Tomsk Polytechnic University

DESCRIPTIVE GEOMETRY
ENGINEERING GRAPHICS

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Lecture 2

Drawing of a Line-Segment.

The Relative Positions of Two Straight Lines

Plan

1. Drawing of a Line-Segment
2. Mutual Positions of a Point and a Line
3. Traces of a Line
4. The Relative Positions of Two Straight Lines

Drawing of a Line-Segment

A line can have different positions relative to the projection planes:

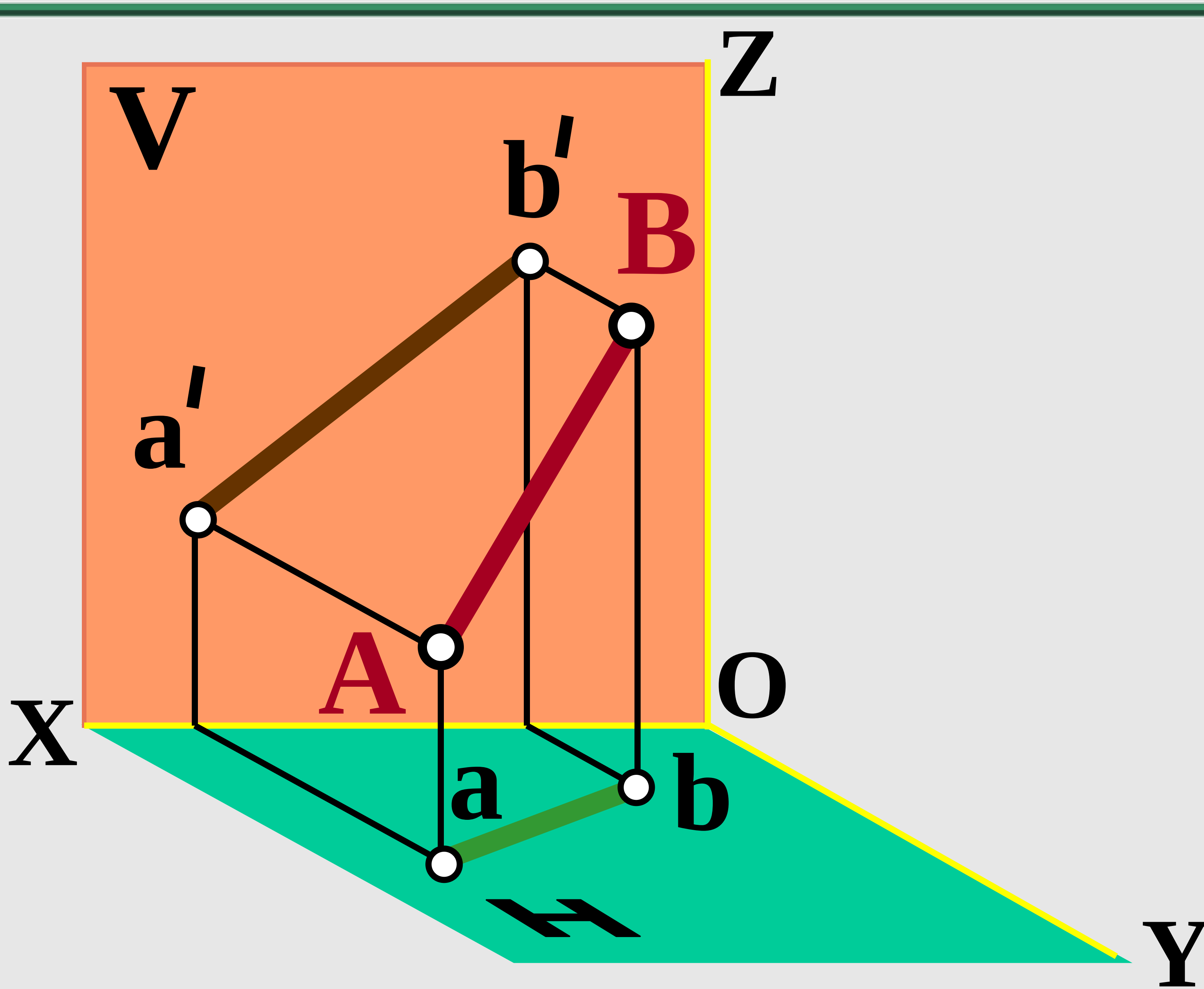
- parallel to neither of the projection planes *H*, *V*, *W* (*the line of general position*);

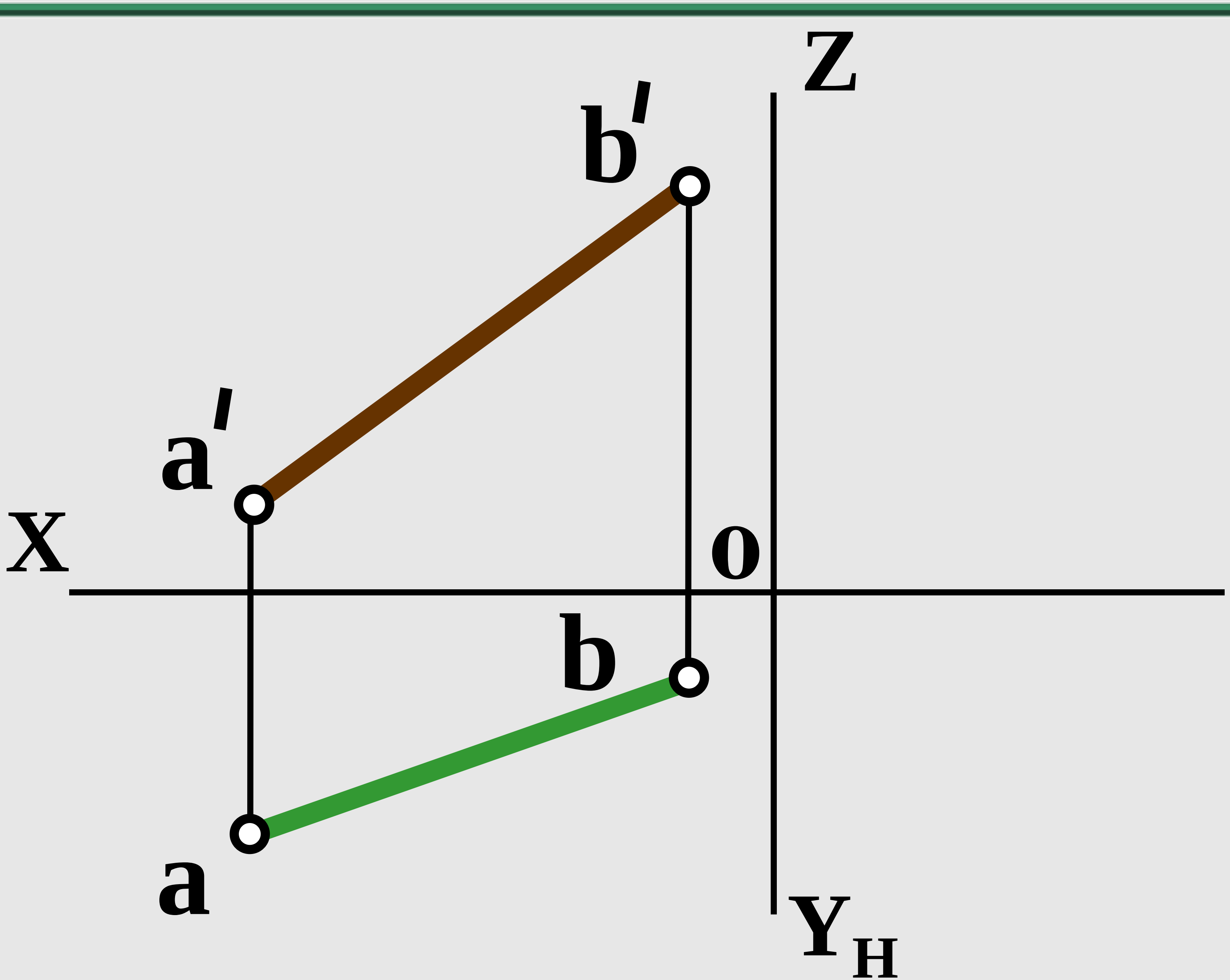
- parallel to one of the projection planes (the line may as well belong to this plane) (*level lines*);

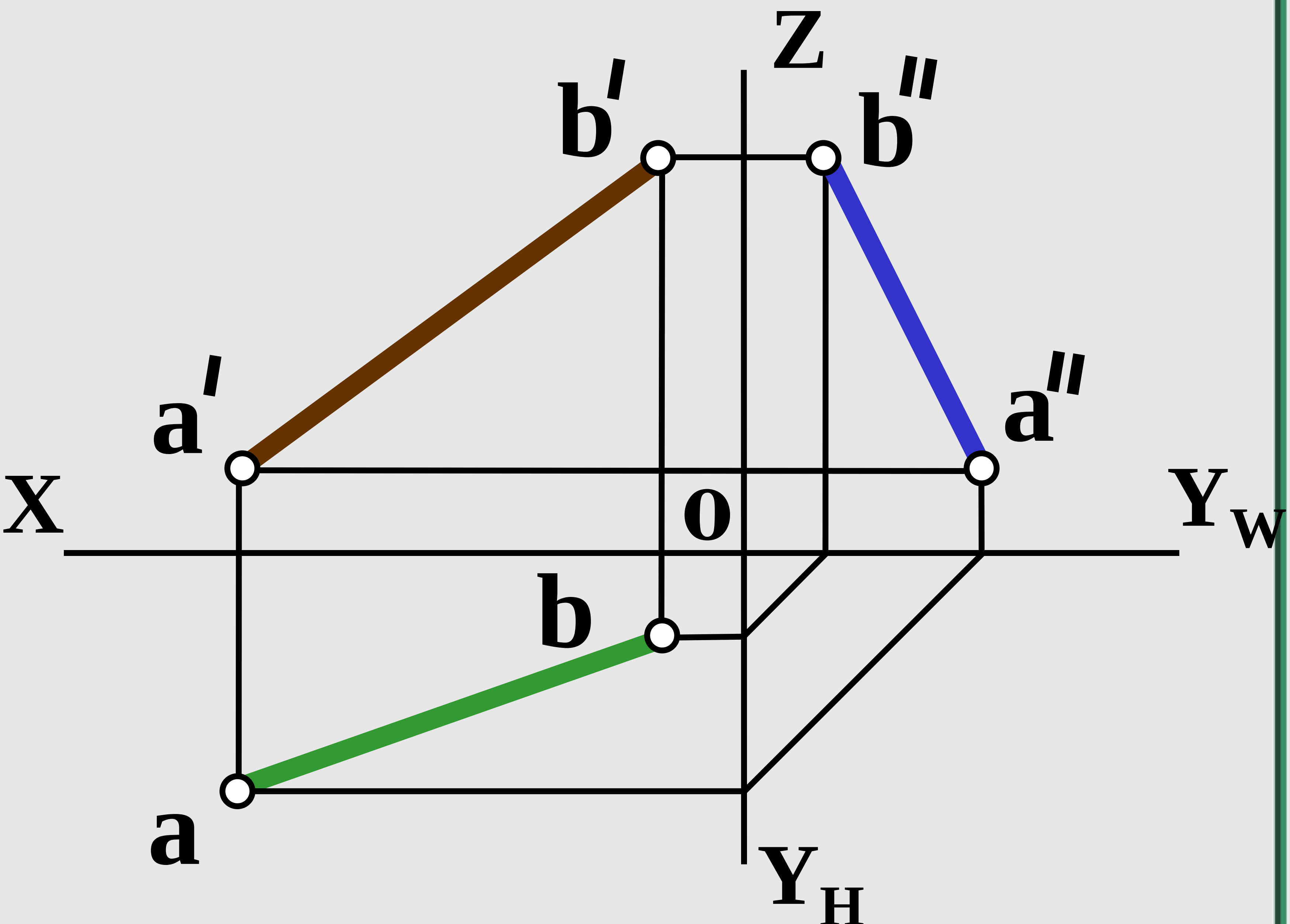
- parallel to two of the projection planes, that is, perpendicular to the third one (*projecting lines*)

The line of general position –

the straight line inclined to all three planes of projections


Z
V
b′
B
a′
A
O
X
a
b
Y


Z
b′
a′
X
0
b
a
Y
H


Z
b′
b″
a′
a″
X
0
Y_W
b
a
Y_H

The line of a particular position –

direct parallel or perpendicular planes of projections

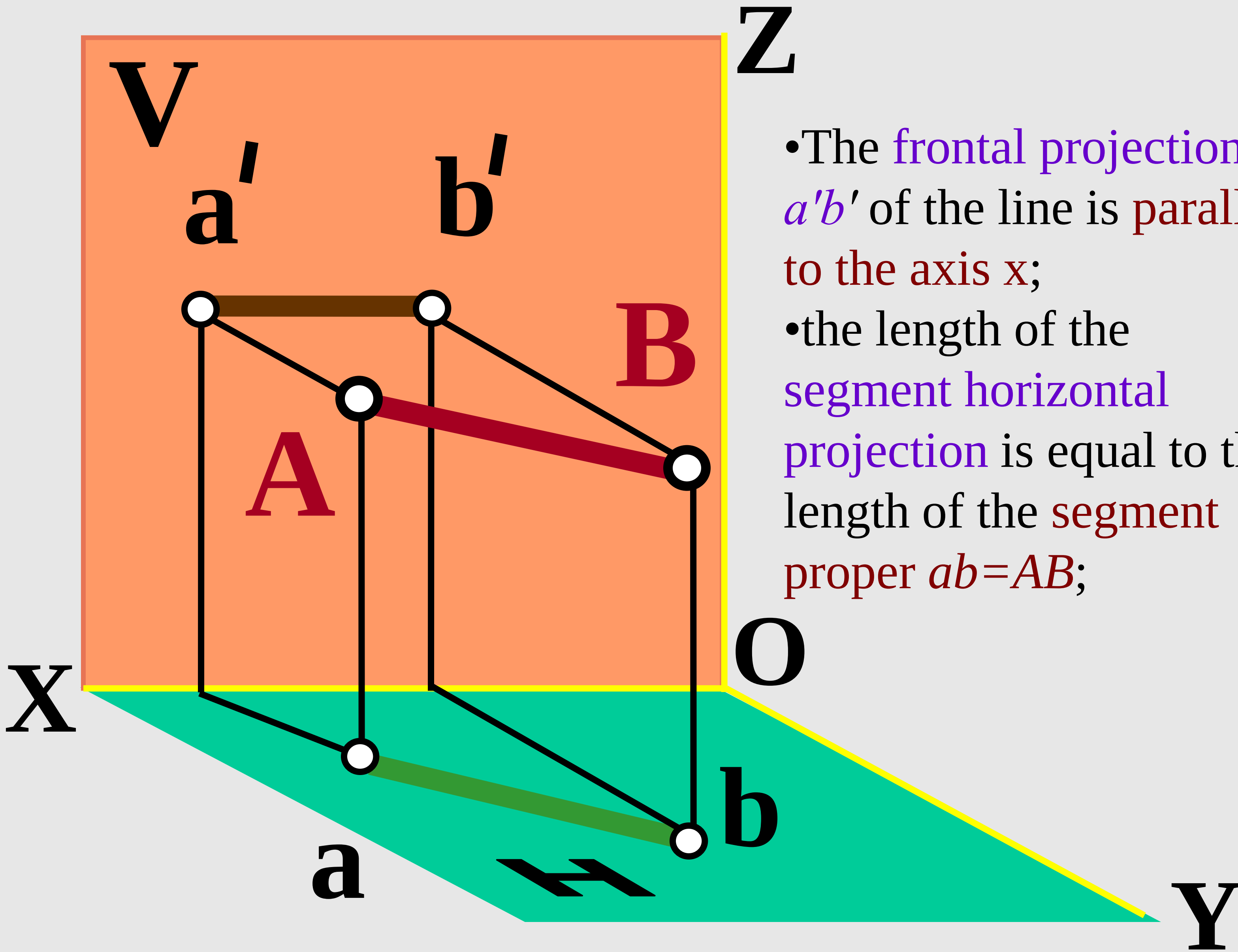
The line of a particular position:

- ***level lines***
- ***projecting lines***

Straight lines parallel planes of a projection

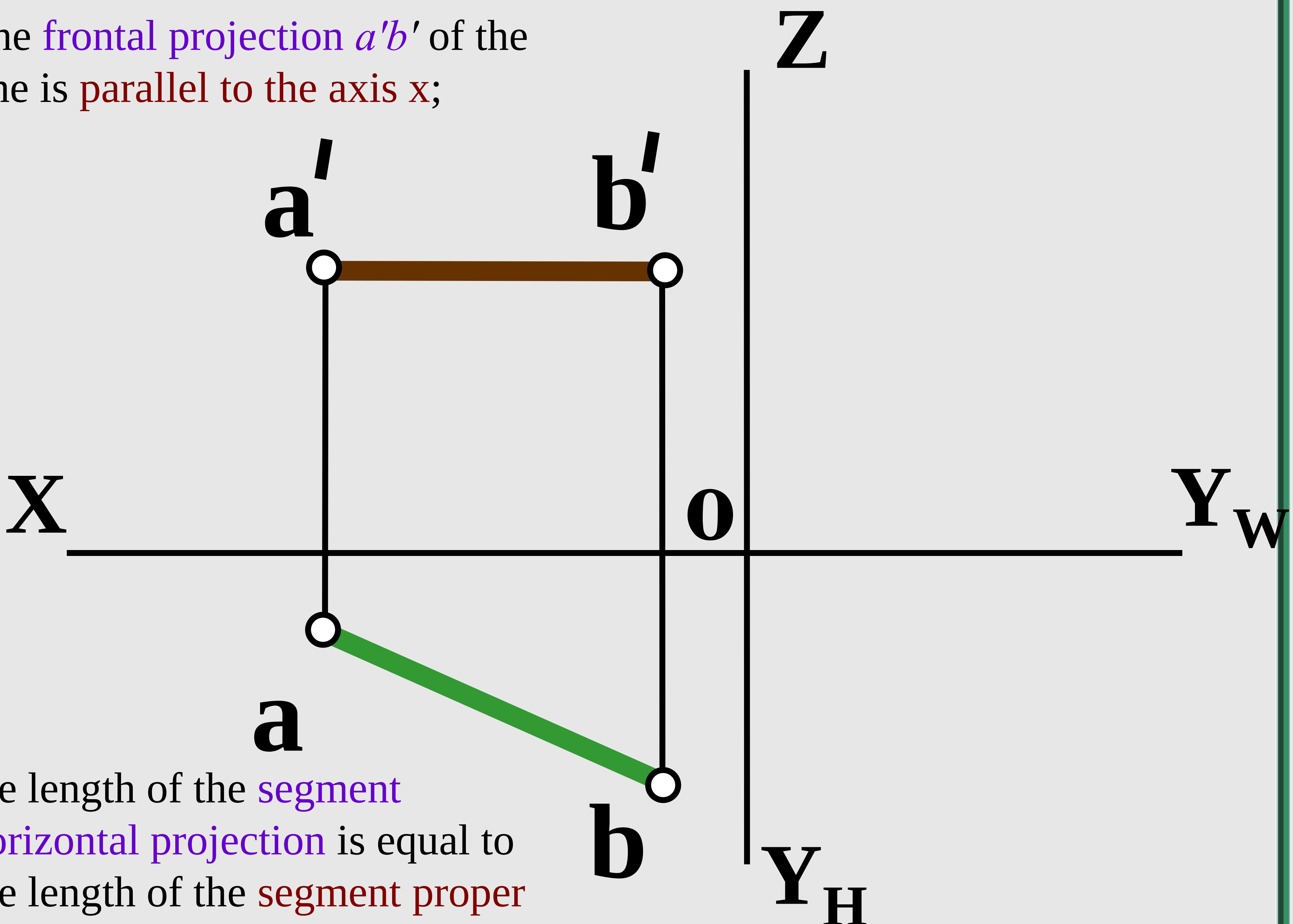
(level lines)

The line AB is parallel to the plane H (it is called a horizontal line)



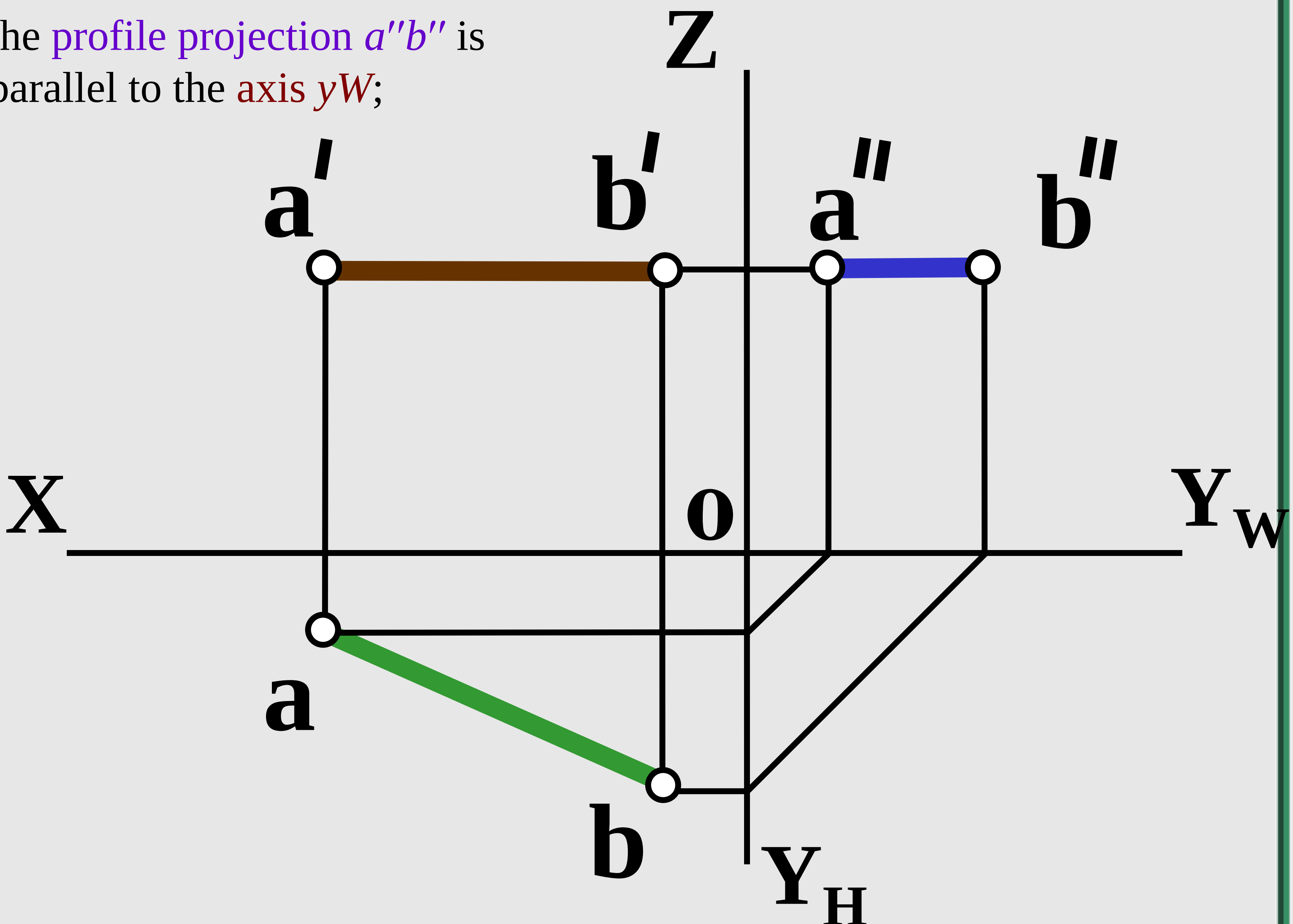
- The frontal projection $a'b'$ of the line is parallel to the axis x;
- the length of the segment horizontal projection is equal to the length of the segment proper $ab=AB$;

The frontal projection *a′b′* of the line is parallel to the axis x;

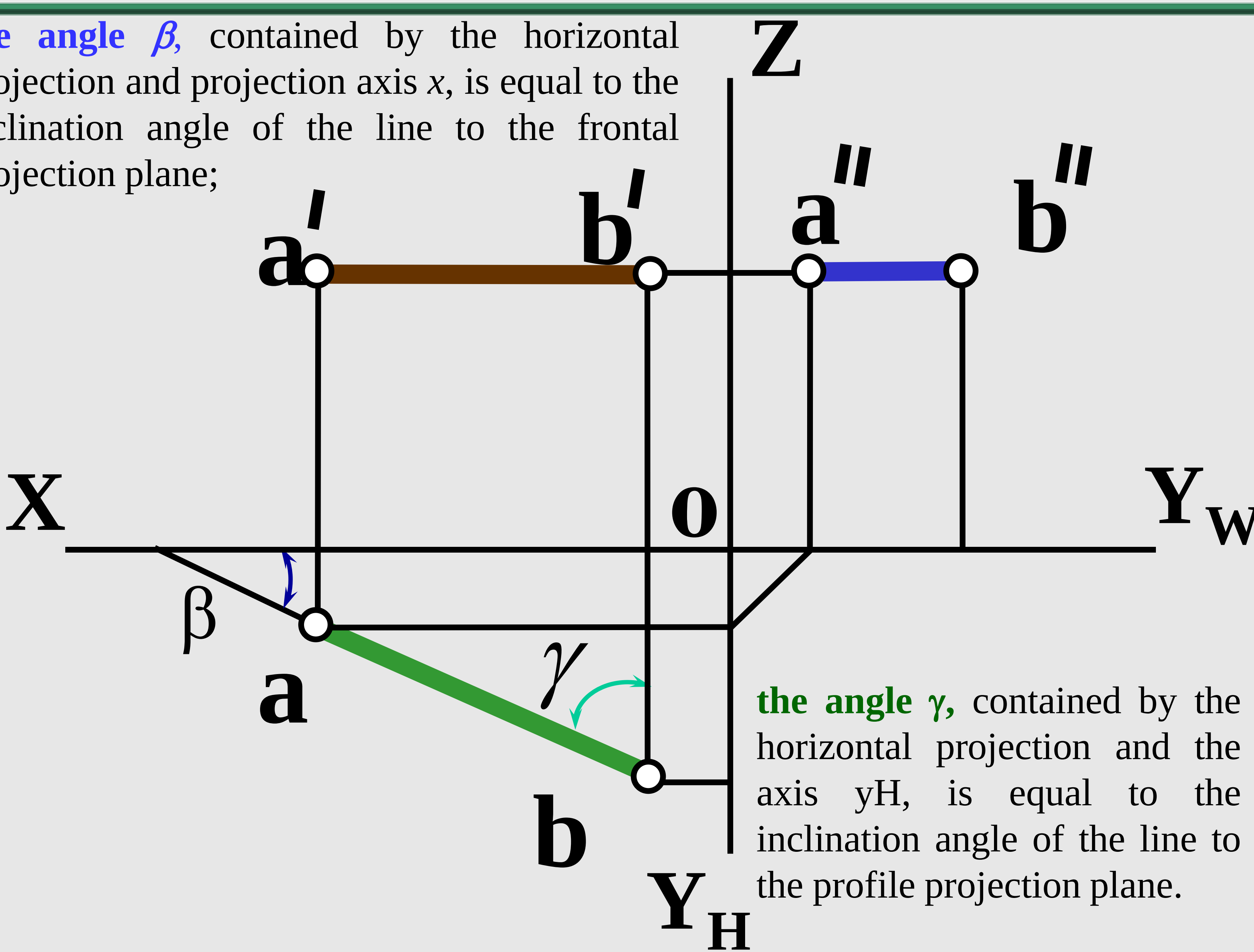


the length of the segment horizontal projection is equal to the length of the segment proper *ab=AB*;

the profile projection *a″b″* is parallel to the axis *yW*;



the angle ***β*,** contained by the horizontal projection and projection axis *x*, is equal to the inclination angle of the line to the frontal projection plane;



the angle γ, contained by the horizontal projection and the axis yH, is equal to the inclination angle of the line to the profile projection plane.

Properties of projections

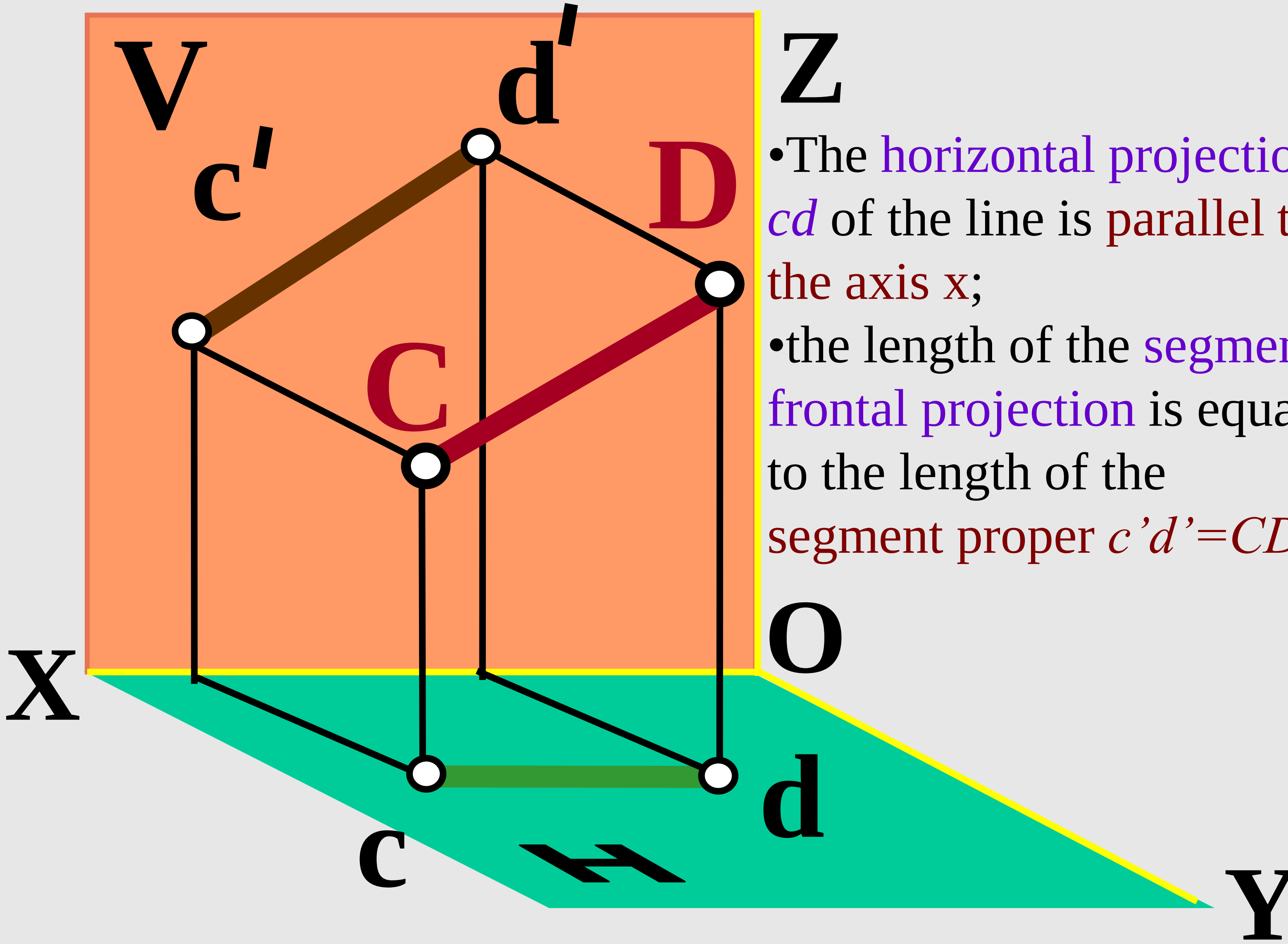
$$|ab| = |AB|;$$

$$(a'b') // (OX), (a''b'') // (OY);$$

$$(AB\hat{\ }V) = (ab\hat{\ }OX) = \hat{\beta};$$

$$(AB\hat{\ }W) = (ab\hat{\ }OY_H) = \hat{\gamma}.$$

The line *CD* is parallel to the plane *V* (it is a frontal line)



- The horizontal projection *cd* of the line is parallel to the axis x;
- the length of the segment frontal projection is equal to the length of the segment proper *c'd'=CD*;

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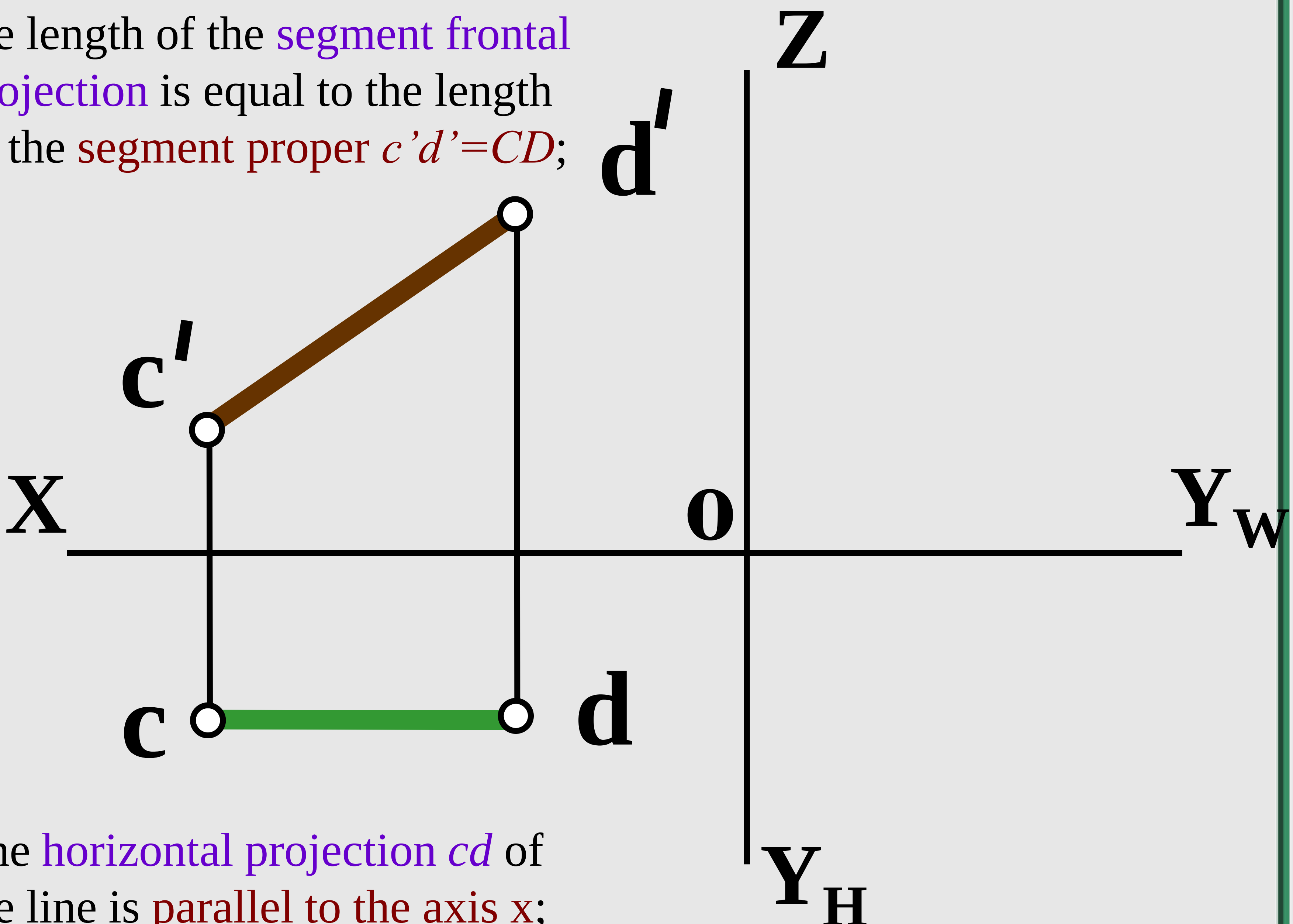
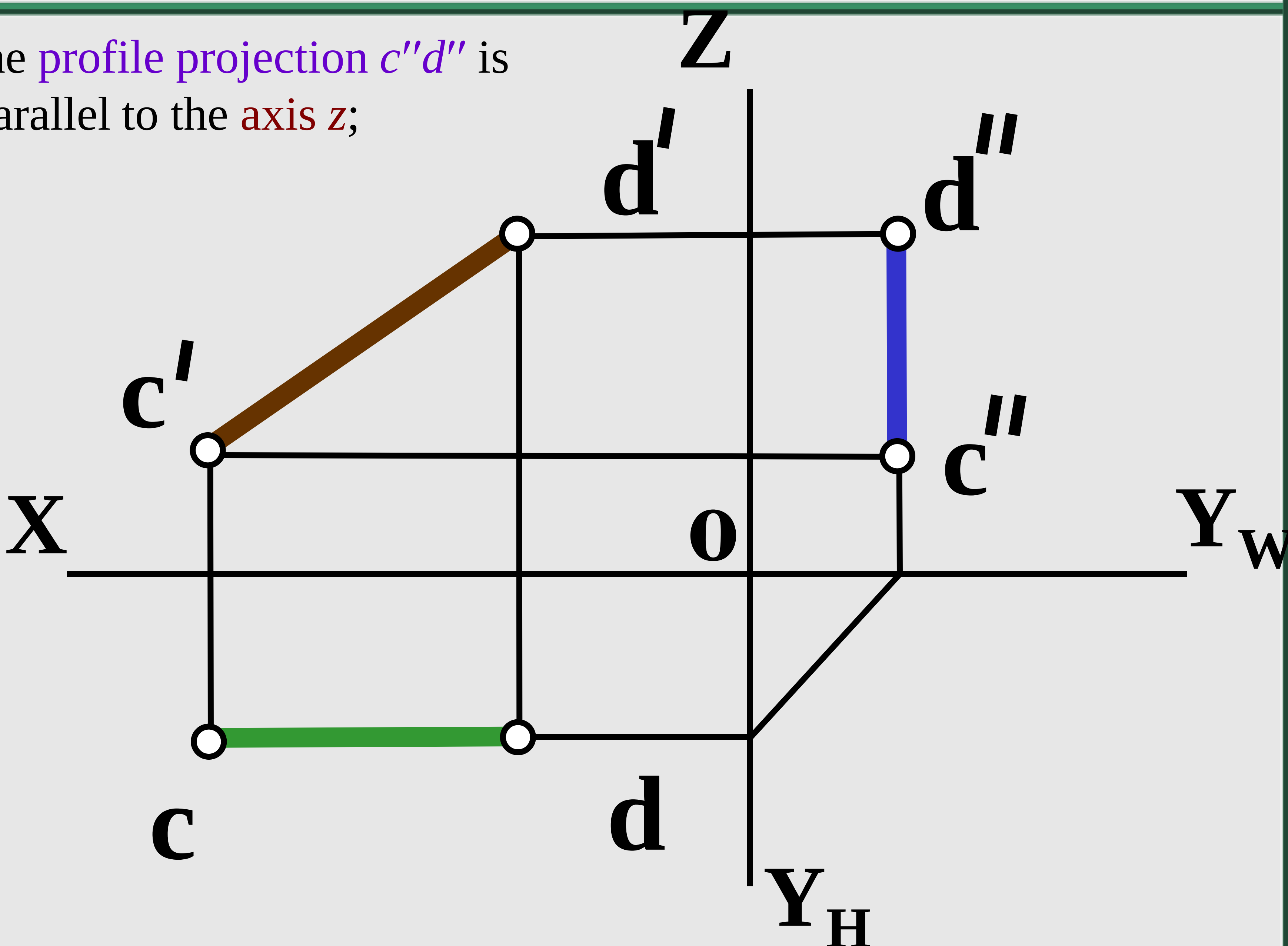
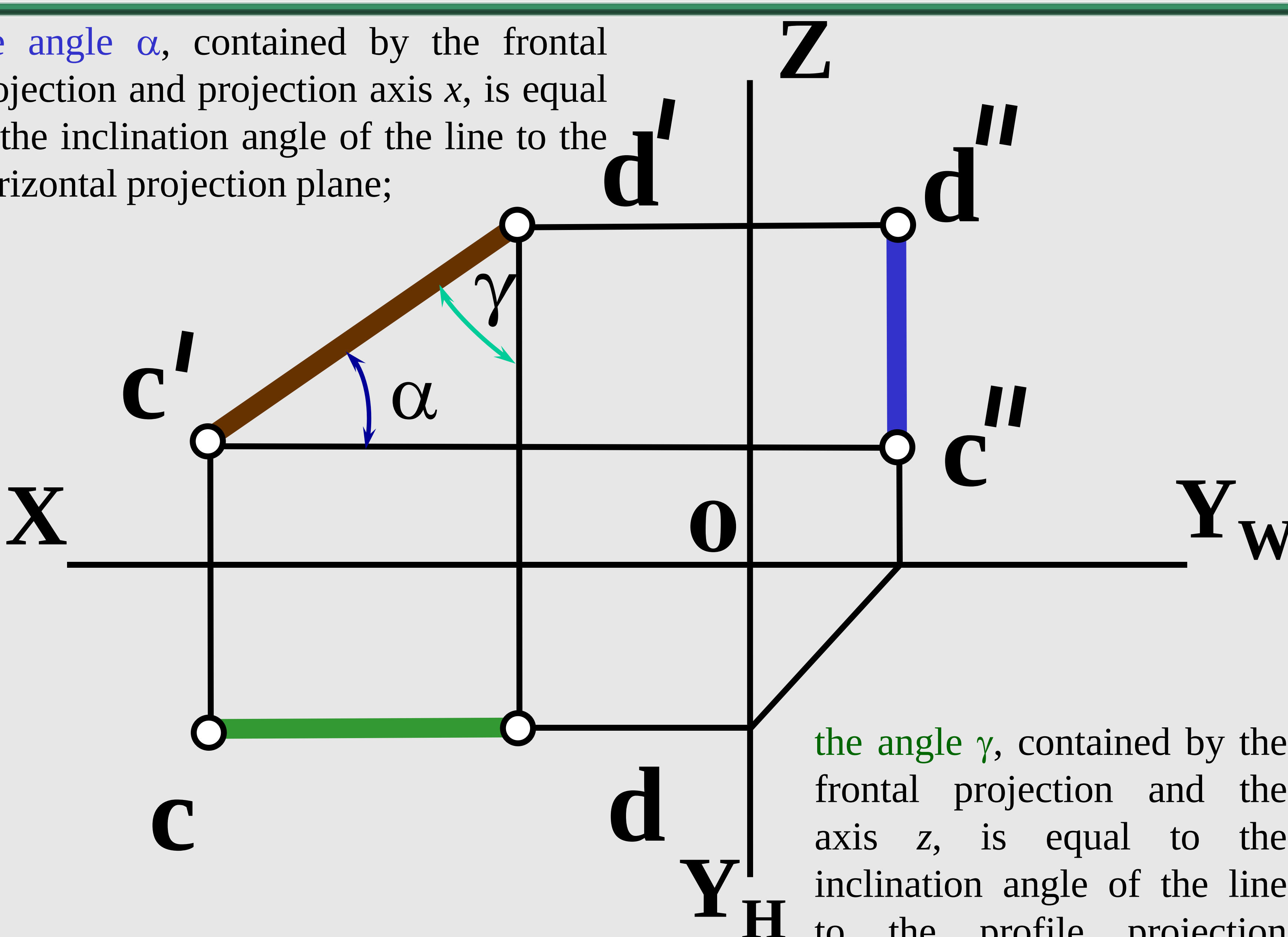


The horizontal projection *cd* of the line is parallel to the axis x;

the profile projection *c''d''* is parallel to the axis *z*;



the angle α, contained by the frontal projection and projection axis *x*, is equal to the inclination angle of the line to the horizontal projection plane;



the angle γ, contained by the frontal projection and the axis *z*, is equal to the inclination angle of the line to the profile projection plane.

Properties of projections

$$|c'd'| = |CD|;$$

$$(cd) \,//\, (OX), (c''d'') \,//\, (OZ);$$

$$(CD \wedge H) = (c'd' \wedge OX) = \hat{\alpha};$$

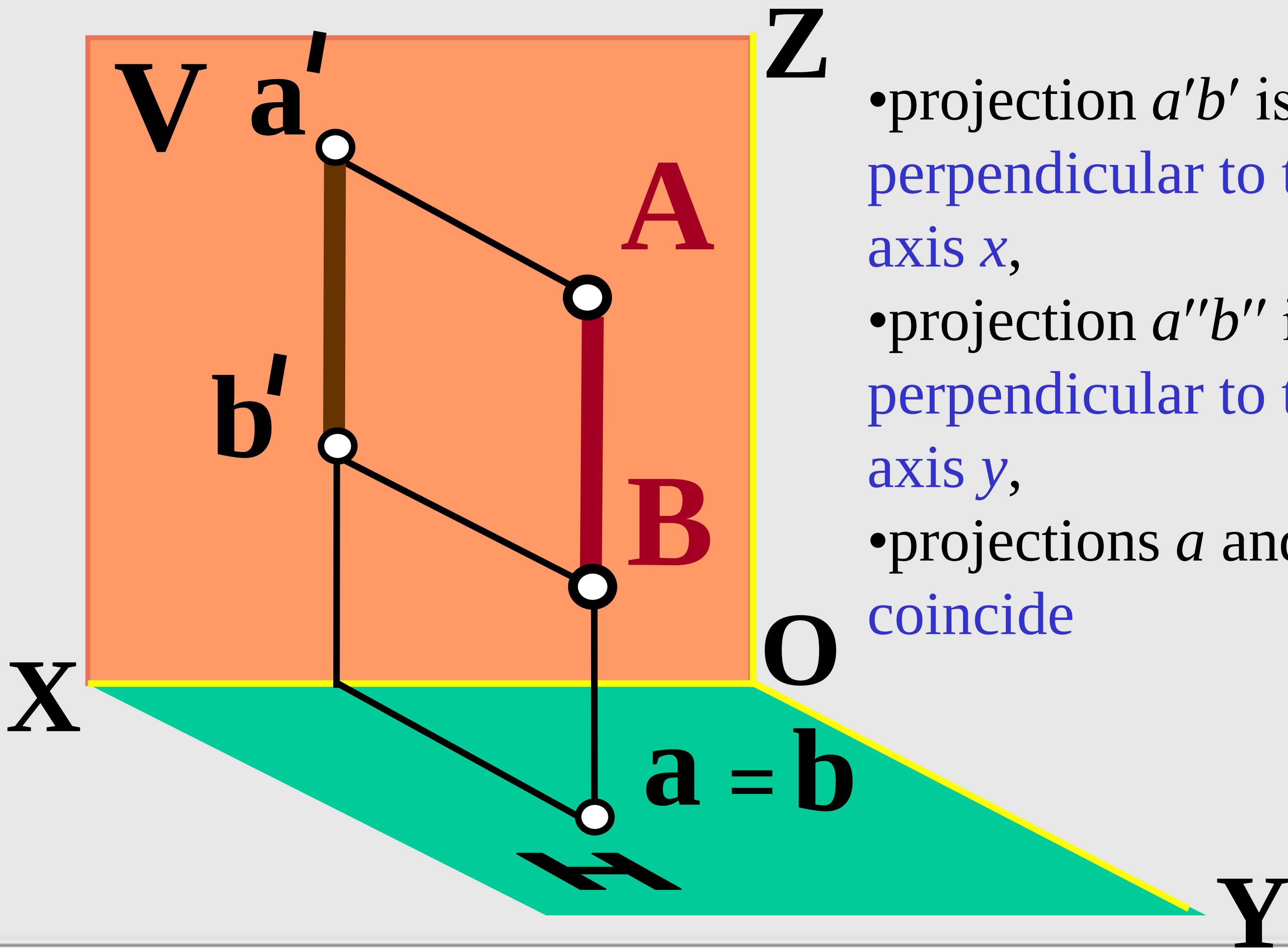
$$(CD \wedge W) = (c'd' \wedge OZ) = \hat{\gamma}.$$

- Each level line projects in true size onto that projection plane to which it is parallel

- The angles contained by this line and two other planes, also project on the above plane in true size

Straight lines perpendicular To planes of projections (projecting lines)

The line *AB* is perpendicular to the plane *H* (horizontal projecting line)



•projection *a′b′* is perpendicular to the axis *x*,

•projection *a″b″* is perpendicular to the axis *y*,

•projections *a* and *b* coincide

Properties of projections

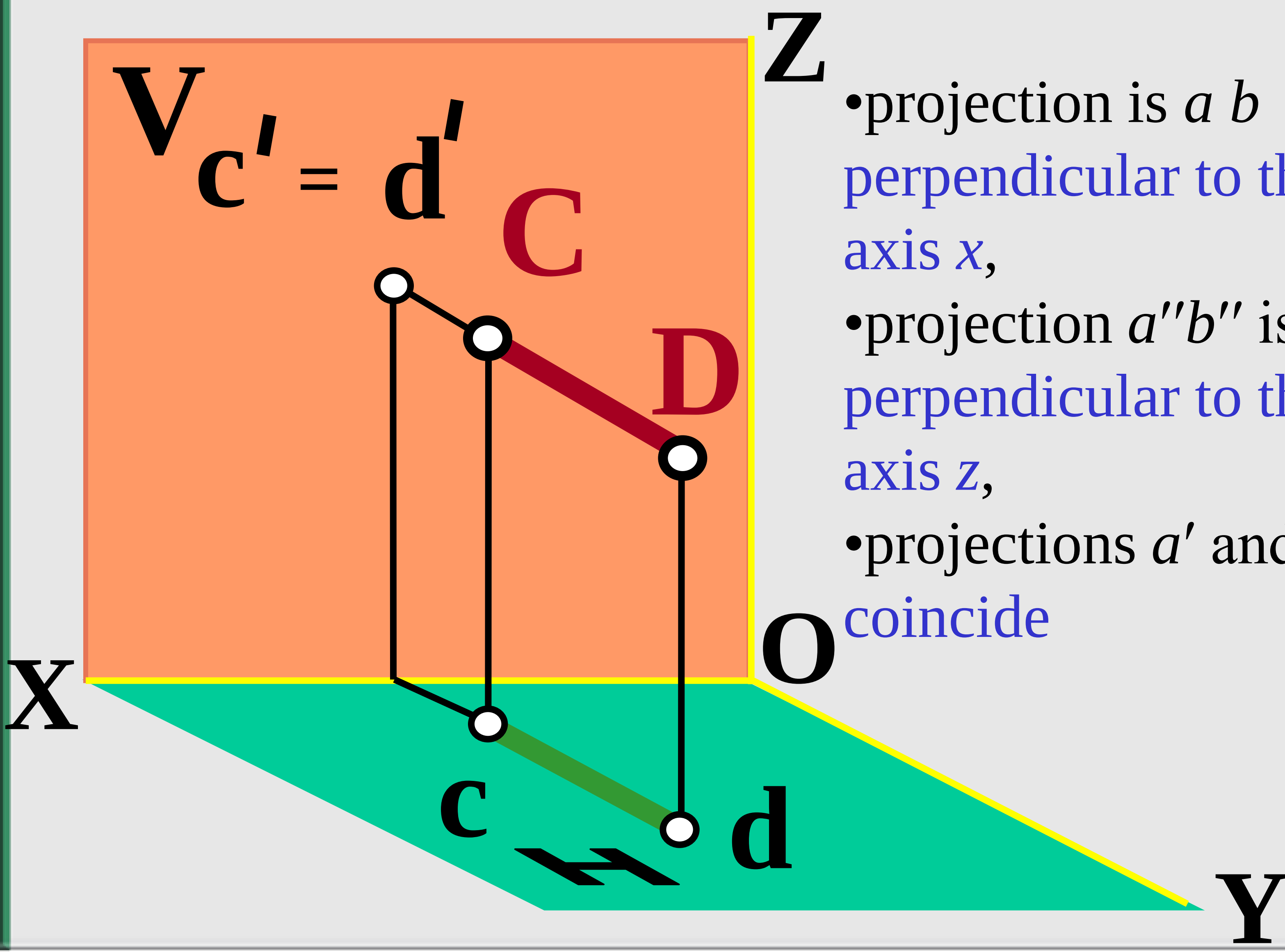
$$(AB)\perp H, (AB)\,//\,V, (AB)\,//\,W;$$

$$ab \text{ - point};$$

$$|a'b'| = |a''b''| = |AB|;$$

$$(a'b')\perp(OX), (a''b'')\perp(OY_W)$$

The line *CD* is perpendicular to the plane *V* (frontal projecting line)



•projection is *a b* perpendicular to the axis *x*,
•projection *a''b''* is perpendicular to the axis *z*,
•projections *a'* and *b'* coincide

Properties of projections

$$(CD)\perp V,(CD)\,//\,H,(CD)\,//\,W;$$

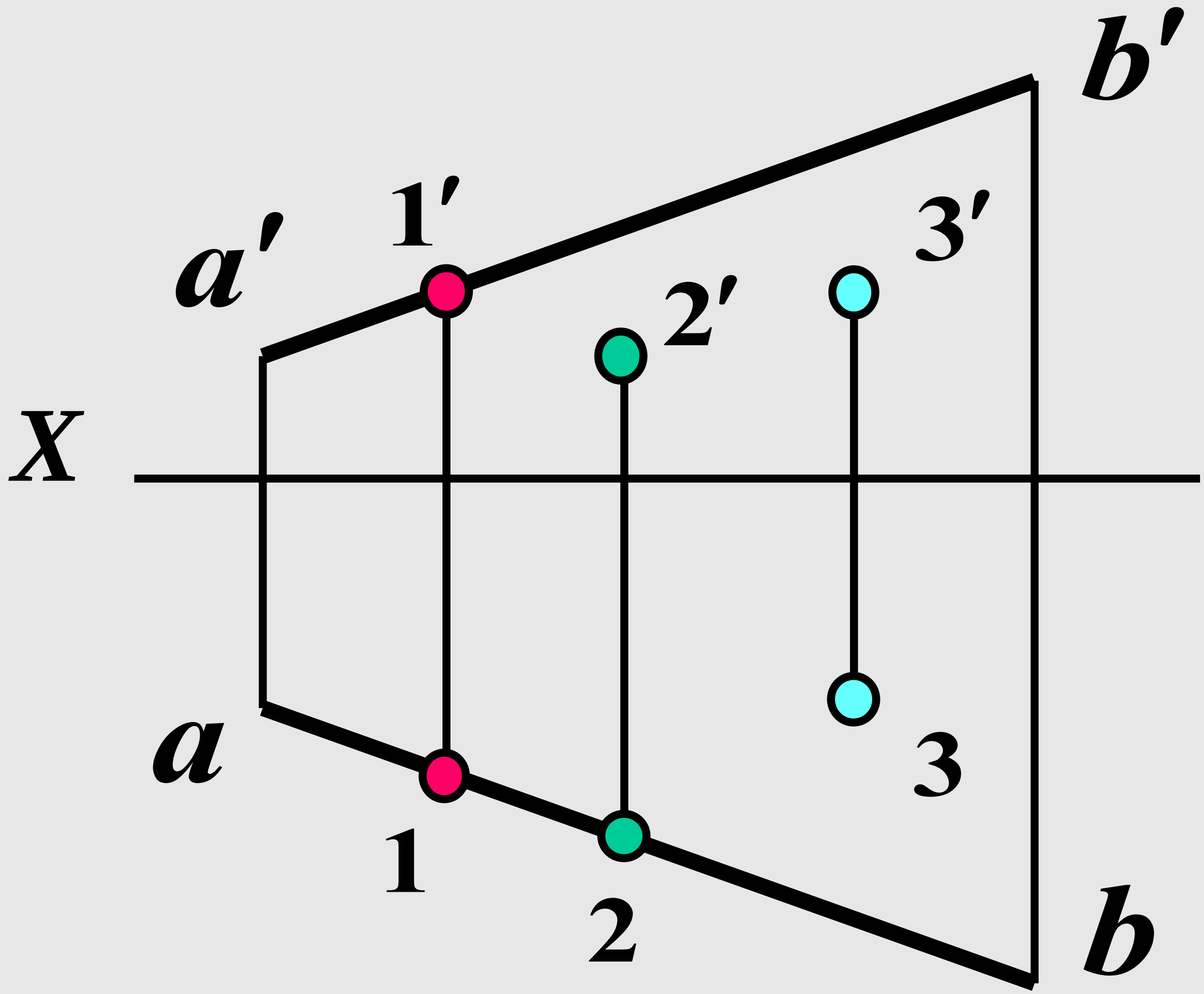
$c'd'$ - point;

$$\left|cd\right|=\left|c''d''\right|=\left|CD\right|;$$

$$(cd)\perp\left(OX\right),(c''d'')\perp\left(OZ\right)$$

The drawing proves that the projecting line is also a level line as it is parallel at the same time to two other projection planes.

Mutual Positions of a Point and a Line



If a point in space belongs to a line, its projections belong to the corresponding projections of the line.

$$(\bullet)1 \in (AB) \Rightarrow (\bullet)1 \in (ab) \wedge (\bullet)1' \in (a'b')$$

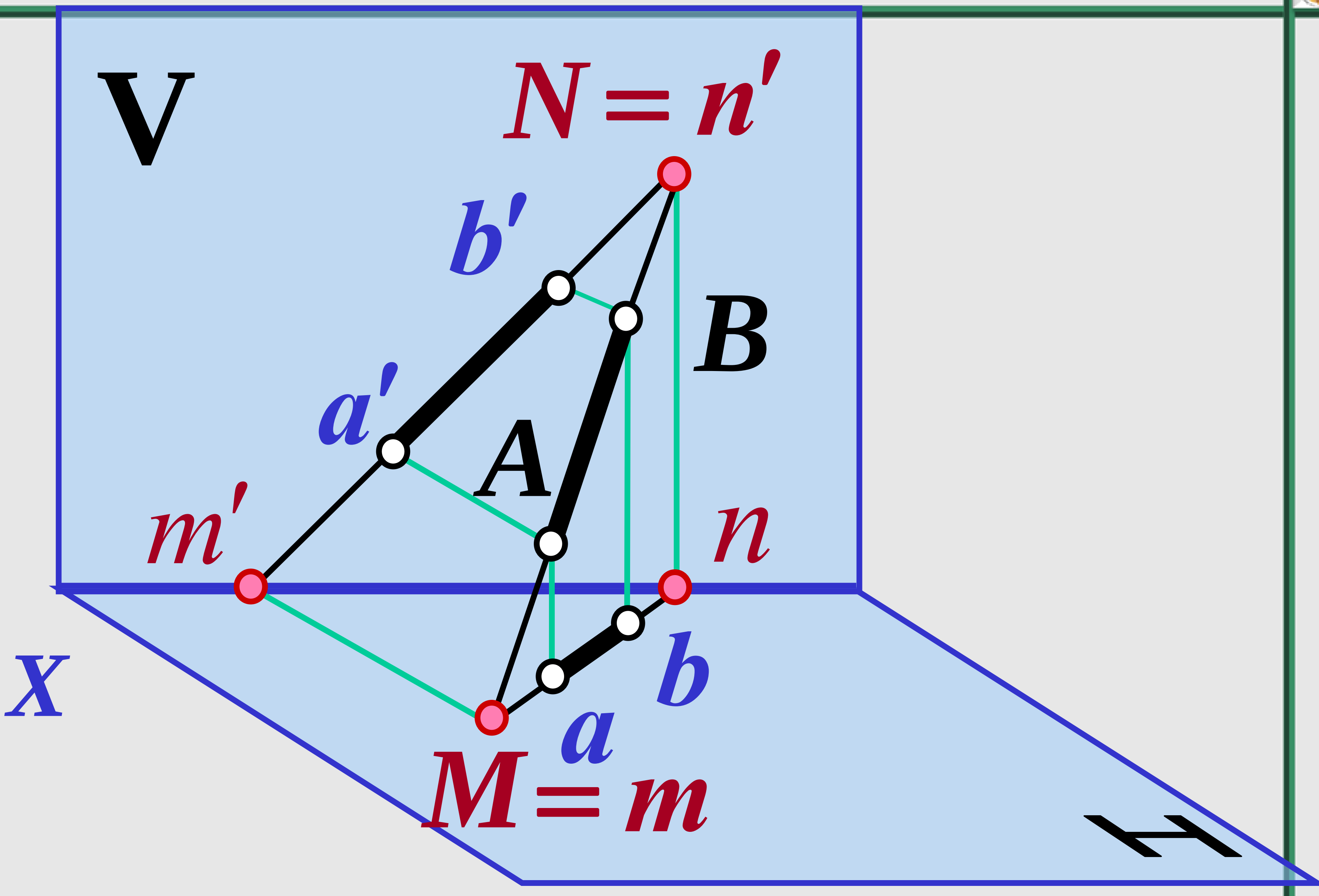
$$(\bullet)2 \notin (AB) \Rightarrow (\bullet)2 \in (ab) \wedge (\bullet)2' \notin (a'b')$$

$$(\bullet)3 \notin (AB) \Rightarrow (\bullet)3 \notin (ab) \wedge (\bullet)3' \notin (a'b')$$

Traces of a Line

The trace of a line is the point at which the line intersects a projection plane.

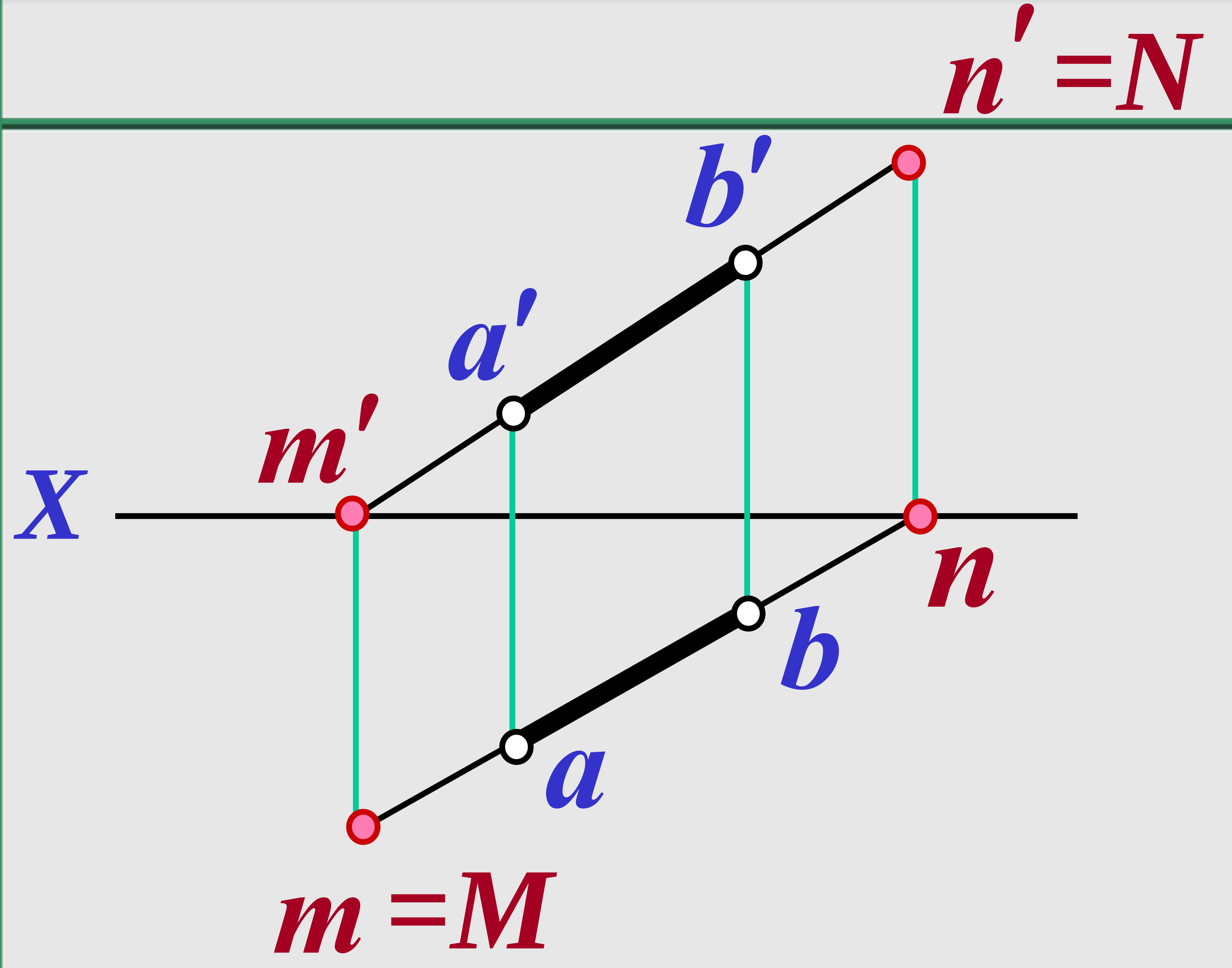
The point *M* is a horizontal trace of the line, the point *N* - a vertical (frontal) one



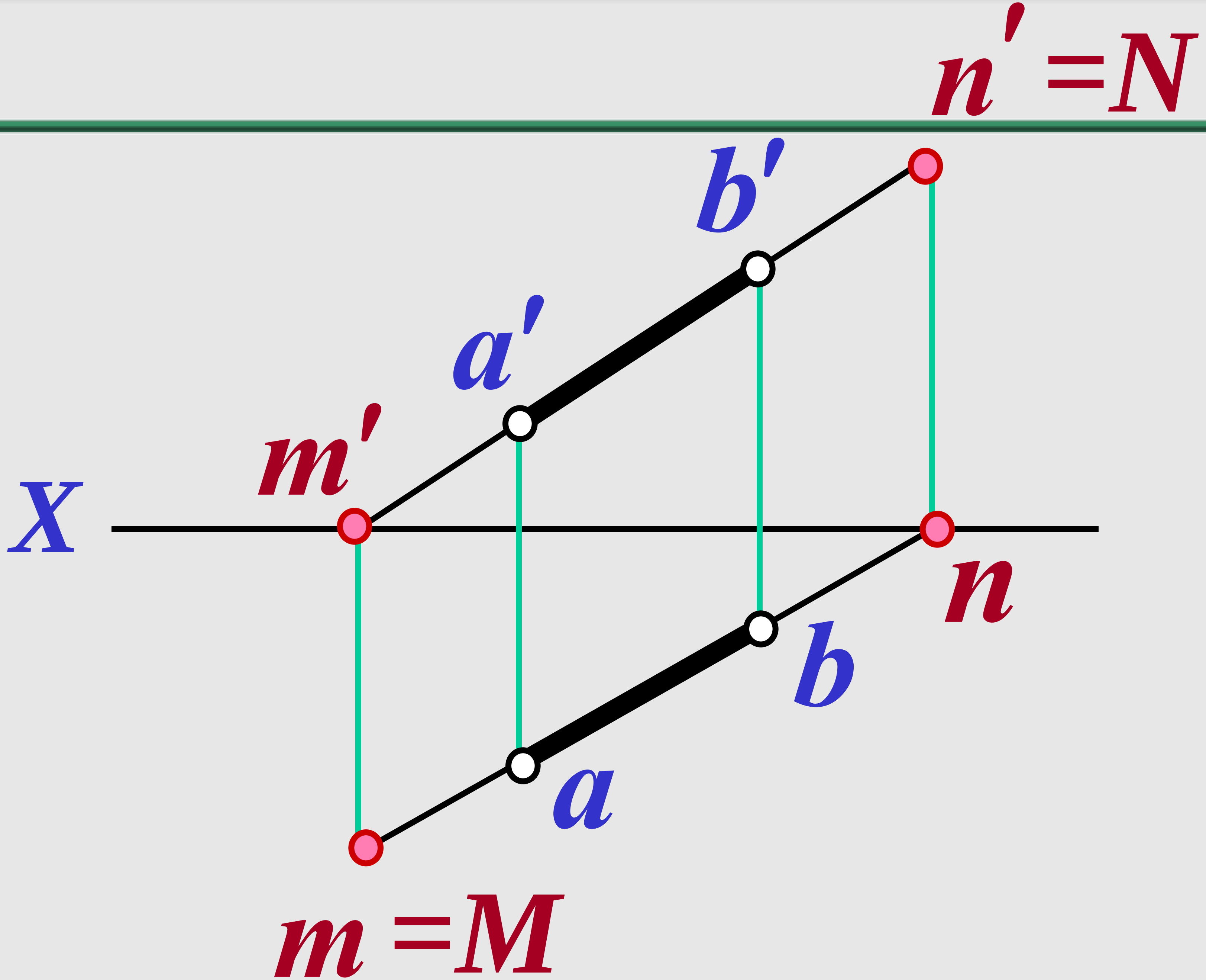
V
N = n′
b′
B
a′
A
m′
n
X
b
a
M = m
H

The horizontal projection *m* of the horizontal trace coincides with the trace proper - the point *M*; the frontal projection *m′* of the trace on the axis *x*. The frontal projection *n′* of the line vertical trace coincides with the vertical trace - the point *N*; the horizontal projection *n* on axis *x*.

To construct the horizontal trace of a line in a plane drawing proceed as follows: prolong the vertical projection *a′b′* of the line to intersect the *x*-axis at the point *m′*. At this point erect a perpendicular to the coordinate axis to intersect the prolongation of the horizontal projection *ab*. The point *m* thus obtained is the horizontal projection of the horizontal trace.


n′ =N
b′
a′
m′
X
n
b
a
m =M

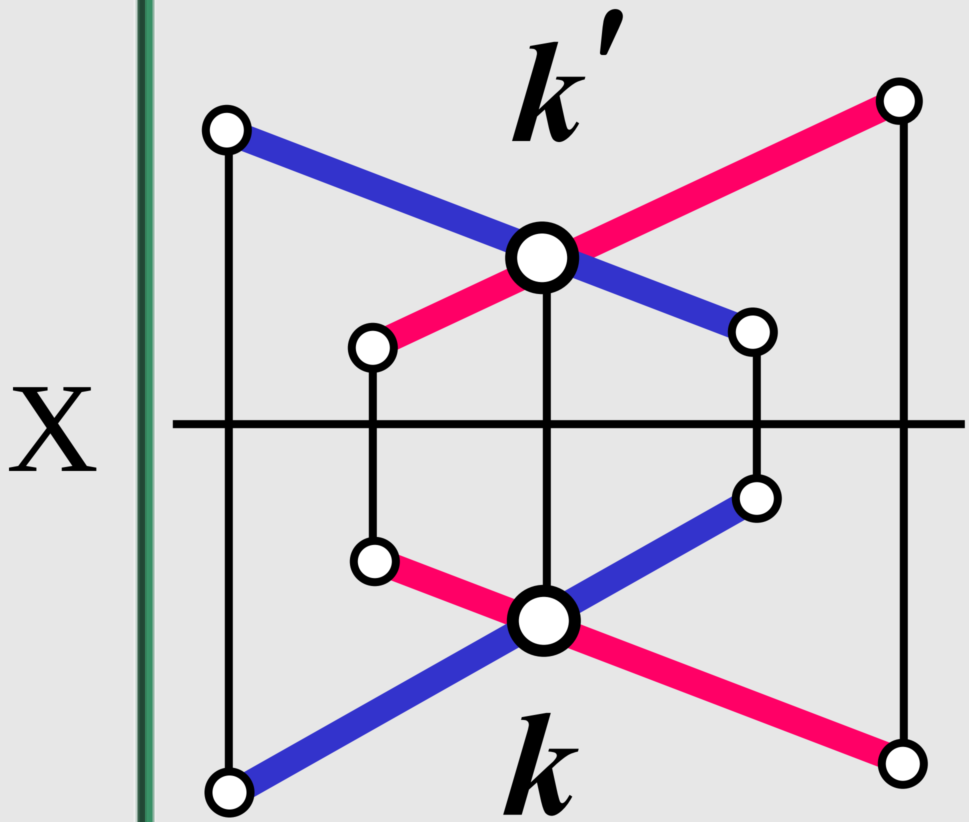
The vertical trace of a line is found in much the same manner: prolong the horizontal projection *ab* of the line to intersect the *x*-axis at the point *n*. At this point erect the perpendicular to the *x*-axis to intersect the prolongation of the frontal projection *a′b′*. The point n’ is the frontal projection of the vertical trace.


n′ =N
b′
a′
m′
X
n
b
a
m =M

The Relative Positions of Two Straight Lines

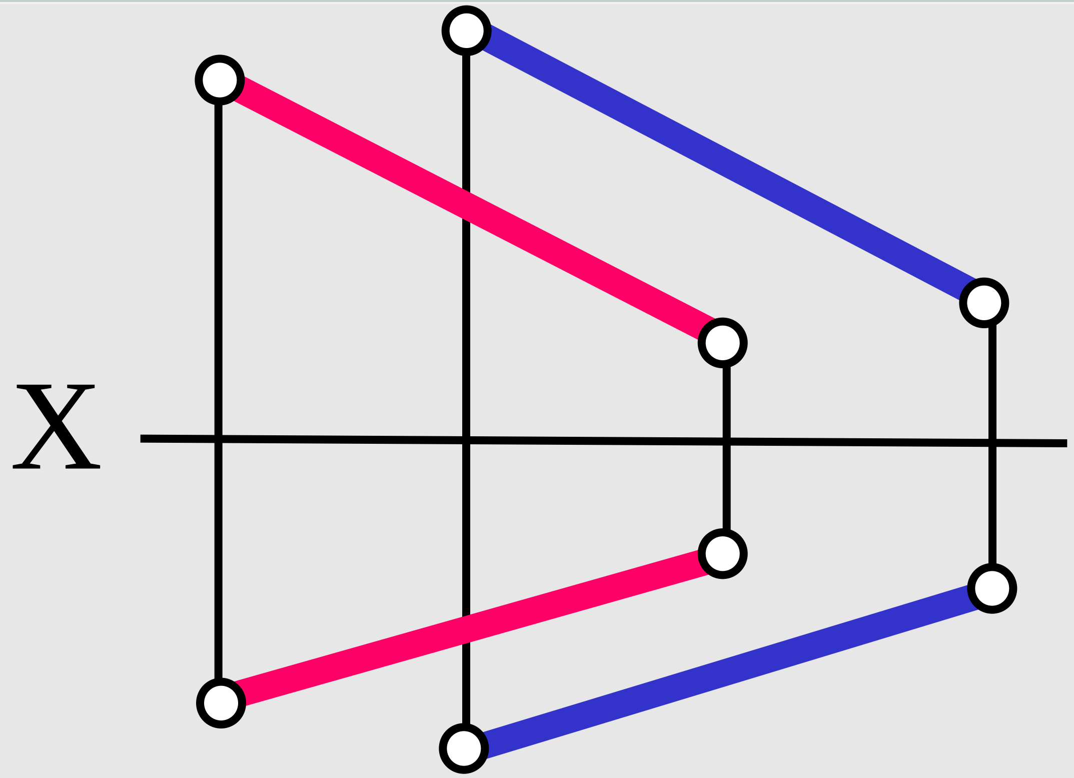
Straight lines in space may have different relative positions:

- to intersect;
- to be parallel;
- to cross, that is to have no common points



Intersecting lines

If the lines intersect, their like projections intersect and the points of intersection lie on one connection line.



Parallel lines. **If the lines in space are parallel, their like projections are also parallel.**

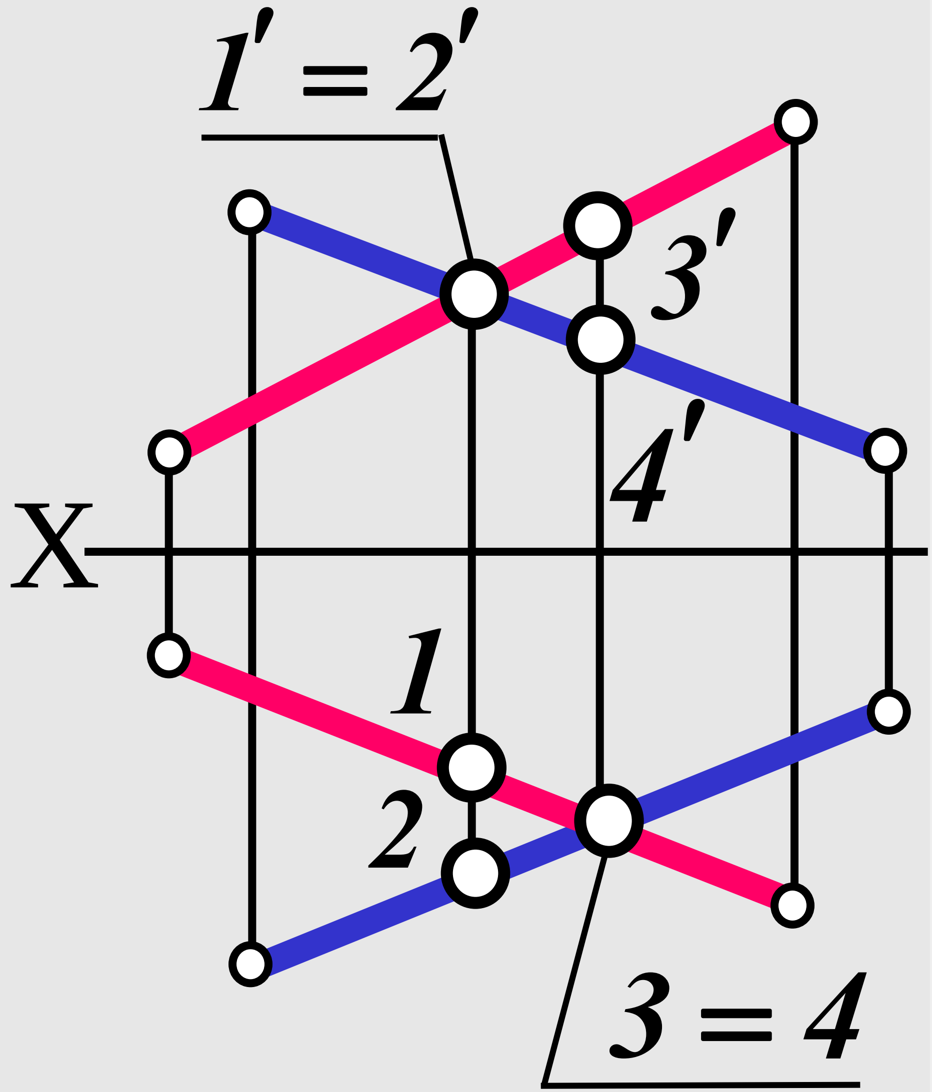
The lines in space are parallel when their like projections on two planes are also parallel, provided:

- for the lines of general position - their like projections are parallel in the system of any two projection planes
- for the lines of particular position - their like projections are parallel to one of the projection axis and their like projections are parallel to each other on that projection plane, to which the above lines are parallel

Skew (to cross) lines.

If the lines in space do not intersect but cross, their like projections in a drawing may intersect , but the intersection points of projections do not lie on one connecting line.

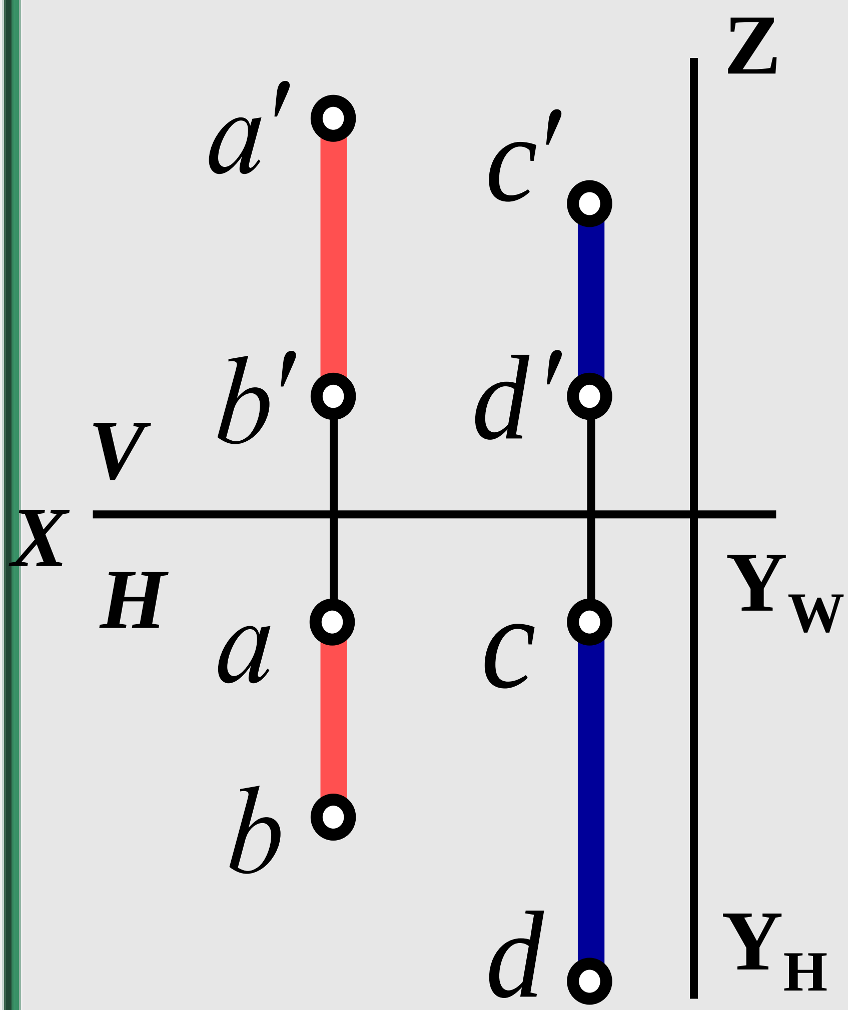
These points are not common to the above lines.



(•)1, (•)2, (•)3, (•)4 -

competitive points.

Considered the above points of skew lines, projections of which coincide on one of the planes, are referred to as competitive points.



Are parallel Whether the data Straight lines?