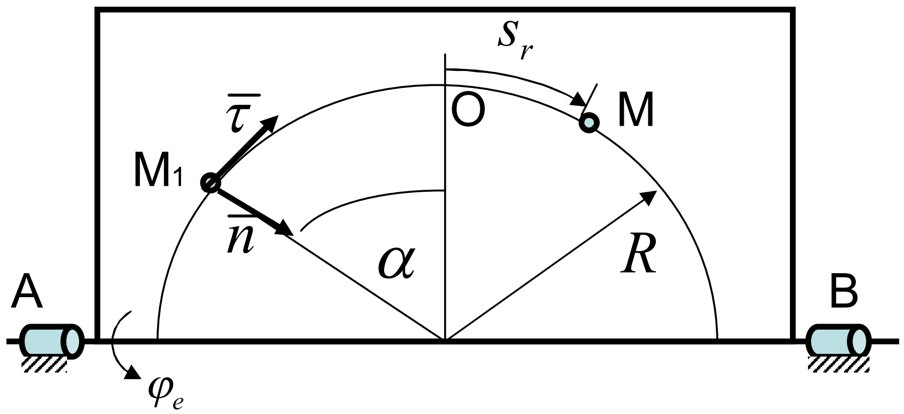
Задача №1

Движение точки M по пластине – относительное движение.

Вращение пластины – переносное движение для точки M.



Дано:

$$\varphi_e = 2t^2 - t^3;$$

$$s_r = OM = \frac{2\pi R}{3}\sin(\frac{7}{6}\pi t).$$

Определить абсолютные скорость и ускорение точки M в момент времени t =1сек. Или др. словами

$$V_a(1) = ?,\quad a_a(1) = ?.$$

Решение:

1). Относительное движение.

$$\varphi_e = const;\quad s_r = s_r(t).$$

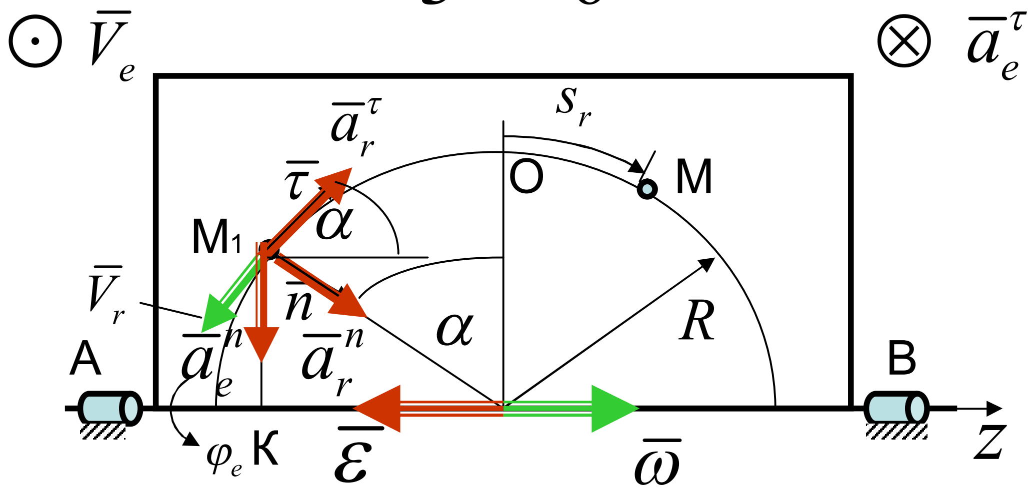
$$s_r(1) = \frac{2\pi R}{3}\sin(\pi + \frac{\pi}{6}) = -\frac{\pi R}{3}.$$

$$\alpha = \frac{|s_r|}{R} = \frac{\pi}{3} = 60^\circ.$$

$$V_{r\tau}(t)=\dot{s}_r=\frac{2\pi R}{3}\cdot\frac{7}{6}\pi\cdot\cos(\frac{7}{6}\pi t).\qquad V_{r\tau}(1)=-\frac{7\sqrt{3}\cdot\pi^2 R}{18}.\qquad V_r=\left|V_{r\tau}\right|$$

$$a_r^\tau(t)=\dot{V}_{r\tau}(t)=-\frac{2\pi R}{3}\cdot(\frac{7}{6}\pi)^2\cdot\sin(\frac{7}{6}\pi t).\qquad a_r^\tau(1)=\tilde{a}_r^\tau=\frac{\pi R}{3}\cdot(\frac{7}{6}\pi)^2.$$

$$a_r^\tau=\left|\tilde{a}_r^\tau\right|=\frac{\pi R}{3}\cdot(\frac{7}{6}\pi)^2;\qquad a_r^n=\frac{V_r^2}{R}.$$



2). Переносное движение. $M_1K=h.$

$s_r(t)=const,\quad \varphi_e=\varphi_e(t).\quad \varepsilon_z(t)=\dot{\omega}_z(t)=4-6t;$

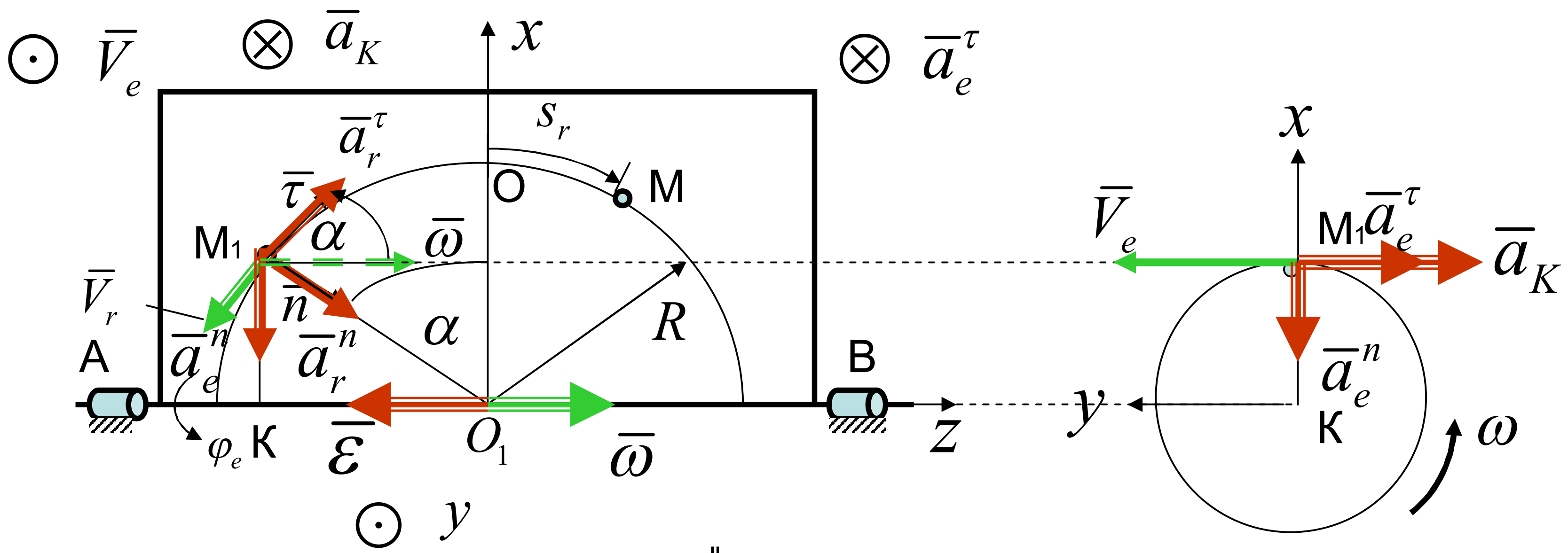
$\omega_z(t)=\dot{\varphi}_e=4t-3t^2;\qquad \tilde{\varepsilon}_z(1)=-2;$

$\omega_z(1)=1;\qquad \omega=\left|\omega_z(1)\right|=1.\qquad \varepsilon=\left|\tilde{\varepsilon}_z(1)\right|=2.$

$V_e=\omega\cdot h;$

$a_e^\tau=\varepsilon\cdot h;$

$a_e^n=\omega^2\cdot h;$



3). Ускорение Кориолиса.

$$\bar{a}_K = 2 \cdot \bar{\omega}_e \times \bar{V}_r;$$

$$a_K = 2\omega \cdot V_r \cdot \sin\alpha;$$

4). Абсолютная скорость.

$$\bar{V}_a = \bar{V}_r + \bar{V}_e$$

$$\bar{V}_r \perp \bar{V}_e$$

$$V_a = \sqrt{V_r^2 + V_e^2};$$

5). Абсолютное ускорение.

$$\bar{a}_a = \bar{a}_r^\tau + \bar{a}_r^n + \bar{a}_e^\tau + \bar{a}_e^n + \bar{a}_K.$$

$$a_{ax} = a_r^\tau \sin\alpha - a_r^n \cos\alpha - a_e^n;$$

$$a_{ay} = -a_K - a_e^\tau;$$

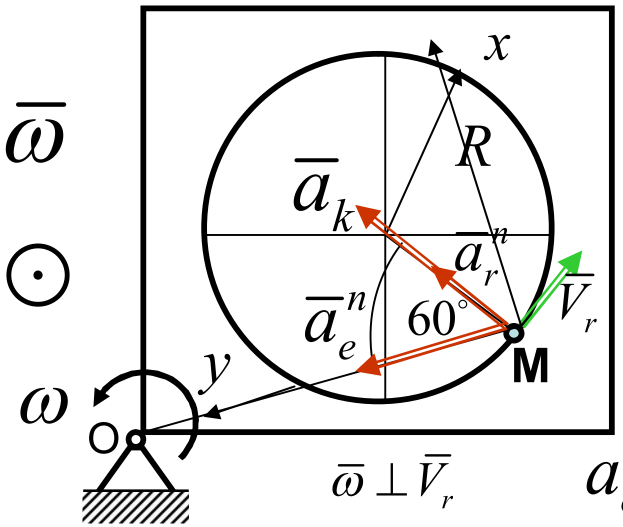
$$a_{az} = a_r^n \sin\alpha + a_r^\tau \cos\alpha.$$

$$a_a = \sqrt{a_{ax}^2 + a_{ay}^2 + a_{az}^2}.$$

Задача №2

Дано: $OM = 2R, \quad \omega = const, \quad V_r = 2R\omega.$

Определить в указанном положении абсолютное ускорение точки М.



Решение:

$$a_r^\tau = \dot{V}_r \equiv 0; \qquad a_r^n = \frac{V_r^2}{R} = 4R\omega^2.$$

$$a_e^\tau = \varepsilon MO = \dot{\omega} MO \equiv 0; \quad a_e^n = MO \cdot \omega^2 = 2R\omega^2.$$

$$\bar{a}_K = 2\bar{\omega}_e \times \bar{V}_r. \qquad \bar{\omega}_e = \bar{\omega}. \qquad a_K = 2\omega \cdot V_r = 4R\omega^2.$$

$$\bar{a}_M = \bar{a}_r^n + \bar{a}_e^n + \bar{a}_K.$$

$$a_{Mx} = (a_r^n + a_K) \cdot \cos 30^\circ = 4R\omega^2\sqrt{3};$$

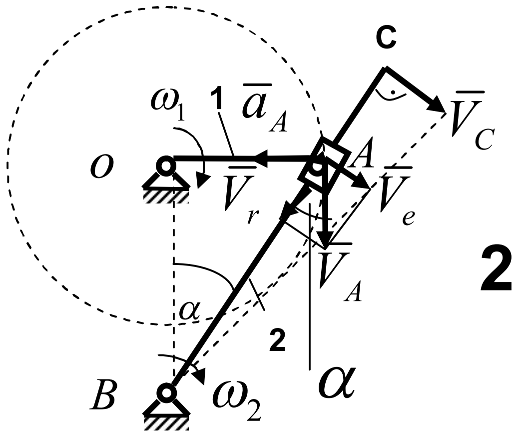
$$a_{My} = a_e^n + (a_r^n + a_K) \cdot \cos 60^\circ = 6R\omega^2;$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2} = 2R\omega^2\sqrt{21}.$$

Пример №3

Дано: $OA = r;\ \alpha = 30^\circ;\ BC = 3r;\ \omega_1 = \omega = const.$

$$V_c = ?;\quad a_c = ?.$$

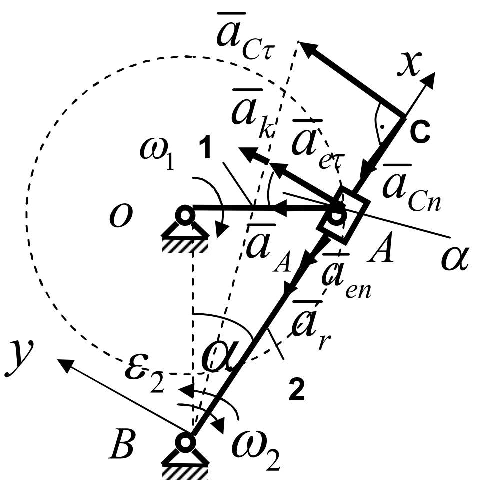


1. $V_A = \omega r;\qquad a_A = a_A^n = \omega^2 r.$

2. $V_r = V_A \cos\alpha = \omega r \dfrac{\sqrt{3}}{2};\quad V_e = V_A \sin\alpha = \omega r / 2.$

$$\omega_2 = V_e / BA = (\omega r/2)/2r = \omega/4.$$

$$V_c = \omega_2 BC = (\omega/4)\cdot 3r = 3\omega r/4.$$



3. $a_{en} = \omega_2^2 BA = \dfrac{\omega^2}{16}\cdot 2r = \dfrac{\omega^2 r}{8};$

$$a_K = 2\omega_2 V_r = \sqrt{3}\omega^2 r/4.$$

4. $\underline{\underline{\bar{a}_A}} = \underset{\parallel BA}{\underline{\bar{a}_r}} + \underline{\underline{\bar{a}_{en}}} + \underset{\perp BA}{\underline{\bar{a}_{e\tau}}} + \underline{\underline{\bar{a}_K}}.\quad \varepsilon_2 = a_{e\tau}/BA = \omega^2\sqrt{3}/8.$

$y \parallel\quad a_A \cos 30^\circ = a_{e\tau} + a_K;\quad a_{e\tau} = a_A\sqrt{3}/2 - a_K = \omega^2 r\sqrt{3}/4.$

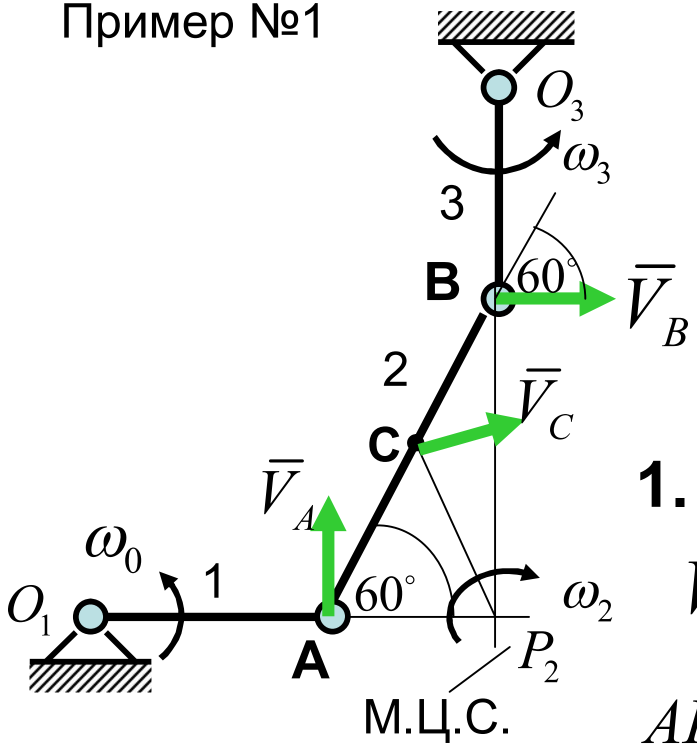
5.

$$a_{C\tau} = \varepsilon_2 BC = \frac{\omega^2\sqrt{3}}{8}\cdot 3r;$$

$$a_{Cn} = \omega_2^2 BC = \frac{\omega^2}{16}\cdot 3r;$$

$$a_c = \sqrt{a_{C\tau}^2 + a_{Cn}^2} = \frac{3\sqrt{13}}{8}\cdot \omega^2 r.$$

Пример №1



Дано: $O_1A = r, \quad AB = 2r, \quad O_3B = r;$

$$AC = AB/2, \quad \omega_1 = \omega_0 = const.$$

$$V_B = ?, \quad V_C = ?, \quad a_B = ?, \quad a_C = ?;$$
$$\varepsilon_2 = ?, \quad \varepsilon_3 = ?.$$

Решение:

1. Определяем скорости.

$$V_A = \omega_1 \cdot O_1A = \omega_0 r; \qquad CP_2 = AC = r.$$

$$AP_2 = AB\cos 60^\circ = r; \quad BP_2 = AB\sin 60^\circ = r\sqrt{3}.$$

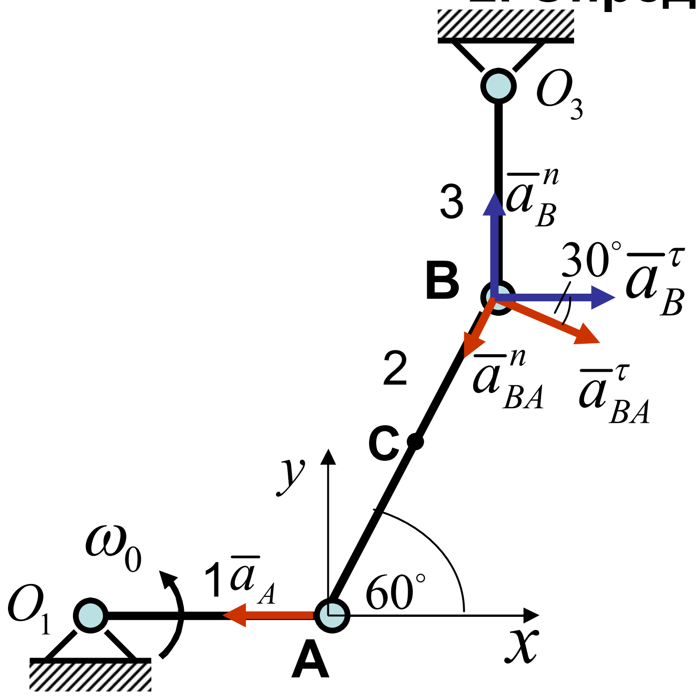
$$\omega_2 = V_A / AP_2 = \omega_0; \quad V_C = \omega_2 CP_2 = \omega_0 r; \qquad V_B = \omega_2 BP_2 = \omega_0 r\sqrt{3}.$$

$$\omega_3 = V_B / O_3B = \omega_0\sqrt{3}.$$

По теореме о проекциях $V_A\cos 30^\circ = V_B\cos 60^\circ.$

$$V_B = V_A\sqrt{3} = \omega_0 r\sqrt{3}.$$

2. Определение ускорения точки B.



$$a_A = a_A^n = \omega_1^2 O_1A = \omega_0^2 r;$$

$$\overline{a}_B = \overline{a}_A + \overline{a}_{BA}^n + \overline{a}_{BA}^\tau; \quad (1)$$

$$\overline{a}_B = \overline{a}_B^n + \overline{a}_B^\tau; \quad (2)$$

$$\underline{\underline{\overline{a}_B^n}} + \underset{\perp O_3B}{\underline{\overline{a}_B^\tau}} = \underline{\underline{\overline{a}_A}} + \underline{\underline{\overline{a}_{BA}^n}} + \underset{\perp AB}{\underline{\overline{a}_{BA}^\tau}}; \quad (3)$$

$$a_B^n = \omega_3^2 O_3B = \omega_0^2 3r;$$

$$a_{BA}^n = \omega_2^2 AB = \omega_0^2 2r;$$

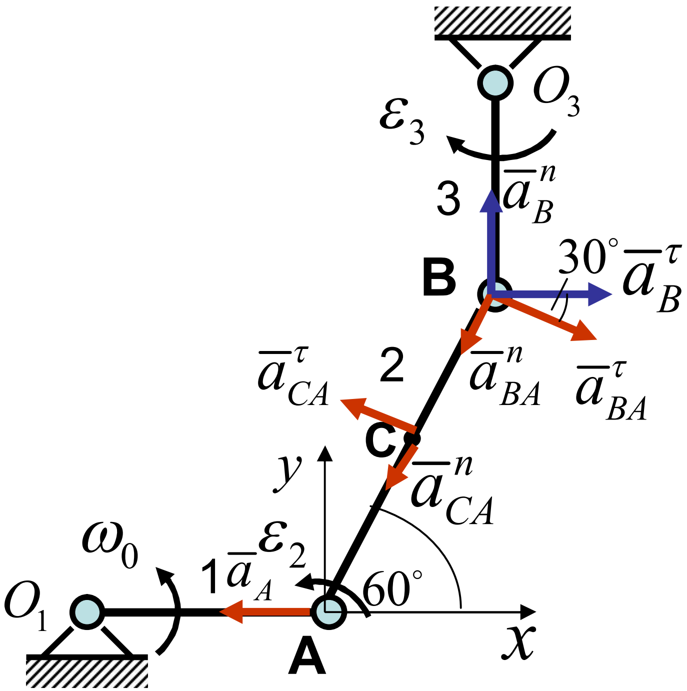
x: $a_B^\tau = -a_A - a_{BA}^n \cos 60^\circ + a_{BA}^\tau \cos 30^\circ;$ (4)

y: $a_B^n = -a_{BA}^n \sin 60^\circ - a_{BA}^\tau \sin 30^\circ.$ (5)

Из (5) $a_{BA}^\tau = -(a_{BA}^n \sin 60^\circ + a_B^n)/\sin 30^\circ = -2r\omega_0^2(3+\sqrt{3}).$

Из (4) $a_B^\tau = -\left[\omega_0^2 r + \omega_0^2 2r/2 + 2\omega_0^2 r(3+\sqrt{3})\sqrt{3}/2\right] = -\omega_0^2 r(5+3\sqrt{3}).$

$$\underline{\underline{a_B = \sqrt{(a_B^n)^2 + (a_B^\tau)^2} = \omega_0^2 r\sqrt{9+(5+3\sqrt{3})^2}.}}$$



3. Определение угловых ускорений.

$$\varepsilon_2 = \left|a_{BA}^\tau\right| / AB = \omega_0^2(3+\sqrt{3}).$$

$$\varepsilon_3 = \left|a_B^\tau\right| / O_3B = \omega_0^2(5+3\sqrt{3}).$$

4. Определение ускорения точки С.

$$\overline{a}_C = \overline{a}_A + \overline{a}_{CA}^n + \overline{a}_{CA}^\tau.$$

$$a_{CA}^n = \omega_2^2 AC = \omega_0^2 r;$$

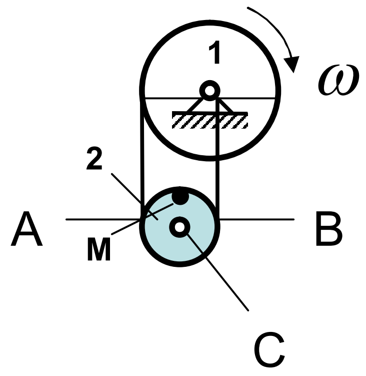
$$a_{CA}^\tau = \varepsilon_2 AC = \omega_0^2 r(3+\sqrt{3}).$$

$$a_{Cx} = -a_A - a_{CA}^n \cos 60^\circ - a_{CA}^\tau \cos 30^\circ;$$

$$a_{Cy} = -a_{CA}^n \sin 60^\circ + a_{CA}^\tau \sin 30^\circ;$$

$$a_C = \sqrt{a_{Cx}^2 + a_{Cy}^2}.$$

Пример №2



Дано: $\omega = const; \quad r_1 = 2r.$

$$V_M = ?, \quad a_M = ?$$