



Tomsk Polytechnic University

**DESCRIPTIVE GEOMETRY
ENGINEERING GRAPHICS**

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Lecture 5

SURFACES



Plan

- 1. Conical and Cylindrical Surfaces.**
- 2. Rotation Surfaces.**
- 3. Rotation Surface Cut by a Plane**



Conical and Cylindrical Surfaces

The position of a plane on a drawing may be specified in one of the following ways:



Conical and Cylindrical Surfaces



Conical Surface

The conical surface is produced by the motion of a **linear generating** line along a **curved directrix**.

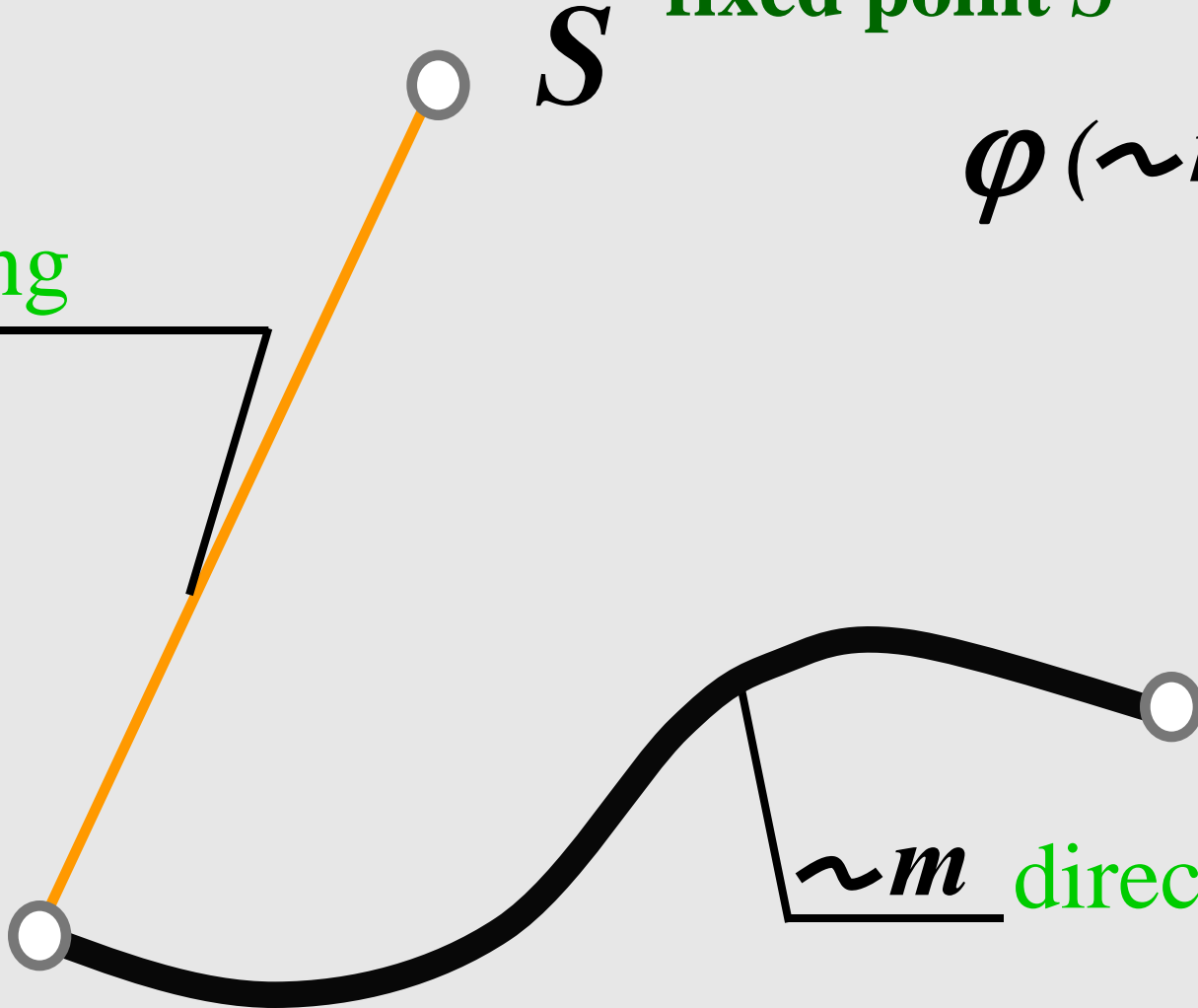
At that, the generatrix passes some fixed point S , referred to as a vertex.



Determinant surface: Directrix and certain fixed point S

$$\varphi(\sim m, S)$$

generating



$\sim m$ directrix

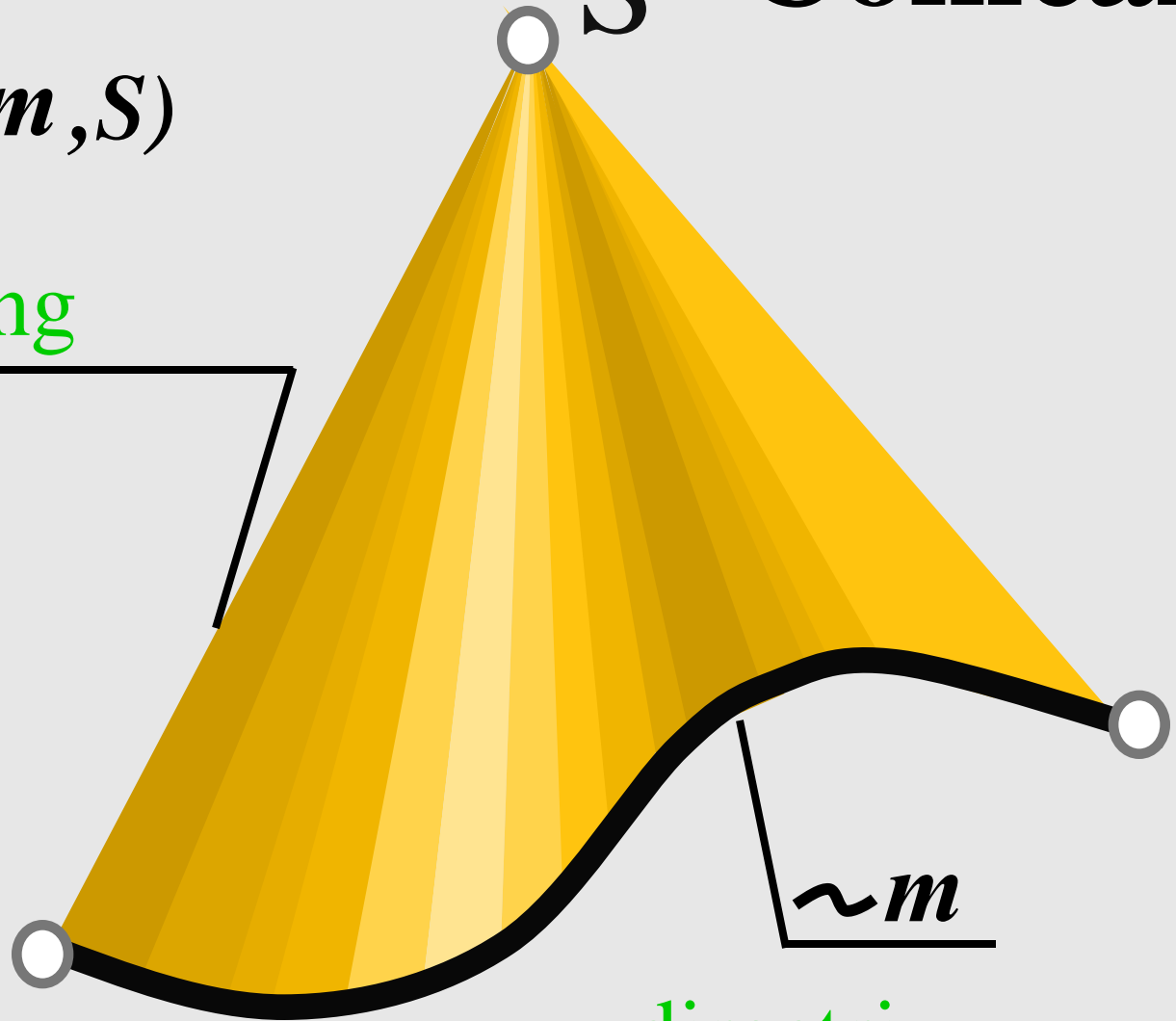


Conical

S

$$\varphi(\sim m, S)$$

generating



$\sim m$

directrix



The cylindrical surface

The cylindrical surface is produced by parallel to a given straight line l motion of a linear generating line along a curved directrix

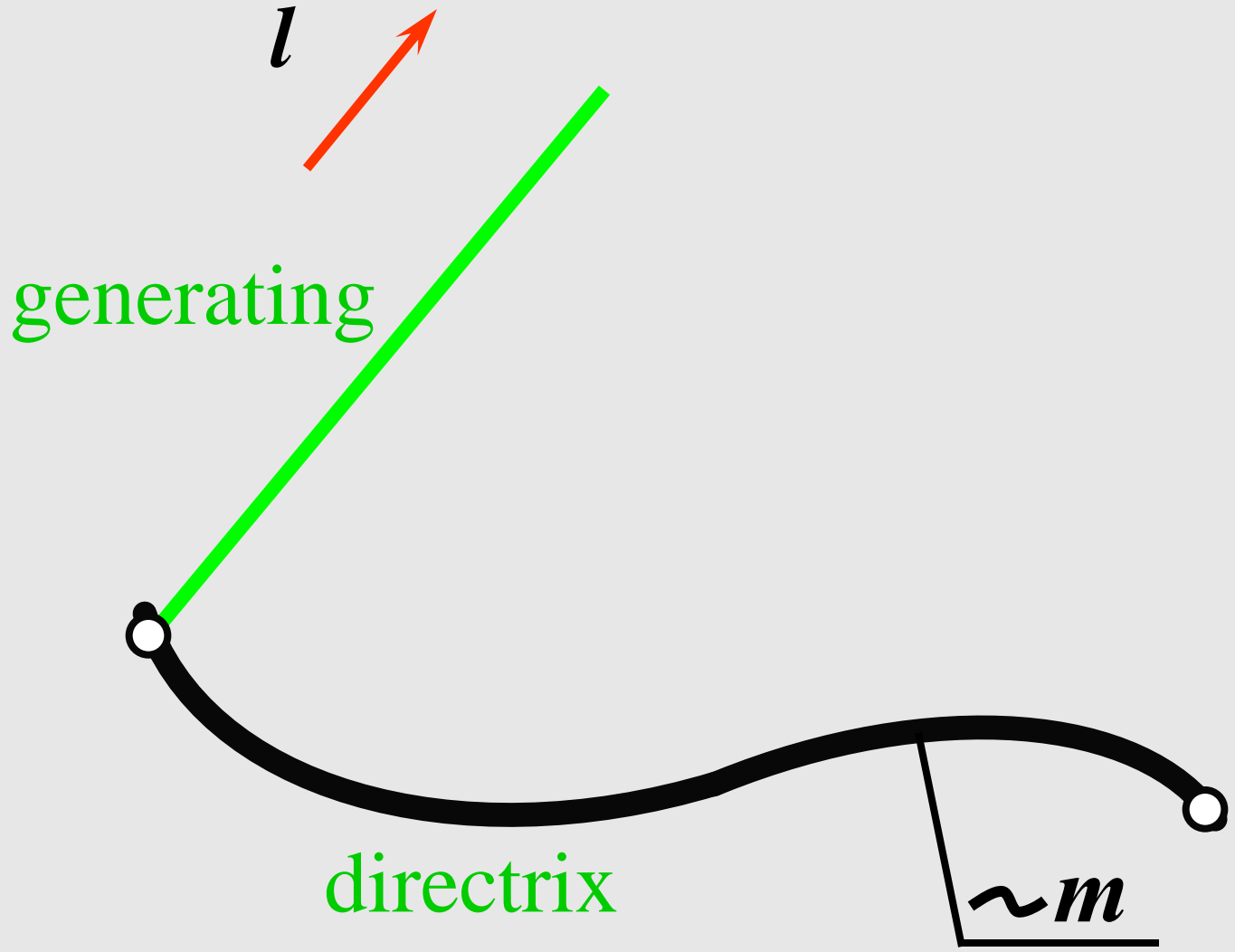


A cylindrical surface is considered to be distinguished if a directrix and a generatrix.



Determinant surface:

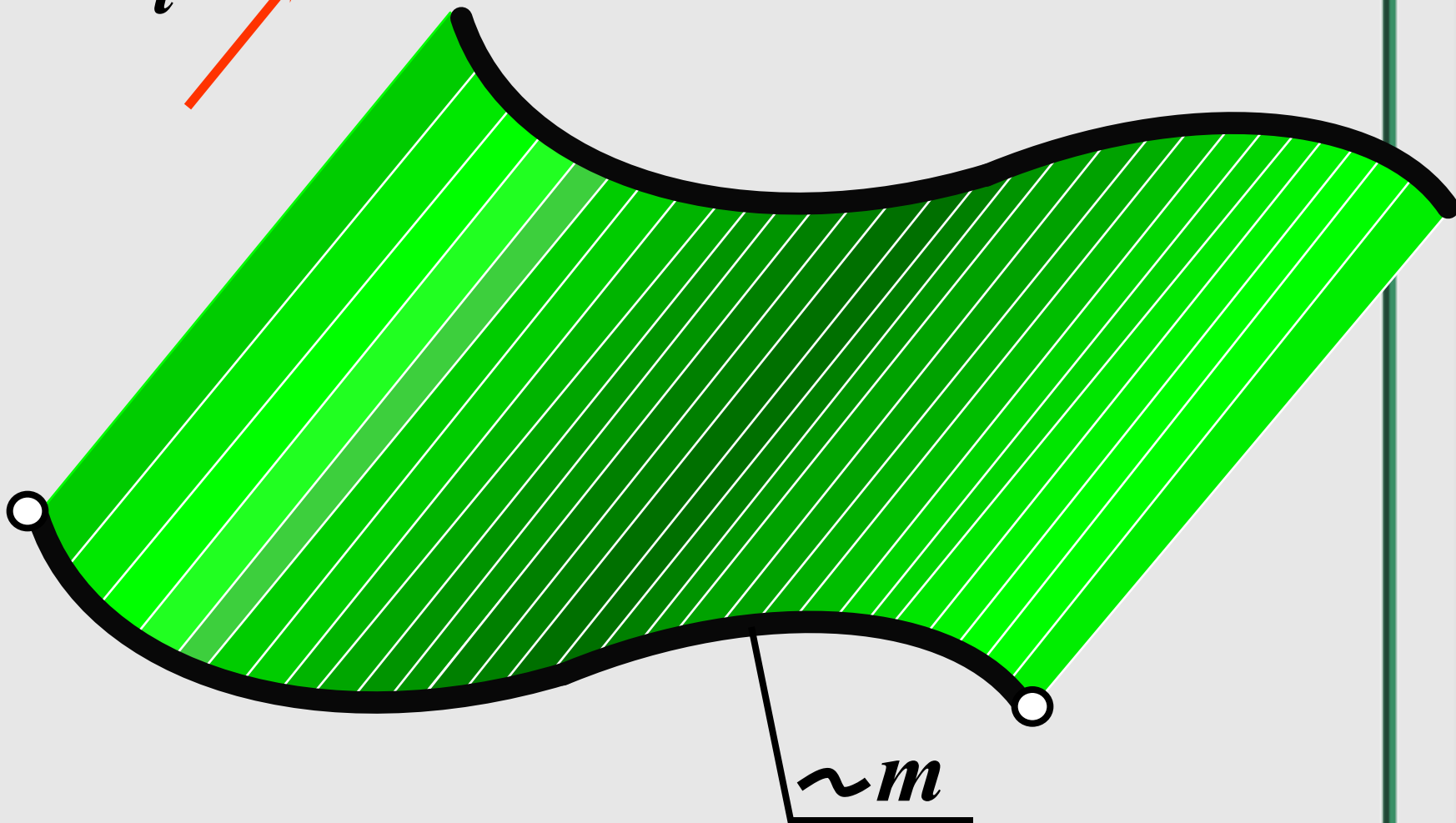
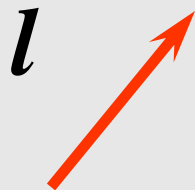
$$\varphi(\sim m, l)$$





$\varphi (\sim m, l)$

cylindrical





Rotation Surfaces

Rotation surface is a surface described by a curve (or a straight line), rotating on its axis



Rotation Surfaces

Cylinder of rotation - this is a surface produced by rotation of the line L round the axis I parallel to it



Cylinder of rotation

I-I AXIS OF ROTATION

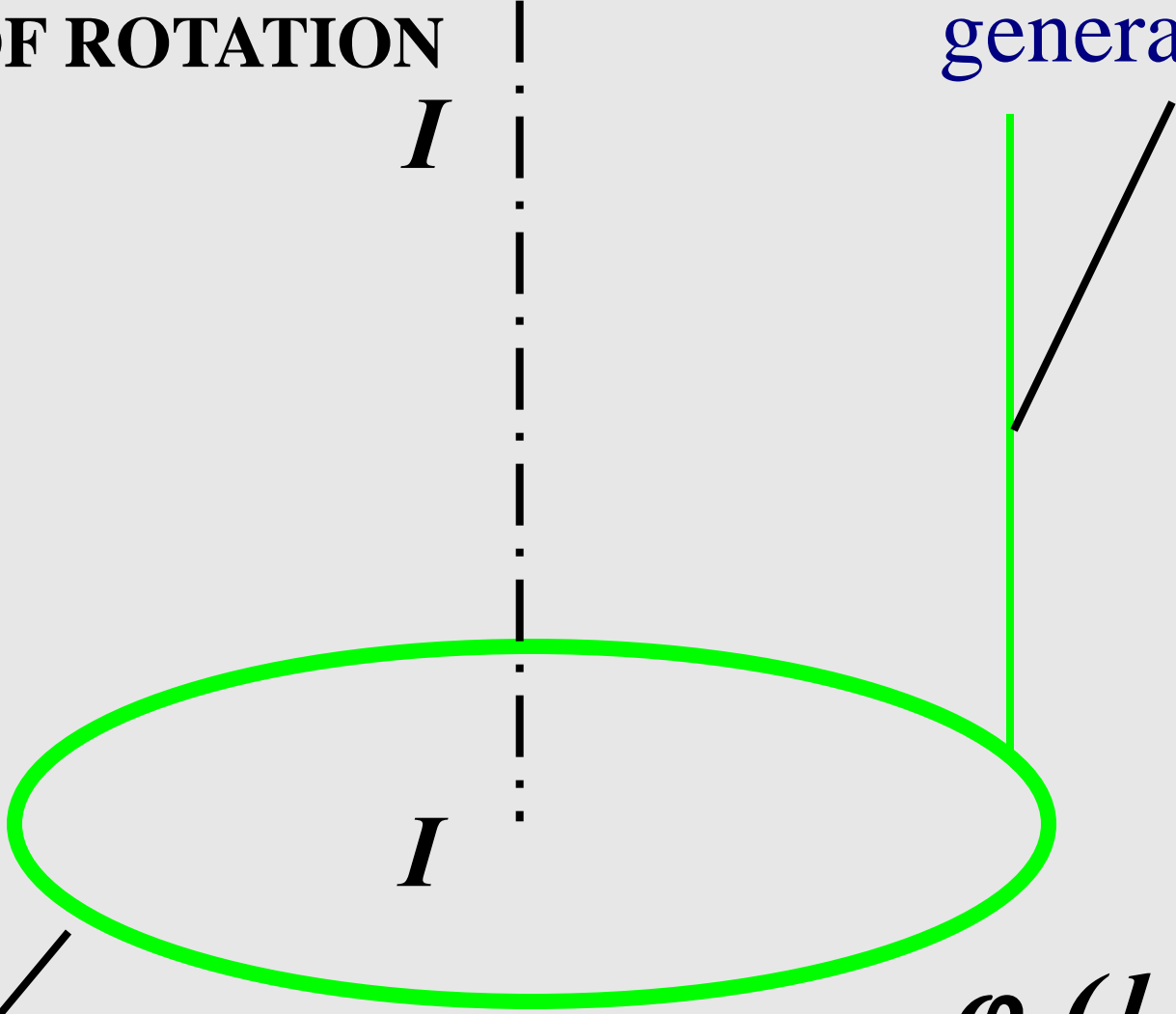
I

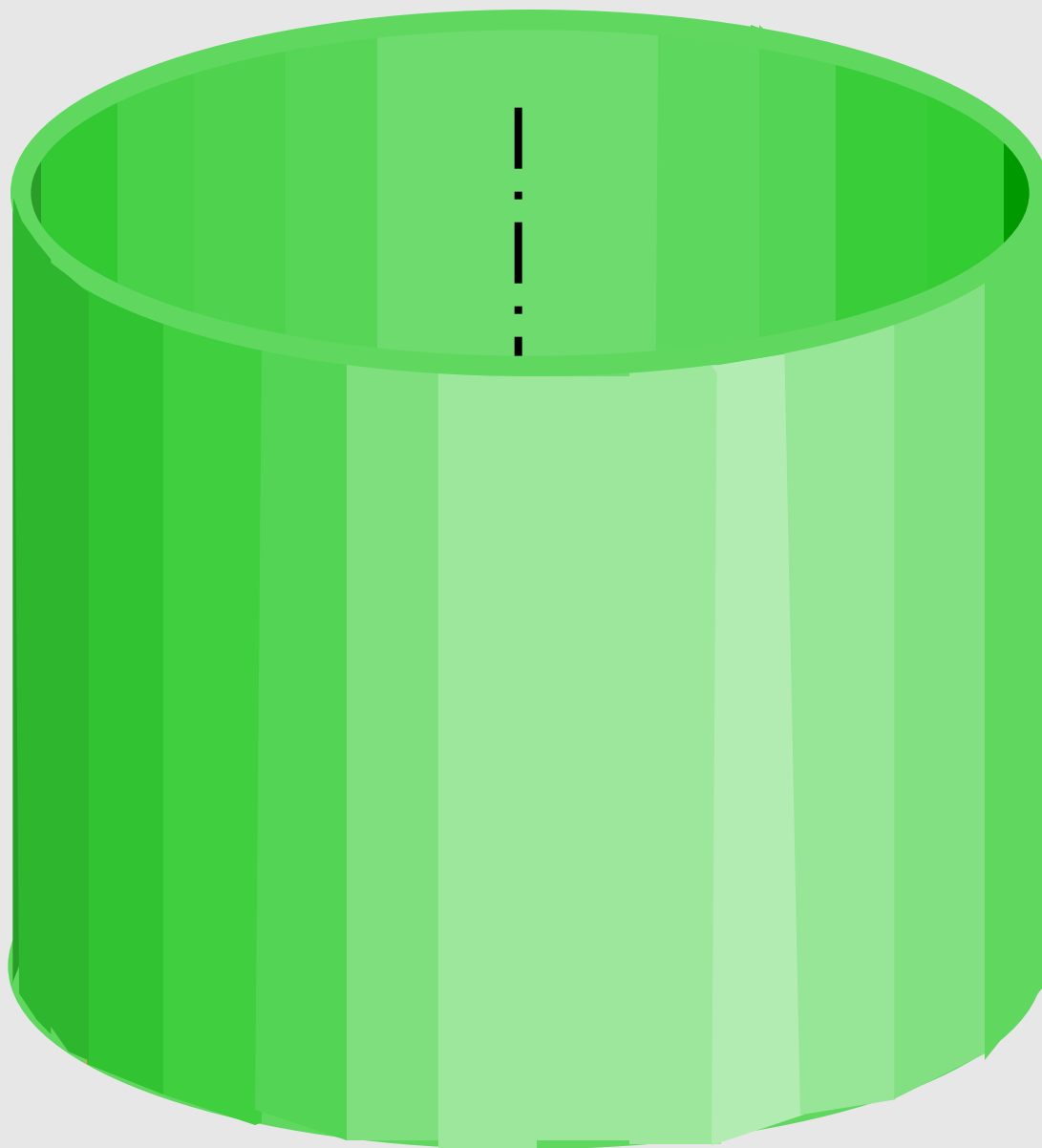
generating

I

CIRCLE

$\varphi (l, II)$

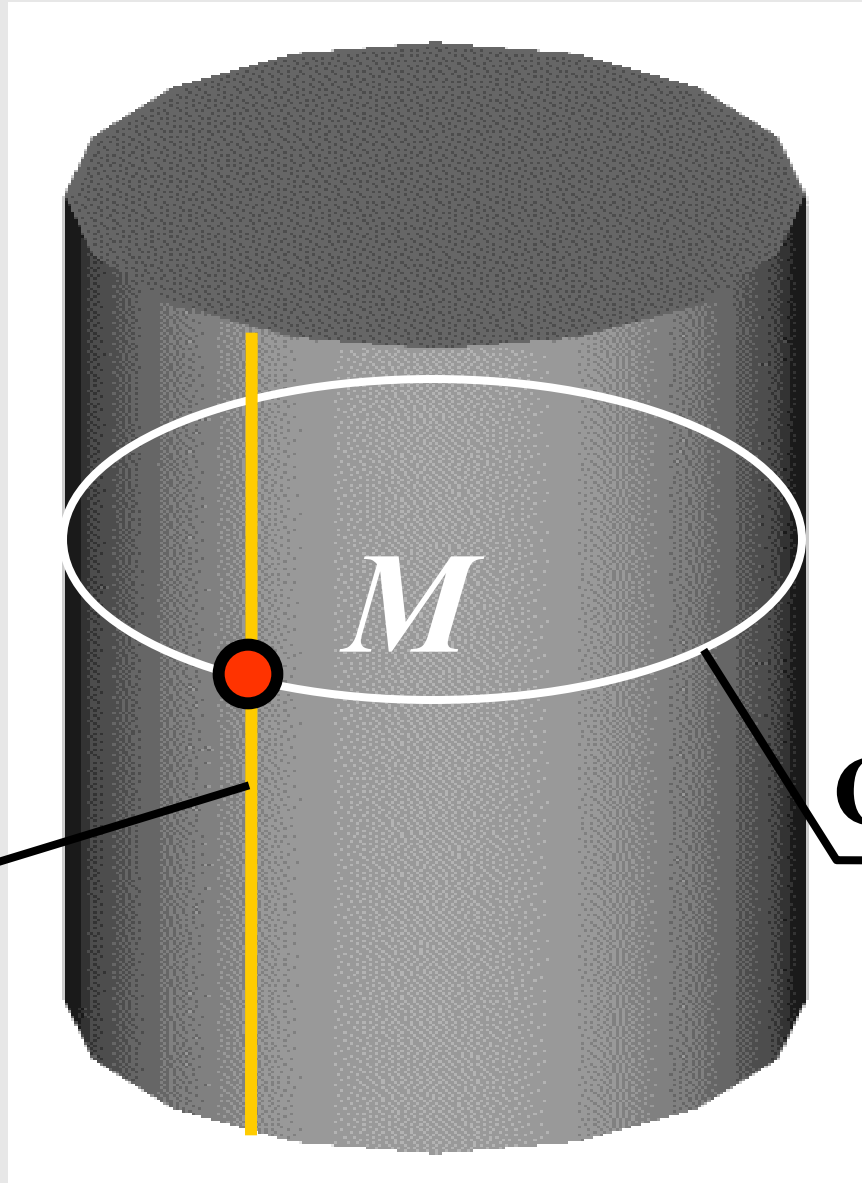






A Point on the Cylinder of rotation

It is necessary to construct a straight line or a circle which belongs to the cylinder and passes through the set point.



generating

CIRCLE



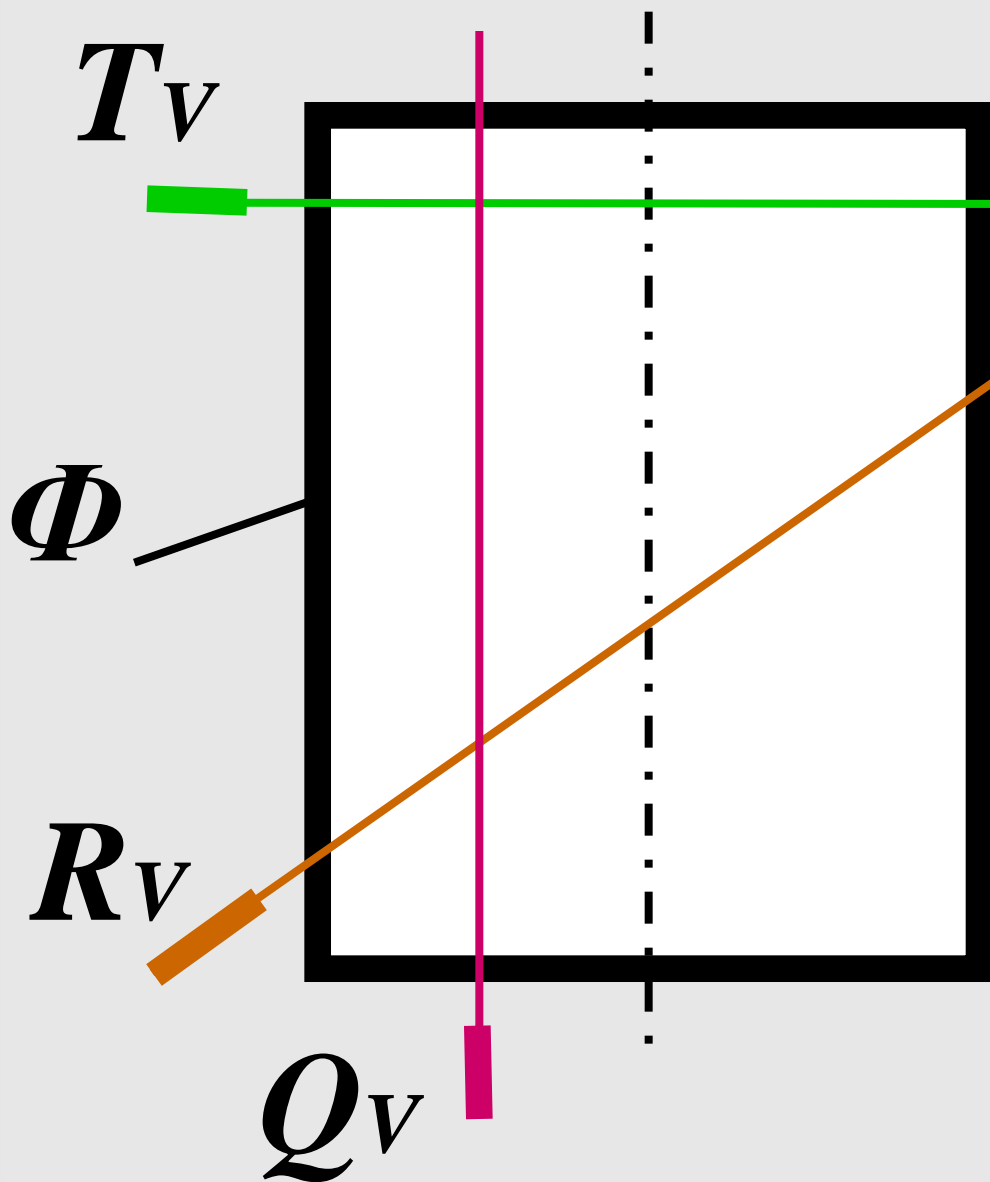
Cylinder of rotation Cut by a Plane



**At crossing the cylinder
Planes it is possible to receive on
a surface**

3 types of lines:

- **Two straight lines**
- **Circle**
- **Ellipse**



$$Q(Q_v) \cap \Phi =$$

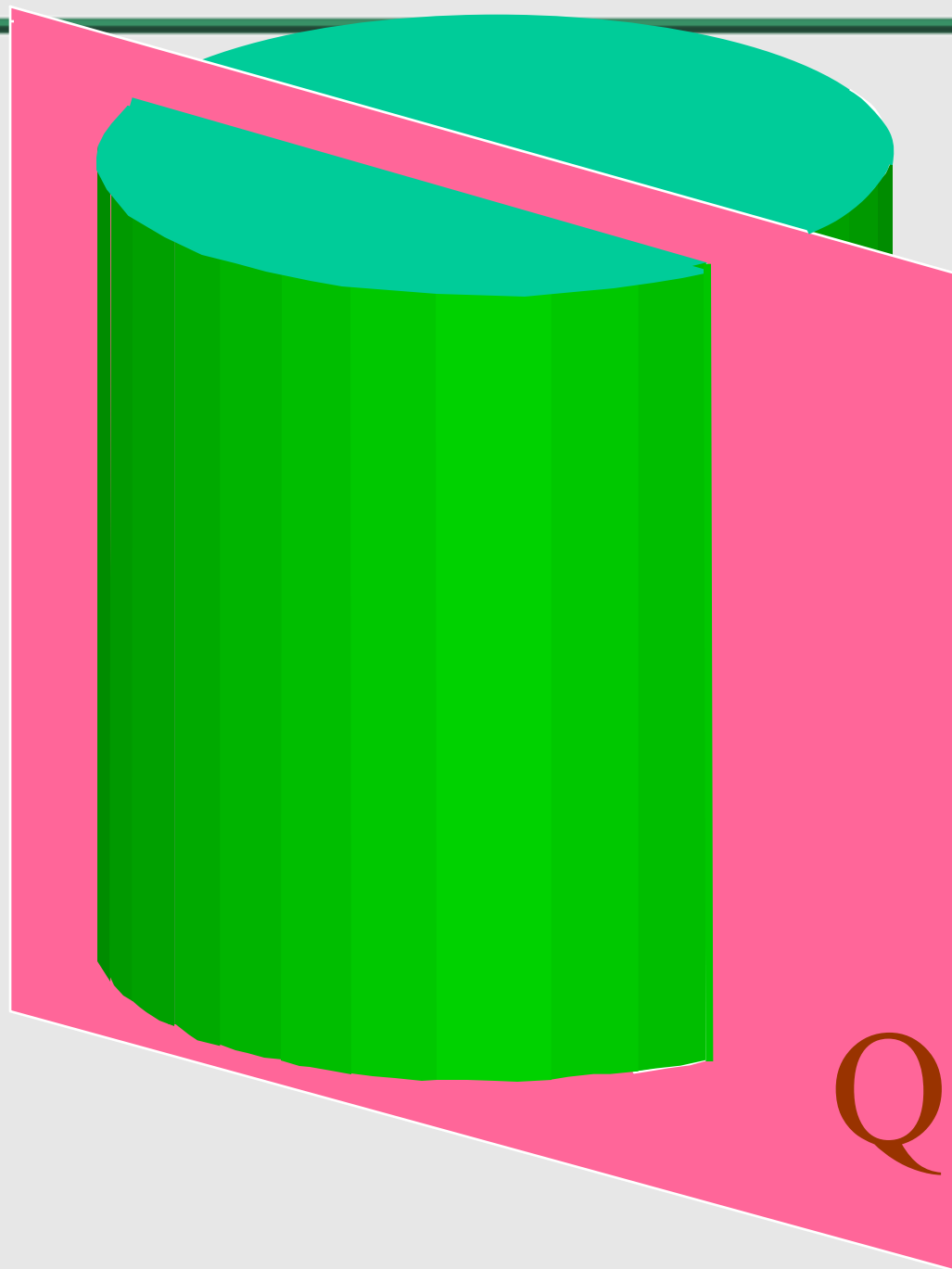
Two straight lines

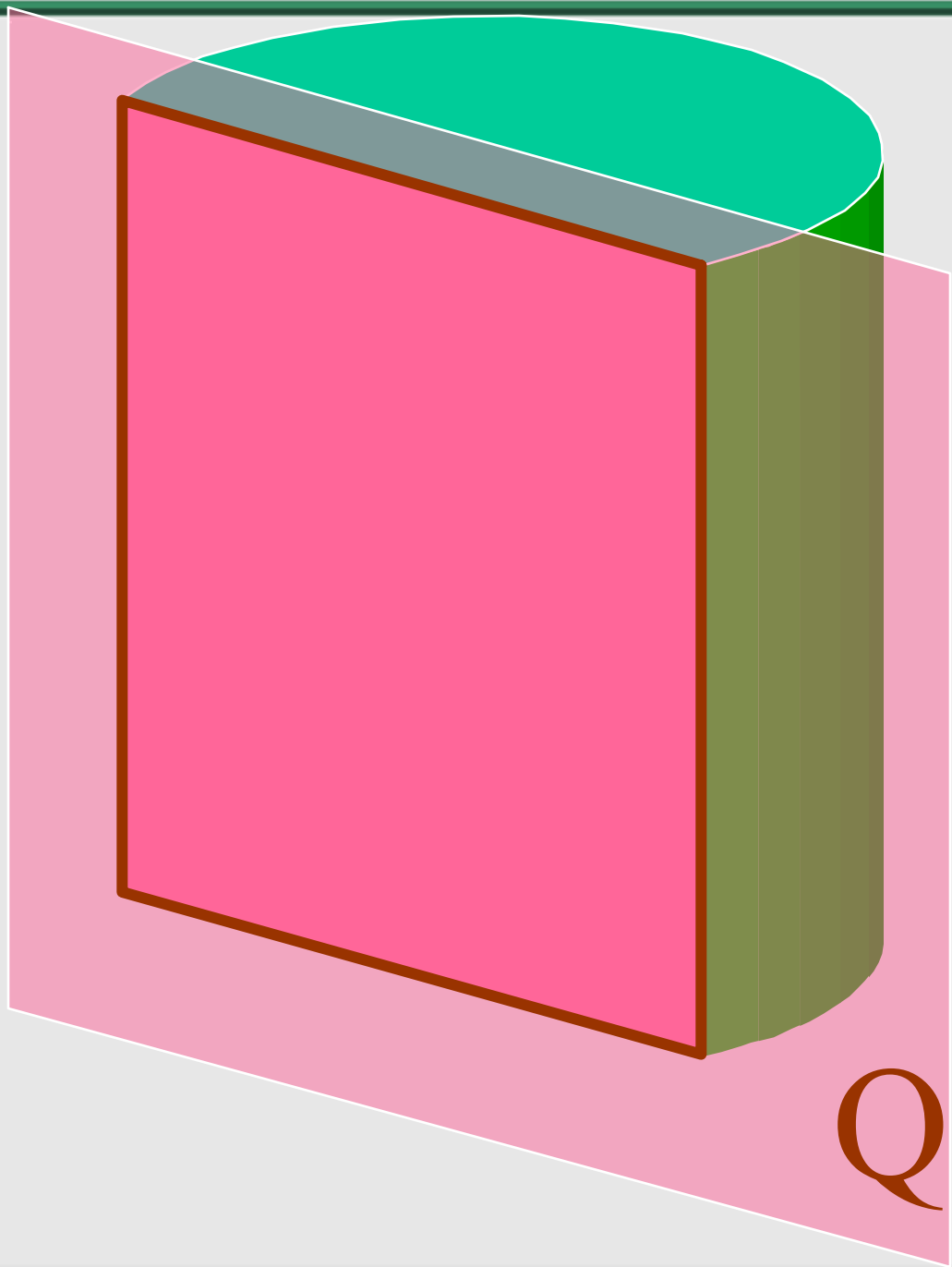
$$T(T_v) \cap \Phi =$$

Circle

$$R(R_v) \cap \Phi =$$

Ellipse



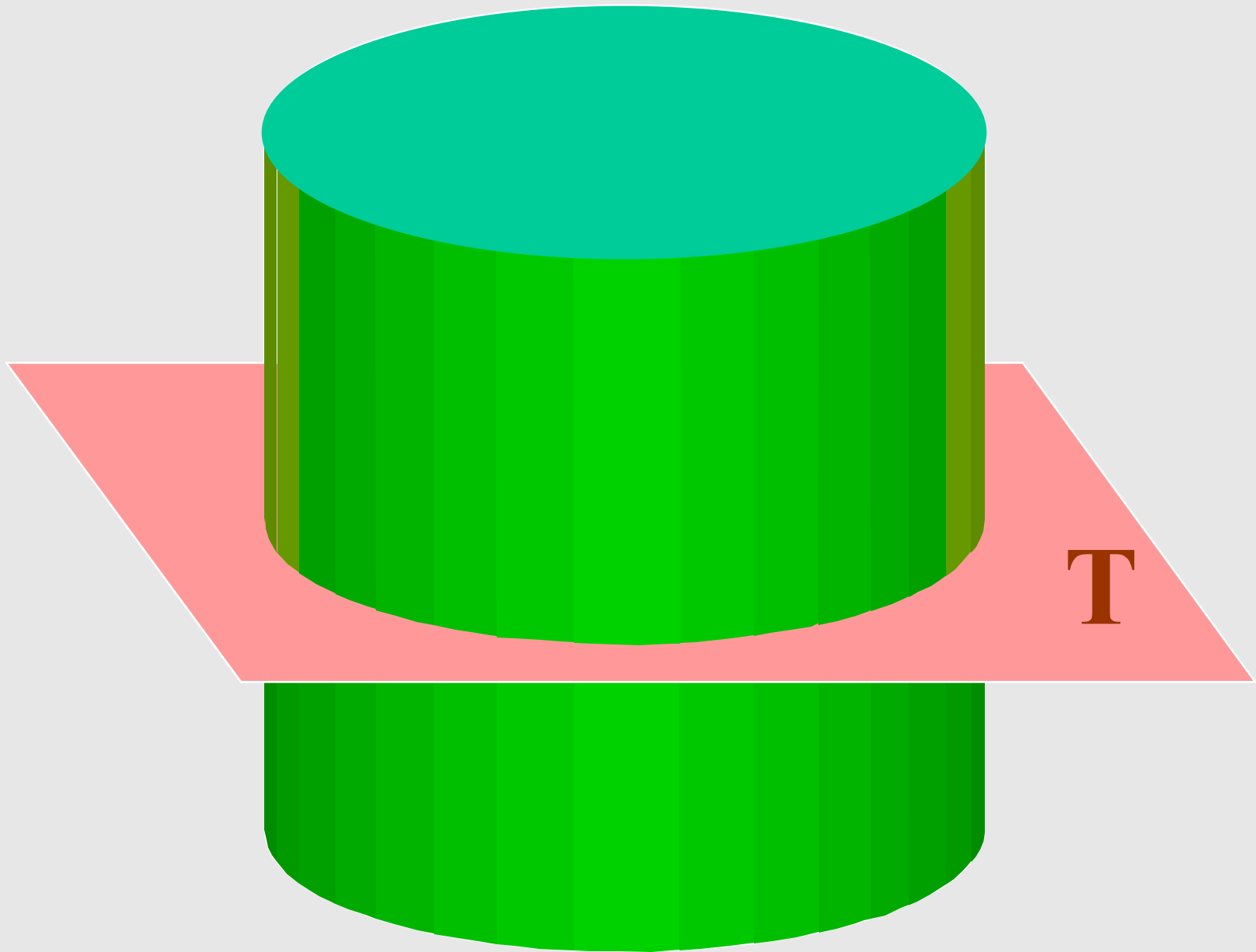


$$Q(Q_v) \cap \Phi =$$

Two straight lines

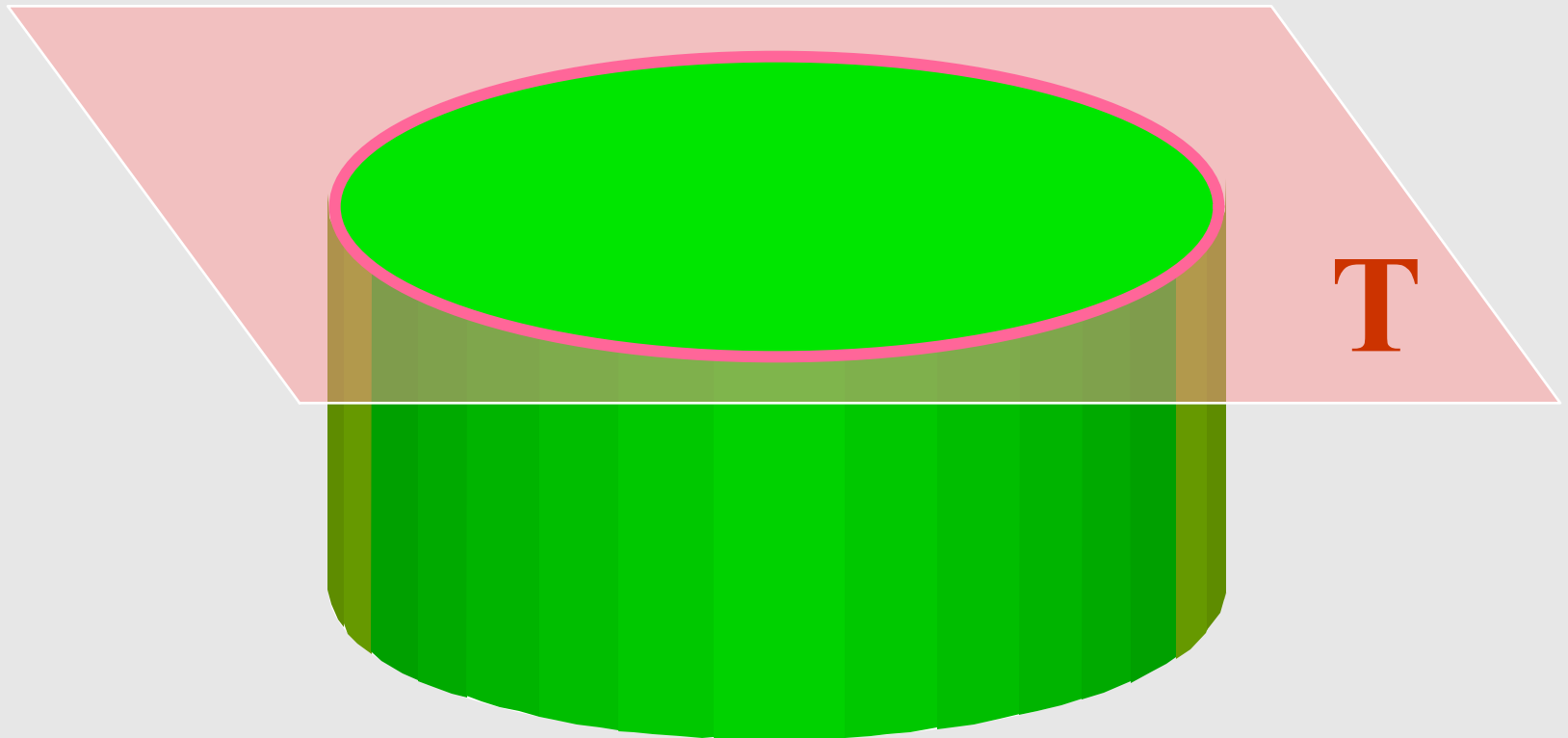
Section - a rectangular

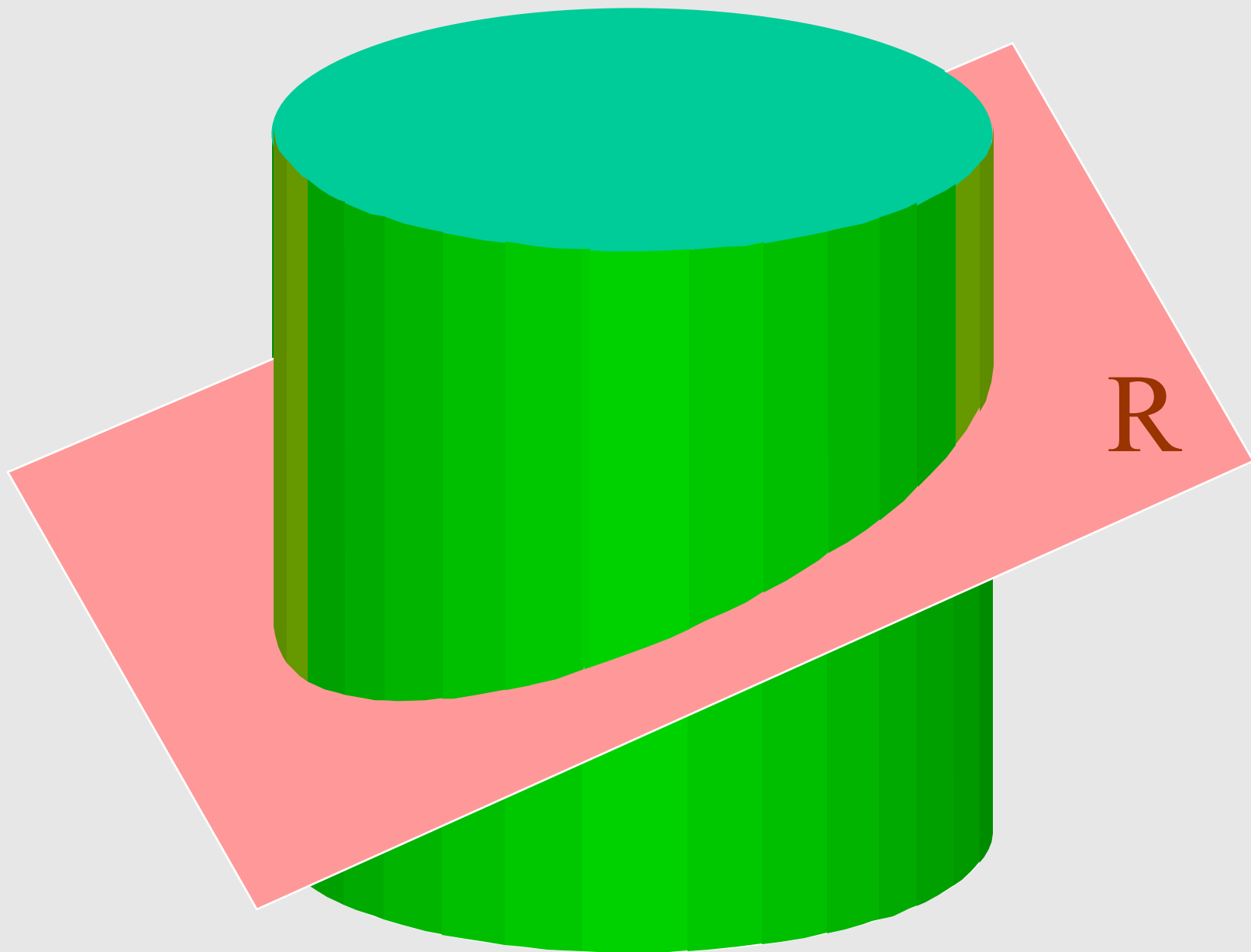
Q





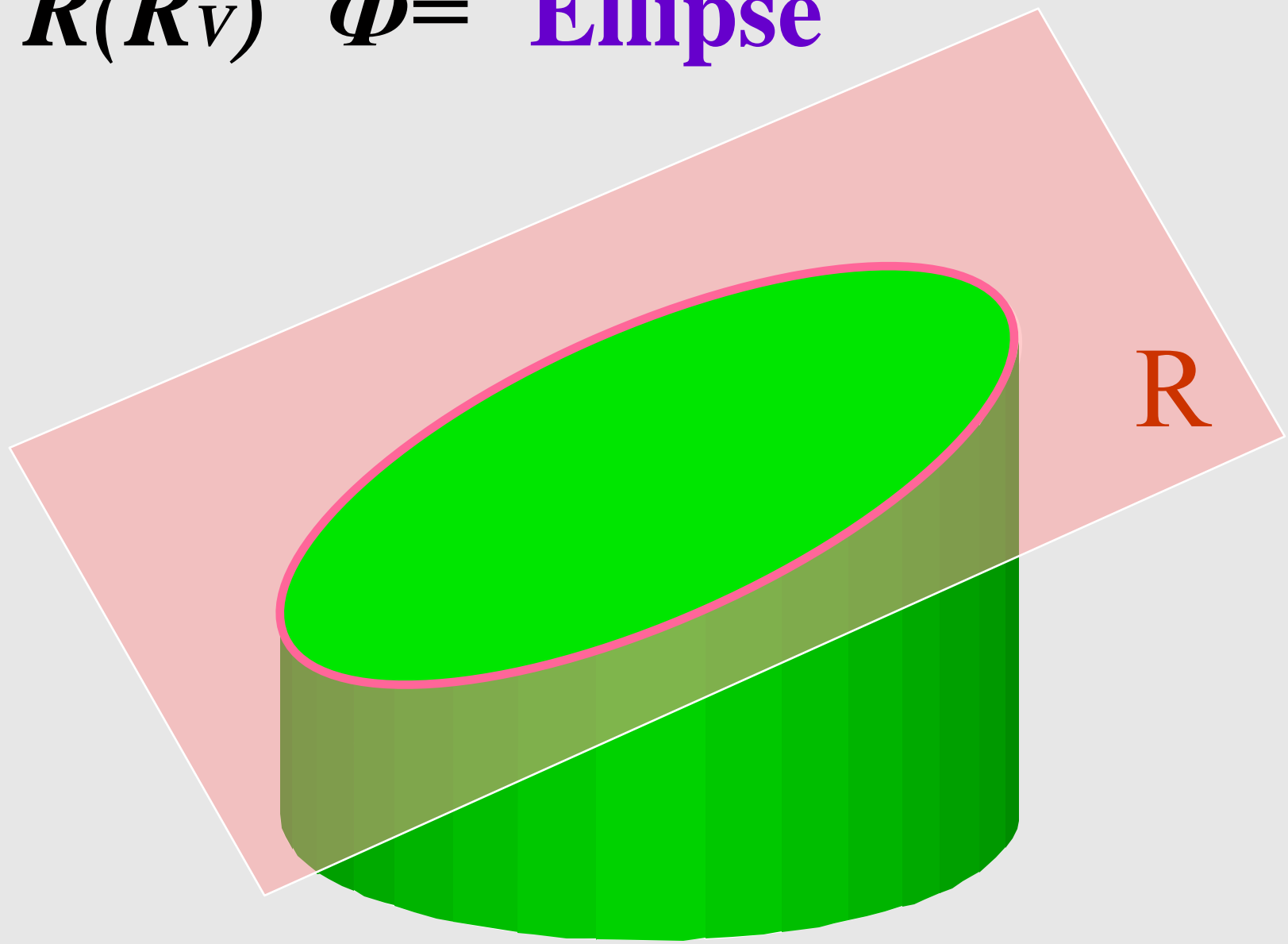
$$T(T_v) \cap \Phi = \text{Circle}$$







$R(R_v) \quad \Phi =$ **Ellipse**



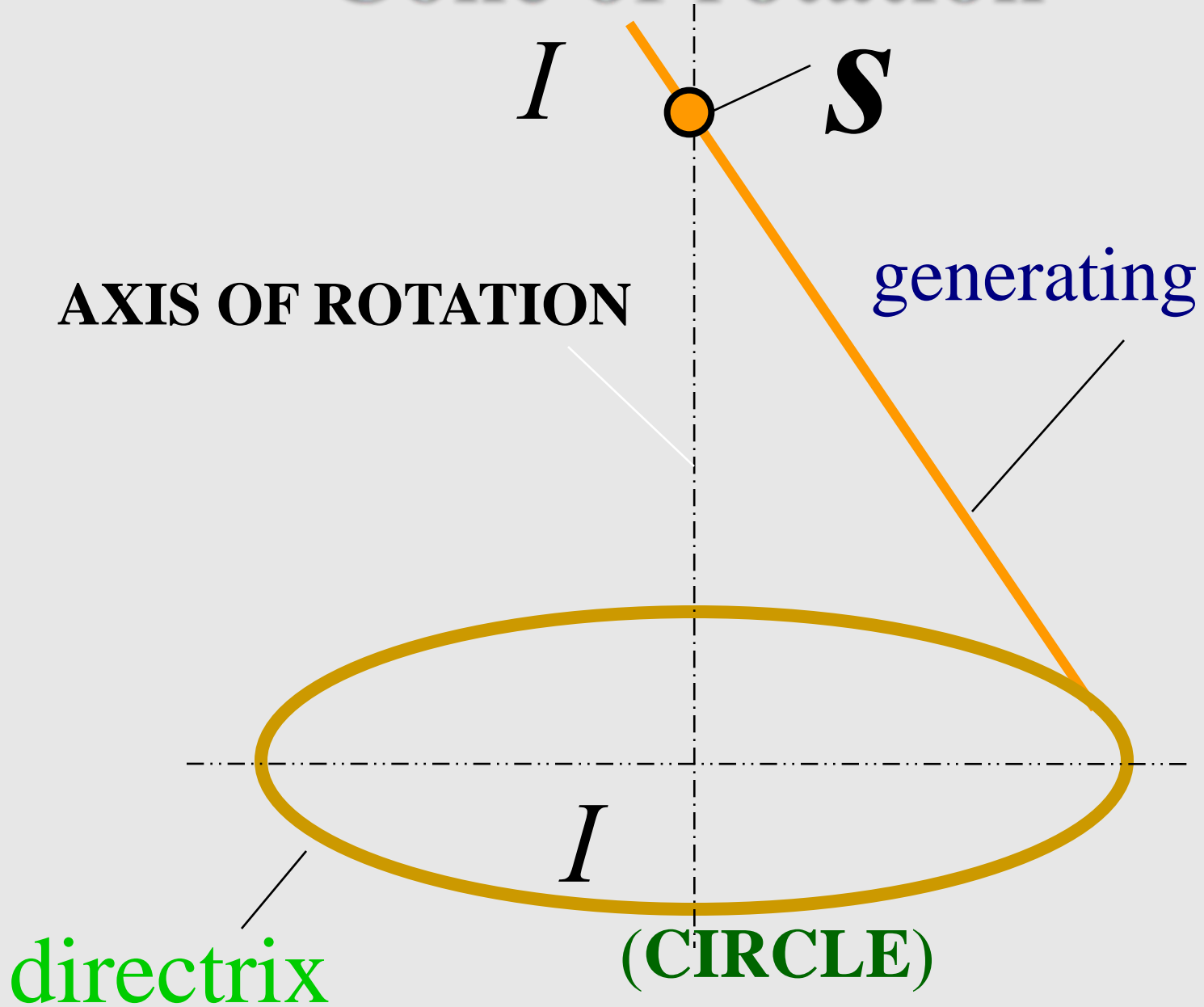


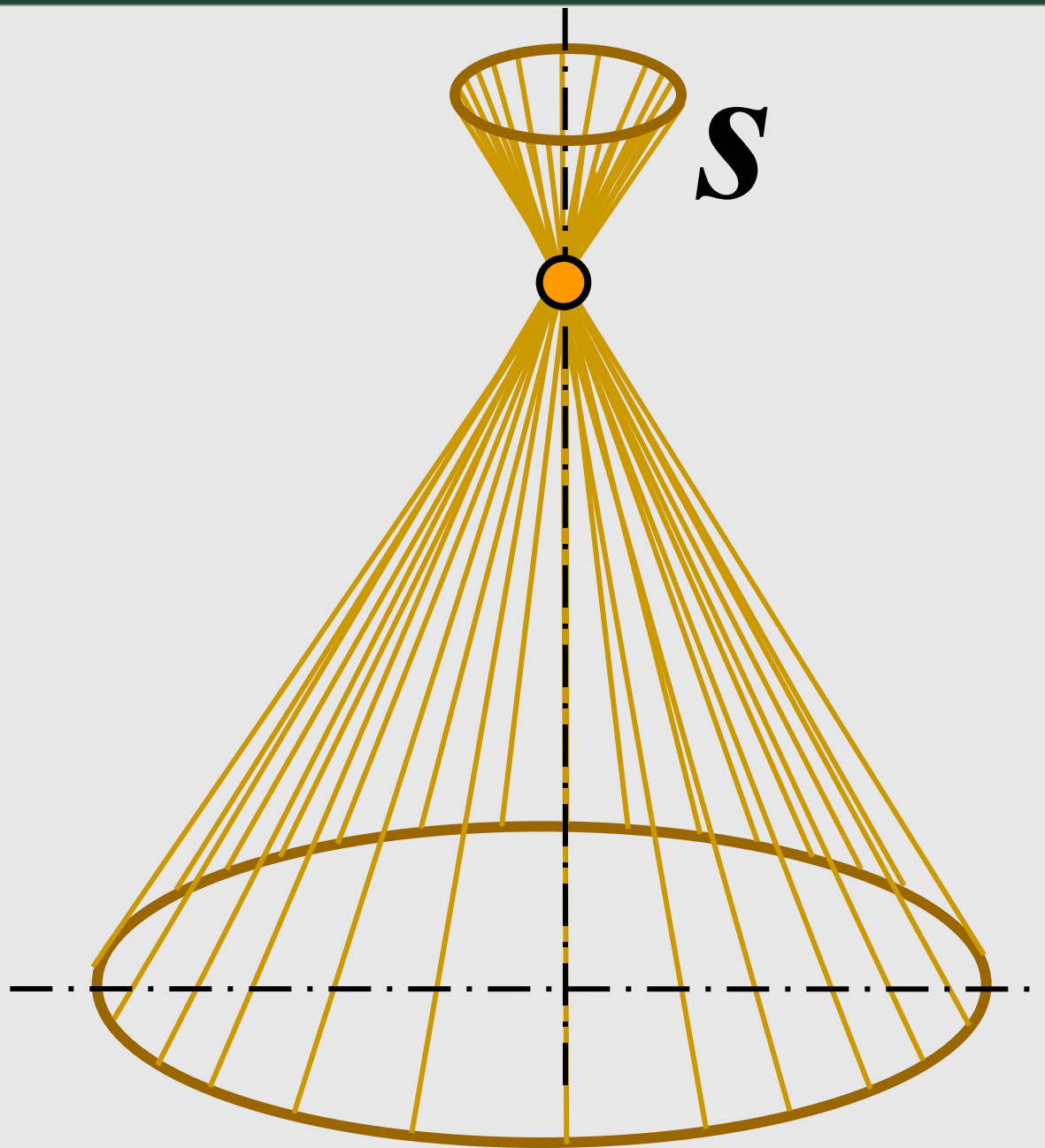
Rotation Surfaces

Cone of rotation - this is a surface produced by rotation of the line L round the axis I intersecting it



Cone of rotation







Cone of rotation **Cut by a
Plane**



Depending on the direction of a cutting plane, different lines, called the lines of conical sections, may be obtained in the section of a rotation cone.

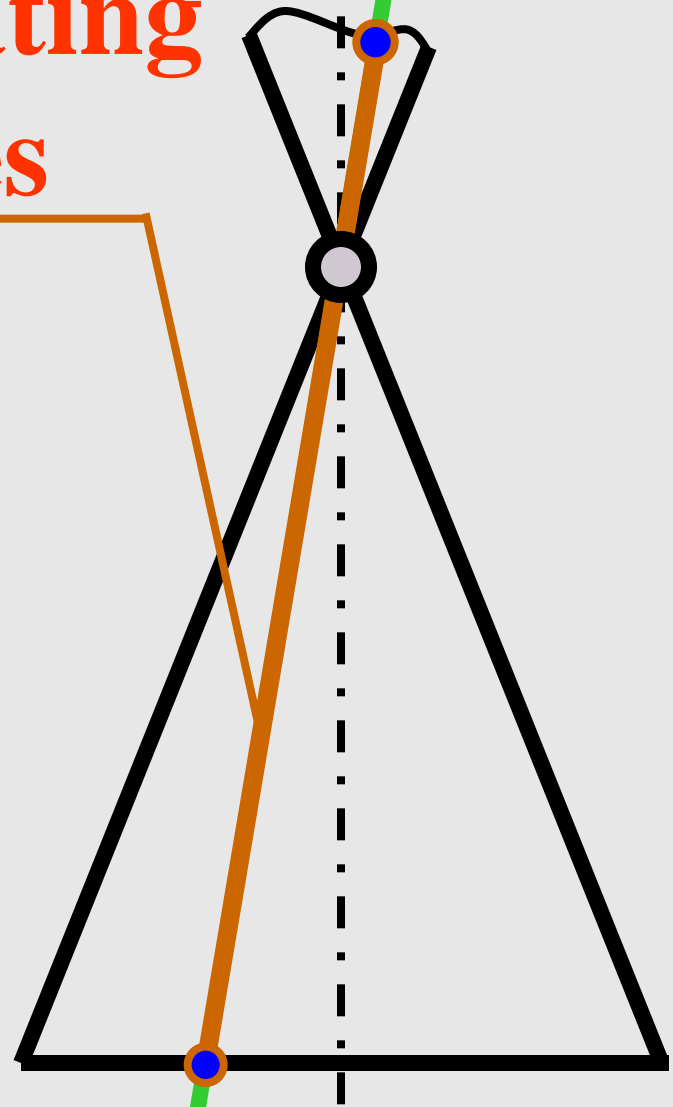
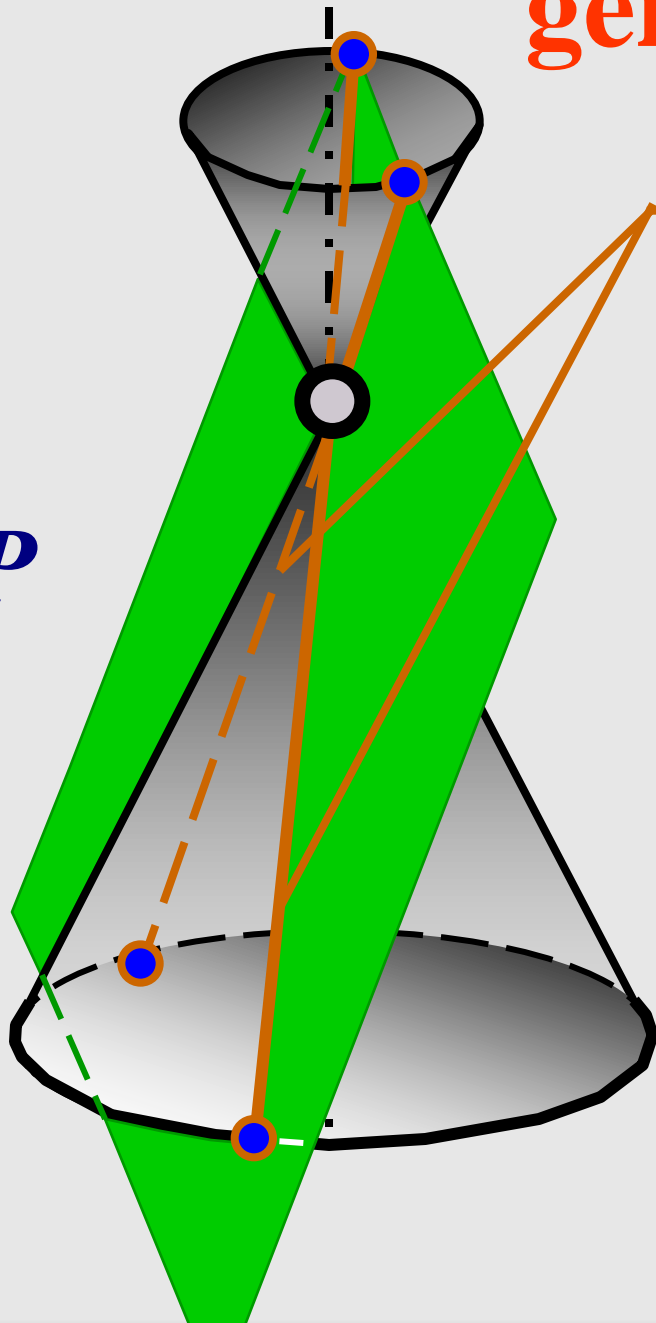


If a cutting plane passes through a vertex of a cone, we get in its section a pair of generating lines (triangle)

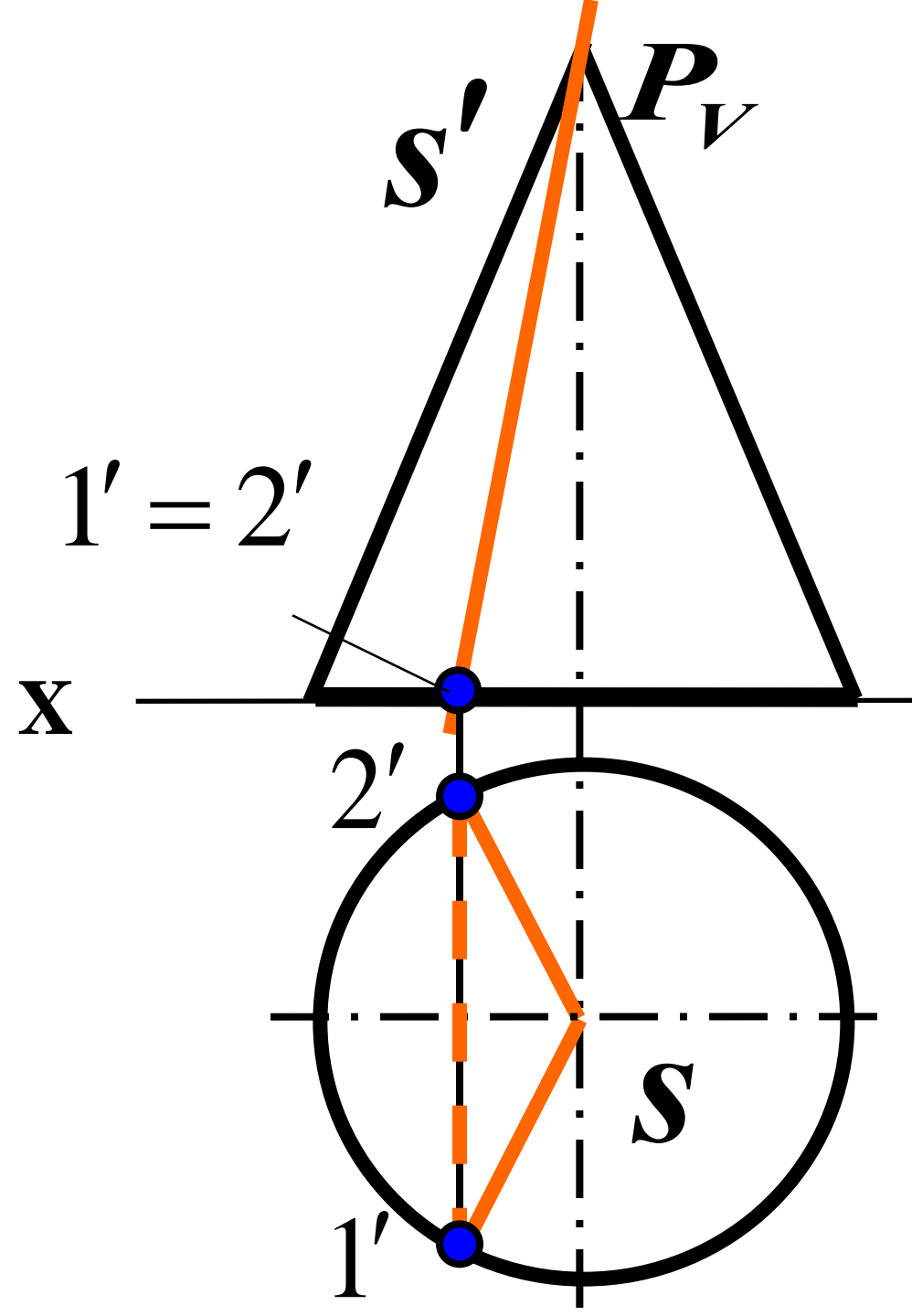


generating
lines

P

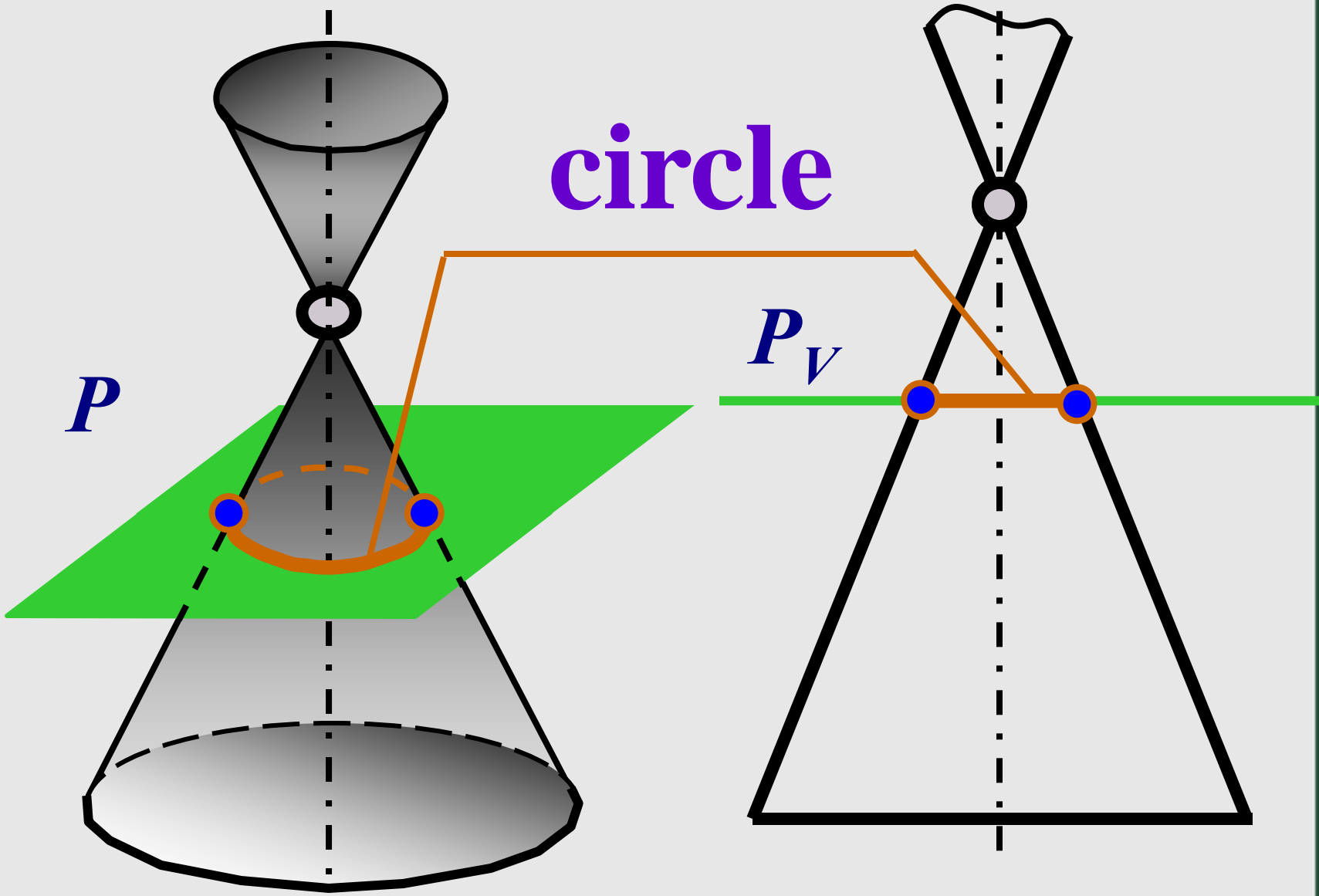


P_V






As a result of intersection of a cone with a plane **perpendicular to the cone axis**, a **circle** is obtained

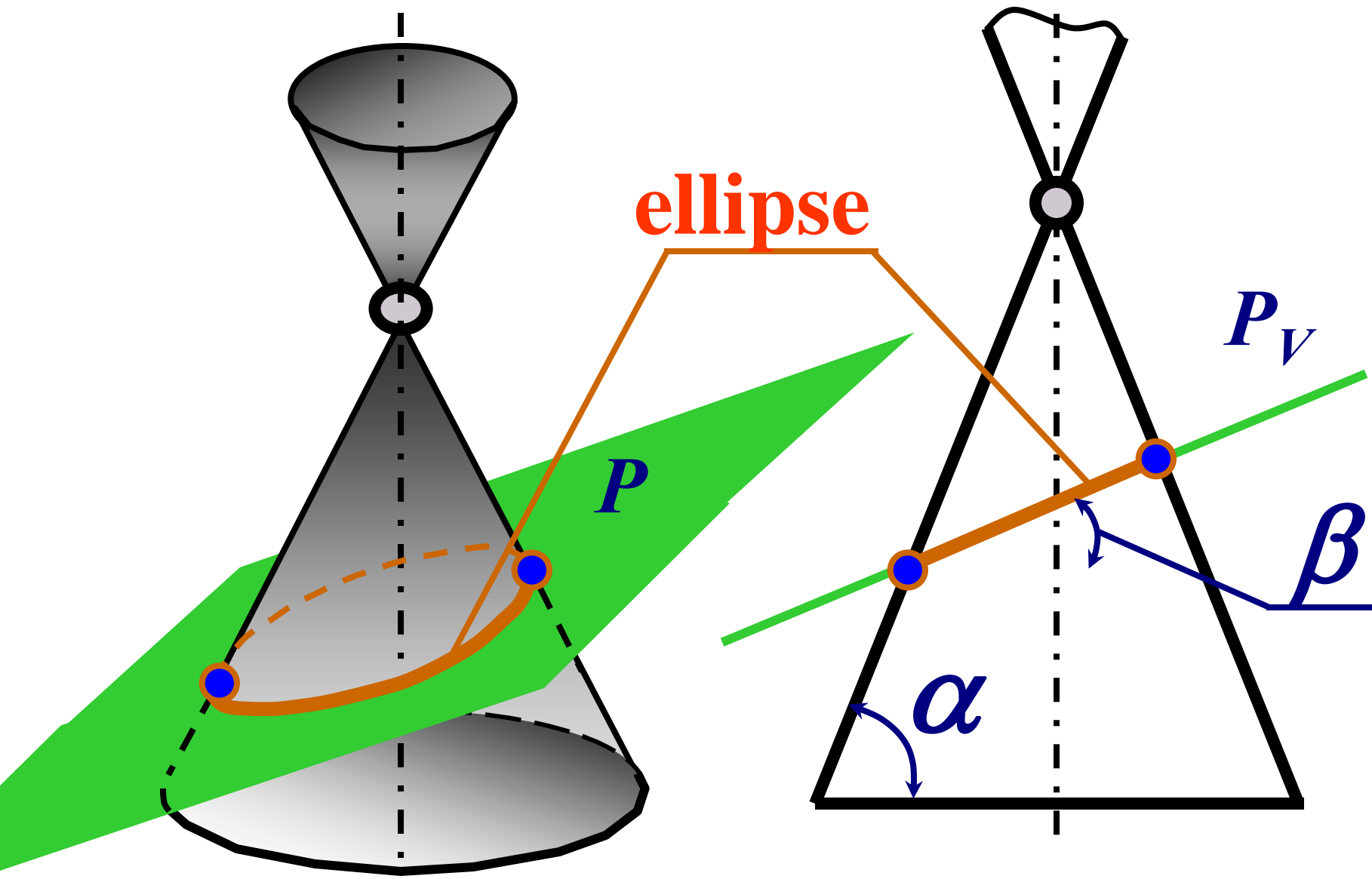




If a cutting plane is **inclined to the rotation axis of a cone** and does not pass through its vertex, **an ellipse**



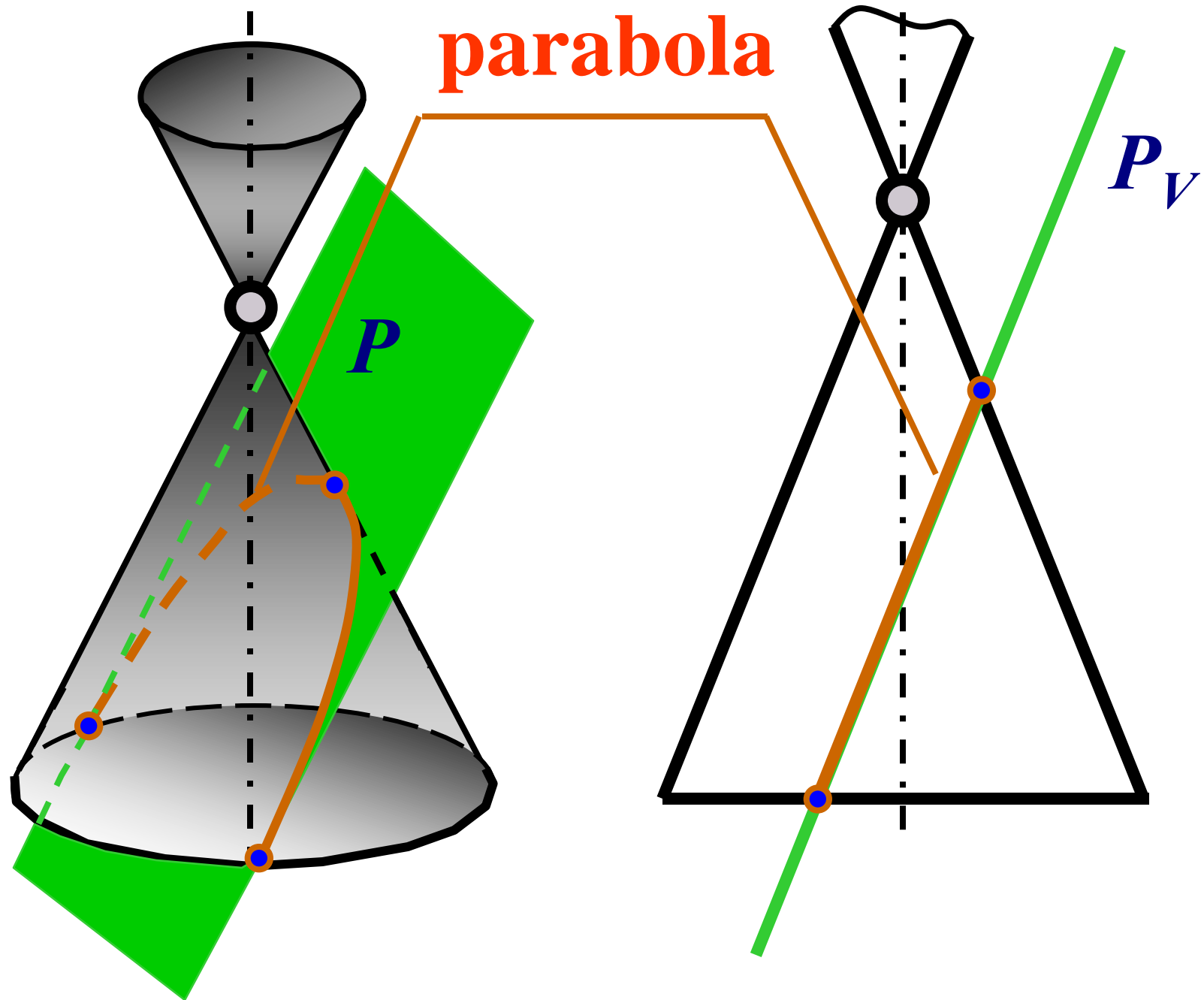
An ellipse is obtained when the inclination angle β is less than the inclination angle α of the cone generatrix to its base ($\beta < \alpha$), that is when a plane cuts all generating lines of a given cone

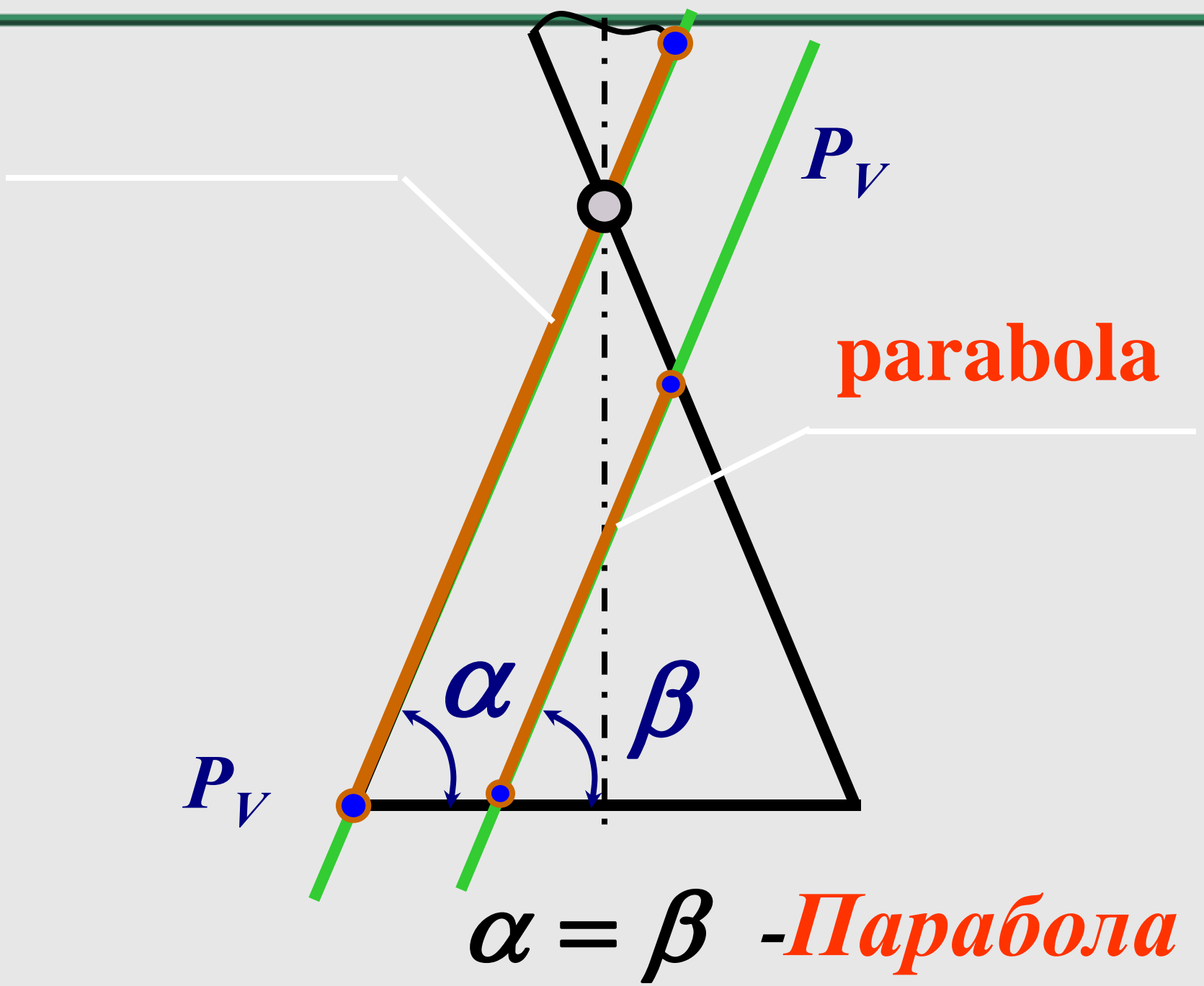


$\alpha > \beta > 0$ - ellipse



In case the angles α and β are equal, i.e. a cutting plane is parallel to one of the generatrices of the cone, a **parabola** is obtained in the section.





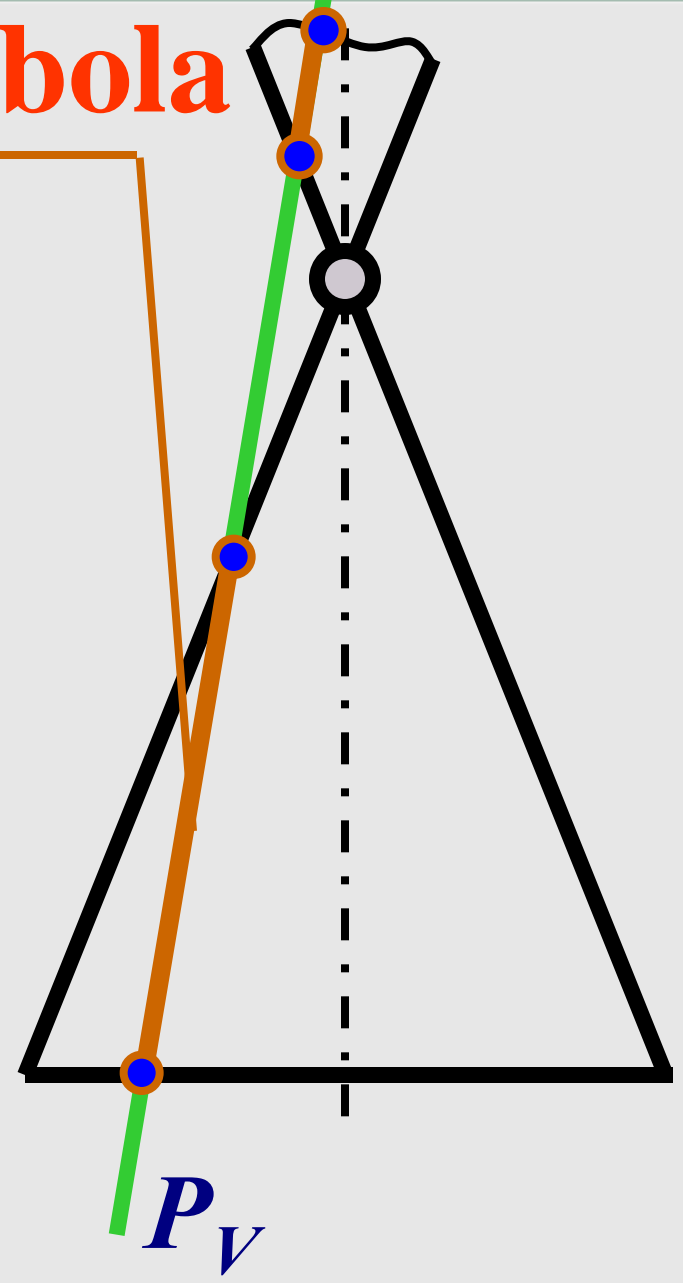
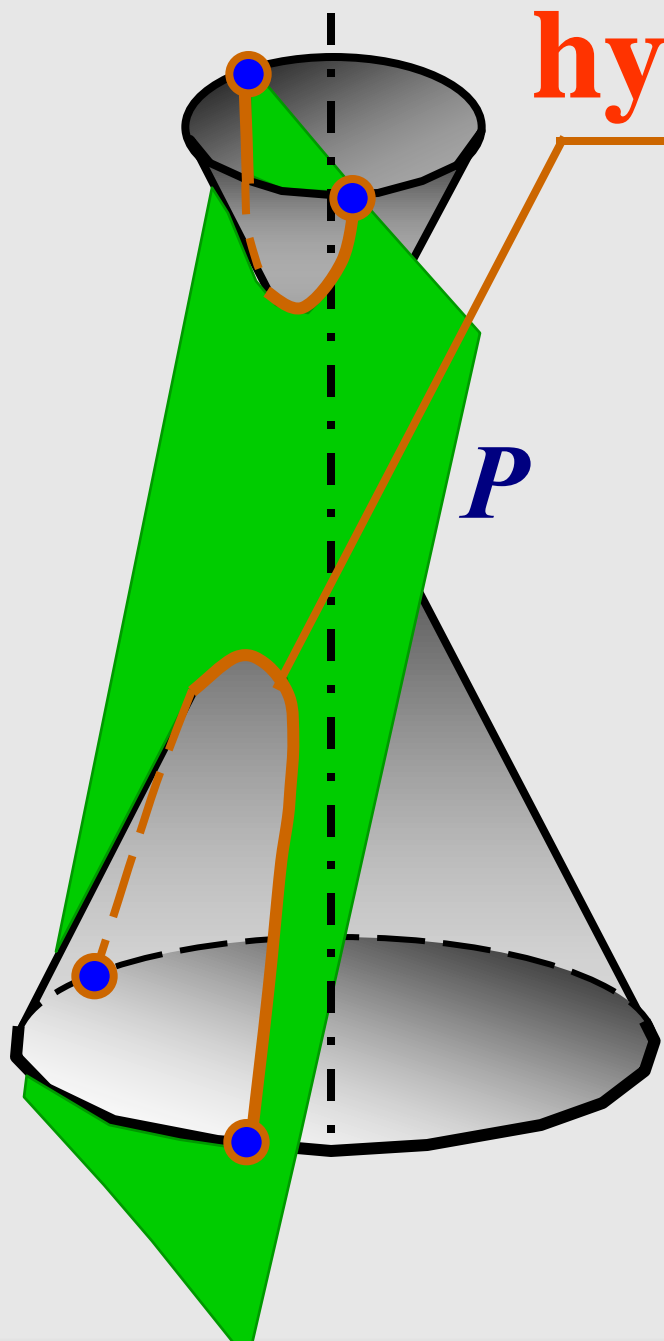


If a cutting plane is inclined at an angle which changes in the following limits - $90^\circ \geq \beta > \alpha$, a **hyperbola** is obtained in the section.

The cutting plane here is parallel to **two generating lines** of the cone.



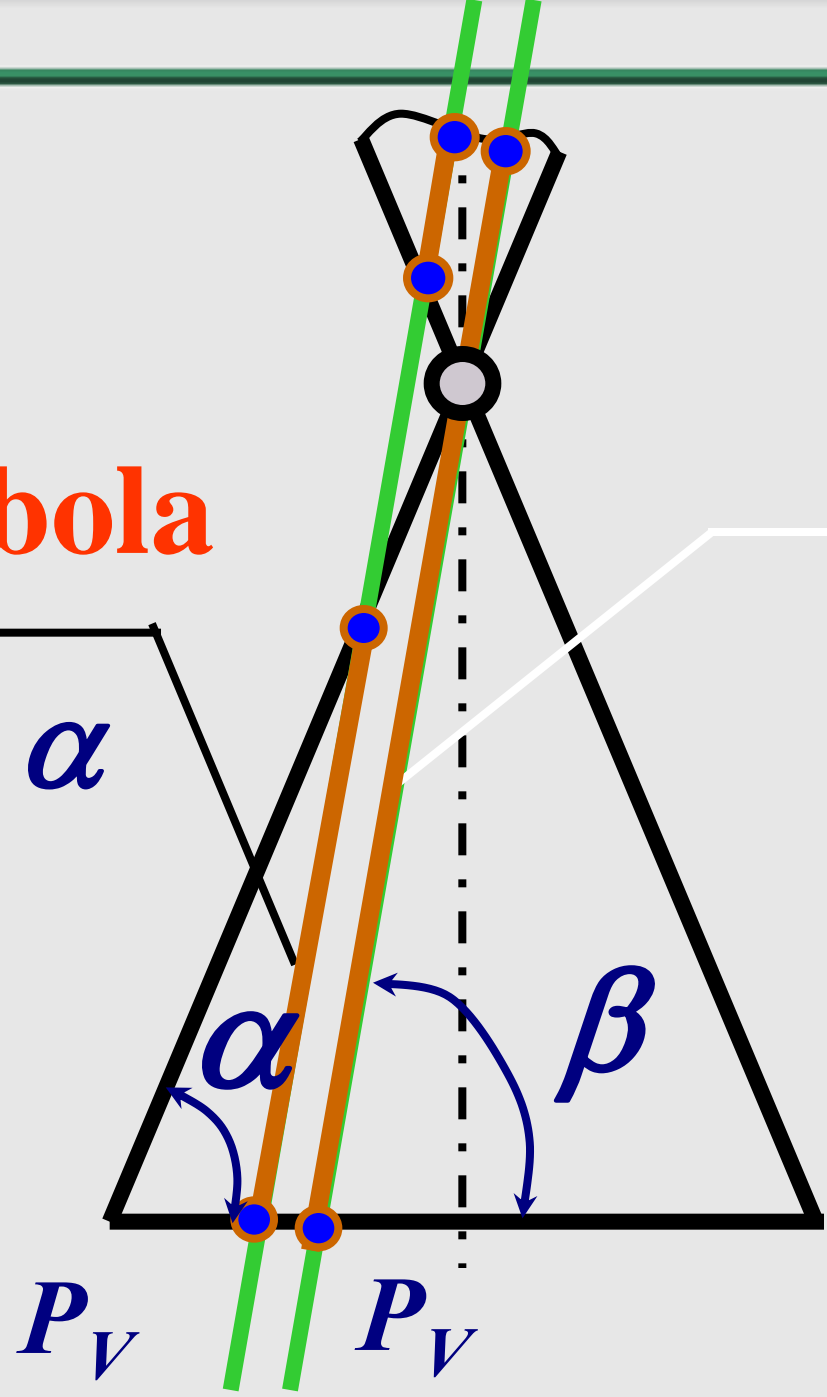
hyperbola





hyperbola

$$90^\circ \geq \beta > \alpha$$



α

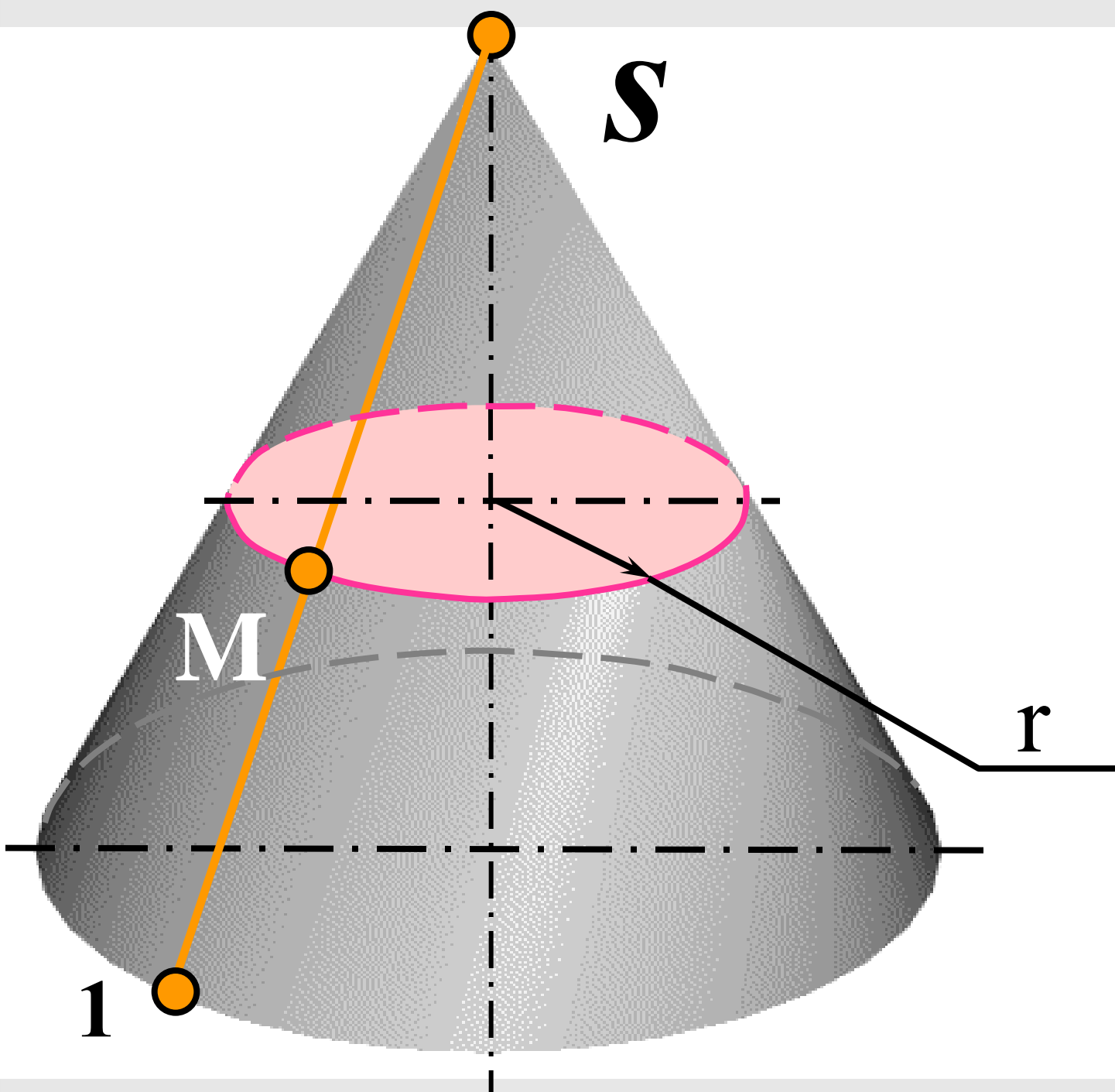
β

P_V

P_V



A Point on the *Cone* of rotation .
If the point belongs to a surface
of a cone it is necessary to
construct a circle or forming
which belong to a cone





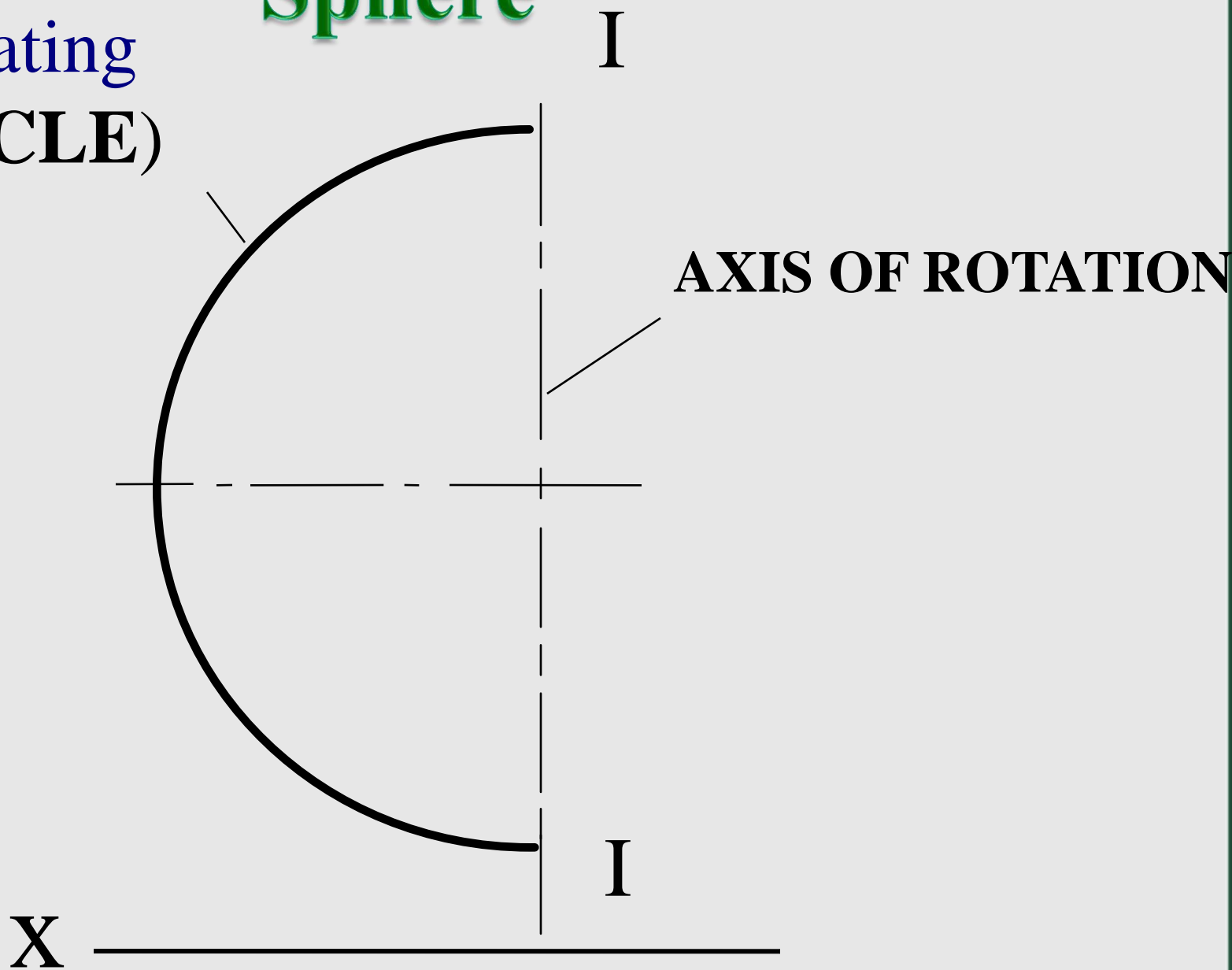
Rotation Surfaces

Sphere - this is a surface produced by rotation of a circle round its diameter



Sphere

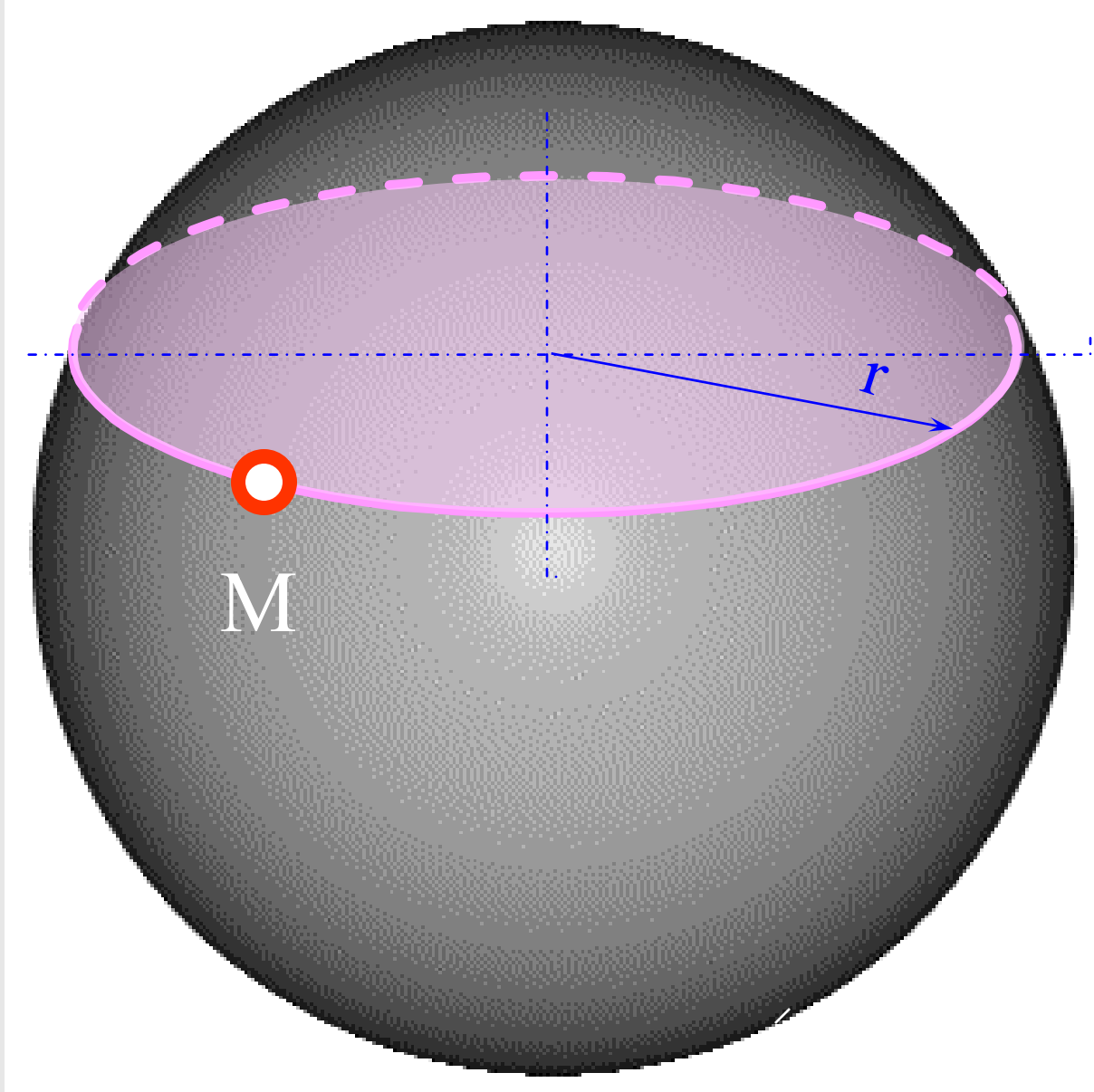
generating
(CIRCLE)

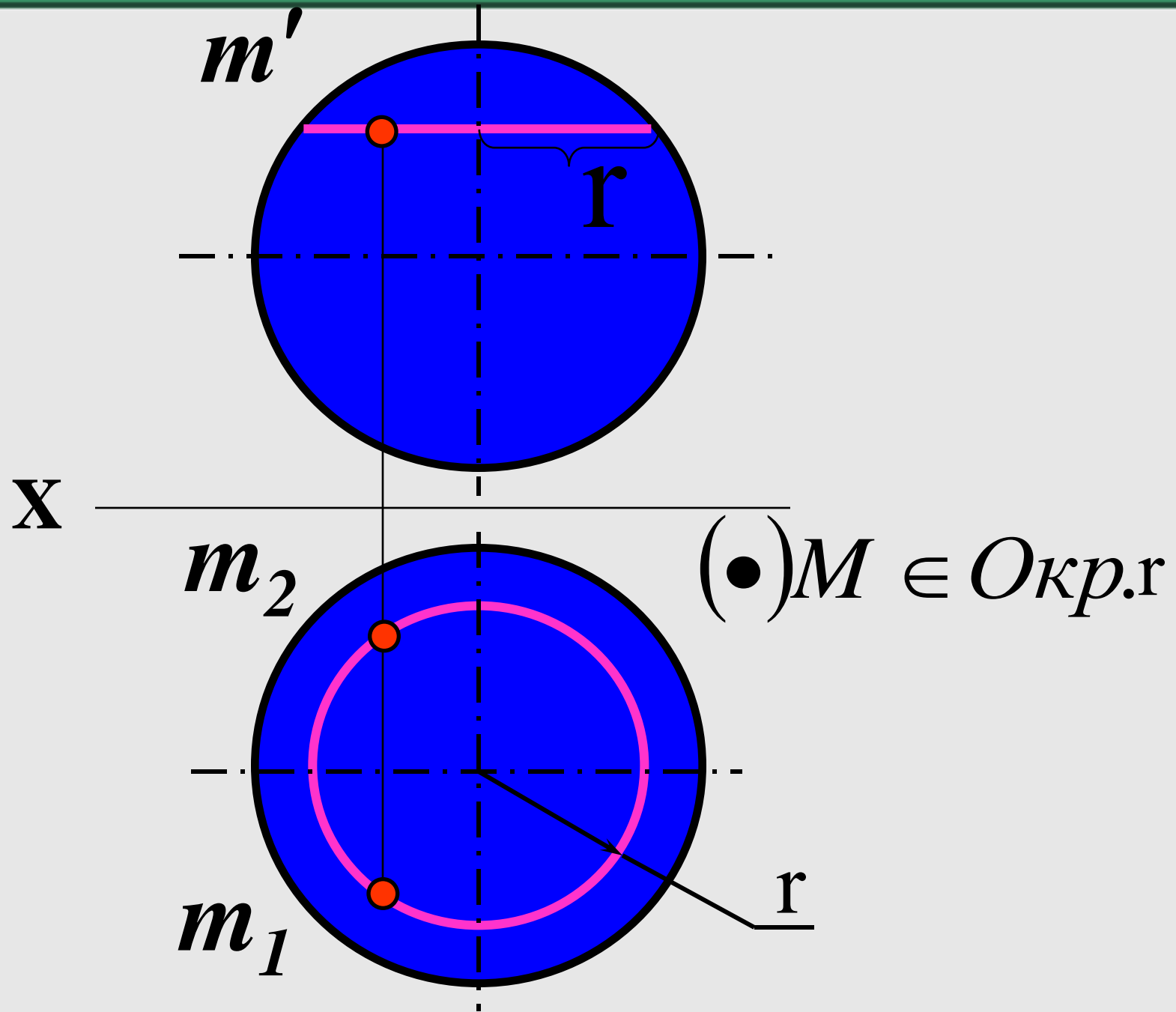




A Point on the Sphere.

If the **point belongs** to a **Sphere** then it is necessary to **construct** a **circle** which **belong** to a **Sphere**







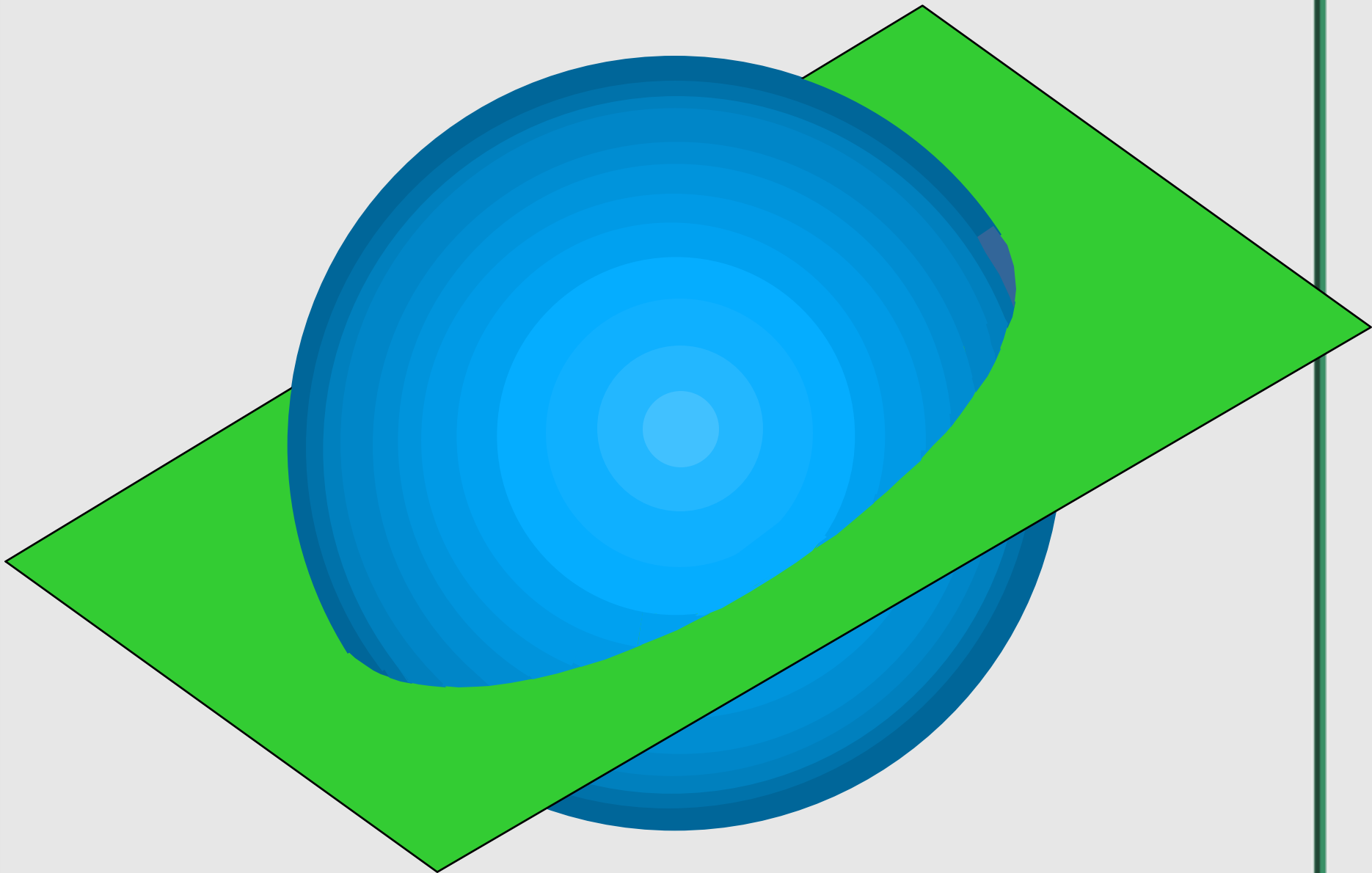
Sphere Cut by a Plane

A plane intersects a sphere always
in a **circle**.



This circle may be projected as:

- *a straight line* if the cutting plane is **perpendicular** to the projection plane
- *a circle* of radius equal in length to distance from the axis of the sphere rotation to the outline if the cutting **plane is parallel** to the projection plane
- *an ellipse* if the cutting plane is **not parallel** to the projection plane





circle

