## ENGINEERING GRAPHICS

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The working book on descriptive geometry and the engineering drawing is developed for first-year students. The writing-book is used for work on lecture employment under the direction of the teacher.

This work book is intended for distance leaning Engineering Graphics for the Certificate of Higher Technical Education.

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## CONTENTS

Preface ..... 5
Chapter 1. From the history of graphic representation ..... 7
Chapter 2. Projection method ..... 9
2.1. Central projection ..... 9
2.2. Parallel projection ..... 11
2.3. Methods of projection drawings supplementation ..... 12
Chapter 3. The point and the straight line ..... 15
3.1. Drawing of point ..... 15
3.2. Mutual positions of two points. Terms of visibility in a drawing ..... 17
3.3. Drawing of a line-segment. Straight lines of a particular position ..... 18
3.4. Mutual positions of a point and a line ..... 22
3.5. Traces of a line ..... 22
3.6. The relative positions of two straight lines ..... 23
3.7. Projecting of plane angles ..... 26
3.8. Determining the true size of a line-segment ..... 26
Chapter 4. The plane ..... 29
4.1. Ways of specifying a plane ..... 29
4.2. The position of a plane relative to the projection planes ..... 30
4.3. The point and the line in the plane ..... 33
4.4. The principal lines of the plane ..... 35
4.5. The relative positions of a line and a plane ..... 37
4.6. Mutual positions of the plane ..... 40
4.7. Method of replacing planes of projection ..... 43
Chapter 5. Surfaces ..... 49
5.1. Determining and specifying surfaces in a drawing. Classification ..... 49
5.2. A point and a line on the surfaces ..... 50
5.3. Polyhedral surfaces and polyhedrons. A polyhedron cut by a plane ..... 51
5.4. Conical and cylindrical surfaces. Torses ..... 54
5.5. Rotation surfaces. Rotation surface cut by a plane ..... 55
5.6. Screw surfaces ..... 64
5.7. Mutual intersection of surfaces ..... 66
Chapter 6. Axonometric projections ..... 74
6.1. The method of axonometric projection. Foreshortening factor ..... 74
6.2. Isometric ..... 76
6.3. Dimetric ..... 78
6.4. Representation of a circle and a sphere in axonometry ..... 80
6.5. Oblique axonometry ..... 83
Chapter 7. The principal rules of a drawing representation ..... 86
7.1. Unified system of engineering papers ..... 86
7.2. Formats and title blocks ..... 87
7.3. Scales ..... 90
7.4. Lines ..... 90
7.5. Lettering ..... 92
7.6. Conventional Representations of Materials ..... 95
Chapter 8. Representations ..... 97
8.1. Basic rules and definitions ..... 97
8.2. The views ..... 102
8.3. Sectional views ..... 104
8.4. Sections ..... 108
8.5. Conventions and simplifications ..... 110
8.6. Removed view ..... 114
Chapter 9. Dimensioning ..... 116
Chapter 10. Joints ..... 134
10.1. Detachable and permanent joints ..... 134
10.2. Thread representation and designation ..... 134
10.3. Threaded products and joints ..... 148
10.4. The keyed and splined joints ..... 161
10.5. Permanent joints. Welded, soldered and adhesive joints ..... 165
Index ..... 172

## PREFACE

Engineers create representation of a detail or a product on a sheet of paper as a drawing before it will be manufactured. Teaching a lot of subjects in high school is linked with studying different devices, machines and technological processes by their representations - drawings. So Engineering Graphics is included in number of subjects for training engineers.

Engineering Graphics contains as elements of Descriptive Geometry (theory of geometric objects drawing construction) as Technical Drawing (compose and reading of products drawings). In the process of study students know with main rules and requirements on a process of creation, designing and use of engineering documents. Moreover solving of geometrical problems develops spatial imagination and logical thinking, Without these skills any technical creation is impossible.

After studying the course "Engineering Graphics" students will:

- learn theoretical principles of drawing the points, lines, planes and some kinds of spatial lines and surfaces on a plane;
- familiarize with particular cases of problems solution on mutual belonging and intersection of geometric figures, as well as determining the true size of plane figures;
- master methods of drawing and corresponding conventions of simple objects;
- be able to determine geometric form of simple details by their image, and construct such images;
- acquaint with representation of some joints and connections, be able to read the drawings of technical devices, also construct such drawings following the standards requirements.

This textbook is intended for studying the course "Engineering Graphics" by students of technical specializations of TPU who go through the Bachelor Degree Program. The course is taught in the first semester.

The textbook is accompanied by an workbook and systematic instructions and tasks.

Workbook includes tasks to be performed by the students to consolidate theoretical material. The content of the workbook is in the same sequence as that in the textbook.

Systematic instructions will help the students complete the offered tasks.

The authors will be thankful to you for your suggestions on perfecting the next edition of this textbook.

The following designations are accepted in the present book:

1. Points of space are usually denoted by Latin capital letters $(A, B, C, \ldots$.$) or figures (1,2,3, \ldots$.
2. Sequence of points (and other elements) - by interlinear indexes $\left(A_{1}, A_{2}, A_{3}, \ldots B_{1}, B_{2}, B_{3}, \ldots\right)$
3. Lines in space - by the points specifying the given line $(A B, C D, \ldots)$
4. Angles - by Greek small letters ( $\alpha, \beta, \gamma, \ldots$ )
5. Planes - by Latin capital letters ( $P, R, Q, \ldots$ )
6. Surfaces - by Greek capital letters ( $\Psi, \Phi, \Omega, \ldots$ )
7. Projection planes:
horizontal - $H$;
frontal $-V$;
profile - $W$
Projection centre - $S$
8. Coordinate axes system - $x y z O$, where:
abscissa axis - $x$;
axis of ordinates $-y$;
applicate axis $-z$;
origin of coordinates - $O$ (capital letter);
new projection axes obtained at planes replacing $-x_{1}, y_{2}, .$.
9. Point projections - by the corresponding lower-case letters ( $a, b$ )
for horizontal projection plane $-a$;
for frontal projection plane $-a^{\prime}$;
for profile projection plane $-a^{\prime \prime}$;
10. Line projections - by projection of the points specifying the line - $a b, a^{\prime} b^{\prime}, a^{\prime \prime} b^{\prime \prime}$.
11. Coincidence, identity - $\equiv$
12. Coincidence, equality - =
13. Parallelism - ||
14. Perpendicularity $\quad-\perp$
15. Crossing - $\div$
16. Representation $-\rightarrow$
17. Belonging of an element (a point) to a set (line, plane, etc.) $-\in$
18. Belonging of a subset (a line) to a set (plane, surface) $-\subset$
19. Intersection of sets $-\cap$

## CHAPTER 1. FROM THE HISTORY OF GRAPHIC REPRESENTATIONS

Graphic representations appeared at the early stages of the development of human society. Judging by those, which have been kept safe till nowadays, we can realise that most of them were connected with trade and handicrafts.

The first representations have been produced by the simplest tools, in the form of drawings outlining only the shape of things. But further development of man's manufacturing activities required more accurate representations of spatial objects.

Construction of fortresses and different fortifications demanded their preliminary imaging on the plane. The remnants of grand antique buildings prove that different plans and other representations of the erecting constructions have been used by the ancient experts.

Together with the development of graphic representations there evolved a science determining the rules and theory of the process. The first manuscripts in this field appeared in 3-5 ages B.C. They were the works by Hippokrates, Pithagoras, Archimedes and others. After them many outstanding scientists continued the development of the field. An Italian scientist, Leon Battista Alberti (1404-1472) presented the basis of the theoretical perspective. An ingenious Italian artist, Leonardo da Vinci (1471 - 1519) filled it up with the doctrine "About Decrease of Colours and Contour Precision". A German artist and engraver Albrecht Durer (1471 1528) contributed the development of perspective. His method of perspective construction, given two orthogonal projections, is widely known. An Italian scientist Gvido Ulbani (1545-1607) can by right be considered a founder of the theoretical perspective, as his works contain the solutions of nearly all principal problems on it. A French architect and mathematician Desargues (1593-1662) was the first to apply the method of coordinates to construct the perspective, and became the founder of axonometric method in descriptive geometry.

At the end of XVIII century a French scientist Jasper Monge (1746 1818) summarised the knowledge on the theory and practice of imaging, and created a clear scientific discipline about rectangular projections. In 1798 he published his work "Descriptive Geometry" in which suggested to consider a plane drawing containing two projections to be a result of coincidence of two mutually perpendicular projection planes. This coincidence is obtained by rotation of the planes round their intersection line. Later the line was called "projection axis".

In ancient Russia the graphics developed intensively but in its own original way. Some ancient drawings produced according particular rules are now available, such as: a plan of Pskov-town (1581), a drawing of the Moscow Kremlin (1600), "Siberian Book of Drawings" compiled by Semyon Remezov in 1701.

Evolution of technics, inventions and discoveries gave a new impulse to the development of representation means. In 1763 I.I.Polzunov produced a drawing of a factory steam machine invented by him. Some drawings of a self-taught mechanic I.P.Kulibin have also been kept. For example, drawings of a single span arch bridge over the Neva river (1773).

When in 1810 the Institute of Railway Engineering Corps was opened in Petersburg, among the other subjects there was taught a course of descriptive geometry. Carl Pottier, one of J.Monge's pupils, was the first lecturer there. Since 1818 the lectures on descriptive geometry have been delivered by Professor Y.A.Sevostyanov (1796-1849). In 1821 he published an original course named "Foundation of Descriptive Geometry". It happened to be the first textbook on descriptive geometry in Russia in the Russian language.

In Tomsk Polytechnic University the graphic disciplines have been taught since 1900. The first lecturer on descriptive geometry was V.Jhons.

Further development of descriptive geometry in Russia is closely connected with the names of M.I.Makarov (1824-1904), V.I.Kurdyumov (1853-1904), E.S.Fyodorov (1853-1919) and other scientists.

Professor V.O.Gordon (1892-1971), Academician N.F.Chetverukhin (1891-1974), Professor I.I.Kotov (1909-1976) and others greatly contributed to scientific researches on graphic representations, also to teaching descriptive geometry and drawing in the colleges and universities of our country.

Diversity of the drawings produced required unification of the rules and conventions of their production. In Russia it is regulated by National Standards of Russia and by international standards of ISO (International Standards Organisation).

Fulfilling the training drawings and other graphical papers students must follow the above standards which will be presented within the course study.

## CHAPTER 2. PROJECTION METHOD

Images of three-dimensional space objects on a plane are made by the projection method.

The projector includes a projectable object, projecting beams and the plane on which the picture of the object appears.

### 2.1 Central Projection

Central projection is the general case when geometric images are projected on a given plane from a selected outside centre. Figure 2.1 - Point $S$ is the Projection Centre, Plane $P$ is the Projection Plane. To make a central projection of a point, draw a projecting line through the point and the projection


Fig.2.1 centre. The intersection point of this line and the projection plane turns out to be the central projection of the given point on the selected plane.

The $a, b, c, d$ points are the central projections of the $A, B, C, D$ points on the $P$ plane.
$B$ and $C$ are two different points which are situated on one projecting line. That is why the central projections $b$ and $c$ coincide. The whole set of points of one projecting line has only one central projection if there is only one projection centre.

So, on the given plane and given projection centre one point in space has one central projection. But one central projection of a point does not allow us to determine the location of the point in space unambiguously, i.e. in central projecting there is no reversibility of drawing and additional conditions are required to provide drawing reversibility.

Central projection provides imaging of any line or surface as a set of projections of all its points. With this, the projecting lines, collectively, drawn through all points of a curved line, either make a projecting conical surface (Fig. 2.2) or can appear on one plane (Fig.2.3).

The projection of a curved line is a line of intersection of the projecting conical surface and projection plane. Fig.2.2 - the projecting conical surface $\Phi$ intersects the projection plane $P$ along an $a b$ curve which is an $A B$ line projection. However, one projection of a line does not determine the line
projected as there can be an infinite quantity of lines projecting to one and the same line in the projection plane.


Fig.2.2


Fig.2.3

The plane serves as a projection surface if a line which does not pass through the projection center is being projected. Figure 2.3 - the projecting plane $Q$ formed by the $S C$ and $S D$ projection lines which pass through the $C$ and $D$ points of a straight line, intersects the projection plane $P$ along the $c d$ line which is the projection of the above line. As the $M$ point belongs to the $C D$ line then its projection - $m$ point - belongs to the $c d$ projection.


Fig.2.4

The construction of projections of lines, surfaces or bodies very often requires the construction of projections of some characteristic points only. For example, to construct a projection of the $A B C$ triangle (Fig.2.4) it is enough to build projections of its three points - $A, B, C$ vertices.

To summarize the material above let us note the following properties of central projection:
I. Under central projection
a) a point projects to a point;
b) a line not passing through the projection center projects to a line (projecting line - to a point);
c) a plane (two-dimensional) figure not belonging to the projecting plane projects to a two-dimensional figure (figures belonging to the projecting plane project to straight lines);
d) a three-dimensional figure projects to a two-dimensional one;
e) a central projection of figures keep mutual belonging, continuity and some other geometric properties
2. Under the given projection center figures of parallel planes are similar.
3. Central projection provides unambiguous similarity of the figure and its projection, for example, projection on a screen or a film.

In spite of visualization, a great positive property, central projections still have some disadvantages such as the complexity of a subject image construction and its real dimension determination. That is why this method is limited in usage. Central projection is used in constructing building perspectives, in painting, etc.

### 2.2 Parallel Projection

Parallel projection is one of the cases of central projection, where the center of projection is infinitely distant ( $S \infty$ ). In this case parallel projecting lines are used, drawn in the given direction relative to the plane of projections. If the direction of projecting is perpendicular to the plane of projections, the last are called rectangular or orthogonal. In other cases they are called oblique projections. In Fig.2.5 the direction of projection is


Fig.2.5 shown by the arrow at an angle $\alpha \neq$ $90^{\circ}$ with the projection plane $P$.

Parallel projection possesses all the properties of central projection and the following new ones:

1. Parallel projections of mutually parallel lines are parallel, and the ratio of those lines segments is equal to the ratio of their projections lengths.
2. A plane figure parallel to a projection plane projects to the same figure on the above plane.
3. A parallel transfer of a figure in space or in projection plane does not change the shape or dimensions of the figure projection.

Using parallel projection of a point and a line, one can draw parallel projections of a surface and a body. Neither parallel nor central projections provide drawing reversibility.

### 2.3. Methods of Projection Drawings Supplementation

In projecting on one projection plane there is no one-to-one correspondence between the figure and its projection. At the given position and set direction, only one projection corresponds to the subject projected.


Fig.2.6

However, the figure obtained can be a projection of an infinite set of other figures, different in size and form. Figure 2.6 shows that m is the only projection of the spatial point $M$ on the plane $P$. At the same time the point $m$ is the projection of a set of points situated on the projecting line ( $M, M_{1}, M_{2}, M_{3}$ ). The segment $m n$ can be a projection not only of the segment $M_{1} N_{1}$ or $M_{2} N_{2}$, but a projection of the curve $M_{3} N_{3}$ and of any plane figure situated in the projecting plane.

Therefore, representation of the figure is not complete. The drawing can be understood clearly only if there are some additional clarifications.
Let us consider a few supplementary methods of the projection representation which provide the opportunity to make it "reversible", i.e. unambiguously similar to the object projected.

## Method of Projection with Digital Marks

This method is primarily used in drawings of landscape plans and some engineering facilities (such as dams, roads, dikes, etc.). The essence of the method is that the position of any point in space is specified by its rectangular projection on a horizontal plane taken for a zero plane (Fig.2.7).


Fig.2.7
Near the projections of the points $(a, b, c)$ their marks are shown, i.e. a number of length units, determining the distance from a point to the projection plane.

## Method of Vector Projections

Academician E.S. Fyodorov suggested the representation of the points altitudes (heights) by means of parallel seg-


Fig.2.8 ments in the projection plane, the beginnings of which are situated on the projections of the respective points. The directions of all altitude segments are arbitrary. If the points are located above the horizontal plane, the altitude segments, as well as digital marks are considered to be positive. In the opposite case (if the points are below the plane) - negative. Positive and negative altitude segments in "Fyodorov's projections" differ in direction. Drawings produced by method of "Fyodorov's projections" are used in geology, mining, topography (Fig.2.8).

## The Method of Orthogonal Projections

Drawing in a system of rectangular projections is constructed by projecting a component on two or three mutually perpendicular planes of projection. This method is a special case of parallel projection, when the direction of projecting is perpendicular to the projection plane. The foot of the perpendicular drawn from a point to the projection plane is called the rectangular (orthogonal) projection of the point.

Jasper Monge was the first to make the projection on two mutually perpendicular planes. Such a kind of projection ensures drawing reversibil-
ity, i.e. an unambiguous determination of a point position in space by its projections. One of the planes is usually arranged horizontally, and it is called the horizontal projection plane $H$, the other is


Fig.2.9


Fig.2.10 perpendicular to it. Such a vertical plane is called the frontal projection plane $V$. The line of intersection of the above planes is called the projection (or coordinate) axis (Fig.2.9).

To obtain a point projection on a plane drop perpendiculars (projecting beams) from the spatial point $A$ to the planes $H$ and $V$. The projecting beams produce the plane $P$ which is perpendicular to the planes $H$ and $V$ and cuts these planes along the lines perpendicular to the projection axis, and the axis proper - in the point $a_{x}$. Consequently, the lines $a a_{x}, a^{\prime} a_{x}$ and the axis $x$ are mutually perpendicular.
Construction of a certain spatial point $A$ from given its two projections (the horizontal one $a$ and the frontal one $a^{\prime}$ ) is shown in Fig.2.10. Find the point $A$ in the intersection of the perpendiculars drawn from the projection $a$ onto the plane $H$ and from the projection $a$ ' onto the plane $V$. The above perpendiculars belong to one plane P perpendicular to $H$ and $V$ planes, and intersect in the only spatial point $A$, which is the desired one.
Thus, two orthogonal projections of a point fully determine its position in space relatively to a given system of mutually perpendicular projection planes.

## Questions to Chapter 2

1. What is the method of construction of the central projection of a point?
2. In what case is the central projection of a straight line represented by a point?
3. What is the essence of the parallel projection method?
4. How is the parallel projection of a line constructed?
5. Can the parallel projection of a line be represented by a point?
6. What are the positions of a point and a line projections if the point lies on the line?

## CHAPTER 3. THE POINT AND THE STRAIGHT LINE

To obtain a clear understanding of the all external and internal forms of the components and their joints as well as to be capable of solving other problems, it is usually necessary to have three or more views of each detail. That is why there can be three or more projection planes.

Into the system of the planes $H$ and $V$, let us introduce one more plane, perpendicular to them. This plane is called the profile projection plane and is denoted by letter $W$ (Fig.3.1). The plane $W$ intersects the planes $H$ and $V$ along the lines $y$ and $z$ (axes of projection). The intersection point of all axes is called origin of co-ordinates and is designated by letter $O$ (the first letter of the Latin word "origo" which means "origin"). The axes $x, y, z$ are mutually perpendicular.


Fig.3.1


Fig.3.2

The three mutually perpendicular planes divide space into eight parts, eight octants, Fig. 3.2 (from Latin "octo" - "eight"). Most of the Continental countries adopted the right-handed system, the so-called "European system of projection positioning". The axis $x$ is directed to the left from the origin of co-ordinates, $y$ - forward (to us), $z$ - upward. The opposite directions of the co-ordinate axes are considered negative.

### 3.1. Drawing of Point

Rectangular projections of a point (in the $H, V$ and $W$ planes) are always obtained as the bases of perpendiculars, dropped from the given point onto each of projection planes:

$$
a-\text { horizontal; } \quad a^{\prime}-\text { frontal; } \quad a^{\prime \prime}-\text { profile } .
$$

Representation of a point in the system of the planes $H, V$ and $W$ shown in Fig.3.3, is too complex and, therefore, is inconvenient for drawing. Let us convert it so that the horizontal and profile planes will coincide with the frontal projection plane, forming one drawing plane (Fig.3.4).


Fig.3.3


Fig.3.4

Such a conversion can be realized by the rotation of the plane $H$ around the axis $x$ down at the angle of $90^{\circ}$, and of the plane $W$ to the right around the axis $z$, at the angle of $90^{\circ}$. As a result of the above coincidence obtain the drawing known as "epure of Monge" or "orthographic drawing of Monge" (from French "epure" - drawing, projection).

Having used the orthographic drawing we have lost the spatial picture of the projection planes and point positioning. But epure ensures representational precision and convenience of dimensioning, at a considerable simplicity of drawing.

For simplicity, epure of Monge as well as projection drawings based on the method of Monge are called drawings (or complex drawings) in this book.

Horizontal and frontal projections of a point ( $a$ and $a^{\prime}$ ) are situated on one perpendicular to the $x$-axis - on the connection line $a a^{\prime}$; frontal and profile projections ( $a^{\prime}$ and $a^{\prime \prime}$ ) are situated on one perpendicular to the $z$-axis on the connection line $a^{\prime} a^{\prime \prime}$.

Construction of a profile point projection, given its frontal and horizontal projections, is shown in Fig.3.4. For the construction one may use either a circular arc passed from the point $O$, or bisectrix of the angle $y_{H} O y_{W}$. The first of the methods is more preferable as it is more precise.
Thus, in a complex drawing consisting of three orthogonal point projections: two projections are situated on one connection line; connection lines are perpendicular to the projection axes;
two projections of a point specify the locus of its third projection;
two projections of a point specify its locus in space.
The location of a point in space is specified by means of its three coordinates (abscissa $x$, ordinate $y$, applicate $z$ ), i.e. of three figures showing the
distance from this point to the co-ordinate projection planes. Recording of the point co-ordinates is made in the following form: $A(x, y, z)$.
In regards to the projection planes a point may have a general (the point $A$ ) or particular (the points $B$ and $C$ ) position (Fig.3.5). If a point is located in a projection plane, its two projections lie on


Fig. 3.5 the projection axes (the point $B$ ). One of the co-ordinates of such a point is equal to zero. A point belonging to two projection planes simultaneously (the point $C$ ) lies on a projection axis. Its two projections coincide, the third one merges with the point $O$, origin of co-ordinates. In this case two of its co-ordinates are equal to zero. A point belonging to three projection planes is situated in the origin of co-ordinates.

Therefore, the sizes of the connection lines segments in a drawing specify the numerical distance from a projection point to a projection plane. The segment $a_{x} a$ shows the distance (depth) from the point to the frontal projection plane, the segment $a_{x} a^{\prime}$ - the distance (height) from the point to the horizontal projection plane, and the segment $a^{\prime} a_{2}$ - the distance to the profile projection plane.

### 3.2 Mutual Positions of Two Points. Terms of Visibility in a Drawing

Consider a model drawing shown in Fig.3.6. Projections of some points coincide as they are situated on one projecting line. For example, images $a$ and $b$ of the vertices $A$ and $B$ on the horizontal projection merged into one point as they lie on one horizontal projecting line. On the frontal projec-


Fig. 3.6 tion representations c and d of the vertices $C$ and $D$ merged into one point as they lie on one frontal projecting line.

The points lying on one projecting line are called competitive points. $A$ and $B$ are horizontal competitive points, $C$ and $D$ - frontal competitive points, etc.

It is obvious that if two points were situated on one projecting line, one of them covers the other. How to determine which of them is visible and which
is invisible one?
Of two horizontal competitive points in the horizontal plane that one is visible which is situated higher in space.

The frontal projection (Fig.3.7) shows that the $z$-co-ordinate of the point $A$ is bigger than that of the point $B$. Hence, the point A is located higher than $B$ and being projected onto the horizontal projection plane it covers the point $B$. On the horizontal projection $A$ is visible, $B$ - invisible. On the frontal one both of the points are visible. Of two frontal competitive points in the frontal plane that one is visible which is situated closer to a viewer facing the frontal projection plane (Fig.3.8).


Fig. 3.7


Fig. 3.8


The horizontal projection helps to determine which of the points is located closer to the viewer. For example, comparing the horizontal projections of the points $D$ and $C$ one can conclude that the point $C$ is visible on the plane $V$, and $D$ - is invisible.

Likewise determine the visible point of two profile competitive ones in the projection plane $W$ - that one is visible which is situated closer to the left side.

So, when projections of the like points in a drawing do not coincide, or only one projection pair coincides, these points do not coincide in space and are located a certain distance from each other.

### 3.3 Drawing of a Line-Segment.

 Straight Lines of a Particular Position and its orthogonal projecting onto the plane $P$. Let us consider orthogonal projecting of the segment $A B$ subject to the properties of parallel projection. The projecting lines $A a$ and $B b$ passed from the points $A$ and $B$ produce the projecting plane $Q$. The cutting line of the planes $Q$ and $P$ passes through the projections
$a$ and $b$ of the points $A$ and $B$ on the projection plane $P$. This very line is the only projection of the above line in the projection plane $P$. Fig. 3.10 presents a visual picture of the segment $A B$ projecting onto two projection planes in the system $H, V$, Fig.3.11 presents a drawing.


Fig. 3.10


Fig. 3.11

If a point belongs to a line, its projection belongs to the line projection. For example, the point $D$ (Fig.3.9) belongs to the line $A B$, so, its projections belong to the line projections (Fig.3.11).

A line can have different positions relative to the projection planes:

- parallel to neither of the projection planes $H, V, W$;
- parallel to one of the projection planes (the line may as well belong to this plane);
- parallel to two of the projection planes, that is, perpendicular to the third one.
A line parallel to neither of the projection planes is referred to as the line of general position (Fig.3.9 through 3.11).

A line parallel to one of the projection planes or to two (that is, perpendicular to the third one) is referred to as the line of a particular position.


Fig.3.12

Fig.3.12-3.14 show the visual pictures and drawings of the lines of a particular position - the lines parallel to the projection planes. Such lines are called level lines.
a) The line $A B$ is parallel to the plane $H$ (it is called a horizontal line). The frontal projection $a^{\prime} b^{\prime}$ of the line is parallel to the axis; the profile projection $a^{\prime \prime} b^{\prime \prime}$ is parallel to the axis $y_{w}$; the length of the segment horizontal projection is equal to the length of the segment proper $a b=A B$; the angle $\beta$, contained by the horizontal projection and projection axis $x$, is equal to the inclination angle of the line to the frontal projection plane; the angle $\gamma$, contained by the horizontal projection and the axis $y_{t}$, is equal to the inclination angle of the line to the profile projection plane.

$$
\begin{aligned}
& |a b /=| A B / ; \quad\left(a b^{\prime}\right) / /(O x) ; \quad\left(a^{\prime \prime} b^{\prime \prime}\right) / /\left(O y_{w}\right) ; \\
& \left(A B^{\wedge} V\right)=\left(a b^{\wedge} O x\right)=\beta ; \quad\left(A B^{\wedge} W\right)=\left(a \hat{}^{\wedge} O y_{H}\right)=\gamma .
\end{aligned}
$$

b) The line $C D$ is parallel to the plane $V$ (it is a frontal line) - Fig.3.13


Fig.3.13
The horizontal projection $c d$ of the line is parallel to the axis $x$; the profile projection $c^{\prime \prime} d^{\prime \prime}$ is parallel to the axis $z$; the length of the segment frontal projection is equal to the length of the segment proper $c^{\prime} d^{\prime}=C D$; the angle $\alpha$, contained by the frontal projection and projection axis $x$, is equal to the inclination angle of the line to the horizontal projection plane; the angle $\gamma$, contained by the frontal projection and the axis $z$, is equal to the inclination angle of the line to the profile projection plane.

$$
\begin{array}{ll}
\left|c^{\prime} d^{\prime}\right|=/ C D / ; \quad(c d) / /(O x) ; \quad\left(c^{\prime \prime} d^{\prime}\right) / /(O z) ; \\
\left(C D^{\wedge} H\right)=\left(c^{\prime} d^{\prime} \cap\right)=\alpha ; & \left(C D^{\wedge} W\right)=\left(c^{\prime \prime} d^{\prime \prime \wedge} O z\right)=\gamma .
\end{array}
$$

c) The line $E F$ is parallel to the plane $W$ (it is a profile line).

The horizontal projection ef of the line is parallel to the axis $y_{n}$; the frontal projection $e^{\prime} f$ ' is parallel to the axis $z$; the length of the segment pro-
file projection is equal to the length of the segment proper $e^{\prime \prime} f^{\prime \prime}=E F$; the angles $\alpha$ and $\beta$, contained by the profile projection and projection axes $y_{w}$ and $z$, is equal to the inclination angles of the line to the horizontal and frontal projection planes respectively.


Fig. 3.14

$$
\begin{aligned}
& \left(e^{\prime \prime} f^{\prime \prime} \mid=/ E F / ; \quad(e f) / /\left(O y_{n}\right) ; \quad\left(e f^{\prime}\right)=(O z) ;\right. \\
& \left(E F^{\wedge} H\right)=\left(e^{\prime \prime} f^{\prime \prime} \cap O y_{w}\right)=\alpha ; \quad\left(E F^{\wedge} V\right)=\left(e^{\prime \prime} f^{\prime \prime} \wedge O z\right)=\beta .
\end{aligned}
$$

Thus, each level line projects in true size onto that projection plane to which it is parallel. The angles contained by this line and two other planes, also project on the above plane in true size.


Fig.3.15
Fig.3.15 shows drawings of the straight lines perpendicular to the projection planes. These lines are called projecting lines.
a) The line $A B$ is perpendicular to the plane $H$ (horizontal projecting line), its projection $a^{\prime} b^{\prime}$ is perpendicular to the axis $x$, projection $a^{\prime \prime} b^{\prime \prime}$ is perpendicular to the axis $y$, projections $a$ and $b$ coincide.
$(A B) \perp H ; \quad(A B) / / V ; \quad(A B) / / W ;$
$a b-$ point; $\left\langle a^{\prime} b^{\prime}\right|=\left|a^{\prime} b^{\prime \prime}\right|=|A B| ; \quad\left(a^{\prime} b^{\prime}\right) \perp(O x) ; \quad\left(a^{\prime} b^{\prime \prime}\right) \perp\left(O y_{w}\right)$.
b) The line $C D$ is perpendicular to the plane $V$ (frontal projecting line), its projection $c d$ is perpendicular to the axis $x$, projection $c^{\prime \prime} d^{\prime \prime}$ is perpendicular to the axis $z$, projections $c^{\prime}$ and $d^{\prime}$ coincide.
$(C D) \perp V ; \quad(C D) / / H ; \quad(C D) / / W$;
$c^{\prime} d^{\prime}-$ point; $\left|c d /=\left|c^{\prime \prime} d^{\prime \prime}\right|=/ C D / ; \quad(c d) \perp(O x) ; \quad\left(c^{\prime \prime} d^{\prime \prime}\right) \perp(O z)\right.$.
c) The line $E F$ is perpendicular to the plane $W$ (profile projecting line), its projection ef is perpendicular to the axis $y_{H}$, projection $e^{\prime} f$ ' is perpendicular to the axis $z$, projections $e^{\prime \prime}$ and $f^{\prime \prime}$ coincide.
$(E F) \perp W ; \quad(E F) / / H ; \quad(E F) / / V$;
$e^{\prime \prime} f^{\prime \prime}-$ point; $\left|e f /=\left|e^{\prime} f^{\prime}\right|=\right| E F / ; \quad(e f) \perp\left(O y_{n}\right) ; \quad\left(e f^{\prime}\right) \perp(O z)$.
The drawing proves that the projecting line is also a level line as it is parallel at the same time to two other projection planes.
So, projecting lines are projected on two projection planes in true shape, i.e. in true size, and on the third plane they are projected as a point.

### 3.4 Mutual Positions of a Point and a Line

A point and a line in space may have different positions relative to each other and to a projection plane.

If a point in space belongs to a line, its projections belong to the corre-


Fig. 3.16 sponding projections of the line.

Fig.3.12 through 3.14 illustrate the above statement.
Let us examine it again in a plane drawing (Fig.3.16).
The point $F$ belongs to the line $A B$ as the horizontal projection $f$ of the point belongs to the line horizontal projection $a b$, and the point frontal projection $f^{\prime}$ belongs to the line frontal projection $a^{\prime} b^{\prime}$.

The point $C$ is located above the line $A B$, the point $D$ is situated under the line $A B$, the point $E$ lies behind the line $A B$.

### 3.5. Traces of a Line

The trace of a line is the point at which the line intersects a projection plane. The point $M$ (Fig.3.17) is a horizontal trace of the line, the point $N$ - a vertical (frontal) one.

The horizontal projection m of the horizontal trace coincides with the trace proper - the point $M$ (Fig.3.17, a); the frontal projection $m^{\prime}$ of the trace
lies on the axis $x$. The frontal projection $n^{\prime}$ of the line vertical trace coincides with the vertical trace - the point $N$; the horizontal projection n lies on the same projection axis.


Fig.3.17
To construct the horizontal trace of a line (points $m$ and $m^{\prime}$ ) in a plane drawing proceed as follows (Fig.3.17, b): prolong the vertical projection $a^{\prime} b^{\prime}$ of the line to intersect the $x$-axis at the point $m^{\prime}$. At this point erect a perpendicular to the coordinate axis to intersect the prolongation of the horizontal projection $a b$. The point $m$ thus obtained is the horizontal projection of the horizontal trace.

The vertical trace of a line (points $n$ and $n^{\prime}$ ) is found in much the same manner: prolong the horizontal projection $a b$ of the line to intersect the $x$ axis at the point $n$. At this point erect the perpendicular to the $x$-axis to intersect the prolongation of the frontal projection $a^{\prime} b^{\prime}$. The point $n '$ is the frontal projection of the vertical trace.

A line may intersect the profile projection plane as well, it means there may be the profile trace, too. This trace on the profile plane merges with its projection, where as, its frontal and horizontal projections lie on the axes $z$ and $y$, accordingly.

### 3.6.The Relative Positions of Two Straight Lines

Straight lines in space may have different relative positions:

- to intersect, that is to have one common point;
- to be parallel if their intersection point is at infinity;
- to cross, that is to have no common points.

Intersecting lines. If the lines intersect, their like projections intersect and the points of intersection lie on one connection line.


Fig.3.18
Fig.3.18, a presents the visual picture of two lines $A B$ and $C D$, intersecting at the point $K$; Fig.3.18, b - the drawing in the system of $H$ and $V$ planes.

If one of the lines in the $H$ and $V$ planes system is a profile line, to find out where the lines intersect, construct their profile projections.

Fig.3.19 - all projections of the point $K\left(k, k^{\prime}, k^{\prime \prime}\right)$ belong to the lines $A B$ and $C D$ simultaneously, it means that the lines $A B$ and $C D$ intersect.


Fig.3.19


Fig.3.20

Fig.3.20 - the profile projection $k^{\prime \prime}$ of the point $K$ (the $C D$ line) does not belong to the profile projection $a^{\prime \prime} b^{\prime \prime}$, hence, the lines $A B$ and $C D$ do not intersect.
Parallel lines. If the lines in space are parallel, their like projections are also parallel. Fig.3.21 - the projecting planes $Q$ and $R$, passed through the parallel lines $A B$ and $C D$, are also parallel. They intersect the plane $P$ with the parallel lines $a b$ and $c d$, which are the projections of $A B$ and $C D$ lines. A drawing of two parallel lines of general position is shown in Fig.3.22, drawings of parallel lines of particular position - in Fig.3.23:
a) horizontal lines;
b) frontal lines;
c) profile lines.

Note. The lines in space are parallel when their like projections on two planes are also parallel, provided:
for the lines of general position - their like projections are parallel in the system of any two projection planes (Fig.3.22);


Fig. 3.21
Fig. 3.22
for the lines of particular position - their like projections are parallel to one of the projection axis and their like projections are parallel to each other on that projection plane, to which the above lines are parallel (Fig.3.23).
Skew lines. If the lines in space do not intersect but cross (Fig.3.24), their like projections in a drawing may intersect, but the intersection points of projections do not lie on one connecting line. These points are not common to the above lines.

Comparing the loci of such points, determine which of the lines in a drawing is situated higher than the other or closer to the viewer.


Fig. 3.23

Fig.3.24, a - the point $E$ (belonging to the line $A B$ ) is situated above the point $K$ (belonging to the line $C D$ ) and if viewed from above, in the direction of the arrow, the point $E$ covers the point $K$. A similar situation is in Fig.3.24, b - the frontal projection $e^{\prime}$ is located higher than the frontal projection $k^{\prime}$, and if viewed from above, in the direction of the arrow $N$, the point $e$, being projected on the plane $H$, covers the point $k$. The straight line $A B$ runs above the line $C D$.


Fig. 3.24
The frontal projections $1^{\prime}$ and $2^{\prime}$ of the lines $A B$ and $C D$ coincide on the plane $V$. If viewed in the direction of the arrow $M$, the point $l$ of the $A B$ line is situated closer to the observer, so, being projected on the plane $V$ this point covers point 2 of the line $C D$. The line $A B$ lies closer to the observer.

Considered the above points of skew lines, projections of which coincide on one of the planes, are referred to as competitive points.

### 3.7. Projecting of Plane Angles



Fig.3.25

Any linear angle is formed by two intersecting lines. It is usually projected onto the projection planes in distortion. However, if both arms of the angle are parallel to one of the projection planes, the angle is projected on this plane without changing it, i.e. in true size. For example, the arms of $A B C$ angle (Fig.3.25) are parallel to the horizontal plane $P$, hence,
the angle $\alpha$ is projected on it in true size. Note. A right angle with at least one side parallel to one of the projection planes is projected on this plane also as a right angle. Consider a theorem on a right angle projecting.
Theorem. A right angle is projected as a right angle, when one of its arms is parallel to a projection plane and the second arm is not perpendicular to it.
Given: the arm $E D$ of the right angle $K E D$ is parallel to the plane $P$, the arm $K E$ is not perpendicular to it (Fig.3.26). Prove that the angle projection ked $=90^{\circ}$

$$
(E D \perp E K) \wedge(E D / / P)(E K \perp P) \Rightarrow(e d) \perp(e k) .
$$



Fig.3.26
proved.

### 3.8 Determining the True Size of a Line-Segment

No segments of the lines of general position are projected on the planes of projection in true size. But the solution of some problems requires determining by a drawing the length of a line-segment and the angle of the inclination of a line to the projection planes.

In this case the method of a right triangle construction is used.
Theorem. The true length of a segment of a line of general position is equal to the hypotenuse of a right triangle one leg of which is a projection of the given line-segment on one of the projection planes, the second leg being equal to the absolute value of the algebraic difference of the distances from the ends of the line-segment to the same plane.

Proof. Fig.3.27 shows that the true length of the line-segment $A B$ is the hypotenuse of the right triangle $A B 1$ one leg of which is equal to the projection of the given segment, the second leg - to the absolute value of the algebraic difference of the distances from the ends of the line-segment to the projection plane.

Determine the true length of the line segment $A B$ and the angle of its inclination to the plane $H$ (angle $\alpha$ ) given two projections of the linesegment (Fig.3.28, a). Construct a right triangle given its two legs: the horizontal projection of the line-segment and a line-segment of length equal to the value of the algebraic difference of the distances from the line-segment ends to the plane $H$ (difference of $z$ coordinates of the points $A$ and $B$ ). The hypotenuse $a b_{0}$ of this triangle will yield the true length of $A B$, and the angle $b a b_{0}$ - its inclination angle to the plane $H$.

Fig.3.28, b shows determination of the true size of the line-segment $A B$ and its inclination angle to the plane $V$ (angle $\beta$ ).


Fig. 3.28

## Questions to Chapter 3

1. What location relative to the projection planes causes a line to be called "a line of general position"?
2. What is the locus of a line in the system of the planes $H, V, W$ given all three projections of the line are equal in length?
3. How do we construct a profile projection of a line of general position given its frontal and horizontal projections?
4. What positions of a straight line in the system of $H, V, W$ planes are considered to be the particular ones?
5. What is the position of a frontal projection of a line-segment given its horizontal projection is equal to the line-segment proper?
6. What is the position of a horizontal projection of a line-segment given its frontal projection is equal to the line-segment proper?
7. What is referred to as "the trace of a straight line on a projection plane"?
8. Which coordinate is equal to zero:
a) for a frontal trace of a line;
b) for a horizontal trace of a line?

## CHAPTER 4. REPRESENTATION OF A PLANE IN A DRAWING

### 4.1 Ways of Specifying a Plane

The position of a plane on a drawing may be specified in one of the following ways (Fig.4.1):
a) by the projection of three points not lying on one line;
b) by a line and a point projection not lying on one line;
c) by projections of two intersecting lines;
d) by projections of two parallel lines;
e) by any plane figure projection;
f) by the plane traces.

a

b

c

d

e

f

Fig.4.1
It is possible to move from one way of specifying a plane to another. For example, passing a line through the $A$ and $B$ points (Fig.4.1, a) we move from specifying the plane by three points to specifying it by a point and a line (Fig.4.1, b), etc.

In particular cases a plane can be represented more visually by means of its intersection lines with the projection planes.

The intersection lines of a plane with the projection planes are called traces of the plane (Fig.4.2):
$P_{V}$ - the frontal trace of the plane $P$;
$P_{H}$ - the horizontal trace of the plane $P$;
$P_{W}$ - the profile trace of the plane $P$.
The points of intersection of a plane with the axes are called the vanishing points of traces (Fig.4.2)

Construct the like traces of two lines lying in a plane to obtain a trace of the plane.


Fig.4. 2

### 4.2 The Position of a Plane Relative to the Projection Planes

A plane may have the following positions relative to the projection planes:

Inclined to all projection planes;
Perpendicular to the projection plane;
Parallel to the projection plane.
A plane which is not perpendicular or parallel to any of the projection planes is called an oblique plane (a plane of general position). The oblique planes are shown in Figures 4.1, 4.2 and 4.3.


Fig. 4.3


Fig. 4.4


Fig. 4.5

A plane which comes higher, the further it gets from a viewer, is referred to as an ascending plane (Fig.4.4). A plane which comes lower, the further it gets from a viewer, is called a descending one (Fig. 4.5).

It is possible to distinguish representations of an ascending and a descending plane after analysis of the triangle the plane is specified by. The
drawing of an ascending plane (Fig.4.4) shows that both of the $A B C$ triangle projections (the plan $a b c$ and the elevation $a^{\prime} b^{\prime} c^{\prime}$ ) are designated clockwise. However, the $A_{l} B_{l} C_{l}$ projections which specify the descending plane in Fig.4.5, are designated in counterpart ways - the plan $a_{1} b_{1} c_{1}$ is designated counter-clockwise, the elevation $a^{\prime}{ }_{1} b^{\prime}{ }_{1} c^{\prime}{ }_{1}$ - clockwise.

The Planes of Particular Position. The planes perpendicular or parallel to the projection planes are called the planes of particular position.

A plane perpendicular to a projection plane is called a projecting plane.

## 1. The horizontal projecting plane $P(A B C D) \perp H$




Fig.4.6
2. The vertical projecting plane $Q(A B C D) \perp V$


Fig.4.7

## 3. The profile projecting plane $T(A B C D) \perp W$



Fig.4.8
A plane projects to a perpendicular projection plane as a straight line. This projection can also be considered as a trace of the plane.

There is an important property of the projecting planes, called a collective one: if a point, a line or a figure are contained in a plane perpendicular to the projection plane, their projections on the above plane coincide with the trace of the projecting plane.

The planes parallel to the projection planes are called the planes of level (the level planes). The level planes are perpendicular to two projection planes simultaneously (double projecting planes).

## 1. The horizontal plane $\operatorname{P}(A B C D) / / H$



Fig.4.9
2. The frontal plane $Q(A B C D) / / V$


Fig.4.10
3. The profile plane $T(A B C D) / / W$


Fig.4.11
Any line or figure contained in a level plane parallel to a projection plane, projects to the last plane in true shape.

### 4.3 The Point and the Line in the Plane

The following problems are considered to be the principal ones among those being solved in the plane: drawing a line in a plane; constructing a point on a plane; constructing a lacking projection of a point contained in a plane; checking of a point belonging to a plane.

Solution of the above problems is based on well-known geometric principles: a line belongs to a plane, if it passes through two points belonging to the plane; or if it passes through one point of the above plane and is parallel to a line contained in the plane.


Fig.4.12


Fig.4.13

## Construction of a straight line in a plane

To construct a straight line in a plane (Fig 4.12) it is necessary to specify two points contained in this plane, say, the points $A$ and $l$, and draw the line $A 1$ through them ( $a 1$ and $a^{\prime} 1^{\prime}$ ).

Fig.4.13 - The line $B 1$ belongs to the plane of the triangle $A B C$ as it passes through the vertex $B$ and is parallel to the side $A C\left(b^{\prime} l^{\prime} / / a^{\prime} c^{\prime}\right.$ and $b 1 / /$ $a c$ ).

## Construction of a point in a plane

A point belongs to a plane if it lies on a line contained in this plane.
To construct a point in a plane draw an auxiliary line in it and specify a point on this line.



Fig.4.15

Fig.4.14
Projections of the auxiliary line $A 1$ ( $a 1$ and $a^{\prime} 1^{\prime}$ ), belonging to the plane specified by the projections of the point $A\left(a\right.$ and $\left.a^{\prime}\right)$ and the line $B C$ ( $b c$ and $b^{\prime} c^{\prime}$ ) (Fig.4.14), are drawn in the above plane. Mark down on it the projections $d$ and $d^{\prime}$ of the point $D$, belonging to this plane.

Construction of a lacking point projection
The plane on Fig.4.15 is specified by the triangle $A B C$ ( $a b c$ and $\left.a^{\prime} b^{\prime} c^{\prime}\right)$.

The point $D$ belonging to this plane is specified by $d^{\prime}$. Find the horizontal projection of the point $D$. It may be found by a construction of an auxiliary line belonging to the plane and passing through the point $D$. To do this draw the frontal projection of the line Al, construct its horizontal projection al and mark off on it the desired horizontal projection $d$ of the point.

Identification if a point belongs to a plane


Use an auxiliary line included in a plane to check whether the point belongs to this plane. The plane on Fig.4.16 is specified by the parallel lines $A B$ and $C D$, the point - by the projections $e$ and $e^{\prime}$. Draw the projections of the auxiliary line to pass through one of the point projections. For example, the frontal projection of the auxiliary line $1^{\prime}-2^{\prime}$ passes through the frontal projection of the point $e^{\prime}$. Construct the horizontal projection of the line 1-2. It is obvious from the drawing that the horizontal projection $e$ of the point does not belong to it. Thus, the point $E$ does not belong to the plane.

Fig. 4.16

### 4.4 The Principal Lines of the Plane

There are a lot of lines belonging to a plane. Those of them which have a special or particular position should be distinguished. They are: $H$ parallels or horizontal lines, $V$ parallels or frontal or vertical lines, profile lines and lines of maximum inclination or the steepest lines. The above lines are referred to as the principal lines of the plane.

H parallels or horizontal lines are lines lying in a given plane and parallel to the horizontal plane of projection (Fig.4.17).

The frontal projection a' 1 ' of the horizontal line is parallel to the $x$ axis, the profile one - to the $y$ axis.
$V$ parallels or frontal or vertical lines are lines lying in a given plane and parallel to the vertical plane of projection (Fig.4.18). The horizontal projection $c l$ of the frontal line is parallel to the $x$ axis, the profile one - to the $z$ axis.

Profile lines are lines lying in a given plane and parallel to the profile plane of projection. The horizontal projection $b 1$ of the profile line is parallel to the $y$ axis, the frontal one - to the $z$ axis (Fig.4.19).


Fig.4.17


Fig.4.18


Fig.4.19

The lines considered above are the lines of minimum inclination to the planes of projections.

Among three lines of maximum inclination or the steepest lines let us mark out the inclination line to the horizontal plane. It is called the steep line.

The steep line is a line lying in a given plane and perpendicular to its horizontal trace or to its $H$ parallel (Fig.4.20). Having constructed the steepest line in a drawing, one may determine the size of the dihedron between the given and projection planes. This angle is equal to the linear angle between the steepest line and its projection on the plane.

Use the method of a right triangle to determine an angle of inclination.


Fig.4.20


Fig.4.21

### 4.5 The Relative Positions of a Line and a Plane

The relative positions of a line and a plane are determined by the quantity of points belonging both to the plane and to the line:
a) if a line and a plane have two common points, the line belongs to the plane;
b) if a line and a plane have one common point, the line intersects the plane;
c) if the point of intersection of a line and a plane is at infinity, the line and the plane are parallel.

The problems in determining


Fig. 4.22 the relative positions of different geometric figures are called positional problems.

A line is parallel to a plane if it is parallel to any line contained in this plane. To construct such a line, specify a line in the plane and draw the required one parallel to it.

Through point $A$ (Fig.4.22) draw the line $A B$ parallel to the plane $P$, which is specified by the triangle $C D E$. To do this through the frontal projection $a^{\prime}$ of the point $A$ draw the frontal projection $a b$ of the required line parallel to the frontal projection of any line contained in the plane, say, the line $C D\left(a^{\prime} b^{\prime} / / c^{\prime} d^{\prime}\right)$. Parallel to $c d$ through the horizontal projection $a$ of the point $A$ pass the horizontal projection $a b$ of the desired line $A B(a b / / c d)$. The line $A B$ is parallel to the plane $P$ specified by the triangle $C D E$.

## Construction of the intersection point of a line and a plane.

The problem of the construction of intersection point of a line and a plane is widely used in descriptive geometry. It is fundamental for the problems of: the intersection of two planes; of a plane and a surface; a line and a surface; and on the mutual intersection of surfaces.

To construct the point of intersection of a line and a plane means to find a point belonging to both, a given line and a plane. Graphically this is a point of intersection of the straight line and a line contained in the plane.

## The plane has a projecting position.

If a plane has a projecting position (for example, it is perpendicular to the frontal plane of projections - Fig.4.23), the frontal projection of the cutting point must belong to both the frontal trace of the plane and the frontal
projection of the line, i.e. to be in the point of their intersection. That is why, first the frontal projection of the point K is determined (the cutting point of the line $A B$ and the frontal projecting plane $Q(\triangle C D E)$, and then its horizontal projection.


Fig.4.23


Fig.4.24

The line has a projecting position.
Fig.4.24 shows the oblique plane $P(\triangle C D E)$ and the frontal projecting line $A B$, cutting the plane at the point $K$. The frontal projection of the point (the point $k$ ) coincides with the points $a^{\prime}$ and $b^{\prime}$. Draw through the point $K$ in the plane $P$ a straight line (say, l-2) to obtain the horizontal projection of the intersection point. First construct its frontal projection, then - the horizontal one. The point $K$ is the point of intersection of the lines $A B$ and l-2. It means that $K$ belongs to both, the line $A B$ and the plane $P$ and, therefore, is the point of their intersection.

The line and the plane have a general position.
In this case a line lying in the plane and intersecting the given line may be obtained as a line of intersection of an auxiliary plane passed through the line with the given plane (Fig.4.25, 4.26).

To determine the point of intersection of a straight line and a plane proceed as follows:

Pass an arbitrary auxiliary plane $P$ through the line $A B$ (the simplest way is to pass a projecting plane);

Find the line $M N$ of intersection of the given $[Q(\triangle C D E)]$ and auxiliary $(P)$ planes;

As the lines $A B$ and $M N$ lie in one plane $P$, the point of their intersection $(K)$ yields the desired point.

Determine the relative visibility of the line $A B$ and the plane $Q$.


Fig.4.25


Fig.4.26

To determine the visible sections of the line $A B$ analyse the position of the points on the skew lines (the competitive points).

The points $M$ and $L$ are situated on the skew lines $A B$ and $C D$ : $M \in C D, L \in A B$. Their frontal projections $m^{\prime}$ and $l^{\prime}$ coincide. The horizontal projection shows that, if the $V$ plane is viewed in the direction of the arrow, the point $L$ (projection $l$ ) is situated in front of the point $M$ (projection $m$ ), i.e. being projected on the frontal plane it covers the point $M$. Therefore, the line $A B$ to the left of the point $K$ is situated in front of the triangle $C D E$, and it is visible on the frontal projection. The triangle $C D E$ covers the line $A B$ to the right of the point $K$ up to the point $N$, that is why the segment $k^{\prime} n^{\prime}$ is shown as an invisible one.

The invisible part of the horizontal projection of the line $A B$ is determined by analysis of the position of the points 1 and $2(1 \in D E, 2 \in A B)$, belonging to the skew lines $A B$ and $D E$. The frontal projection shows that, if the $H$ plane is viewed in the direction of the arrow, the point $l$, which lies above the point 2 , is visible first. On the horizontal projection the point 1 covers the point 2 . In this section the line $A B$ is covered by the triangle $D E F$ as far as the intersection point $K$ (the projection section $k 2$ ).

### 4.6 Mutual Positions of the Planes

A general case of the mutual positions of planes is their intersection. In the particular case when the intersection line is at infinity, the planes become parallel. The parallel planes coincide when the distance between them is shortened to zero.

## The parallel planes

The planes are considered to be parallel if two intersecting lines of one plane are relatively parallel to two intersecting lines of the other (Fig.4.27)


Fig.4.27

To pass through the point $D$ a plane parallel to a given plane ( $\triangle A B C$ ), draw two lines through the point, parallel to any two lines contained in the given plane, say, the triangle sides.

## Intersecting planes

The line of intersection of two planes is determined by two points, each belonging to both planes; or by one point, belonging to both planes, plus a given direction of the line. In both cases the problem is to find the point common to both planes.

## Intersection of two projecting planes

If the planes are of a particular position (say, a horizontal projecting one, like in Fig.4.28), the projection of the intersection line on the plane of projections to which the given planes are perpendicular (in this case, to the horizontal one) comes to be a point. The frontal projection of the line of intersection is perpendicular to the projection axis.


Fig.4.28

## Intersection of a rojecting plane and an oblique plane

In this case, one projection of the line of intersection coincides with the projection of the projecting plane on that projection plane to which it is perpendicular. Figure 4.29 shows the construction of the projections of the intersection line of the frontal projecting plane specified by traces; Figure 4.30 - of the horizontal projecting plane specified by the triangle $A B C$ with the plane of general position specified by the triangle $D E F$.
$\downarrow$ to H


Fig.4.29


Fig.4.30

In the point of intersection of the plane $P_{v}$ trace with the sides $D E$ and $D F$ of the triangle $D E F$ on the frontal projection (Fig.4.29), find the frontal projections $m^{\prime}$ and $n^{\prime}$ of the intersection line. Drawing the connection lines find the horizontal projections of the points $M$ and $N$ of the intersection line.

Viewing the plane $H$ in the direction of the arrow, one can see (by the plan) that a part of the triangle to the left from the cutting line $M N\left(m^{\prime} n\right.$ ) is above the plane $P$, it means it is visible on the horizontal projection plane. The other part is under the plane $P$, i.e. it is invisible.

Find the intersection lines of the planes of Fig. 4.30 in a similar fashion.

## Intersection of the oblique planes

The method of drawing the intersection lines of such planes consists of the following:

Introduce an auxiliary plane (intermediary) and draw the lines of intersection of this plane with the two given ones (Fig.4.31). The intersection of the drawn lines shows the common point of the above planes. To find the other common point use another auxiliary plane.

In solving such kinds of problems, it is better to use projecting planes as intermediaries.

Fig.4.32 shows the construction of the intersection line of two triangles. The solution is as follows:


Fig.4.31
Fig.4.32

Draw two auxiliary frontal projecting planes - the plane $P$ through the side $A C$ and the plane $Q$ through the side $B C$ of the triangle $A B C$. The plane $P$ cuts the triangle $D E F$ along the line $1-2$. In the intersection point of the plans $1-2$ and $a c$ find the plan of the point $M(m)$ of the intersection line. The plane $Q$ cuts the triangle $D E F$ along the line 3-4. In the intersection point of the horizontal projections 3-4 and $b c$ find the plan of the point $N(n)$ of the intersection line. Pass the connecting lines to find the frontal projections of the above points and, therefore, of the intersection line.

Analysis of the mutual visibility of the triangles on the projection planes do with the help of competitive points.

To determine visibility on the frontal projection plane compare the frontal competitive points 1 and 5 lying on the skew lines $A C$ and $D E$. Their frontal projections coincide. The horizontal projection shows that on the plane $V$, viewed in the direction of the arrow, the point 5 is situated closer to a viewer and that is why it covers the point 1 . So, the segment of the line $A C$ to the left of the point $M$ is visible on the frontal projection plane.

To determine visibility on the horizontal projection plane compare the horizontal competitive points 6 and 7 lying on the skew lines $A C$ and $D F$. Their horizontal projections coincide. The plane $H$, viewed in the direction of the arrow, shows that the point 6 and the line $A C$ are situated above the point 7 and the line $D F$. So, the segment $A M$ of the line $A C$ is visible on the horizontal projection plane.

### 4.7. Method of Replacing Planes of Projection

Different methods of transformation of orthogonal projections are used to make the solution of metric and positional problems simpler. After such transformations the new projections help to solve the problem by minimal graphic means.

The method of replacing planes of projection consists in the substitution of a plane with a new one. The new plane should be perpendicular to the remaining one. The position in space of the geometric figure remains unchanged. The new plane should be positioned so that the geometric figure has a particular position to it, convenient for solving the problem.

Fig.4.33 shows a spatial drawing of the $A B$ line-segment of general position and its projection on the planes $H$ and $V$. Replace the plane $V$ with a new vertical plane $V_{l}$, parallel to the line-segment $A B$, to obtain a new system of two mutually perpendicular planes $V_{I}$ and $H$, relatively to each the segment $A B$ has a particular position $\left(A B / N_{l}\right), x_{l}$ is a new coordinate axis. The new projection of the line-segment $A B\left(a_{1}^{\prime} b_{1}^{\prime}\right)$ is equal in length to its
true size, and the angle $\alpha$ is equal to the true size of the inclination angle between $A B$ and the plane $H$.

When replacing the frontal plane of projection (Fig.4.33), z coordinates are constant, i.e. the distance between the points and the horizontal projection plane $H$ remains unchanged. Therefore, to construct a new projection of the line-segment (Fig.4.34) proceed as follows:


Fig. 4.33


Fig.4.34

- at any distance pass the new axis $x_{1}$ parallel to the horizontal projection of the line-segment $A B$;
- through the horizontal projections $a$ and $b$ perpendicular to the axis $x_{1}$ pass the connection lines;
- from the point of intersection of the connection lines with the axis $x_{I}$ lay off $z$-co-ordinates of $A$ and $B$ points;
- connect the obtained points $a_{1}{ }_{1} b_{1}$ with a straight line.

When replacing the horizontal plane $H$ with a new one, $y$-co-ordinates remain unchanged, which means that the distance between the point and the frontal projection plane does not change if the point is projected on the new plane, perpendicular to the plane $V$.

## Four principal problems solved by replacing the projection planes

1. Transform a line of general position into a line parallel to one of the projection planes. Such a transformation helps to determine the true size of the line-segment and its inclination angles contained by the projection planes (Fig.4.35)

To solve the problem, draw a new plane, say, $V_{l}$ (Fig.4.35), parallel to the segment. In this case the new coordinate axis passes parallel to the horizontal projection of the given line. Draw, through the horizontal projections $a$ and $b$ perpendicular to the new axis, the connection lines. Lay off on them $z$-co-ordinates of the points (the distance from the $x$ axis to the frontal projection of the points). The new projection $a_{1}^{\prime} b_{1}^{\prime}$ is equal to the true size of the segment, and the angle $\alpha$ is equal to the inclination angle contained by the segment and the plane $H$.


Fig.4.35


Fig.4.36

When replacing the horizontal projection with a new one, draw this plane parallel to the line-segment $A B$ and determine the true size of the segment and its inclination angle with the plane $V$ - the angle $\beta$ (Fig.4.36).

In this case pass the coordinate axis of the new plane parallel to the frontal projection of the line $a^{\prime} b^{\prime}$, and take the co-ordinates $y$ from the horizontal projection plane.
2. Transform a line parallel to one of the projection planes into a projecting line, i.e. position it perpendicular to the projection plane, to project the line on this plane as a point (Fig.4.37).

As the given line is parallel to the horizontal plane, to transform it into a projecting line replace the frontal plane V with a new $V_{l} \perp H$, drawing the plane $V_{l}$ perpendicular to $A B$. As a result the given line is projected on the plane $V_{l}$ as a point $\left(a_{1}^{\prime}=b_{1}^{\prime}\right)$.

To transform the general position line $A B$ (Fig.4.38) into a projecting one, make two replacements, i.e. solve both problems, the first and the second ones, successively. First transform a general position line into a line parallel to a projection plane (a line of level), then the last is transformed into a projecting one.
3. Transform the oblique plane $P(\triangle A B C)$ into a projecting one (Fig.4.39), i.e. positioned perpendicular to one of the projection planes.


Fig.4.37


Fig.4.38

Replace, for example, the plane $V$ with a new plane $V_{l}$, which is perpendicular to the plane $H$ and the plane $P$. The plane $V_{l}$ is perpendicular to the plane $P$ if it is drawn perpendicular to one of the lines of the plane. Let us take $H$ parallel here (the line parallel to the horizontal projection plane). Draw in the plane $P$ the $H$ parallel $C l$ and pass a new plane $V_{l}$ perpendicular to it. Pass the axis $x_{1}$ in any place perpendicular to the horizontal projection of the $H$ parallel $\left(x_{1} \perp c l\right)$. Now construct a new frontal projection of the plane $P$. The $H$ parallel is projected onto the new plane as a point $\left(c_{l}^{\prime}=l_{1}{ }_{1}\right)$, the plane $P(\triangle A B C)$ - as the line $a^{\prime}{ }_{1} c^{\prime}{ }_{1} b^{\prime}{ }_{1}$.

To transform the plane P into a horizontal projecting plane, replace the plane $H$ with a new one, perpendicular to the plane $V$ and to the $V$ parallel of the plane $P$ (which has been drawn in this plane previously).


Fig. 4.39
4. Transform the plane $P(\triangle A B C)$ from $a$ projecting plane into a level plane (a plane parallel to one of the projection planes). In this case we determine the true size of the plane figure (Fig.4.40 and 4.41).

Fig. 4.40 shows the frontal projecting plane. Replace the horizontal plane $H$ with a new one, positioning it perpendicular to the plane $V$ and parallel to the plane $P$. Pass the new axis $x_{1}$ parallel to the frontal projection $a^{\prime} b^{\prime} c^{\prime}$, and new connection lines - perpendicular to $x_{1} . Y$ coordinates remain unchanged as the horizontal plane was changed. Carry the coordinates onto the new plane. As a result obtain a new horizontal projection of the triangle equal to the true size of $\triangle A B C$.


Fig. 4.40


Fig.4.41

The problem is solved in a similar fashion if the plane $P(\triangle A B C)$ is a horizontal projecting plane (Fig.4.41). In this case the frontal plane $V$ is replaced with the new one $V_{l}$ which is drawn perpendicular to the plane $H$ and parallel to the plane $P$. The axis $x_{I}$ is passed parallel to the line $a b c$. With such a replacement the coordinates $z$ remain unchanged, lay them off on the connection lines, from the new axis $x_{1}$.

To transform an oblique plane into a plane, parallel to one of the projection planes, two replacements are necessary (Fig.4.42); that is to solve the third and the forth problems successively.


Fig. 4.42

## Questions to Chapter 4

1. What are the ways of specifying a plane figure?
2. What are "traces of the plane"?
3. What plane is called a projecting plane?
4. What is the level plane?
5. Under what conditions does a line belong to a plane?
6. Under what conditions does a point belong to a plane? What lines are referred to as the principal lines of the plane?
7. What are the terms of a line and a plane to be parallel?
8. How can you find the meeting point of a line and a plane?
9. What are the relative positions of the planes?
10. What determines mutual parallelism of two oblique planes in a drawing?
11. What is the way of drawing an intersection line of two planes?
12. What is the gist of the replacing planes of projection method?
13. What mutual relations must the old and new planes of projections have?
14. What actions are necessary to obtain the following transformations: of a general position line into a projecting one; of an oblique plane into a level plane?

## CHAPTER 5. SURFACES

### 5.1 Determining and Specifying Surfaces in a Drawing.

## Classification

In descriptive geometry surfaces are referred to as a set of consecutive locations of a moving line. This method of a surface formation is called the kinematic method.

A line (a curved or a straight one) moving in space and producing a surface is called a generatrix (a generating line). As a rule, this line moves along another one, called a directrix (a directional line).

Except mentioned above kinematic method, the surface may be specified by: the analytical method, i.e. presented in a mathematical expression; and the frame method which is used for specifying complex surfaces subjected to no rules.

In this case, to specify a surface, a number of its parallel sections (frame) is required which may be referred to as the locations of a variable generatrix. This method is used in lorry body manufacturing, in aircraft industry, shipbuilding, etc.

Method of specifying a surface by a frame, for instance, by intersection lines of a surface with level planes, is applied in topography, mining, road making. Projections of a level line on a projection plane with the corresponding marks represent a relief landscape map. A surface referred to as the earth one, is called topographic surface.

To specify a surface in a complex drawing it is necessary to present in it only those elements of a surface which give the opportunity to construct each of its points. A collection of these elements is called surface determinant. Surface determinant consists of two parts: a geometric part, including constant geometric elements (points, lines), which form the surface; and an algorithm part, specifying the principle of generating line motion, the nature of its form modification.

When a surface $\Omega$ is projected on the projection plane $P$ in a parallel way, projecting lines tangential to $\Omega$, form a cylindrical surface (Fig. 5.1). These projecting lines contact the surface $\Omega$ in certain points, forming the line $M$ which is called a level line.

Projection of the level line $M$ on the plane $P-m_{p}$, is called the surface outline.

To simplify understanding of a drawing, the draughtsmen represent not only outline of the surface but also its most significant lines and points.


Fig. 5.1

## Classification of the Surfaces

There is a great quantity of different surfaces. But some of them are considered to be the most significant. Classification of them depends on generatrix form, also on the form, number and location of directrices:

1. Regular and nonregular surfaces;
2. Ruled surfaces (formed by a travel of a straight line) and nonruled (curve-lined) surfaces;
3. Developable surfaces (or torses) and nondevelopable ones;
4. Surfaces with generatrix of a constant form and of a variable form;
5. Surfaces with translational, rotary and helical motion of generatrix.

### 5.2 A Point and a Line on the Surface

A point belongs to a surface when it belongs to a line of the surface.
A line belongs to a surface when it passes through the points of the surface. A straight line belongs to a surface when it passes through two points belonging to the surface.

Hence, if a point belongs to a surface, its projections belong to the like projections of the surface line.

To construct the points lying on a surface, use the prime lines of this surface.

### 5.3 Polyhedral Surfaces and Polyhedrons.

## A Polyhedron Cut by a Plane

A polyhedral surface is a surface formed by a travel of a linear generating line along a polygonal directrix. Polyhedral surfaces are divided into two kinds: pyramidal (Fig. 5.2) and prismatic (Fig. 5.3) surfaces.


Fig. 5.2


Fig. 5.3

A pyramidal surface is a surface obtained by a travel of a linear generatrix along a polygonal directrix. Note: All generating lines pass through a certain fixed point $S$.

A prismatic surface is a surface obtained by a travel of a linear generatrix along a polygonal directrix. Note: All generating lines are parallel to a certain given direction $l$.

The points $M$ and $N$ belong to a pyramidal and prismatic surfaces respectively, as they belong to the straight lines contained in these surfaces.

A part of space bounded in all directions by a surface is called a body.
A body bounded by plane polygons is called a polyhedron. Among all polyhedrons only prisms and pyramids are considered in this textbook.

Prism is a polyhedron with the bases being equal mutually parallel faces and the sides being parallelograms. If the edges of the sides are perpendicular to the base, the prism is a right prism.

To specify a prism it is necessary to specify its base and a lateral edge (Fig.5.4).


Fig. 5.4


Fig. 5.5

Construct the edges $D L, B F$ and $C Q$ parallel and equal in length to the given edge $A E$ to determine the second base and, hence, all the prism faces (Fig. 5.5).

To construct a lacking projection point lying on a polyhedron face draw a straight line through the above point. E.g. Given: horizontal projection of the point M belonging to the face $B C Q F$. To construct its frontal projection draw the line $K N$ through the given point. The point $\mathrm{m}^{\prime}$ is a desired point belonging to the projection $k^{\prime} n^{\prime}$.

Pyramid is a polyhedron (Fig. 5.6), one face of which is an arbitrary polygon $A B C D$ taken for the base, the other faces (lateral) are the triangles with the common vertex $S$ being called the vertex of pyramid.

To specify a pyramid it is necessary to specify its base and vertex. To construct projections of a point on a pyramid surface pass a line through this point like it was shown above (Fig. 5.5) for a prism.


## A Polyhedron Cut by a Plane

When polyhedral surfaces are cut by planes we obtain polygons in the section, whose vertices are determined as the points of intersection of the polyhedron edges with a cutting plane.

A polygon obtained by cutting may be determined in two ways:

1. Its vertices may be found in the points of intersection of straight lines (the edges) with a cutting plane;
2. Its sides may be distinguished as the lines of intersection of polyhedron planes (the faces) with a cutting plane.


Fig. 5.7

Consider an example of construction of a pyramid cutting by a frontal projecting plane $P$ (Fig. 5.7).

The cutting plane is a frontal projecting plane, therefore, all lines lying in this plane, including the section figure, coincide with the frontal trace $P_{V}$ of the plane $P$. Thus, the intersection of the frontal projections of the pyramid edges with the trace $P_{V}$ yields the frontal projection of the section figure $1^{\prime}-2^{\prime}-3^{\prime}$. Find the horizontal projections of the points $1,2,3$ by means of the connection lines on the horizontal projections of the corresponding edges.

## A Pyramid with a Notch

As an example of drawing a polyhedron cutting by a few planes consider construction of a pyramid with a notch produced by three planes ( $P, R$ and $T$ ) (Fig. 5.8).

The plane $P$, parallel to the horizontal projection plane, intersects the pyramid surface along a pentagon $1-2-3-K-6$. The pentagon sides are parallel to the sides projections of the pyramid base on the horizontal projection plane. After the horizontal projection of the pentagon has been constructed, denote the points 4 and 5 .

The frontal projecting plane $R$ cuts the pyramid along the pentagon 1-2-7-8-9. To find the horizontal projections of the points 8 and 9 pass through them additional generatrices $S M$ and $S N$. First do it on the frontal projection $-s^{\prime} m^{\prime}$ and $s^{\prime} n^{\prime}$, then on the horizontal projection - $s m$ and $s n$.

The frontal projecting plane $T$ cuts the pyramid in pentagon 4-5-10-9-8. Having constructed the horizontal notch projection, draw its profile projection.

5.4 Conical and Cylindrical Surfaces. Torses


Fig. 5.9


The conical surface is produced by the motion of a linear generating line along a curved directrix. At that, the generatrix passes some fixed point $S$, referred to as a vertex (Fig. 5.9).

The cylindrical surface is produced by parallel to a given straight line $l$ motion of a linear generating line along a curved directrix (Fig. 5.10).

The point $N$ belongs to the given surfaces as it belongs to their generatrix $B$.
A conical surface is considered to be distinguished in a drawing when a directrix (by form and position) and a vertex are specified. Depending on the directrix form the conical surface may be closed or not. A body bounded by a conical surface and a plane is called a cone. If the base of a cone is a circle, the cone may be circular.

A cylindrical surface is considered to
be distinguished if a directrix (by form and position) and a generatrix (by position) are specified. To draw a cylindrical surface It is advisable to take as a directrix a line of intersection of this surface with a projection plane or another plane parallel to it.

Cylindrical surface may also be closed or not. A body bounded by a cylindrical closed surface and two parallel planes is called a cylinder. Cylindrical surfaces are distinguished by the view of its normal section, i.e. a curve obtained by cutting the surface with a plane perpendicular to its generating lines.

## Torse

Torse (a surface with a cuspidal edge) is obtained by the motion of a


Fig. 5.11

### 5.5 Rotation Surfaces. Rotation Surface Cut by a Plane

Rotation surface is a surface described by a curve (or a straight line), rotating on its axis (Fig. 5.12). This surface is represented in a drawing by specifying its generatrix and rotation axis.


Each point of a generatrix describes, rotating, a circle with the centre on the axis. These circles (say, circle l) are called parallels. The largest of them (circle 2 ) is called an equator, the smallest (circle 3 ) - a gorge circle.

The curves obtained by cutting a rotation body by the planes, passing through the axis, are called meridians. Meridian 4, parallel to the frontal projection plane, is referred to as the principal meridian. All meridians are equal to each other.

In a drawing the axis of a surface rotation is positioned perpendicular to one of the projection planes, say, horizontal one. Then all parallels are projected on this plane in true size, and the equator and gorge circle determine a horizontal outline of the surface. A meridian located in the frontal plane, that is the principal meridian, is considered to be a frontal outline.

The points on a rotation surface are constructed by means of parallels (i.e. the circles on the surface).

Let us consider some bodies and rotation surfaces.

## 1. Surfaces obtained by rotation of a straight line:

a) cylinder of rotation - this is a surface produced by rotation of the line $L$ round the axis I parallel to it (Fig. 5.13);
b) cone of rotation - this is a surface produced by rotation of the line $L$ round the axis $I$ intersecting it (Fig. 5.14);
c) one sheet hyperboloid of rotation - this is a surface produced by rotation of the line $L$ round the skew axis $I$ (Fig. 5.15).


Fig. 5.13


Fig. 5.14


Fig. 5.15

The point $A$, lying on a perpendicular to the rotation axis and generating line, describes the smallest circle, which is the gorge of hyperboloid. One sheet hyperboloid may also be obtained by rotation of hyperbola on its conjugate axis.
2. Surfaces obtained by rotation of a circle round a fixed axis:
a) sphere - this is a surface produced by rotation of a circle round its diameter (Fig. 5.16);
b) torus - this is a surface produced by rotation of a circle round the axis I lying in the plane of this circle but not passing through its centre (Fig. 5.17 through 5.20). If the axis of rotation passes beyond a circle, the surface is called an open torus or a ring-torus (Fig. 5.17); if the axis is tangential to a circle, it is a closed torus (Fig.5.18); if the axis intersects a circle - it is self-intersecting torus (Fig. 5.19, 5.20).

The torus presented by Fig. 5.19 is also called an apple-torus, the one from Fig. 5.20-a lemon-torus.


Fig.5. 20
3. Rotation surfaces obtained by the curves of the second order:

a) ellipsoid of rotation - this is a surface produced by rotation of an ellipse on its axis (Fig.5.21). Surface obtained by rotation on its major axis is called an oblong ellipsoid of rotation, on its minor axis - an oblate one;
b) paraboloid of rotation - this is a surface produced by rotation of a parabola on its axis (Fig.5.22);
c) two sheet hyperboloid of rotation - this is a surface produced by rotation of a hyperbola on its axis (Fig.5.23).

## Drawing a Projection of an Intersection Line of a Cylinder Cut by a Plane

When a cylinder of rotation is cut by a plane parallel to the rotation axis, a pair of straight lines (generatrices, Fig.5.24) appears in the section. If a section plane is perpendicular to the axis of rotation, the cutting results in a circle (Fig. 5.25). Generally, when a cutting plane is inclined to the rotation axis of a cylinder, an ellipse is obtained by cutting (Fig.5.26).

Let us consider an example of drawing an intersection line of a cylinder and a frontal projecting plane $Q$, when an ellipse is obtained in the section (Fig.5.27).

The frontal projection of the section line, in this case, coincides with the frontal trace of the plane $Q_{V}$, the horizontal one - with the horizontal projection of the cylinder surface (a circle). The profile projection of the line is constructed by two projections available, the horizontal and frontal ones.


Fig.5.24
On the whole, drawing a line of intersection of a surface with a plane consists in distinguishing the common points, belonging to both, a cutting plane and a surface.


Fig.5.27

The method of auxiliary cutting planes is usually applied to find the above points:

1. Pass an auxiliary plane;
2. Construct the intersection lines of this plane with the surface and with the given plane;
3. Determine the intersection points of thus obtained lines.

The auxiliary planes should be drawn so, that they cut the surface along the prime (most simple) lines.

Determining the points of intersection start with determining the characteristic or control points. They are:

1. Upper and lower points;
2. A left and a right points;
3. Points of visibility bounds;
4. Characteristic points of a given intersection line (for an ellipse points of major and minor axes).
To make the construction of the intersection line more accurate it is necessary to draw additional (passing) points as well.

In the example above points 1 and 8 are the lower and the upper points. For the horizontal and frontal projections point $l$ is a left point, point 8 is a right one. For the profile projections points 4 and 5 are the points of visibility bounds: the points of the line of intersection located lower than 4 and 5 are visible, all the rest are invisible.

Points 2, 3 and 6, 7 - are additional, used for the drawing accuracy. The profile projection of the section figure is an ellipse, the minor axis of which is the segment $1-8$, the major one $-4-5$.

## Drawing the Intersection Lines Projections of a Cone Cut by a Plane

Depending on the direction of a cutting plane, different lines, called the lines of conical sections, may be obtained in the section of a rotation cone.

If a cutting plane passes through a vertex of a cone, we get in its section a pair of generating lines (triangle) (Fig.5.28, a). As a result of intersection of a cone with a plane perpendicular to the cone axis, a circle is obtained (Fig.5.28, b). If a cutting plane is inclined to the rotation axis of a cone and does not pass through its vertex, an ellipse, parabola or hyperbola may be obtained in the section (Fig.5.28, c, d, e) - it depends on the size of inclination angle of the cutting plane.


Fig.5.28

An ellipse is obtained when the inclination angle $\beta$ is less than the inclination angle $\alpha$ of the cone generatrix to its base $(\beta<\alpha)$, that is when a plane cuts all generating lines of a given cone (Fig.5.28, c).

In case the angles $\alpha$ and $\beta$ are equal, i.e. a cutting plane is parallel to one of the generatrices of the cone, a parabola is obtained in the section. Here the cutting plane intersects all the generating lines except one to which it is parallel (Fig.5.28, d). If a cutting plane is inclined at an angle which changes in the following limits $-90^{\circ} \geqslant \beta>\alpha$, a hyperbola is obtained in the section. The cutting plane here is parallel to two generating lines of the cone. The obtained hyperbola has two branches as the conical surface is of two sheets (Fig.5.28, e).

It is well-known that a point belongs to a surface if it belongs to any line of this surface. The prime lines of a cone are straight (generating) lines and circles. So, if a problem statement requires to find the horizontal projections of the points $A$ and $B$, belonging to a cone surface, it is necessary to pass one of the prime lines through those points.

Find the horizontal projection of the point A by means of a generating line. To do that pass an auxiliary frontal projecting plane $P\left(P_{V}\right)$ through the point $A$ and the cone vertex $S$. This plane intersects the cone along two generatrices $S M$ and $S N$, the frontal projections of which coincide. Construct the horizontal projections of the generating lines, pass a connection line through the point $a^{\prime}$ and determine the horizontal projection of the point. There are two variants of answers in this problem - the points $a_{1}$ and $a_{2}$ (Fig.5.29).


Fig.5.29

Find the horizontal projection of the point B by construction a circle on which it is located. For that pass through the point the horizontal plane $T\left(T_{V}\right)$. The plane intersects the cone in a circle of radius $r$. Construct the horizontal projection of this circle. Through the point $b^{\prime}$ pass a connection line to meet the circle. There are also two answers to this problem - the points $b_{1}$ and $b_{2}$.

Now let us consider an example of drawing the projections of an intersection line of the cone cut by the frontal projecting plane $P\left(P_{V}\right)$, when there is an ellipse in the section (Fig.5.30).


The frontal projection of the intersection line coincides with the frontal trace of the plane $P_{V}$.

To simplify the solution designate the utmost generatrices of the cone and determine the characteristic or control points.

The lower point $l$ lies on the generatrix $A S$, the upper point 2 - on the generatrix $B S$. These points specify the value of the cone major axis. The minor axis is perpendicular to the major one. To find the minor axis bisect the segment 1-2. Points 3 and 4 specify the cone minor axis. Points 5 and 6 located on the generating lines $C S$ and $D S$ are the points of visibility bound of the profile projection plane. The projections of points $1,2,5$ and 6 are situated on the corresponding projections of the generating lines. To find the projections of points 3 and 4 pass an auxiliary cutting plane $T\left(T_{V}\right)$, which cuts the cone in a circle of radius $r$. The projections of the given points are located on this circle. On the horizontal projection plane the circle is projected in true size, so, if we draw the connection line, we find the horizontal projections of
points 3 and 4. Lay off the co-ordinates of points 3 and 4 from the cone axis $y$ on the connection line to find the profile projections (Fig.5.30).

To make an accurate construction of an ellipse it is not enough to find the points mentioned above, hence, determine additional (arbitrary) points. Find the projections of these points in a similar fashion with points 3 and 4, or by passing generating lines through these points. Having found the projections of all points, join them subject to visibility. All points lying on the cone surface are visible on the horizontal projection. On the profile one - only the line passing throgh points $5,3,1,4,6$ are visible, the rest - are not.

## Ball Surface (a Sphere)

Ball surface is a surface obtained by rotation of a circle round an axis of its diameter. A plane intersects a sphere al-


Fig. 5.31 ways in a circle. This circle may be projected as:

- a straight line if the cutting plane is perpendicular to the projection plane;
- a circle of radius equal in length to distance from the axis of the sphere rotation to the outline (e.g. a circle of radius $r$ (Fig.5.31) if the cutting plane is parallel to the projection plane;
- an ellipse if the cutting plane is not parallel to the projection plane.

To construct the projections of a point lying on a sphere surface it is necessary to pass through the point a cutting plane parallel to the projection plane and draw the circle on which the point is located.

## Cutting a Sphere by a Plane

Intersect a sphere by the frontal projecting plane $Q$. To begin with, determine the characteristic points. Points 1 and 2 are located on the principal meridian. These points are the ends of the ellipse minor axis, also are the highest and the lowest points. Construct their horizontal and profile projections by means of the frontal projections. Points 3 and 4 are situated on a profile meridian and specify visibility on the profile projection plane.

Find the horizontal projections of the points by their profile projections. Points 5 and 6 are situated on the equator and are the points of visibility bounds on the horizontal projection. Find the profile projections of the points by their horizontal ones. Points 7 and 8 belong to the ends of the ellipse longer axis. Construct them in the following way: First find the middle point of the line-segment $l^{\prime}-2^{\prime}$ - this is the frontal projection of the point $O^{\prime}$, the
circle centre of the section. Then find its horizontal projection, the point $O$. The line-segments $O^{\prime} l^{\prime}$ and $O^{\prime} 2^{\prime}$ on the frontal projection are equal to the true size of the circle radius. On the horizontal projection the diameter of the circle is projected without shortening, hence, lay off the segments $O 7$ and $O 8$ equal in length to $O^{\prime} l^{\prime}$. To construct more accurate section line it is necessary to find a few additional points. Use auxiliary cutting planes for that, as shown in Fig.5.32. Then join the points thus obtained in a smooth curve subject to its visibility.


Fig.5.32

### 5.6 Screw Surfaces

A screw surface is a surface described by a generatrix at its helical motion.

If a generatrix of a screw surface is a straight line the surface is referred to as a ruled screw surface or a helicoid (helice (Fr.)-a spiral, a spiral staircase). A helicoid may be right or oblique depending on the generating line being perpendicular or inclined to the helicoid axis.

A few kinds of a ruled screw surface:

1. A right helicoid is produced by the motion of the linear generatrix $l$ along two directrices, one of which is the cylindrical screw line $m$, the other is its axis $i$. Note that in all its positions the line 1 is parallel to the plane perpendicular to the axis $I$ (called a plane of parallelism). Usually one of the projection planes is taken for the plane of parallelism (Fig.5.33). The generatrix $l$
of the right helicoid intersects the screw axis $I$ at a right angle. The right helicoid may be referred to as one of the conoids and called a screw conoid.


Fig.5.33


Fig.5.34


Fig.5.35
2. An oblique helicoid is distinguished from a right one by its generatrix $l$ intersecting the helicoid axis at a constant angle $\alpha$ different from the right angle. In other words, the generatrix $l$ of an oblique helicoid slides along two directrices, one of which is the cylindrical screw line $m$, the other - its axis $I$. Note that in all its positions the line $l$ is parallel to the generating lines of a certain cone of rotation. The angle of this cone included between the generating line and the axis parallel to the helicoid axis, is equal to $\varphi$. It is called a director cone of an oblique helicoid.

Fig.5.34 shows the construction of the oblique helicoid projections. The cylindrical screw line m and the axis $I$ are the directinal lines of the helicoid. They are parallel to the corresponding generating lines of the director cone.
3. A developable helicoid is produced by motion of the linear generatrix $l$ tangential in all its positions to the cylindrical screw line $m$, the last being the helicoid cuspidal edge (Fig.5.35). The developable helicoid being a ruled surface with a cuspidal edge is considered to be one of the torses.

The surface of a developable helicoid is bounded by the cuspidal edge $m$ and the line $a$ obtained by intersection of helicoid surface with a surface of coaxial cylinder of a larger diameter (than the diameter of the screw line $m$ ).

If the generating line $l$ intersects the surface axis, the helicoid is referred to as a closed one (Fig.5.33 and 5.34), if not - as an open one (Fig.5.35).

### 5.7 Mutual Intersection of Surfaces

The line of intersection of two surfaces is the locus of the points belonging to both surfaces.


Fig. 5.36

A general method of drawing the points belonging to a curve of mutual intersection of surfaces is the method of auxiliary surfaces-mediators. This method is similar to the method of construction intersection lines of surfaces cut by planes and consists in the following:

Take some intersecting surfaces $\Phi$ and $\Psi$ (Fig.5.36). Introduce an auxiliary plane $P$ intersecting the surfaces along the lines $l$ and $k$ which yields the points $M$ and $N$ belonging to the intersection curve.

As the surfaces-mediators very often planes or ball surfaces (spheres) are used. Depending on mediators the following main methods of construction an intersection line of two surfaces are distinguished:
a) method of auxiliary cutting planes;
b) method of auxiliary spheres.

Like in construction an intersection line of surfaces with planes, to draw a line of mutual intersection of surfaces it is necessary to draw, first, control points of a curve, as these points present the bounds of intersection line, between which the passing (arbitrary) points should be determined.

## Method of Auxiliary Cutting Planes

The following example illustrates how to apply the method of auxiliary cutting planes for construction of an intersection line of a sphere with a cone of rotation (Fig.5.37).

To construct an intersection line of the given surfaces it is advisable to introduce the frontal plane $P$ and a number of horizontal planes $(S, T, R)$ as the auxiliary surfaces.

Start with determination of the characteristic points projections. Draw the frontal plane $P\left(P_{H}\right)$. This plane intersects the surfaces along the outlines. Find the frontal projections of the highest and the lowest points ( $1^{\prime}$ and $2^{\prime}$ ) as the points of intersection of the outlines. Pass the connection lines to determine the horizontal projections $l$ and 2.

The auxiliary horizontal planes cut the sphere and the cone in circles.


Fig. 5.37
Find the projections $3^{\prime}$ and $4^{\prime}$ of the points lying on the sphere equator with the help of horizontal plane $T\left(T_{V}\right)$, which passes through the centre of the sphere. The plane intersects the sphere along the equator and the cone in the circle of radius $r$. The horizontal projections intersection of the latest yields the horizontal projections 3 and 4 . The horizontal projections of points 3 and 4 are the points of visibility bounds of the intersection line on this projection. Determine the passing points ( $5,6,7,8$ ) by means of auxiliary horizontal planes $S\left(S_{V}\right)$ and $R\left(R_{V}\right)$. Join the points thus obtained in a smooth curve subject to visibility.

## Method of Auxiliary Spheres

This method is widely used for solution of problems on construction the intersection lines of rotation surfaces with intersecting axes.

Before studying this method let us consider a particular case of intersection of rotation surfaces, the axes of which coincide, i.e. a case of intersection of coaxial surfaces of rotation.


Fig. 5.38
Coaxial surfaces intersect in a circle, the plane of which is perpendicular to the axis of rotation surfaces. At that, if the axis of rotation surfaces is parallel to the projection plane, the intersection line projects onto this plane as a line-segment (Fig.5.38).

This property is used for construction a line of mutual intersection of two rotation surfaces by means of auxiliary spheres. Here may be used both, concentric (constructed from one centre) and eccentric (drawn from different centres) spheres. We are going to consider application of auxiliary concentric spheres, those with a constant centre.

Note: if a plane of rotation surface axes is not parallel to the projection plane, the circles in which the surfaces intersect, are projected as ellipses and this make the problem solution more complicated. That is why the method of auxiliary spheres should be used under the following conditions:
a) intersecting surfaces are the surfaces of rotation;
b) axes of the surfaces intersect and the intersection point is taken for the centre of auxiliary spheres;
c) the plane produced by the surfaces axes (plane of symmetry) is parallel to one of the projection planes.

Using this method it is possible to construct the line of intersection of the surfaces on one projection.

Let us consider an example of drawing an intersection line of a cylinder and a cone of rotation (Fig.5.39).

The points $1,2,3,4$ are determined as the points of level generatrices of the surfaces belonging to the plane of axes intersection (the plane of symmetry $P\left(P_{H}\right)$ ). Find the other points by method of auxiliary spheres.

From the intersection point of the given surfaces (point $O^{\prime}$ ) draw an auxiliary sphere of an arbitrary radius. This sphere is simultaneously coaxial to the cone and the cylinder and cuts them along the circumferences, the planes of which are perpendicular to the corresponding rotation axes. The frontal projections of those circles are line-segments. The sphere constructed intersects the cone along the circumference of diameter $A B\left(a^{\prime} b^{\prime}\right)$, and the cylinder - along the circumferences $C D\left(c^{\prime} d^{\prime}\right)$ and $E F\left(e^{\prime} f^{\prime}\right)$. In the intersection points of the circle $A B$ with the circles $C D$ and $E F$ obtain points 9-10 and 1112 respectively, which belong to the intersection line.


In such a way it is possible to construct a certain amount of points of the desired intersection line. Students should note that not all the spheres may be used for the problem solution. Consider the limits of the auxiliary spheres usage.

The maximal radius of a cutting sphere is equal to the distance from the centre O to the farthest intersection point of the level generatrices (from $O^{\prime}$ to $2^{\prime}$ and $4^{\prime}$ ). The minimal cutting sphere is a sphere, which contacts one
surface (the larger one) and cuts another (the smaller one). In the example above the minimal sphere contacts the cone surface in the circle $M N\left(m^{\prime} n^{\prime}\right)$ and intersects the cylinder along the circumferences $K L\left(k^{\prime} l^{\prime}\right)$ and $S T\left(s^{\prime} t^{\prime}\right)$. Meeting each other the circles $M N$ and $K L$ yield the points of intersection line $5\left(5^{\prime}\right)$ and $\sigma\left(6^{\prime}\right)$, and the circles $M N$ and $S T$ yield the points $7\left(7^{\prime}\right)$ and $8\left(8^{\prime}\right)$. These are the deepest points of the intersection line.

If an auxiliary sphere cuts only one given surface, this sphere is not proper for the problem solution.

$$
R_{\max }>R_{\text {pass }}>R_{\min }
$$

To construct the second projection of the intersection line one may use the circles obtained at cutting the cone by auxiliary spheres, or to draw the additional sections of the surface. Points 13-14 and 15-16 lying on the level generatrices of the cylinder, are the points of visibility bounds of the intersection line on the horizontal projection.

## Possible Cases of Intersection of Curved Surfaces

There are four variants of two surfaces meeting:

1. Penetration. All generating lines of the first surface (cylinder) intersect the other surface, but not all generatrices of the second surface intersect the first one. In this case the intersection line of the surfaces decomposes into two closed curves (Fig.5.40).


Fig.5.40
2. Cutting-in. Not all generatrices of both surfaces intersect each other. In this case the intersection line is one closed curve (Fig.5.41)


Fig.5.41
3. Unilateral contact. All generating lines of one surface intersect the other surface, but not all generatrices of the second surface intersect the first
one. There is a common tangent plane in one point of the surfaces (the point K, Fig.5.42). The intersection line decomposes into two closed curves meeting in the point of contact.


Fig.5.42


Fig.5.43
4. Bilateral contact. All generating lines of both surfaces intersect each other. The intersecting surfaces have two common tangent planes. In this case the intersection line decomposes into two plane curves which meet in the points of contact (Fig.5.43).

## Intersection of the Surfaces of the Second Order

Generally two surfaces of the second order intersect along a spatial curve of the forth order. But in some special relative positions the surfaces of the second order may meet along the plane curves of the second order, that is the spatial curve of intersection decomposes into two plane curves.

1. If two surfaces of the second order have two common points through which two common tangent planes may be passed to them, the line of their mutual intersection decomposes into two plane curves of the second order, and the planes of the above curves pass through the straight line connecting the tangent points.


Fig.5.44
Fig.5.44 shows the construction of the intersection line of the surfaces of a right circular cylinder and an elliptic cone. At the common points $K$ and $K 1$ these surfaces have common frontal projecting tangent planes $P\left(P_{V}\right)$ and $R\left(R_{V}\right)$. The lines of intersection (ellipses) lie in the profile projecting planes $S$ and $T$, passing through the line $K K_{l}$ which connect the tangent points.
2. Monge theorem. If two surfaces of the second order may be inscribed into the third one or described around it, the line of their mutual intersection decomposes into two plane curves. The planes of those curves pass through a straight line connecting the intersection points of the tangent lines.

Fig.5.45, 5.46 and 5.47 present the examples of construction of the surfaces intersection lines on the basis of Monge theorem, where two cylinders, a cylinder and a cone and two cones are described around a sphere.


Fig.5.45


Fig.5.46


Fig.5.47

The Fig.5.48 illustrates construction of intersection line of two oblate ellipsoids of rotation which inscribed info sphere.


Fig.5.48

## Questions to Chapter 5

1. What is "surface"?
2. What is the meaning of the expression "To specify a surface in a drawing'"?
3. What surfaces are called "ruled surfaces"?
4. What is the difference between the polyhedral surfaces and polyhedrons?
5. What is the condition of a point belonging to a surface?
6. How do we obtain the surfaces of rotation?
7. What lines on a surface of rotation are referred to as parallels and meridians?
8. How is a surface of helicoid formed?
9. What lines are produced by intersection of a rotation cylinder with the planes?
10. What lines are produced by intersection of a rotation cone with the planes?
11. How to pass a plane to obtain a circle in a torus section?
12. What is the general method of drawing the intersection line of surfaces?
13. In what cases do we use projection planes, spheres as mediators for the construction of intersection lines of surfaces?
14. What points of an intersection line are referred to as control ones?
15. Give the formulation of Monge theorem and introduce the example of its application in practice.

## CHAPTER 6. AXONOMETRIC PROJECTIONS

### 6.1 The Method of Axonometric Projection Foreshortening factor

A complex drawing is rather simple and easily measured, although it is hard sometimes to imagine an object in space by means of it. It is often necessary to have in addition to it a drawing of pictorial view, which may be obtained by projecting an object and its co-ordinate axes onto one plane. Then one projection will provide a visual and metrically distinguished image of the object. Such kinds of an object representation are called the axonometric projections.

The method of axonometric projection consists in the following: a given figure and the axes of rectangular co-ordinates to which the figure is related in space are projected on a plane referred to as a plane of projections (it also called a picture plane).

Depending on the distance between the centre of projection and the picture plane all axonometric projections are classified as: the central projections - the centre is located at a finite distance from the plane; and the parallel projections - the centre is at infinity.

Only parallel axonometric projections are considered in this chapter.
The word "axonometry" is derived from the Greek words "axon" which means "axis" and "metro" meaning "I measure', so it can be translated as "the measurement by the axes". That is, an axonometric representation provides the opportunity to measure an object both by the co-ordinate axes $x$, $y, z$ and by the directions parallel to them.

Let us construct an axonometric projection of the point $A$ related to three mutually perpendicular projection planes (Fig.6.1).


Fig.6.1

The $x, y, z$ co-ordinate axes are called the natural axes. Take a scaled line-segment $e$ (natural scale), lay it off on the axes and designate it as $e_{x}, e_{y}$, $e_{z}\left(e=e_{x}=e_{y}=e_{z}\right.$ ). Now project (by parallel beams) the point $A$, the projections $a, a^{\prime}, a^{\prime \prime}$, the co-ordinate axes and the scaled segments $e_{x}, e_{y}, e_{z}$ onto the picture plane $Q$.

Let us introduce some designations:
$Q \quad-\quad$ axonometric projection plane (picture plane);
$S \quad-\quad$ direction of projecting;
$\alpha \quad-\quad$ angle of inclination of the direction $S$ to the plane $Q$;
$x_{1}, y_{1}, z_{1}-$ axonometric co-ordinate axes or axonometric axes;
$A_{1} \quad$ - axonometric projection of the point $A$;
$a_{1}, a_{1}{ }^{\prime}, a_{1}{ }^{\prime \prime}$ - secondary projections of the point $A$;
$e_{x}, e_{y}, e_{z}-\quad$ scaled line-segments;
$e_{x l}, e_{y l}, e_{z 1}$-axonometric (secondary) projections of the scaled linesegments;
Depending on the position of the planes $H, W, V$, the axonometric projection plane $Q$ and the direction of projecting $S$ in the space, the point coordinates will be projected with different distortions.

The ratio of the length of the axonometric projection segment to its true size is referred to as the foreshortening factor on an axis.

Let us designate the above coefficients:
on the axis $x$

$$
m=\frac{e_{x 1}}{e_{x}}
$$

on the axis $y$

$$
n=\frac{e_{y 1}}{e_{y}}
$$

on the axis $z$

$$
k=\frac{e_{z 1}}{e_{z}}
$$

Depending on the ratio of the foreshortening factor on the axes, the following axonometric projections are distinguished:

1. Isometric projections when $m=n=k$;
2. Dimetric projections when $m=k \neq n$ or $m=n \neq k$;
3. Trimetric projections when $m \neq n \neq k$.

The names of the projections are derived from Greek: "isos" - equal (the isometric projection is a projection of the equal coefficients of distortion on all axes); "di" - double (dimetric projection is a projection of the equal coefficients of distortion on two axes); "treis" - three (trimetric projection is a projection of different coefficients of distortion on all axes).

Depending on the direction of projecting relative to the axonometric projection plane $Q$, axonometric projections are classified as rectangular (the projection angle $\alpha=90^{\circ}$ ) and oblique ( $\alpha \neq 90^{\circ}$ ).

The relationship between the coefficients of distortion, depending on the direction of projecting, may be expressed in the following equations:
for oblique axonometry
for orthogonal axonometry

$$
\begin{aligned}
& m^{2}+n^{2}+k^{2}=2+\cot 2 \alpha ; \\
& m^{2}+n^{2}+k^{2}=2 .
\end{aligned}
$$

Depending on the location of the co-ordinate axes, the axonometric projection planes and on the directions of projecting, the vast majority of axonometric projections may be different in the direction of the axonometric axes and in their scales.

The principal theorem of axonometry was declared by the German mathematician K.Pohlke in 1853: "Any three line-segments passing from one point on a plane can be referred to as the parallel projections of three equal and mutually perpendicular line-segments in the space". The first generalisation and proof of this theorem was provided by another German mathematician G.Schwarz in 1864. Since then, the theorem is called the Polke Schwarz theorem.

The definition of axonometry may be derived from it:
Axonometry is a representation of an object on a plane related to a certain coordinate system and completed to a certain scale subject to the coefficients of distortion.

### 6.2 Isometric

In the Isometric is widely used in the practice of technical drawing. In rectangular isometric projection (Fig.6.2) the axonometric axes $x, y, z$ are at $120^{\circ}$ to each other and the coefficients of distortion are similar in all three axes $(m=n=k)$, and are equal to $0.82\left(m^{2}+n^{2}+k^{2}=2 ; m=n=k=\sqrt{ } 2 / 3=0.82\right)$. However, to simplify an isometric projection, a reduction of the coefficients of distortion is usually applied ( $m=n=k=\overline{1}$ ). In this case, the representation obtained is enlarged by 1.22 .


Fig. 6.2

The axis $z$ is positioned vertically, the axes $x_{1}$ and $y_{1}-$ at $30^{\circ}$ to the horizontal direction.

Example: To construct an isometric projection of the point $A$ given its orthogonal projections (Fig.6.3), draw the axonom etric axes at $120^{\circ}$ to each other (Fig.6.4). Then from the origin of co-ordinates $O_{l}$ on the axis $x_{l}$ lay off the line-segment $o_{l} a_{x l}$ equal to the co-ordinate $x_{A}$ of the point $A$. The coordinate $x_{A}$ is taken from the complex drawing (Fig.6.3).


Fig.6.3


Fig.6.4

From the point $a_{x l}$ pass a straight line parallel to the axis $y_{l}$ and on it lay off a segment equal to the co-ordinate $y_{A}$ of the point $A$ to obtain the point $a_{1}$. From this point $\left(a_{1}\right)$ draw a line-segment parallel to the axis $z_{1}$ and equal to the co-ordinate $z_{A}$ of the point $A$. The point $A_{l}$ thus obtained is an isometric projection of the point $A$.


Fig. 6.5

Constructing an isometry of a pentagonal pyramid by its drawing is shown in Fig.6.5. First, determine the co-ordinates of all points of the pyramid base. Then according to $x$ and $y$ co-ordinates construct an isometry of
five points - the vertices of the pyramid base. For example, to construct an isometric projection of the point $A$ from the origin $O_{l}$ on the axis $x_{l}$ lay off a segment equal to the co-ordinate $x_{A}=a^{\prime} d .^{\prime}$ From the end of the segment pass a line parallel to the axis $y_{1}$, on which lay off a segment equal to the second coordinate of the point $y_{A}=a^{\prime} a$. Now construct the pyramid height and find its vertex $S_{l}$. Joining the point $S_{l}$ with the points $A_{l}, B_{l}, C_{l}, D_{l}, E_{l}$ obtain the pyramid isometry.

Fig.6.6 shows the construction an isometry of a hexagonal prism.



Fig. 6.6

### 6.3 Dimetric

In the dimetric the axis $z_{l}$ is vertical, the axis $x_{1}$ is at $7^{\circ} 10^{\prime}$ and the axis $y_{l}$ is at $41^{\circ} 25^{\prime}$ to the horizontal line (Fig.6.7). The coefficients of distortion on the axes $x_{l}$ and $z_{l}$ are assumed to be equal ( $m=k$ ), those on the axis $y_{l}$ twice less ( $n=1 / 2 m$ ). Then:

$$
m^{2}+k^{2}+n^{2}=m^{2}+m^{2}+(1 / 2 m)^{2}=2 ; \quad m=\sqrt{8 / 9}=0.94 ; \quad n=0.47
$$



In practice the reduction of dimetry is usually used with the coefficients of distortion $m=k=1, n=0.5$. In this case the representation is enlarged by 1.06 .

To construct a dimetric projection of the point $A$, given its orthogonal projection (Fig.6.8), pass the axonometric axes at a given angle (Fig.6.9).

On the axis $x_{1}$ from the origin of coordinates lay off the line-segment $o_{1} a_{x l}$
equal to the coordinate $x_{A}$ of the point $A$. From the point $a_{x l}$ draw a straight line parallel to the axis $y_{1}$, and on it lay off a segment equal to the half length (as the foreshortening factor on the axis $y_{l}$ is 0.5 ) of the coordinate $y_{l}$ of the point $A$. From the point a1 pass the segment $a_{1} A_{l}$ equal to the coordinate $z_{A}$. The point $A_{l}$ thus obtained is the desired dimetric projection of the point $A$.


Fig.6.8


Fig.6.9

Construction of a dimetry of a prism with a prismatic hole (Fig.6.10) is shown in Fig.6.11.

To uncover the inside form of the detail, the axonometric projection is completed with a notch $1 / 4$ (the angle contained by the cutting planes is drawn open for a viewer). As the detail is a symmetric one, introduce the origin of the co-ordinates (the point $O$ ) in the prism centre and construct the axes $x, y$, $z$ (Fig.6.10).


To draw the axonometric projection proceed as follows:

Construct the axonometric axes and the plane figures obtained by cutting the detail with the planes $x O z$ and $y O z$ (Fig.6.111)


Рис. 6.10

Designate the vertices of the lower base (the points $1,2,3,4$ ) and construct the axonometric projections of the points 2,3 , 4. Now pass from the obtained points the segments parallel to the axis $z_{1}$ and lay off on them the prism height $z_{H}$ to construct the upper base of the prism (Fig.6.11-2).

Designate the vertices of the prismatic opening in the upper base (points $5,6,7,8$ ) and construct the axonometric projections of the points $6,7,8$. From the obtained points pass the lines parallel to the axis $z_{1}$ and lay off on them the depth of the opening $z_{B}$. Join the points thus obtained with the thin lines (Fig.6.11-3).

Outline the visible lines of the drawing and take away the auxiliary constructions. Draw the cross-hatching lines (Fig.6.11-4)

The cross-hatching lines in axonometric projections are drawn parallel to one of the diagonals of the squares lying in the corresponding co-ordinate planes, the sides of which are parallel to the axonometric axes (Fig.6.12-for isometry, Fig.6.13 - for dimetry).


Fig.6.11


Fig.6.12


Fig.6.13

### 6.4 Representation of a Circle and a Sphere in Axonometry

A circle in axonometry is generally projected in an ellipse. When constructing an ellipse, it is necessary to know the direction of its axes and their dimensions. Note: the minor axis of an ellipse is always perpendicular to the major one.

When a circle projection is constructed (the circle lies in one of the coordinate planes), the minor axis of the ellipse is directed parallel to the axonometric axis which does not participate in the formation of the plane the drawing is in.

## Isometric Projection of a Circle

When an accurate axonometry of a circle is constructed, the length of the ellipse major axis is equal to the diameter of the above circle. When a reduced axonometry is drawn, the dimensions are enlarged by 1.22 , so the ellipse major axis' length makes $1.22 D$, the minor one's $-0.71 D$. Fig. 6.14 presents a graphical method of determination of the ellipse axes' dimensions. Draw a circle of the diameter $D$, the chord $A B=0.71 D$ (the length of the ellipse minor axis). Assuming the points $A$ and $B$ as the centre, with the radius equal to $A B$ draw the arcs to meet each other in $E$ and $F$. Join the obtained points with a straight line. $E F=1.22 D$ (the length of the ellipse major axis).


Fig.6.14


Fig.6.15

Construct the axonometric axes $z_{l}, y_{1}, z_{l}$ and specify in the plane $x_{1}, o_{l}$, $z_{1}$ the directions of the major and minor axes (Fig.6.15). Lay off the segments equal in length to the major $E F$ and the minor $A B$ axes, to meet in the centre of the ellipse - the point $O_{2}$. Through this point pass the lines parallel to the axes $x_{1}$ and $z_{1}$ generating the given plane. On the lines, lay off the values equal to the diameter $D$ of the circle. Join the obtained 8 points to get an ellipse. Another method may also be used for ellipse construction.

Construction of an ellipse in the other planes is similar, only the directions of the axes change.

## Dimetric Projection of a Circle

In contradiction to the isometry where the sizes of the ellipse major and minor axes remain equal whatever the plane of the circle is, in dimetry
only the length of the major axis is always constant (1.06D). The size of the minor axis in the horizontal $(H)$ and profile $(W)$ planes makes $0.35 D$, in the frontal ( $V$ ) plane it makes $0.94 D$.

To determine the size of an ellipse axes by means of the graphical method let us construct a right triangle (Fig.6.16) given the legs ( 100 mm and 35 mm ) and the hypotenuse ( 106 mm ). If we lay off the segment $A B$ equal to the circle diameter $D$ on the longer leg, the segment $D C$ will make $0.35 D$, i.e. will be equal to the length of the minor ellipse axis on the planes $H$ and $W$.


Fig. 6.16
The line-segment $A C$ is equal to $1.06 D$, that is to the length of the major ellipse axis. If we lay off the length of the diameter $D$ (the segment $A K$ ) on the hypotenuse and then drop a perpendicular from the point $K$ to the longer leg of the triangle, the segment $A E$ will be equal to $0.94 D$, i.e. to the length of the ellipse minor axis on the plane $V$.

The representation of a circle in the rectangular dimetric projection is shown in Fig.6.17.


Fig. 6.17
Draw the lines parallel to the axes $x_{1}$ and $z_{1}$ and lay off on them the segments equal to the circle diameter; then draw a line parallel to the axis $y_{1}$
and lay off on it a segment of $0.5 D$. Construct the major and minor axes of the ellipse. Join the points thus obtained with a smooth line.

In rectangular parallel axonometry, a sphere is represented as a circle. When a sphere is constructed by the true values of distortion, its axonometric projection is a circle of the diameter equal to the diameter of the sphere. When a sphere is constructed by reduction, the diameter of the circle enlarges in conformity with the reduction coefficient: in isometry it is 1.22 ; in dimetry - 1.06 .

Fig. 6.18 shows an isometric projection of a torus produced by means of the auxiliary spheres inscribed in it.


Fig.6.18

### 6.5 Oblique Axonometry

## The Frontal Isometric Projection

The position of the axonometric axes is shown in Fig.6.19.
It is admissible to apply the frontal isometric projections with the angle of inclination of the axis $y_{1}$ of $30^{\circ}$ and $60^{\circ}$. The frontal isometric projection is completed without distortion on the axes $x_{1}, y_{l}, z_{l}$.


Fig.6.19


Fig.6.20

The circles lying in the planes parallel to the frontal projection plane $V$, are projected on the axonometric plane as circles. The circles lying in the planes parallel to the planes $H$ and $W$, are projected as ellipses (Fig.6.20).

The major axis of ellipses 2 and 3 makes $1.3 D$, the minor one $-0.54 D$ ( $D$ is the diameter of the circle). The major axis of ellipses 2 and 3 is directed along the acute bisectrix between the lines parallel to the axonometric axes and passing through the ellipse centres.


Fig.6.21
A detail in the frontal isometry should be positioned relative to the axes so that the complex plane figures, circles and arcs of the plane curves are located in the planes parallel to the frontal projection plane (Fig.6.21). In this case their representations are distortionless and the drawing work is simpler to do.

## The Frontal Dimetric Projection

The location of the axonometric axes is similar to that of the frontal isometric projection (Fig. 6.22).

It is admissible to apply the frontal dimetric projections with the angle of inclination of the axis $y_{1}$ of $30^{\circ}$ and $60^{\circ}$.


Fig.6.22


Fig.6.23

The distortion coefficient on the axis $y_{1}$ is 0.5 , on the axes $x_{1}$ and $z_{1}$ it is 1 .

The circles lying in the planes parallel to the frontal projection plane $V$ are projected on the axonometric plane as circles, those lying in the planes parallel to $H$ and $W$ planes - as ellipses (Fig.6.23-6.24).

The major axis of ellipses 2 and 3 is 1.07 , the minor one -0.33 of the circle diameter. The major axis $A_{l} B_{l}$ of ellipse 2 is inclined to the horizontal axis $x_{I}$ at the angle of $7^{\circ} 14^{\prime}$, the major axis of ellipse 3 - at the same angle to the vertical axis $z_{l}$.


Fig.6.24

## Questions to Chapter 6

1. What is the essence of the method of axonometric projection?
2. Formulate the principal theorem of axonometry.
3. What is the foreshortening factor?
4. How are the coefficients of distortion related to each other?
5. How are the axonometric projections classified according to the direction of projecting and the comparable value of the coefficients of distortion?
6. What is the way of determining the direction of the major and minor axes of an ellipse, if ellipse is the isometric and dimetric projection of a circle?

## CHAPTER 7. THE PRINCIPAL RULES OF A DRAWING REPRESENTATION

### 7.1. Unified System of Engineering Papers

The unified rules on presentation of drawings and other technical papers are regulated by Unified System of Engineering Papers (USEP).

The goal of USEP standards is to adopt unified regulations for the completion, presentation and legalisation of engineering papers to achieve the following:

- the opportunity to exchange engineering papers among organisations and productions without their re-presentation;
- the stabilisation of integrity to avoid duplication and the development of unnecessary manufacturing documents;
- the unification of the design of projects for industrial production;
- the simplification of forms of engineering papers and graphical representations;
- the mechanisation and automation of technical papers and information processing;
- the improvement of the operational conditions of industrial products;
- prompt preparation of papers for efficient field re-adjustment.

USEP standards for the development, legalisation and circulation of documents regulate all engineering papers; recording and registering documents; technical part lists and technological documentation; scientific and technical literature and textbooks (unless specially regulated).

USEP standards must be assumed to be the foundation of different methodical and manufacturing papers and technical instructions.

Any object or set of objects produced at a manufacturing plant is referred to as a product. Products are classified as parts (details), assembly units, complexes and sets.

A Detail is a product manufactured without assembly operations from a specific homogeneous material (e.g. a drill, a bolt, a nut, etc.).

Parts of a detail having a certain destination are called its elements, e.g. a chamfer, groove, fillet, etc.

An Assembly unit is a product the parts of which are to be joined by mounting operations, e.g. a machine, telephone, etc.

Complex - two or more products (consisting of two or more parts each) not assembled by the manufacturer but designated to fulfil interrelated operational functions, e.g. shop-automaton, drilling rig, etc.

Set - two or more products not assembled by the manufacturer, being a collection of auxiliary items for an operation, e.g. a set of spare parts, etc.

Engineering papers are the graphical and text documents specifying (separately or as a set) the construction of a product, and including all necessary data for its manufacture, control, operation and repair.

There are 25 types of engineering papers settled by standards in accordance with the contents. Among them: the drawing of a detail, outline drawing, assembly drawing, part lists and others.

The Drawing of a detail is a document containing a detail representation and other data necessary for its manufacture and control.

An Assembly drawing is a document containing an assembly unit's representation and other data necessary for its assembly (manufacture) and control.

An Outline drawing is a document specifying the design of a product and the interactions of its parts, and clarifying the principles of the product's functioning.

A Part list is a document specifying the composition of an assembly unit, a complex or a set.

### 7.2. Formats and Title Blocks

Format (from Latin "forma" - looks, appearance) - the size of drawing sheets and other engineering papers. The regulations establish the following formats and their designations:

| Format designation | A0 | A1 | A2 | A3 | A4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Dimensions of a for- | $841 \times$ | $594 \times$ | $420 \times$ | $297 \times$ | $210 \times$ |
| mat, mm | 1189 | 841 | 594 | 420 | 297 |

If it is necessary it is permitted to use size $A 5,148 \mathrm{~mm} \times 210 \mathrm{~mm}$.

The square of size A0 is $1 \mathrm{~m}^{2}$, the ratio of sides is $1: \sqrt{2}$. Solving of the above two equations determines the dimensions of a format's sides. Each next smaller format is obtained by division (parallel to its smaller side) of the previous format into halves.

It is permissible to use auxiliary formats, produced by means of enlarging the short sides of the basic formats by multiplying by an integral number. For example: size $A 0 \times 2$ is $1189 \times 1682$, size $A 4 \times 3$ is $297 \times 630$, etc.

The format of paper sheets corresponds to the dimensions of the drawing external frame passed with a thin line (the line of clipping a drawing after its completion). The frame of drawing is produced with the base-line. The distance between the edge of paper and the frame is 5 mm and more space is on the left side ( 20 mm ). A specimen lay out (with title blocks, $185 \times 55$, and an additional box, $70 \times 14$ ) is shown in Fig.7.1.


Size A4


Sizes A0...A3


Fig.7.1
For size A4 the title blocks (or boxes) are spread out along the bottom of the paper. For the other sizes - in the lower right-hand corner close to the frame.

Position an additional box in the upper left-hand corner of size A4, and spread it out at the top along the longer side of other formats (Fig.7.1).

The title blocks are drawn in base and thin lines (Fig.7.2). The information in the title blocks is arranged in the following way:

Box 1 - Name of item (of your task) Lettering №7,
Box 2 - Drawing Designation (Lettering №7)
DGG I. XXXXXX. 001
a b c d
a - Department of Descriptive Geometry and Graphics;
b - Test number;
c - Classification characteristics of the item (for a detail or an assembly unit drawing) or XXXXXX (for other drawings);
d - The last figure of the student's card number;
Box 3 - Material of the item;
Box 4 - "T" (training drawing);
Box 6 - Scale (is not denoted on a sketch)


Fig.7.2 Title Blocks (Form 1)
Box 7 - Sheet number (if the task consists of one sheet, the box is not filled in)
Box 8 - Total number of sheets in the test (is filled in only on the first sheet of the test)
Box 9 - TPU (Tomsk Polytechnic University), CDL (Center of Distance Learning), Group number
Box 10 - Student's Name
Box 12 - Student's Signature
Box 13 - Date
Other boxes are not filled in for training drawings.
A drawing designation turned by the angle of $180^{\circ}$ is inserted in an additional box for size A4, and the larger formats if the title blocks spread out along the longer side of a sheet. If in these formats the title blocks are spread along the shorter side, a drawing designation should be turned by the angle of $90^{\circ}$.

### 7.3. Scales

Scale is a ratio of a representation's dimensions to the natural dimensions of a product.

According to the drawing's complexity and the dimensions of the items being represented, select the scales from the following table:

| Reduction <br> scales | $1: 2 ; 1: 2.5 ; 1: 4 ; 1: 5 ; 1: 10 ; 1: 15 ; 1: 20 ; 1: 25 ; 1: 40 ; 1: 50 ;$ <br> $1: 75 ; 1: 100$, etc. |
| :--- | :--- |
| Nominal scale | $1: 1$ |
| Increase scales | $2: 1 ; 2.5: 1 ; 4: 1 ; 5: 1 ; 10: 1 ; 20: 1 ; 40: 1 ; 50: 1 ; 100: 1 ;$ etc. |

Choose a scale to make working with a drawing easier.
The Scale is inserted in the box "Scale" of the title blocks. When dimensioning a drawing, the scale different from that given in the title blocks, is placed just above the representation near the representation sign.

Example: for extension elements, auxiliary and detail views - $\mathrm{A}(2: 1)$ for sectional views and sections - A-A(2:1)

### 7.4. Lines

The Regulations establish design and principle applications of lines on drawings for all production and construction industries.

Thickness $s$ of a continuous base-line must be $0.5-1.4 \mathrm{~mm}$, depending on the size and complexity of the representation and on the drawing size (format). The lines of one drawing must be of equal thickness on all elevations drawn to one scale. You are recommended to draw continuous baselines $0.8-1 \mathrm{~mm}$ thick.

The standard establishes the minimal thickness of lines and the minimal distance between adjacent lines depending on a drawing size; it also specifies the outlining of representations:

* the length of dashes of dashed and dash-and-dot lines depends on the dimensions of representations;
* dashes should be approximately equal in length;
* spaces between dashes of each line must be approximately equal;
- dash-and-dot lines must intersect and end in dashes;
* dash-and-dot lines applied as centre lines should be replaced by continuous thin lines, if a circle diameter or dimensions of other geometric figures on representation is less than 12 mm ;
- for complex sectional views and sections it is permitted to connect the ends of a broken line by a thin chain line.

The Lines and Their Applications


Fig.7.3 presents the dimensions recommended for some types of lines: dashed, chain, chain thick and double-dotted chain.


Fig.7.3

### 7.5. Lettering

All notes on drawings and other technical papers are completed by the drawing lettering. The lettering for technical papers of all production and construction fields is regulated by a standard.

The principal lettering parameters (Fig.7.4):
lettering size $\boldsymbol{h}$ - height of capital letters in mm, measured along a line perpendicular to the basis of line;
height of small letters $\boldsymbol{c}$ (without branches $-k$ );
width of a letter $\boldsymbol{g}$ - maximal width of a letter;
thickness of a lettering line $\boldsymbol{d}$ depends on type and height of lettering.
There are the following lettering types:
type A without sloping ( $d=1 / 14 h$ ); type $A$ with sloping of about $75^{\circ}$ ( $d=1 / 14 h)$; type $A$ with sloping of about $75^{\circ}(d=1 / 10 h)$;
type $B$ without sloping ( $d=1 / 10 h$ );
type $B$ with sloping of about $75^{\circ}(d=1 / 10 h)$, with the parameters shown in the table.

The following lettering sizes are specified by the standard:
(1.8); 2.5; 3.5; 5; 7; 10; 14; 20; 28; 40

Lettering of Type B

| Parameter | Des-n | Relative size |  | Dimensions, mm |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lettering size <br> Height of capi- <br> tal letters | $c$ | $(10 / 10) h$ | $10 d$ | 2.5 | 3.5 | 5.0 | 7.0 | 10 |
| $(7 / 10) h$ | $7 d$ | 1.8 | 2.5 | 3.5 | 5.0 | 7.0 |  |  |
| Distance be- <br> tween letters | $a$ | $(2 / 10) h$ | $2 d$ | 0.5 | 0.7 | 1.0 | 1.4 | 2.0 |
| Minimal pace <br> of lines | $b$ | $(17 / 10) h$ | $17 d$ | 4.3 | 6.0 | 8.5 | 12 | 17 |
| Minimal space <br> between words | $e$ | $(6 / 10) h$ | $6 d$ | 1.5 | 2.1 | 3.0 | 4.2 | 6.0 |
| Thickness of <br> lettering lines | $d$ | $(1 / 10) h$ | $d$ | .3 | .4 | 0.5 | 0.7 | 1.0 |

Notes: 1. Distance $\boldsymbol{a}$ between the letters the neighbouring lines of which are not parallel to each other (e.g. $T A, A T$ ), may be reduced by a half, i.e. by the thickness $\boldsymbol{d}$ of the lettering lines.
2. Minimal distance $\boldsymbol{e}$ between the words, separated by punctuation marks, is the distance between a punctuation mark and a word following it.

Width of Letters and Figures

| Letters and Figures |  | Relative size |  | Lettering dimensions, mm |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2.5 | 3.5 | 5.0 | 7.0 | 10 |
| $$ | 1 |  |  | (3/10)h | 3d | 0.8 | 1.1 | 1.5 | 2.1 | 3 |
|  | 4 | (4.5/10)h | 4.5d | 1.1 | 1.6 | 2.3 | 3.2 | 4.5 |
|  | 2,3,5,6,7,8,9,0 | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | No | (10/10)h | 10d | 2.5 | 3.5 | 5 | 7 | 10 |
|  | Г,Е,3,С | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | Б,В,И,Й,К,Л,Н,О,П | (6/10)h | 6d | 1.5 | 2.1 | 3 | 4.2 | 6 |
|  | $Р, Т, У, Ц, Ч, Ь, Ь, Э, Я$ | (6/10)h | 6d | 1.5 | 2.1 | 3 | 4.2 | 6 |
|  | А,Д,М, , $^{\text {, }}$, | (7/10)h | 7d | 1.8 | 2.5 | 3.5 | 4.9 | 7 |
|  | Ж,Ф,Ш,Щ | (8/10)h | 8d | 2 | 2.8 | 4 | 5.6 | 8 |
|  | c | (4/10)h | 4d | 1 | 1.4 | 2 | 2.8 | 4 |
|  | 3 | (4.5/10)h | 4.5d | 1.1 | 1.6 | 2.3 | 3.2 | 4.5 |
|  | $а, б, в, г, \partial, e, u, \kappa, л, н$ | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | $o, n, p, y, x, u, \downarrow, ъ, b, э, Я$ | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | м, ¢ $^{\text {, }} \ldots$ | (6/10)h | 6d | 1.5 | 2.1 | 3 | 4.2 | 6 |
|  | $\varkappa, m, \phi, u, w$ | (7/10)h | 7d | 1.8 | 2.5 | 3.5 | 4.9 | 7 |
|  | I | (1/10)h | d | 0.3 | 0.4 | 0.5 | 0.7 | 1.0 |
|  | $J$ | (4/10)h | 4d | 1 | 1.4 | 2 | 2.8 | 4 |
|  | $C, E, F, L$ | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | $B, D, G, H, K, N, O$ | (6/10)h | 6d | 1.5 | 2.1 | 3 | 4.2 | 6 |
|  | $P, Q, R, S, T, U, Z$ | (6/10)h | 6d | 1.5 | 2.1 | 3 | 4.2 | 6 |
|  | $A, M, V, X, Y$ | (7/10)h | 7 d | 1.8 | 2.5 | 3.5 | 4.9 | 7 |
|  | W | (9/10)h | 9d | 2.2 | 3.1 | 4.5 | 6.3 | 9 |
|  | $i$ | (1/10)h | d | 0.3 | 0.4 | 0.5 | 0.7 | 1.0 |
|  | $l$ | (2/10)h | 2d | 0.5 | 0.7 | 1.0 | 1.4 | 2.0 |
|  | $j$ | (3/10)h | 3d | 0.8 | 1.1 | 1.5 | 2.1 | 3.0 |
|  | $c, f, r, t$ | (4/10)h | 4d | 1 | 1.4 | 2 | 2.8 | 4 |
|  | $a, b, d, e, g, h, k, n$ | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | $o, p, q, s, u, v, x, y, z$ | (5/10)h | 5d | 1.3 | 1.8 | 2.5 | 3.5 | 5 |
|  | $m, w$ | (7/10)h | 7d | 1.8 | 2.5 | 3.5 | 4.9 | 7 |



Fig.7.4

## АБВГДЕХЗИИККМНОПРСТУ

## фХцчшШЪыьэюя

## аঠВгдежзиклмнопрсту

## фхцчшயьыь эюя

## ABCDEFGHIJKLMNOPQRST

## UVWXYZ

## abcdefghijklmnopqrst

## uvwxyz

# $12345678903 N^{\circ}$ 

Fig.7.5

Standard drawing lettering consists of the Russian, Latin and Greek alphabets, and the Arabian and Roman figures and symbols. A specimen of the Russian and Latin lettering is presented in Fig.7.5.

To give notes on drawings, students must know not only the design of letters and figures, but also the most rational sequence of their outlining. As a rule, when outlining letters and figures, it is recommended to draw all vertical and sloping elements downwards, the horizontal ones - rightwards.

An auxiliary gauze, into which the letters are inscribed, may be applied for writing. The pace of auxiliary lines of the griddepends on the thickness $d$ of the lettering.

### 7.6. Material Designations

Brands of materials are designated in the engineering papers in accordance with the standards for these materials. They are specified by alphanumeric designations.

Graphical conventions of materials in the sections depend on the type of material from which a detail is manufactured.

| Material | Conventions |
| :--- | :---: |
| 1. Metals and hard alloys |  |
| 2. Non-metallic materials including fiber, monolith |  |
| and extruded materials (except named below) |  |
| 3. Timber |  |
| 4. Natural stone |  |
| 5. Ceramic and silicate material for masonry |  |
| 6. Concrete |  |
| 7. Glass and other transparent materials |  |
| 8. Liquid |  |
| 9. Natural ground |  |
| 10. Grid |  |

The distance between the parallel hatching lines is from 1 to 10 mm , depending on the hatched area, and should be similar for all sections of a given part drawn to one scale.

The hatching lines should normally be drawn at $45^{\circ}$ to the frame of the drawing sheet. No restrictions for the direction of hatching but it should be similar for one and the same part in all sections of one drawing. If the shape or position of the section would bring $45^{\circ}$ sectioning parallel or nearly parallel to the outlines or axis lines, another angle may be chosen (usually it is $30^{\circ}$ or $60^{\circ}$ to a horizontal side of the frame).

Cross-hatching of adjacent parts is drawn in the opposite directions (one - rightwards, the other - leftwards). When there are three or more adjacent sections, the direction and also space of the hatching are changed (Fig.7.6).

Sections, the width of which on a drawing is less than 2 mm , are represented blackened with the clearances from 0.8 mm between the adjacent sections (Fig.7.7).


Fig.7.6


Fig.7.7

If a long and narrow section ( $2-4 \mathrm{~mm}$ wide on a drawing) is represented, cross-hatch it only on the ends and at the outline of the holes; the rest section area is presented as shown in Fig.7.8. In such cases, the section lines are inclined at $15^{\circ}-20^{\circ}$ to the longer side of the section outline.


Fig.7.8


Fig.7. 9

## Questions to Chapter 7

1. What are the dimensions of size A 4 of a drawing sheet?
2. What format has the side of 594 mm ?
3. What is the application of dash-and-dot line?
4. What is the recommended thickness of section lines?
5. How thick should the broken line be?
6. What are the lettering dimensions according to standards?
7. What value determines the lettering size?
8. How to determine the height of small letters?

## CHAPTER 8. REPRESENTATIONS

### 8.1 Basic Rules and Definitions

The previous chapters were concerned with the elements of descriptive geometry, which is considered to be the basis of technical drawing. The representations of geometric bodies and the simplest objects were constructed by parallel orthogonal projecting on two or three basic mutually perpendicular projection planes and on the auxiliary planes.

But rather often, three basic planes are not enough for technical drawings of details, devices and other apparatuses. To simplify the process of drawing, to lessen the number of drawing sheets and to save visualisation and unambiguity of a drawing understanding, there are some rules and conventions.

Representations of objects in technical drawing are completed by the method of rectangular (orthogonal) projections. Six faces of a cube are taken as the basic projection planes. The orthogonal projections are completed either in first angle projection (the E-method, "European", applied in Russia and in most of the continental countries) or in third angle projection (the Amethod, "American", applied in the USA, England, Holland).

To distinguish what method is used, the International Standards Organisation (ISO) recommends applying special symbols (Fig.8.1). They are placed above the main inscriptions. If a drawing is completed in the European system, the symbol may be omitted.


Fig.8.1

## Projecting in First Angle Projection (E-Method)

When this method is used, imagine an object to be placed inside a cube and projected on the interior surfaces of its faces. The projecting beams are directed from the viewer to the cube faces. Six faces of the cube are taken as the basic projection planes, they coincide with a drawing plane, as shown in Fig.8.3 and 8.4. The image on the frontal projection plane is assumed to be the principal one. An object is located relative to the frontal projection plane
so that the image on it represents the form and dimensions of the object with sufficient clarity.


Fig.8.2


Fig.8.3

Fig.8.2 shows the directions determining the corresponding views of an object on the projection planes (the cube faces). The rear face is taken for the frontal projection plane. All the rest are coincided with it by rotation around their intersection lines (Fig.8.3). The drawing thus obtained includes six projections (Fig.8.4).


Fig.8.4
Positioning of the images (views) relative to the principal view (the front elevation), after the development (unfolding) of the projection planes in one, is the following: 1 - front elevation; 2 - plan; 3 - left-side elevation; 4 -right-side view; 5 - bottom view, 6 - rear elevation (the last one may be located to the left from the right-side view).

Third Angle Orthographic Projection (A-Method)
When this method is applied, the projection planes are assumed to be transparent and located between the viewer and the object being represented.


Fig.8.5
Fig.8.6
Fig.8.5 shows the directions of viewing the object.
When the projection planes were coincided, we obtain the system of elevations, in which the plan takes the position of the bottom view and the left-side elevation - the position of the right-side view (Fig.8.6).


Fig.8.7
Fig.8.7 demonstrates the positioning of the elevations relative to the principal view (the front one) after development of the projection planes; the names of the elevations is similar to the E-system.

All drawing representations are classified into views, sections and sectional views.

View (elevation) - is a representation of a visible, facing the viewer, part of an object surface. To lessen the number of representations, it is admissible to show the necessary hidden parts in short dashes (Fig.8.8)


Fig.8.8
Sectional View - is a representation of an object with imaginary cut by one or several planes (the imaginary cutting corresponds only to a given section but none of the other views are affected. They keep their normal full outline). When a section is projected, the remaining visible features which can be seen on the other side of the cutting plane are also drawn on the sectional view. But it is admissible to draw not all of them if the construction of the object is clear (e.g. stiffening ribs, Fig.8.10).


Fig.8.9


Fig.8.10

Section - is a figure representation obtained by imaginary cutting of an object by one or several planes. On the section we draw only the elements obtained in the cutting plane.

A cylindrical surface, then developed into a plane, may be applied as a cutting plane (Fig.8.12).


Fig.8.11


Fig.8.12

The number of representations (elevations, sections, sectional views) should be minimal, still providing a clear pictorial view of an object.

Application of different designations allows us to lessen the number of representations on a drawing. In the simplest cases, it is enough to draw only one view and the corresponding designation. The designation $\varnothing$ (Fig.8.13) means that a cylinder is drawn, $\square$ (Fig.8.14) - a bar of a square section. The letters are also used to save time and space on drawings: $s$ denotes the thickness of a drawn object (Fig.8.15), $l$ - its length (Fig.8.16). These letters are written on the shelf of an extension line. The extension line ends in a dot inside the outline of the representation. To distinguish a sphere from the other surfaces, $t 1 Q$ word "Sphere" or the symbol are marked dOvn preceding a dimension, e.g. $\quad \varnothing 25$ (Fig.8.17). The diameter of the sphere symbol is equal to the size of the dimensioning figures in a drawing.


Fig.8.13
Fig.8.14
Fig.8.15

### 8.2. The Views

There are principal, auxiliary and detail views.
The principal views are the views obtained by projecting an object on the six principal projection planes.

If the top, left-side, right-side, rear views have no direct projection link with the principal representation (an elevation or sectional view on the frontal projection plane), an arrow should be drawn to point in the direction that the view is projected. One and the same capital letter should be written above the arrow and above the view (Fig.8.8)

Similar presentation of a drawing is used when the views mentioned above are separated from the principal view by the other views or located on the other sheets.

If some part of the object cannot be shown in true shape on the above elevations, the auxiliary views are applied. The auxiliary views are obtained on the planes not parallel to the principal projection planes (Fig.8.18). Thus, an auxiliary view is an object (or its part) representation on a plane, which is parallel to none of the projection planes.


Fig.8.18
When there is a direct projection link between the auxiliary view and the corresponding representation, the arrow and the designation are not drawn (Fig.8.18, a).

If there is no projection link between the principal and the auxiliary views, the later should be marked with a capital letter, and the linked (with the auxiliary view) representation with an arrow pointing in the direction of the view, plus the corresponding lettering (the arrow A, Fig.8.18, b, c).

An auxiliary view may be rotated (but the position of the object, as a rule, corresponds to the principal view), then the view is marked with a special symbol of rotation - $\bigcirc$ (Fig. 8.18, c).

If necessary, the angle of rotation may also be shown after the symbol. Several similar auxiliary views of one object are designated by one letter and only one view is drawn. If, in this case, the parts of the object, linked with
the auxiliary view, are positioned at different angles, the symbol of rotation $\bigoplus$ is not shown.

a

b

c

Fig.8.19

Fig.8.19 presents the dimensions of the arrow, showing the direction of view (Fig.8.19, a), and the symbols of the words "rotated" (Fig.8.19, b) and "unfolded" (or "developed") (Fig.8.19, c). The size of the capital letter placed at the arrow must exceed the size of dimensions on a drawing by approximately two times.

Representation of a certain limited part of an object surface is referred to as a detail (partial) view (views A and B, Fig.8.20). A detail view may be terminated with a continuous irregular line in the possible minimal size (Fig.8.20); or it may not be terminated (view B, Fig.8.20). A detail view must be marked on a drawing in a similar fashion as an auxiliary view.

If a detail view is to a scale which differs from the scale of the other representations in a drawing, its scale is shown in brackets next to the lettering of the view.


Fig.8.20

### 8.3 Sectional Views

According to a cutting plane's position relative to the horizontal projection plane, the sectional views are classified as:



Fig.8.21


Fig.8.22
horizontal - a cutting plane is parallel to the horizontal projection plane (e.g. sectional view $A-A$, Fig.8.21; sectional view $B-B$, Fig.8.23);
vertical - a cutting plane is perpendicular to the horizontal projection plane (e.g. sectional view on the place of the left-side view, Fig.8.22);
oblique - the cutting plane is inclined to the horizontal projection plane at an angle different from a right one (e.g. sectional views $A-A$ and $C$ $C$, Fig.8.23).


Fig.8. 23
A vertical sectional view is called a frontal one if a cutting plane is parallel to the frontal projection plane, or it is called a profile one if a cutting plane is parallel to the profile projection plane (Fig.8.22).

Depending on the number of the cutting planes, the sectional views are classified as:

Simpel - one cutting plane (Fig.8.21-8.23);
Complex - several cutting planes.
$A-A$


Fig.8.24
The complex sectional views may be step-type when the cutting planes are parallel (e.g. step-type frontal section $A-A$ at Fig.8.24); and polygonal, when the cutting planes intersect (e.g. $A-A$ sectional view at Fig.8.25). If the cutting planes are directed along the object's length or height (Fig.8.24), the sectional views are referred to as lengthwise. If the cutting planes are directed perpendicular to the object's length or height, they are referred to as transverse (Fig.8.22).


Fig.8.25


Fig.8.26

The position of the cutting plane is denoted by the section line.
The section line in a drawing is a broken line, (1...1.5)s thick. It is thick at the ends and at changes of direction. The ends of the line are marked with arrows showing the direction of view. The arrows are drawn at a distance of $2-3 \mathrm{~mm}$ from the ends (Fig.8.26).

The ends of the broken line must not intersect the outline of the corresponding image.

When a complex sectional view is drawn, a change of cutting planes is conventionally not shown.

Fig.8.27 presents a case when the direction arrows are placed on one line.


Fig.8.27
The same Latin capital letter is marked down at the ends of a section line and, if necessary, at the intersection points of the cutting planes. The letters are located near the direction arrows, so that the arrows are between the letter and the view.

Every sectional view should be marked with two letters written with a dash, eng. " $A-A$ ".

In a case when a cutting plane coincides with the symmetry plane of the object, and the corresponding views are located on one and the same sheet in a direct projecting link and there are no other views between them, the position of the cutting plane of the horizontal, frontal and profile sectional views is not marked and there is no lettering (e.g. sectional view on the place of the left-side view, Fig.8.22).

The horizontal, frontal and profile sectional views are located, as a rule, on the drawing space of the corresponding principal views.

The vertical sectional view (when a cutting plane is not parallel to the frontal or profile projection planes) and the oblique one are to be constructed in conformity with the direction pointed by the arrows on the section line.

Such sectional views may be positioned in any drawing space (sectional view B-B, Fig.8.23), and they may also be revolved to correspond the position of the object in the principal view. But then a conventional graphic representation must be added to the lettering (sectional view A-A, Fig.8.23).

It is often necessary to show a small section showing the true shape across an object (polygonal sectional view). Then the revolved section is
obtained by revolving the section in the position and breaking the outline to accommodate the section (Fig.8.28-8.30). The revolving direction may not coincide with the direction of view (Fig.8.29).


Fig.8.28


Fig.8.29


Fig.8.30

The elements located on the other side of the cutting plane are drawn in the way they are projected on the corresponding plane (Fig.8.28, the flange B).

Remembering that sections are used only to clarify a drawing, it is quite likely that you will come across a case where only a small part of the drawing needs to be sectioned to clarify a point. In this case a part or scrap sectional view is permitted.

A scrap cut is distinguished on a drawing by a continuous irregular line (Fig.8.29, 8.33, 8.34).


Fig.8.31


Fig.8.33


Fig.8.32


Fig.8.34

A part of a view and a part of a corresponding section may be joined together and separated by a continuous irregular line or by a continuous polygonal line (Fig.8.32). If a half view and a half section are each a symmetric figure, the axis of symmetry serves as a separating line (Fig.8.34). It is also permitted to use a dash-and-dot thin line (Fig.8.35), which
coincides with the plane of the symmetry trace (not of the whole object, but of its part representing the revolving body).


Fig.8.35


Fig.8.36

It is also possible to join a quarter of a view and the quarters of three


Fig.8.37 sectional views: a view quarter, a quarter of one section and a half of the other, etc., provided each of these representations is a symmetrical one (Fig.8.36).
If an edge representation of a symmetric part coincides with the axis of symmetry, its half view cannot be joined to its half section. A part or scrap sectional view is applied here. And it should be constructed in such a way that the edge is visible (Fig.8.37).

### 8.4 Sections

The sections not included in the sectional view are classified as removed (Fig.8.38) and covering (Fig.8.39).


Fig.8.38


Fig.8.39


Fig.8.40

The removed section should be used in preference to the covering one if there is room on the drawing. It may be located in the break between the parts of one view (Fig.8.40).

The outline of a removed section as well as of that included in a sectional view, is drawn in continuous thick lines, the outline of a covering section - in continuous thin lines. Note: the outline of a view under a covering section is not broken (Fig.8.39).

The axis of symmetry of a removed or covering section (Fig.8.39) is denoted with a dash-and-dot thin line without any lettering or arrows, and the section line is not drawn.

Fig.8.40 shows that on a symmetric section figure the section line is not drawn.

In all other cases the section line is drawn with a broken line. The arrows show the direction of the view. The line is designated with the same Latin capital letters. There is a lettering like, e.g., "A-A" of the section (Fig.8.38).


Fig.8.41


1g.8.42

The section line of the asymmetric sections located in a break (Fig.8.41) or of the covering ones (Fig.8.42) is marked with arrows, no lettering.


Fig.8.43

The construction and position of a section should correspond with the direction [-C pointed by the arrows (Fig.8.43). The location of the section is not restricted by any rules.

The section line of several similar sections of one object is denoted by one letter, and only one section is drawn (Fig.8.43, section A-A and $\mathrm{B}-\mathrm{B}$ ).

If, in such a case, the cutting planes are inclined at
different angles, the graphical convention $\bigcirc$ is not applied. If the positioning of the similar sections is precisely determined by the representation or dimensions, it is admissible to draw only one section line and to show the number of sections above their representation.


Fig.8.44


Fig.8.45

The cutting planes should be chosen so that one obtains the normal


Fig.8.46 cross sections (Fig.8.44).

If a cutting plane passes through an axis of a rotation surface which limits a hole or a recess, the outline of the hole or the recess behind the cutting plane is drawn in the section too (Fig.8.45).

If a section consists of separate independent parts then the sectional views should be drawn (Fig.8.46).

### 8.5 Conventions and Simplifications



If a view, sectional view or a section represent a symmetric figure, only half of the representation may be drawn (Fig.8.20), or a bit more than half, but in this case a break line should be passed.

If in constructing a sectional view, you join a half view and a half section, each being a symmetric figure, the axis of symmetry serves as a separating line. The sectional view here should be drawn to the right or below this axis (Fig.8.47).

Fig.8.47

If an object consists of several similar elements, located evenly, only two or three such elements are represented on a drawing (for example, one or two holes), the rest of the elements are to be shown simplified or conventionally (Fig.8.49).

It is also possible to draw a part of an object (Fig.8.50) with the appropriate designation of the number and location of the elements.


Fig.8.48


Fig.8.49

Projections of the intersection lines of surfaces are also simplified on sections and sectional views, unless their precise constructions are required. For instance, the arcs of circles and straight lines are drawn instead of French curves (Fig.8.51). The smooth transition between surfaces is denoted conventionally or not at all (Fig.8.52).


Fig.8.50


Fig.8.51

## Transition Line


b
Fig.8.52


Fig.8.53

Simplifications, similar to those shown in Fig.8.53, are permitted.
There are some engineering details that, if sectioned, lose their identity or create a wrong impression and these items are never shown sectioned. They are: screws, rivets, keys, shafts, spindles, connecting rods, handles, balls, nuts, washers, etc.


Fig.8.54
Such elements like spokes of flywheels, pulleys, pinions, thin walls, say, of stiffening ribs, are not hatched if a cutting plane is directed along the axis or long side of the element (Fig.8.54).

If in such elements there is a local drilling, a recess and the like, a scrap section should be drawn.

The plates, as well as elements of details (holes, chamfers, slots, recesses, etc.) dimensioned on a drawing 2 mm or less, are constructed as larger size than the scale of the whole drawing.

Small angles of taper or slopes may also be represented with an increase.


Fig.8.55


Fig..8.56

On representations of the elements with an indistinctly disclosed slope or taper (the plan in Fig.8.55), only one line is passed, corresponding to the least dimension of an element with a slope or to the smaller cone base.

If it is necessary to mark out the plane surfaces of an object on a drawing, draughtsmen pass the diagonals with the continuous thin lines (Fig.8.56).

Objects or elements with a permanent or regularly changing cross section (shafts, chains, rods, structural shapes, connecting rods, etc.) may be drawn with breaks (Fig.8.57).

The partial or broken representations are terminated with one of the following lines:
a) a continuous thin line with a break. The line may extend from 2 mm to 4 mm past the outline. This line may be inclined to the contour line;


Fig.8.57
b) a continuous irregular line connecting the corresponding lines of the outline (Fig.8.57).
If you deal with a drawing of entirely gauze, wicker-works, ornaments, knurling, etc., you may draw these elements partially, with all possible simplifications (Fig.8.56).


Fig.8.58

To simplify the drawings or lessen the number of representations it is permitted:
a) to draw an object part located between a viewer and a cutting plane with a chain dot line which is thickened at the proper section (the covering projection, Fig.8.58);
b) to apply complex sections (Fig.8.24 and 8.25);
c) to show only the outline of a hole (Fig.8.48) or a slot (Fig.8.53) instead of the full representation of a detail (when drawing the hub holes of pinions, pulleys and the like, as well as key slots);
c) to draw the sectional views of the holes situated on a round flange which is not in the cutting plane (Fig.8.27).


Fig.8.59
If the plan is not required and a drawing is compiled from the frontal and profile projection planes, the section line and the lettering of a step-type section are completed as shown in Fig.8.59.

Conventions and simplifications permitted for permanent joints, drawings of electric and radio technical devices, etc. are established by the corresponding standards.

### 8.6 Removed view

An extension element is an auxiliary independent representation (usually enlarged) of a certain part of an object which requires the graphic and other designations to clarify the object form, dimensions and other data.

An extension element may include the details not shown on the corresponding representation and may differ from it in contents (e.g. the representation may be a view, where as the extension element - a sectional view).

When an extension element is drawn, the corresponding space on a view, sectional view or a section is marked with a closed continuous thin line (a circle, an oval, etc.) with the extension element designation (a capital letter) on an extension line. The designation and the scale are denoted above the extension element representation (Fig.8.60).


Fig.8.60
The extension element is usually placed as close as possible to the corresponding part of an object representation.

## Questions to Chapter 8

What are the principal views? How are they positioned on a drawing? What are the rules of designating a view having no projecting link with the principal view?
What representation is called an auxiliary view, a detail view? In what cases are they applied and how are they denoted?
When is it permitted to apply a break of a representation?
What representation is called a sectional view? How are the sectional views classified depending on a cutting plane position relative to the horizontal projection plane or relative to the object; depending on a number of the cutting planes?

What sectional view is referred to as a scrap one?
In what cases are the sectional views not designated?
What letters denote the sectional views?
How are the complex sections classified?
What are the peculiarities of drawing a complex step-type sectional view?

When is it permitted to join a half view and a half sectional view?
What line separates a scrap section from the view and how is it drawn?
What elements of an object are not hatched on a section?
What simplifications are used when the projections of the intersection lines of surfaces are drawn?

Are the small angles of taper and slopes shown in all drawings?
How is knurling drawn?
What is a covering projection and what are the rules of its construction?
What is an extension element?
How are the removed view denoted on drawings?

## CHAPTER 9. DIMENSIONING

When a drawing is made, dimensioning is of vital importance since one can determine the size of an object represented only by its dimensioning, whatever the scale is and however accurately the drawing is completed. When dimensioning a drawing, it is very important to specify the dimensions correctly in accordance with an object's application and the conditions of its manufacture. This chapter deals with the principal rules of dimensioning.

## Dimension and Extension Lines. Dimension Figures

Dimensioning of a drawing is completed with dimension figures (numerical values of certain dimensions) and dimension lines, showing the directions and boundaries of measurements. Dimension lines should be parallel to the line-segments dimensions which they show (Fig.9.1 and 9.2).

The dimension line terminates with arrowheads which must touch the boundaries of measurements. The following lines may serve as the boundaries of measurements: extension lines, level lines, axis or centre lines.


Fig.9.1


Fig.9.2


Fig.9.3

Extension lines, as well as axis or centre lines, should extend to $2 \ldots 3$ mm past the arrowheads of dimension lines.

The length of arrowheads depends on the thickness $s$ of the base-line of a given drawing. The recommended arrowhead's length is (4-5) s, Fig.9.3. The length of arrowheads should be approximately equal for all dimensions of a given drawing.

The distance between the parallel dimension lines should be not less than 7 mm . The distance between a dimension line and parallel to it contour (axis or centre) line - not less than 10 mm , depending on the size of a representation and free space on a drawing.


Fig.9.4
Contour (axis, centre, extension) lines must never be used as dimension lines. Dimension lines must never be used as a continuation of contour (axis, centre, extension) lines. Dimension lines should be drawn outside the outline, whenever possible, and should be kept well clear of the intersection points of lines, the conjugate points of arcs, and arcs and lines on a drawing.

To avoid misunderstanding, if there is not enough space for an arrowhead because of the close positioning of a contour or extension line, the latter may be broken (Fig.9.5).


Fig.9.5


Fig.9.6


Fig.9.7

Engineering drawings are usually dimensioned in millimetres, and it is not necessary to put on the abbreviation for the units used (mm). If, to some reasons, the linear dimensions are specified in other units, the abbreviation
for the units used must be given at the dimension figure (e.g. $20 \mathrm{~cm} ; 3 \mathrm{~mm}$ ) or in the corresponding notes of a drawing (technical requirements).

A dimension figure is placed on top of the dimension line, parallel to it and, as a rule, closer to its centre.

If parallel dimensions are to be shown, it avoids confusion if the smallest dimension is put closest to the outline, and the others arranged in ascending order. If several parallel dimension lines, symmetric to the common axis, are to be drawn, it is easier to read the drawing if they are arranged chequer-wise (Fig.9.4, b).

The distance between a dimension line and a dimension figure is about $1-1.5 \mathrm{~mm}$. If the lines are vertical or inclined, dimension figures are given in the way shown in Fig.9.6. If a dimension line is in a hatched area (Fig.9.6), the corresponding dimension figure is placed on the shelf of an extension line (Fig.9.7).

Dimension figures must never be crossed or separated by any lines of a drawing. Dimension figures must never touch any line. If it is necessary to given a dimension figure on an axis line or in a hatched section, the axis/hatching line may be broken (Fig.9.8). No other lines may be broken in such a case. No breaks are permitted in the point of intersection of the centre or axis lines (Fig.9.9, a).


Fig.9.8

a

b

Fig.9.9
When dimensioning angles, draw the dimension lines with a compass; the point of the compass should be on the point of the angle (Fig.9.10). The angles are dimensioned in degrees, minutes and seconds, the units should be designated (Fig.9.10 and 9.11). In the area above the horizontal axis line dimension figures are placed on top of dimension lines from the side of convexity, in the area below the horizontal axis line dimension figures are placed from the side of concavity. If a dimension is to be placed in a hatched area (Fig.9.11),it is placed on the leader line shoulder Likewise dimension the small angles, if there is not enough space, in whatever area they are situated (Fig.9.11, angle $5^{\circ}$ ).


Fig.9.10


Fig.9.11

When the outline is broken, the dimension line is full (Fig.9.12, dimension a).


Fig.9.12


Fig. 9.13


Fig.9.14

When the distance between the boundaries of measurements is less than the length of three arrowheads, counter-arrowheads are used (Fig.9.13, dimensions 5 and 10). The arrowheads of adjacent dimension lines may be taken for the counter-arrowheads (Fig.9.14, dimension b). The lines between the counter-arrowheads are never broken. If there is too little space and it is impossible to place any dimension figure, the variants shown in Fig.9.15 and 9.16 may be used.


Fig.9.15


Fig.9.16

When denoting a group of adjacent small dimensions, replace the arrowheads by clearly denoted dots or hatching lines on the extension lines (Fig.9.17). The slash lines are drawn at $45^{\circ}$ to the dimension lines. The dots and hatching lines must never be placed on the contour lines; in this case, dimensioning is produced as shown in Fig.9.18.


Fig.9.17


Fig.9.18

If there is only one boundary of measurements (for example, when a view is connected to a sectional view (Fig.9.19, a), or when a view with a break takes place (Fig.9.19, b), the dimension lines are broken (Fig.9.19, a, dimensions $\varnothing 10, \varnothing 20$ and $90^{\circ}$; Fig.9.19, b, dimension 20). Note: the dimension line is broken behind the symmetry axis of the representation, at a distance of $6-10 \mathrm{~mm}$ from the axis. The dimension figure is put as close as possible to the symmetry axis and, preferably, between the axis and the contour or extension line. In the given examples, dimensions $\varnothing 10, \varnothing 20,90^{\circ}$ and 20 specify the distance between two symmetrically positioned elements, and not the distance from the elements to the symmetry axis of the representation.


b

c

Fig.9.19
It is also permitted to break the dimension line when a circle diameter is dimensioned, even if the circle is not fully represented (Fig.9.19, c). Note: the dimension line should be broken outside the circle centre.

The extension and dimension lines should be mutually perpendicular. However, if the angle contained by the extension and contour lines is very small (Fig.9.20, dimensions $\varnothing 25$ and $\varnothing 28$ ), the extension line is passed to the dimension line at an angle different from a right one. In this case, as usually, the dimension line is parallel to the dimensioned segment.


Fig.9.20
To simplify a drawing, dimensioning of cylindrical surfaces of a complex configuration is produced as shown in Fig.9.21, a. If it is necessary to specify the co-ordinates of vertices of a rounded-off angle (Fig.9.21, b), the angle vertex is drawn with continuous thin lines.

The arcs of circles may be used as the extension lines (Fig.9.10, a). In this case, the dimension line must coincide with the direction of the arcs' radius.

## Conventional Symbols

A list of the most frequently used conventional symbols is shown below:

| $\varnothing$ | - | diameter |
| :--- | :--- | :--- |
| R | - | radius |
| $\cap$ | - | arc of a circle |
| $\square$ | - | square |
| $\angle$ | - | slope |
| $\triangleright$ | - | taper |



Fig.9.21
Diameter. Diameter is denoted by the symbol $\varnothing$ placed in front of the dimension, e.g. $\varnothing 50$. There are no other symbols or signs between the symbol $\varnothing$ and a dimension. Neither are there any spaces. Diameter may be dimensioned either on a representation where a circle or its part is projected in
true size (Fig.9.22, $\varnothing 40$ ), or on a representation where it is projected as a straight line (Fig.9.22, Ø20).


Fig.9.22
Fig.9.23 presents several examples of diameter dimensioning. Some of the dimensions are placed on the shelves of the extension lines. The shelves are passed horizontally and are produced by means of a break of the dimension line proper. Note: The break should be located at a distance from an arrowhead.




Fig.9.23
Radius. Radius is denoted by the letter R placed in front of the dimension, e.g. R25. There are no other symbols or signs between the letter R and a dimension. The radius dimension line is placed on that representation where the arc is projected in true shape. The dimension line should follow the direction of the true radius and terminate with one arrowhead touching the outline (Fig.9.24, a), or the corresponding extension line (Fig.9.24, b). If a radius is less than 6 mm (on a drawing), the arrowhead should be placed from the external side of the arc (Fig.9.25, a, b).

a

b

a

b

c

d

Fig.9.24
Fig.9.25
The arcs of radius 1 mm and less are not drawn but the dimensions are shown (Fig.9.25, c).


Fig.9.26


Fig.9.27


Fig.9.28

If it is necessary to specify the centre of an arc, it is usually marked by the centre lines crossing (Fig.9.24, b) or by the intersection of the extension lines (Fig.9.26). When drawing an arc of a large radius (no need to mark its centre), the dimension line may be broken (Fig.9.27, R40).

If the centre of an arc of a large radius must be marked, it may be drawn nearer to the outline and the dimension line is broken (Fig.9.28). In this case, both parts of the broken dimension line should be parallel to each other, the line of the break - at $90^{\circ}$ to them. If several dimension lines of radii are passed from one centre, they are not placed on one straight line (Fig.9.29, a). If the values of several radii are equal, they may be shown on one shelf (Fig.9.29, b, c).

a

b

c

Fig.9.29
Note: Radii specify the arcs which round-off the outline, and also most of the arcs are of $180^{\circ}$ and less. Full circles and arcs of more than $180^{\circ}$ are specified only by diameters, even though they may have breaks (Fig.9.30).


Fig.9.30

If the symbol of a diameter or radius is applied to dimension a sphere and it is hard to distinguish the sphere from other surfaces, the word "Sphere" or the sign $\bigcirc$ may precede the symbol, e.g. $\bigcirc$ R12, "Sphere $\varnothing 20$ ", "Sphere R12".

An arc of a circle. The relative position of two points on an arc of a given radius (Fig.9.31) may be determined by the length of the chord $a$, the angle $\alpha$ or by the arc length $l$.


Fig.9.31


Fig.9.32


Fig.9.33

The dimension line specifying the arc length is passed concentrically to the arc, the extension lines - parallel to the angle bisectrix. The symbol $\cap$ is placed above the dimension figure (Fig.9.31). The length of an arc is specified in metric units, and the dimension figure is denoted in the usual way, along the dimension line.

Square. Squares are denoted by the symbol $\square$ placed on the view where the square is projected as a line (Fig.9.32). If a square is dimensioned on a view of its natural shape, dimensions should be shown as in Fig.9.33.

Slope. A slope of a line toward the principal direction is measured in an angle tangent or the angle itself. For example, a slope of the line $A B$ to the line AC is $\tan \alpha$ (Fig.9.34).


Fig.9.34


Fig.9.35


Fig.9.36

Hence, to construct a given slope, say 1:5, draw a right triangle with the legs 10 and 50 mm , or with the legs 11 and 55 mm , etc. The hypotenuse of the triangle yields the desired slope (Fig.9.35).

Slope is denoted by the symbol $\angle$, the acute angle of which points in the direction of the slope. The symbol and the dimensions are placed on the shelf of an extension line with the arrowhead (Fig.9.36). A slope may also be specified in $\%$, e.g. $\angle 5 \%$.

Taper. Taper is a ratio of algebraic difference of the diameters of a truncated cone bases to its height: $K=\left(D_{l}-D_{2}\right) / H$; or the ratio of diameter of a full cone base to its height. 'Taper' is denoted by the symbol $\triangleright$, the vertex of which points in the direction of the cone vertex (Fig.9.37). The symbol and the dimensions are placed on the shelf of an extension line or above the cone axis. Taper may also be specified in $\%$, e.g. $\triangleright 10 \%$.


Fig.9.37
Chamfers dimensioning. When dimensioning cone chamfers (Fig.9.38), the dimension line is passed parallel to the cone axis. The first figure shows the truncated cone height, the second - the inclination angle of the cone generatrix. This simplification is permitted only if the inclination angle of the cone generatrix is $45^{\circ}$; at any other value of the angle two dimensions are given - a linear one and an angular one (Fig.9.39).

If the chamfer height (height of a truncated cone) on a drawing is 1 mm or less, the chamfer is not drawn but is dimensioned (Fig.9.40).


Fig.9.38


Fig.9.39


Fig.9.40


Fig.9.41

Dimensioning of parts is closely connected to the parts' operational usage in an assembly and with their manufacture.

All dimensions may be divided in two groups:
Forming. Dimensions determining the form of any element of a detail;
Coordinating. Dimensions of the arrangement of a detail's elements (distance between holes centres; from edges and frames to holes centres; distances to walls of slots, grooves, flanges and other elements).

It is recommended to group the dimensions of one constructive element in one drawing space of the element view on which this element is represented most completely (Fig.9.42-9.43).


Fig. 9.42
When dimensioning a step-type hole (Fig.9.43, a), its diameter is shown together with its depth. The number of the holes is printed once at the smallest diameter.


Fig. 9.43
There are some more rules about how to dimension a drawing properly:

- dimension lines should be drawn outside the outline, whenever possible;
- unless unavoidable, do not dimension hidden details. It is usually possible to dimension the same detail on another view.
- avoid intersection of the dimension and extension lines, and the crossing of the latter with the outline;
- external dimensions should be placed on other side of a representation than internal dimensions (internal ones - near the sections, ex-ternal- near the views)
The total number of dimensions on a drawing should be minimal but sufficient for manufacturing and monitoring a part. Dimensions should not be repeated. It is necessary to put a dimension on only once, however many views are drawn.


## Dimensioning According to The Datum

Dimensioning an engineering drawing is subject to a number of factors: the geometric form of a part, the peculiarities of its construction and the destination, technology and manufacturing accuracy. To make a part perfect for operation in an assembly unit, it is necessary to adjust the dimensions of the elements and related surfaces. Co-ordination is achieved by putting on dimensions from a certain place taken for the datum.

Surfaces, lines or points co-ordinating the position of a part in an assembly unit or in the process of its manufacturing are referred to as the Datum. There are four types of dimension datum: functional, manufacturing, measurement and assembling, each having its own assignment.

A Functional datum is a surface, line or point determining the position of a part in an assembly unit. Other elements of the part or other parts of the assembly unit are oriented relative to this datum. All adjacent dimensions are specified in accordance with it. Dimensioning according to the functional datum is not connected with the process of manufacturing a part.

A Manufacturing datum is a surface, line or point relative to which each of the surfaces processed during a part manufacturing are oriented. They are selected subject to the sequence of the mechanical processing of the parts. All free dimensions are usually specified according to this datum.

A Measurement datum is a surface, line or point from which the dimensions are indicated during the measurement of ready-to-use parts.

An Assembling datum is a set of surfaces, lines or points relative to which the parts are oriented during their assembly.

Each datum type may be set along three principal directions (a part's length, width and height), sometimes along oblique surfaces.

A part may have several datums, one taken as the principal datum, others as auxiliary. Every auxiliary datum is oriented according to the principal one.

Dimensioning according to the datum should be applied, if possible, to one view (to avoid mistakes).

The manufacturing datum is usually used in training drawing, as it is impossible to determine the position of a certain part in an assembly unit.

The following elements may be taken for a dimension datum: processed ends; shoulders of work-pieces; edges; bearing, supporting and other surfaces; axes of symmetry; centre and other lines; points from which it is convenient to produce dimensioning. Methods of dimensioning depend on the selection of the dimension datum.

There are three methods of dimensioning most frequently applied in practice: chaining, from a datum and combinatorial (Fig.9.44).

The chaining method consists of sequential printing of dimensions pretending to make a chain to determine the sequence of the processing of different elements of a part (Fig.9.44, a). In this case, each dimension is specified by a new datum. This method is applied to show inter-centre distances, dimensions of step-type parts, etc.

The from a datum consists of dimensioning according to one datum (Fig.9.44, b). It is very convenient for production engineering. Dimensions, specified according to one datum, are the co-ordinates showing the distance from this datum to the planes, lines and points of the part. Accuracy here depends only on the technology of the part's manufacturing and does not depend on the accuracy of the other dimensions.


Fig.9.44

The combinatorial method consists of a combination of the chaining and the from a datum (Fig.9.44, c). In this case, not only the principal dimension datum, but also the auxiliary ones are used. This method is widely applied, especially in complex drawings. Whatever the method, the chain of dimensions must never be closed, as it may prevent the accurate manufacturing of a detail. For example, when an overall dimension is to be shown on a drawing and becides it is obtained by means of the chaining method, one of the intermediate dimensions should be omitted.

## Referential and Other Dimensions

Reference dimensions are the dimensions which cannot be completed on a given drawing. But they should be shown to make the drawing easier to use. They are usually denoted by the symbol * and the following note printed in the technical requirements: "*Reference Dimensions".

Among the referential dimensions you may find one of the dimensions of a closed dimensional chain, dimensions from the drawings of blanks, etc.

If one of the dimensions of a dimension chain is marked as referential, the chain may be closed (Fig.9.44, d, e).

Overall dimensions are the dimensions of the external (or internal) limits of an item outline. The overall dimensions are primarily placed on drawings of original parts. Drawings of standard parts do not usually have overall dimensions.

## Simplifications in Dimensioning

There are some simplifications allowing us to reduce the number of dimensions on a drawing (sometimes the number of representations also). For example, dimensions of two symmetrically positioned elements (except holes) are put on only once (no numbering shown), grouped in one place (Fig.9.45). Dimensions of chamfers and grooves are not included in a dimension chain.


Fig.9.45

When several elements, similar in the form and size, are dimensioned, the dimensions of only one of them is shown and the number of the elements. This technique may be used provided the elements lie in one plane (Fig.9.46, a) or are located on a common axis (Fig.9.46, d, e). The number of the elements is placed on the shelf of an extension line (Fig.9.46, a, b, d, e) in front of the dimensional figure. It may also be placed under the shelf (Fig.9.46, c).

If the elements on a drawing are evenly positioned on a circle (holes, recesses and the like), the angle dimensions specifying their mutual location are not printed, only the number of the elements is shown. But it is necessary to denote the angle of $45^{\circ}$ (Fig.9.46, e), as it specifies the positioning of four holes $\varnothing 10$ relative to the slot.


Fig.9.46

When dimensioning the distances between similar evenly positioned elements of an object (Fig.9.47-distances btw 15 holes of diameter 8), instead of the chaining method it is recommended to do as follows. Print the dimension between two neighbouring elements (Fig.9.47, dimension 11) and the product of the distance size by the number of distances, as the dimension between the utmost elements (Fig.9.47, dimension 14x11=154).

If a large number of dimensions is placed according to one common line (base) as shown in Fig.9.48, it is permitted to replace independent dimension lines by one common line passed from the mark 0 : for linear dimensions (Fig.9.49, a), for angle dimensions (Fig.9.49, b).



Fig.9.48

Fig.9.47

b

Fig.9.49

Similar elements located in different parts of an object (for example, holes) are considered as one element provided there is no space between them (Fig.9.50, a) or they are connected by continuous thin lines (Fig.9.50, b). If there are not the above conditions, the total number of the elements must be shown (Fig.9.51, two holes $\varnothing 10$ and Fig.9.52, six holes $\varnothing 8$ ).


Fig.9.50

If similar elements are situated on different surfaces of an object and are shown on different representations, the number of these elements is printed separately for each surface. For example, in Fig.9.52 ten holes $\varnothing 8$ are divided into two groups: 6 and 4.

Dimensions of similar elements or their groups lying on one surface may be repeated only if they are located at a significant distance from each other and are not correlated with each other by dimensions (Fig.9.53).


Fig.9.52


Fig.9.53

If the radii of rounds and fillets, bends and the like on a drawing are similar or if one of them is predominant, they are not dimensioned on a representation. Their dimensions are noted in the technical requirements on a drawing margin by the following signs: "Radii of rounds and fillets 4 mm ", "Internal radii of bends 10 mm ", "Non-mentioned radii 8 mm ".

## Questions to Chapter 9

1. Is the scale taken into account when the dimension figures are put on a drawing?
2. How many millimetres past should an extension line extend from the arrowheads of a dimension line?
3. What are the limits of the distance between parallel dimension lines?
4. How is a dimension line of an angle passed?
5. In what cases are the counter-arrowheads of dimension lines applied?
6. How is a slope denoted on a drawing?
7. What is taper and how is it denoted on a drawing?
8. What does the sign " $2.5 \times 45^{\circ}$ " mean?
9. How should the dimension figures be arranged when several parallel dimension lines are drawn symmetrically relative to the common axis?
10. When are dimensions of several similar, in form and size, elements of an object denoted only once (no numbering)?
11. What are the rules of printing of linear dimensions?
12. What are the recommendations of dimensioning between evenly positioned similar elements of an object?
13.What are the methods of dimensioning?
14.Discuss dimensioning of one functional element.

## CHAPTER 10. JOINTS

### 10.1 Detachable and Permanent Joints

The joints of parts in apparatus, assemblies and machines vary in designated purpose, design form, and technology of manufacture.

Joints are classified into detachable and permanent.
Temporary joint are the joints, recurrent assembly and disassembly of which cause no damage to their parts. Among such kind of joints are: threaded, keyed, splined, pinned, splint-pinned joints, etc.

Permanent joints are not designed for disassembly, hence, it damages the parts of a joint. Among permanent joints are: welded, soldered, adhesive, riveted joints, etc.

Temporary joint are also divided into movable and fixed ones.
The movable temporary joint are the joints, in which one part may move relative to another, e.g. a joint of a movable nut with a screw of a lathe carriage.

The fixed temporary joint are the joints, the parts of which cannot move relative to each other, e.g. a joint by means of a screw or a bolt and a nut.

### 10.2 Thread Representation and Designation

Thread is a surface made by a spiral movement of a plane contour along a cylindrical or conical surface. By such a movement, the plane contour produces a screw protrusion of a corresponding profile, limited by the screw's cylindrical or conical surfaces.

Threads are classified according to the following (Fig.10.1):
The threaded surface's form (cylindrical, conical);
The profile form (triangular, square, trapezoidal, round, etc.);
The screw handedness (right-hand and left-hand);
The number of the thread starts (single-start and multistart);
The thread's location (external and internal);
The designated purpose (fastening, sealing, translational, special threads, etc.).


All threads are divided into two groups: standard and non-standard. Parameters of the standard threads (profile, pitch and diameter) are specified by standards. Parameters of non-standard threads do not correspond to any standards.

A Thread type depends on its profile, i.e. the contour obtained in the cutting plane, which passes through the thread axis.

If the screw motion is made by a point, the spatial curve produced by it is referred to as a screw line (Fig.10.2, a).


Fig.10.2
The principal parameters of thread:
the major (nominal) diameter of thread d,D - diameter of an imaginary cylinder or cone, described around the vertices of the external thread or the root of internal thread;
the minor diameter of thread $d_{l}, D_{l}$ - diameter of an imaginary cylinder or cone, described around the root of external thread or the vertices of internal thread;
the thread profile - the contour of a thread cut by a plane, which passes through its axis (for example, Fig.10.2, b, c shows the triangular profile);
the angle $\alpha$ - is the angle contained by the adjacent lateral profile sides;
the thread pitch $P$ - the distance (parallel to the thread axis) between two corresponding flanks;
the thread lead $P_{h}$ - the axial distance between adjacent corresponding flanks that belong to the same helical surface The axis travel of a screw or a nut per one revolution is equal to one lead of a thread.

A lead of a single-start thread is equal to a thread pitch, a lead of a multistart thread is equal to the product of the pitch P by the number of starts n: $P_{h}=n \times P$

All the diameters of the conical threads are specified in the principal plane. The principal plane is the plane of a reference section, perpendicular to the thread axis and positioned at the given distance $l$ from the cone base (Fig.10.2, d). The pitch of the conical thread is the projection of a linesegment, connecting the neighbouring vertices of the thread profile, onto the axis.

Depending on the screw direction there are right-handed or lefthanded threads. The right-handed thread is produced by the contour, which rotates clockwise and travels along the axis away from the viewer. The left-handed thread is produced by the contour, which rotates counterclockwise and travels along the axis away from the viewer.

The screw thread is designed for the fixed fastening of parts. Both, metric and inch threads can be used as the screw threads.

The screw-sealing threads are designed mostly for the close, hermetically sealed fastening of parts. The pipe and conical threads belong to the screw-sealing threads.

The lead threads are applied to transmit the axis power and the motion of lead screw of lathes, hoisting apparatus, etc. The trapezoidal, buttress, square and round threads can be applied as the lead ones. They may be single-start and multistart.

## Designing and Technological Elements of Threads

The external thread is cut by special machines or by threading tools, thread milling cutters and threading dies. The internal thread is usually cut by threading tools or screw taps. Dies and taps are used for threading preliminary machined parts.

Thread run-out is the areas with an incomplete, evenly reduced profile. The taper is a non-operational part of thread, and it must be considered when determining the length of a threaded detail part.

When an internal thread is cut, the material is first drilled a little deeper than is actually required. The diameter of the hole is the same as the root diameter of the thread and is called the tapping diameter. The screw
thread is then cut with a tap, but the tap cannot reach right to the bottom of the hole and some of the thread run-out is left.

To cut full profile of external or internal thread, an additional circular groove is cut.


Fig.10.3
The circular groove is cut on a shaft (its diameter $d_{\mathrm{e}}$ is smaller than the minor diameter of the thread being cut, Fig.10.3, a) or in a hole (its diameter is larger than the major diameter, Fig.10.3, b) to provide over travel of the threading tool.

An important designing element of the threaded parts is chamfer - an edge of a cylindrical or conical screw body or hole, cut in the way of a truncated cone (Fig.10.3, c).

## Drawing Screw Threads

Drawing a screw thread properly is a tedious task. There are conventions for drawing threads which make life very much easier.

The thread body is represented by continuous thick lines along the external diameter of the thread and by continuous thin lines along the internal diameter (Fig.10.4). On the representations obtained by projecting onto a plane, parallel to the body axis, a continuous thin line (equal to the thread length minus taper) is drawn along the internal thread diameter. On the representations obtained by projecting onto a plane, perpendicular to the body axis, an arc is passed along the internal thread diameter. The arc is drawn $3 / 4$ of the circle, broken at any place. The line specifying the thread boundary is drawn at the end of the complete thread profile (in front of the tapering). It is a continuous base-line, passed up to the line of the external thread diameter (Fig.10.4, a). If the thread is represented as invisible, its boundary is drawn with short dashes up to the line of external diame$\operatorname{ter}$ (Fig.10.4, c, d).

The thread hole is represented by continuous thick lines along the internal diameter of the thread and by continuous thin lines along the external diameter (Fig.10.4).

$a$

b

c

On the representations obtained by projecting onto a plane, perpendicular to the hole axis, an arc is passed along the external thread diameter. The arc is drawn $3 / 4$ of the circle, broken at any place. The line of the thread boundary in the hole is drawn with a continuous base-line, passed up to the line of the external thread diameter (Fig.10.5, b).

The cross-hatching lines on sectional views and sections are passed up to the lines of the external diameter of the screw body, and up to the lines of the internal diameter of the hole, i.e. in both cases cross-hatching is drawn up to the continuous base-lines (Fig.10.4 and 10.5).

If the thread hole is represented as invisible, the thread is drawn with parallel hatching lines of similar thickness. The thread of a hole on a sectional view is more pictorial. The continuous thin line of the thread is passed at a distance of 0.8 mm minimum and the pitch value maximum, from the continuous base-line (approx. 0.85d).

The chamfers of no designated purpose are not represented on the above projection (Fig.10.4, a and 10.5, a). The continuous thin line of the screw body thread must intersect the line of the chamfer's boundary (Fig.10.4, a).


Fig. 10.5
The taper is not considered when dimensioning the thread of a shank or a hole (Fig.10.6, a, b). If it is necessary, proceed as shown in Fig.10.6, c, d.


Fig.10.6
The hole ends with a cone, obtained by drilling (the sharpened end of a drill is of a conical form). Unless otherwise required, the thread is drawn to the bottom of the hole (Fig.10.7, a), the conical end is not shown (Fig.10.7, b).

Table 10.1

| $\begin{gathered} \text { Type } \\ \text { of } \\ \text { Thread } \end{gathered}$ | $\begin{aligned} & \text { Profile } \\ & \text { offead } \\ & \text { Thread } \end{aligned}$ | Conventional Representations of Trread |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | External Trread |  | Interal ITread |  |
|  |  | ammmate | 为 | smater bom | nrumatme |
| $\begin{aligned} & \text { Metric } \\ & \text { (coarse pitch) } \end{aligned}$ |  |  | $\underset{m b}{\infty}$ |  | $\theta^{M 16}$ |
| $\begin{gathered} \text { Metric } \\ \text { (fine pitcth } \end{gathered}$ |  | $\square$ |  |  |  |
| $\begin{aligned} & \text { Parallel inch } \\ & \text { tread } \end{aligned}$ |  |  | $\stackrel{\oplus}{12^{\prime \prime}}$ | $H$ | $\overbrace{}^{1 / 2}$ |
| $\begin{aligned} & \text { Taper inch } \\ & \text { tread } \end{aligned}$ |  | K |  |  |  |
| $\begin{gathered} \text { Pipe } \\ \text { Cilindrical } \end{gathered}$ |  |  |  |  |  |
| $\begin{gathered} \text { Pipe } \\ \text { conical } \end{gathered}$ |  |  |  |  |  |
| Trapezoidal |  | 蓳 | $\overbrace{41762}$ |  |  |
| Buttress |  | 管 | $8$ | $\stackrel{4}{4}$ | $\overbrace{516 \times 2}$ |
| Stuare |  | $\pi^{A}$ |  | $)_{1}^{8}$ |  |

When drawing on the plane parallel to the thread axis, the thread of the hole of a threaded joint is shown only in the part not covered by the shank thread (Fig.10.8, a, b).

One of the methods of representation of thread with a non-standard profile is shown in Fig. 10.9 (with all necessary dimensions and extreme tolerances, dimensions are in letters). The additional data (number of starts, whether the thread is left-handed, etc.) is presented if necessary, accompanied with the word "Thread". Table 10.1 provides conventional representations of thread of different types according to standards.


Fig. 10.8
Fig. 10.7

b


Fig.10.9

## Thread Designations

Metric thread has a triangular profile with the profile angle of $60^{\circ}$ (Fig.10.10). The profile and the basic dimensions of metric thread, also the diameters and pitches, are specified by standards. Metric threads for diameters from 1 to 600 mm are divided into two types: coarse pitch thread (diameters 1-68 mm) and fine pitch thread (diameters 1-600 mm). Starting
from 6 mm , several pitches are specified for each diameter of fine thread, Table 10.2.

The conventional designation of the coarse pitch thread includes the letter M, presenting the thread profile and the nominal (external) diameter of thread in millimetres. For example, M56 indicates that the thread is metric, coarse pitch, nominal diameter 56 mm .

The designation of fine pitch thread additionally includes the pitch dimension in millimetres, e.g. $M 56 \times 3$.

If it a right-hand thread, it is not shown in the convention. If it is a left-hand one, it is designated by letters LH, e.g. $M 56 \times 3 L H$.

Table 10.2

| External <br> diameter <br> of thread | Thread Pitch |  |
| :---: | :---: | :--- |
|  | Coarse | Fine |
| 6 | 1 | $0.75 ; 0.5$ |
| 8 | 1.25 | $1 ; 0.75 ; 0.5$ |
| 10 | 1.5 | $1.25 ; 1 ; 0.75 ; 0.5$ |
| 12 | 1.75 | $1.5 ; 1.25 ; 1 ; 0.75 ; 0.5$ |
| 14 | 2 | $1.5 ; 1.25 ; 1 ; 0.759 ; 0.5$ |
| 16 | 2 | $1.5 ; 1 ; 0.75 ; 0.5$ |
| 18 | 2.5 | $2 ; 1.5 ; 1 ; 0.75 ; 0.5$ |
| 20 | 2.5 | $2 ; 1.5 ; 1 ; 0.75 ; 0.5$ |
| 22 | 2.5 | $2 ; 1.5 ; 1 ; 0.75 ; 0.5$ |
| 24 | 3 | $2 ; 1.5 ; 1 ; 0.75$ |
| 27 | 3 | $2 ; 1.5 ; 1 ; 0.75$ |
| 30 | 3.5 | $3 ; 2 ; 1.5 ; 1 ; 0.75$ |

Conventional representations of standard threads are supplemented by designations to indicate their complete characteristics. Designations of all threads except conical and pipe cylindrical, refer to the external diameter and are printed on the extension and dimension lines (Fig.10.10).


Fig. 10.10

The designations of tapered and parallel pipe threads are shown on the shelves of the extension lines passed from the thread representation (Fig.10.11 and 10.12).

Metric thread may be cut as multistart. Below is an example of the designation of a multistart metric thread:
$M 24 \times 6(P 2)$ - three - start metric thread with the nominal diameter 24 mm , the pitch 2 mm , the lead 6 mm .

Inch (Whitworth) thread is standardised for fastening parts but, as a rule, is applied for manufacture of spare parts.

The profile of the inch thread is an isosceles triangle with the profile angle $\alpha=55^{\circ}$.

The diameter is measured in inches, one inch ( $1^{\prime \prime}$ ) is equal to 25.4 mm . Inch thread is evaluated in terms of the number of threads (or threads of a screw) per an inch of its length.

When designating the inch thread, only its external diameter is shown (in inches). For example: inch thread of diameter $d=1 / 2^{\prime \prime}$ is designated $1 / 2^{\prime \prime}$.

The Pipe cylindrical thread has a triangular profile with the angle $\alpha=55^{\circ}$ and rounded-off crests and roots. Each size of the thread is evaluated in terms of a certain integrate number of pitches per one inch of its length, i.e. per 25.4 mm . The profiles of external and internal threads completely coincide, which guarantees hermetic sealing of threaded joints. Such a thread is applied in pipe joints, also in joining of an internal cylindrical thread with an external conical one. This thread's designation contains the letter $G$, specifying thread type and size. The size of thread is equal to the internal diameter of the pipe, on which this thread is cut. The external diameter is equal to the internal one of this pipe plus two dimensions of thickness of its wall. For example: G1 reads that it is a pipe cylindrical thread cut on a pipe with the internal diameter 25.4 mm (i.e. 1 inch). The external diameter is 33.249 mm (Fig.10.11, a).


Fig. 10.11
Designation of the thread in Fig.10.11, b reads: a pipe cylindrical thread of a hole into which a pipe of the internal diameter 25.4 mm (1inch)
is screwed. Designation of the left-handed thread is completed with the letters $L H$ placed after the thread dimension. Example: GII/2LH - a pipe cylindrical thread, size $11 / 2^{\prime \prime}$, left-handed.

A metric conical thread with the angle $\alpha=60^{\circ}$, angle of taper 1:16 (the inclination angle of generatrices to the geometric axis of the cone $1^{\circ} 47^{\prime} 24^{\prime \prime}$ ) and a nominal diameter from 6 to 60 mm is applied for the conical threaded joints and joints of the external conic thread with the internal cylindrical thread.

Designation: $\quad M K 24 \times 1.5$ (Fig.10.11, c)
For the left-handed thread: $M K 24 \times 1.5 \mathrm{LH}$
For the joints of the external conic thread with the internal cylindrical one:

$$
M / M K 24 \times 1.5
$$

Inch conical thread has a triangular profile with the angle $\alpha=60^{\circ}$. It is cut on the conical surfaces of parts with the angle of taper 1:16. Its designation is K and the nominal diameter in inches, e.g. $\mathrm{K} 11 / 2^{\prime \prime}$.

A pipe conical thread has a triangular profile with the angle $\alpha=55^{\circ}$ and rounded-off crest. It is cut on the conical surfaces of parts with the angle of taper 1:16. Its dimensions on the principal plane correspond to the dimensions of the pipe cylindrical thread.

Its designation is: $R$ - for the external thread, $R_{c}$ - for the internal one, and the thread's size (nominal diameter in inches), e.g. R1 $1 / 2^{\prime \prime}$ - a pipe external conical thread with the nominal diameter $11 / 2^{\prime \prime}$ (Fig.10.12).


Fig. 10.12
A trapezoidal single-start thread has the profile of an isosceles trapezoid with the angle, contained by the lateral sides, $30^{\circ}$ (Fig.10.13, a). Each diameter is specified, as a rule, with three pitches. Designation: Tr , external diameter and pitch, e.g. $\operatorname{Tr} 32 \times 6$ (Fig.10.13, b).

Designation of a multistart trapezoidal thread: Tr, nominal external thread diameter, numeric lead value, and in brackets the letter P for pitch and numeric pitch value. There is the sign x between the nominal diameter and the lead value, e.g. $\operatorname{Tr} 20 \times 4(P 2)$. The letters LH are added for the lefthanded thread: $\operatorname{Tr} 80 \times 40($ P10 $)$ LH, $\operatorname{Tr} 32 \times 6$ LH (Fig.10.13, c).

a

b

c

Fig.10.13
Buttress thread: profile of non-isosceles trapezoid with the angle of a working side $3^{\circ}$, non-working - $30^{\circ}$ (Fig.10.14, a). The profile roots are rounded-off. Both, trapezoidal and buttress threads may have different pitches for one diameter.

Designation: $S$, nominal external diameter and pitch, e.g. $S 50 \times 8$. For the left-handed thread the letters $L H$ are added: $S 50 \times 8 L H$.

Designation for multistart thread: $S$, nominal external diameter, lead value and letter $P$ in brackets with the pitch value, e.g. $S 50 \times 24(P 8)$ (Fig.10.14, c).


The square thread is applied to prevent self-unscrewing under loading. As the profile of this thread is not standardised, all data necessary for its manufacture must be shown on a drawing (Fig.10.15).


Fig. 10.15
There are also the other types of standard threads for special purposes. Special threads are of two types:

1) with the standard profile and non-standard sizes of pitch or diameter. Designations of such threads include the letters $S p$ which means «special», the profile, dimensions of external diameter and pitch. Example: $S p M 60 \times 3.5$ - special metric thread with the external diameter 60 mm and fine non-standard pitch 3.5 mm

2 ) with non-standard profile. The profile of these threads is drawn to an increased scale as an extension element, and all dimensions and other data are placed on it. A square thread, for example, belongs to nonstandard threads (Fig.10.15).

Table 10.1 presents designations of threads of different types.
Dimensions of Design and Technological Elements of Threads
Design and technological elements of threads (chamfers, tapers, tapping holes, turnings) are specified by concrete forms and dimensions. For most standardised threads they are determined in accordance with a thread pitch.

Turned elements (grooves and recessings) are usually drawn simplified, but the drawing must be completed with an extension element. Fig. 10.17 and 10.18 show an example of a representation of a turning, type 1, for the external and internal threads (dimensions in letters).

Conical chamfers on external surfaces and in holes with metric thread are shaped as a truncated cone with an angle $90^{\circ}$ (Fig.10.16).


Fig.10.16

The designing and technological elements are often represented according to dimension relationships, but in accordance with standards regulations.

Thus, the following relationships are used for the metric thread with the pitch P (Fig.10.17 and 10.18):

The width of turnings for the metric thread with the pitch $P$ is assumed to be equal to the length of the corresponding tapping hole, i.e. $b_{e} \approx$ $3 P$ and $b_{i} \approx 4 P$ (Fig.10.3, a, b). The diameter $d_{e}$ of the external turning is drawn a bit shorter than the internal thread diameter $d_{\mathrm{i}}$, likewise the diameter $d_{i}$ of the internal turning - a bit longer than the external thread diameter. A(4:1)


Fig. 10.17


Fig.10.18

### 10.3 Threaded Products and Joints

Threaded joints are widely used in engineering. Usually they are divided into two types:
a) joints obtained by direct screwing of the parts being joined;
b) joints obtained by means of special joining details, such as bolts, screws, studs, fittings, etc.

The most wide spread threaded products are bolts, screws, studs, nuts and the like. All the products are standardised.

Bolts. A bolt is a cylindrical body with a head and a threaded end for a nut. Bolt heads are different in shape. According to the accuracy of manufacture, the bolts are classified as bolts of a normal, long and coarse accuracy rate.

The most frequently used bolts are the ones with a hexagonal head.

The hexagon bolts may have metric thread with the coarse or fine pitch. Fig. 10.19 presents three variants of representation of such a bolt:


Fig. 10.19
To strengthen the bolt, the so-called, fillet is produced - roundingoff of the radius $R$. The term 'length of bolt' means the length of the shank, dimension $l$. To avoid the thread's face and to simplify screwing a nut on, the threaded bolt end is usually turned to a cone (the chamfer is removed).

An example of a bolt designation:
Bolt $M 10 \times 50$ is a bolt of version I (the version is not to be shown in designations) with the metric thread of the diameter $\mathrm{d}=10 \mathrm{~mm}$, with a coarse pitch (which is not to be shown in designations) and the bolt length $\mathrm{l}=50 \mathrm{~mm}$.

Screws. A screw is a cylindrical body with a head and a threaded end. Screws are divided into fastening and set screws (forcing, adjusting, etc). Fastening screws are classified as the screws for metal and those for wood or plastics (wood screws).

Fastening screws for metal work pieces are widely used in machine building (Fig.10.20). They are:

- round-head screw;
- button-head screw;
- countersunk screw;
- semicounter-sunk screw;

Round-head screws have one version of manufacture - with a straight slot.

Button-head, countersunk and semicounter-sunk screws have two versions of manufacture - with a straight and Philips slot.


Fig. 10.20
The length of the screw body is referred to as the screw length l. The length $l$ of countersunk and semicounter-sunk screws includes the body length and the head height $K$.
Examples of designations:
Screw M10x50 - is a screw of version 1 (the version is not to be shown in designations) with the metric thread of diameter 10 mm , with a coarse pitch (which is not to be shown in designations) and the screw length 50 mm .

Screw 2 M 10 x 50 , where

## 2 - version;

M- metric thread;
10- thread diameter in mm;
50-screw length in mm
Wood screws are the screws for fastening wooden and plastic parts, also for fastening the later with metal ones (Fig.10.21). They have a pointed body end and a special profile of thread.

There are four versions of manufacture of the button-head, countersunk and semicounter-sunk wood screws; and only one version of a hexagon wood screw.


Fig.10.21
Designation example:
Wood screw 4-3x20, where
4 - version;
3 - diameter in mm;
20- screw length in mm
Set screws are manufactured with different types of heads and points - flat-point, cone-point, cylinder-point screws, etc. (Fig.10.22). The designation is similar to that of fastening screws.


Fig. 10.22
Studs. A stud is a fastening detail. It is a cylindrical body with two threaded ends (Fig.10.23). The design and dimensions of the studs are standardised. General-purpose studs are used for fastening parts with both, threaded and smooth holes. They are manufactured in two versions, two
accuracy classes. The studs of version 2 have the body diameter $d_{1}$ approximately equal to the average thread diameter.


Fig. 10.23
Designations of the stud elements:
$d$ - nominal diameter of thread;
$l$ - length of stud (length of tie end);
$l_{l}$ - length of driven threaded end;
$d_{1}$ - diameter of body;
$l_{0}$ - length of nut's end.
The length $l_{1}$ of the driven threaded end of a stud depends on the material of the part, the stud is driven into. If it is steel, brass or bronze, $l_{l}=1.25 d$ or $l_{l}=1.6 d$. If it is a fusible alloy, $l_{l}=2 d$ or $l_{l}=2.5 d$.

It is permissible to manufacture the stud with different pitches of the ends.

Designation Examples:

* Stud M20x150 - is a stud of version I (the version is not to be shown in designations) with the thread diameter 20 mm , with a coarse pitch $\mathrm{p}=2.5 \mathrm{~mm}$, and the length $\mathrm{l}=150 \mathrm{~mm}$.

$$
\text { StudM } 20 \times \frac{1.5}{2.5} \times 160,
$$

this is a stud of version I with the thread diameter $d=20 \mathrm{~mm}$, with a fine pitch $p=1.5 \mathrm{~mm}$ on the driven in end and with a coarse pitch $p=2.5 \mathrm{~mm}$ on the nut's end, and the length $l=160 \mathrm{~mm}$.

Nut. A nut is a detail with a threaded hole, which is screwed on a bolt or a stud to join two or more pieces together securely but not permanently. The nuts are classified according to:

* the surface shape;
* the way of manufacture;
* the accuracy of manufacture.

Depending on the surface shape, the nuts may be hexagon (Fig.10.24, a), round, wing (butterfly) (Fig.10.24, b), castle and slotted, which are used on bolted joints of version 2 (Fig.10.24, c), square and the like.

The most popular are the hexagon nuts, the accuracy rating $\mathrm{B}, \mathrm{A}, \mathrm{C}$ (normal, long and coarse precision). The accuracy rating specifies the surface finish class. The nuts of the A and B accuracy classes have metric thread with coarse or fine pitch, where as the nut of the C class has the thread with the coarse pitch.

The hexagon nuts are divided into common (Fig.10.24, a), slotted and castle, normal, thin, thick and especially thick.

There are three versions of the hexagon nuts:
Version 1 - with two external conical chamfers;
Version 2 - with one external conical chamfer;
Version 3 - with a cylindrical or conical flange on one end face, no external chamfers. The nuts of version 3 may be only of the accuracy rate $B$ and C.


Version 2 Version 3

a

b
c
Fig. 10.24

The choice of the nut type depends on its application and operational conditions. Thick and especially thick nuts are used, if the threaded joint is often disassembled during operation, or if it is subjected to a large axial force. When the axial forces are not significant, thin nuts are used.

For the joints subjected to vibrations, the slotted and castle nuts with pins are used (Fig.10.24, c).

Designations:
Nut M16 - a nut of version 1, the thread diameter is 16 mm , coarse pitch of thread.

Nut 2 M16 $\times 1,5-$ a nut of version 2 , the thread diameter is 16 mm , with a fine pitch $1,5 \mathrm{~mm}$.

Washers. The washer is usually placed between a nut and a detail surface to increase the supporting surface and protect the detail from damage when a nut is tightened, also to exclude the possibility of selfunscrewing of the fastening parts. The washers are classified as round (Fig.10.25), spring (Fig.10.26), oblique (Fig.10.27), lock (Fig.10.28), etc.

Version 1


Fig. 10.25


Version 2


Fig. 10.26


Version 2


Fig. 10.27
Fig. 10.28

Round washers may be normal, large and small. There are two versions of normal washers: version 1 without a chamfer, version 2 with chamfer (Fig.10.25). Their dimensions correspond to the fastening parts with the thread from 1 to 48 mm .

An example of designations for a washer of version 1 intended for a fastening part with the diameter 12 mm :

Washer 12.
For a similar washer but of version 2 :
Washer 2.12
Spring washers prevent the loosening of nuts in joints subjected to vibrations and kicks (Fig.10.26).

Spring washers are divided into the following types: light (L), normal (N), hard (H) and especially hard (EH). The designation of spring washers contains the above letters (except N ), which are placed after the diameter of the fastening part.

For example: Washer 12L - it means that the spring washer is light, version 1, produced for a fastening part of diameter 12 mm .

Threaded joints for pipes. Any pipe-line consists of pipes and special joining parts called fittings.

The principal parameter for pipes and fittings is the nominal bore $D_{n}$, which is approximately equal to the internal nominal diameter of the pipe. Nominal bores are standardised. If it is necessary, the pipes may be completed with couplings.

Designations: "Pipe", coupling and coating (if available), nominal diameter, metric length, designation of standard.

If the pipes are strengthened or, vice visa, light, the corresponding letters $S$ or $L$ are added to the designations after the word 'Pipe'. The pipes of long precision are marked with the letter P placed after the nominal bore.

Examples:

1. The common pipe of normal precision, with the nominal bore 40 mm , non-metric length, the wall thickness is $3,5 \mathrm{~mm}$, no thread, no coupling:

Pipe $40 \times 3,5$
2. The same but with a coupling:

Pipe M-40 $\times 3,5$
3. The same but of metric length 8 m , with thread:

Pipe $P-40 \times 3,5-8000$.

The fittings provide the opportunity to join several pipes, to make branches at different angles, to change from one diameter to another and so on.

To obtain the required stiffness of the fittings, they are manufactured with beads; the couplings - with several lateral ribs. Fig.10.29, b shows a right short coupling, Fig.10.29, c - a right elbow, Fig.10.29, d - a right Tjoint.


Fig. 10.29
The designations of fittings include their names, the nominal bore in mm , the number of standard.

Examples:
Coupling short 40 - a short coupling for a pipe with $D_{n}=40$ mm .

Elbow 40 - an elbow for a pipe with $\mathrm{D}_{\mathrm{n}}=40 \mathrm{~mm}$.
If a coupling is used for joining pipes of different diameters, both diameters are designated:

Coupling $32 \times 25-$ a transition coupling, no coating, for the pipes of $D_{n}=32 \mathrm{~mm}$ and $\mathrm{D}_{\mathrm{n}}=25 \mathrm{~mm}$.

## Representation of Detachable Threaded Joints

The components of joints are represented by one of the following methods: detailed, simplified or conventional.

By the detailed method the parts are represented with all details and nominal dimensions taken from the corresponding standards. In training practice it is permitted to complete a detailed drawing of the fastening threaded parts according to the conventional relations of dimensions.

The choice of representation depends on the purpose and scale of the drawing. If the drawing is to a significantly large scale, and the representations of bolted, screw or stud joints are not very small, they are drawn simplified. If the diameters of fastening parts in the drawing are 2 mm or less (a reduced scale), the conventional method is used.

There are the following peculiarities of simplified representations of the fastening parts:
a) dimensions for drawing are determined by the external thread diameter d;
b) chamfers on shank ends of bolts, screws, studs and fillets, as well as chamfers on nuts, bolt heads and washers are not drawn;
c) thread is conventionally shown along the whole length of bolt (screw, stud) shank;
d) internal thread diameter is assumed to be 0.85 d ;
e) thread of bolt or stud on the end elevation is not shown;
f) clearance between a bolt, stud, screw and the hole wall of a fastened part (parts) is not represented;
g ) thread boundary of the full profile in a blind hole is not shown on a sectional view, and the thread is conventionally drawn up to the hole bottom which is represented flat on the level of the screw body end face;
h) on the elevations, obtained by projecting on the plane parallel to the screw axis the screwdriver slot is drawn along the screw axis with a line 2 s thick. On the elevations, obtained by projecting on the plane perpendicular to the screw axis, the slot is drawn at $45^{\circ}$ to the drawing frame, also with a line 2 s thick. If, in this case, the slot coincides with the centre line or is close to it, it is drawn at $45^{\circ}$ to the centre line (Fig.10.30);
j) when drawing hexagonal nuts and bolt heads, situated on flanges, plugs and similar details, their biggest dimension is to be coincided with the radial centre line. It is convenient for drawing hexagons with compasses (Fig.10.31).


Fig. 10.30


Fig.10.31

## Bolted Joints

There usually must be a clearance between a bolt and the bottom of the hole the bolt is driven in. Joining is obtained by tightening a nut.


Fig.10.32

Fig.10.32, a shows a detailed representation of a bolted joint. The bolt, the nut and the washer are drawn according to the nominal dimensions taken from the corresponding standards. Fig.10.32, b presents a detailed representation of the above bolted joint according to the conventional relations of dimensions, pointed on the drawing. Fig.10.32, c contains a simplified representation of the same joint, drawn according to the conventional relations of dimensions, too.

In the conventional relationship the external thread diameter d of a bolt is assumed to be the principal one. The length of the bolt is determined in accordance with the sum of thickness' $\left(H_{1}+H_{2}\right)$ of the joined parts, washer thickness $S$, nut height H and the value K of the minimal bolt protrusion past the nut. Compare the total shank length thus obtained with the data of the corresponding standard and take the closest larger standard length.

## Stud Joints

Fig.10.33, a shows a detailed representation of joining by a stud, a nut and a washer, which are drawn according to the true dimensions taken from the corresponding standards. Fig.10.33, b presents a detailed representation of the above joint according to the conventional relations of dimensions, pointed on the drawing of a bolted joint.


Fig. 10.33
The seat depth is determined due to the formula $l_{3}=l_{1}+l_{2}$, where $l_{1}=d$ - for steel, $l_{l}=1.25 d$ - for cast iron, $l_{1}=2 d$ - for aluminium, and $l_{2}=6 P$,
where P - pitch of thread, $l_{4}=4 P$. When drawing a detailed representation of a stud joint, the boundary line of the thread of the screwed-in end is conventionally coincided with the line of the parts' detachment. In the upper part of the blind hole, occupied by the stud, the lines of the hole thread change for the lines of the stud thread. The outline generatrices of the hole cone must be spaced from the base-lines of the cylindrical hole. The hatching lines in the section are passed up to the base-lines of the thread on the stud and in the seat.

Fig.10.33, c presents a simplified representation of a stud joint, which is also drawn according to conventional relationships.

If stud length $l$ is determined to be similar to that of the bolt, then it is correlated with the standards and the closest value is to be chosen.

## Screw Joints

A screw joint is applied for joining together two or more parts. The screws are driven into the basic part. A hole for a screw may be blind or through. The characteristic dimensions of the joint are the thickness of the parts joined and the diameter of the screw shank.

a

b

Fig. 10.34
The depth of driving screws and studs into similar material is approximately the same. Sizes of screws, their thread diameters and the form of heads are taken from the corresponding standards, subject to the designated purpose of the joined parts.

Fig.10.34, a shows a detailed representation of a screw joint with a cylindrical head, drawn according to the nominal dimensions taken from
the corresponding standards, and Fig.10.34, b - a simplified representation of this joint. When determining dimensions according to conventional relations, proceeding from the external thread diameter d , assume that:

$$
\begin{aligned}
& d_{0}=1,1 d \\
& l_{1}=d-\text { for steel } \\
& l_{1}=1,25 d \text { - for cast-iron; } \\
& l_{1}=2 d-\text { for aluminium; } \\
& l_{2}=(0,3-0,5) d ; \\
& l_{3}=(0,5 \ldots 1) d ;
\end{aligned}
$$

$l_{0}$ and $l$ are selected from standards.
In the detailed representation a clearance between a screw and a hole is to be shown, also an end of a blind screw hole. Unlike the stud joint, the boundary line of thread on the screw must be $2 \ldots 3$ pitches higher than the line of the parts' detachment. Otherwise, it will be impossible to tighten the details joined.

### 10.4. Keyed and Splined Joints

Keys are applied to transmit the torque from one part (shaft) to another (gear-wheel).

The most frequently used keys are: sunk, tapered and Woodruf (semicircular).


Fig. 10.35
Sunk keys are available as square and rounded.
Tapered keys are manufactured with a head and without it.

Sunk keys are manufactured in three versions (Fig.10.35). The lateral faces of these keys are working, there is a clearance above the upper one.

A key section depends on shaft diameter, length - on transmission torque and the design peculiarities of the joint.

In the general case, designations of sunk keys include the word "Key", version (except the first one), the parameters $b \times h \times l$ and key standards.
e.g. Key 2-18×11×100, where

2 - version (except version1);
18 - width $b$;
11 - height $h$;
100 - length $l$.


Version $2 \quad$ Version $3 \quad$ Version 4
Version 3


Fig. 10.36

Tapered keys are manufactured in 4 versions (Fig.10.36). They are used in slowly rotating components. Their working surfaces are the upper and the lower faces. There are clearances between the key lateral faces and the slot.

Designation is similar to that of the sunk keys.
Key 4-18x11x100, where
4 - version;
18 - key width;
11 - key height;
100 - key length.
Woodruf keys have two versions (Fig.10.37). They are used for transmitting small torque (as the deep slot weakens the shaft) and at the ends of shafts with small diameters ( $\mathrm{d} \leqslant 55 \mathrm{~mm}$ ).

Version 1


Version 2

$h_{1}=0.8 \mathrm{~h}$

Fig.10.37
Designation:
Key5x6.5, where
5 - width, 6.5 - height of the key (version 1 ).
Fig. 10.38 presents the examples of dimensioning key slots.


Fig. 10.38

Splined joints. These joints are called multikeyed the keys in such joints are made as a single whole with the shaft, which allows to transmit larger torque in comparison with key joints.

The splined joints are manufactured straight-sided, involute and triangular form. The teeth (protrusions) of the joint enter into the corresponding tooth spaces of the same profile of the joined part.


Fig.10.39
The most popular are the splined joints with a rectangular profile of teeth. The rectangular splined joints differ in the method of alignment of the hub relative to the shaft, which is made by the major diameter D , by the minor diameter d or by the sides b (Fig.10.39).

The tooth surfaces of shafts and holes on drawings are represented simplified. Particularly, the circles and generating surfaces of protrusions are drawn with the continuous base-lines, the circles of tooth spaces - with the continuous thin lines (Fig.10.39, a). On the lengthwise (axis) section the surface of the tooth spaces and crests is shown with the continuous base-lines. Note: the splines on the shaft are conventionally shown not cut (Fig.10.39). On the lateral section the circle of roots is drawn with the continuous thin line. On the representations obtained by projecting onto the plane, perpendicular to the axis of the shaft or the hole, the profile of one tooth (protrusion) and two roots without chamfers, slots and round-offs is shown. The boundary of the tooth surface and the boundary between the teeth of full profile and the tapering are shown with the continuous thin line. The teeth length of the full profile $l_{I}$ up to the tapering is printed on the drawing. It is also permitted to print the full teeth length $l$ or the longest radius $R_{\max }$ of a device or the tapering length $l_{2}$ (Fig.10.40, a). On the
lengthwise section of a tooth joint that surface part of the hole protrusions, which is not covered by the shaft is shown (Fig.10.41).


Fig. 10.40
The designation of the splined joint is subject to the method of alignment:
a) alignment by the internal diameter d (Fig.10.40, a):
d $-6 \times 28 \times 34 \times 6$, where $\mathrm{d}-$ the way of alignment, $6-$ number of teeth; 28 - internal diameter; 34 - external diameter; 6 - tooth width;
b) alignment by the external diameter D (Fig.10.40, b):


Fig. 10.41

D $-8 \times 32 \times 38 \times 8$, where $\mathrm{D}-$ the way of alignment, $8-$ number of teeth; $32-$ internal diameter; 38-external diameter; 8 - tooth width.

On training drawings it is usually enough to show the number of teeth, dimensions of internal and external diameters and tooth width: $8 \times 36 \times 40 \times 7$ (Fig.10.41).

### 10.5 Permanent Joints.

## Welded, Soldered and Adhesive Joints

The permanent welded, soldered (or brazed) and adhesive joints are applied in technological equipment, electronic devices, radio, computation technique, apparatus of automatics and TV mechanics. There are a lot of them and they are quite different. So, let us consider only some of them.

## Representation and Designation of Welds

Independent of welding method, all welds are conventionally drawn in the following way: visible - with the continuous base-line, s thick; invisible - with short dashes, s thick (Fig.10.42).

The conventional representation of visible individual weld spots, independent of the method of welding, is the symbol "+", drawn with continuous base-lines. Invisible solitary points are not showed.

In Russia to designate the location of a joint weld, the extension line with a single-sided arrowhead is used, drawn with the continuous thin line, $\mathrm{s} / 2 \ldots \mathrm{~s} / 3$ thick (Fig. 10.42). The recommended inclination of this extension line to the weld is $30^{\circ} \ldots 60^{\circ}$. The horizontal shelf of this line is of the same thickness. If it is necessary, the extension line may be broken. It is preferable to pass it from the representation of a visible weld.

The ISO standards of the weld designation on a drawing differ from Russian. They are as follows:

The elementary designations, reminding the form of welds, are used to characterise different kinds of welded joints (see Table 10.3).

Designation of a weld on a drawing is made by means of a line with an arrow, pointing to the joint (the arrow must contact the weld), the shelf of an extension line with the weld convention on it, including a certain number of dimensions and symbols. The arrow location relative to the weld is, usually, of no importance. However, the arrow of welded joints 4, 6, 8 (Table 10.3) must be directed to the bevel (Fig.10.43).

When applying the E-method of projection, depending on the arrow location, the weld designation is placed on the shelf, if the extension line is passed on the face of a weld (Fig.10.42, a), and under the shelf, if the extension line is passed on the back of a weld (Fig.10.42, b).


Fig.10.42

Table 10.3

| NoNo | Item | Illustration | Designation |
| :---: | :---: | :---: | :---: |
| 1 | Flange (butt) weld. flanged edges are completely melted | Bermoser |  |
| 2 | Nonbevel-butt weld | MTITIS |  |
| 3 | V-double bevel-butt-weld | MIITM | V |
| 4 | V-single bevel-butt-weld | Mulles | $V$ |
| 5 | $V$-double bevel-butt-weld, wide root face | Mrmus | $Y$ |
| 6 | $V$-single bevel-butt-weld. wide root face | Man | $V$ |
| 7 | $V$-double bevel-butt-weld. sigle-sided curvature | ITIITs | $\varphi$ |
| 8 | $V$-single bevel-butt-weld. single-sided curvature |  | F |
| 9 | Back-up weld. back-up plate (USA) | Muntun | $\square$ |
| 10 | Fillet weld |  |  |
| 11 | Slot weld |  | $\checkmark$ |
| 12 | Spotted weld |  | $0$ |
| 13 | Seam weld |  | 5 |

When applying the A-method, the weld designation is placed under the shelf, if the surface of weld is located on the arrow side, and above the shelf, if the surface of weld is on the reverse side (Fig.10.43).


Fig. 10.43

## Soldered and Adhesive Joints

Soldered and brazed joints are obtained by joining the metal parts with melted metal (solder), the melting point of which is lower than the melting point of the details joined. Soldering is used for hermetic joining, to produce a coating inhibiting corrosion (tin-plating), when fastening parts under small load, etc. Sometimes soldering is better than welding, for example, it is widely used in radio electronics and instrument making.

There is a great number of soldering methods, for example (according to the heating source): with a soldering iron (the simplest method), torch brazing, by metal dip brazing, laser and others.

The solders are:

* according to the melting point - fine solder (up to $145^{\circ} \mathrm{C}$ ), quick solder (up to $450^{\circ} \mathrm{C}$ ), medium (up to $1100^{\circ} \mathrm{C}$ ), hard (above $1850^{\circ} \mathrm{C}$ );
* according to the basic component - pewter solder (PS), pew-ter-lead solder (PLS); brazing spelter (BS), copper-zinc (brass) solder (BS), silver solder (SS), etc.

The solders are manufactured as wire, bars, sticks, etc.
The solders' designations:

PLS40 (no range of products) where 40 - percentage of tin (the rest is lead)

Solder SS70-70\% of silver, $26 \%$ of copper, $4 \%$ of zinc
The solder PLS40 is a soft solder, SS70 is a hard one.
The solder joints (Fig.10.44) are divided into:

- butt joints;
- lap joints;
- fillet joints;
- tee joints (T-joint);
- contact joints.


b

e

c
a

d

Fig. 10.44
Independent of the method of soldering, the joints on sections and sectional views are drawn with a continuous line, 2s thick (Fig.10.45). The convention of soldering (semicircle) is placed on an extension line (drawn with a thin line and started from the joint representation with a doublesided arrow). The semicircle is drawn with a base-line.

If the joint is produced along a closed line, it is designated with the symbol $O$ (circle of diameter $3 \ldots 5 \mathrm{~mm}$ ).

a

b

c

d

Fig. 10.45

The type of joint is printed on the shelf of the extension line in sketches and technical projects, sometimes in working engineering papers, too. For example:


The adhesive joints are obtained by means of glues of different compositions. This method of joining wooden, plastic and metal products is widely used in production. There are the cases when the adhesive joint is the only one possible, e.g. when joining plastic parts.

The designation rules are similar to those of the soldering joints, described above, only the symbol of soldering is changed for the symbol of adhesiolK (Fig.10.46).

a

b

c

Fig. 10.46
Designation of an adhesive substance is printed in the technical notes, in the simplest cases - on shelf of the extension line.

## Questions to Chapter 10

1 . What is thread?
2. What are the characteristic parameters of thread?
3. What is referred to as the thread profile?
4. What is pitch, lead of thread? How are they related?
5. What is the basic difference between the metric and pipe threads, between trapezoidal and buttress threads?
6. How is thread representation on the shank and in the hole on the elevations obtained by projecting onto the planes, parallel and perpendicular to the axis of shank and hole?
7. How is thread drawn in joints?
8. In what cases and in what way is the thread profile drawn?
9. What data (in what sequence) are printed in standard threads' designations in the general case?
10.What is "right-hand" ("left-hand") thread? How are these terms shown in thread designation?
11.What is the difference between designations of threads with coarse and fine pitch?
12.What are the peculiarities of the pipe thread designation?
13. In what cases are the letters " Sp " present in thread designation, what does it mean?
14.Discuss detachable and permanent joints. Give examples.
15. What is a bolt, screw, stud? Give examples of designations.
16.What is a bolted joint?
17.What is a screw joint?
18. What is the difference between detailed and simplified representations of a detachable threaded joint?
19.Discuss the keys (types, manufacture versions and designations).
20.What is splined joint? Discuss conventional representations and designations.
21.Discuss permanent joints.
22.Discuss welded joints, their conventional representations and designations.
23.Discuss soldered joints, their conventional representations and designations.
24.Discuss adhesive joints, their conventional representations and designations.

## INDEX

## A

Abscissa, x-coordinates 15
Angle 26
Apure of Monge 16
Axis

- coordinate 15,17
- projection 15, 17
- rotation 55

Axonometric projections 74

- prism 77
- pyramid 76
- circle 80, 83
- oblique-angled 82
- orthogonal 76


## B

Ball surface 63
Body (solid) of revolution

- ball 57, 63
- cone 56,60
- cylinder 58


## C

Coaxial surfaces 68
Coefficients of distortion 75
Complex drawings 16
Cone 56, 6062
Connection line 16
Contact

- bilateral 71
- unilateral 71

Coordinate 15
Coordinate axes 15
Coordinate system 15
Cross-hatching lines 80
Curve line 54
Cylinder 58

## D

Descriptive geometry 7
Dimetric 78
Drawing

- line-segment 19
- point 15


## E

Ellipse 59, 62
Ellipsoid 58
Equator 63
Evidence 18

## G

Generating line, (generator) 51
Gorge circle 55
Graphical representation 16
Guide line 51, 54

## H

Height 15
Helices 54
Helicoids 54
Hyperbola 54
Hyperboloid 58

## I

Isometric 75

## L

Length 15
Level line 19
Line

- general position 18
- intersecting 23
- parallel 24
- particular position 18
- skew 25


## M

Meridian 55
Method, (way, procedure)

- concurrent points 25
- replacing planes of projection 43


## 0

Ordinate, y- coordinate 15
Origin of coordinates 15
Outline of surface 55

## P

Parabola 58, 60
Paraboloid 58
Parallel 56
Pictorial, (picture) plane 74
Plane

- plane of general position 31
- plane of particular position 31
- projection 9

Plane compartment 29
Polyhedron 52, 54
Principal lines of the plane

- frontal 36
- horizontal 36
- profile line 36

Prism 51, 52
Projection

- axonometric 74
- central 9
- $\quad$ line 12,18
- orthographic 15
- parallel 11
- point 16
- secondary 75

Projection centre 9
Pyramid 52

## R

Reversibility of a drawing 13
Right angle 26

## S

Screw motion 56
Section

- of ball 63
- of cone 61
- of cylinder 58
- of pyramid 53

Side, (face) 51, 55
Size 26

- natural 27
- true 27

Sphere 57, 63, 66
Spiral line 64
Spiral staircase 64
Surface

- polyhedral surface 50
- $\quad$ spiral surface 54
- $\quad$ surface of cone 56,60
- surface of cylinder 56, 58
- surface of prism 51
- surface of pyramid 51
- surface of rotation 55
- surface of sphere 63
- tore 57
- torso 55


## T

Tore 57
Torso 55
Trace

- plane 29
- $\quad$ line 22

Trimetric 74

## V

Vertex 51
Visibility 24

## Z

z-axis, z-coordinates 15

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