Tomsk Polytechnic University

DESCRIPTIVE GEOMETRY

ENGINEERING GRAPHICS

G.F.Vinokurova, R.G. Dolotova, S.P. Burkova

Lecture 5

AXONOMETRIC PROJECTIONS

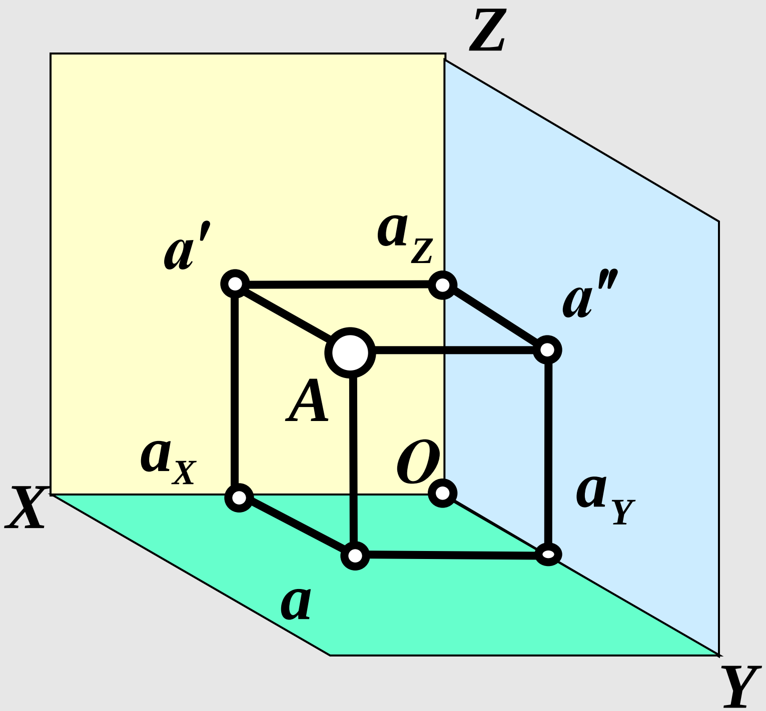
Plan

1. The Method of Axonometric Projection
2. Rectangular Parallel Isometry and Dimetry
3. Representation of a Circle and a Sphere in Isometry and Dimetry

The Method of Axonometric Projection

The word **“axonometry”** is derived from the Greek words **“*axon*”** which means **“axis”** and **“*metro*”** meaning “I measure’, so it can be translated as **“the measurement by the axes”.**

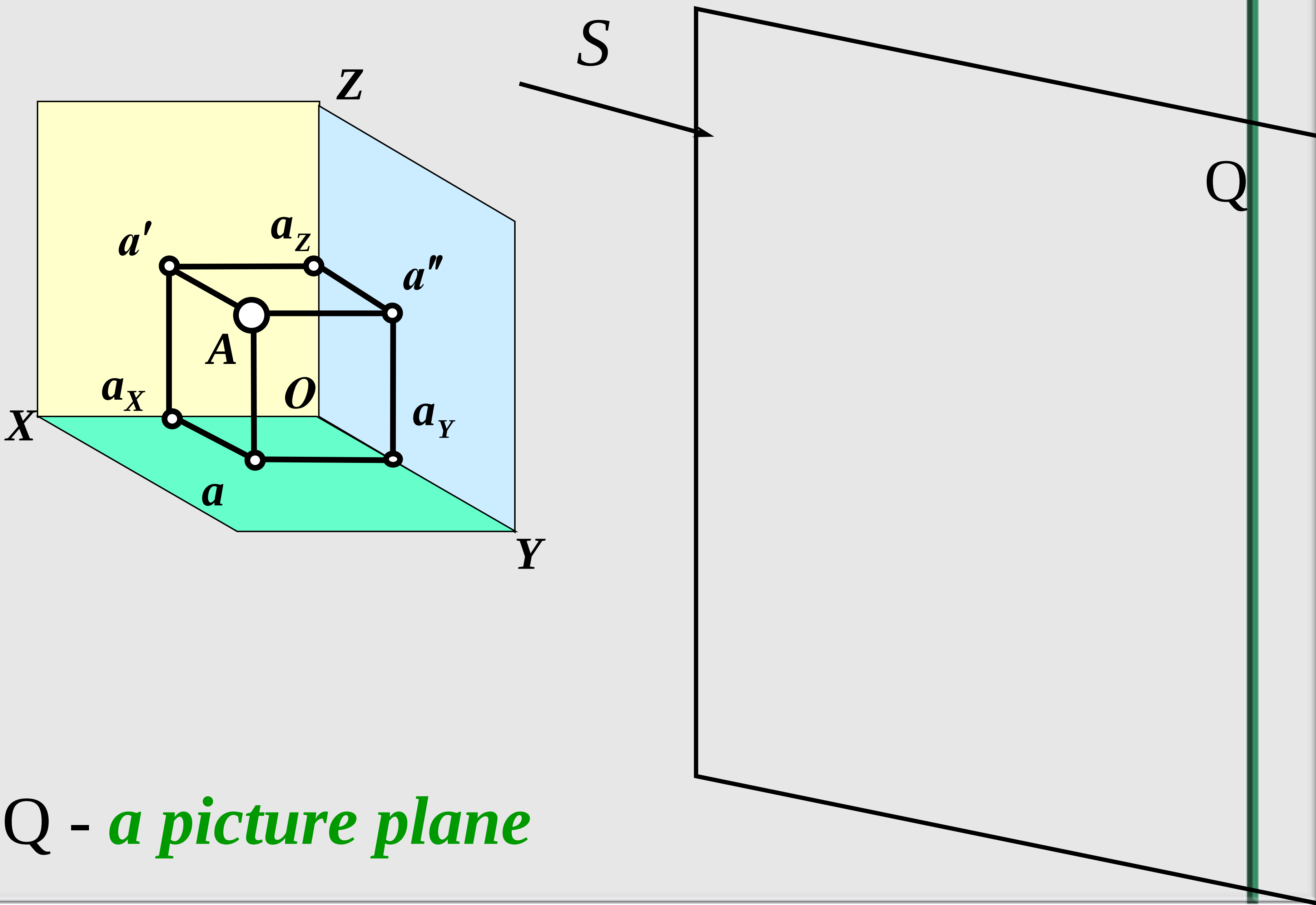
The method of axonometric projection consists in the following: **a given figure and the axes of rectangular co-ordinates to which the figure is related in space are projected on a plane referred to as a *plane of projections*** (it also called ***a picture plane***).


Z
a′
a_Z
a″
A
a_X
O
a_Y
X
a
Y

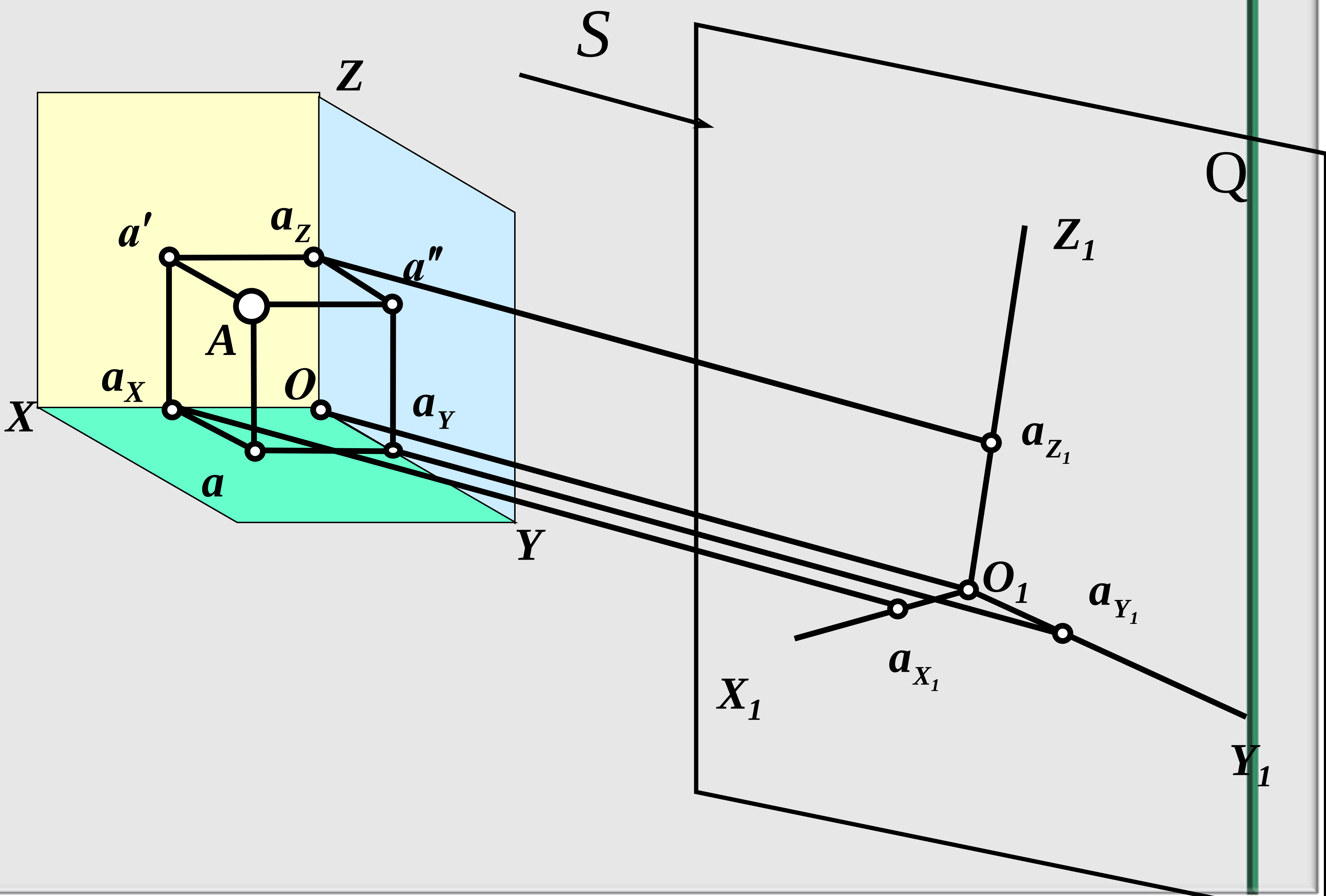
Depending on the distance between the centre of projection and the picture plane all axonometric projections are classified as:

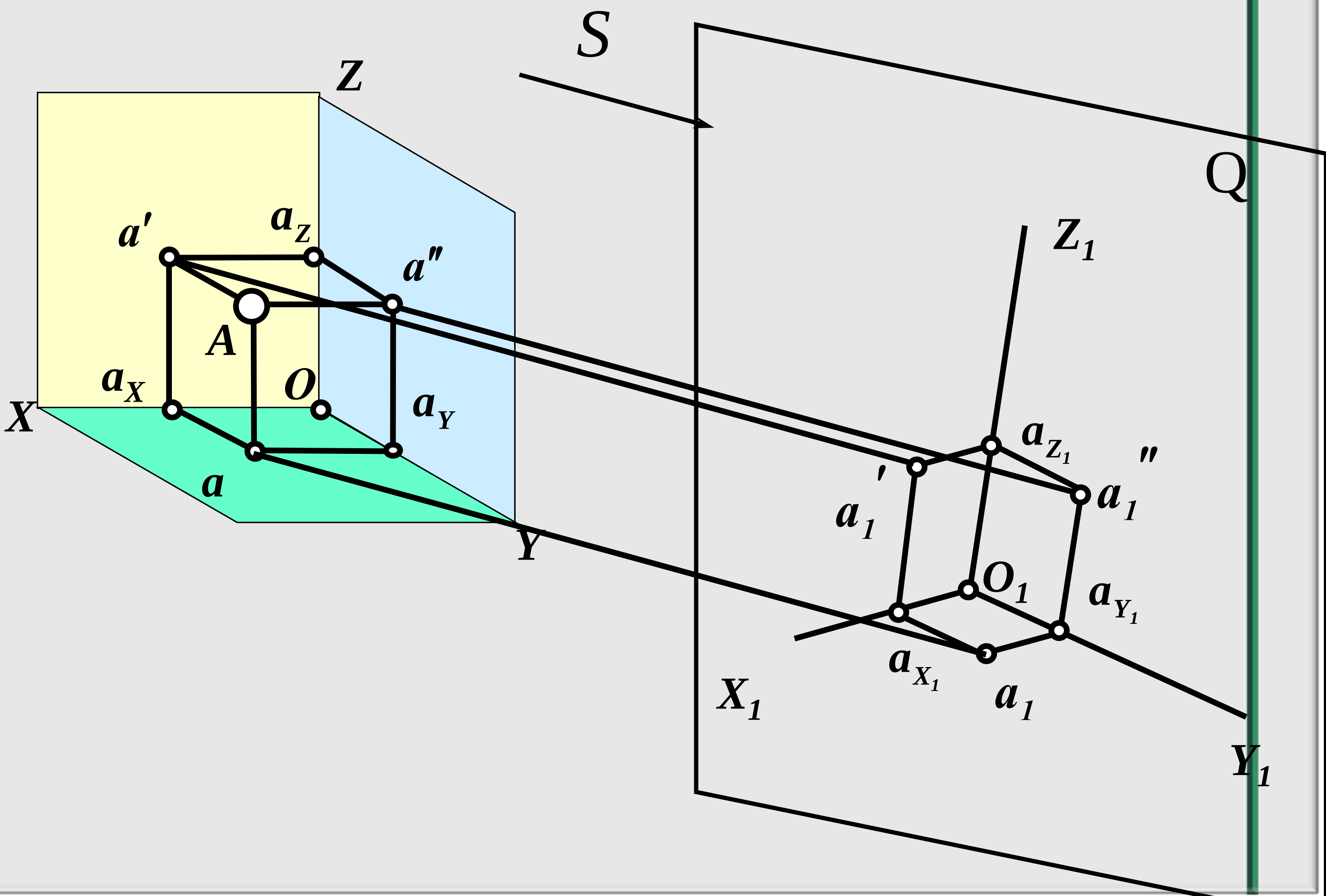
the *central projections* - the centre is located at a finite distance from the plane;

the *parallel projections* - the centre is at infinity.

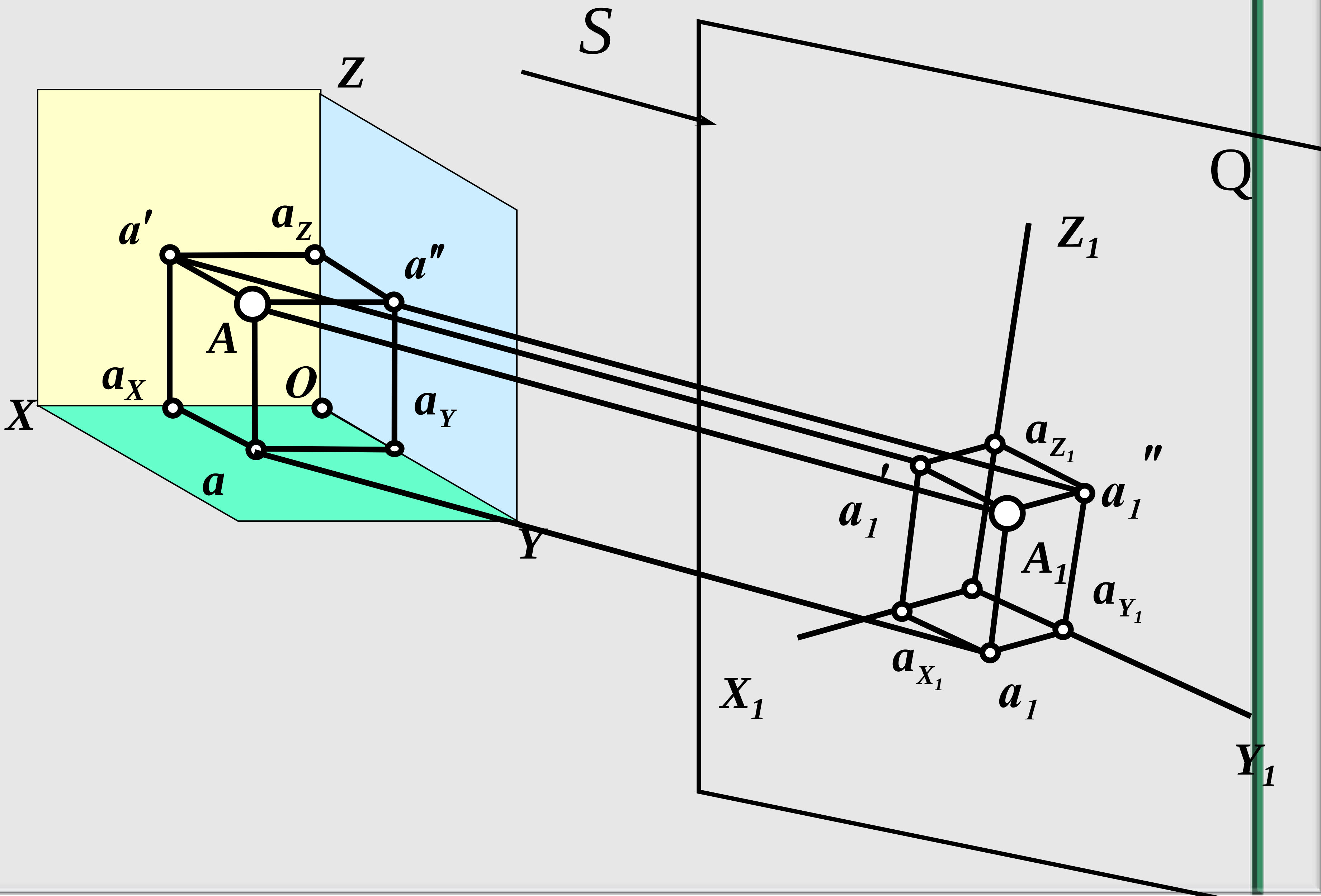


Q - ***a picture plane***


S
Z
Q
a′
a_Z
a″
A
a_X
O
a_Y
X
a
Y
Z_1
a_{Z_1}
O_1
a_{Y_1}
a_{X_1}
X_1
Y_1



S
Q
Z
a′
a_Z
a″
A
a_X
O
a_Y
X
a
Y
Z_1
a_Z1
″
′
a_1
a_1
O_1
a_Y1
a_X1
X_1
a_1
Y_1

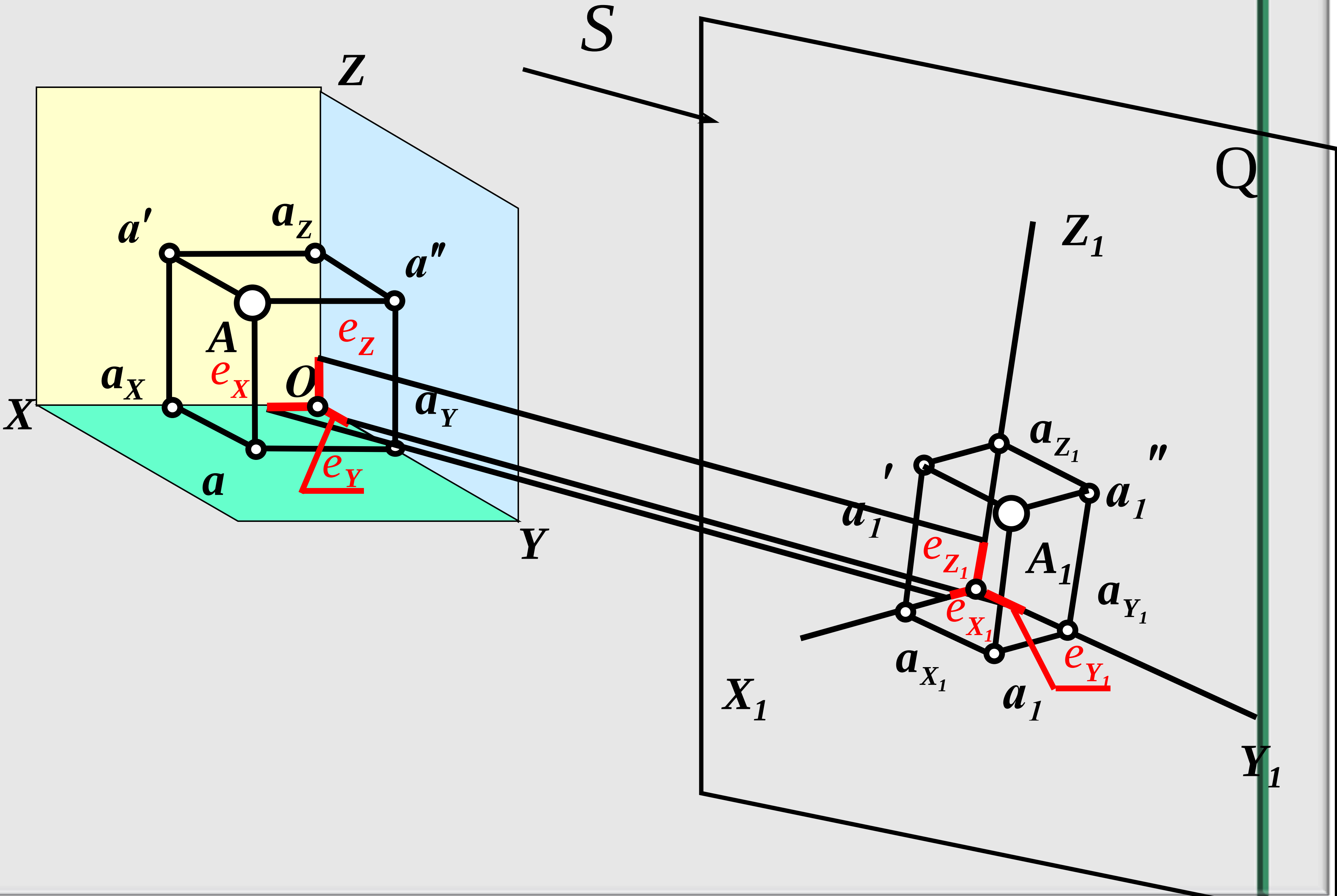


S
Q
Z
a'
a_Z
a"
A
a_X
O
a_Y
X
a
Y
Z_1
a_{Z_1}
a_1'
a_1"
A_1
a_{Y_1}
a_{X_1}
a_1
X_1
Y_1

X_1O_1, Y_1O_1, Z_1O_1 - axonometric co-ordinate axes or axonometric axes

A_1 - axonometric projection of the point *A*;

a_1, a_1', a_1'' - secondary projections of the point *A*;

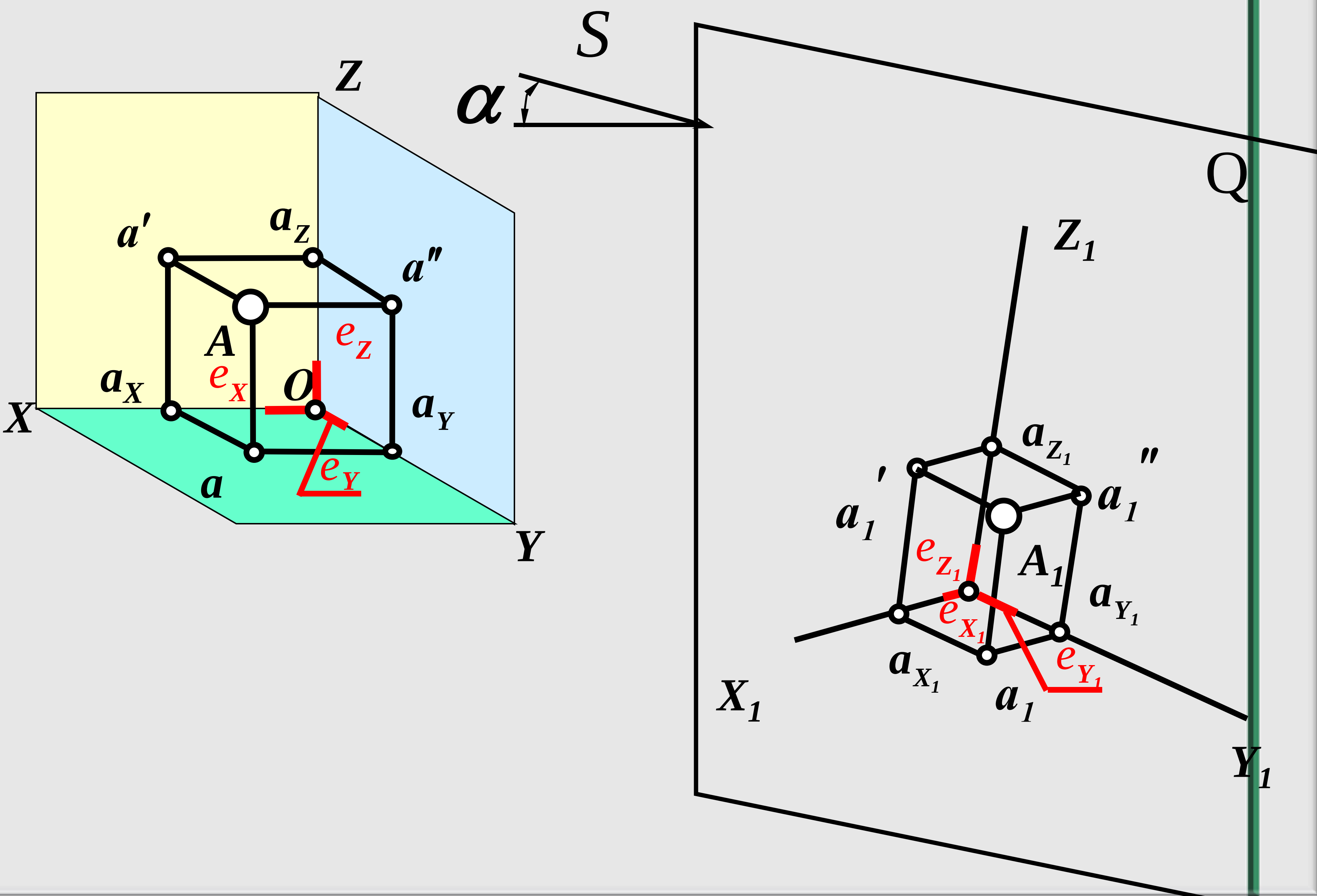


S
Q
Z
a′
a_Z
a″
A
e_Z
a_X
e_X
O
a_Y
X
e_Y
a
Y
Z_1
a_{Z_1}
′
″
a_1
a_1
e_{Z_1}
A_1
a_{Y_1}
e_{X_1}
a_{X_1}
e_{Y_1}
X_1
a_1
Y_1

e_X, e_Y, e_Z – scaled line-segments

$e_{X_1}, e_{Y_1}, e_{Z_1}$ –

axonometric (secondary) projections of the scaled line-segments



S
α
Z
a′
a_Z
a″
A
e_Z
a_X
e_X
O
a_Y
X
e_Y
a
Y
Q
Z_1
a_{Z_1}
a_1″
a_1′
e_{Z_1}
A_1
a_{Y_1}
e_{X_1}
a_{X_1}
e_{Y_1}
X_1
a_1
Y_1

The ratio of the length of the axonometric projection segment to its true size is referred to as the **coefficient of distortion on an axis.**

Coefficient of distortion on an axis

$$\frac{e_{X_1}}{e_X} = m, \quad \frac{e_{Y_1}}{e_Y} = n, \quad \frac{e_{Z_1}}{e_Z} = k$$

Oblique axonometric projection $(\alpha \neq 90^\circ)$

$$m^2 + n^2 + k^2 = 2 + ctg^2\alpha$$

Rectangular axonometric projection $(\alpha = 90^\circ)$

$$m^2 + n^2 + k^2 = 2$$

Axonometry is a representation of an object on a plane related to a certain coordinate system and completed to a certain scale subject to the coefficients of distortion.

$$m = n = k$$ *Isometric projections*

$$m = n \neq k$$
$$m = k \neq n$$
Dimetric projections

$$m \neq n \neq k$$ *Trimetric projections*

Rectangular Parallel Isometry and Dimetry

Rectangular Parallel Isometry

In rectangular isometric projection the axonometric axes *x*, *y*, *z* are at 120° to each other; The coefficients of distortion

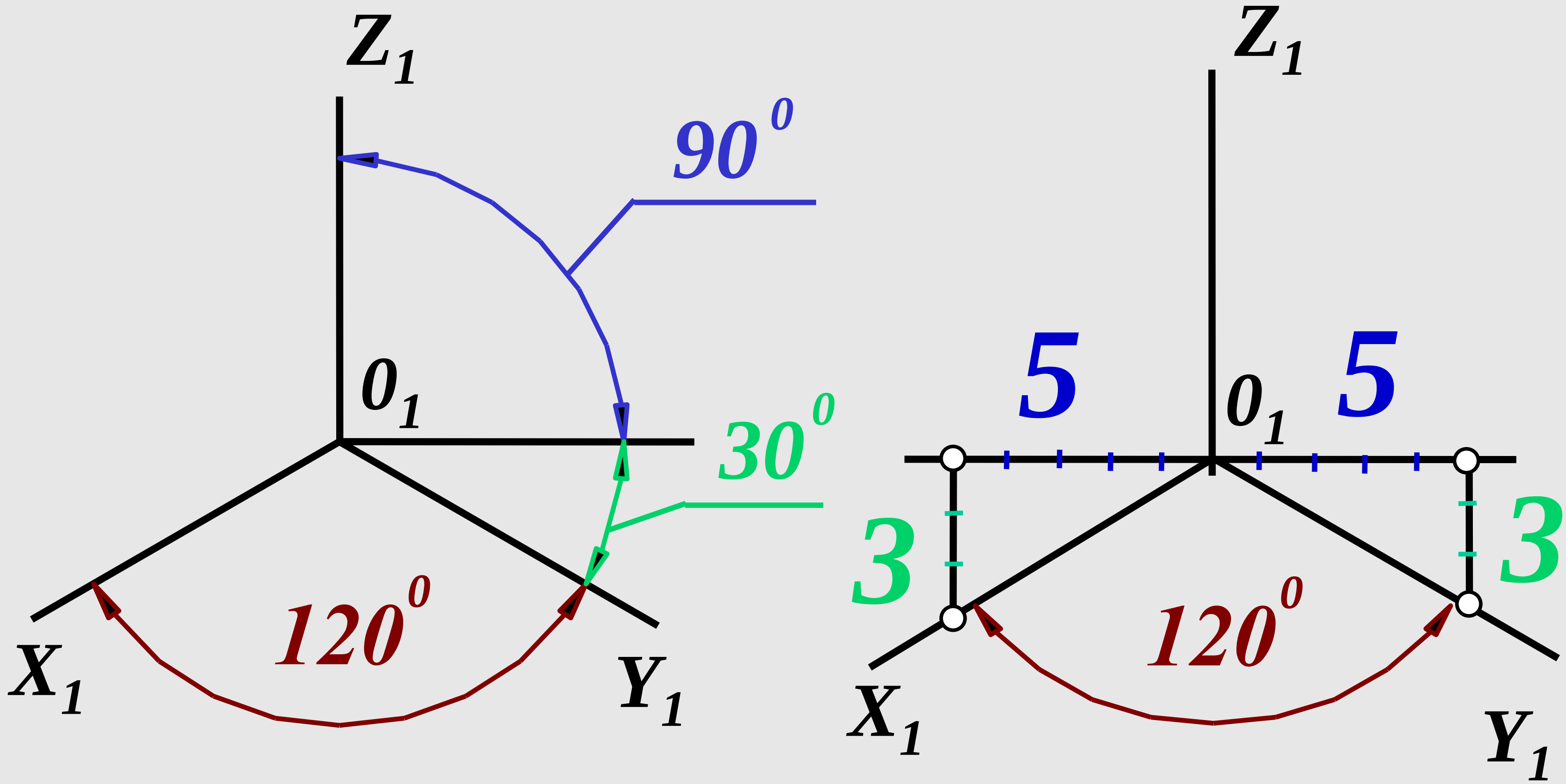
$$m = n = k = 0{,}82$$

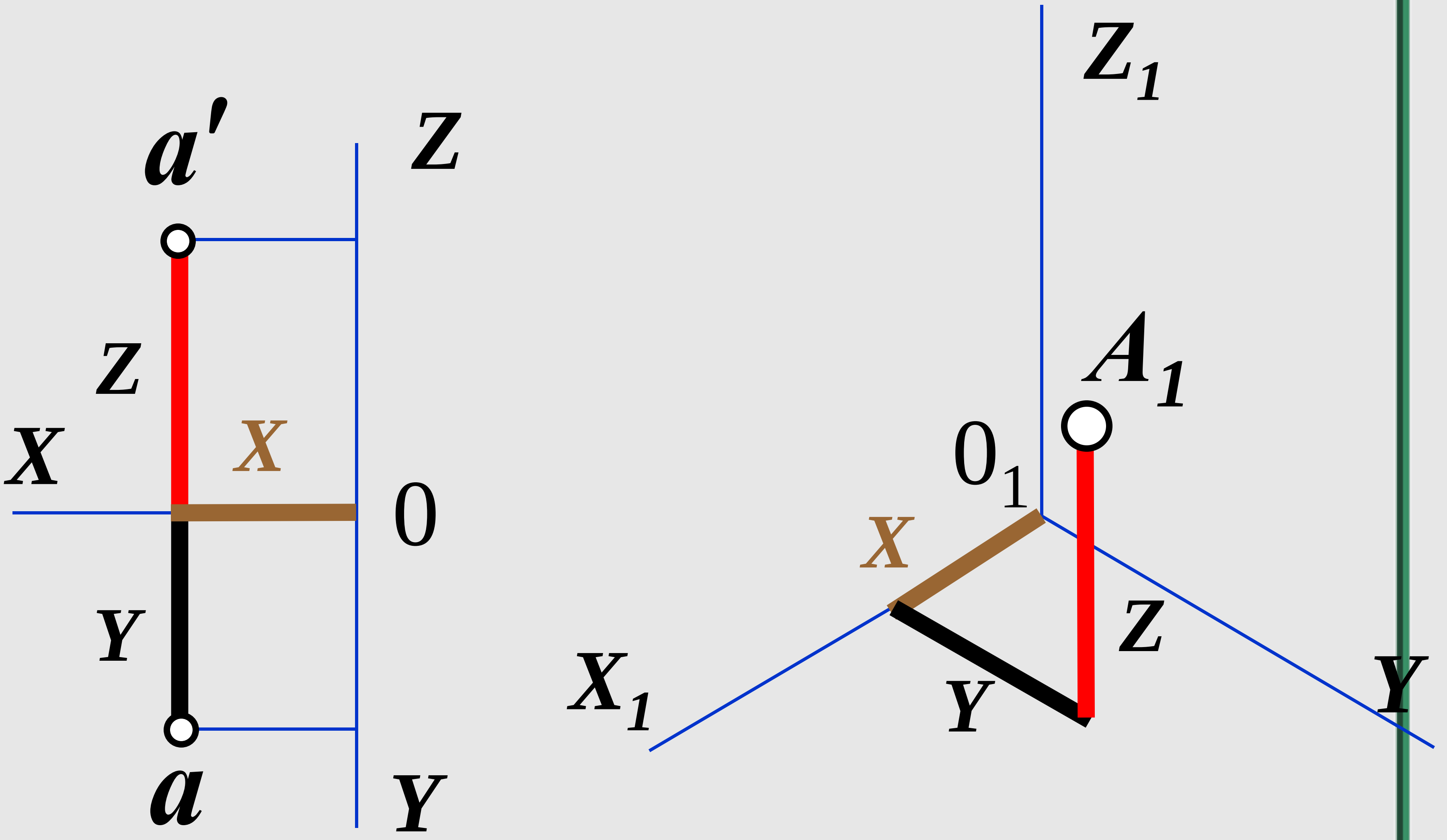
A reduction of the coefficients of distortion is usually applied

$$m = n = k = 1$$

In this case, the representation obtained is enlarged by 1.22.

Axes in a rectangular isometric





a'
Z
Z
X
X
0
Y
a
Y
Z_1
A_1
0_1
X
Z
X_1
Y
Y

Rectangular Parallel Dimetry

In the rectangular dimetry the **axis *z1* is vertical**, the **axis *x1* is at 7°10′ and the axis *y1* is at 41°25′** to the horizontal line.

The coefficients of distortion on the axes *x1* and *z1* are assumed to be equal (*m*=*k*), those on the axis *y1* - twice less (*n*=*1/2m*).

$$m = k = 0{,}94 \qquad n = (1/2)m = 0{,}47$$

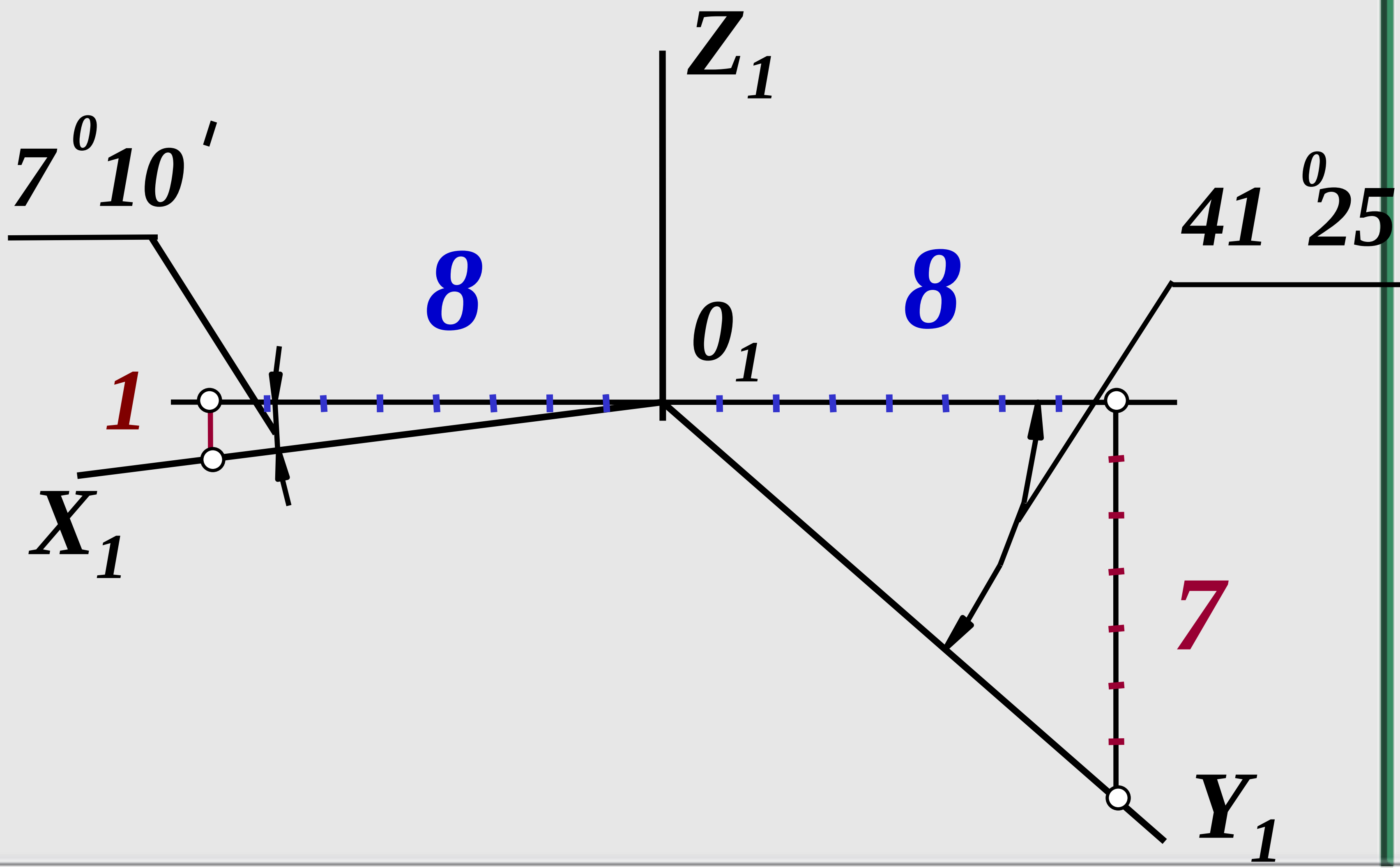
Rectangular Parallel Dimetry

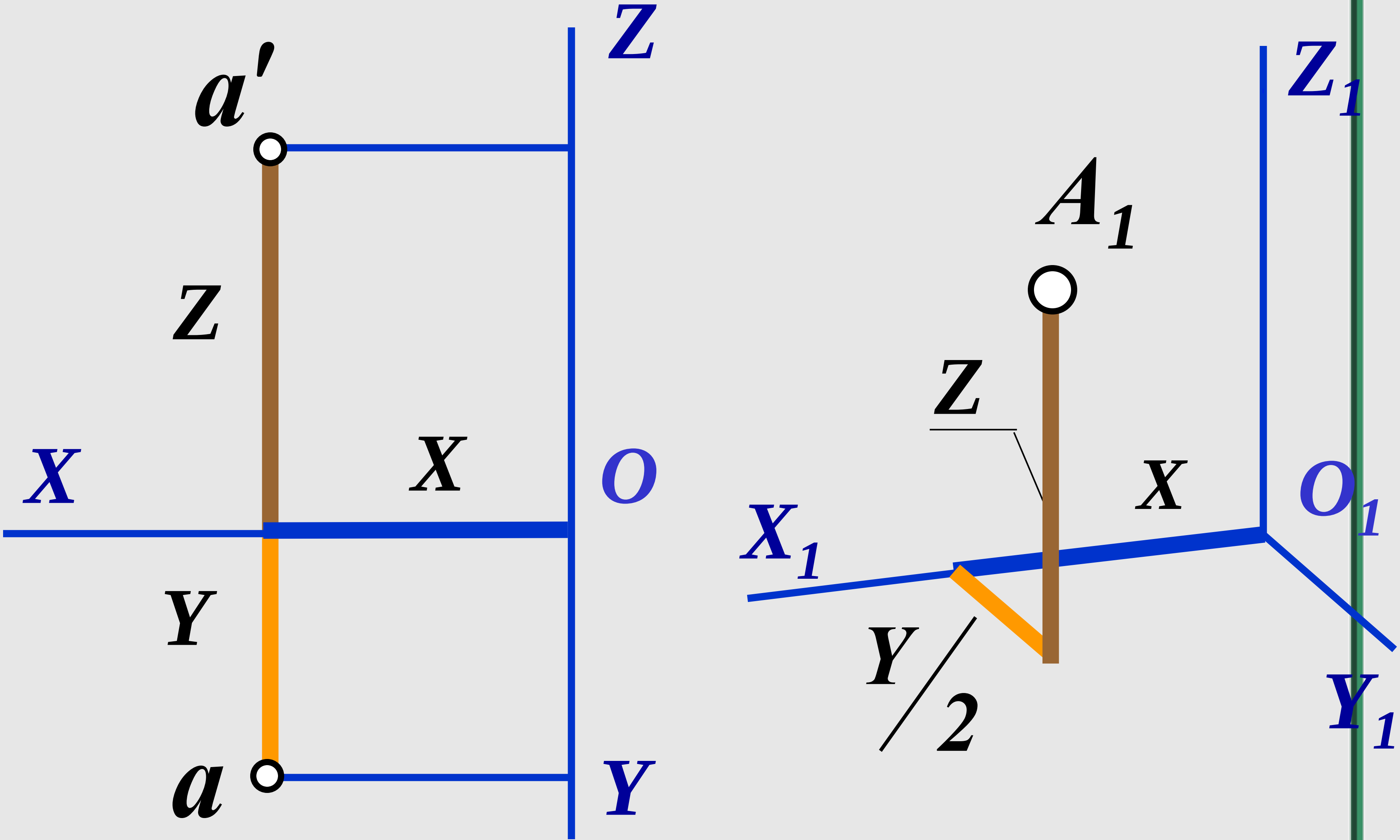
In practice the reduction of dimetry is usually used with the **coefficients of distortion**

$$m = k = 1 \qquad n = (1/2)m = 0{,}5$$

In this case the representation is **enlarged by *1.06*.**

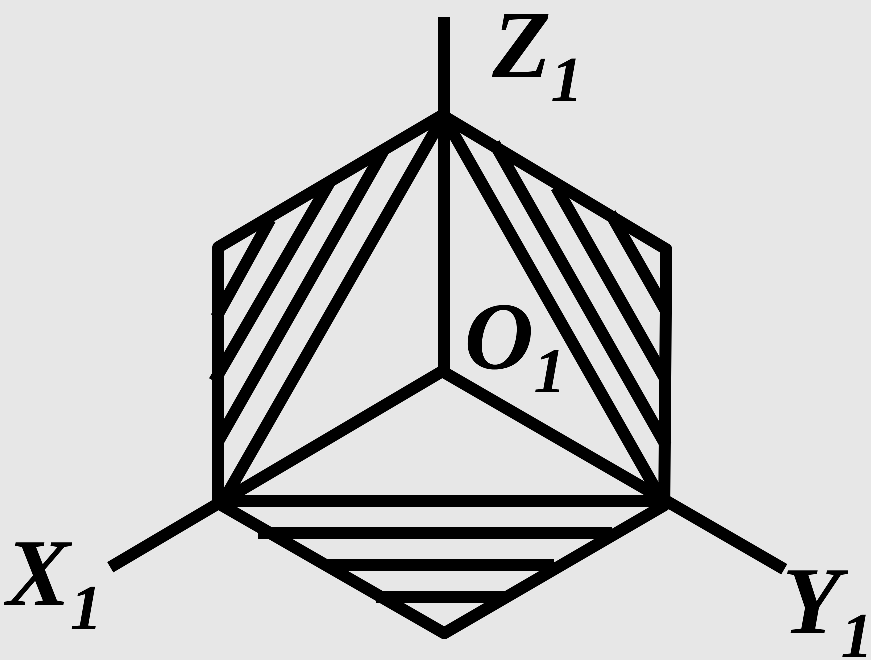
Axes in a rectangular Dimetry



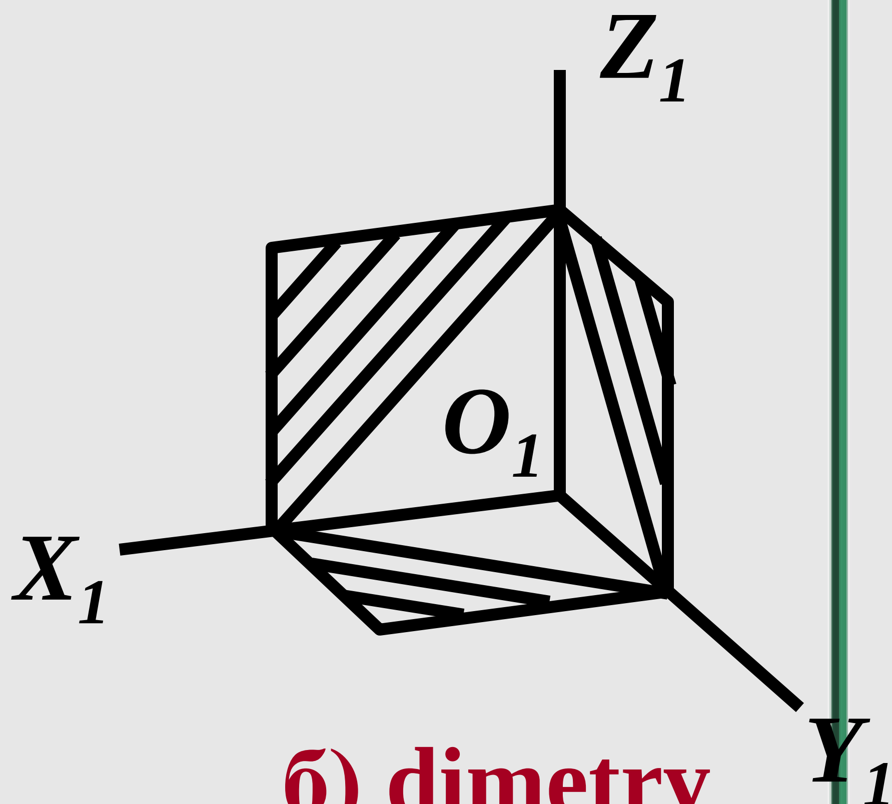


a′
Z
Z
X
X
O
Y
a
Y
Z1
A1
Z
X
O1
X1
Y/2
Y1

The cross-hatching lines in axonometric projections are **drawn parallel to one of the diagonals of the squares lying in the corresponding co-ordinate planes**, the sides of which are **parallel to the axonometric axes**



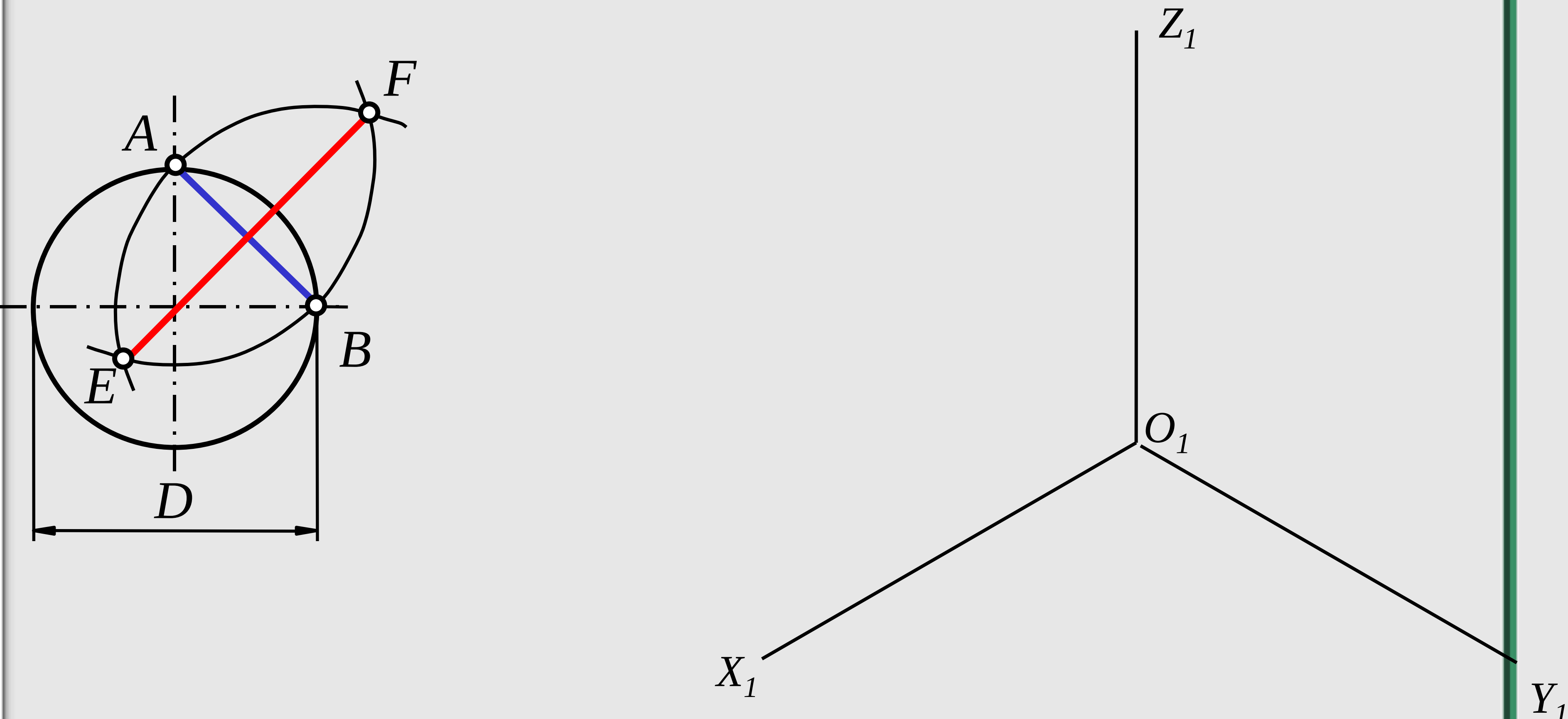
a) isometry



б) dimetry

Representation of a Circle and a Sphere in Isometry and Dimetry

Representation of a Circle in Isometry



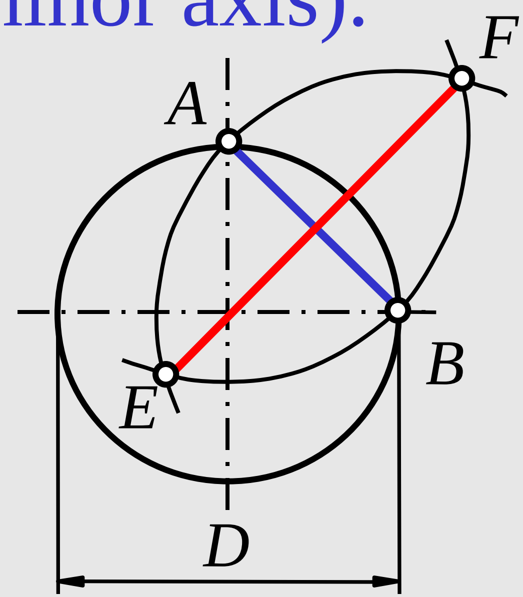
A circle in axonometry is generally projected in an ellipse. When constructing an ellipse, it is necessary to know the direction of its axes and their dimensions.

Note: the minor axis of an ellipse is always perpendicular to the major one.

When a circle projection is constructed (the circle lies in one of the co-ordinate planes), **the minor axis of the ellipse is directed parallel to the axonometric axis** which does not participate in the formation of the plane the drawing is in.

Representation of a Circle in Isometry

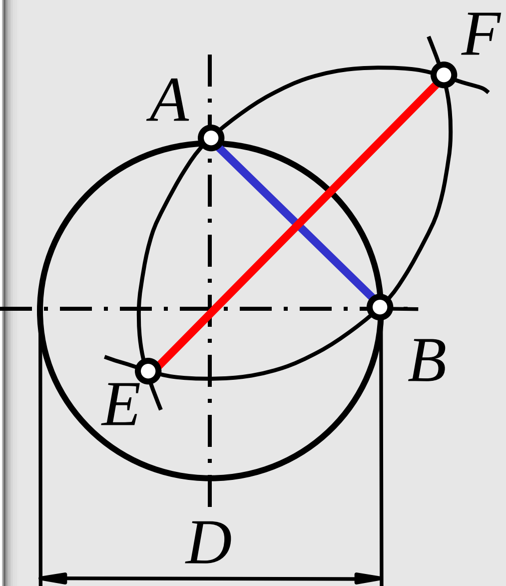
A graphical method of determination of the ellipse axes' dimensions. Draw a circle of the diameter *D*, the chord ***AB=0.71D*** (the length of the ellipse minor axis).

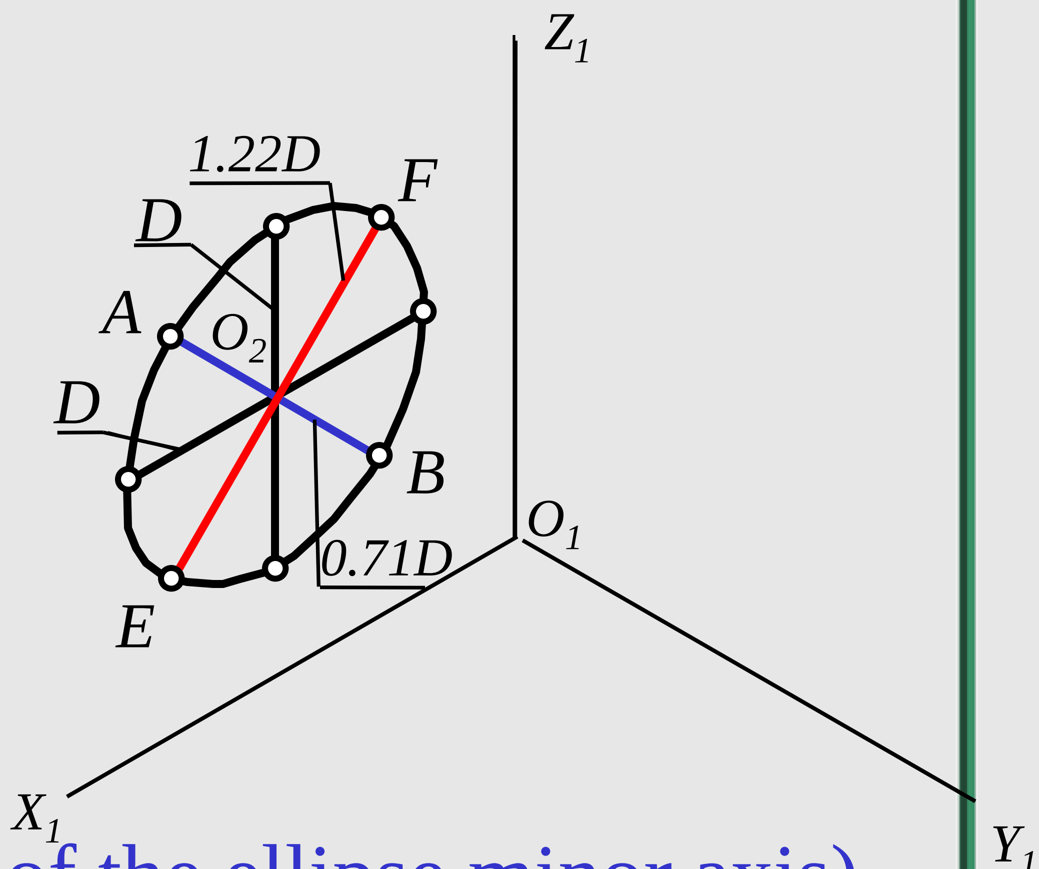


Assuming the points *A* and *B* as the centre, with the radius equal to *AB* draw the arcs to meet each other in *E* and *F*.

Join the obtained points with a straight line. ***EF=1.22D*** (the length of the ellipse major axis)

Representation of a Circle in Isometry





AB=0.71D (the length of the ellipse minor axis)

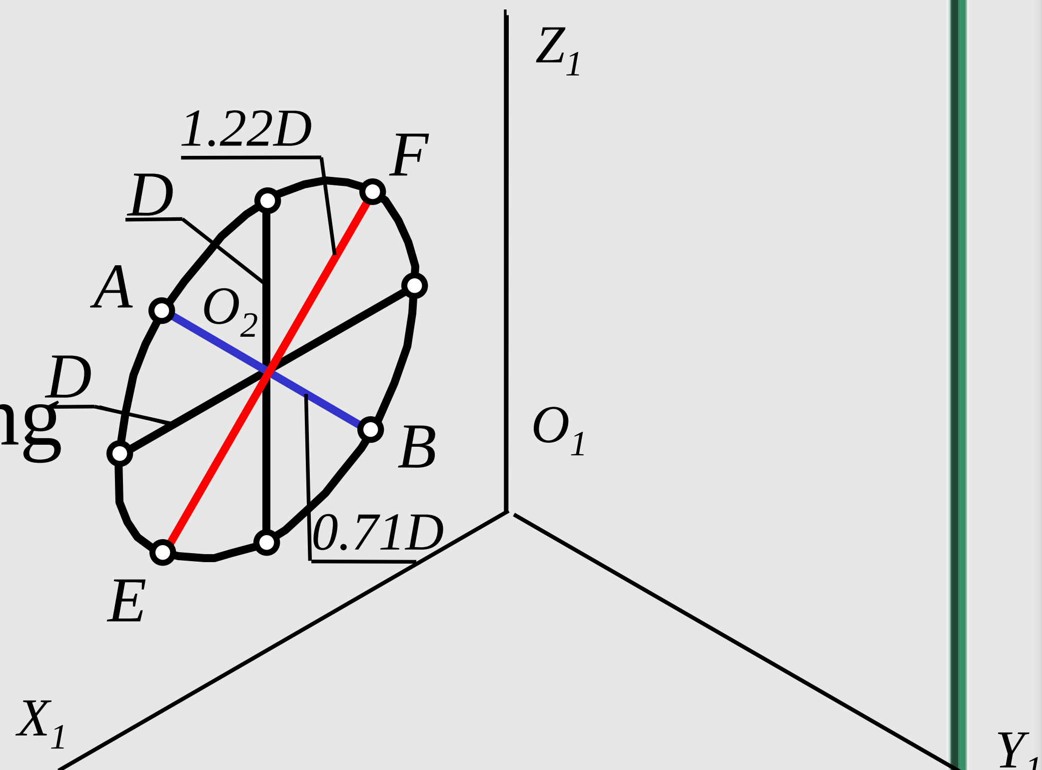
EF=1.22D (the length of the ellipse major axis)

Representation of a Circle in Isometry

Lay off the segments equal in length to **the major *EF* and the minor *AB* axes**, to meet in the centre of the ellipse - **the point *O2*.**

Through this point pass the lines **parallel to the axes *x1* and *z1*** generating the given plane.

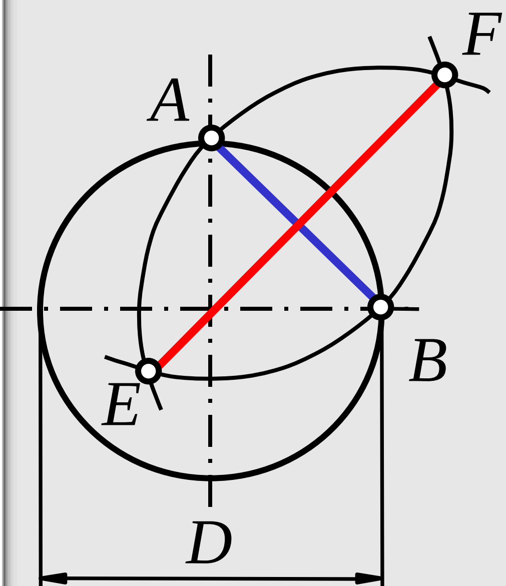
On the lines, lay off the values equal to the **diameter *D* of the circle**.

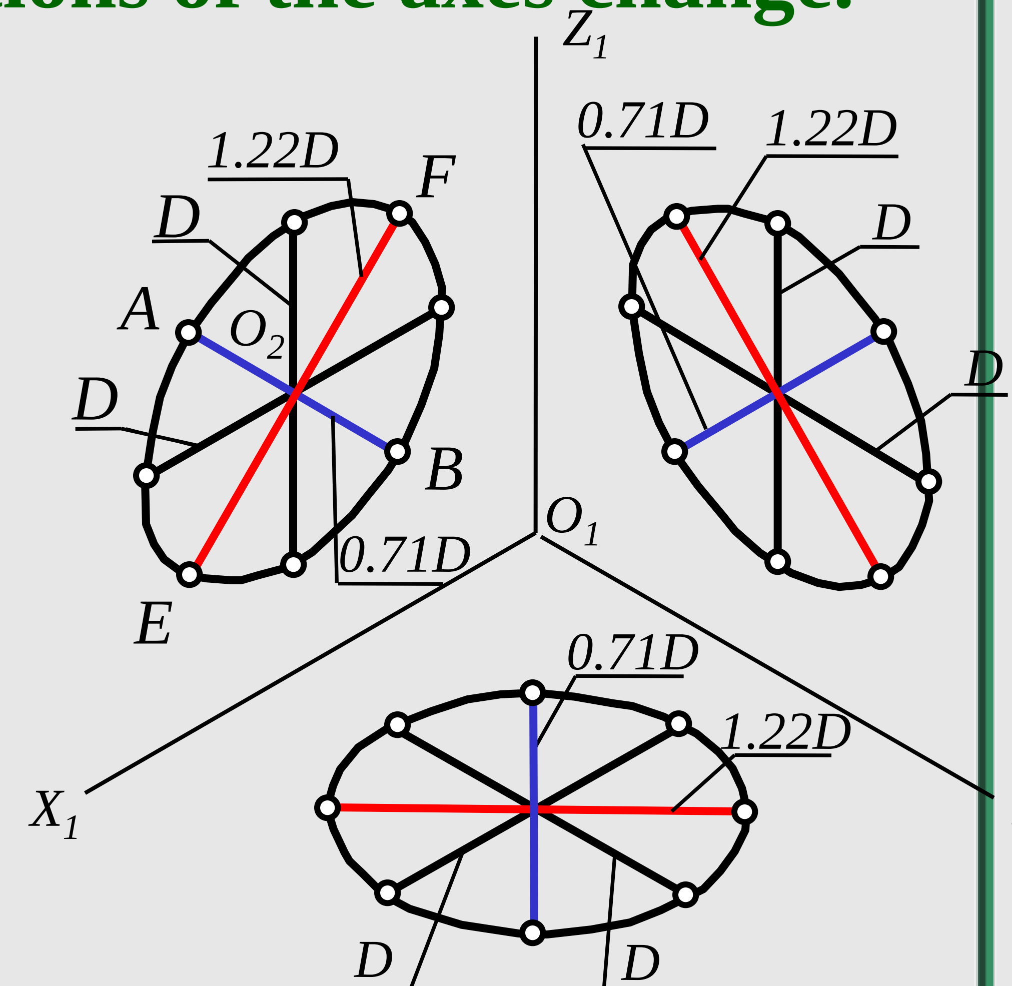


Join the obtained ***8* points to get an ellipse**.

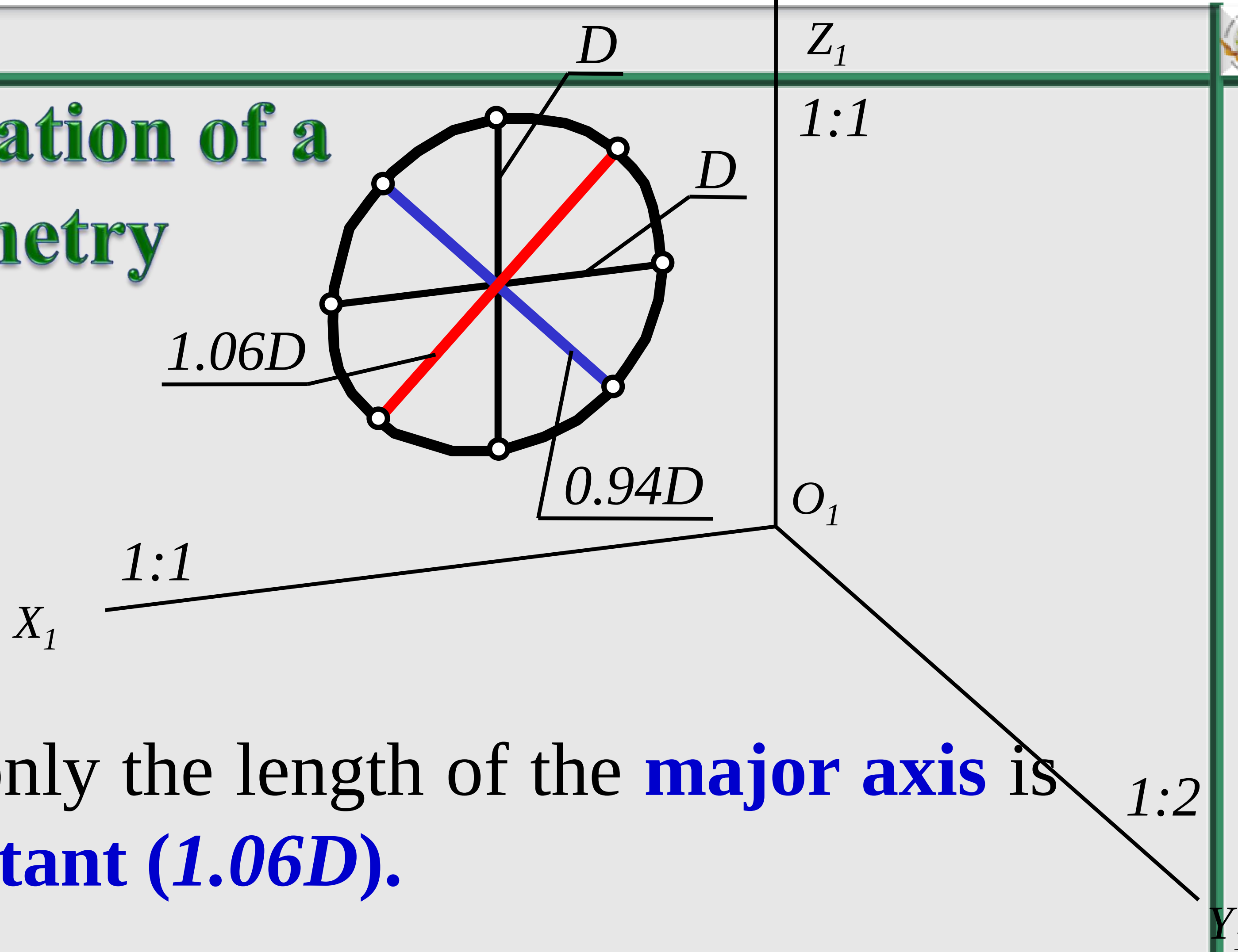
Representation of a Circle in Isometry

Construction of an ellipse in the other planes is similar, only the directions of the axes change.





Representation of a Circle Dimetry



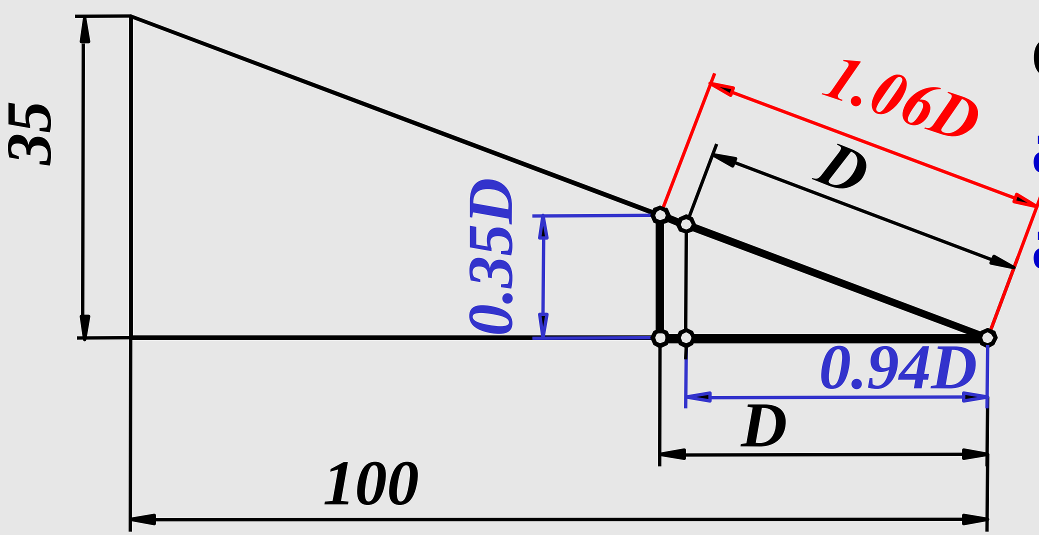
In dimetry only the length of the **major axis** is always **constant (*1.06D*).**

The size of the **minor axis** in the horizontal **(*H*) and profile (*W*) planes makes *0.35D***, in **the frontal (*V*) plane it makes *0.94D***.

Representation of a Circle Dimetry

To determine the size of an ellipse axes by means of the graphical method let us construct a right triangle given the legs **(100 mm and 35 mm)** and the hypotenuse **(106 mm).**

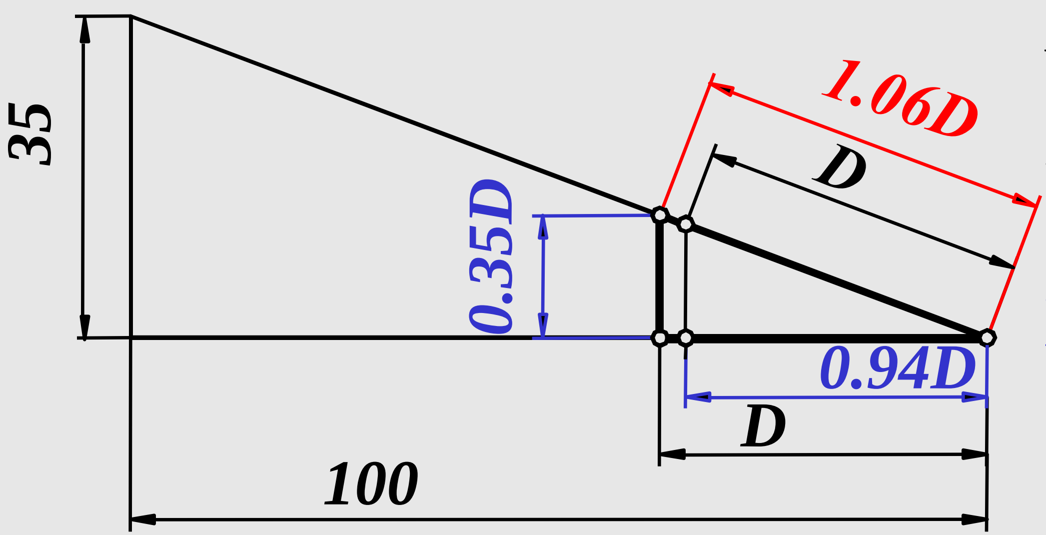
If we lay off the segment equal to the circle diameter *D* on the longer leg, the legs will make ***0.35D***, i.e. will be equal to the length of the minor ellipse axis on the planes *H* and *W*.



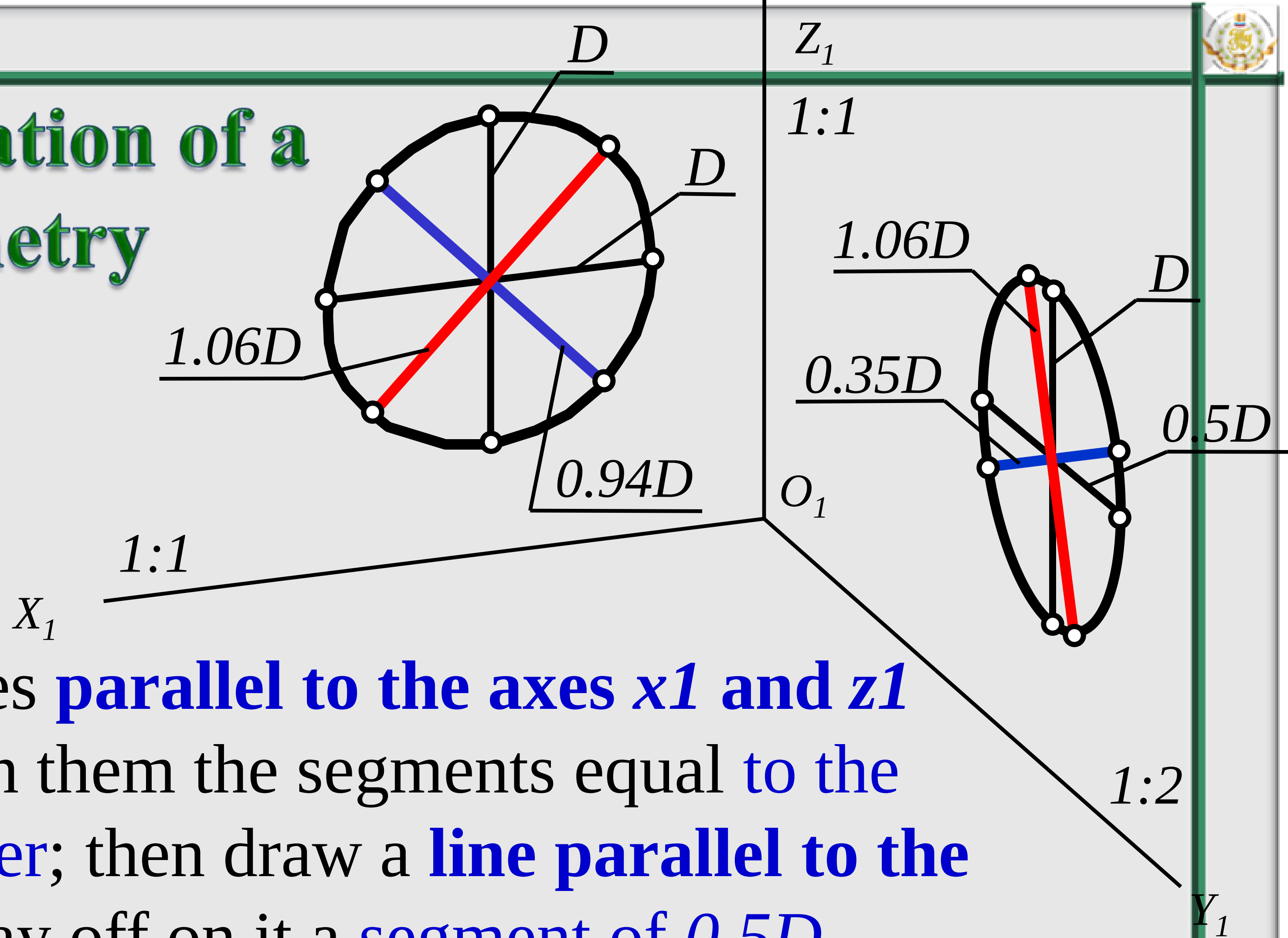
Representation of a Circle Dimetry

If we lay off the segment equal to the circle diameter *D* on the longer leg, the legs will make ***0.35D***, i.e. will be equal to the length of the minor ellipse axis on the planes *H* and *W*.

The hypotenuse is equal **to *1.06D***, that is to the length of the major ellipse axis. If we lay off the length of the diameter *D* on the hypotenuse and then drop a perpendicular to the longer leg of the triangle, the segment will be equal **to *0.94D***, i.e. to the length of the ellipse minor axis on the plane *V*.

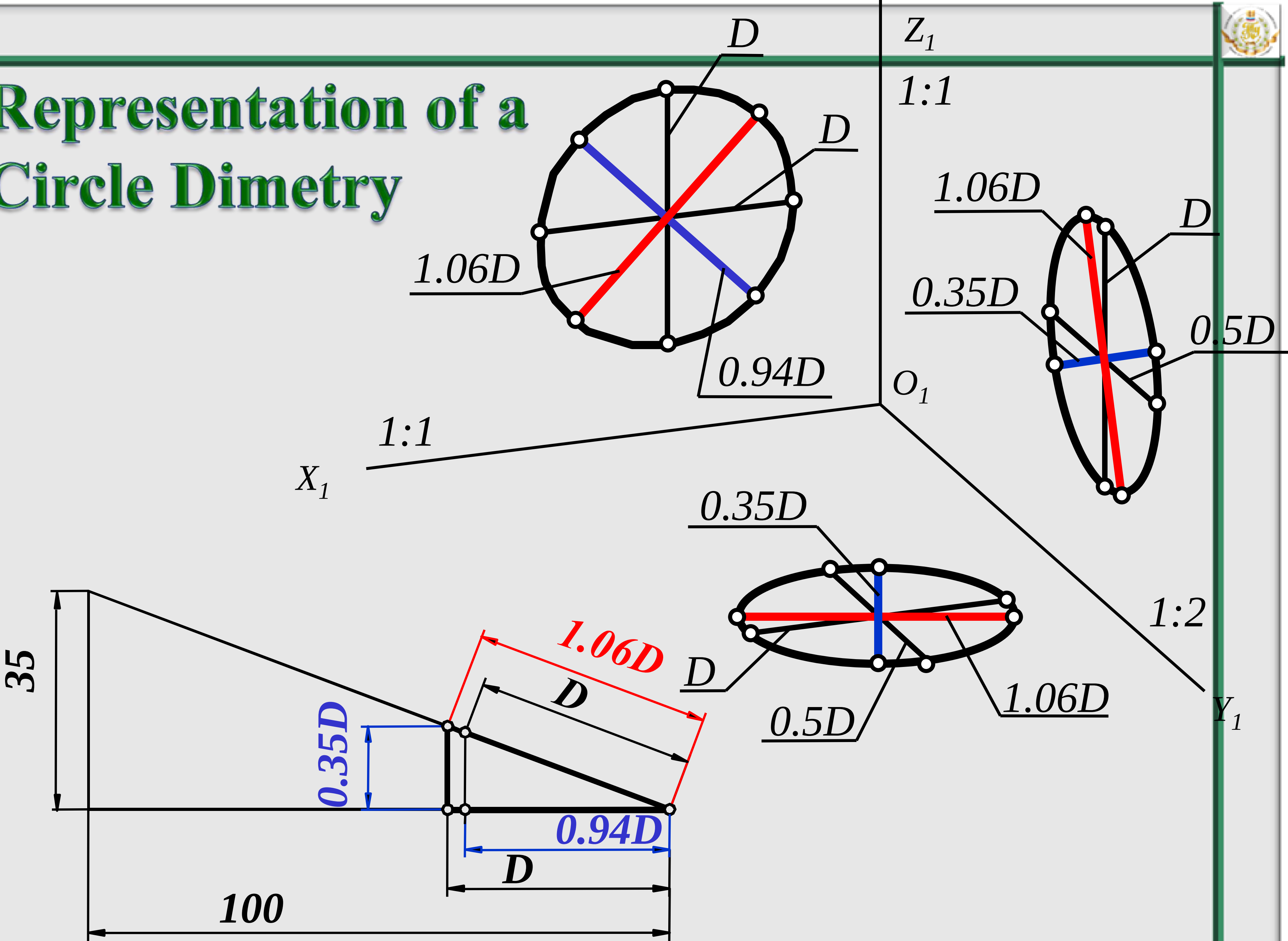


Representation of a Circle Dimetry



Draw the lines **parallel to the axes *x1* and *z1*** and lay off on them the segments equal to the circle diameter; then draw a **line parallel to the axis *y1*** and lay off on it a segment of *0.5D*. Construct the major and minor axes of the ellipse. Join the points thus obtained with a smooth line.

Representation of a Circle Dimetry



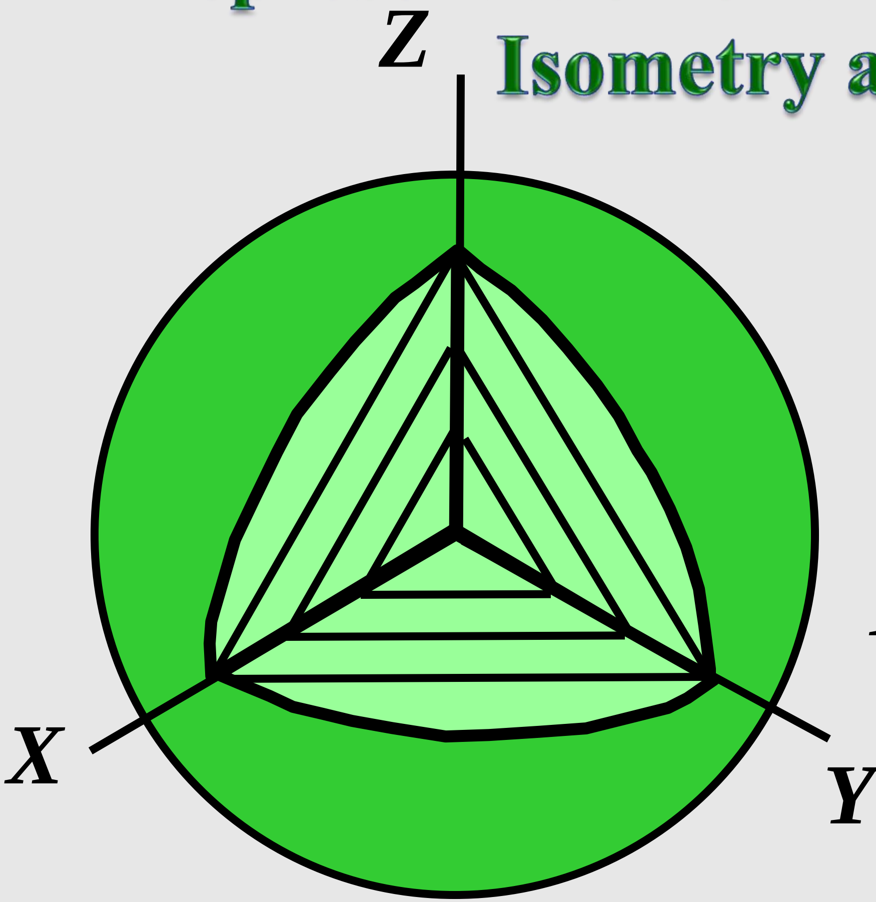
Representation of a Circle and a Sphere in Isometry and Dimetry

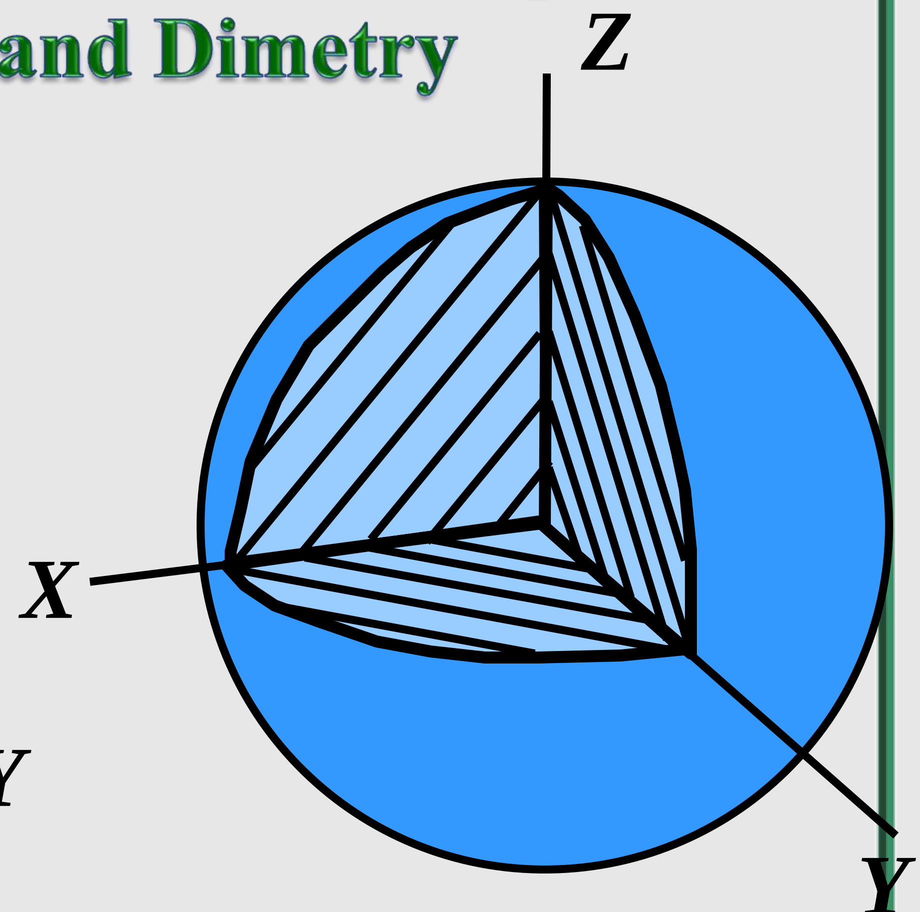
In rectangular parallel axonometry, **a sphere is represented as a circle.**

When a sphere is constructed by the true values of distortion, **its axonometric projection is a circle of the diameter equal to the diameter of the sphere.**

When a sphere is constructed by reduction, the diameter of the circle enlarges in conformity with the reduction coefficient: **in isometry it is 1.22; in dimetry - 1.06.**

Representation of a Circle and a Sphere in Isometry and Dimetry





Isometry

Dimetry