.9688. This value may be considered as highly satisfactory for a planetary gear train of this type. However, this type of train can be designed only for relatively small speed ratios, having a maximum absolute value that is approximately of the same magnitude as may be obtained by two pairs of gears in a conventional gear train.

**Modified Differential System:** In another type of differential planetary gear train, Fig. 9, the sun gears are in the form of internal gears *A* and *K*. In this particular train, gear *A* is fixed and *K* is connected to the output shaft. Planet cage *G* is connected to input shaft *D*.

For this train, as in the arrangement shown in Fig. 7a, speed ratio is given by Equation 20. These two differential systems also have the same expressions for  $\eta_t$  and  $R$ . Here again, two principal cases must be considered: (1)  $d_A d_J / d_B d_K < 1$  and (2)  $d_A d_J / d_B d_K > 1$ . In the first case, gear *K* is the driver in the equivalent train with the planet cage stopped, and Equations 23, 24 and 25 must be used for efficiency calculations. For the second case, gear *A* becomes the driver and Equations 28 and 29 are pertinent.

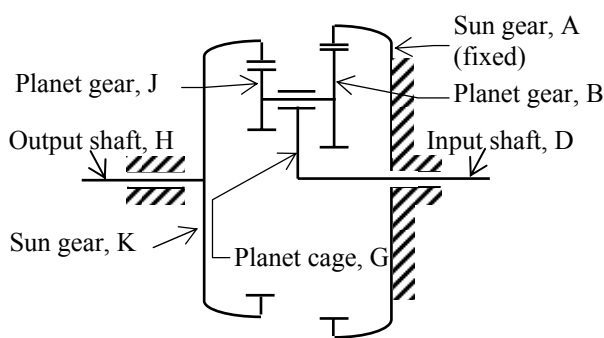


Fig. 9-Modified differential system in which the sun gears are internal gears

The only difference between the two train arrangements, Figs. 7a and 9, lies in the values for  $\Delta_1$ ,  $\Delta_2$ , and  $\eta_t$  of the train. The losses in internal gears, if all other factors remain the same, are somewhat lower than for comparable external gear pairs. Therefore, the value of  $\eta_t$  may be considerably smaller for the

internal gear arrangement, Fig. 9, than for the external gear system, Fig. 7a. Total efficiency for a planetary train of this type will thus be higher for a given speed ratio if internal gears are used.

## Summary

Many different planetary gear arrangements can be obtained by varying the positions and size of the driving, driven, and fixed gear members. In each case, tooth mesh loss, efficiency, and power developed by the gears can be readily determined by the approach just discussed. Analysis of various planetary train types shows that efficiencies of all simple and compound systems may be expressed by one of four basic formulas; Equations 14, 23, 28 and 33.

Two basic conditions determine which one of these expressions must be used for a given case: (1) Whether the input shaft of a planetary train is connected to one of the sun gears or to the planet cage and (2) whether the position of the input shaft of the planetary train coincides or does not coincide with the position of the driving member of the equivalent train obtained by stopping the planetary cage.

This last condition depends upon the train design and, in some cases such as the differential systems, upon the diameter ratios of the gears constituting the given train. The magnitude of  $R$  in these expressions is a function of the type of planetary train and gear size relationships. This value can be positive or negative. In all cases, the absolute value of  $R$  must be used for calculation of tooth mesh losses and efficiency. In the expressions developed here, losses due to oil churning, as well as those introduced by the support members, have not been considered.

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## EXERCISES TO THE CHAPTER "PLANETARY GEAR"

### 1. Learn these words and phrases.

1.	appreciably	- заметно, ощутимо
2.	abscissa	- абсцисса
3.	angular velocity	- угловая скорость
4.	approximation	- приближенное значение
5.	assume	- предполагать
6.	ball bearing	- шариковый подшипник
7.	calculation	- вычисление, расчет
8.	chart	- диаграмма
9.	comparative evaluation	- сравнительная оценка
10.	conventional gear train	- последовательная зубчатая передача
11.	equation	- уравнение
12.	equilibrium	- равновесие
13.	fixed gear	- неподвижное колесо
14.	gear train	- сложная зубчатая передача
15.	geometry	- размер, геометрия
16.	idler	- промежуточное зубчатое колесо
17.	infinity	- бесконечность
18.	input power	- входная мощность
19.	input shaft	- входной вал
20.	key	- закреплять клином
21.	magnitude	- значение
22.	mean coefficient of friction	- средний коэффициент трения
23.	numerator	- числитель
24.	oil churning	- разбрызгиватель
25.	output shaft	- выходной вал
26.	overall efficiency	- общий результирующий КПД
27.	pitch-line velocity	- скорость по делительной прямой
28.	planet cage	- планетарное водило
29.	planet carrier	- водило планетарного ряда
30.	planet pinion	- сателлит планетарный
31.	plot	- график
32.	product	- произведение
33.	reverse power	- обращенная мощность
34.	ring gear	- зубчатое колесо внутреннего зацепления
35.	roller bearing	- роликовый подшипник
36.	sliding friction	- трение скольжения

- 37. solution - решение
- 38. sun gear - солнечное, центральное колесо
- 39. support bearing - опорный подшипник
- 40. tangential force - касательная (окружная) сила
- 41. utilize - использовать
- 42. velocity - скорость

**2. Fill in the following table.**

<i>Noun</i>	<i>Adjective</i>	<i>Verb</i>
Development		
	determinate	
		suggest
	representative	
Function		
	characteristics	
Illustration		
		compare
	dependent	
Slide		

**3. Find the words which mean the opposite to these:**

Outside; resemble; insignificant; first; simple; external; unreliable; inappreciably; useless; vanish; refuse; these; dependent.

**4. Find the words which mean the same:**

Reject; dependable; transfer; though; substantial; suggest; varied; appropriate; overall; modify; determine.

**5. Give Russian equivalents to the following words.**

1. conventional gear train	11. contributing source
2. sliding friction	12. velocity
3. input power	13. pitch-line velocity
4. chart	14. abscissas
5. gear train	15. plot
6. significant effect	16. appreciably
7. ball bearing	17. ring gear
8. product	18. planet pinion
9. planet cage	19. planet carrier
10. magnitude	20. input gear

**6. Give English equivalents to the following words.**

1. потеря мощности	11. средний коэффициент трения
2. использовать	12. опорный подшипник
3. водило планетарного ряда	13. приближенное значение
4. предполагать	14. решение
5. солнечное зубчатое колесо	15. неподвижное колесо
6. роликовый подшипник	16. закреплять клином
7. произведение	17. входной вал
8. разбрызгиватель	18. уравнение
9. числитель	19. сравнительная оценка
10. входная мощность	20. режим нагрузки

**7. Match the notions and their definitions.**

1. Bearing	a. speed in a certain direction
2. Friction	b. to believe (something to be true without actually having proof that it is suppose
3. Power	c. generally the curved surface on a shaft or in a bore, or the part assembled onto one or into the other to permit relative rotation with minimum wear and friction
4. Equation	d. the resistance to motion between two bodies in contact with each other
5. Velocity	e. it is the rate of doing work .The unit of power is usually foot-pounds per minute or foot-pounds per second
6. Gear	f. to change to the opposite
7. Assume	g. a statement that two quantities are equal
8. Reverse	h. an apparatus especially one consisting of a set of toothed wheels

**8. Find the endings to the given beginnings.**

1. Power losses become a function of...	a. depends upon the same factors as for pair of gears with fixed centers.
2. Tooth mesh loss of a pair of gears in a planetary train...	b. has small influence on gear efficiency.
3. In a planetary gear the velocity of engagement is...	c. the power developed in the gearing.
4. Most of the analyses are based on the...	d. it is important to know which gear in the train with the stopped planet cage is driver.
5. Pitch-line velocity...	e. principles of equilibrium of moments

6. To derive an expression for the efficiency of the planetary train...	first suggested by Massot.
7. Ring gear is...	f. always larger than sun gear. g. affected by the relative motion of the planet cage.

**9. Tick the statements which are true:**

1. Power losses become a function of the power transmitted in the gearing.
2. Tooth mesh losses in external gears are less than in internal gears.
3. One of the main sources of power losses is oil-churning
4. Ring gear is always smaller than sun gear.
5. Pitch-line velocity has great influence on gear efficiency.
6. A mean coefficient of friction is equal to 0, 09.

**10. Translate the following verb forms.**

Methods have been proposed; gear arrangements are analyzed; general concepts should be rewired; expressions have been developed; to be best suited; conclusions can be made; must be taken into account; analyses are based; symbols are defined; gear pairs will be considered.

**11. Read the paragraph “Planetary gear drives” again and tell it in English using the following words and word combination:**

Planetary gear drive; significant effect; efficiency; power losses; power developed; principle of equilibrium; equivalent conventional gear; kinematic characteristics; sliding friction; geometry of tooth profiles; comparative evaluation; pitch-line velocity; coefficient of friction.

**12. Discuss the theme “Simple planetary system” using the following as phrase-openings.**

1. The problem is
2. I want to tell about
3. The main idea of the chapter is
4. As far as I know
5. The key problem of the article
6. Next I have to point out
7. I want to note
8. We have to keep in the mind
9. At last it’s necessary to

**13. Suggest your own headings for the paragraphs “Simple planetary system” and “Compound systems”. Compare ideas with your partner.**

**14. Make up your own presentation about compound systems.**

## VI. CAMS AND CAM DESIGN

**Classes of Cams.**— Cams may, in general, be divided into two classes: uniform motion cams and accelerated motion cams. The uniform motion cam moves the follower at the same rate of speed from the beginning to the end of the stroke; but as the movement is started from zero to the full speed of the uniform motion and stops in the same abrupt way, there is a distinct shock at the beginning and end of the stroke, if the movement is at all rapid. In machinery working at a high rate of speed, therefore, it is important that cams are so constructed that sudden shocks are avoided when starting the motion or when reversing the direction of motion of the follower.

The uniformly accelerated motion cam is suitable for moderate speeds, but it has the disadvantage of sudden changes in acceleration at the beginning, middle and end of the stroke. A cycloidal motion curve cam produces no abrupt changes in acceleration and is often used in high-speed machinery because it results in low noise, vibration and wear. The cycloidal motion displacement curve is so called because it can be generated from a cycloid which is the locus of a point of a circle rolling on a straight line.<sup>2</sup>

**Cam Follower Systems.**—The three most used cam and follower systems are radial and offset translating roller follower, Figs. 1a and 1b; and the swinging roller follower, Fig. 1c. When the cam rotates, it imparts a translating motion to the roller followers in Figs. 1a and 1b and a swinging motion to the roller follower in Fig. 1c. The motion of the follower is, of course, dependent on the shape of the cam; and the following section on displacement diagrams explains how a favorable motion is obtained so that the cam can rotate at high speed without shock.

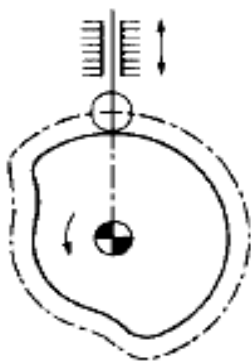


Fig. 1a. Radial Translating Roller Follower

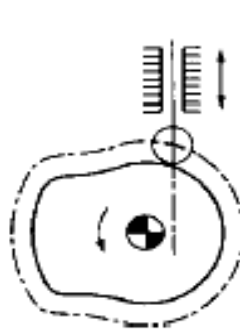


Fig. 1b. Offset Translating Roller Follower

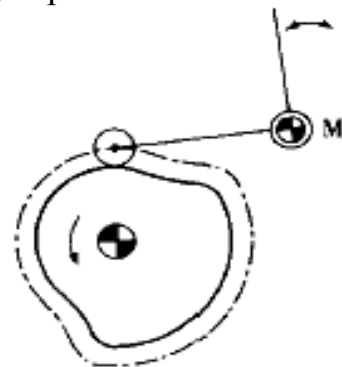


Fig. 1c. Swinging Roller Follower

<sup>2</sup> Jensen, P. W., *Cam Design and Manufacture*, Industrial Press Inc.



The arrangements in Figs. 1a, 1b, and 1c show open-track cams. In Figs. 2a and 2b the roller is forced to move in a closed track. Open-track cams build smaller than closed-track cams but, in general, springs are necessary to keep the roller in contact with the cam at all times. Closed-track cams do not require a spring and have the advantage of positive drive throughout the rise and return cycle. The positive drive is sometimes required as in the case where a broken spring would cause

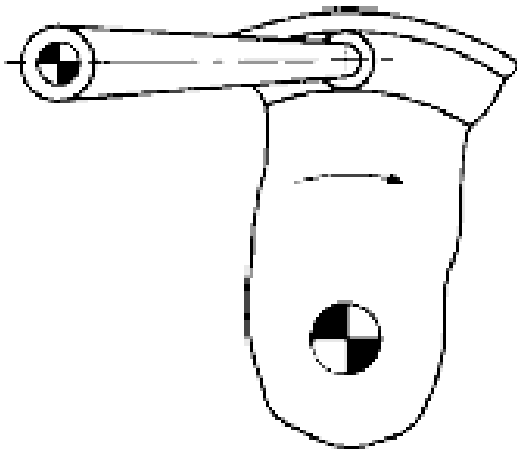


Fig. 2a. Closed-Track Cam

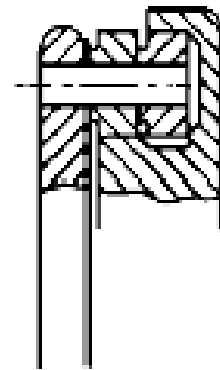


Fig. 2b. Closed-Track Cam with Two Rollers

serious damage to a machine.

**Displacement Diagrams.**—Design of a cam begins with the displacement diagram. A simple displacement diagram is shown in Fig. 3. One cycle means one whole revolution of the cam; i.e., one cycle represents  $360^\circ$ . The horizontal distances  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$  are expressed in units of time (seconds); or radians or degrees. The vertical distance,  $h$ , represents the maximum “rise” or stroke of the follower.

The displacement diagram of Fig. 3 is not a very favorable one

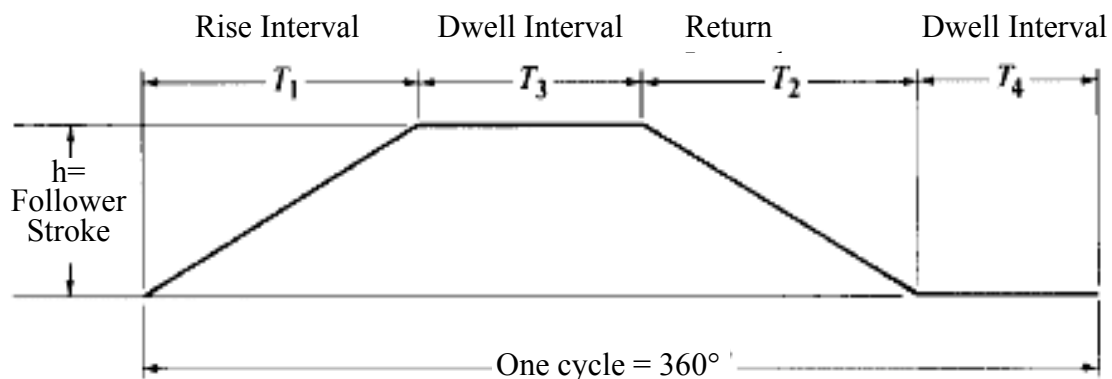


Fig. 3. A Simple Displacement Diagram

because the motion from rest (the horizontal lines) to constant velocity takes place instantaneously and this means that accelerations become infinitely large at these transition points.

*Types of Cam Displacement Curves:* A variety of cam curves are available for moving the follower. In the following sections only the rise portions of the total time-displacement diagram are studied. The return portions can be analyzed in a similar manner. Complex cams are frequently employed which may involve a number of rise-dwell-return intervals in which the rise and return aspects are quite different. To analyze the action of a cam it is necessary to study its time-displacement and associated velocity and acceleration curves. The latter are based on the first and second time-derivatives of the equation describing the time-displacement curve:

$$y = \text{displacement} = y = f(t) \text{ or } y = f(\phi)$$

$$v = \frac{dy}{dt} = \text{velocity} = \omega \frac{dy}{d\phi}$$

$$a = \frac{d^2y}{dt^2} = \text{acceleration} = \omega^2 \frac{d^2y}{d\phi^2}$$

*Meaning of Symbols and Equivalent Relations:*  $y$ =displacement of follower, inch

$h$ =maximum displacement of follower, inch

$t$  =time for cam to rotate through angle  $\phi$ , sec, =  $\phi/\omega$ , sec

$T$ =time for cam to rotate through angle  $\beta$ , sec, =  $\beta/\omega$ , or  $\beta/6N$ , sec

$\phi$  =cam angle rotation for follower displacement  $y$ , degrees

$\beta$  =cam angle rotation for total rise  $h$ , degrees

$v$ =velocity of follower, in./sec

$a$ =follower acceleration, in./sec<sup>2</sup>

$t/T = \phi/\beta$

$N$ =cam speed, rpm

$\omega$  =angular velocity of cam, degrees/sec =  $\beta/T = \phi/t = d\phi/dt = 6N$

$\omega_R$  =angular velocity of cam, radians/sec =  $\pi\omega/180$

$W$ =effective weight, lbs

$g$  = gravitational constant = 386 in./sec<sup>2</sup>

$f(t)$  = means a function of  $t$

$f(\phi)$  = means a function of  $\phi$

$R_{\min}$  = minimum radius to the cam pitch curve, inch

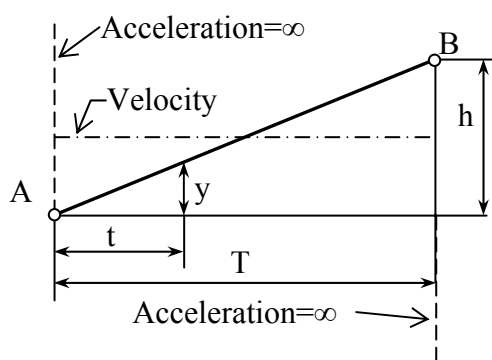


Fig. 4. Cam Displacement, Velocity, and Acceleration Curves for Constant Velocity Motion

$R_{\max}$  = maximum radius to the cam pitch curve, inch

$r_f$  = radius of cam follower roller, inch

$\rho$  = radius of curvature of cam pitch curve (path of center of roller follower), inch

$R_c$  = radius of curvature of actual cam surface, in., =  $\rho - r_f$  for convex surface; =  $\rho + r_f$  for concave surface.

Four displacement curves are of the

greatest utility in cam design.

1. Constant-Velocity Motion: (Fig. 4)

$y = h \frac{t}{T}$ or $y = \frac{h\phi}{\beta}$	(1a)	} $0 < t < T$
$v = \frac{dy}{dt} = \frac{h}{T}$ or $v = \frac{h\omega}{\beta}$	(1b)	
$a = \frac{d^2y}{dt^2} = 0^*$	(1c)	

\* Except at  $t = 0$  and  $t = T$  where the acceleration is theoretically infinite.

This motion and its disadvantages were mentioned previously. While in the unaltered form shown it is rarely used except in very crude devices, nevertheless, the advantage of uniform velocity is an important one and by modifying the start and finish of the follower stroke this form of cam motion can be utilized. Such modification is explained in the section Displacement Diagram Synthesis.

2. Parabolic Motion: (Fig. 5)

For $0 \leq t \leq T/2$ and $0 \leq \phi \leq \beta/2$	For $T/2 \leq t \leq T$ and $\beta/2 \leq \phi \leq \beta$
$y = 2h(t/T)^2 = 2h(\phi/\beta)^2$ (2a)	$y = h[1 - 2(1 - t/T)] = h[1 - 2(1 - \phi/\beta)^2]$ (2d)
$v = 4ht/T^2 = 4h\omega\phi/\beta^2$ (2b)	$v = 4h/T(1 - t/T) = (4h\omega/\beta)(1 - \phi/\beta)$ (2e)
$a = 4h/T^2 = 4h(\omega/\beta)^2$ (2c)	$a = -4h/T^2 = -4h(\omega/\beta)^2$ (2f)

Examination of the above formulas shows that the velocity is zero when  $t = 0$  and  $y = 0$ ; and when  $t = T$  and  $y = h$ .

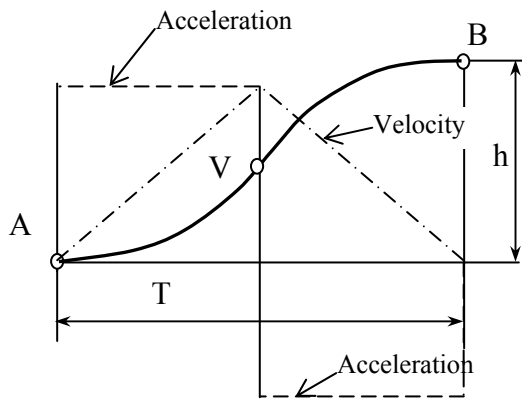


Fig. 5. Cam Displacement, Velocity, and Acceleration Curves for Parabolic Motion

The most important advantage of this curve is that for a given angle of rotation and rise it produces the smallest possible acceleration. However, because of the sudden changes in acceleration at the beginning, middle, and end of the stroke, shocks are produced. If the follower system were perfectly rigid with no backlash or flexibility, this would be of little significance. But such systems are mechanically impossible to build and a certain amount of impact is caused at each of these changeover points.

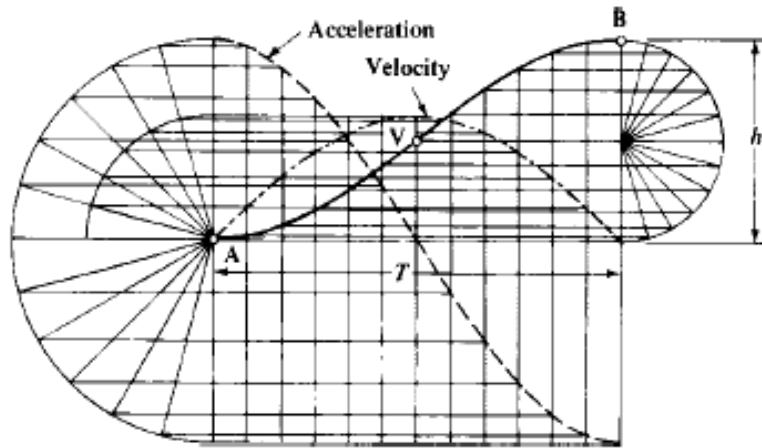


Fig. 6. Cam Displacement, Velocity, and Acceleration Curves for Simple Harmonic Motion

### 3. Simple Harmonic Motion: (Fig. 6)

$y = \frac{h}{2} \left[ 1 - \cos\left(\frac{180^\circ t}{T}\right) \right] \quad \text{or} \quad y = \frac{h}{2} \left[ 1 - \cos\left(\frac{180^\circ \phi}{\beta}\right) \right] \quad (3a)$	$\left. \vphantom{\begin{matrix} (3a) \\ (3b) \\ (3c) \end{matrix}} \right\} 0 < t < T$
$v = \frac{h}{2} \cdot \frac{\pi}{T} \sin\left(\frac{180^\circ t}{T}\right) \quad \text{or} \quad v = \frac{h}{2} \cdot \frac{\pi \omega}{\beta} \sin\left(\frac{180^\circ \phi}{\beta}\right) \quad (3b)$	
$a = \frac{h}{2} \cdot \frac{\pi^2}{T^2} \cos\left(\frac{180^\circ t}{T}\right) \quad \text{or} \quad a = \frac{h}{2} \cdot \left(\frac{\pi \omega}{\beta}\right)^2 \cos\left(\frac{180^\circ \phi}{\beta}\right) \quad (3c)$	

Smoothness in velocity and acceleration during the stroke is the advantage inherent in this curve. However, the instantaneous changes in acceleration at the beginning and end of the stroke tend to cause vibration, noise, and wear. As can be seen from Fig. 6, the maximum acceleration values occur at the ends of the stroke. Thus, if inertia loads are to be overcome by the follower, the resulting forces cause stresses in the members. These forces are in many cases much larger than the externally applied loads.

**Cam Profile Determination.**—In the cam constructions that follow an artificial device called an *inversion* is used. This represents a mental concept which is very helpful in performing the graphical work. The construction of a cam profile requires the drawing of many positions of the cam with the follower in each case in its related location. However, instead of revolving the cam, it is assumed that the follower rotates around the *fixed* cam. It requires the drawing of many follower positions, but since this is done more or less diagrammatically, it is relatively simple. As part of the inversion process, the direction of rotation is important. In order to preserve the correct sequence of events, the artificial rotation of the follower must be the reverse of the cam's prescribed rotation. Thus, in Fig. 7 the cam rotation is counterclockwise, whereas the artificial rotation of the follower is clockwise.

*Radial Translating Roller Follower:* The time-displacement diagram for a cam with a radial translating roller follower is shown in Fig. 7(a). This diagram is read from left to right as follows: For 100 degrees of cam shaft rotation the follower rises  $h$  inches ( $AB$ ), dwells in its upper position for 20 degrees ( $BC$ ), returns over 180 degrees ( $CD$ ), and finally dwells in its lowest position for 60 degrees ( $DE$ ). Then the entire cycle is repeated.

Fig. 7(b) shows the cam construction layout with the cam pitch curve as a dot and dash line. To locate a point on this curve, take a point on the displacement curve, as 6" at the 60-degree position, and project this horizontally to point 6" on the 0-degree position of the cam construction diagram. Using the center of cam rotation, an arc is struck from point 6" to intercept the 60-degree position radial line which gives point 6" on the cam pitch curve. It will be seen that the smaller circle in the cam construction layout has a radius  $R_{\min}$  equal to the smallest distance from the center of cam rotation to the pitch curve and, similarly, the larger circle has a radius  $R_{\max}$  equal to the largest distance

to the pitch curve. Thus, the difference in radii of these two circles is equal to the maximum rise  $h$  of the follower.

The cam pitch curve is also the actual profile or working surface when a knife-edged follower is used. To get the profile or working surface for a cam with a roller follower, a series of arcs with centers on the pitch curve and radii equal to the radius of the roller are drawn and the inner envelope drawn tangent to these arcs is the cam working surface or profile shown as a solid line in Fig. 7(b).

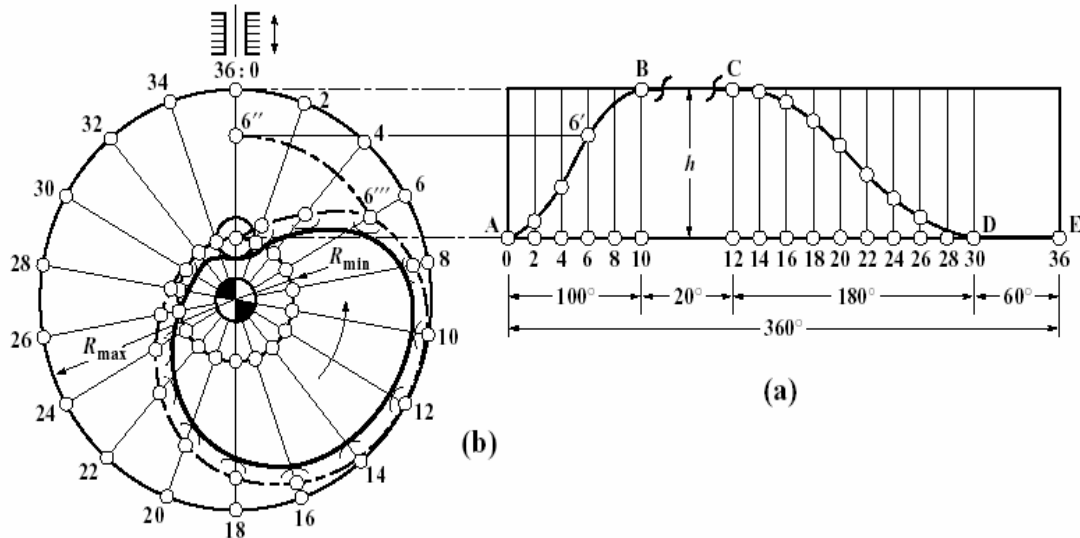


Fig. 7. (a) Time-Displacement Diagram for Cam to be Laid Out; (b) Construction of Contour of Cam With Radial Translating Roller Follower

*Swinging Roller Follower:* Given the time-displacement diagram Fig. 8(a) and the length of the swinging follower arm  $L_f$ , it is required that the displacement of the follower center along the circular arc that it describes be equal to the corresponding displacements in the time-displacement diagram. If  $\phi_0$  is known, the displacement  $h$  of Fig. 8(a) would be found from the formula  $h = \pi\phi_0 L_f / 180^\circ$ ; otherwise the maximum rise  $h$  of the follower is stepped off on the arc drawn with  $M$  as a center and starting at a point on the  $R_{\min}$  circle. Point  $M$  is the actual position of the pivot center of the swinging follower with respect to the cam shaft center. It is again required that the rotation of the cam be counterclock-wise and therefore  $M$  is considered to have been rotated clockwise around the cam shaft center, whereby the points 2, 4, 6, etc., are obtained as shown in Fig. 8(b). Around each of the pivot points, 2, 4, 6, etc., circular arcs whose radii equal  $L_f$  are drawn between the  $R_{\min}$  and  $R_{\max}$  circles giving the points 2', 4', 6', etc. The  $R_{\min}$  circle with center at the cam shaft center is drawn through the lowest position of

the center of the roller follower and the  $R_{\max}$  circle through the highest position as shown. The different points on the pitch curve are now located. Point 6''', for instance, is found by stepping off the  $y_6$  ordinate of the displacement diagram on arc 6' starting at the  $R_{\min}$  circle.

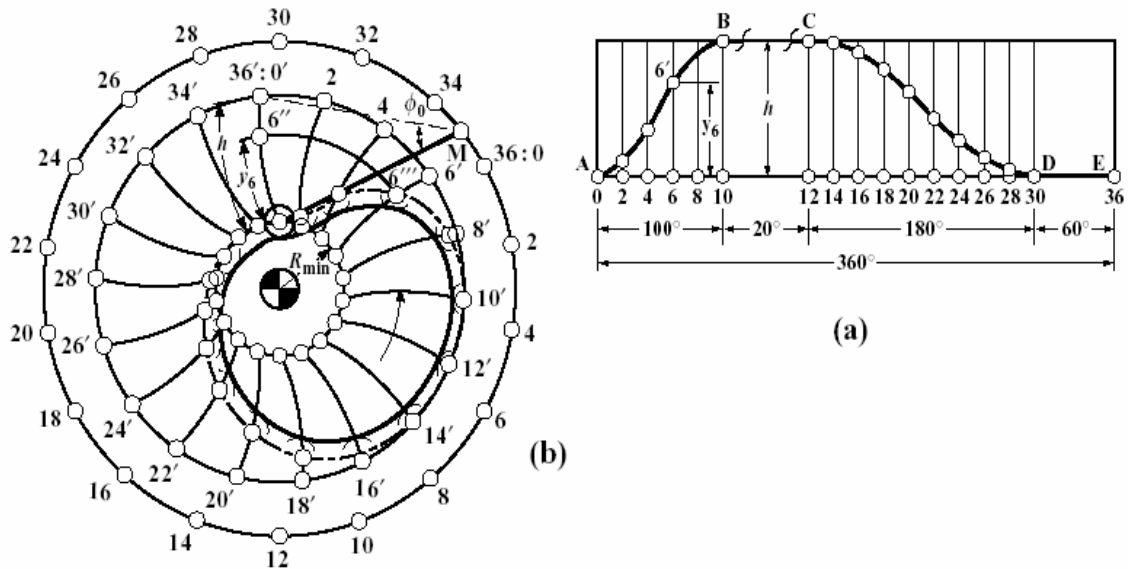


Fig. 8. (a) Time-Displacement Diagram for Cam to be Laid Out;  
(b) Construction of Contour of Cam With Swinging Roller Follower

**Pressure Angle and Radius of Curvature.**— The pressure angle at any point on the profile of a cam may be defined as the angle between the direction where the follower wants to go at that point and where the cam wants to push it. It is the angle between the tangent to the path of follower motion and the line perpendicular to the tangent of the cam profile at the point of cam-roller contact. The size of the pressure angle is important because:

- 1) Increasing the pressure angle increases the side thrust and this increases the forces exerted on cam and follower.
- 2) Reducing the pressure angle increases the cam size and often this is not desirable because:
  - A) The size of the cam determines, to a certain extent, the size of the machine.
  - B) Larger cams require more precise cutting points in manufacturing and, therefore, an increase in cost.
  - C) Larger cams have higher circumferential speed and small deviations from the theoretical path of the follower cause additional acceleration, the size of which increases with the square of the cam size.

D) Larger cams mean more revolving weight and in high-speed machines this leads to increased vibrations in the machine.

E) The inertia of a large cam may interfere with quick starting and stopping.

The maximum pressure angle  $\alpha_m$  should, in general, be kept at or below 30 degrees for translating-type followers and at or below 45 degrees for swinging-type followers. These values are on the conservative side and in many cases may be increased considerably, but beyond these limits trouble could develop and an analysis is necessary.

In the following, graphical methods are described by which a cam mechanism can be designed with translating or swinging roller followers having specified maximum pressure angles for rise and return. These methods are applicable to any kind of time-displacement diagram.

**Determination of Cam Size for a Radial or an Offset Translating Follower.**—Fig. 9 shows a time-displacement diagram. The maximum displacement is preferably made to scale, but the length of the abscissa,  $L$ , can be chosen arbitrarily. The distance  $L$  from 0 to 360 degrees is measured and is set equal to  $2\pi k$  from which

$$k = \frac{L}{2\pi}$$

$k$  is calculated and laid out as length  $E$  to  $M$  in Fig. 10. In Fig. 9 the two points  $P_1$  and  $P_2$  having the maximum angles of slope,  $\tau_1$ , and  $\tau_2$ , are located by inspection. In this example  $y_1$  and  $y_2$  are of equal length. Angles  $\tau_1$  and  $\tau_2$  are laid out as shown in Fig. 10, and the points of intersection with a perpendicular to  $EM$  at  $M$  determine  $Q_1$  and  $Q_2$ . The measured distances

$$MQ_1 = k \tan \tau_1 \quad \text{and} \quad MQ_2 = k \tan \tau_2$$

are laid out in Fig. 11, which is constructed as follows:

Draw a vertical line  $R_u R_o$  of length  $h$  equal to the stroke of the roller follower,  $R_u$  being the lowest position and  $R_o$  the highest position

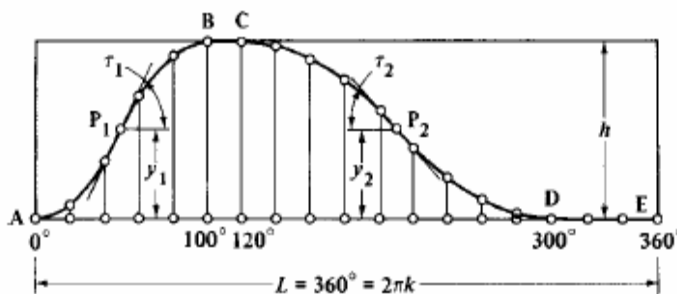


Fig. 9. Displacement Diagram

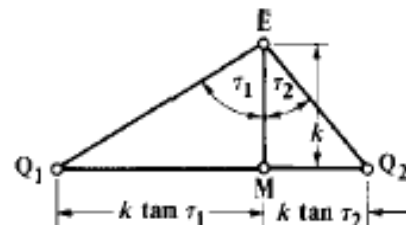


Fig. 10. Construction to Find  $k \tan \tau_1$  and  $k \tan \tau_2$



of the center of the roller follower. From  $R_u$  lay out  $R_u R_{y1} = y_1$  and  $R_u R_{y2} = y_2$ ; these are equal lengths in this example. Next, if the rotation of the cam is counterclockwise, lay out  $k \tan \tau_1$ , to the left,  $k \tan \tau_2$  to the right from points  $R_{y1}$  and  $R_{y2}$ , respectively,  $R_{y1}$  and  $R_{y2}$  being the same point in this case.

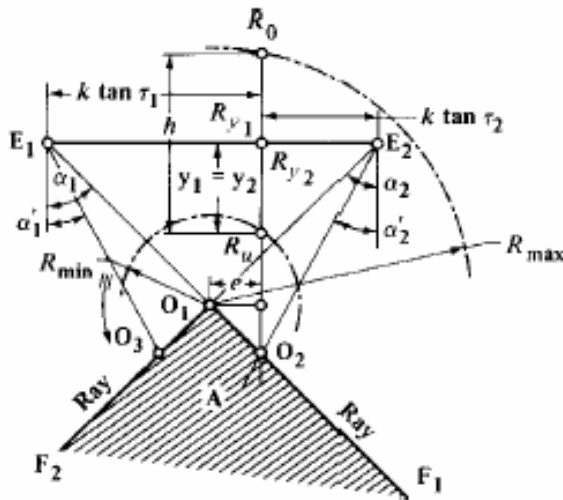


Fig. 11. Finding Proportions of Cam; Offset Translating Follower

The specified maximum pressure angle  $\alpha_1$  is laid out at  $E_1$  as shown, and a ray (line)  $E_1 F_1$  is determined. Any point on this ray chosen as the cam shaft center will proportion the cam so that the pressure angle at a point on the cam profile corresponding to point  $P_1$ , of the displacement diagram will be exactly  $\alpha_1$ .

The angle  $\alpha_2$  is laid out at  $E_2$  as shown, and another ray  $E_2 F_2$  is determined. Similarly, any point on this ray chosen as the cam shaft center will proportion the cam so that the pressure angle at a point on the cam profile corresponding to point  $P_2$  of the displacement diagram will be exactly  $\alpha_2$ .

Any point chosen within the cross-hatched area A as the cam center will yield a cam whose pressure angles at points corresponding to  $P_1$  and  $P_2$  will not exceed the specified values  $\alpha_1$  and  $\alpha_2$  respectively. If  $O_1$  is chosen as the cam shaft center, the pressure angles on the cam profile corresponding to points  $P_1$  and  $P_2$  are exactly  $\alpha_1$  and  $\alpha_2$ , respectively. Selection of point  $O_1$  also yields the smallest possible cam for the given requirements and requires an offset follower in which  $e$  is the offset distance.

If  $O_2$  is chosen as the cam shaft center, a radial translating follower is obtained (zero offset). In that case, the pressure angle  $\alpha_1$  for the rise is unchanged, whereas the pressure angle for the return is changed from  $\alpha_2$  to  $\alpha'_2$ . That is, the pressure angle on the return stroke is reduced at the point  $P_2$ . If point  $O_3$  had been selected, then  $\alpha_2$  would remain unchanged but  $\alpha_1$  would be decreased and the offset,  $e$ , increased.

Fig. 12 shows the shape of the cam when  $O_1$  from Fig. 11 is chosen as the cam shaft center, and it is seen that the pressure angle at a point on the cam profile corresponding to point  $P_1$  is  $\alpha_1$  and at a point corresponding to point  $P_2$  is  $\alpha_2$ .

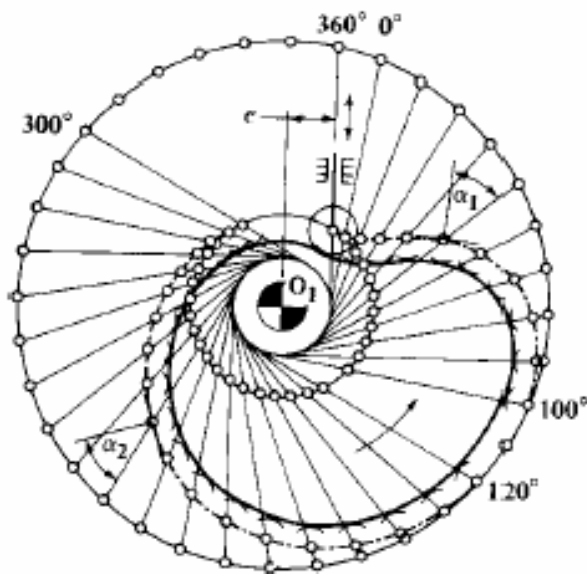


Fig. 12. Construction of Cam Contour; Offset Translating Follower

In the foregoing, a cam mechanism has been so proportioned that the pressure angles  $\alpha_1$  and  $\alpha_2$  at points on the cam corresponding to points  $P_1$  and  $P_2$  were obtained. Even though  $P_1$  and  $P_2$  are the points of greatest slope on the displacement diagram, the pressure angles produced at some other points on the actual cam may be slightly greater.

However, if the pressure angles  $\alpha_1$  and  $\alpha_2$  are not to be exceeded at any point — i.e., they are to be maximum pressure angles — then  $P_1$  and

$P_2$  must be selected to be at the locations where these maximum pressure angles occur. If these locations are not known, then the graphical procedure described must be repeated, letting  $P_1$  take various positions on the curve for rise ( $AB$ ) and  $P_2$  various positions on the return curve ( $CD$ ) and then setting  $R_{\min}$  equal to the largest of the values determined from the various positions.

## EXERCISES TO THE CHAPTER “CAMS AND CAM DESIGN”

### 1. Learn these words and phrases

- |                                       |   |
|---------------------------------------|---|
| 1. uniform motion cam                 | - кулачок с равномерным движением толкателя |
| 2. accelerated motion cam             | - кулачок с ускоренным движением толкателя  |
| 3. stroke                             | - ход, шаг                                  |
| 4. cycloid motion curve cam           | - циклоидальный закон движения              |
| 5. locus                              | - местоположение                            |
| 6. cam follower system                | - система кулачок-толкатель                 |
| 7. radial translating roller follower | - центральный толкатель с роликом           |

8. offset translating roller follower	- смещенный толкатель с роликом
9. swinging roller follower	- качающийся роликовый толкатель
10. impart	- передавать, сообщать движение
11. open-track cam	- кулачок без геометрического замыкания цепи
12. closed-track cam	- кулачок с геометрическим замыканием цепи
13. instantaneously	- мгновенно, немедленно
14. cam displacement curve	- закон (диаграмма) перемещения толкателя
15. rise interval	- участок подъема
16. dwell interval	- интервал выстоя
17. return interval	- интервал возврата
18. acceleration curve	- диаграмма ускорения
19. crude device	- механизмы низкой точности
20. changeover point	- точка поворота (перемены)
21. impact	- удар
22. inertia load	- инерционная нагрузка
23. artificial device	- условный, искусственный
24. inversion	- инверсия, обращенный
25. counterclockwise	- против часовой стрелки
26. dwell	- задерживаться, останавливаться, выстой, останов
27. intercept	- пересечение, отсекал, задерживать
28. knife-edged follower	- толкатель с заостренной кромкой (ножевой)
29. inner envelope	- внутренняя огибающая
30. solid line	- сплошная линия
31. swinging follower arm	- плечо качающегося (коромыслового) толкателя
32. pivot center	- точка опоры (поворота)
33. radius	- радиус
34. exert	- действовать (о силе)
35. circumferential speed	- окружная скорость
36. deviation	- отклонение
37. interfere	- создавать помехи, препятствовать
38. cross-hatched area	- поперечно-заштрихованная поверхность
39. yield	- приводить к..., давать
40. foregoing	- упомянутый выше

## 2. Answer the following questions

1. How many classes of cams do you know?
2. What kinds of machinery are cycloid motion curve cams used in?
3. What are the main cam follower systems?
4. What does the design of a cam begin with?
5. What is necessary to know in analyzing the action of a cam?
6. What is an inversion?
7. How is the pressure angle of a cam defined?
8. Why is the size of the pressure angle so important?
9. What is maximum pressure angle equal to?
10. Name the main reasons for a wide application of cam mechanisms in modern automatic devices.
11. Draw schemes of roller follower cam.
12. Draw the scheme of closed-track cam.
13. In what conditions and under what circumstances can the impact appear?
14. Enumerate the design stages of cam mechanisms?
15. Draw a diagram of follower motion.
16. How is the position of the revolution center of a cam in a mechanism with translating roller follower determined?
17. Where should the axis of cam rotation be placed if it is necessary to get a cam of smallest size?
18. What is the graphical way of constructing the profile of plane cams?

## 3. Discuss in pairs.

1. Laws of follower motion.
2. Define the sizes of cam mechanisms.
3. Define the possible position of the revolution cam center.

## 4. Study this table of verbs and related nouns of change in order to describe graphs.

Direction	Verb	Noun
<b>Up</b>	climb	-
	go up	-
	increase	increase
	rise	rise
<b>Down</b>	decline	decline
	decrease	decrease
	dip	dip

	drop	drop
	fall	fall
	go down	-
<b>Level</b>	not change	no change
	remain constant	-

This adjectives and adverbs are used to describe the rate of change:

<b>Adjective</b>	<b>Adverb</b>
slight	slightly
gradual	gradually
steady	steadily
steep	steeply
sharp	sharply
sudden	suddenly
fast	fast

**5. Make up reports according to the theme “Cams and Cam design” with the usage of graphs.**

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