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ELEMENTARY PARTICLE PHYSICS AND FIELD THEORY

INTERFERENCE OF THE TRANSIENT RADIATION FIELDS PRODUCED BY AN ELECTRIC CHARGE AND A MAGNETIC MOMENT

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The conditions for the occurrence of interference of the transient radiation fields produced by an electric charge and a magnetic moment are analyzed. The contribution of the transient radiation of a charge – magnetic moment system, depending on the orientation of the magnetic moment, to the total output has been calculated for the case of a particle escaping from a titanium target in vacuum for a photon energy of 454 eV. The possibility of harnessing the effect under consideration in developing a new type of polarimeter has been evaluated.

Keywords: transient radiation, magnetic moment, electric charge, interference, asymmetry of radiation, polarimeter.

Analysis of the spin states of an electron beam is so far a complicated experimental problem. For these purposes, the dispersion of electrons in the Coulombian field of a nucleus has been generally harnessed (Mott polarimeters) [1, 2]. The spin-orbit interaction has the result that the effective cross section for scattering at a given angle appears different for electrons with opposite spins. The resulting left-right asymmetry of scattering, $A_{\mathrm{l.r}}$ is used for measuring the polarization of an electron beam in the energy range up to 1 MeV.

Experimentally, the polarization of a beam is determined from the relation

$$P_0 = A_{\mathrm{l.r}} / S_{\mathrm{eff}}, \tag{1}$$

where S_{eff} is the analyzing ability of Mott scattering, which can be calculated theoretically with rather high accuracy.

It should be noted that the main disadvantage of Mott polarimeters is the restricted range of electron beam energies (10 eV – 1 MeV). For today many experimenters in high energy physics use beams of moderately relativistic polarized electrons and positrons with energies up to 10 MeV that are analyzed with the use of a Mueller polarimeter [3], which is also not efficient for this energy range. Therefore, search for new physical mechanisms which could be harnessed in developing more efficient polarimeters based on detecting asymmetry in the electromagnetic radiation of polarized particles is rather promising.

It seams that the polarimeters whose operation is based on detection of the electromagnetic radiation of a charge + spin system (charge + magnetic moment in the classical consideration) will extend experimental capabilities and will make it possible to calculate the analyzing ability of the process with any desired accuracy by using the well developed methods of quantum (classical) electrodynamics.

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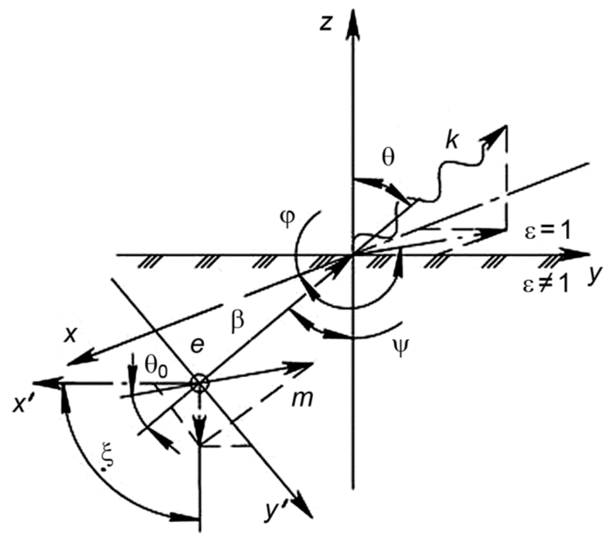

Fig. 1. Sketch of the generation of TR by a charged particle possessing an intrinsic magnetic moment.

We use a quasiclassical approximation to consider the transient radiation (TR) of polarized electrons (positrons) as the radiation of the point charge and spin magnetic moment of an electron.

To define explicitly the interference term and the conditions of its existence, we consider the geometry shown in Fig. 1, such that a particle with a charge e, possessing an intrinsic magnetic moment $\boldsymbol{m}$, escapes with a velocity $v = \beta c$ from a medium of permittivity ε through an oblique interface in vacuum ($\varepsilon = 1$).

In the coordinate system where the z-axis is directed normal to the interface and the y-axis lies in a plane passing through the normal and the particle velocity vector [4], the components of the Hertz vectors for the magnetic moment and for the electric charge, obtained by the method of images [5], can be written as

$$\boldsymbol{\Pi}^{m}_{\omega p} = \frac{ic\boldsymbol{e}_z \exp\left(i\dfrac{\omega}{c}R\right)(1-\varepsilon)n_z}{\pi\omega^2 R(\varepsilon n_z+\sigma)(1-n_z^2)} \times\left[\frac{B_z(1-n_z^2)[(1-\beta_y n_y)^2-\beta_z^2-\beta_z\sigma(1-\beta_y n_y)]+(B_x n_x+B_y n_y)[\beta_z^2 n_z^2\sigma+\beta_z(1-\beta_y n_y)(1-n_z^2)]}{[(1-\beta_y n_y)^2-\beta_z^2 n_z^2](1-\beta_y n_y-\beta_z\sigma)}\right], \tag{2}$$

$$\boldsymbol{\Pi}^{m}_{\omega n} = -\frac{ic\beta_z^2 \exp\left(i\dfrac{\omega}{c}R\right)(1-\varepsilon)n_z}{\pi\omega^2 R(n_z+\sigma)(1-n_z^2)}\left[\frac{\boldsymbol{e}_x B_x(1-n_z^2)+\boldsymbol{e}_y B_y(1-n_z^2)+\boldsymbol{e}_z(B_x n_x+B_y n_y)n_z}{[(1-\beta_y n_y)^2-\beta_z^2 n_z^2](1-\beta_y n_y-\beta_z\sigma)}\right], \tag{3}$$

$$\boldsymbol{\Pi}^{e}_{\omega p} = \frac{ev_z\boldsymbol{e}_z \exp\left(i\dfrac{\omega}{c}R\right)(1-\varepsilon)n_z}{\pi\omega^2 R(\varepsilon n_z+\sigma)(1-n_z^2)}\left[\frac{(1-n_z^2)[1-\beta_y n_y-\beta_z^2-\beta_z\sigma]+\beta_z\beta_y n_y\sigma}{[(1-\beta_y n_y)^2-\beta_z^2 n_z^2](1-\beta_y n_y-\beta_z\sigma)}\right], \tag{4}$$

$$\boldsymbol{\Pi}^{e}_{\omega n} = \frac{ev_y\beta_z^2 \exp\left(i\dfrac{\omega}{c}R\right)(1-\varepsilon)n_z}{\pi\omega^2 R(n_z+\sigma)(1-n_z^2)}\left[\frac{\boldsymbol{e}_y(1-n_z^2)+\boldsymbol{e}_z n_y n_z}{[(1-\beta_y n_y)^2-\beta_z^2 n_z^2](1-\beta_y n_y-\beta_z\sigma)}\right], \tag{5}$$

where $\boldsymbol{e}_i$ is the ith unit vector, $\sigma = \sqrt{\varepsilon - (1-n_z^2)}$, $\chi_x = \frac{\omega}{v} n_x$, $\chi_y = \frac{\omega}{v} n_y$, $\beta = 1 - \frac{\gamma^{-2}}{2}$, γ is the Lorentz factor,

$$B_x = \gamma^{-1}\left(\chi_y m_z - \gamma^{-1}\frac{\omega}{v} m_y\right),$$

$$B_z = (\chi_x m_y - \chi_y m_x)\cos\psi - \gamma^{-1}\left(\gamma^{-1}\frac{\omega}{v} m_x - \chi_x m_z\right)\sin\psi, \tag{6}$$

$$B_y = \gamma^{-1}\left(\gamma^{-1}\frac{\omega}{v} m_x - \chi_x m_z\right)\cos\psi + (\chi_x m_y - \chi_y m_x)\sin\psi.$$

We shall indicate the quantities corresponding to the waves polarized in the incidence plane by the symbol p (“parallel”) and those corresponding to the waves polarized in the normal plane by the symbol n (“normal”).

Expressions (2)–(5) can be written in terms of the projections of the unit wave vector

$$\boldsymbol{n} = \frac{\boldsymbol{k}c}{\omega} = \{n_x, n_y, n_z\} = \{\cos\theta_x, \cos\theta_y, \cos\theta_z\} = \{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\} \tag{7}$$

and of the vector $\boldsymbol{\beta} = \{0, \beta_y, \beta_z\} = \beta\{0, \sin\psi, \cos\psi\}$.

The projections of the magnetic moment vector can be written in terms of the polar angle θ_0 and azimuthal angle ξ in the intrinsic coordinate system of the magnetic moment:

$$\begin{aligned} m_x &= |\boldsymbol{m}|\cdot\sin\theta_0\cdot\cos\xi, \\ m_y &= |\boldsymbol{m}|\cdot\sin\theta_0\cdot\sin\xi, \\ m_z &= |\boldsymbol{m}|\cdot\cos\theta_0. \end{aligned} \tag{8}$$

The z'-axis of the system K' (the rest system for the magnetic moment) coincides in direction with the vector **β.** Thus, the system K' is turned relative to the system K (the system related to the target) by an angle ψ (see Fig. 1).

Assuming that ε is a complex quantity, we shall seek the total Hertz vectors in the form

$$\boldsymbol{\Pi}_\omega = \left[\boldsymbol{n}\times(\boldsymbol{\Pi}_{\omega p} + \boldsymbol{\Pi}_{\omega n})\right] = A(\varepsilon)\cdot\left(\boldsymbol{P}'_\omega + i\boldsymbol{P}''_\omega\right). \tag{9}$$

Extracting the complex common factor $A(\varepsilon)$, the real (primed) and imaginary (doubly primed) parts in the total Hertz vectors, we find

$$A(\varepsilon) = \frac{c\exp\left(i\frac{\omega}{c}R\right)(1-\varepsilon)n_z}{\pi\omega^2 R(n_z+\sigma)(\varepsilon n_z+\sigma)(1-n_z^2)[(1-\beta_y n_y)^2 - \beta_z^2 n_z^2](1-\beta_y n_y - \beta_z\sigma)}, \tag{10}$$

$$\begin{aligned} \boldsymbol{P}'^{e}_\omega &= e\beta_z(1-n_z^2)\left[\boldsymbol{e}_x(\beta_z\beta_y n_z\tau'_1 + n_y\tau'_2) - \boldsymbol{e}_y n_x\tau'_2 - \boldsymbol{e}_z\beta_z\beta_y n_x\tau'_1\right], \\ \boldsymbol{P}''^{e}_\omega &= e\beta_z(1-n_z^2)\left[\boldsymbol{e}_x(\beta_z\beta_y n_z\tau''_1 + n_y\tau''_2) - \boldsymbol{e}_y n_x\tau''_2 - \boldsymbol{e}_z\beta_z\beta_y n_x\tau''_1\right], \end{aligned} \tag{11}$$

$$
\begin{aligned}
\boldsymbol{P}'^{m}_{\omega} &= \boldsymbol{e}_x(\beta_z^2 n_z\left[n_y(B_x n_x + B_y n_y) - (1-n_z^2)B_y\right]\tau_1'' - n_y\tau_3'') \\
&+\boldsymbol{e}_y(\beta_z^2 n_z\left[(1-n_z^2)B_x - n_x(B_x n_x + B_y n_y)\right]\tau_1'' + n_x\tau_3'') - \boldsymbol{e}_z\beta_z^2(1-n_z^2)(B_x n_y - B_y n_x)\tau_1'', \\
\boldsymbol{P}''^{m}_{\omega} &= \boldsymbol{e}_x(n_y\tau_3' - \beta_z^2 n_z\left[n_y(B_x n_x + B_y n_y) - (1-n_z^2)B_y\right]\tau_1') \\
&-\boldsymbol{e}_y(\beta_z^2 n_z\left[(1-n_z^2)B_x - n_x(B_x n_x + B_y n_y)\right]\tau_1' + n_x\tau_3') + \boldsymbol{e}_z\beta_z^2(1-n_z^2)(B_x n_y - B_y n_x)\tau_1'.
\end{aligned}
\tag{12}
$$

For simplification of the writing, complex quantities τ_i have been introduced into expressions (11) and (12):

$$
\tau_1 = \varepsilon n_z + \sigma, \quad \tau_2 = (n_z + \sigma)\left[(1-\beta_y n_y)(1-\beta_z\sigma) - \beta_z^2 - \beta_z\beta_y n_z n_y\right],
$$

$$
\tau_3 = (n_z + \sigma)\begin{bmatrix} B_z(1-n_z^2)((1-\beta_y n_y)^2 - \beta_z^2 - \beta_z(1-\beta_y n_y)\sigma) \\ +(B_x n_x + B_y n_y)(\beta_z^2 n_z^2\sigma + \beta_z(1-\beta_y n_y)(1-n_z^2)) \end{bmatrix}. \tag{13}
$$

From (9)–(12) we can obtain the spectral-angular distribution of the transient radiation for the charge – magnetic moment system by using the relation

$$
\frac{dW}{d\omega d\Omega} = \frac{\omega^4 R^2}{c^3}\left|A(\varepsilon)\right|^2\left|\boldsymbol{P}'^{e}_{\omega} + i\boldsymbol{P}''^{e}_{\omega} + \boldsymbol{P}'^{m}_{\omega} + i\boldsymbol{P}''^{m}_{\omega}\right|^2 = \frac{dW_{e+m}}{d\omega d\Omega}\left(1 + \sum_i m_i F_i\right). \tag{14}
$$

The last term in (14), linear in the magnetic moment components m_i, arises due to the interference of the fields induced by the charge and magnetic moment TR.

The next valid simplification is the normal incidence approximation $\{\beta_y = 0,\ \beta_z = \beta,\ n_z = \cos\theta\}$ for the case of ultrarelativistic energies (γ >> 1).

The existence of the interference term strongly depends on the orientation of the magnetic moment. Thus, if the magnetic moment is oriented along the velocity vector ($\theta_0 = 0°$, see (8)), no interference occurs, i.e., $F_z = 0$, and the total intensity of the system TR is the sum of the intensities of the charge and magnetic moment radiations:

$$
\frac{dW}{d\omega d\Omega} = \frac{\omega^4 R^2}{c^3}\left|A(\varepsilon)\right|^2\left[\boldsymbol{P}'^{e2}_{\omega} + \boldsymbol{P}''^{e2}_{\omega} + \boldsymbol{P}'^{m2}_{\omega} + \boldsymbol{P}''^{m2}_{\omega}\right] \tag{15}
$$

$$
= \frac{e^2\beta_z^2\left|1-\varepsilon\right|^2 n_z^2\left|1-\beta_z\sigma-\beta_z^2\right|^2(1-n_z^2)}{\pi^2 c\left[1-\beta_z^2\right]^2\left|(\varepsilon n_z+\sigma)(1-\beta_z\sigma)\right|^2} + \frac{m^2\omega^2\gamma^{-2}\beta_z^4\left|1-\varepsilon\right|^2 n_z^2(1-n_z^2)}{\pi^2 c v^2\left[1-\beta_z^2\right]^2\left|(n_z+\sigma)(1-\beta_z\sigma)\right|^2}.
$$

Note that for a neutral particle ($e = 0$) in the ultrarelativistic limit (γ >> 1, θ^2 << 1), on the assumption that $1-\varepsilon \approx \frac{\omega_p^2}{\omega^2}$ << 1, formula (15) coincides with the formulas that describe the spectral-angular distribution of the TR of a magnetic moment [6, 7]:

$$
\frac{d^2W_m}{d\omega d\Omega} = \frac{\gamma^{-2}m^2\omega_{p1}^4}{\pi^2 c^3\omega^2} \times \frac{\theta^2}{\left(\theta^2 + \gamma^{-2} + \omega_{p1}^2/\omega^2\right)^2\left(\theta^2 + \gamma^{-2}\right)^2}. \tag{16}
$$

For the traverse orientation of the magnetic moment, an interference contribution exists. In this case, it is necessary to discriminate between the situation where the magnetic moment is located in the incidence plane (passing through the electron momentum vector and the normal to the target surface) (ξ = 90°) and the situation where the magnetic moment is orthogonal to the plane (ξ = 0°). In what follows we restrict our consideration to the traverse orientation of the magnetic moment relative to the incidence plane (θ_0 = 90°, ξ = 0°) in the plane defined by the equation $n_x = 0$ $\left(n_y = \sin\theta,\ n_z = \cos\theta\right)$. This plane contains only the contribution from the component F_x (m_y = mz = 0).

It should be noted that the case θ_0 = 90°, ξ = 90° must be considered for the plane n_y = 0. For this case, the spectral-angular distributions can be derived from the previous expressions by formally replacing n_y by $-n_x$.

Hence, the spectral-angular distribution of the TR from a charge – magnetic moment system, in view of the interference term, has the following form:

$$\frac{d^2W_{e+m}}{d\omega d\Omega} = \frac{e^2\beta^2 |1-\varepsilon|^2 \cos^2\theta \left|1-\beta\sigma-\beta^2\right|^2 \sin^2\theta}{\pi^2 c[1-\beta^2\cos^2\theta]^2 |(\varepsilon\cos\theta+\sigma)(1-\beta\sigma)|^2} \tag{17}$$

$$+\frac{m^2\omega^2 |1-\varepsilon|^2 \left|\gamma^{-2}\beta[\beta\sigma\cos^2\theta+\sin^2\theta]-\sin^2\theta[1-\beta^2-\beta\sigma]\right|^2 \cos^2\theta}{\pi^2 cv^2 \sin^2\theta[1-\beta^2\cos^2\theta]^2 |(\varepsilon\cos\theta+\sigma)(1-\beta\omega)|^2},$$

$$F_x = 2\frac{\boldsymbol{P}'^e_\omega \boldsymbol{P}'^m_\omega + \boldsymbol{P}''^e_\omega \boldsymbol{P}''^m_\omega}{\boldsymbol{P}'^{e2}_\omega + \boldsymbol{P}''^{e2}_\omega + \boldsymbol{P}'^{m2}_\omega + \boldsymbol{P}''^{m2}_\omega} = \frac{2e\beta\omega}{v} \frac{\tau''_2\tau'_4 - \tau'_2\tau''_4}{e^2\beta^2\sin^2\theta|\tau_2|^2 + \left(\frac{m\omega}{v}\right)^2 |\tau_4|^2}. \tag{18}$$

The quantity τ_2 in expression (18) is determined from expression (13) with ψ = 0° and τ_4 is determined from the relation

$$\tau_4 = (\cos\theta+\sigma)\left\{\gamma^{-2}\beta\left[\beta\sigma\cos^2\theta+\sin^2\theta\right]-\sin^2\theta\left[1-\beta^2-\beta\sigma\right]\right\}. \tag{19}$$

For a particle with a charge e and a magnetic moment m_{B} = $9.27\cdot 10^{-24}$ J/T the relative contribution of the interference to the radiation intensity is described as

$$\frac{d^2W_m}{d\omega d\Omega}\bigg/\frac{d^2W_e}{d\omega d\Omega} \sim \left(\frac{m\omega}{ev}\right)^2 = \left(\frac{\hbar\omega}{2\beta m_e c^2}\right)^2, \tag{20}$$

$$m_x F_x \sim \frac{m\omega}{ev} = \frac{e\hbar c}{2m_e c^2}\frac{\omega}{ev} = \frac{\hbar\omega}{2\beta m_e c^2}, \tag{21}$$

where m_e is the electron mass.

Thus, the contribution of the magnetic moment to the total TR intensity can be neglected, as for various values of the TR photon energy and of the Lorentz factor ($\gamma \geq 10$) this contribution is not above $10^{-5}-10^{-6}$.

From expression (18) it follows that the interference of the radiation fields produced by a charge and by a magnetic moment is possible only in the case that the Hertz vectors of the radiation fields have both the real and the imaginary parts. Thus, the medium permittivity should be a complex quantity $\varepsilon = \varepsilon' + i\varepsilon''$, otherwise the Hertz vectors are real for the charge and pure imaginary for the magnetic moment, and this holds for the case of a perfect conductor.

For the x-ray frequency band, the medium permittivity corresponding to the frequencies near absorption edges is described by the following expression [8]:

$$\varepsilon(\omega) = \left[1 - \left(\frac{\hbar\omega_p}{\hbar\omega}\right)^2 \frac{1}{2Z}\left(f'(\omega) + if''(\omega)\right)\right]^2 = 1 - \delta(\omega) + i\eta(\omega), \qquad (22)$$

where $f'(\omega) + if''(\omega)$ is the anomalous factor and ω_p is the frequency of the target material plasmon, which is determined by the electron concentration n_e in the target material:

$$\omega_p = \sqrt{4\pi n_e r_0 c^2} = \sqrt{4\pi \frac{Z}{A} N_0 \rho r_0 c^2}. \qquad (23)$$

Here r_0 is the classical electron radius; Z and A are the charge and the atomic mass of the target atoms, respectively; N_0 is the Avogadro number, and ρ is the density.

In view of the above simplifications, formulas (17) and (18) yield

$$\frac{d^2W_e}{d\omega d\Omega} = \frac{e^2}{\pi^2 c}\frac{\theta^2}{\left(\gamma^{-2} + \theta^2\right)^2}\left|\frac{\delta - i\eta}{\gamma^{-2} + \theta^2 + \delta - i\eta}\right|^2, \qquad (24)$$

$$F_x = \frac{\omega}{2ve}\frac{\theta\eta\left(\gamma^{-2} + \theta^2\right)}{\left[e^2\theta^2 + \left(\frac{m\omega}{v}\right)^2\left(\gamma^{-2} + \theta^2\right)^2\right]\left|1 - (\delta - i\eta)\right|^2}. \qquad (25)$$

Expression (25) obviously indicates how the type of permittivity affects the interference term, namely:

(i) if ε is a real number, then $\eta = 0$ and, hence, there is no interference ($F_x = 0$);

(ii) in the case of an ideal conductor, i.e., as $\eta \to \infty$, we have $F_x \to 0$ and the spectral-angular distribution of the TR for the charge is described by the Ginzburg–Frank formula [5, 6]

$$\frac{d^2W_e}{d\omega d\Omega} = \frac{e^2}{\pi^2 c}\frac{\theta^2}{\left(\gamma^{-2} + \theta^2\right)^2}. \qquad (26)$$

Let us estimate the interference contribution for the case where a particle with a charge e and a magnetic moment $m = \frac{e\hbar c}{2m_e c^2}$ escapes from a titanium target in vacuum. Figure 2 shows the behavior of the real and imaginary parts of the permittivity of Ti [8, 9] at the L absorption edge of interest (454 eV). For the photon energy equal to 454 eV both the real and the imaginary part of ε abruptly increase.

Figure 3 gives spectral-angular distributions of the TR of a charge and of the interference term for the "forward" direction (formulas (24), (25)).

As follows from Fig. 3, the TR asymmetry is an alternating-sign quantity varying in a narrow angular interval.

The process asymmetry is estimated in magnitude as $|m_x F_x| \sim 10^{-3}$, which is much lower than that in a Mott polarimeter. However, the probability of transient radiation to occur in the soft x-ray band is several orders of magnitude greater than the probability of Mott scattering, and this can result in comparable values of the effective analyzing ability $\left(\frac{d\sigma_{\mathrm{TR}}}{d\Omega}\right)^2 m_x F_x$ of the proposed polarimeter.

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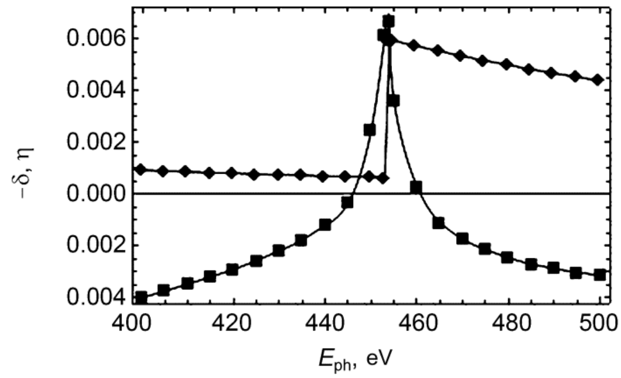

Fig. 2. The real (−δ) and imaginary (η) parts of the permittivity of titanium in the L absorption band (454 eV). Both the real (■) and the imaginary part (♦) experience a jump at the photon energy $E_{ph} = 454$ eV.

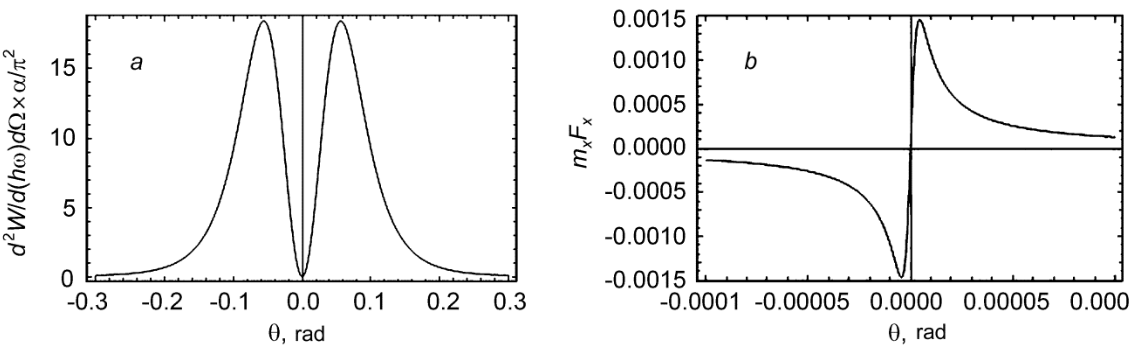

Fig. 3. Spectral-angular distribution of the transient radiation produced by an electric charge + magnetic moment system for $\gamma = 10$: the total spectral-angular distribution (a) and the relative interference contribution (b).

REFERENCES

1. T. J. Gay and F. B. Dunning, Rev. Sci. Instrum., **63**, 1635 (1992).
2. V. N. Petrov, M. S. Galaktionov, and A. S. Kamochkin, Zh. Tekh. Fiz., **71**, 79 (2001).
3. J. Arrington *et al.*, NIMA, **311**, 39 (1992).
4. A. P. Potylitsyn, Russ. Phys. J., No. 3, 304 (2001).
5. V. E. Pafomov, Trudy FIAN, **XLIV**, 28 (1969).
6. V. L. Ginzburg and V. L. Tsitovich, Transient Radiation and Transient Scattering [in Russian], Nauka, Moscow (1984).
7. M. Sakuda, Phys. Rev. Lett., **72**, 804 (1994).
8. B. L. Henke, E. M. Gullikson, and J. C. Davis, At. Data Nucl. Tables, **54**, 181 (1993).
9. W. Knulst, M. J. van der Wiel, and O. J. Luiten, Appl. Phys. Lett., **83**, 4050 (2003).