Tomsk Polytechnic University

# DESCRIPTIVE GEOMETRY
# ENGINEERING GRAPHICS

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# Lecture 3

# REPRESENTATION OF A PLANE IN A DRAWING

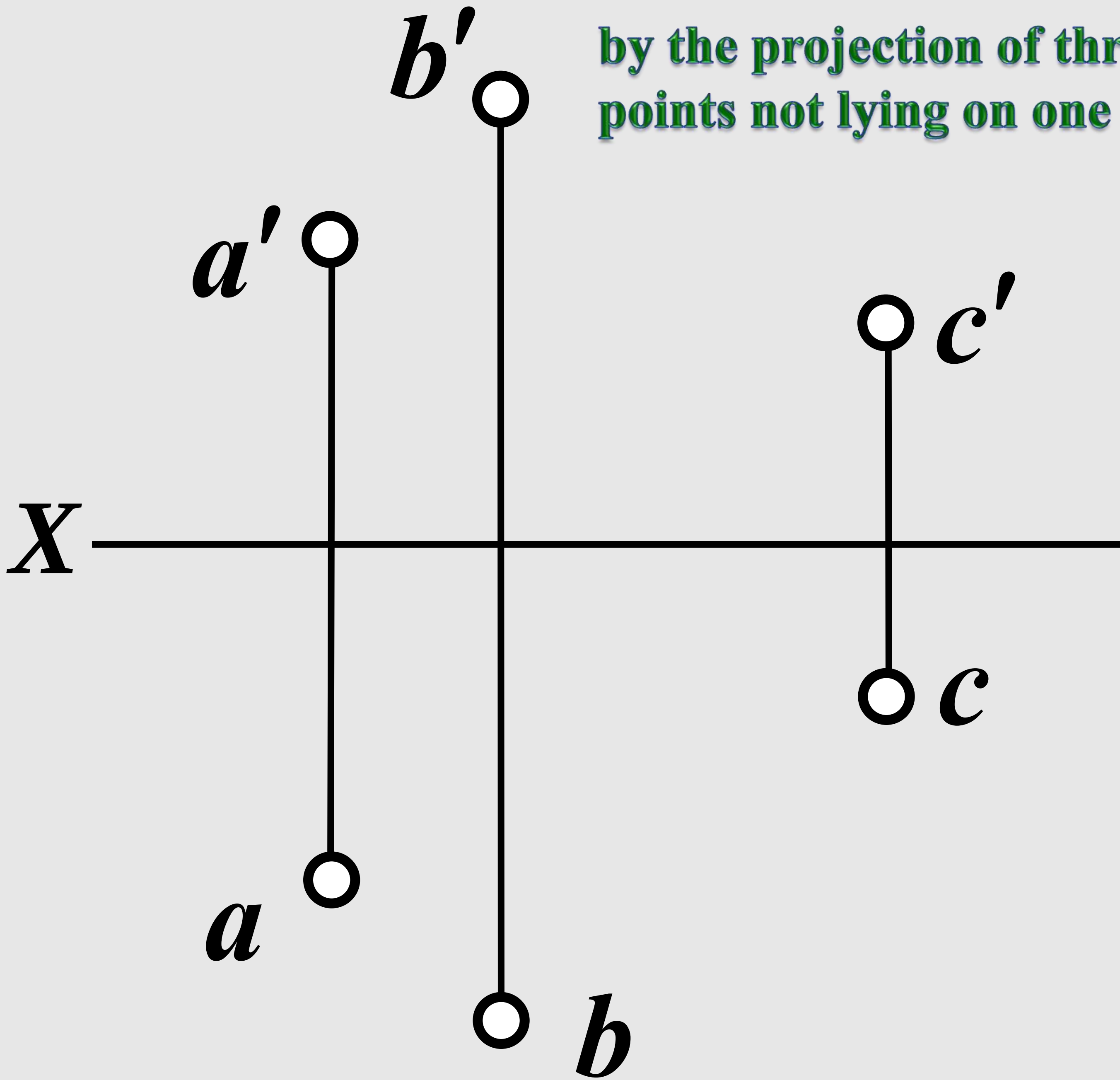
# Plan

1. Ways of Specifying a Plane
2. The Point and the Line in the Plane
3. The Position of a Plane Relative to the Projection Planes
4. The Principal Lines of the Plane
5. The Relative Positions of a Line and a Plane
6. Mutual Positions of the Planes

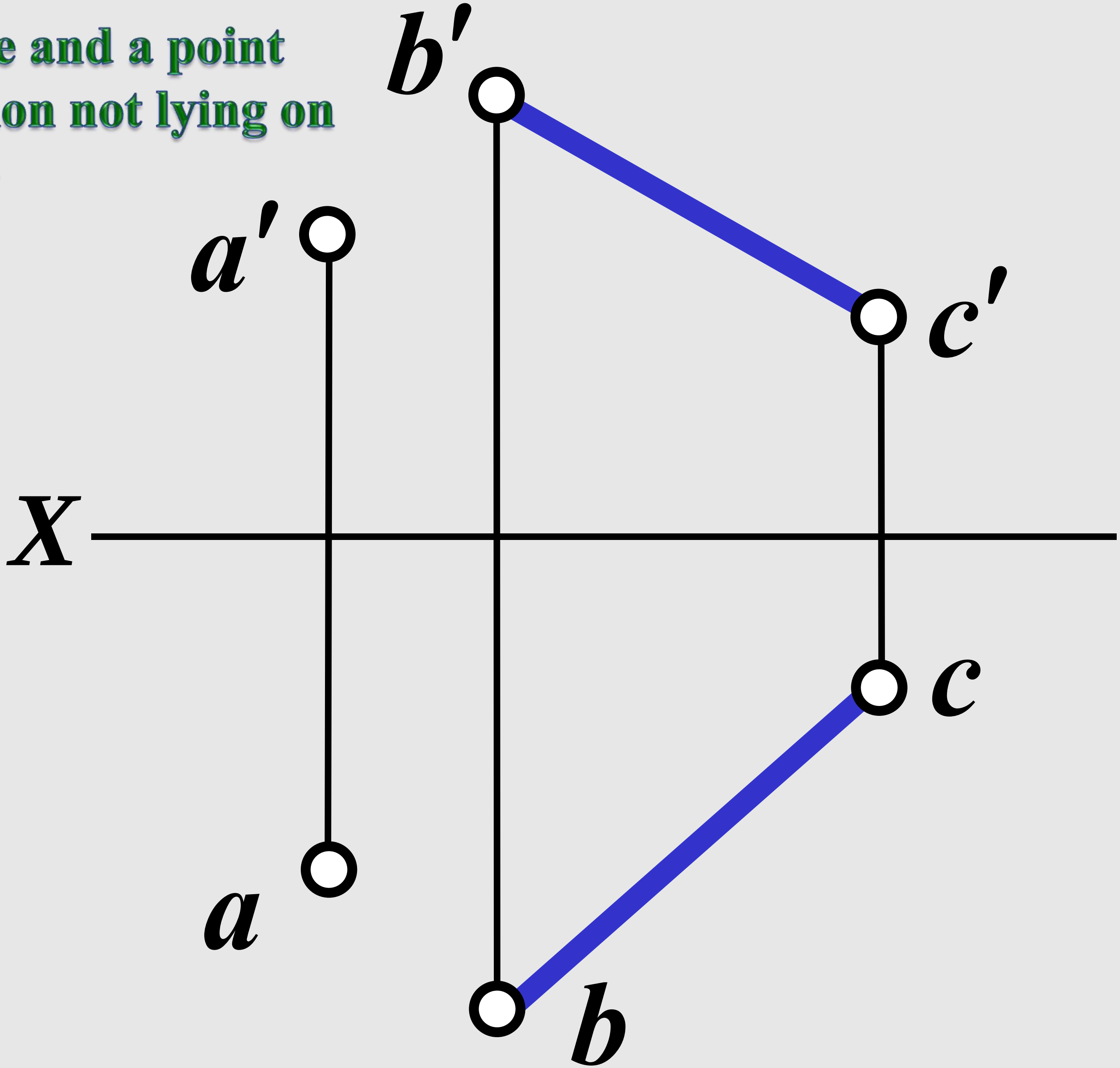
# Ways of Specifying a Plane

**The position of a plane on a drawing may be specified in one of the following ways:**

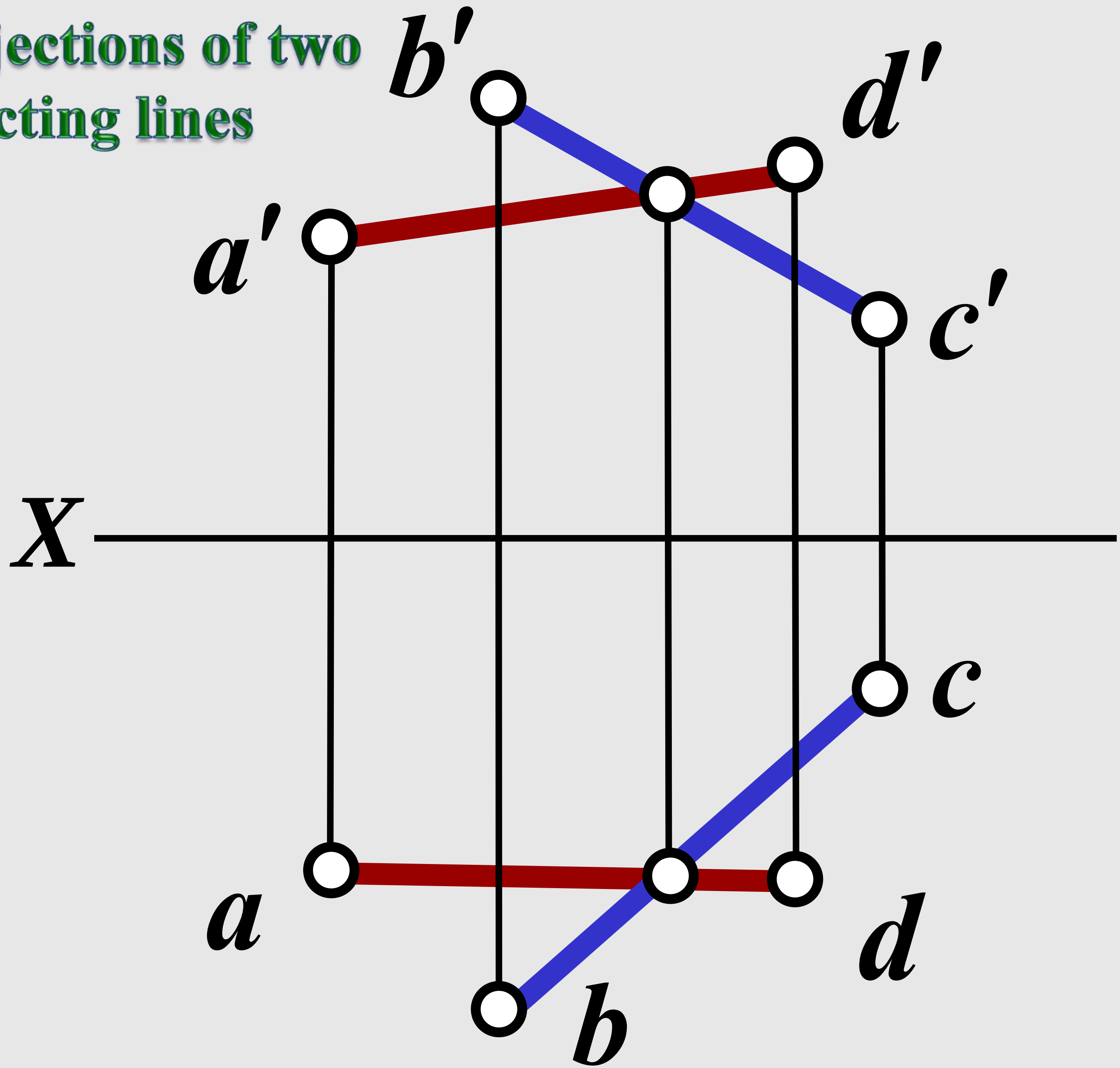
by the projection of three points not lying on one line;



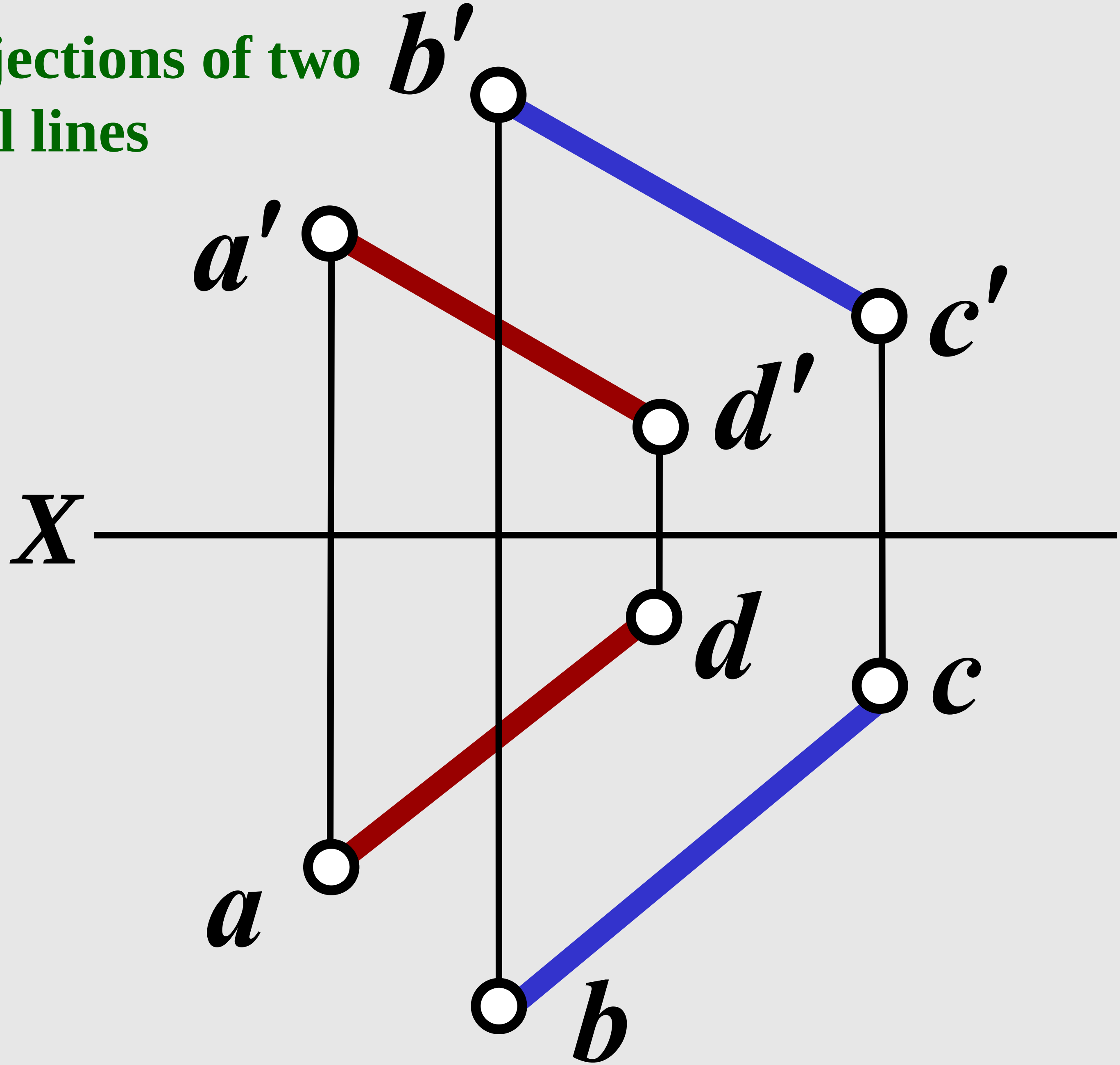
**by a line and a point projection not lying on one line**



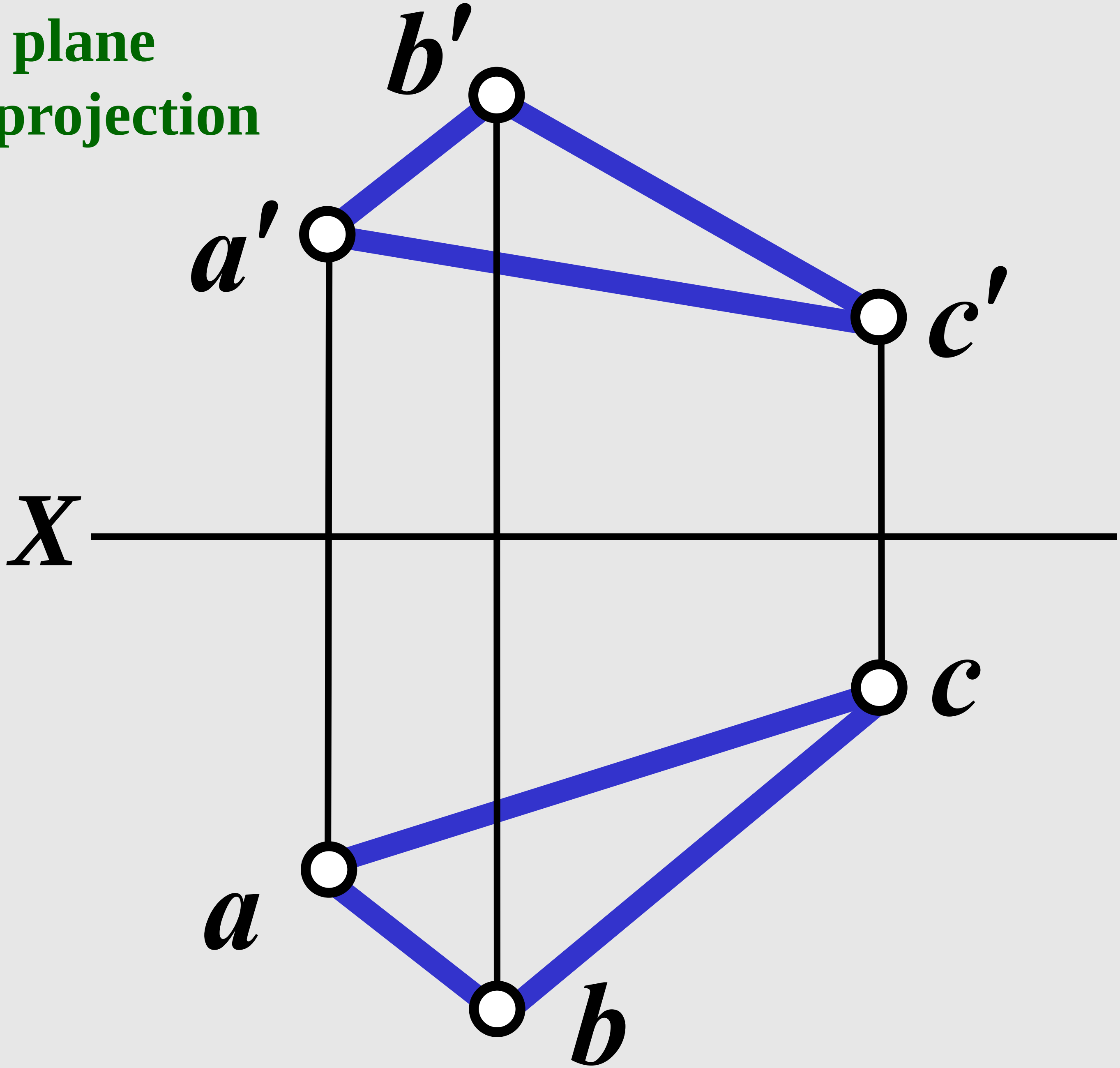
**by projections of two intersecting lines**



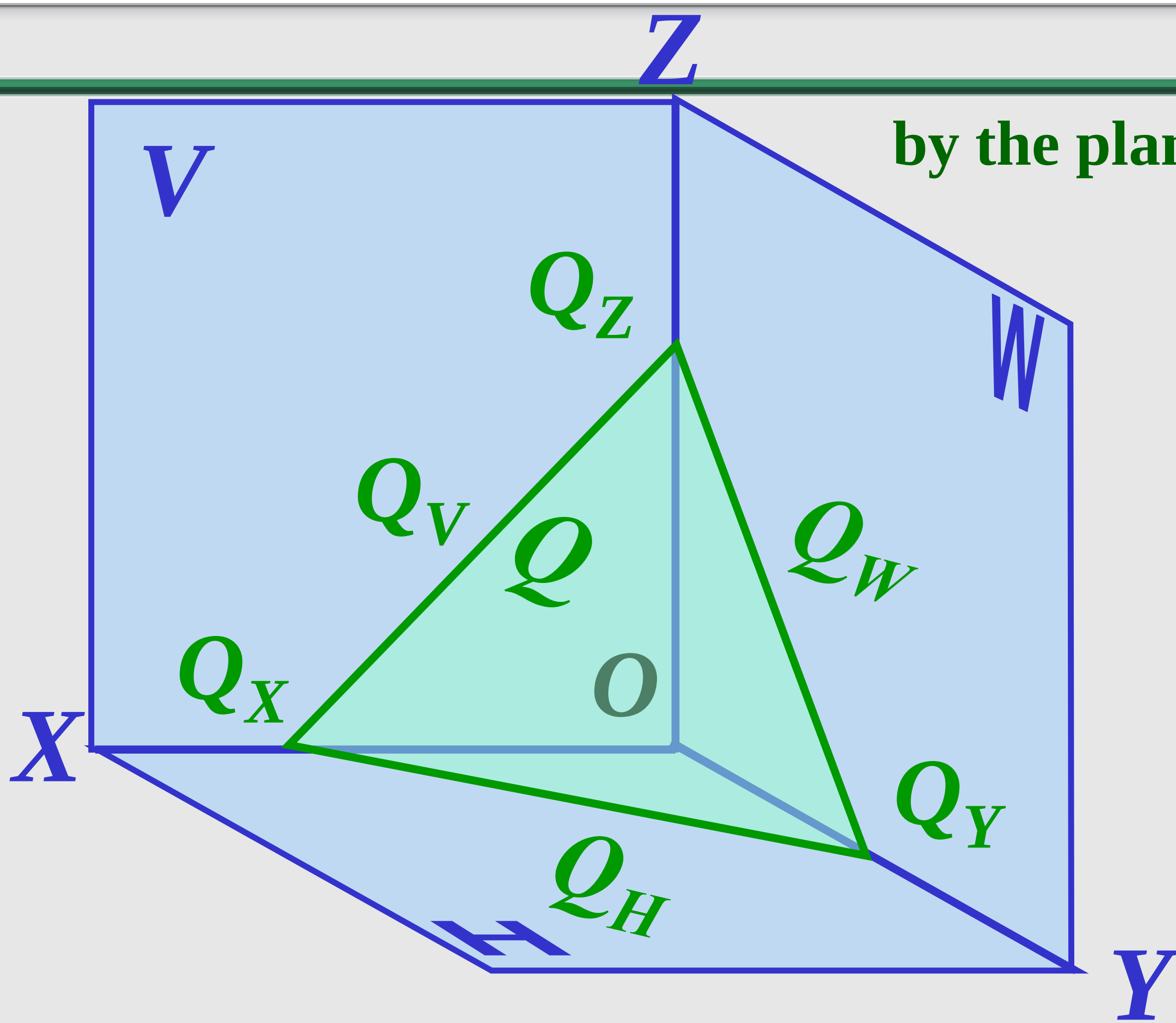
**by projections of two parallel lines**



**by any plane figure projection**



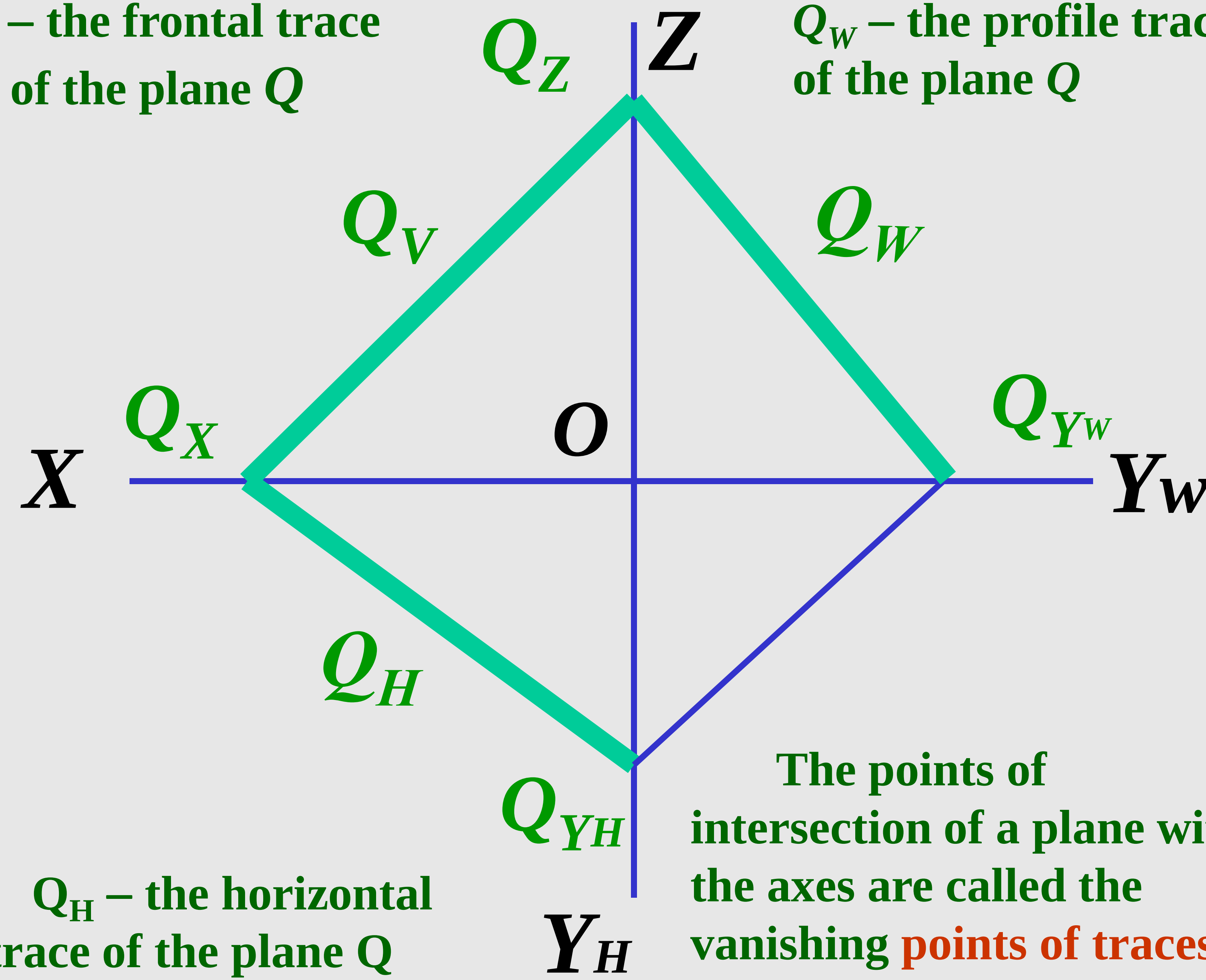
by the plane traces




# Trace of a plane-
# Line of crossing of a plane

**$Q_V$ – the frontal trace of the plane *Q***

**$Q_W$ – the profile trace of the plane *Q***




**$Q_H$ – the horizontal trace of the plane Q**

**The points of intersection of a plane with the axes are called the vanishing points of traces**

# The Point and the Line in the Plane

**A point belongs to a plane if it lies on a line contained in this plane.**

$$(\bullet)K \in \left(AB\right) \subset Q \Rightarrow (\bullet)K \in Q$$

**The straight line belongs to a plane, if:**

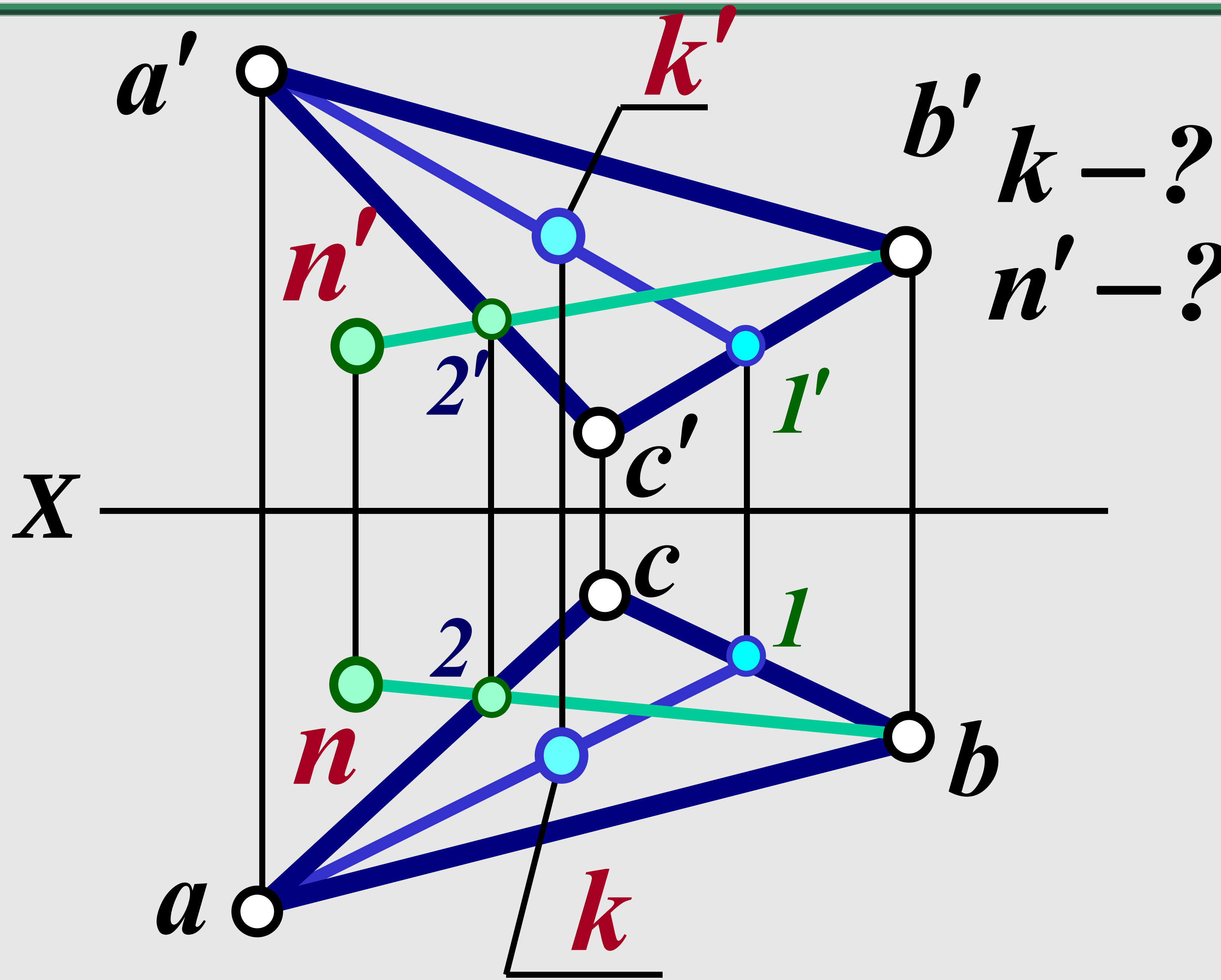
**1) It passes through two points laying in the given plane**

$$(\bullet)A \in Q \wedge (\bullet)B \in Q \Rightarrow (AB) \subset Q$$

**2) It passes through a point belonging to a plane, in parallel any straight line laying in this plane**

$$(\bullet)A \in Q \wedge \left(AB // CD\right)\ \left(CD \subset Q\right) \Rightarrow$$

$$\Rightarrow \left(AB\right) \subset Q$$



a′
k′
b′
k – ?
n′ – ?
n′
2′
1′
c′
X
c
1
2
n
b
a
k

# The Position of a Plane Relative to the Projection Planes

A plane may have the following positions relative to the projection planes:

- Inclined to all projection planes;
- Perpendicular to the projection plane;
- Parallel to the projection plane.

# The Position of a Plane Relative to the Projection Planes

**A plane which is not perpendicular or parallel to any of the projection planes is called an oblique plane (a plane of general position).**

**The planes perpendicular or parallel to the projection planes are called the planes of particular position.**

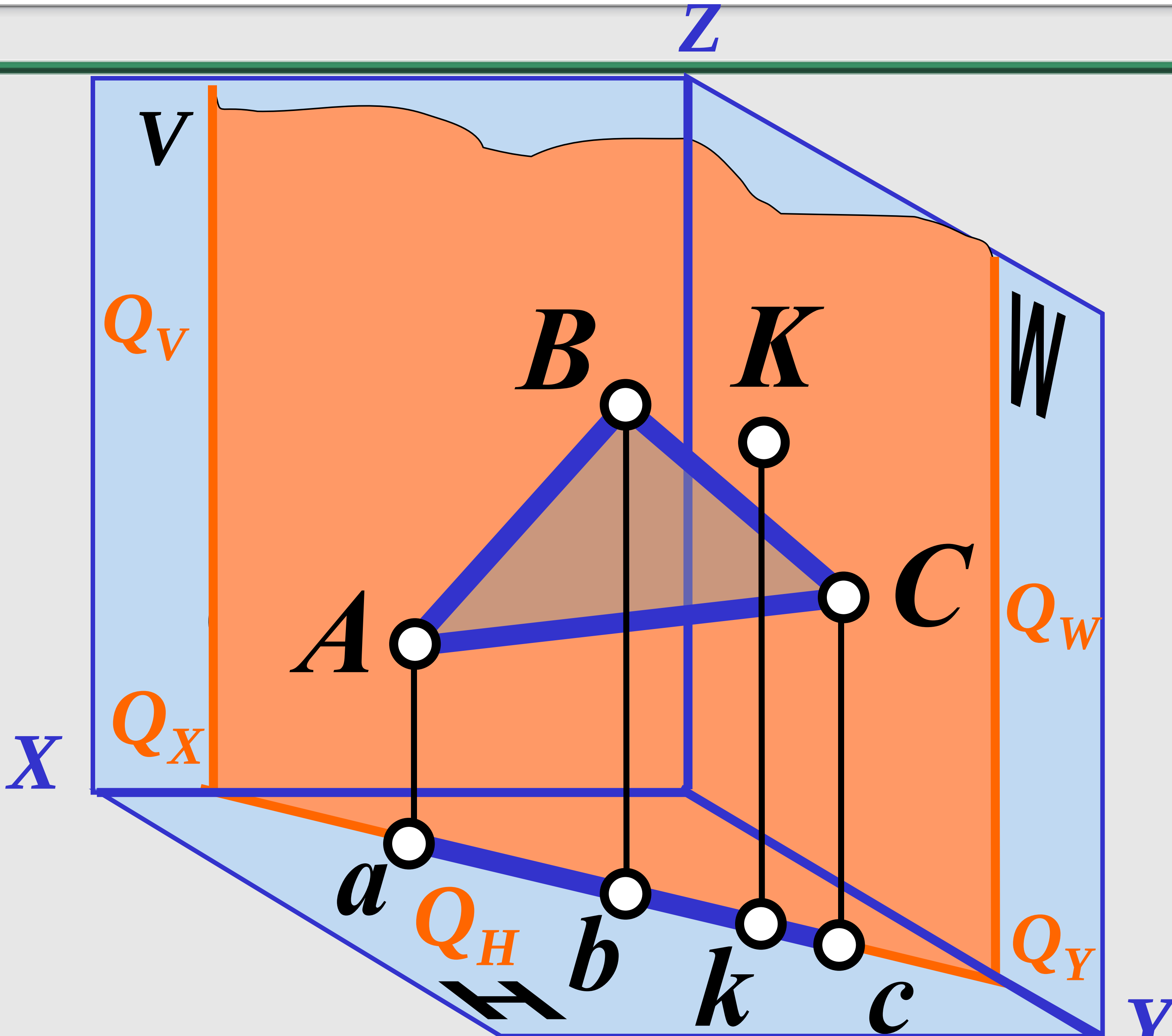
# Planes of particular position

*Projecting plane*

*Level planes*

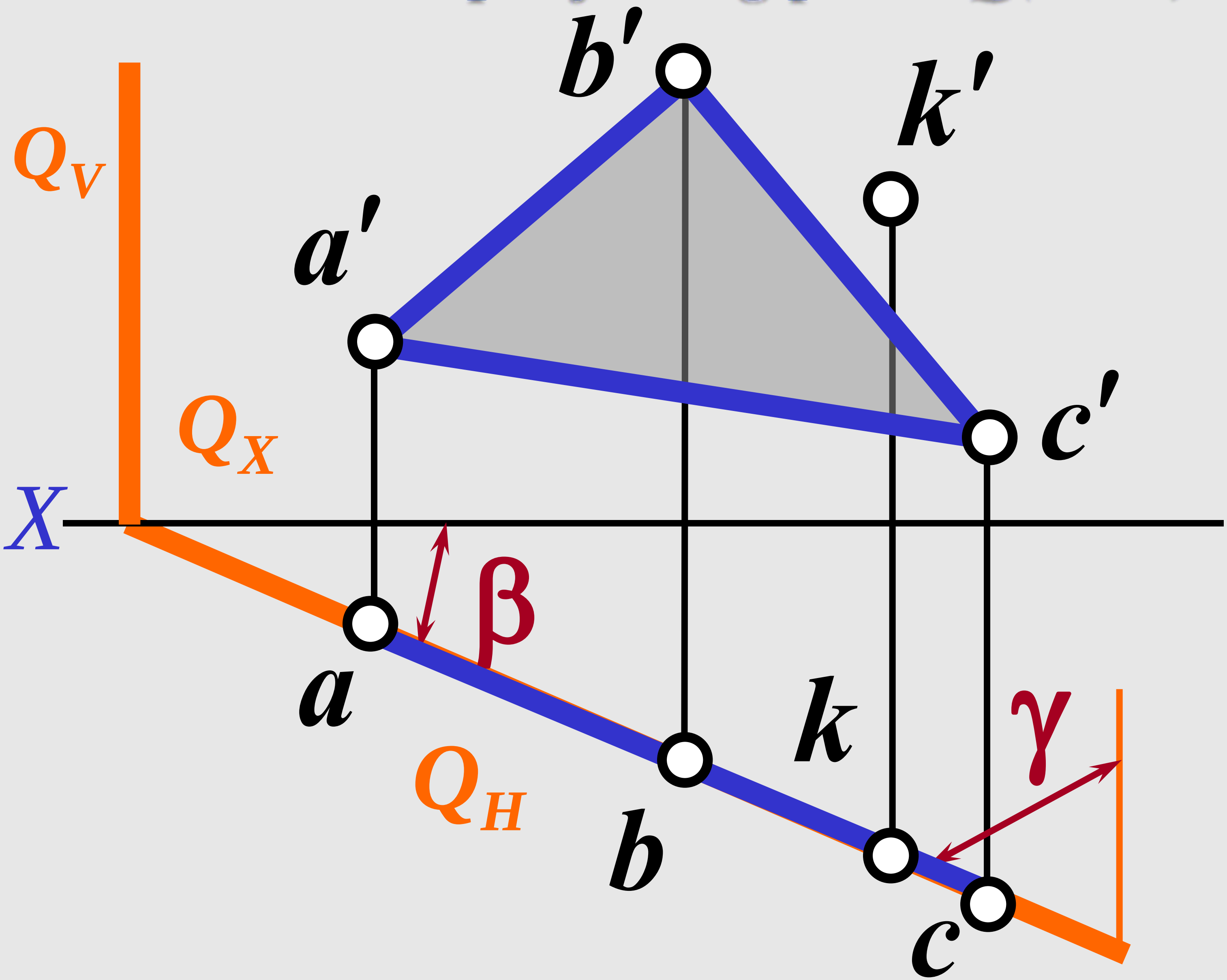
# *Projecting plane:*

- **The horizontal projecting plane**
- **The vertical projecting plane** $Q(ABC) \perp V$
- **The profile projecting plane** $T(ABC) \perp W$

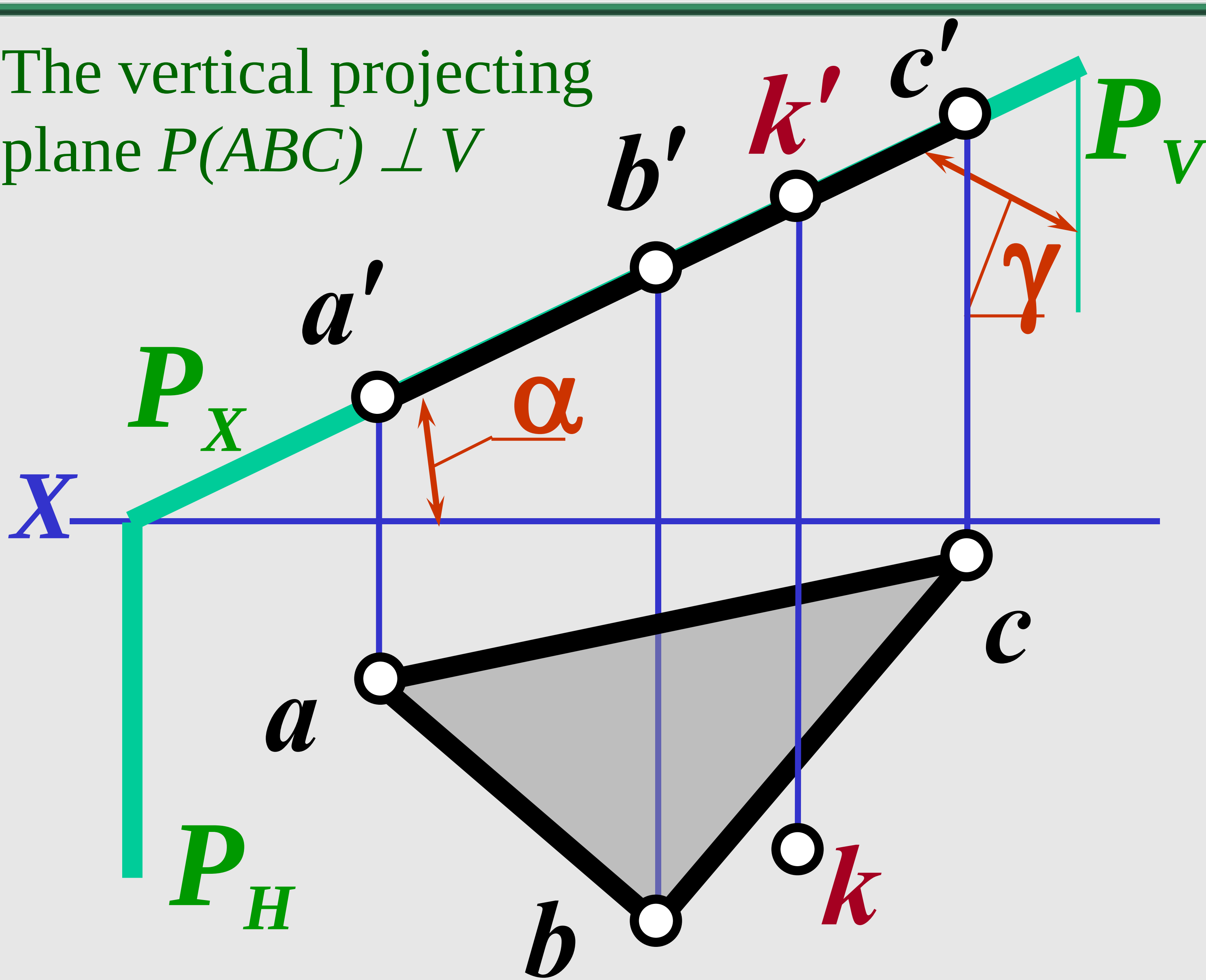


Z
V
W
X
Y
H
$Q_V$
$Q_X$
$Q_W$
$Q_H$
$Q_Y$
B
K
A
C
a
b
k
c

# The horizontal projecting plane *Q(ABC)* ⊥ *H*



The vertical projecting plane *P(ABC)* ⊥ *V*

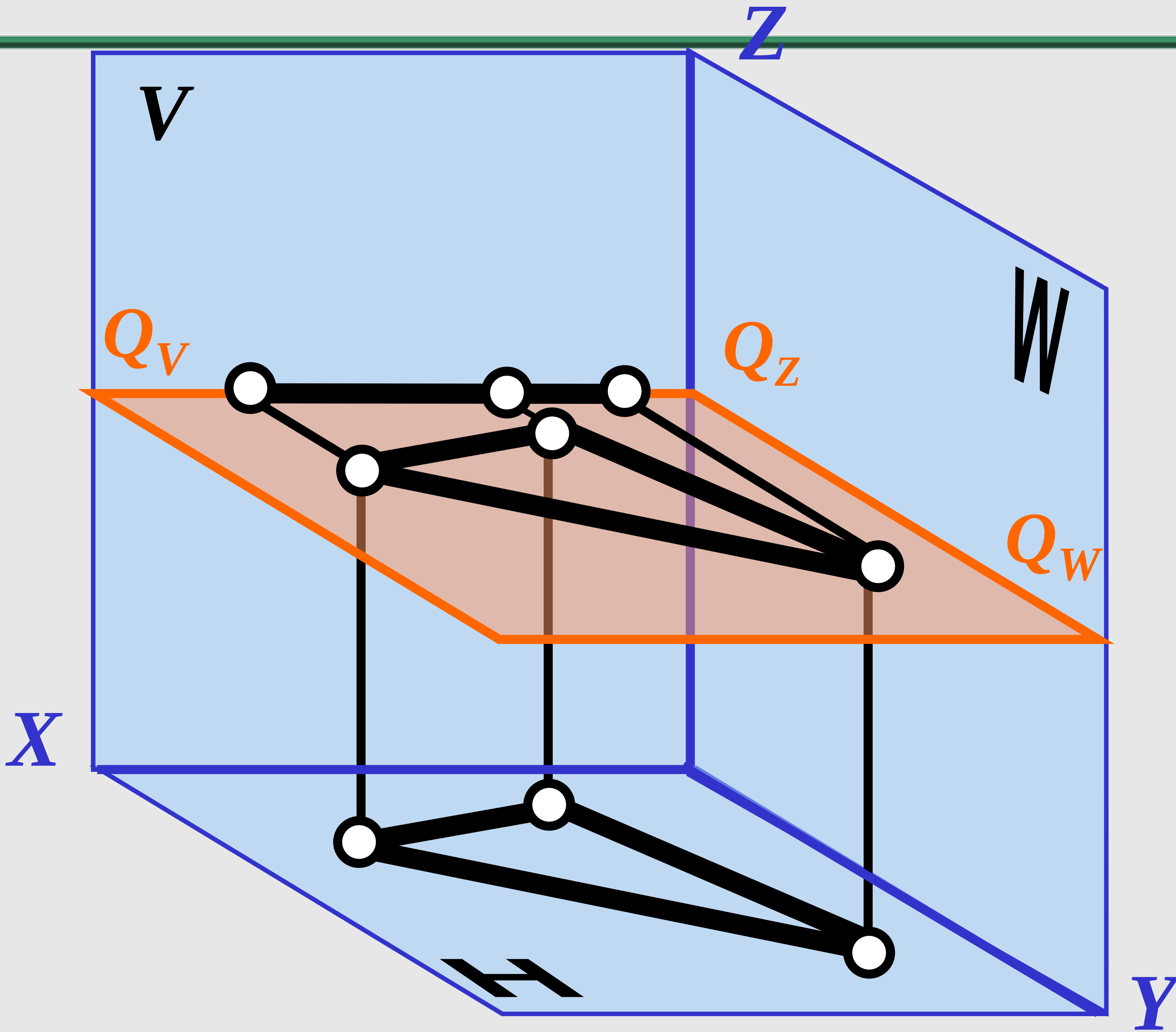


If the figure is perpendicular planes of projections on this plane it is projected in a line;

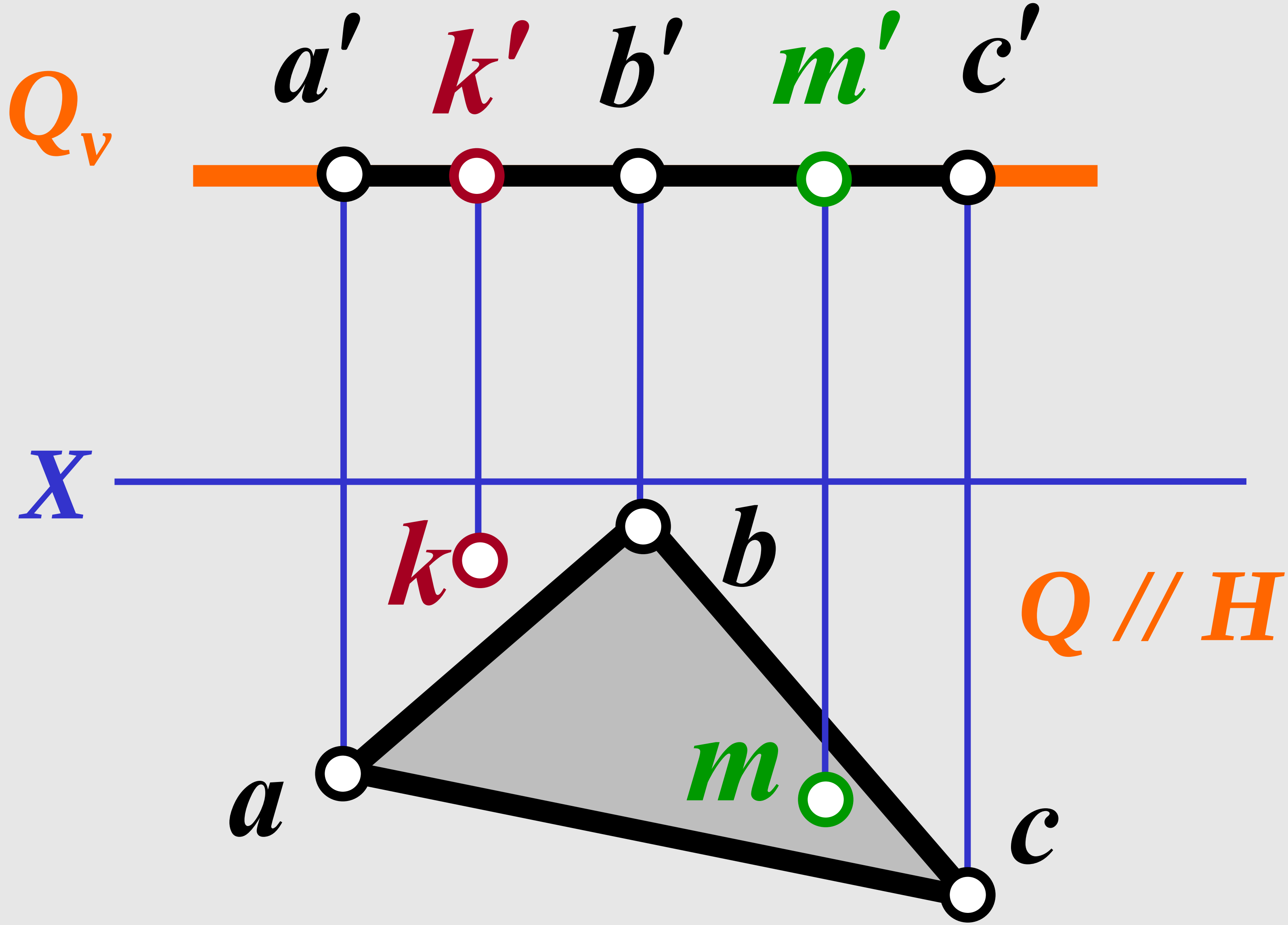
There is an important property of the projecting planes, called a collective one: if a point, a line or a figure are contained in a plane perpendicular to the projection plane, their projections on the above plane coincide with the trace of the projecting plane.

# Level planes:

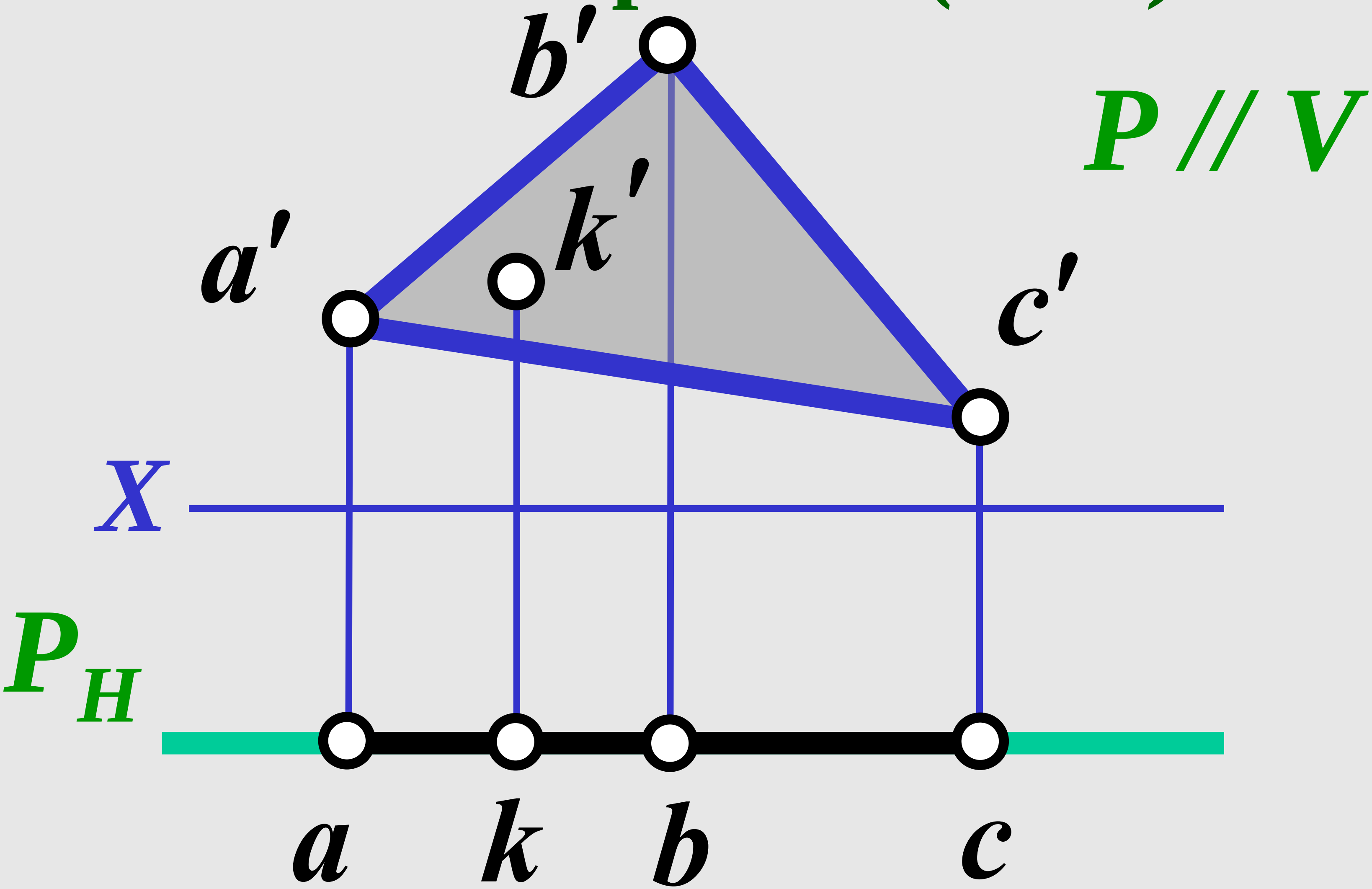
- The horizontal plane
- The frontal plane
- The profile plane


Z
V
W
QV
QZ
QW
X
H
Y

# The horizontal plane *Q(ABC) // H*



# The frontal plane *P(ABC)* V



Any line or figure contained in a level plane parallel to a projection plane, projects to the last plane in true shape.

# The Principal Lines of the Plane

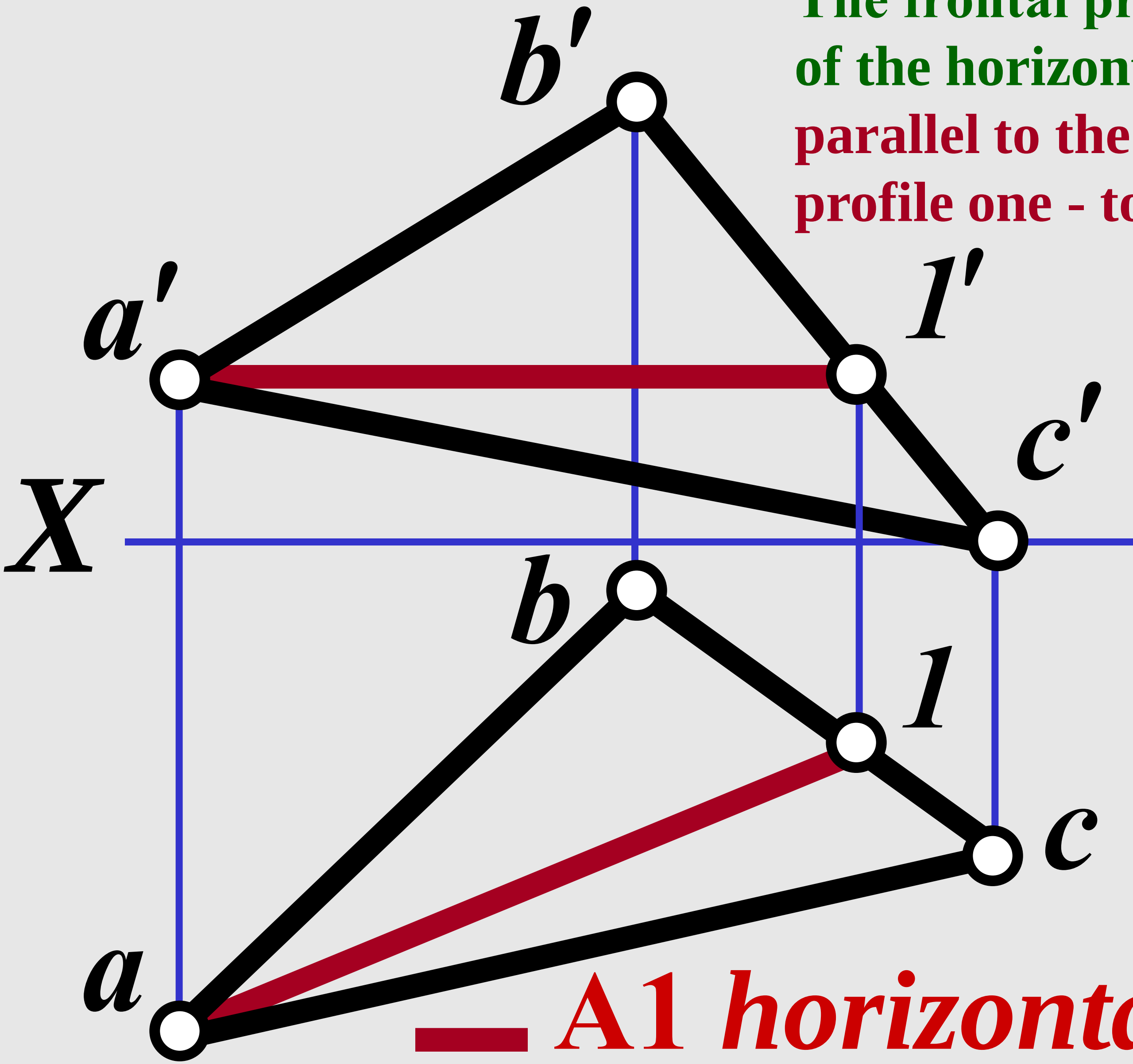
**Lines of a level**

**Lines of maximum inclination or the steepest lines**

# Lines of a level:

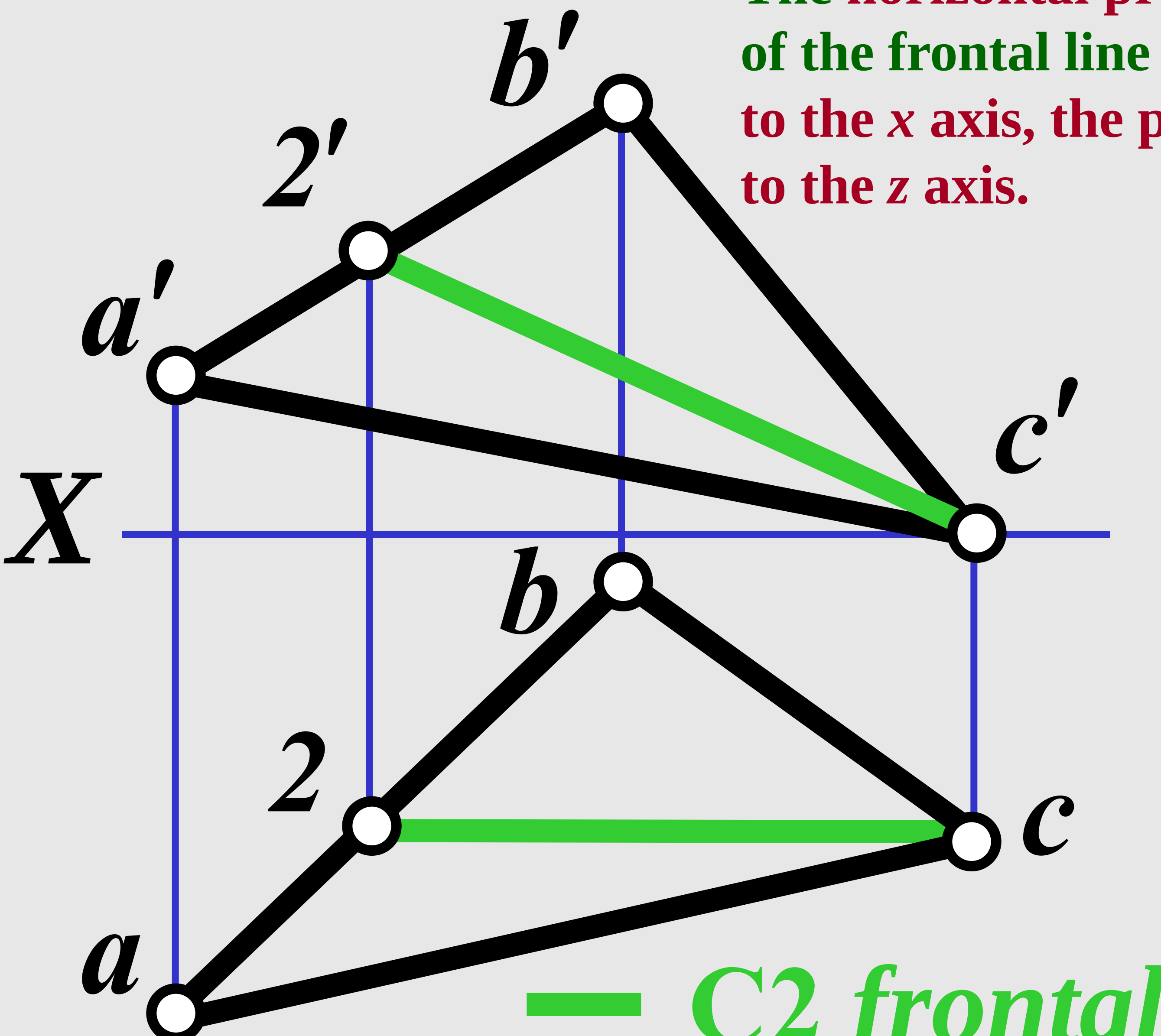
- ***H parallels or horizontal lines*** **are lines lying in a given plane and parallel to the horizontal plane of projection**
- ***V parallels or frontal or vertical lines*** **are lines lying in a given plane and parallel to the vertical plane of projection**
- ***Profile lines*** **are lines lying in a given plane and parallel to the profile plane of projection.**

The frontal projection a’1’ of the horizontal line is parallel to the *x* axis, the profile one - to the *y* axis

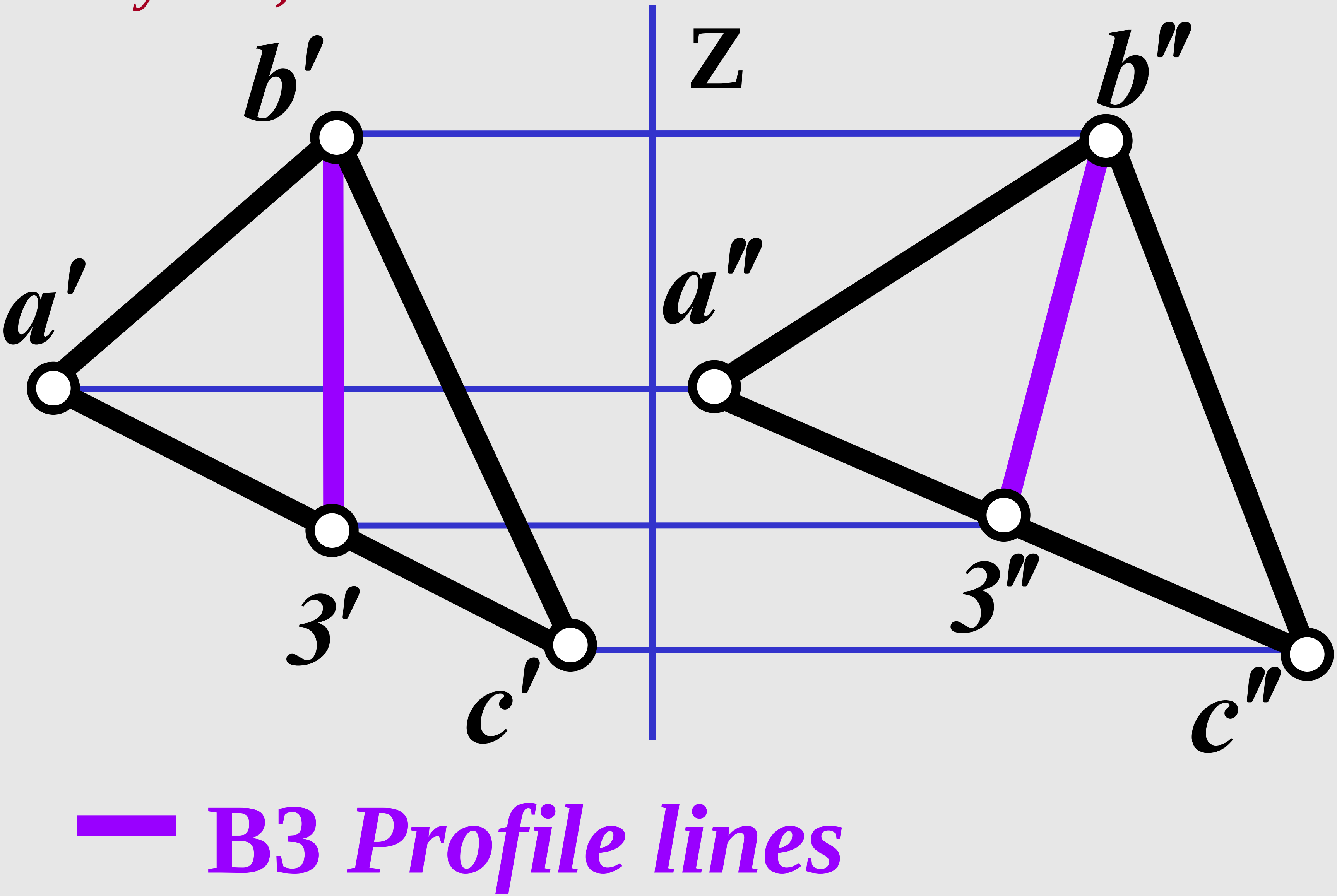



— A1 *horizontal*

**The horizontal projection *c1* of the frontal line is parallel to the *x* axis, the profile one - to the *z* axis.**



The horizontal projection *b1* of the **profile line is parallel to the *y* axis, the frontal one - to the *z* axis**



# The Relative Positions of a Line and a Plane

The relative positions of a line and a plane are determined by the quantity of points belonging both to the plane and to the line:

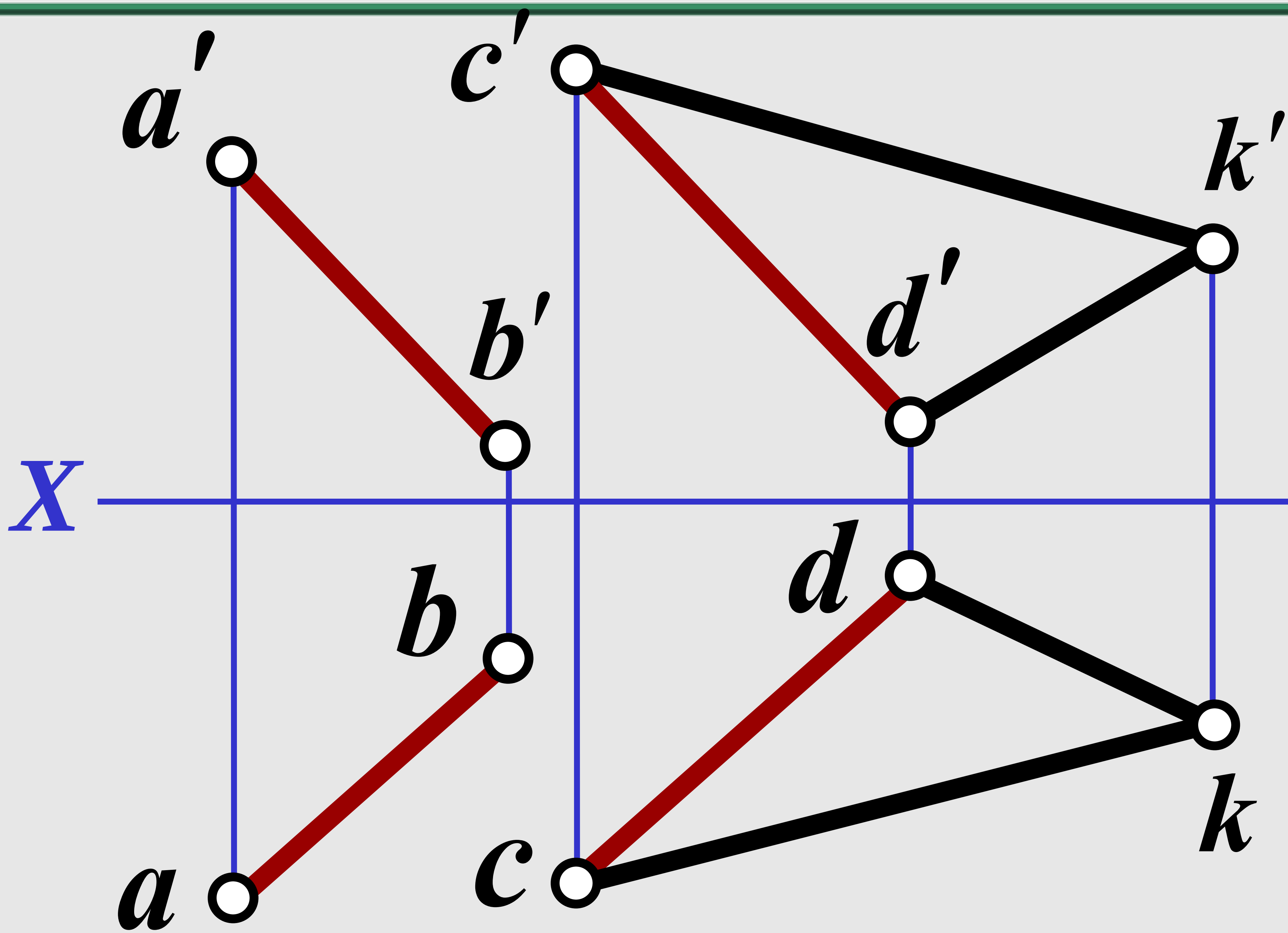
- if a line and a plane have two common points, the line belongs to the plane;
- if a line and a plane have one common point, the line intersects the plane;
- if the point of intersection of a line and a plane is at infinity, the line and the plane are parallel.

# *A line is parallel to a plane*:

1. *A line is parallel to a plane* **if it is parallel to any line contained in this plane.**

**To construct such a line, specify a line in the plane and draw the required one parallel to it.**

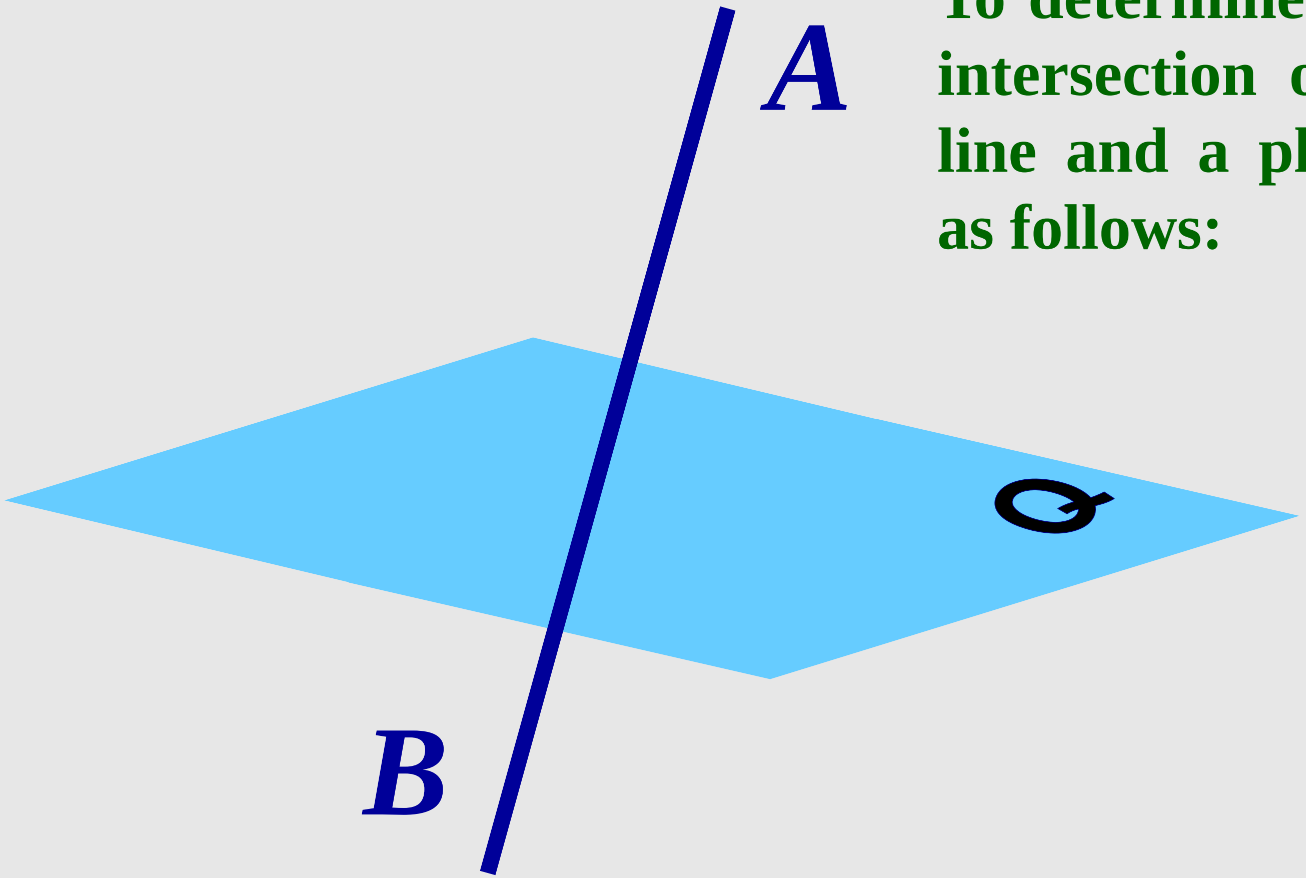
$$(AB)//(CD)\subset Q \Rightarrow (AB)//Q$$



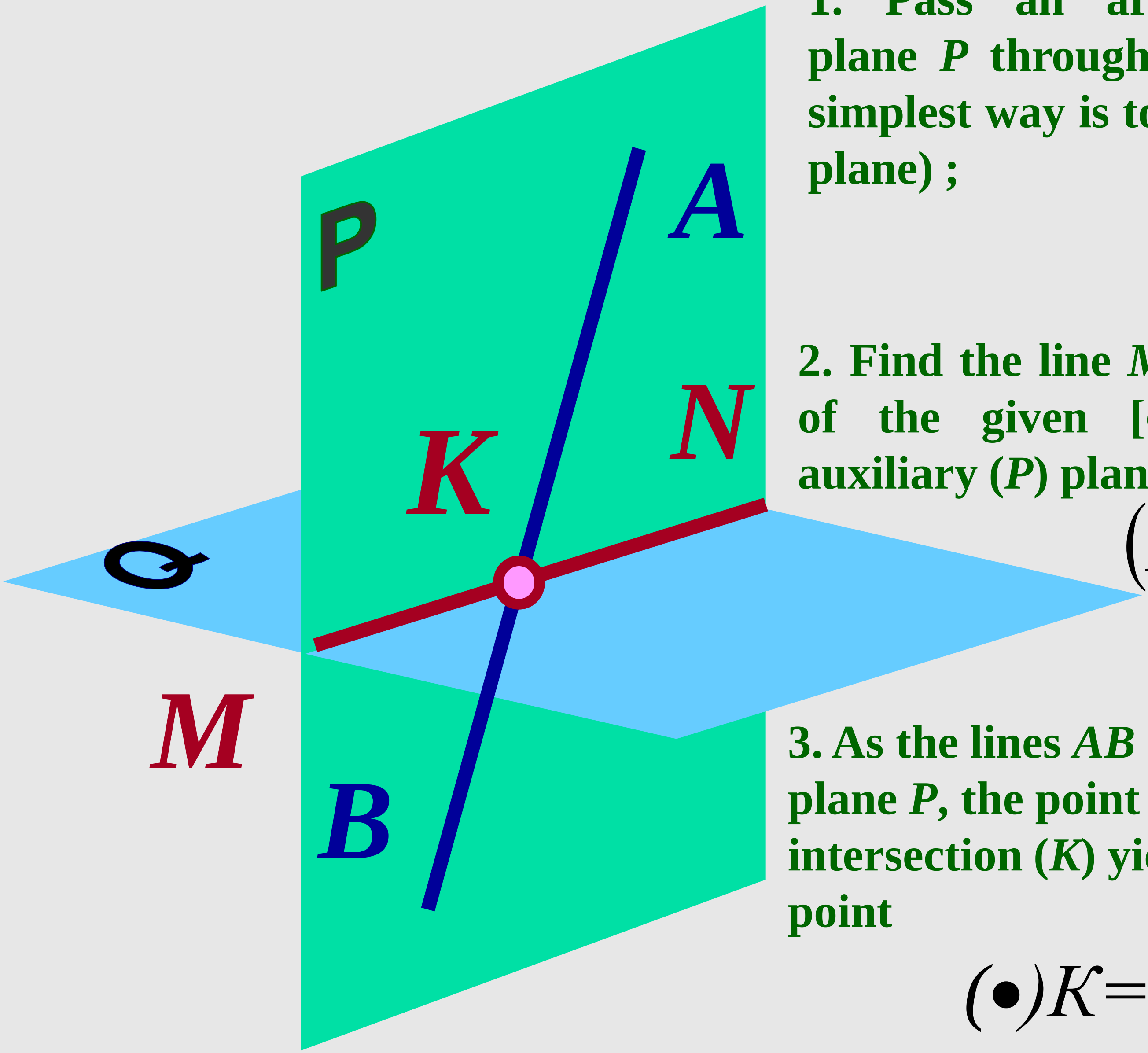
a′
c′
k′
b′
d′
X
b
d
k
a
c

# Line intersects the plane

# Construction of the intersection point of a line and a plane.




**To determine the point of intersection of a straight line and a plane proceed as follows:**




**1. Pass an arbitrary auxiliary plane *P* through the line *AB* (the simplest way is to pass a projecting plane) ;**

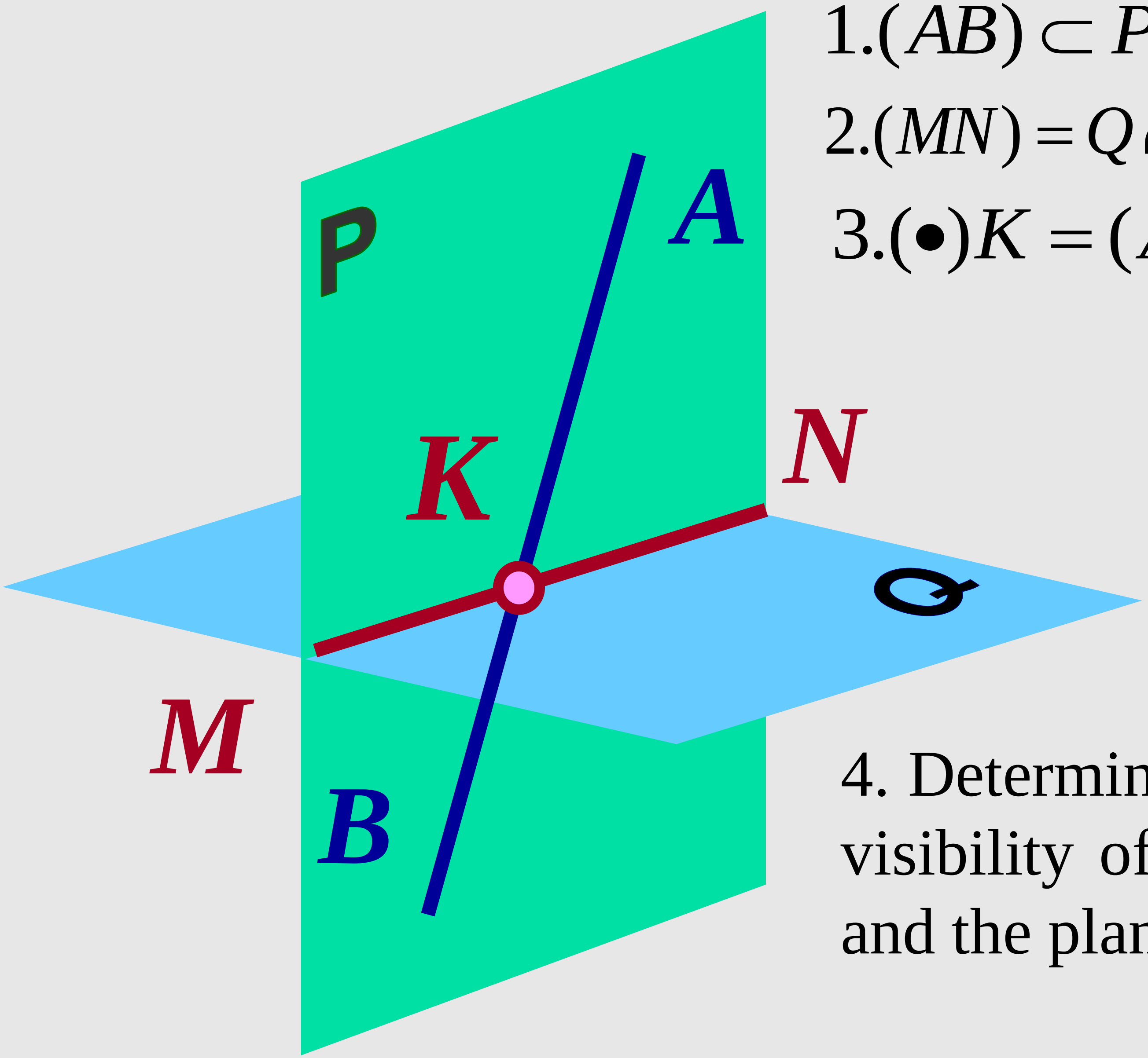
$$(AB) \subset P$$

**2. Find the line *MN* of intersection of the given [*Q* (*ΔCDE*)] and auxiliary (*P*) planes;**

$$(MN) = Q \cap P$$

**3. As the lines *AB* and *MN* lie in one plane *P*, the point of their intersection (*K*) yields the desired point**

$$(\bullet)K = (AB) \cap (MN)$$




$1.\ (AB) \subset P$

$2.\ (MN) = Q \cap P$

$3.\ (\bullet)K = (AB) \cap (MN)$

4. Determine the relative visibility of the line AB and the plane Q.

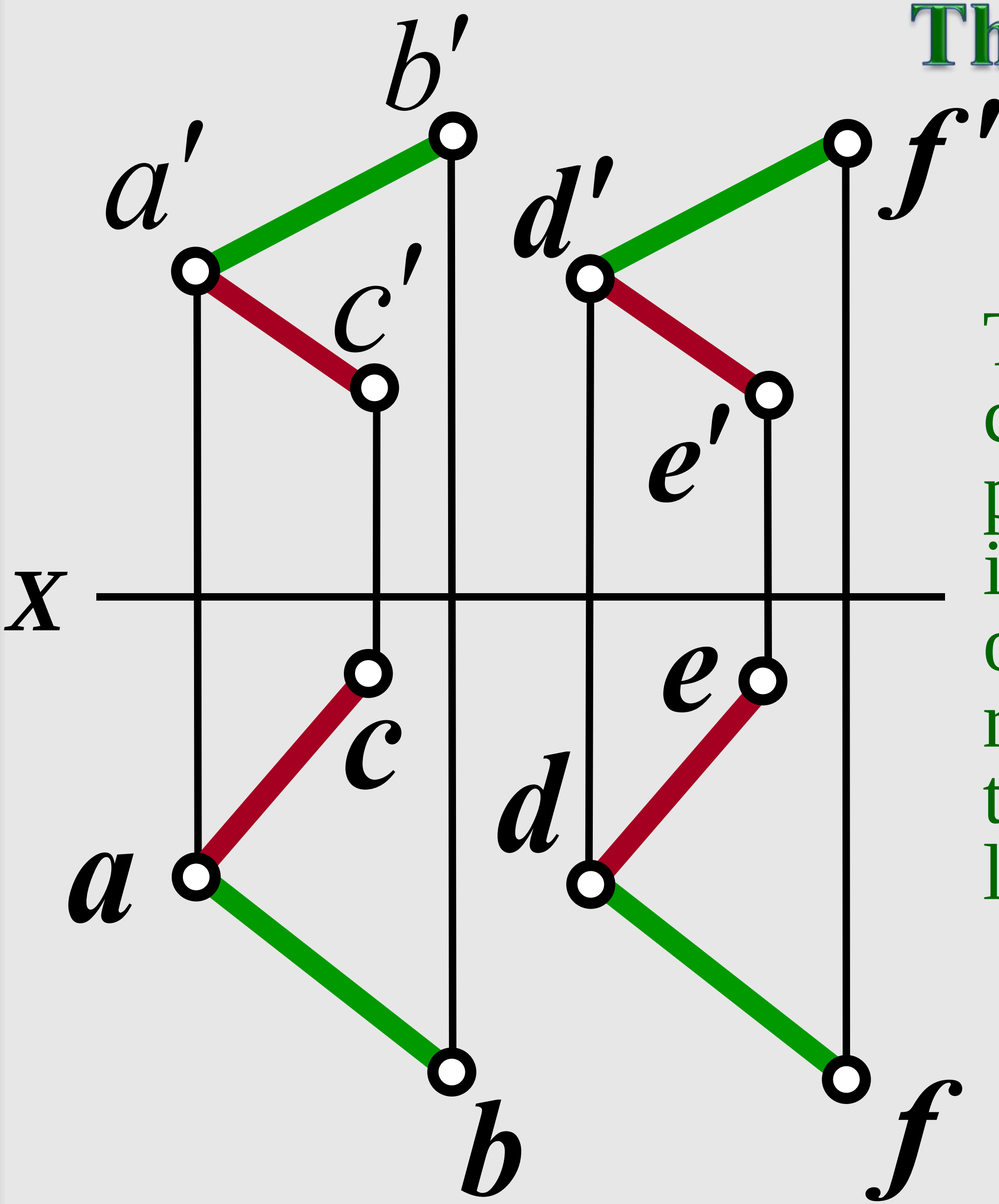
# Mutual Positions of the Planes

***The parallel planes***

***Intersecting planes***

A general case of the mutual positions of planes is their intersection.
In the particular case when the intersection line is at infinity, the planes become parallel.
The parallel planes coincide when the distance between them is shortened to zero.

# The parallel planes



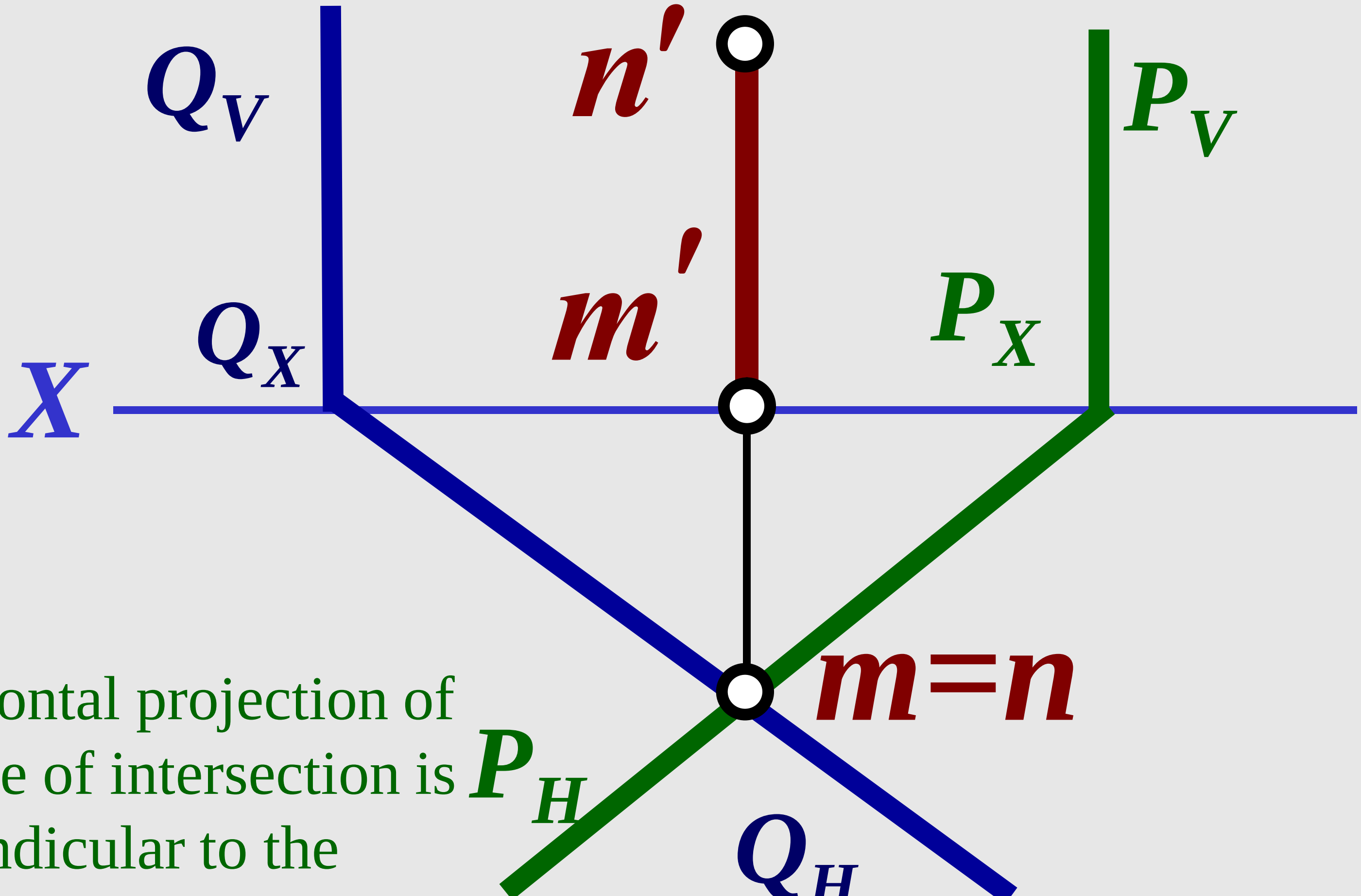

The planes are considered to be parallel if two intersecting lines of one plane are relatively parallel to two intersecting lines of the other

# Intersecting planes

The line of intersection of two planes is determined by two points, each belonging to both planes; or by one point, belonging to both planes, plus a given direction of the line.
In both cases the problem is to find the point common to both planes.

# Intersection of two projecting planes

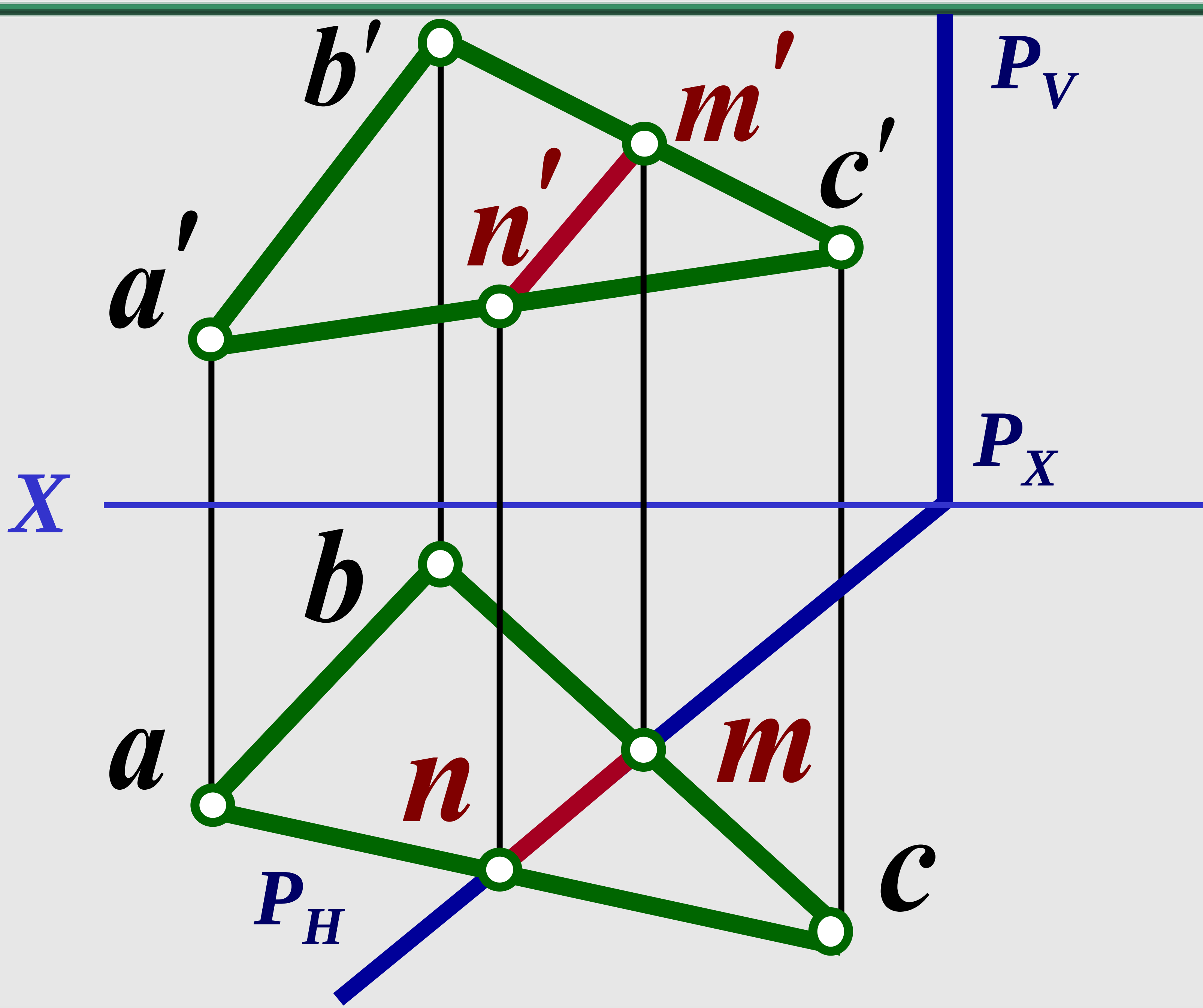
If the planes are of a particular position, the projection of the intersection line on the plane of projections to which the given planes are perpendicular comes to be a point.




The frontal projection of the line of intersection is perpendicular to the projection axis.

# Intersection of a projecting plane and an oblique plane

**In this case, one projection of the line of intersection coincides with the projection of the projecting plane on that projection plane to which it is perpendicular.**



b'
m'
$P_V$
c'
n'
a'
$P_X$
X
b
m
a
n
c
$P_H$

# Intersection of the oblique planes

The method of drawing the intersection lines of such planes consists of the following:

Introduce an auxiliary plane (intermediary) and draw the lines of intersection of this plane with the two given ones.

The intersection of the drawn lines shows the common point of the above planes. To find the other common points use another auxiliary plane.

In solving such kinds of problems, it is better to use projecting planes as intermediaries.

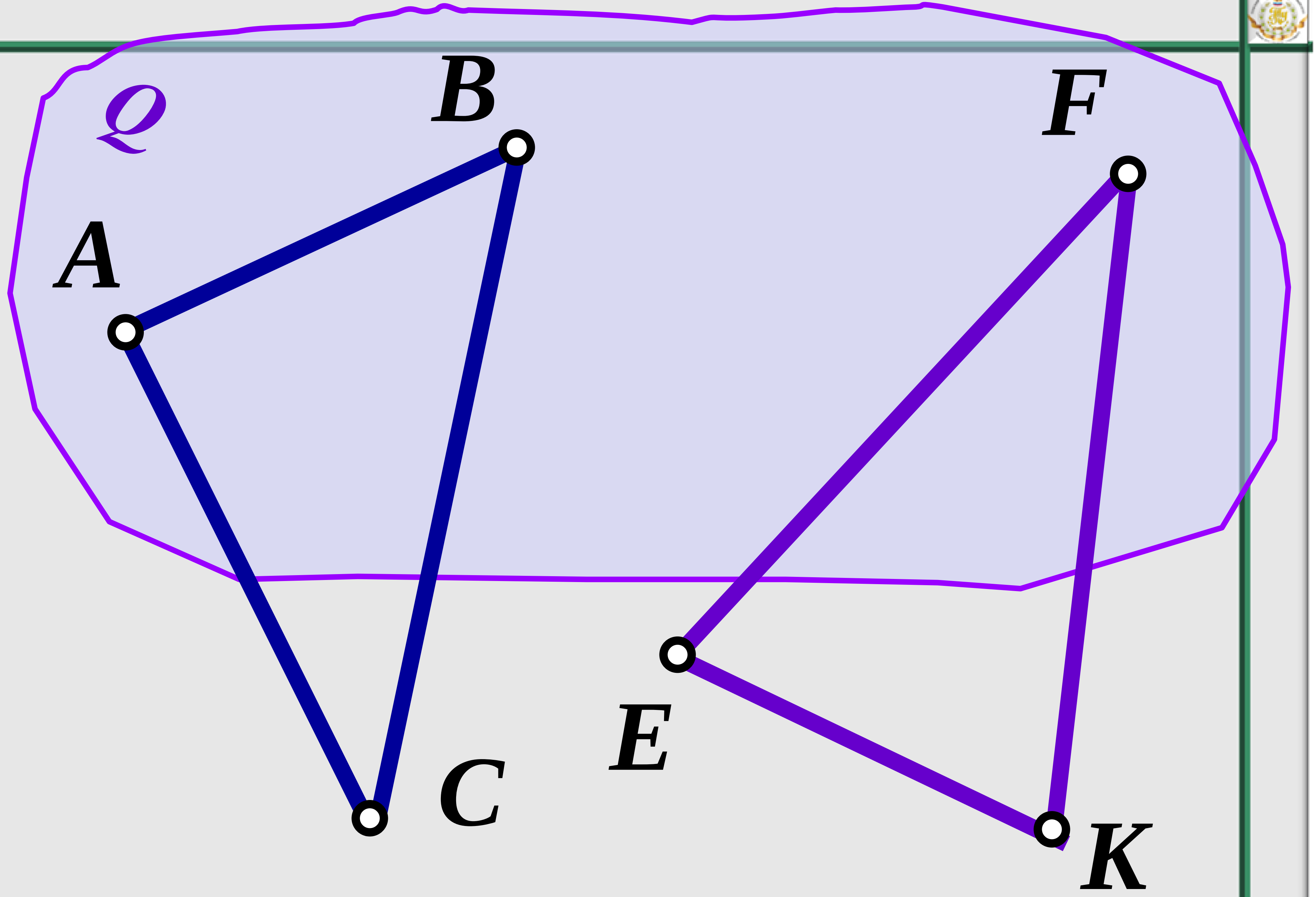
# The solution is as follows:

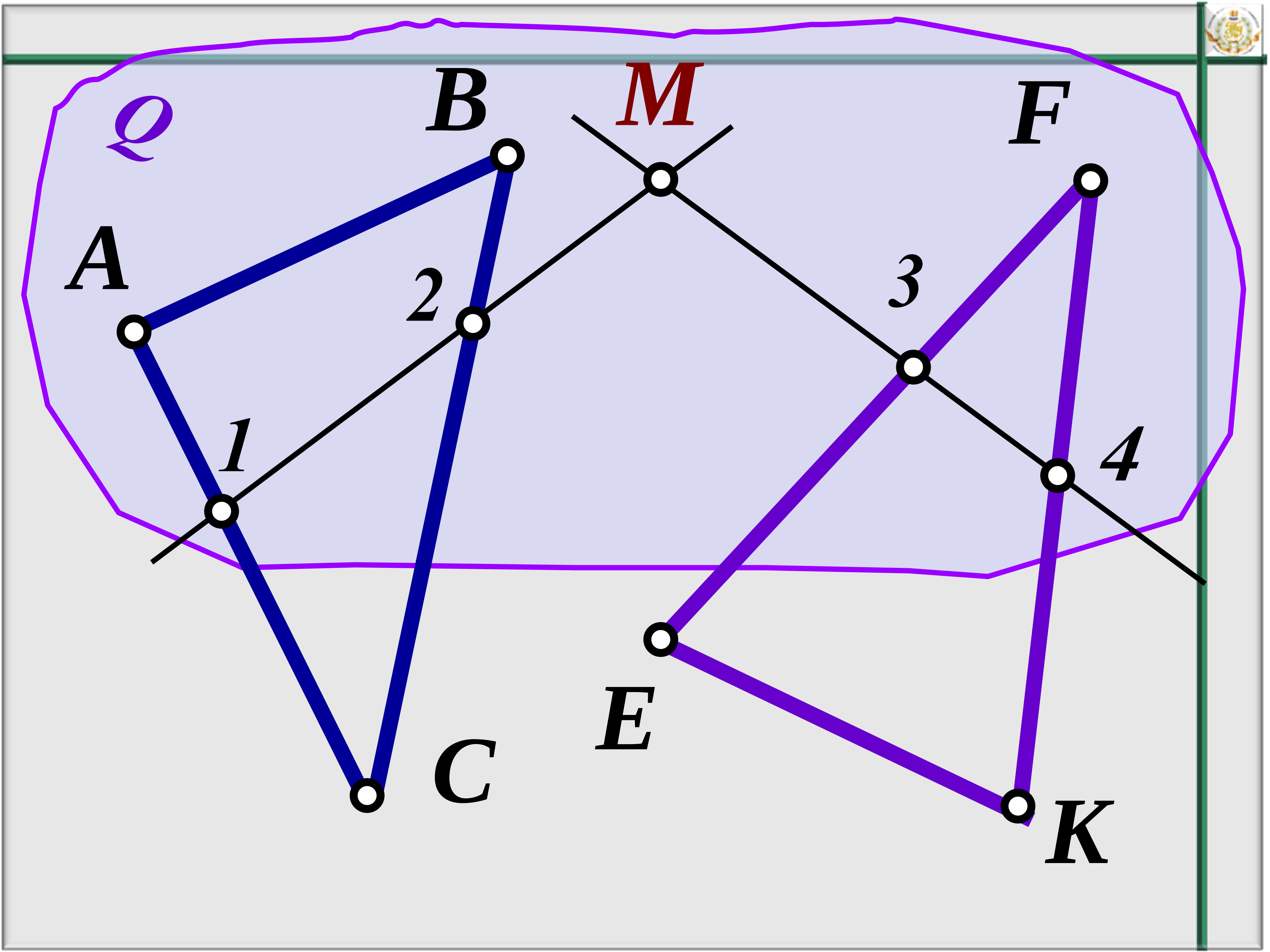
Draw two auxiliary frontal projecting planes Q.

The plane *Q* cuts the triangle *ABC* along the line *1*-*2*.

The plane *Q* cuts the triangle *EFK* along the line 3-*4*.

The intersection of the drawn lines shows the common point M of the above planes


Q
B
F
A
E
C
K

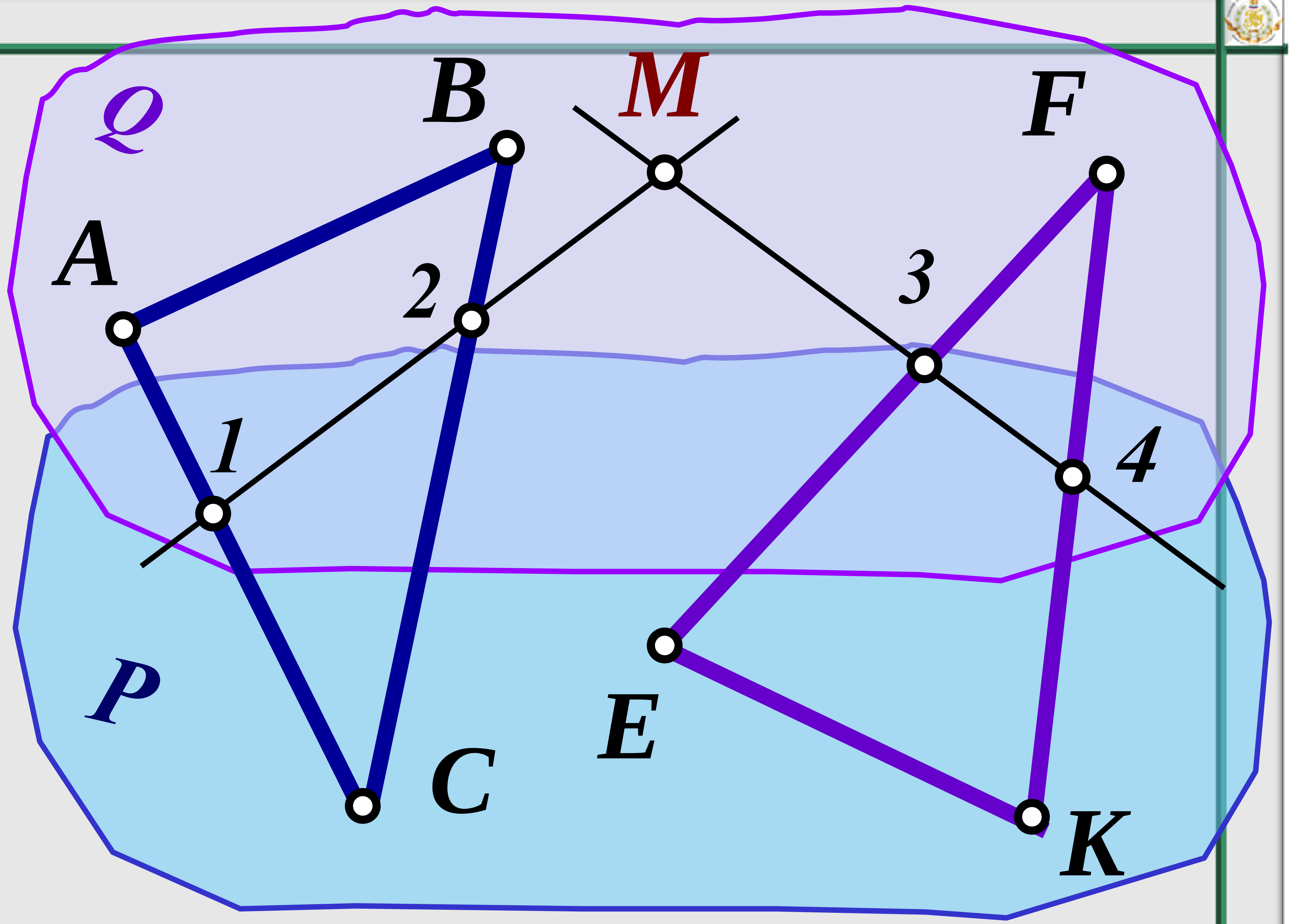

Q
B
M
F
A
2
3
1
4
E
C
K

$$(12) = Q \cap \Delta ABC$$

$$(34) = Q \cap \Delta EFK$$

$$(\bullet) M = (12) \cap (34)$$

Draw two auxiliary frontal projecting planes P.
The plane *P* cuts the triangle *ABC* along the line *5*-*6*.
The plane *P* cuts the triangle *EFK* along the line 7-*9*.
The intersection of the drawn lines shows the common point N of the above planes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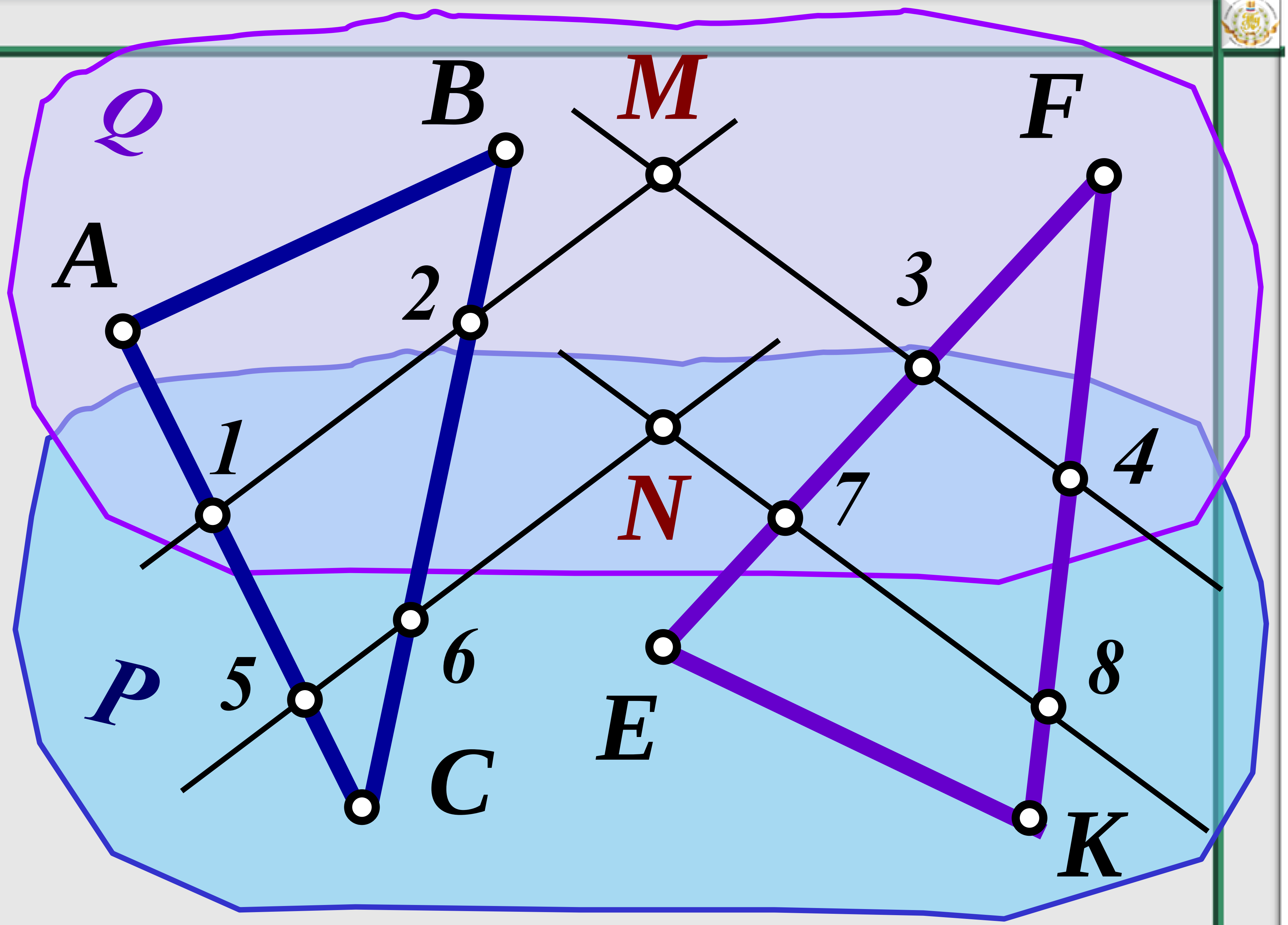

Q
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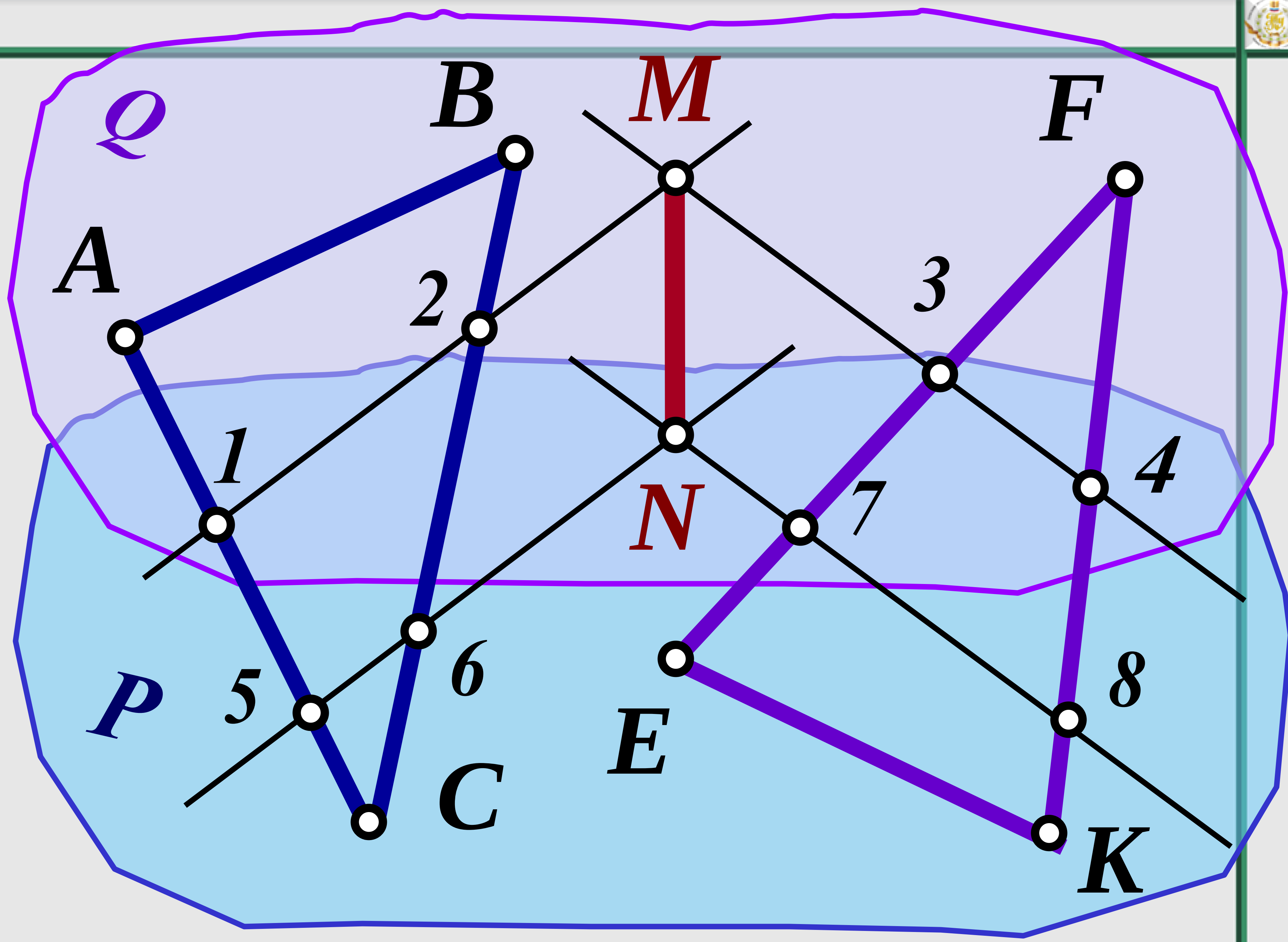
6
5
P
E
8
C
K

$$(56) = P \cap \Delta ABC$$

$$(78) = P \cap \Delta EFK$$

$$(\bullet) N = (56) \cap (78)$$

**We connect points M and N**



Q
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M
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3
1
N
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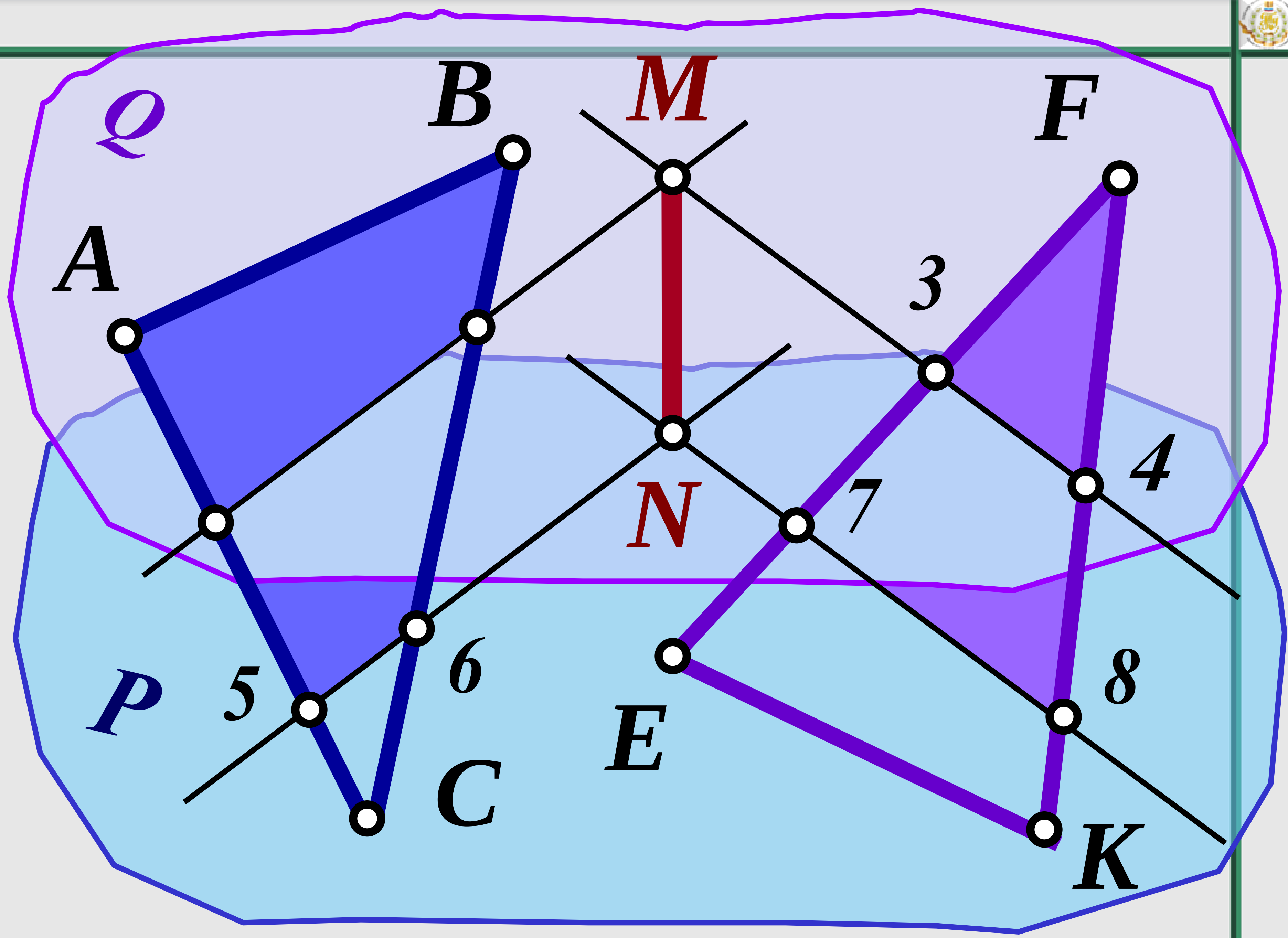
7
6
P
5
E
8
C
K


Q
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C
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