Tomsk Polytechnic University

DESCRIPTIVE GEOMETRY
ENGINEERING GRAPHICS

G.F.Vinokurova, R.G. Dolotova, S.P. Burkova

Lecture 6

Mutual Intersection of Surfaces

Plan

1. Intersection of Surfaces
2. Method of Auxiliary Cutting Planes
3. Intersection of coaxial surfaces
4. Method of Auxiliary Spheres
5. Monge theorem
6. Theorem of a double contact

Intersection of Surfaces

The line of intersection of two surfaces is the locus of the points belonging to both surfaces.

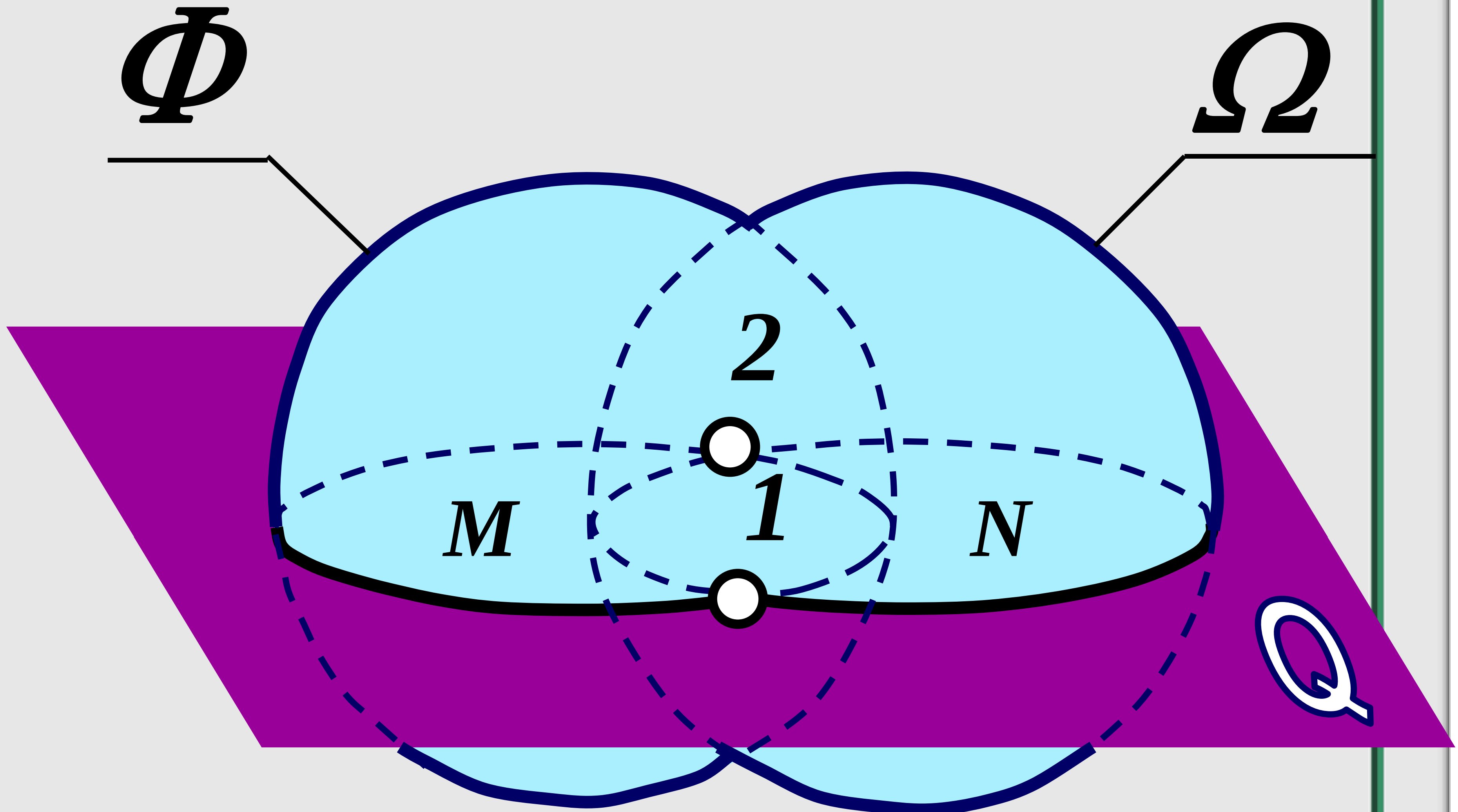
To construct a line of crossing of two surfaces it is necessary to find the common points.

A general method of drawing the points belonging to a curve of mutual intersection of surfaces is the method of auxiliary surfaces-mediators.

This method is similar to the method of construction intersection lines of surfaces cut by planes and consists **in the following:**

Take some intersecting **surfaces** $\boldsymbol{\Phi}$ **and** $\boldsymbol{\Omega}$.

Introduce an auxiliary plane *Q* intersecting the surfaces along the lines *m* and *n* which yields the points *1* and *2* belonging to the intersection curve.



Φ
Ω
2
1
M
N
Q

As the surfaces-mediators very often planes or ball surfaces (spheres) are used.

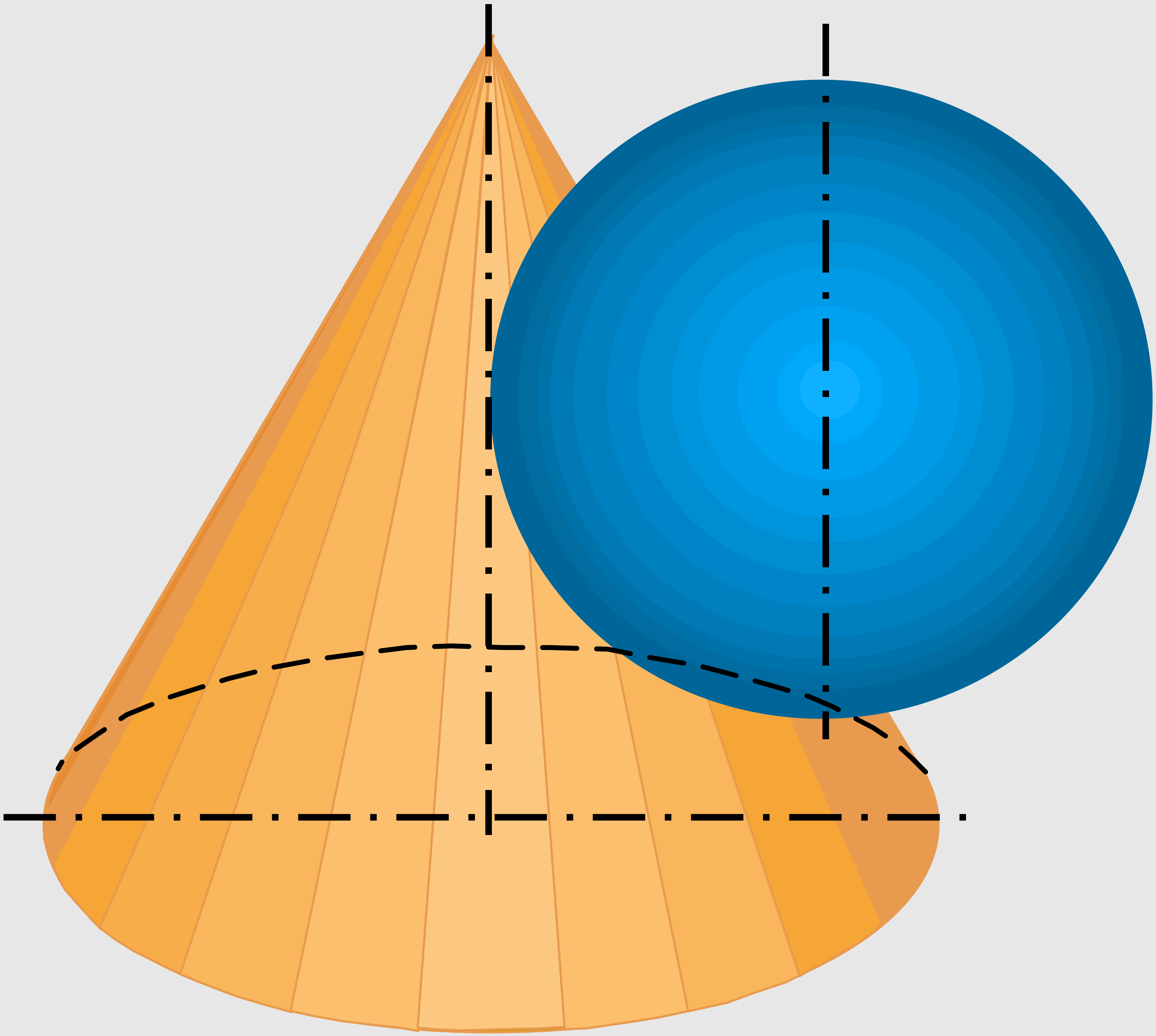
Depending on mediators the following ain methods of construction an intersection line of two surfaces are distinguished:

a) method of auxiliary cutting planes;

b) method of auxiliary spheres.

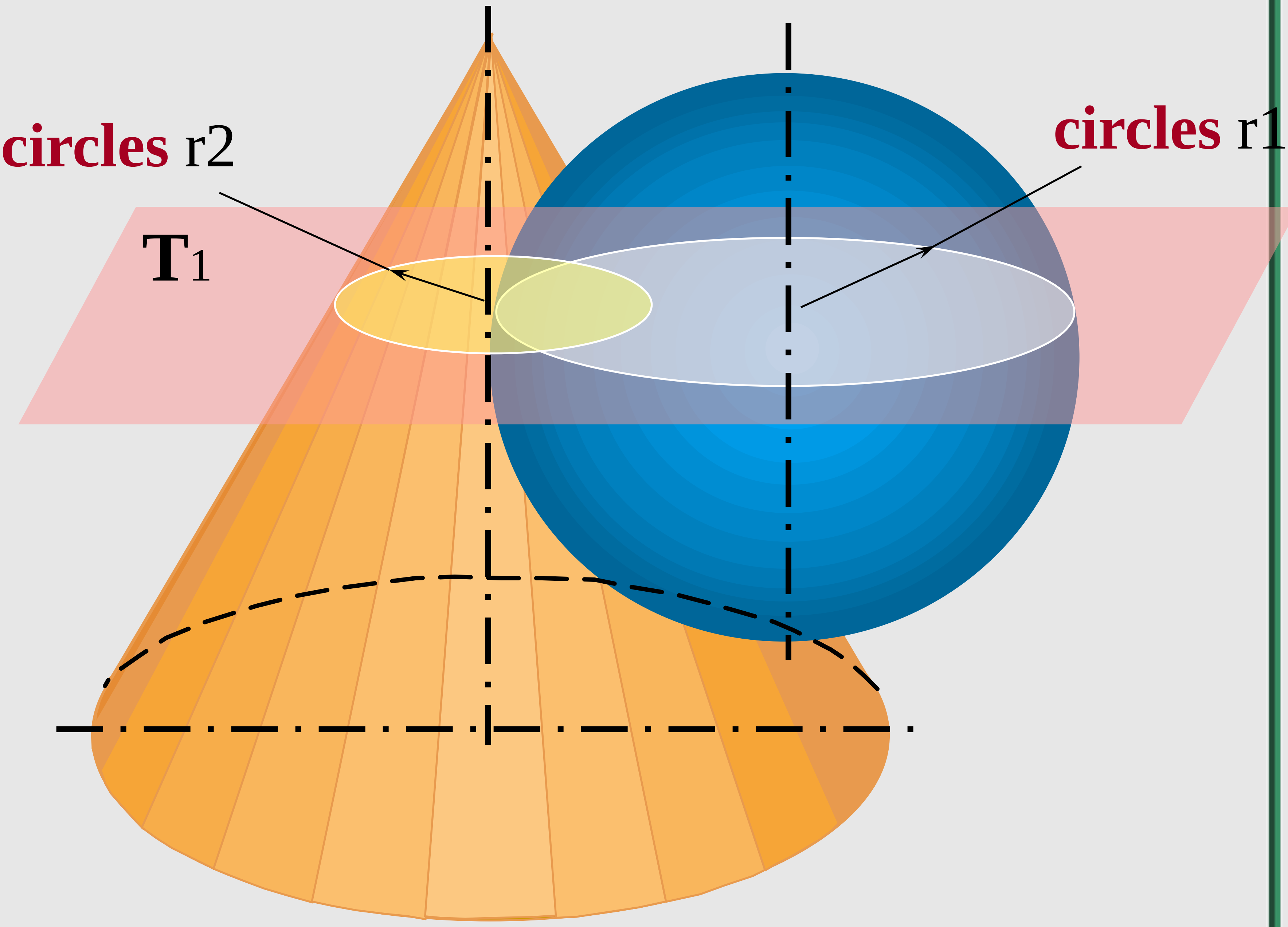
Method of Auxiliary Cutting Planes

Construction of an intersection line of a sphere with a cone of rotation

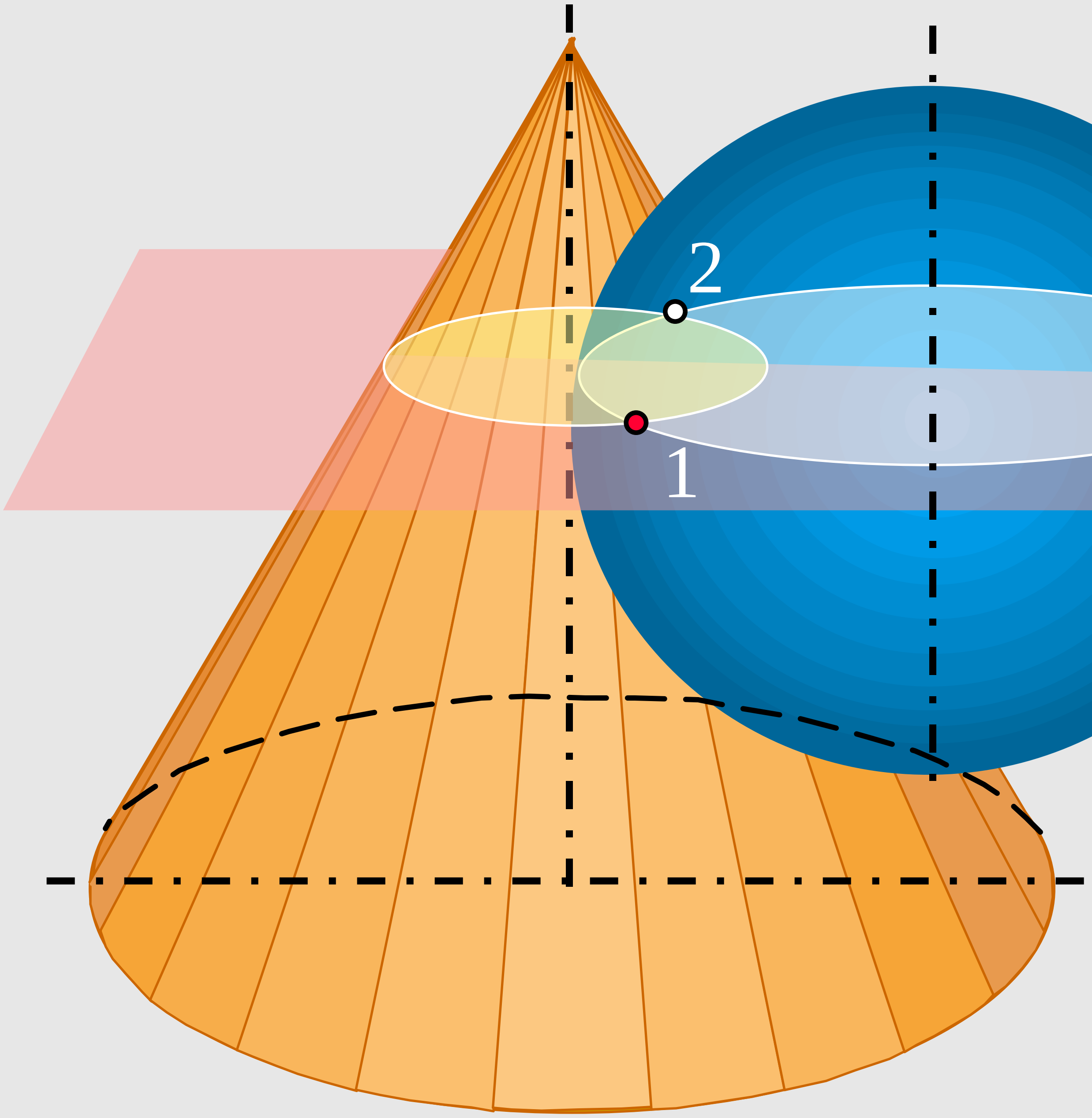


To construct an intersection line of the given surfaces it is advisable to introduce the frontal plane *P* and a number of **horizontal planes *T***, as the auxiliary surfaces.

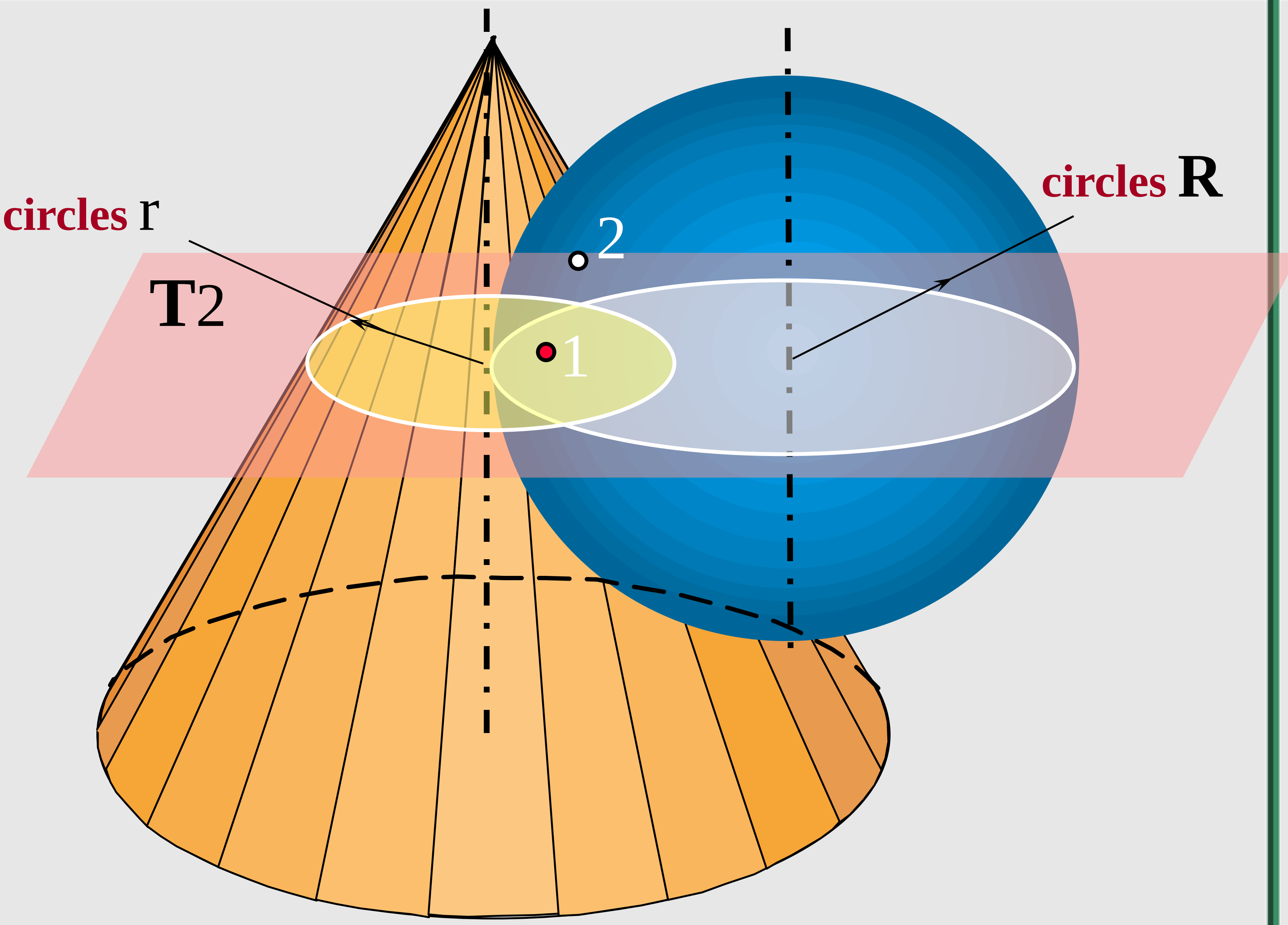
The auxiliary horizontal plane T1 cut the sphere and the cone in circles



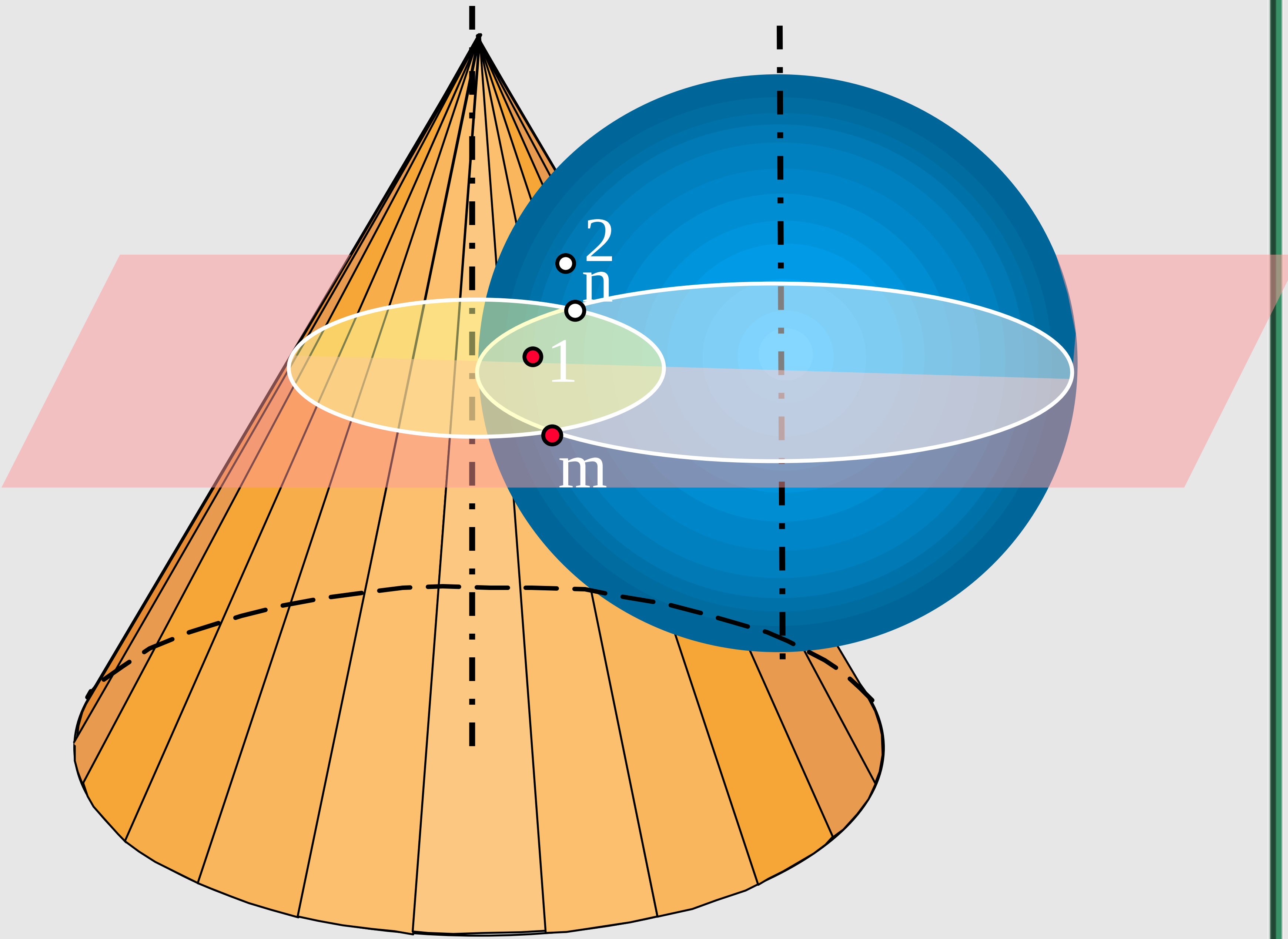
which yields the points *1* and *2* belonging to the intersection curve.



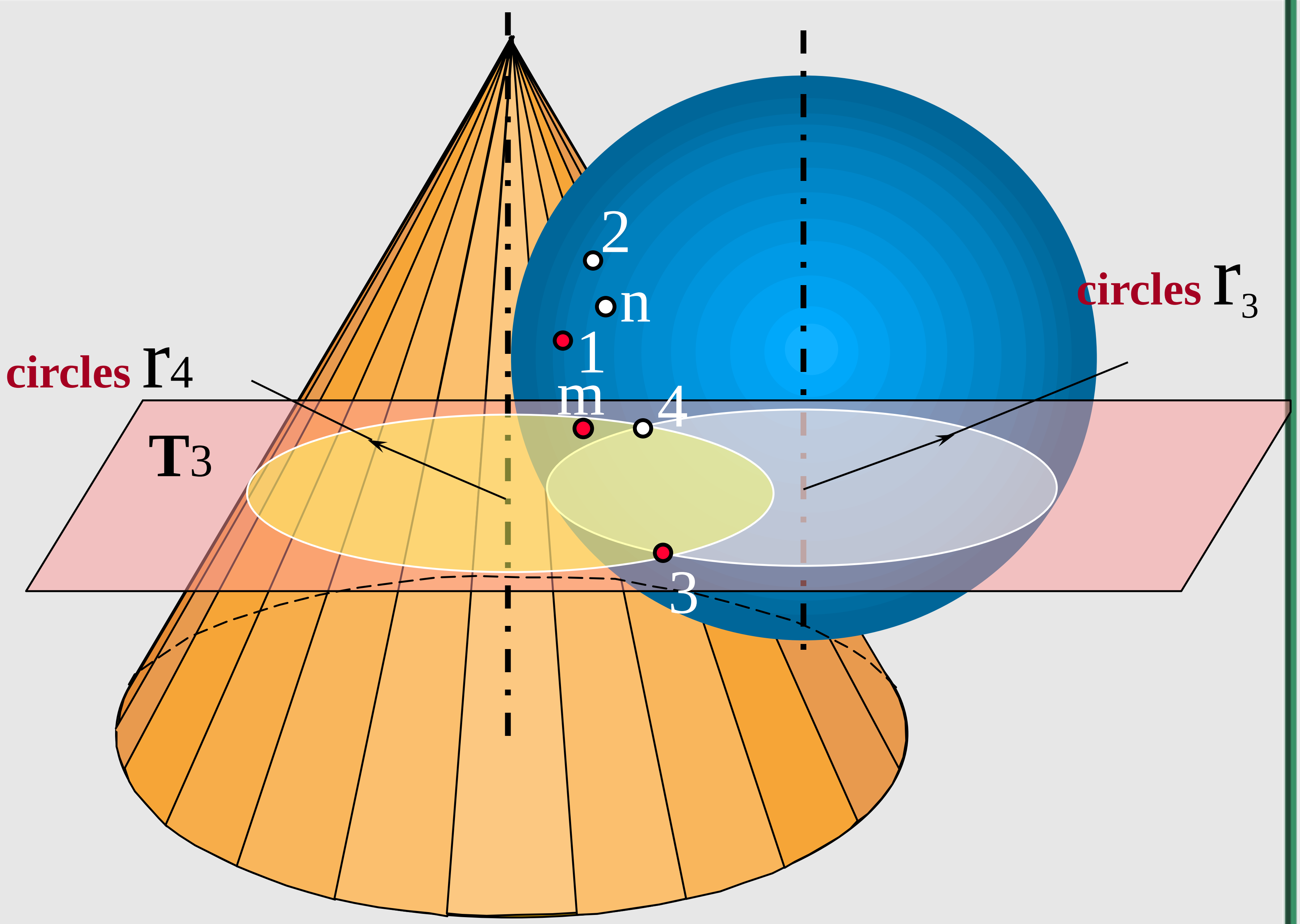
The auxiliary horizontal plane T_2 cut the sphere and the cone in circles



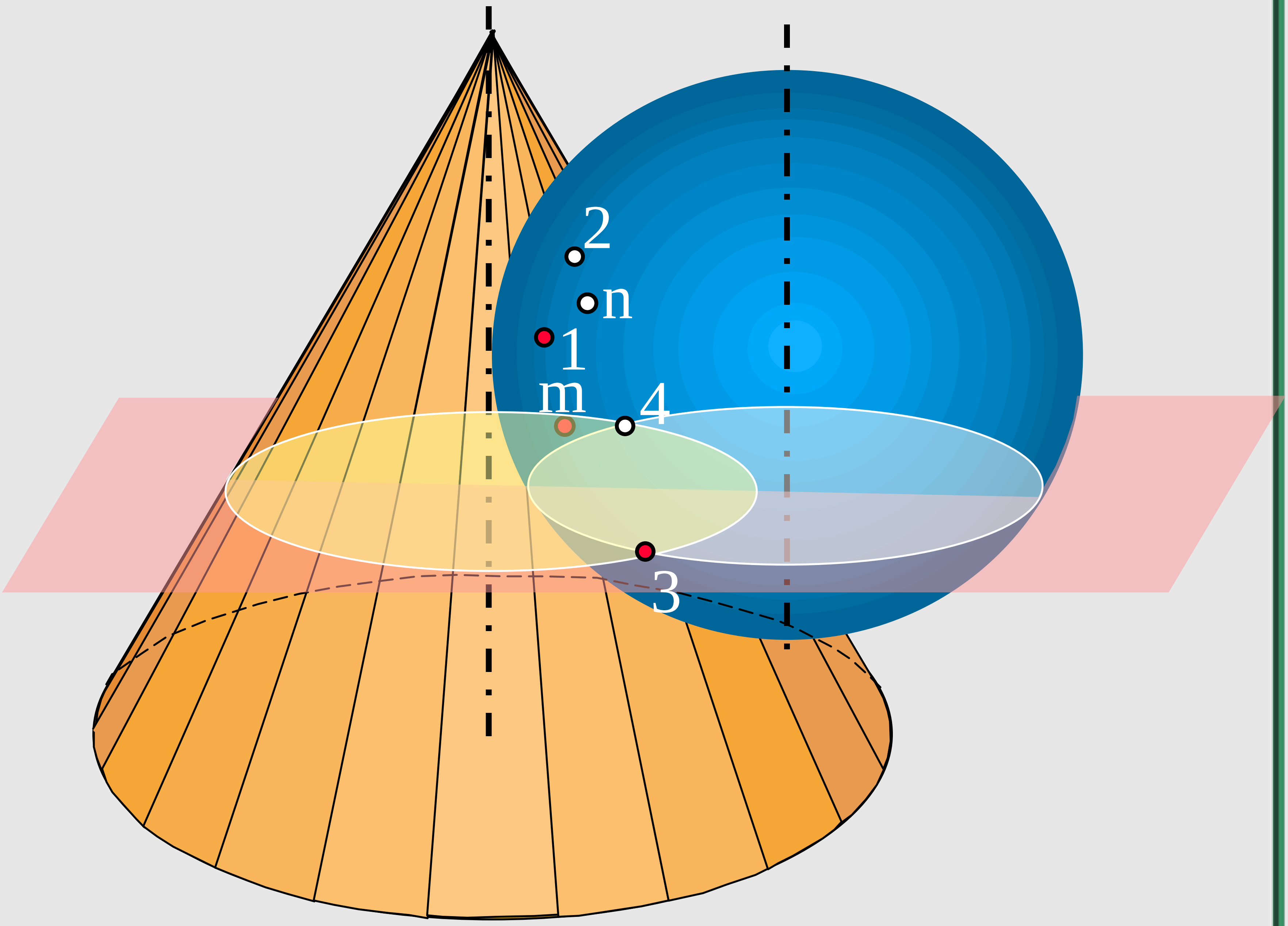
which yields the points *n* and *m* belonging to the intersection curve.



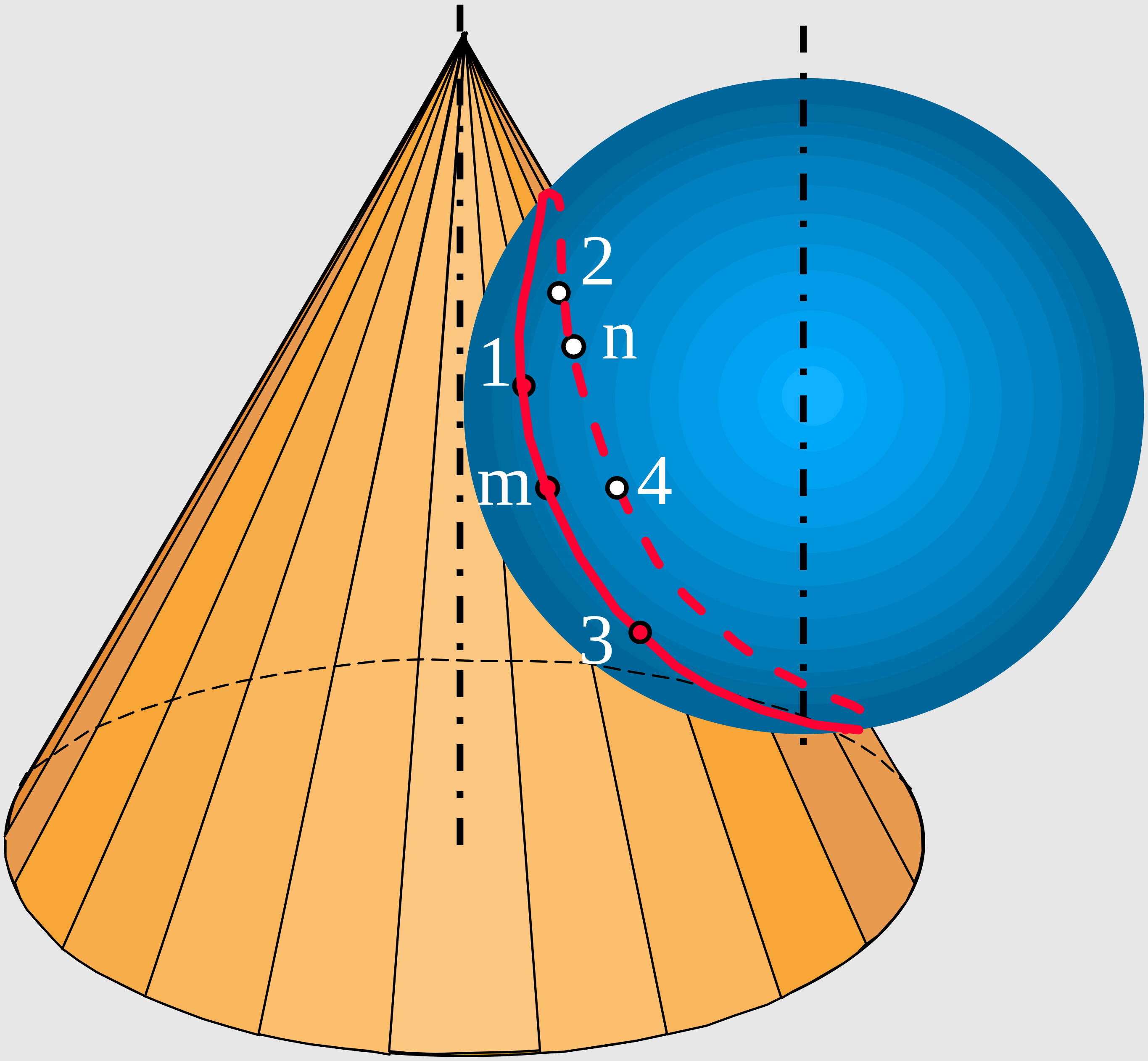
The auxiliary horizontal plane T_3 cut the sphere and the cone in circles



which yields the points *3* and *4* belonging to the intersection curve.

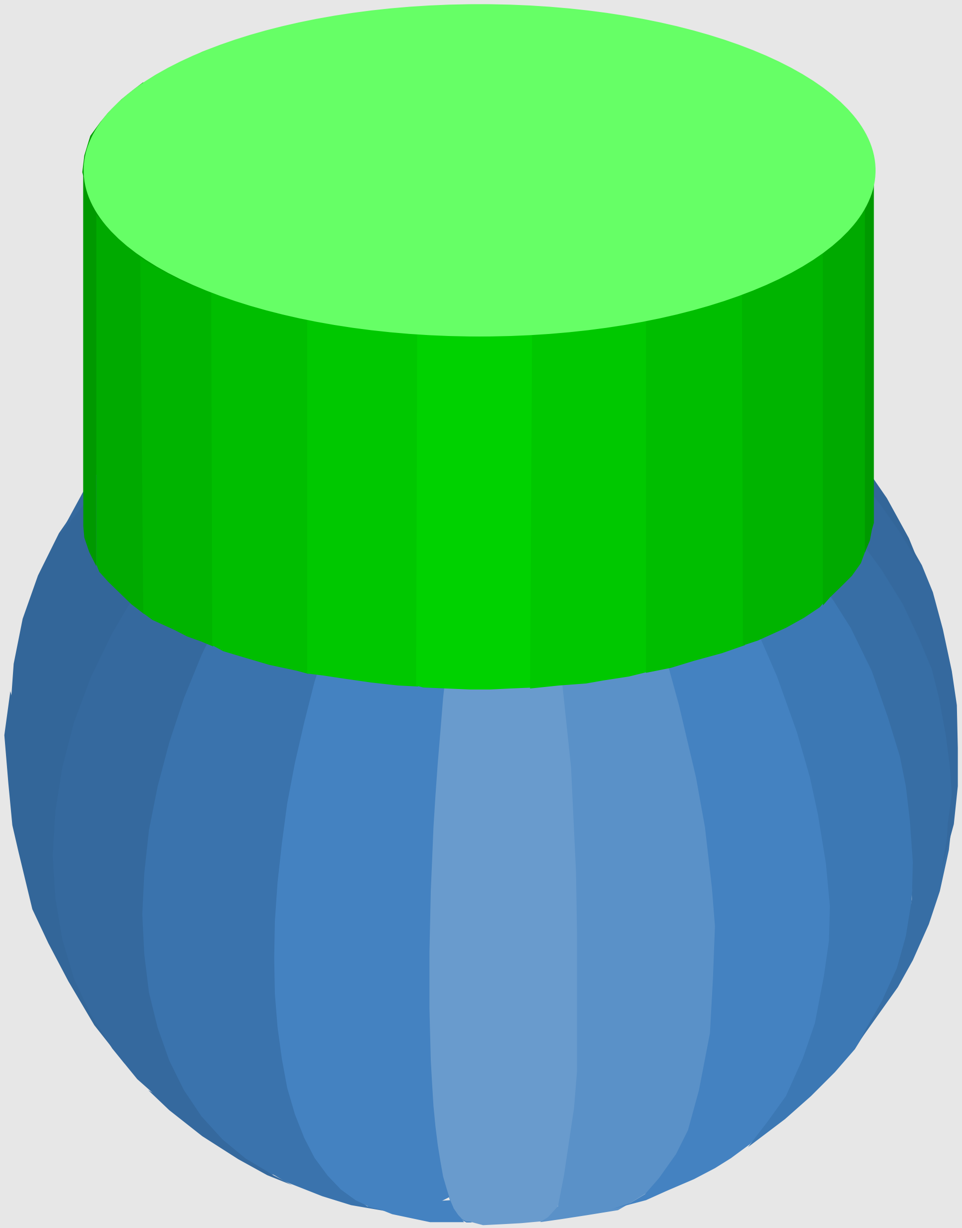


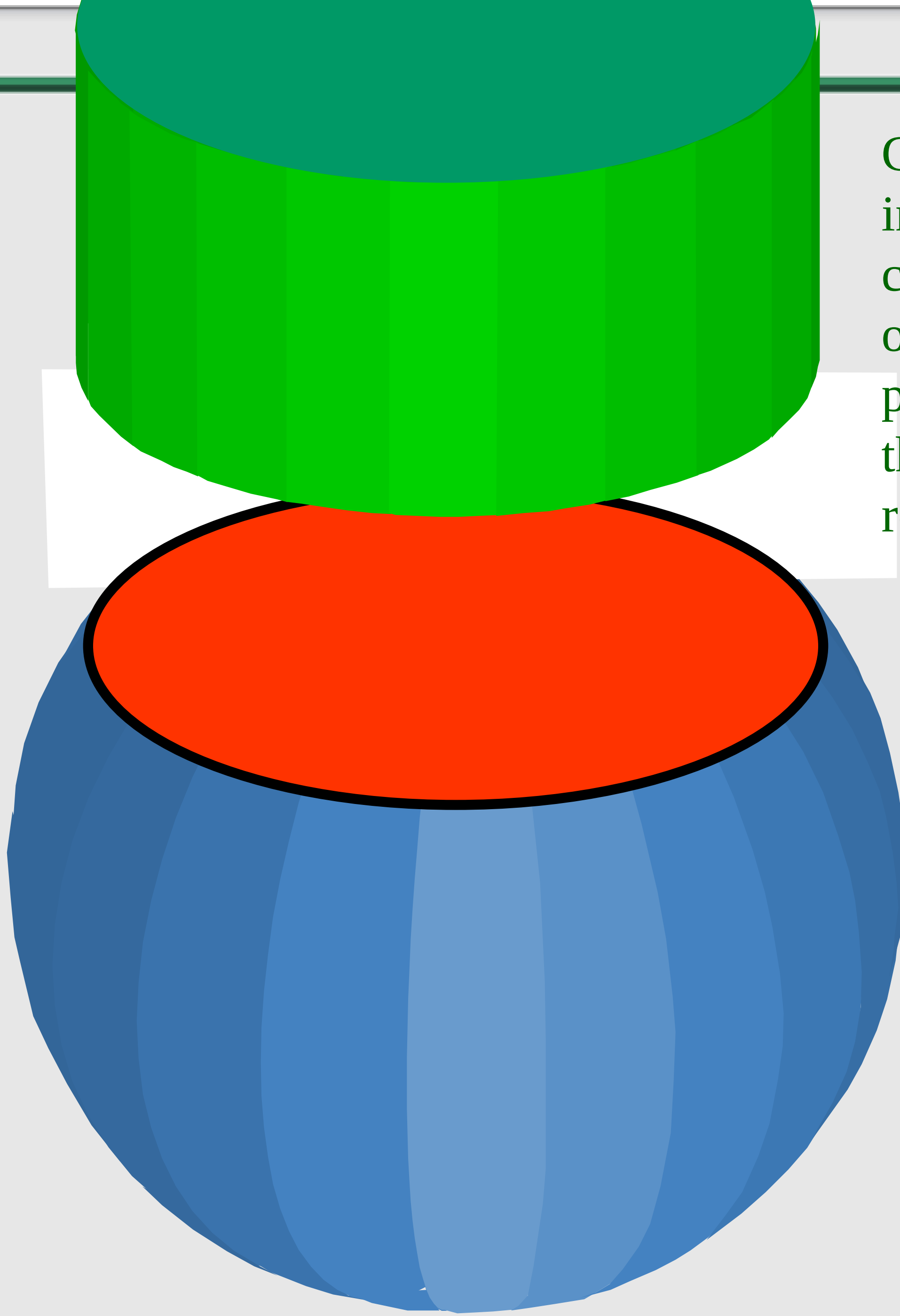
Join the points thus obtained in a smooth curve subject to visibility



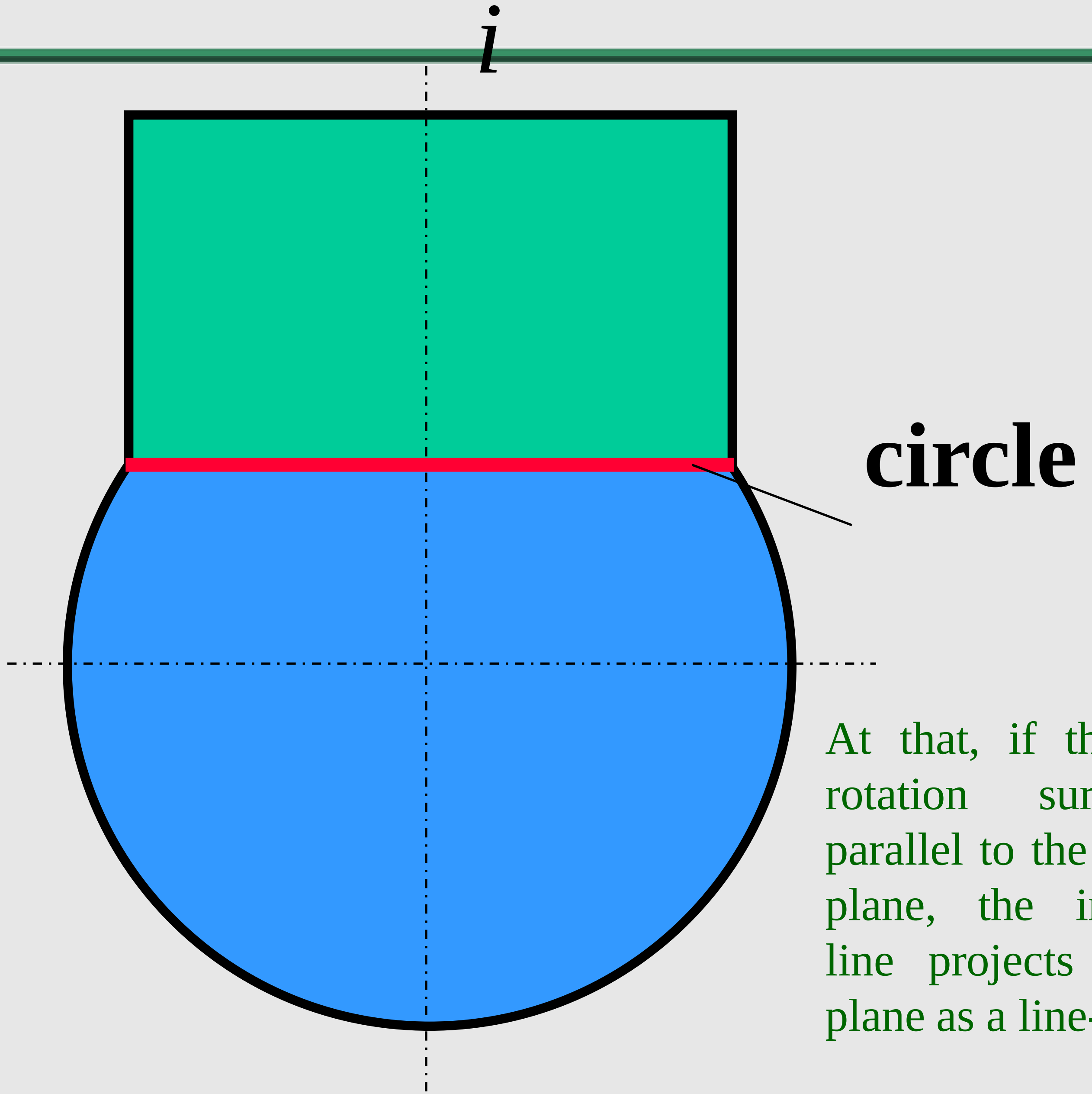
Intersection of coaxial surfaces

Particular case of intersection of rotation surfaces, **the axes of which coincide**, i.e. a case of intersection of coaxial surfaces of rotation.

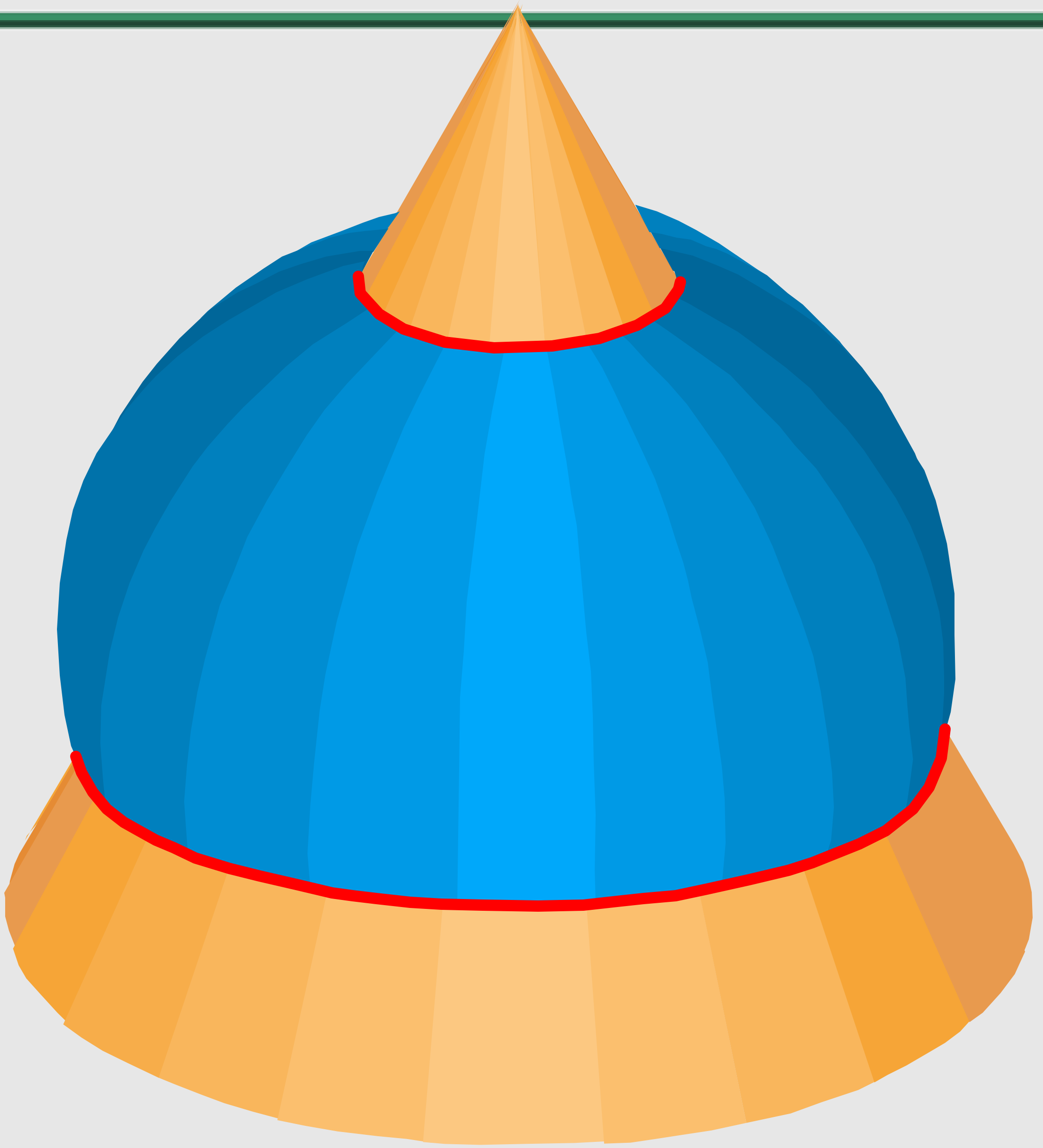


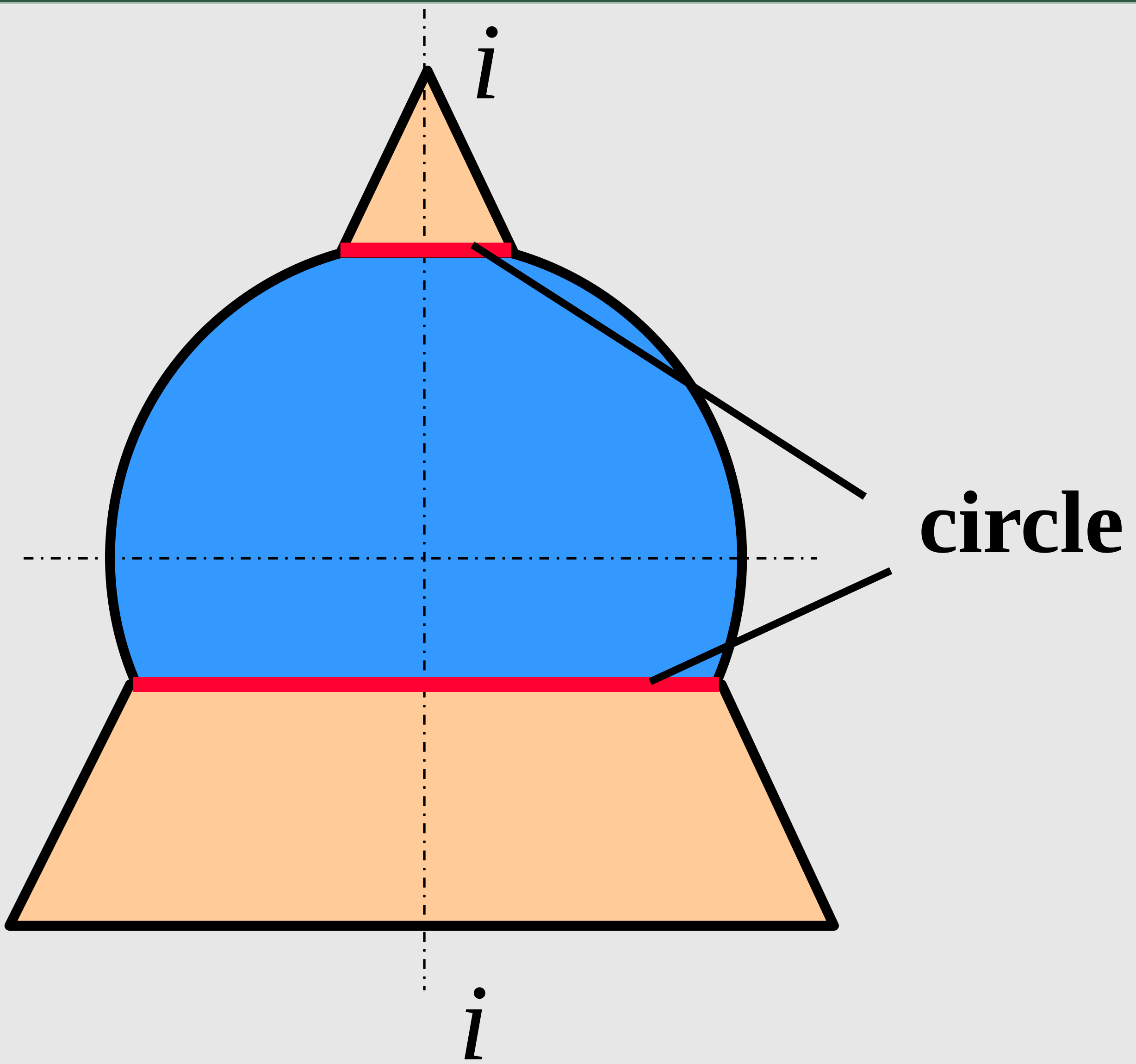


Coaxial surfaces intersect in a circle, the plane of which is perpendicular to the axis of rotation surfaces.



At that, if the axis of rotation surfaces is parallel to the projection plane, the intersection line projects onto this plane as a line-segment

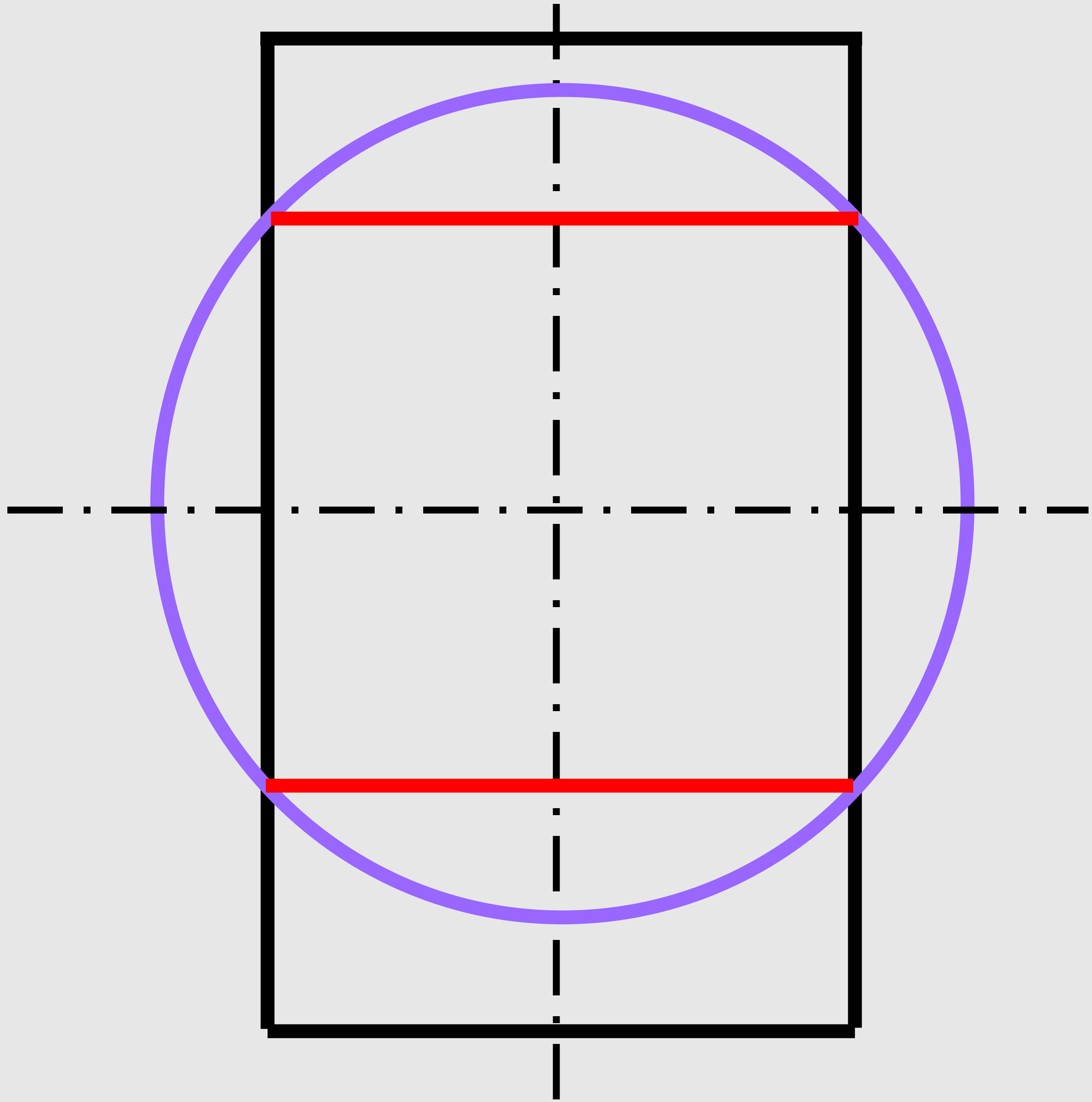



i
circle
i

Coaxial to surfaces **refer to**
The surfaces having
The common axis of rotation

Two coaxial surfaces Are crossed on circles, Laying in planes, Perpendicular axes Rotations of surfaces

Number of circles equally to number of crossings the main meridians



Method of Auxiliary Spheres

In a method of spheres in quality
The surface - intermediary
The sphere gets out.
Thus two variants are possible:

1. **Spheres are carried out**
From one center
(a method of concentric spheres)

2. Spheres are carried out
From the different centers
(a method eccentric spheres)

Method of concentric spheres

Note: if a plane of rotation surface axes is not parallel to the projection plane, the circles in which the surfaces intersect, are projected as ellipses and this make the problem solution more complicated.

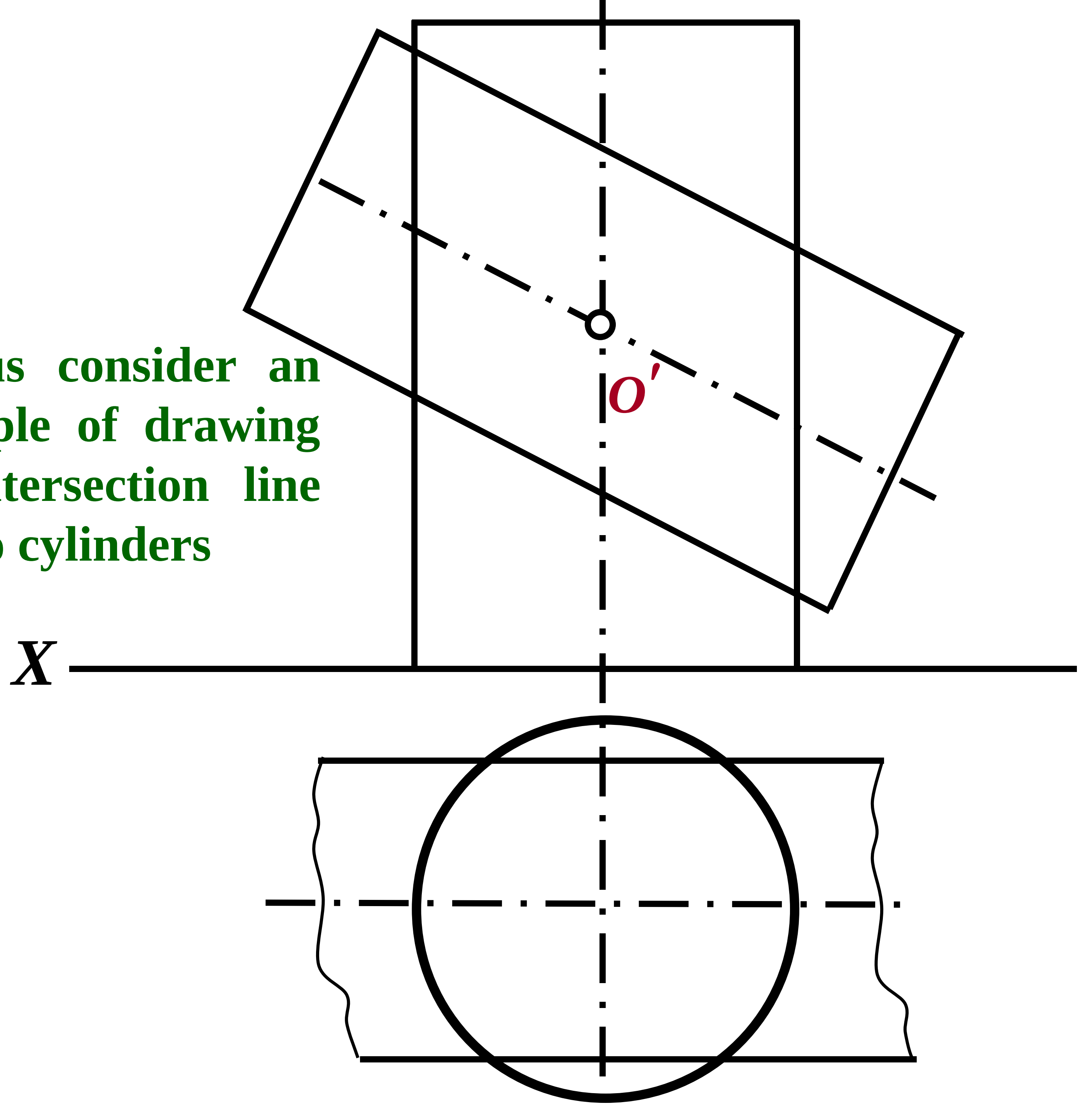
That is why the method of auxiliary spheres should be used under the following conditions:

a) intersecting surfaces are the **surfaces of rotation;**

b) axes of the surfaces intersect and **the intersection point is taken for the centre of auxiliary spheres;**

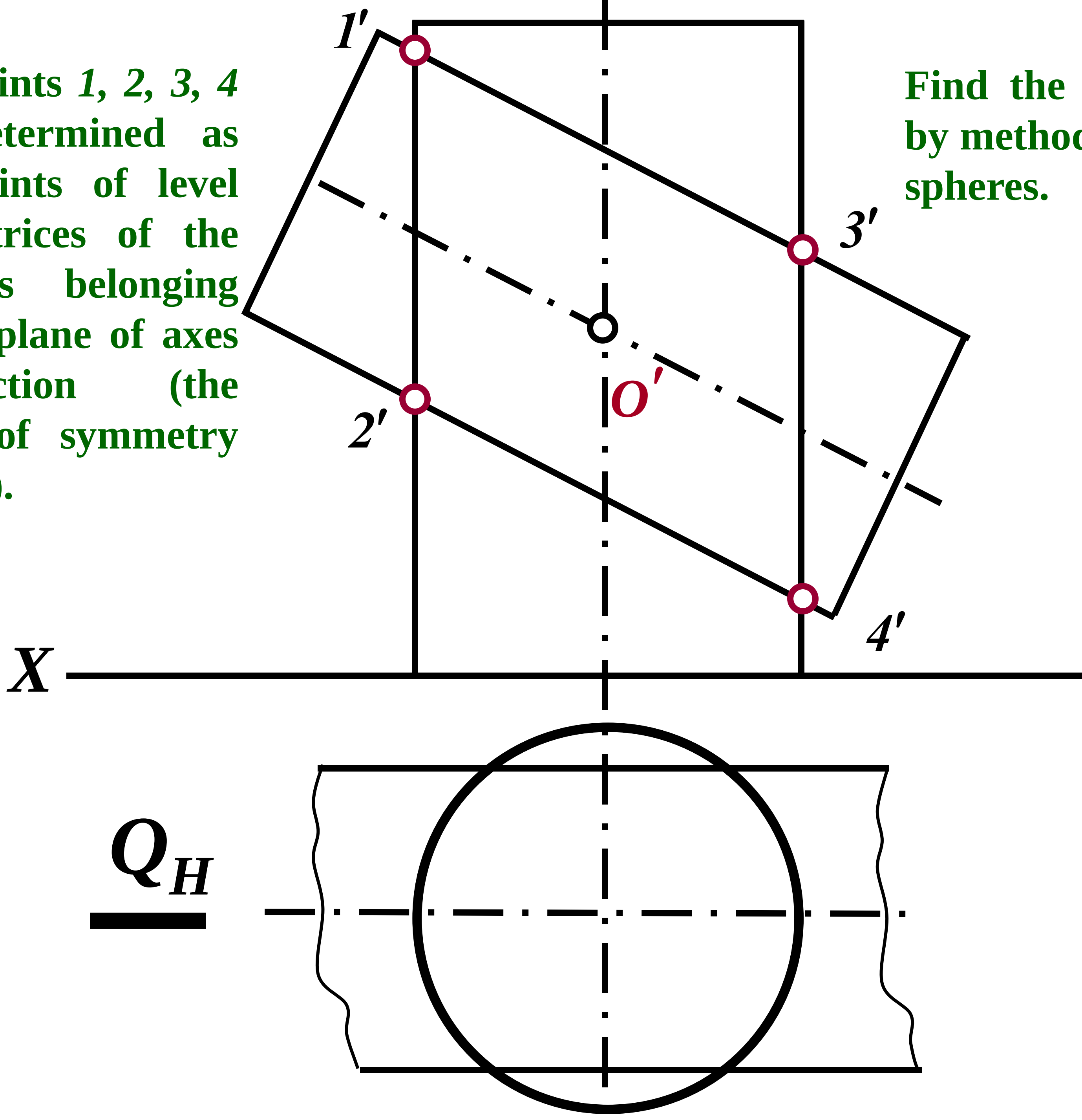
c) the plane produced by the surfaces axes (plane of symmetry) **is parallel to one of the projection planes;**

Let us consider an example of drawing an intersection line of two cylinders



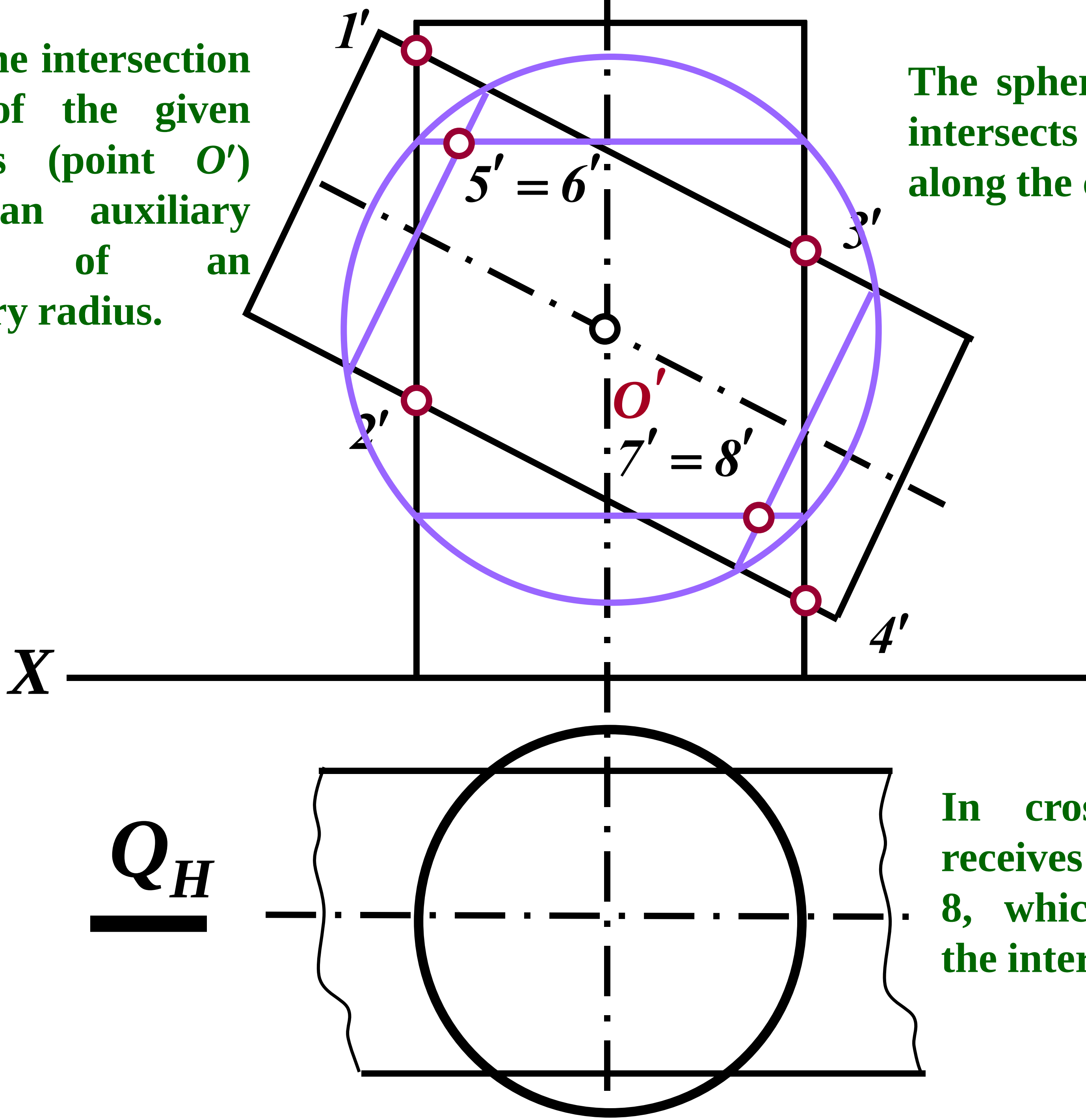
The points *1*, *2*, *3*, *4* are determined as the points of level generatrices of the surfaces belonging to the plane of axes intersection (the plane of symmetry *Q*(*QH*)).

Find the other points by method of auxiliary spheres.



From the intersection point of the given surfaces (point *O′*) draw an auxiliary sphere of an arbitrary radius.

The sphere constructed intersects the cylinders along the circles.



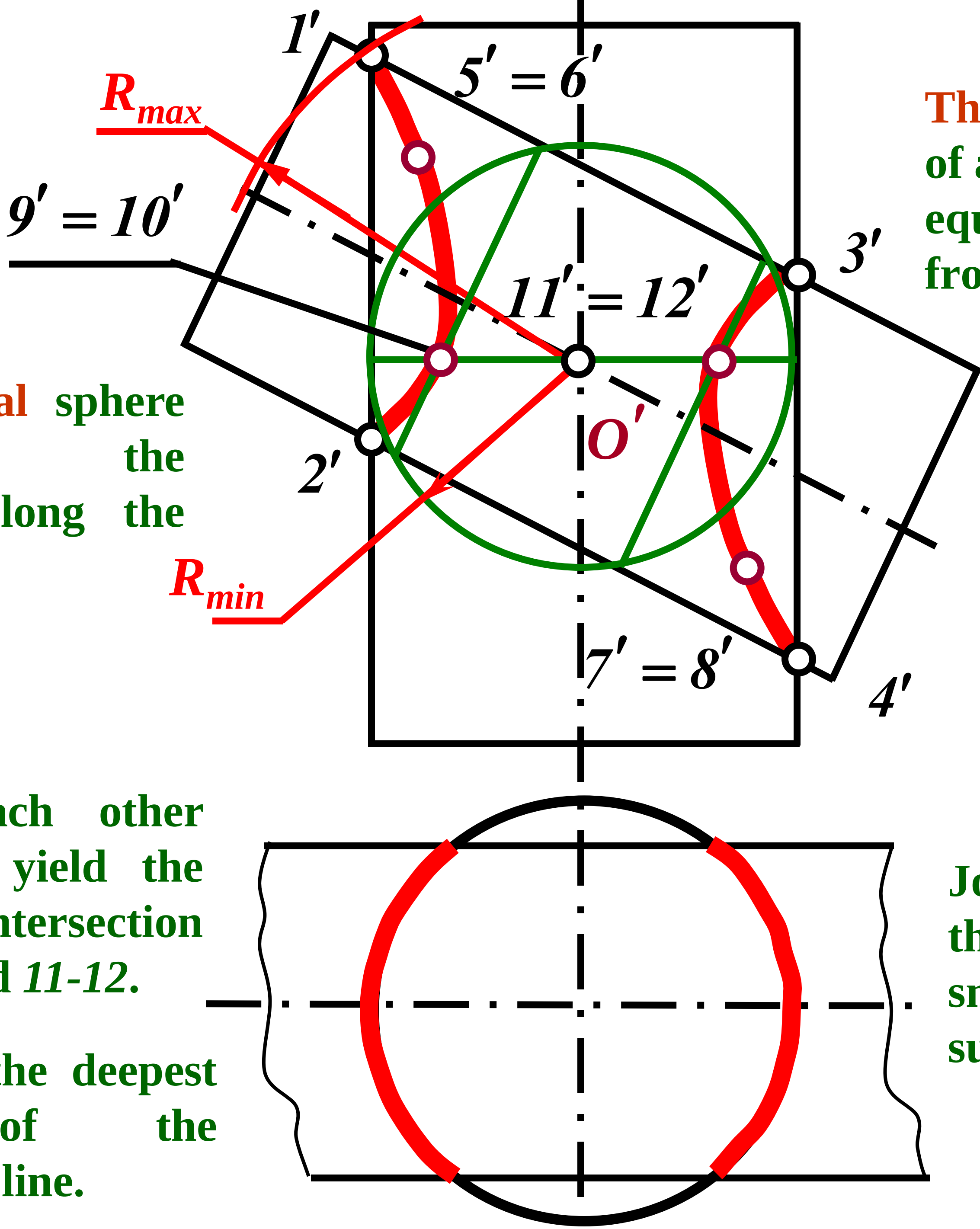
In crossing circles receives points 5-6, 7-8, which belong to the intersection line.

In such a way it is possible to construct a certain amount of points of the desired intersection line.

Consider the limits of the auxiliary spheres usage.

The minimal cutting sphere is a sphere, which contacts one surface (the larger one) and cuts another (the smaller one).

The maximal radius of a cutting sphere is equal to the distance from the centre O to the farthest intersection point of the level generatrices (from *O′* to *1′* and *4′*).



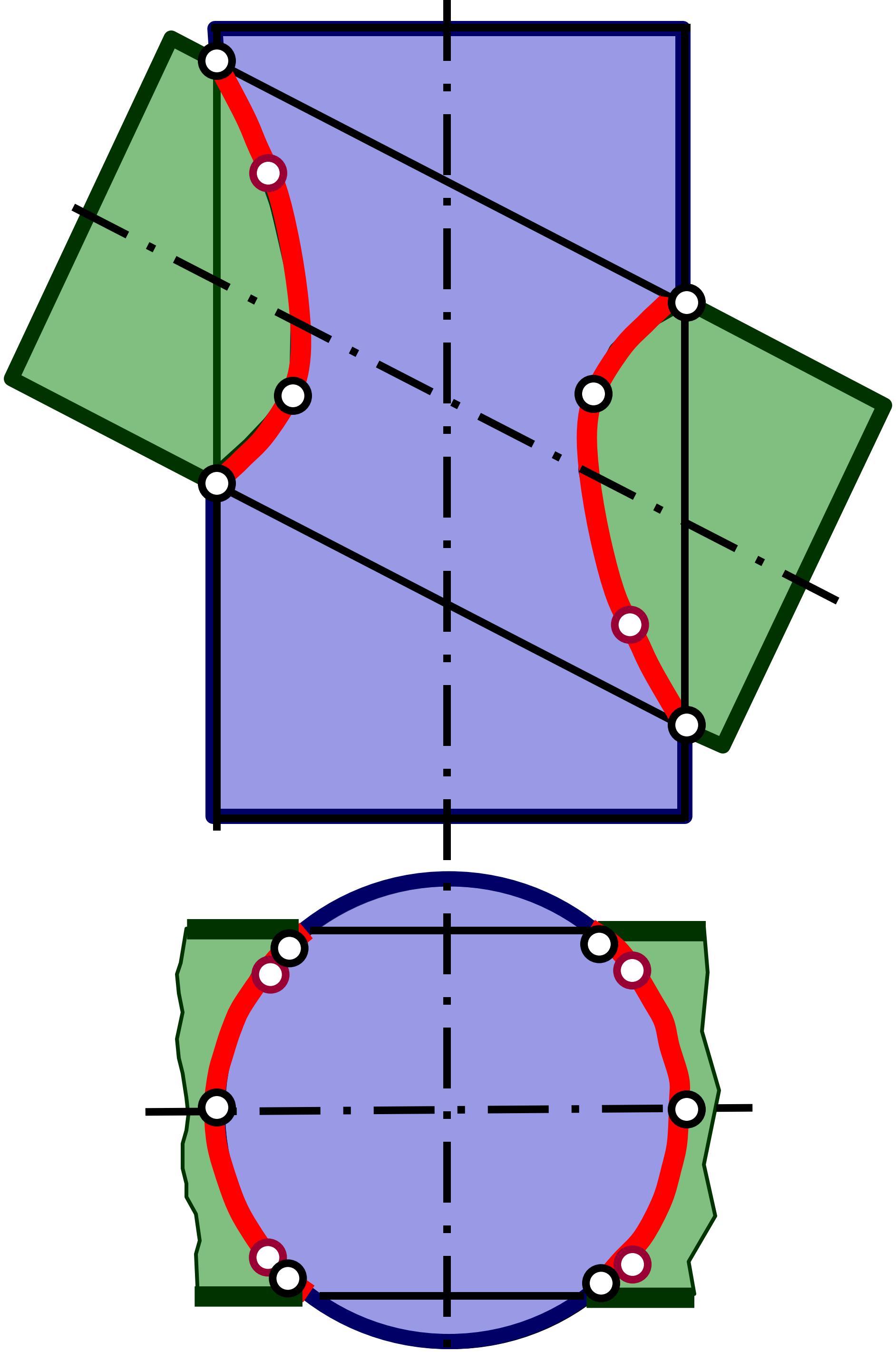
The maximal radius of a cutting sphere is equal to the distance from the centre O to the farthest intersection point of the point of the level generatrices (from *O′* to *1′* and *4′*).

The minimal sphere intersects the cylinders along the circles.

Meeting each other the circles yield the points of intersection line 9-10 and *11-12*.

These are the deepest points of the intersection line.

Join the points thus obtained in a smooth curve subject to visibility

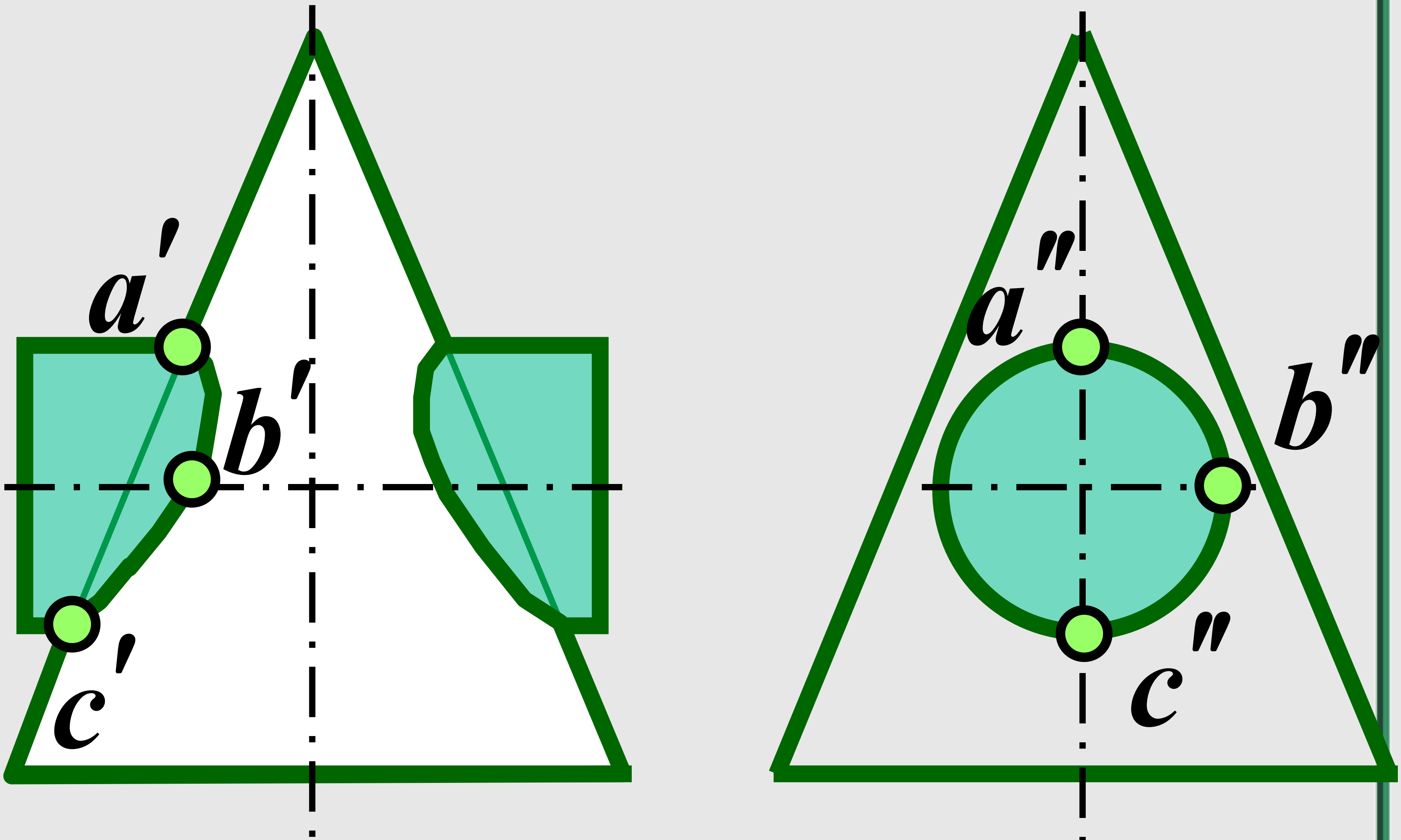


Possible Cases of Intersection of Curved Surfaces

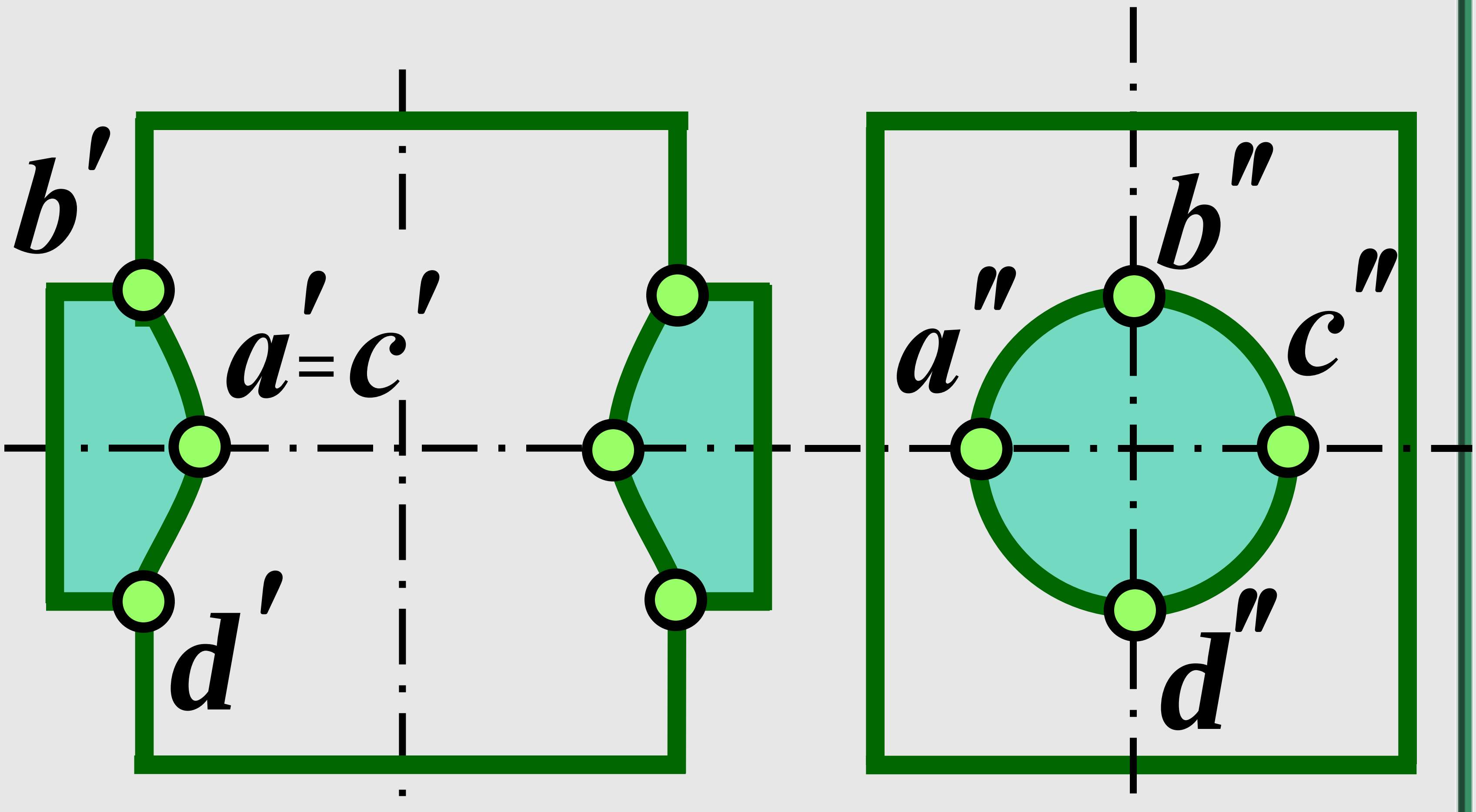
Permeability.

All generating lines of the first surface (cylinder) intersect the other surface, but not all generatrices of the second surface intersect the first one. In this case the intersection line of the surfaces decomposes into two closed curves

Permeability



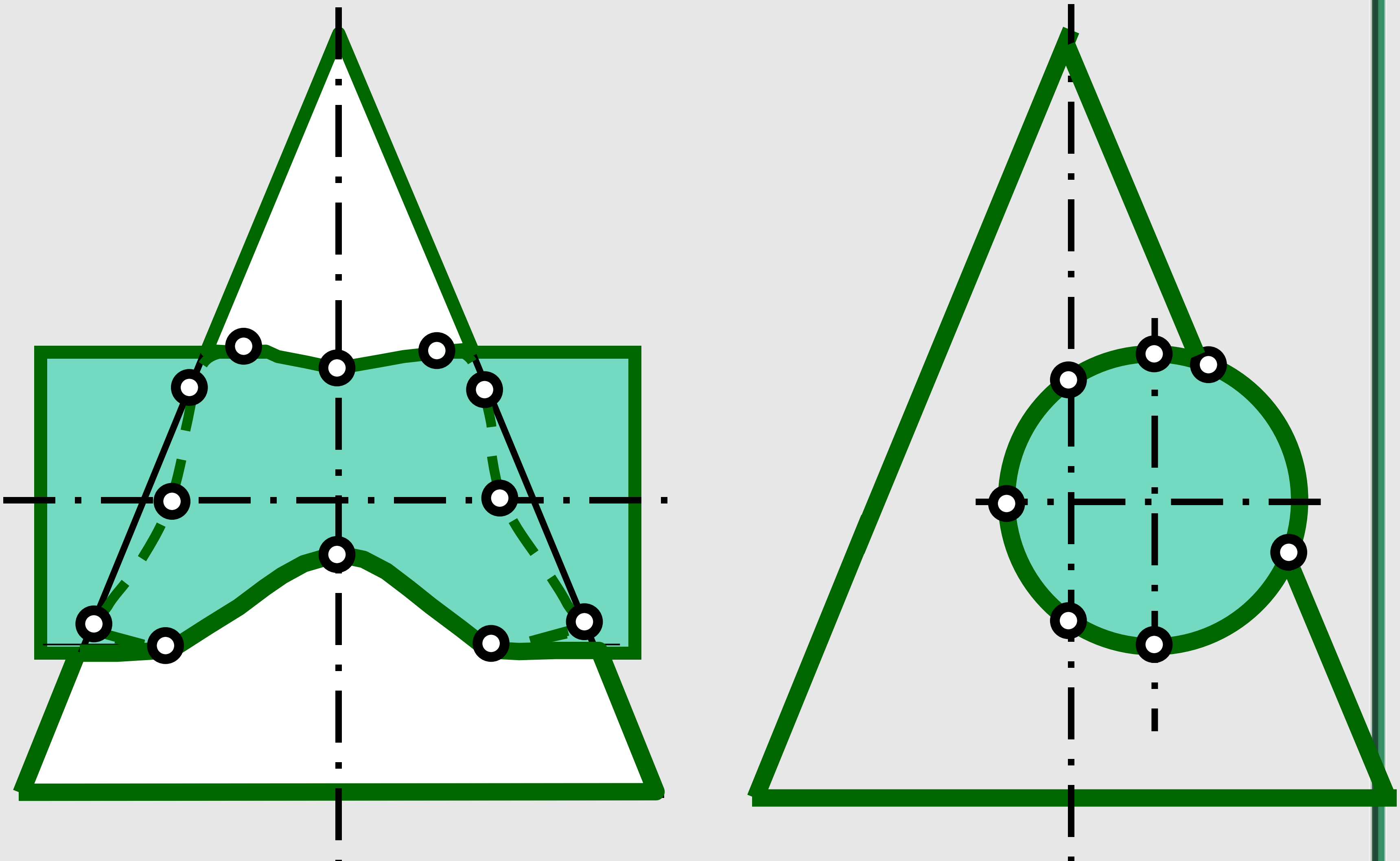
Permeability



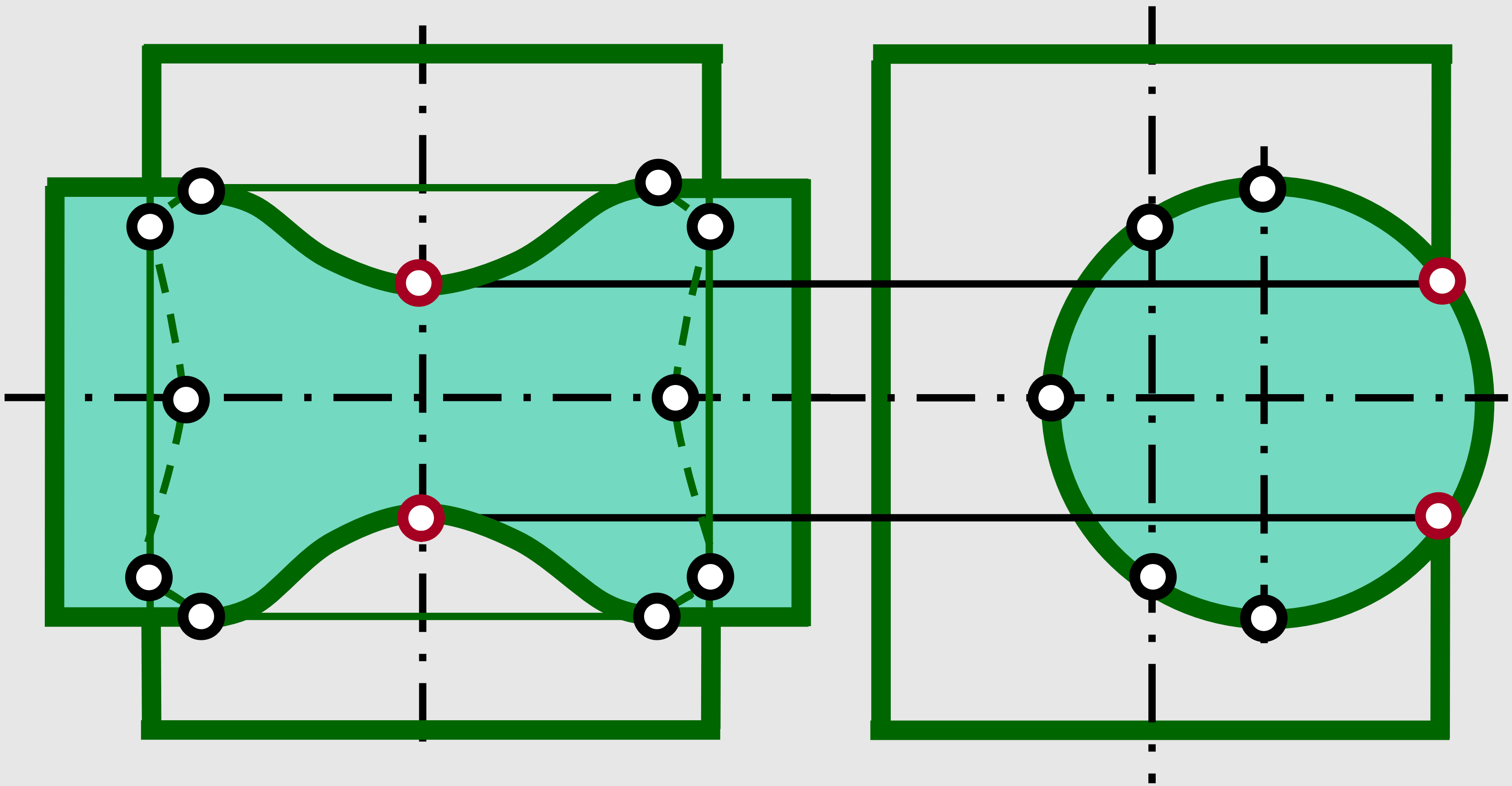
Cutting-in

Not all generatrices of both surfaces intersect each other. In this case the intersection line is one closed curve

Cutting-in



Cutting-in



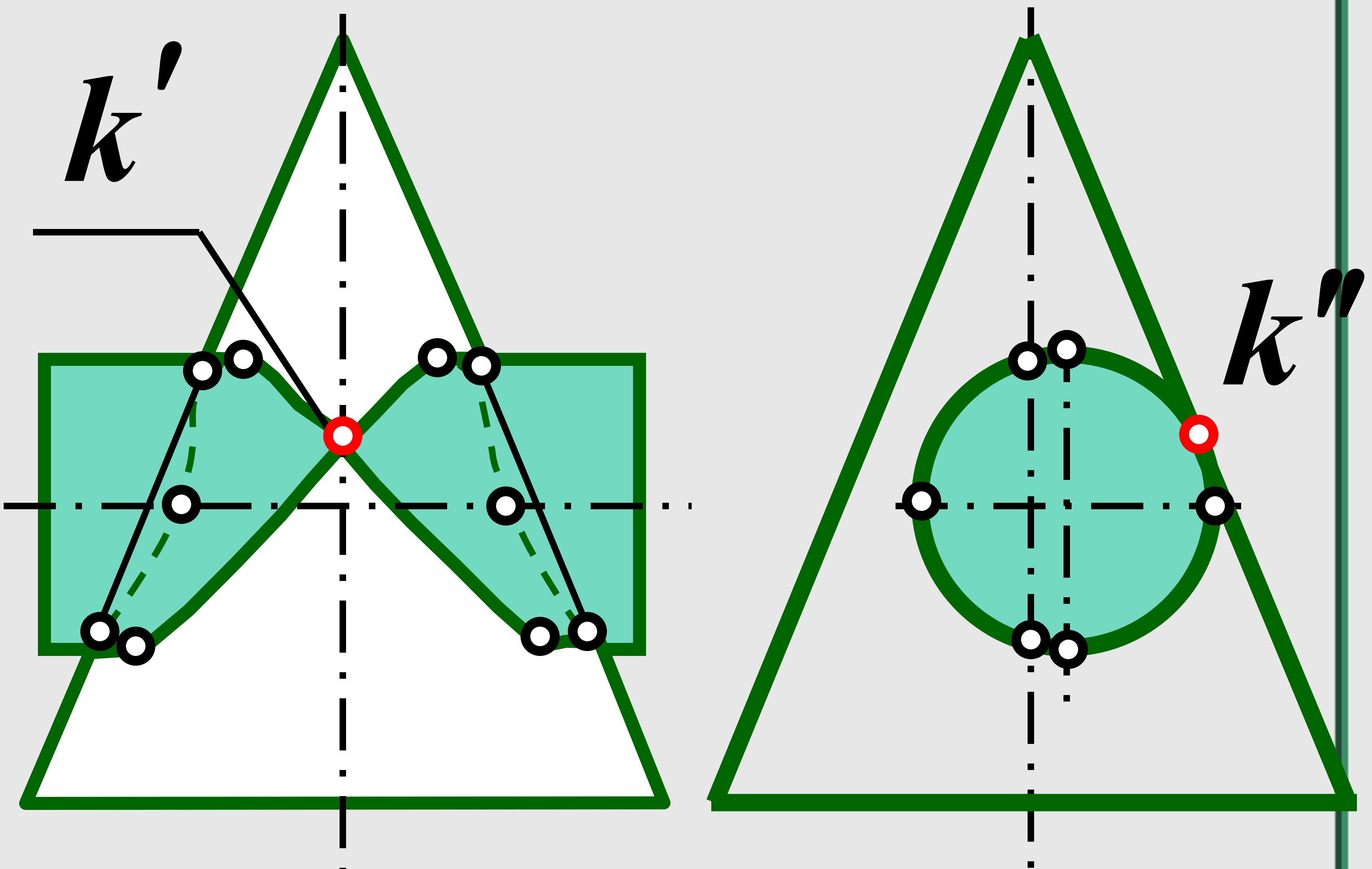
Unilateral contact

All generating lines of one surface intersect the other surface, but not all generatrices of the second surface intersect the first one.

There is a common tangent plane in one point of the surfaces.

The intersection line decomposes into two closed curves meeting in the point of contact.

Unilateral contact



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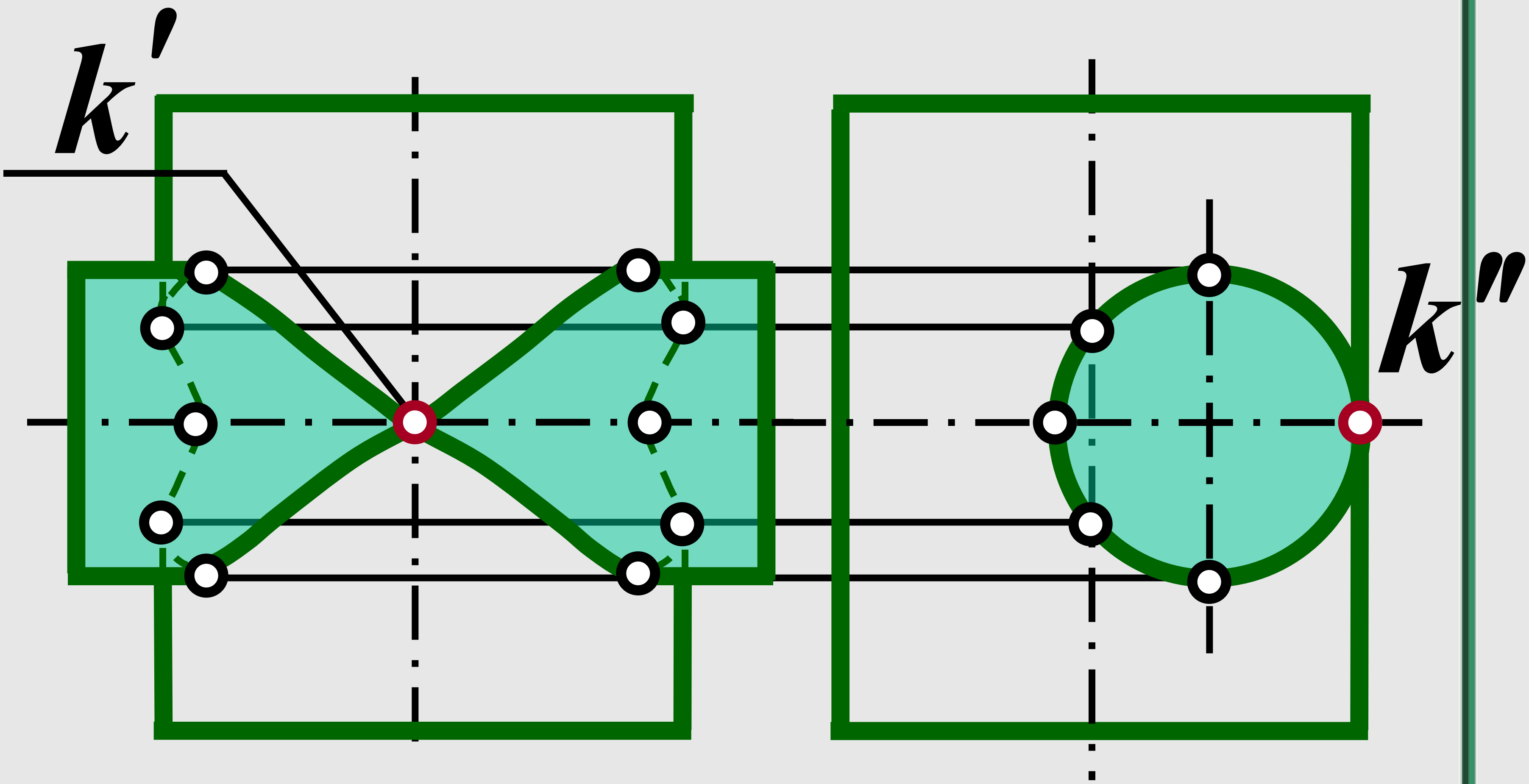
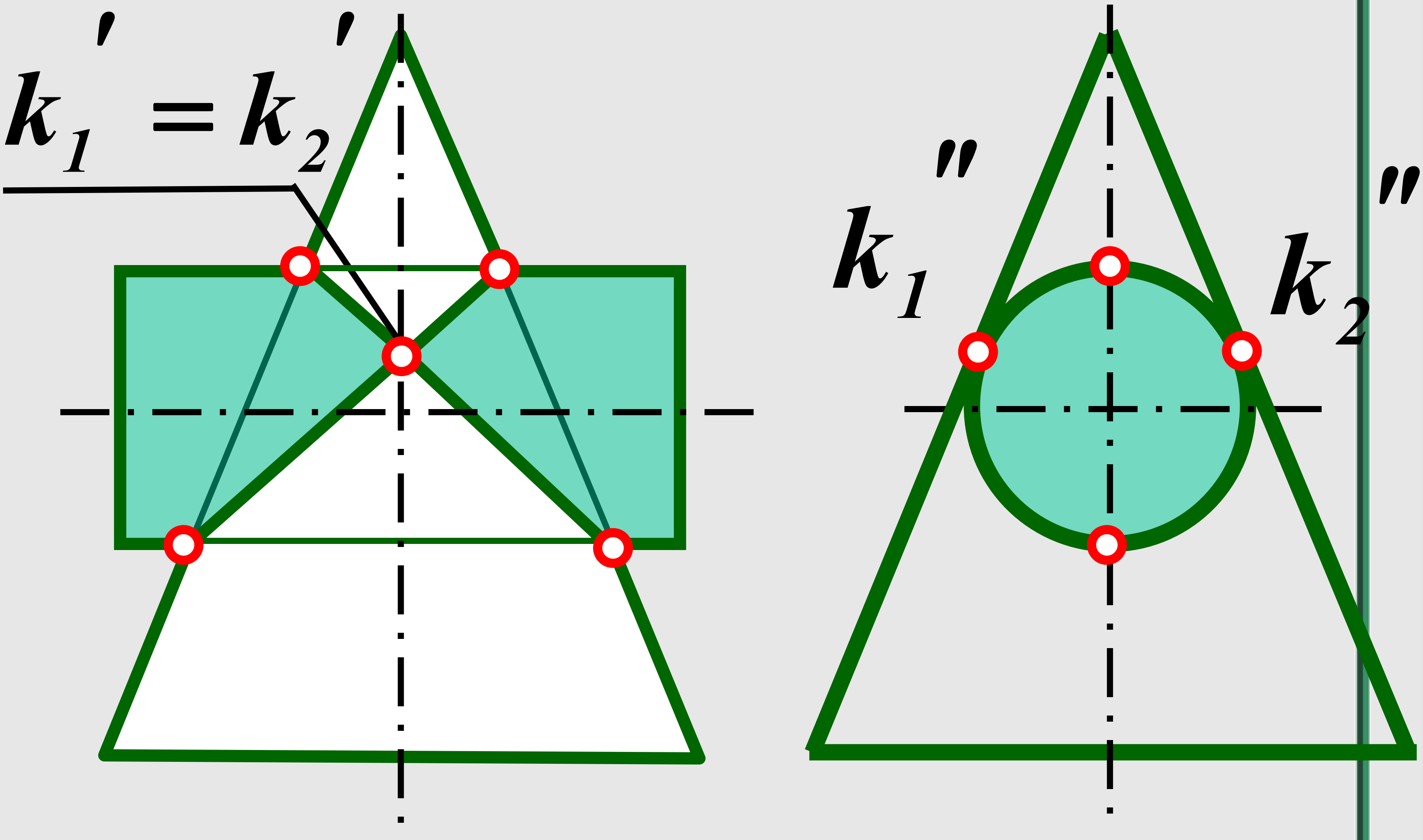


Bilateral contact

All generating lines of both surfaces intersect each other.
The intersecting surfaces have two common tangent planes.
In this case the intersection line decomposes into two plane curves which meet in the points of contact

Bilateral contact



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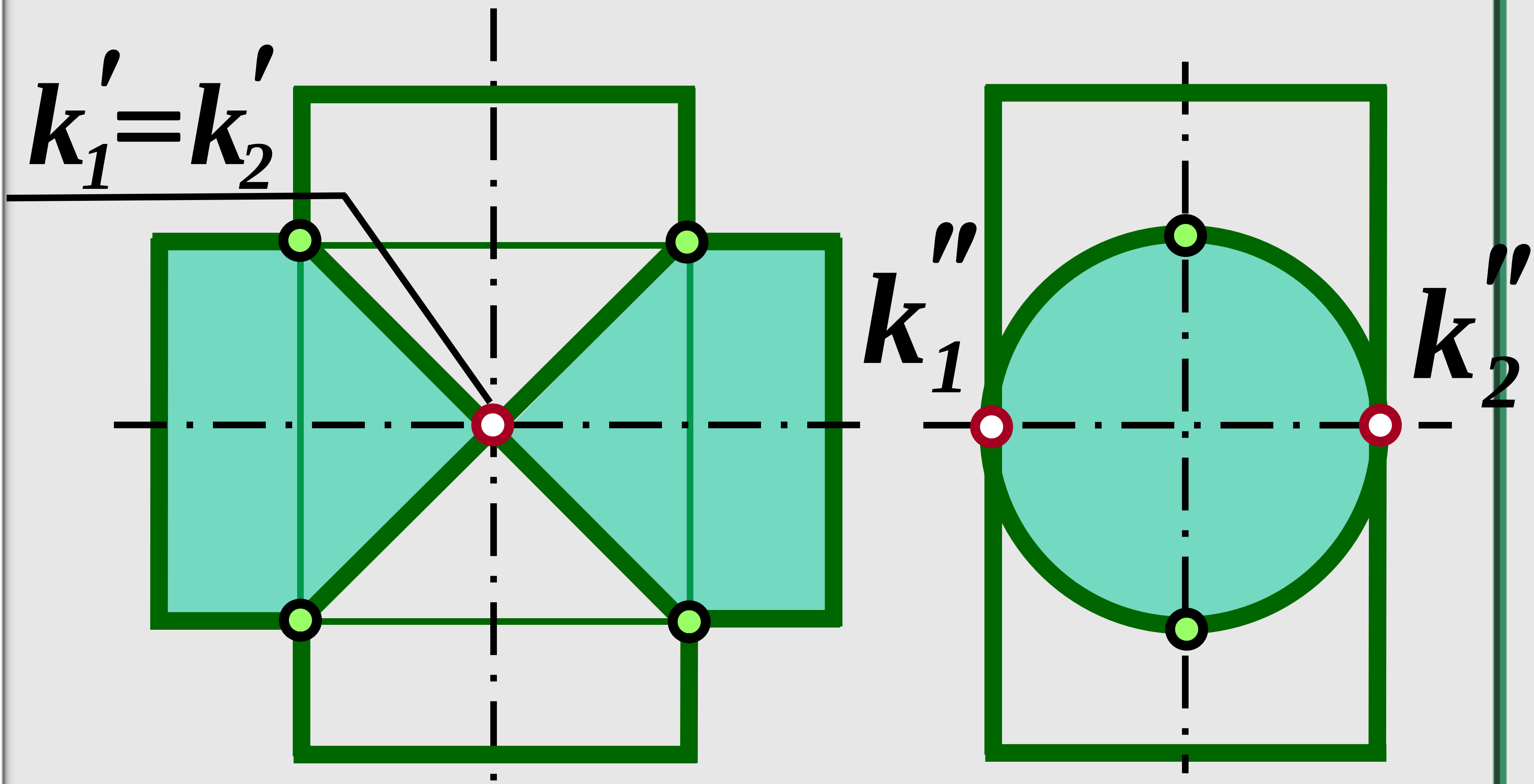
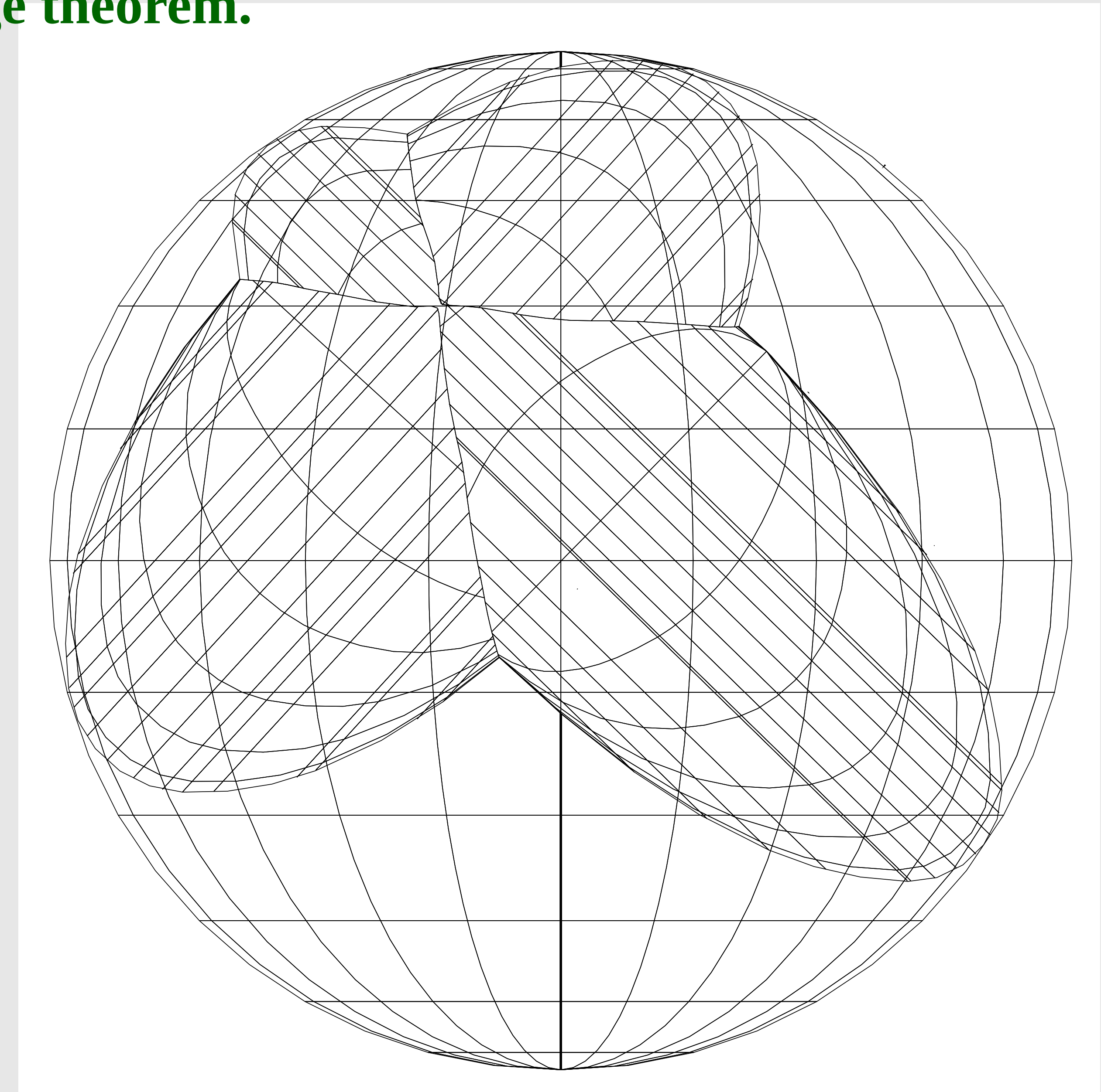


Monge theorem

If two surfaces of the second order may be inscribed into the third one or described around it, the line of their mutual intersection decomposes into two plane curves.

The planes of those curves pass through a straight line connecting the intersection points of the tangent lines

The examples of construction of the surfaces intersection lines on the basis of Monge theorem.



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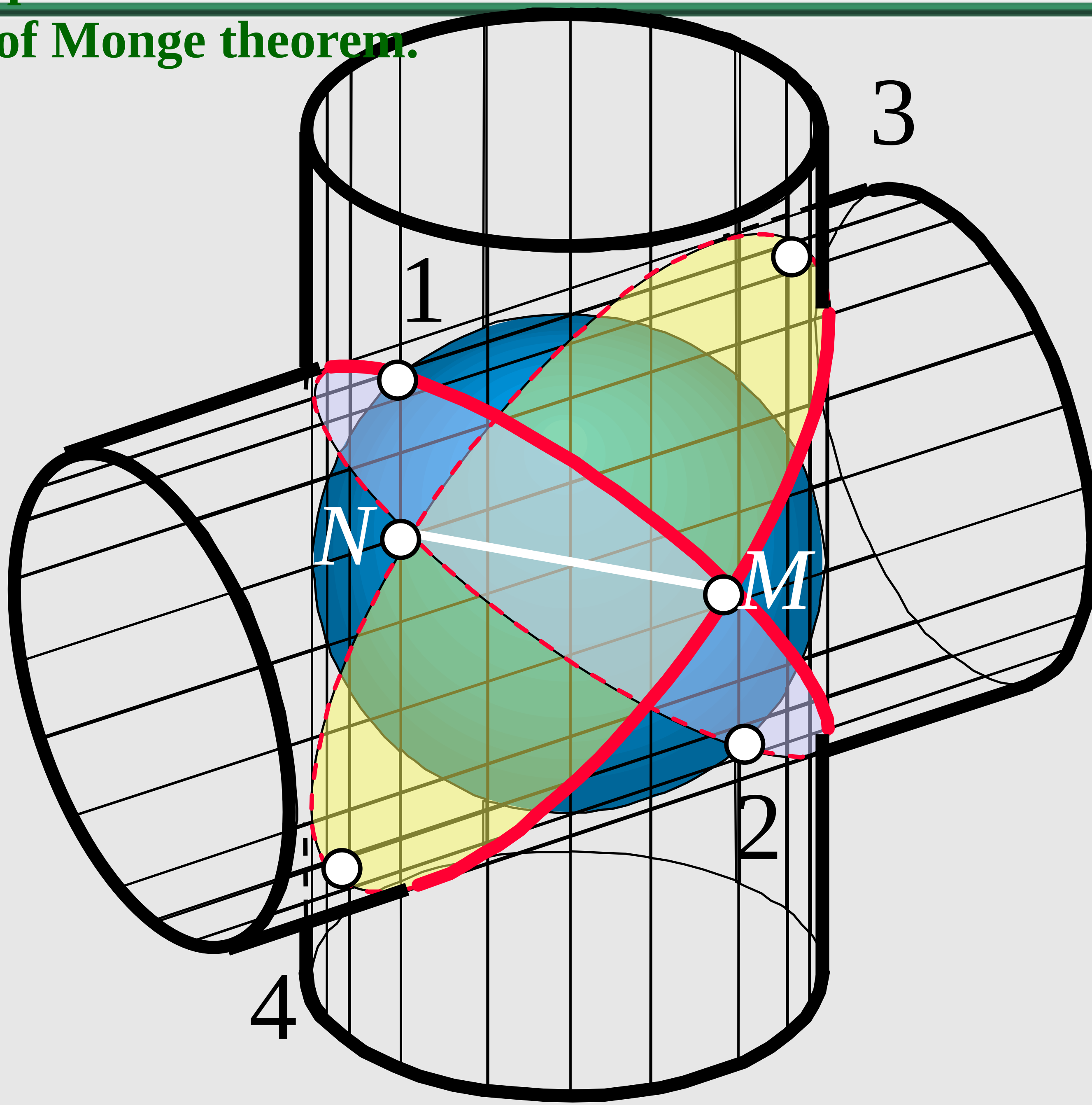

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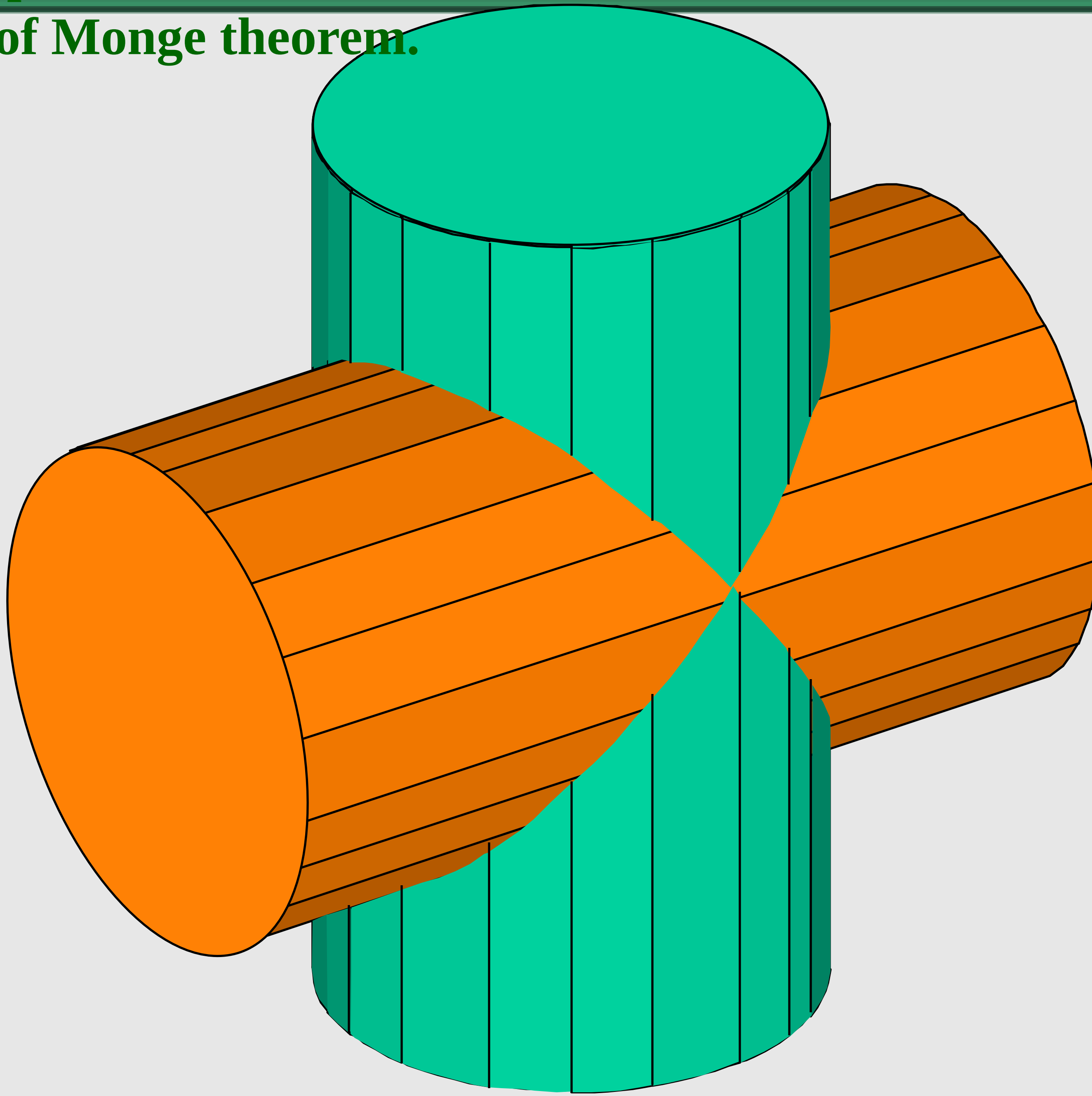

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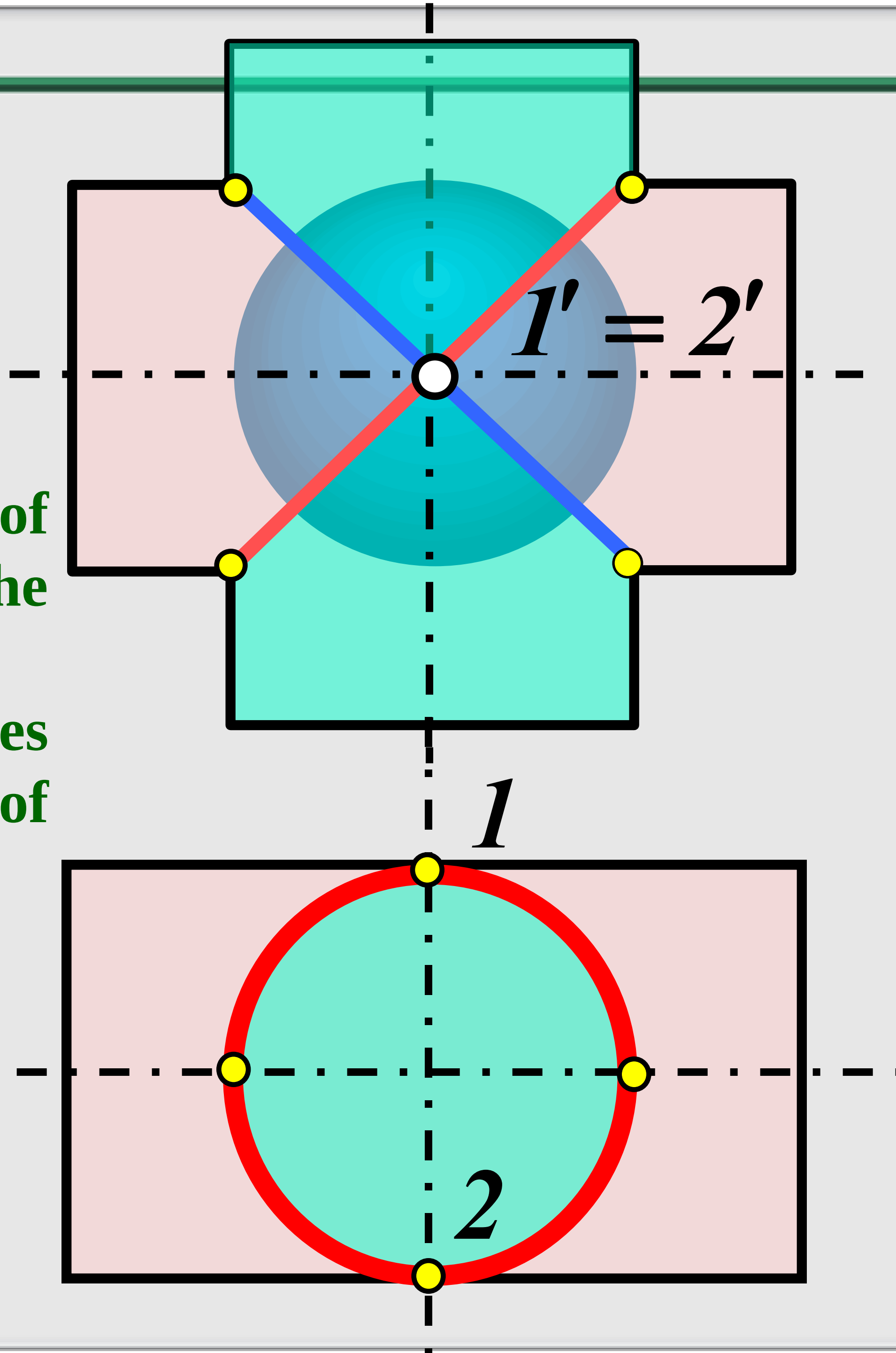

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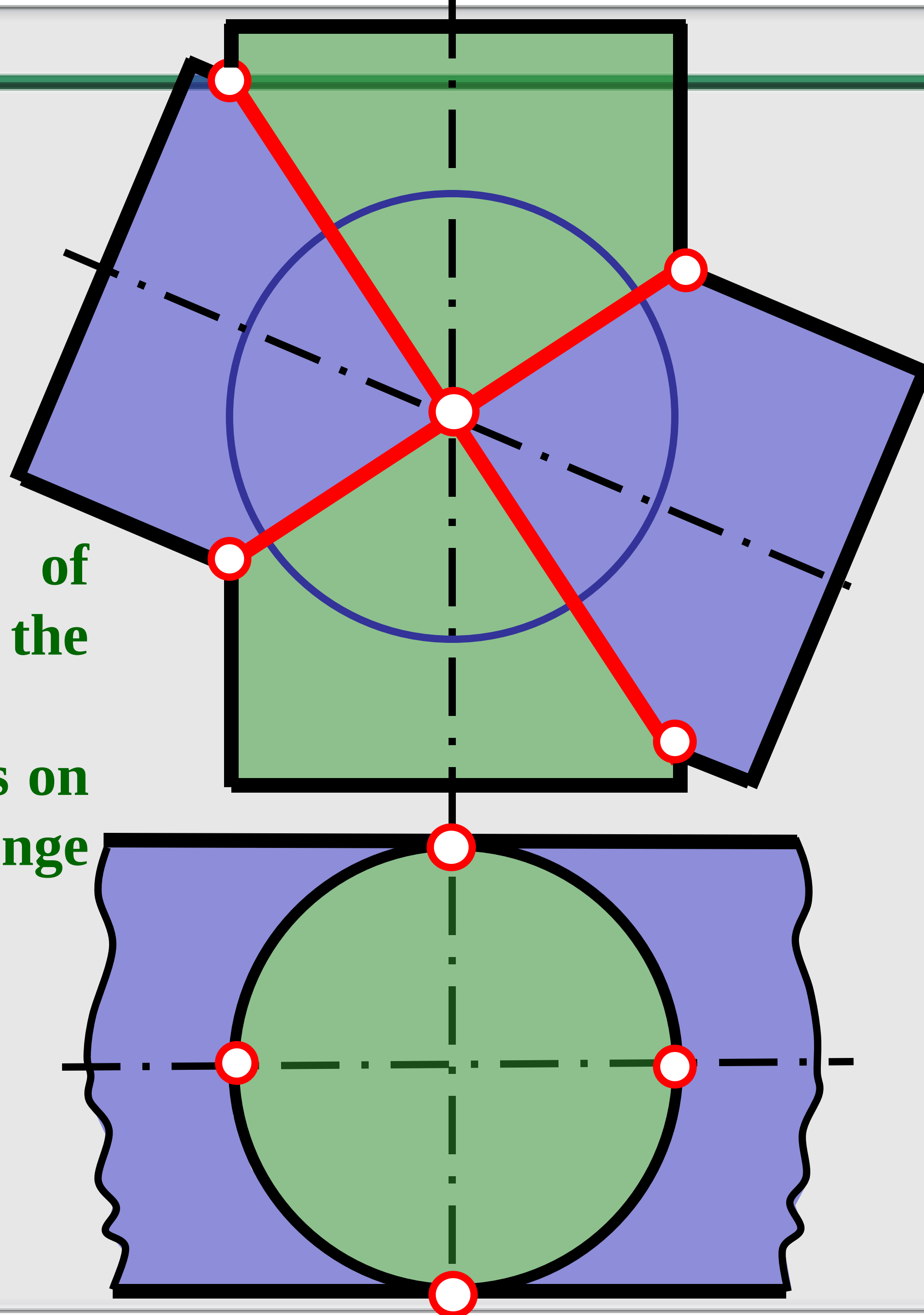
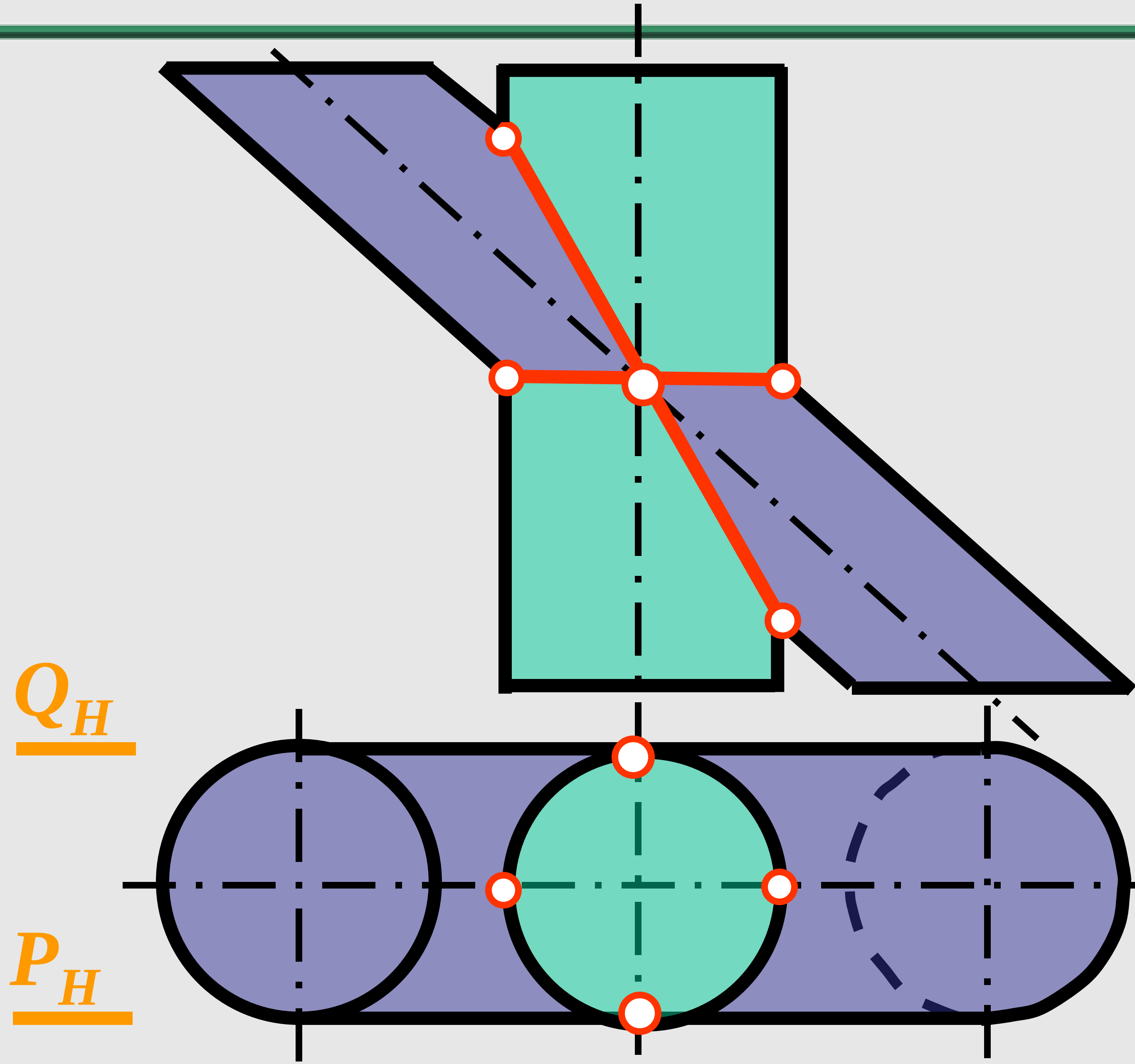


Theorem of a double contact

If two surfaces of the second order have two common points through which two common tangent planes may be passed to them, the line of their mutual intersection decomposes into two plane curves of the second order, and the planes of the above curves pass through the straight line connecting the tangent points.



Q_H
P_H