Three-axis Laser Method for Measuring the Diameter of Cylindrical Objects

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***Abstract*—The paper presents a three-axis method for measuring the diameter and ovality of cylindrical objects in the divergent laser beam. The mathematical expressions to calculate the diameter using measuring devices based on this method have been obtained. The research results can be used to develop and manufacture noncontact optical devices for in-process control of the outer diameter of products in cable, pipe and other industries.**

***Keywords*-component; three-axis method, in-process diameter measurement, divergent beam, multi-element receivers, diameter calculation.**

I. INTRODUCTION

Various geometric and electrical parameters are used to assess the quality of cable products. The rejects can be minimized through the improved technology to control the product major characteristics, namely capacitance per unit length [1, 2], eccentricity [3], outer diameter [4-6], insulation integrity [7] and others parameters [8-12].

Noncontact gauges for measuring the diameter of circular long-length objects such as cables, ropes, pipes, etc., which are based on the shadow measuring method using the divergent luminous flux, exhibit a number of undeniable advantages since they do not use the elements of lens and mirror optics. In particular, the measurement accuracy obtained by these devices using a combination of diffraction methods and statistical data processing may be of micrometer fractions [13,14]. However, when measurement is performed using the divergent light flux, the shadow dimensions of the measured object change in case the object is displaced within the test zone. This requires the use of complex ratios to calculate the true diameter of the object [15]. Two-coordinate measuring gauges provide approximate values of the ovality estimated on the basis of the difference between the diameters of the two measuring axes, which are directly dependent on the object orientation within the measuring field. In this case, particular positions of the object do not allow the ovality fixation. Fig. 1 shows an example of this orientation for an object with an ellipsoidal cross-section. The cross-section images *Dx* and *Dy* in the multi-element receivers of measuring channels *X* and *Y* are equal and they cannot provide the ovality parameters.

The measurements performed in the three-axis laser systems with a divergent laser beam can partially solve the above-stated problems. Three-axis measuring gauges exhibit the following advantages when compared with those of two-axis devices: three synchronized measuring axes on the same plane; detection of any deviation from the circular cross-section regardless of the object orientation in the measured zone; determination of the circumference and area of the cross-section.

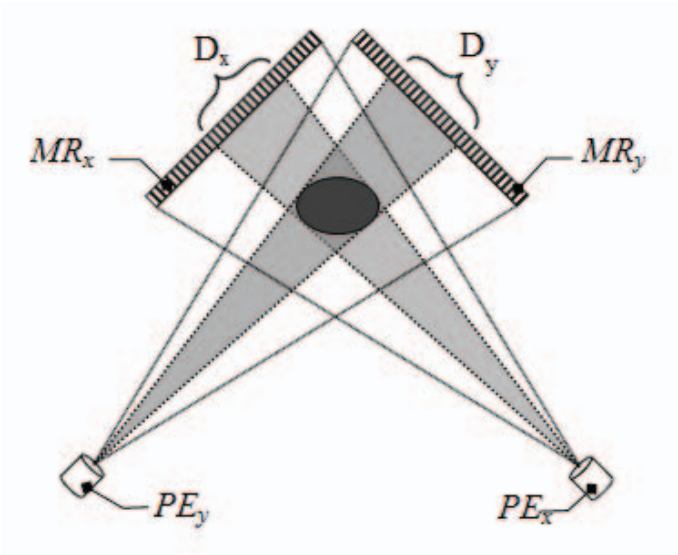

Fig. 1. Example of orientation of the object with a non-circular cross-section, in which the calculation of ovality parameters is impossible. *MRx*, PEx and *MRy*, *PEy* are multi-element receivers and point emitters of measuring axes *Y* and *X*, respectively. *Dx* and *Dy* are images of the object shadows in the corresponding photoreceivers.

II. DIAMETER CALCULATION

Since the true diameter is a calculated value in both three- and two-axis laser systems, the derivation of mathematical expressions for its calculation is required.

Fig. 2 shows the schematic diagram of the considered three-axis measuring gauge. The device comprises three measuring channels, with *X*, *Y* and *Z* axes arranged at 120º angle to each other. Each channel has a point emitter (semiconductor laser) generating a divergent luminous flux and a multi-element photoreceiver measuring the size of the shadow falling on the object. The measuring object is found in the test zone; it is illuminated by three emitters and form three shadows in photoreceiver of the corresponding measuring channels.

The known design parameters of the device are as follows:

Distances H_x, H_y and H_z from the emitter center to the plane of the receiver of each channel: (S_{X0}; B_{X0}), (S_{Y0}; B_{Y0}) and (S_{Z0}; B_{Z0}) segments.

Distances H_{X0}, H_{Y0} and H_{Z0} from the center of the emitter to the center of the test zone, i.e. the point (*0*), which is an imaginary point for intersection of the measuring channel symmetry axes.

The step of the photoreceiver cells allows measurement of the shadow, i.e. its width, in the number of cells shadowed by the object, and the displacement of the shadow center relative to the "zero" cell of the receiver.

The known data are used to measure the object radii by the three channels (R_X, R_Y, R_Z) and to determine the average radius R_{av} and the object ovality. To perform special measurements, data on the object position is required, (displacement of its *Z* center) relative to the center of the gauge test zone ("*0*" point), i.e. the E_X, E_Y and E_Z values.

The problem is solved in three steps:

Step 1 – determination of the position of the *Z* object center projection in the plane of the "*X*", "*Y*" and "*Z*" receivers.

The position of the center projection is characterized by the X_0, Y_0 and Z_0 values, which are (B_{X0}; X_0), (B_{Y0}; Y_0) and (B_{Z0}; Y_0) segments. According to the triangle (S_{X0}; B_{X0}; X_0), it is known that:

$$tg(\alpha+\alpha_1)=\frac{X_0}{H_X}. \text{ Hence: } X_0=H_X\cdot tg(\alpha+\alpha_1) \quad (1)$$

The value α1 in expression (1) can be found through the triangle (S_{X0}; B_{X0}; X_1):

$$tg\alpha_1=\frac{X_1}{H_Y};\ \alpha_1=arctg\frac{X_1}{H_X}. \quad (2)$$

The angle α is determined through the triangle (S_{X0}; B_{X0}; X_2):

$$tg(\alpha+\alpha+\alpha_1)=\frac{X_2}{H_X};\ (2\alpha+\alpha_1)=arctg\frac{X_2}{H_X}.$$

With regard to expression (2), the ratio of the angle α takes the form:

$$\alpha=\frac{1}{2}(arctg\frac{X_2}{H_X}-arctg\frac{X_1}{H_X}) \quad (3)$$

Substitution of (3) and (2) in (1) and transformations result in:

$$X_0=H_X\cdot tg\frac{1}{2}(arctg\frac{X_2}{H_X}+arctg\frac{X_1}{H_X}) \quad (4)$$

The same procedure is performed for Y_0 and Z_0 values:

$$Y_0=H_Y\cdot tg\frac{1}{2}(arctg\frac{Y_2}{H_Y}+arctg\frac{Y_1}{H_Y}) \quad (5)$$

$$Z_0=H_Z\cdot tg\frac{1}{2}(arctg\frac{Z_2}{H_Z}+arctg\frac{Z_1}{H_Z}) \quad (6)$$

Similarity of the triangles (S_{X0}; B_{X0}; X_0) and (S_{X0}; 0_X; Z) provides:

Step 2 – determination of the *Z* object center displacement relative to the center of the test zone. The displacement is characterized by E_x, E_y and E_z values (Fig. 1).

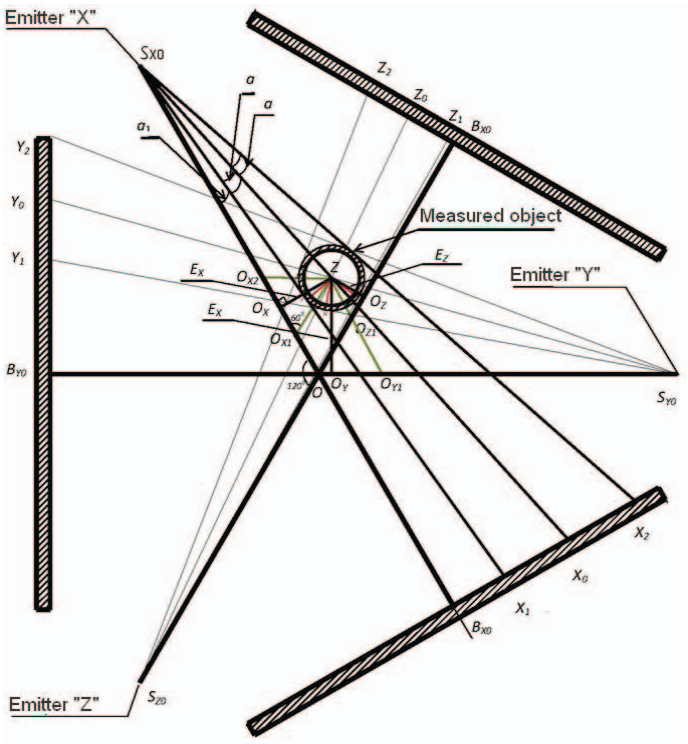

Fig. 2. Calculation model for the device with three measuring axes.

$$\frac{E_X}{X_0}=\frac{(S_{X0};0_X)}{H_X}. \text{ Similarly: } \frac{E_Y}{Y_0}=\frac{(S_{Y0};0_Y)}{H_Y};\quad \frac{E_Z}{Z_0}=\frac{(S_{Z0};0_Z)}{H_Z} \quad (7)$$

Find the values for the segments (S_{X0}; 0_X), (S_{Y0}; 0_Y) and (S_{Z0}; 0_Z) in (7), provided the segment (*0*; 0_X) is:

$$(0;\ 0_X)=(Z;\ O_{Y1})-(O_{X2};\ O_X)=\frac{E_Y}{\cos(\frac{\pi}{6})}-\frac{E_X}{tg(\frac{\pi}{3})};$$

$$\text{segment } (0;\ 0_Y)=(Z;\ O_{X2})-(O_Y;\ O_{Y1})=\frac{E_X}{\cos(\frac{\pi}{6})}-\frac{E_Y}{tg(\frac{\pi}{3})}.$$

$$\text{segment } (0;\ 0_Z)=(Z;\ O_{X1})+(O_Z;\ O_{Z1})=\frac{E_X}{\cos(\frac{\pi}{6})}+\frac{E_Z}{tg(\frac{\pi}{3})}.$$

Then:

$$(S_{X0};\ 0_X) = H_{X0} - \frac{E_Y}{\cos(\frac{\pi}{6})} + \frac{E_X}{tg(\frac{\pi}{3})}$$

$$(S_{Y0};\ 0_Y) = H_{Y0} - \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Y}{tg(\frac{\pi}{3})}\ ; \qquad (8)$$

$$(S_{Z0};\ 0_Z) = H_{Z0} + \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Z}{tg(\frac{\pi}{3})}$$

With regard to expression (8), ratios (7) take the form:

$$\frac{E_X}{X_0} = \frac{H_{X0} - \frac{E_Y}{\cos(\frac{\pi}{6})} + \frac{E_X}{tg(\frac{\pi}{3})}}{H_X};$$

$$\frac{E_Y}{Y_0} = \frac{H_{Y0} - \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Y}{tg(\frac{\pi}{3})}}{H_Y}; \qquad (9)$$

$$\frac{E_Z}{Z_0} = \frac{H_{Z0} + \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Z}{tg(\frac{\pi}{3})}}{H_Z}$$

Expressions (9) are a system of three equations with three unknowns: E_X, E_Y and E_Z. Solve the equations and derive the expressions to calculate E_X, E_Y and E_Z, in which the X_0, Y_0 and Z_0 values are calculated by (4), (5) and (6) ratios. This paper does not provide the extended expressions for calculation of the system of equations in letter values.

Step 3 – calculation of the object radii R_X, R_Y and R_Z.

The schematic diagram in Figure 1 indicates that the R_X radius, i.e. the segment (F;Z), is the side of the right triangle (S_{X0}; Z; F). According to this triangle:

$$R_X = (S_{X0}; Z) \cdot \sin\alpha \qquad (10)$$

The other right triangle (S_{X0}; Z; 0_X) is used to find the segment (S_{X0}; Z):

$$(\mathrm{S_{X0}}, \mathrm{Z}) = \sqrt{(0_X; Z)^2 + (0_X; S_{X0})^2} = \sqrt{E_X^2 + (H_{X0} - \frac{E_y}{\cos(\frac{\pi}{6})} + \frac{E_x}{tg(\frac{\pi}{3})})^2} \cdot$$

Substitute the value of the segment (S_{X0}; Z) in (10) and use expression (3) for the angle α to find the radius R_X.

$$R_X = \sqrt{E_X^2 + (H_{X0} - \frac{E_y}{\cos(\frac{\pi}{6})} + \frac{E_x}{tg(\frac{\pi}{3})})^2} \cdot \sin\frac{1}{2}(arctg\frac{X_2}{H_X} - arctg\frac{X_1}{H_X}) \qquad (11)$$

Similarly:

$$R_Y = \sqrt{E_Y^2 + (H_{Y0} - \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Y}{tg(\frac{\pi}{3})})^2} \cdot \sin\frac{1}{2}(arctg\frac{Y_2}{H_Y} - arctg\frac{Y_1}{H_Y}) \qquad (12)$$

$$R_Z = \sqrt{E_Z^2 + (H_{Z0} + \frac{E_X}{\cos(\frac{\pi}{6})} + \frac{E_Z}{tg(\frac{\pi}{3})})^2} \cdot \sin\frac{1}{2}(arctg\frac{Z_2}{H_Z} - arctg\frac{Z_1}{H_Z}) \qquad (13)$$

The values of the R_X and R_Y radii being known, the average radius Rav of the object and object ovality (O) can be found.

$$R_{av} = \frac{|R_X| + |R_Y| + |R_Z|}{3};\quad O = (R_{\max} - R_{\min}) \qquad (14)$$

III. METHOD EVALUATION

A three-axis optical device for measuring the diameter of cylindrical objects has been simulated to verify the above suggested expressions (11–13). A computer model presented in Fig. 2 comprises three identical measuring channels arranged at 120º to each other. The initial geometric parameters to simplify calculations are the same for all the measuring channels:

- The distance from the emitter center to the plane of the receiver H_x=H_y=H_z=300 (mm).
- The distance from the emitter center to the test zone, i.e. the center of intersection of the measuring channel axes H_{X0}=H_{Y0}=H_{Z0}=220 (mm).

Such values of the optical devices geometric parameters are typical for the measurement area sufficient for most applications. In this model, the length of the working surface of multiple-receiver does not exceed 100 mm. All the initial geometrical parameters and virtual positions of shadows in the radiation receiver planes in the corresponding channels (Fig. 2) X_1, X_2, Y_1, Y_2, Z_1 and Z_2, were determined in the computer model with 1 μm error. Round objects with different diameters and different location in the virtual measuring zone were simulated to perform testing. When substituting the values obtained during the simulation into expressions (11–13), the measuring object diameters were calculated in the corresponding measuring axes X, Y and Z. These calculated diameters D_x, D_y, D_z for three arbitrary circular objects with the diameter D are presented in Table 1.

TABLE I.

Diameter values and the error for three axes							
Measuring diameter D (mm)	*Diameter D_x (mm)*	*Error ΔD_x (μm)*	*Diameter D_y (mm)*	*Error ΔD_x (μm)*	*Diameter D_z (mm)*	*Error ΔD_x (μm)*	*Error ΔD_{av} (μm)*
10.000	10.003	-3	10.002	-2	9.998	2	-1
17.000	16.997	3	16.999	1	17.002	-2	0.67
25.000	24.999	1	25.001	-1	25.000	0	0

The absolute error of the diameter calculation in the measuring axes ΔD_x, ΔD_x and ΔD_x, as well as the mean error ΔD_{av} do not exceed 3 μm, that is acceptable with regard to the error in the initial parameter determination. In practice, the diameter calculation accuracy strongly depends on the accuracy of initial parameters. The mechanisms to solve the above-stated problem are considered in [16,17]. Further simulation of the three-axis device is to be performed on a real prototype using the methods of digital signal processing. Due to this, the error in the diameter calculation may be of micrometer fractions.

IV. CONCLUSION

This paper proposes a method for three-axis measurement of the diameter using a divergent laser beam. Expressions (9, 11–14) have been derived to calculate the precise value of the object radii, the coordinates of the object center position, and the average radius and the precise value of its ovality in the maximum and minimum values. Realized computer simulation three-axis transducer, which confirmed the correctness of the proposed conversion function. The described optical method and mathematical support can be used to design a primary measuring transducer for the device to test the diameter and ovality of long-length products.

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