CHAPTER 10

ALTERNATING CURRENT

10.1 ALTERNATING QUANTITIES

As mentioned earlier, an alternating quantity is one which reverses its direction periodically, being in one direction (say the 'positive' direction) at one moment and in the opposite direction (the 'negative' direction) the next moment. The frequency of alternations can be as low as once every few seconds or as high as once every few nanoseconds (1 ns = 10^{-9} s or $\frac{1}{1\,000\,000\,000}$ s).

The **frequency** of the UK alternating power supply is 50 cycles per second or 50 hertz (Hz), so that the time for one complete cycle (known as the **periodic time** of the supply) is

$$T = \frac{1}{f} \text{ seconds} = \frac{1}{50} = 0.02 \text{ s or } 20 \text{ ms}$$

Whilst many countries have adopted 50 Hz as the supply frequency, other countries such as the US, use a frequency of 60 Hz, having a periodic time of

$$T = \frac{1}{60} \text{ s} = 0.01667 \text{ s or } 16.67 \text{ ms}$$

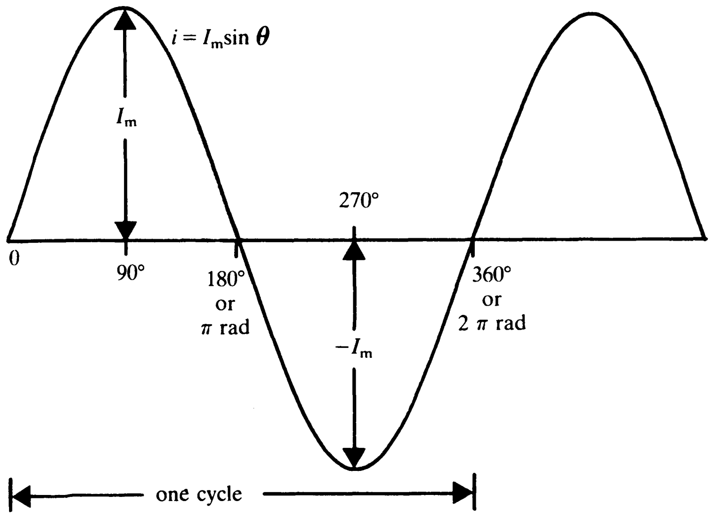
Radio transmissions use a much higher frequency, and a frequency of 10 MHz has a periodic time of

$$\frac{1}{(10 \times 10^6)} = 0.1 \times 10^{-6} \text{ s or } 0.1\ \mu\text{s}$$

The electricity power supply has a sinusoidal waveform of the type in Figure 10.1. The waveform shown is that of a current wave which follows the equation

$$i = I_m \sin\theta \qquad (10.1)$$

fig 10.1 *a sinusoidal current waveform*



where i is the instantaneous value of the current at angle θ, and I_m is the **maximum value** or **peak value**, which is reached at an angle of 90°. The current follows the sinewave through zero, and then increases to its maximum negative value of $-I_m$ at an angle of 270°. The cycle repeats itself every 360°. As mentioned in Chapter 8, the *angular frequency* of a wave is given in radians per second, where

$$\text{angular frequency, } \omega = 2\pi \text{ rad/s} \qquad (10.2)$$

so that the angular frequency corresponding to a frequency of 50 Hz is

$$\text{angular frequency, } \omega = 2\pi \times 50 = 100\pi = 314.2 \text{ rad/s}$$

It was shown in Chapter 8 that an angle in radians is given by

$$\text{angle in radians} = \text{angle in degrees} \times \frac{2\pi}{360}$$

so that

$$\text{for } \theta = 90°\text{, radian angle} = 90 \times \frac{2\pi}{360} = \frac{\pi}{2} \text{ rad}$$

$$\text{for } \theta = 180°\text{, radian angle} = 180 \times \frac{2\pi}{360} = \pi \text{ rad}$$

A complete cycle (360°) is therefore equivalent to

$$\text{radian angle} = 360 \times \frac{2\pi}{360} = 2\pi \text{ rad}$$

If the angular frequency of the waveform is ω rad/sec, then the radian angle, θ, turned through after a time t seconds is

$$\text{radian angle, } \theta = \text{angular velocity (rad/s)} \times \text{time (s)}$$

$$= \omega t \text{ rad}$$

If the supply frequency is 50 Hz (or $\omega = 2\pi f = 100\,\pi$ rad/s), then the angle turned through when $t = 5$ ms after the start of the sinewave is

$$\theta = \omega t = 100\pi \times (5 \times 10^{-3}) = \frac{\pi}{2} \text{ rad (or } 90°)$$

The above equation allows eqn (10.1) to be rewritten in the form

$$\text{current, } i = I_m \sin\theta = I_m \sin\omega t$$

So far we have discussed only a current wave. If the sine wave is that of a voltage, the equation can be written as follows

$$\text{voltage, } v = V_m \sin\theta = V_m \sin\omega t \qquad (10.3)$$

where v is the *instantaneous voltage* of the wave at angle θ after the commencement of the wave (or at time t after the start of the wave), V_m is the *maximum value* or *peak value* of the wave, and ω is the angular frequency of the wave.

10.2 MEAN VALUE OR AVERAGE VALUE OF A SINE WAVE

The strict meaning of the average value of a waveform is

$$\text{average value} = \frac{\text{total area under one complete wave}}{\text{periodic time of the wave}}$$

However, an alternating waveform *has equal positive and negative areas*, so that the **total area under the wave taken over a complete cycle is zero**. That is, the average value of an alternating wave taken over one complete cycle is zero!

The *electrical engineering interpretation of the average value* or *mean value* therefore differs from this and is given by

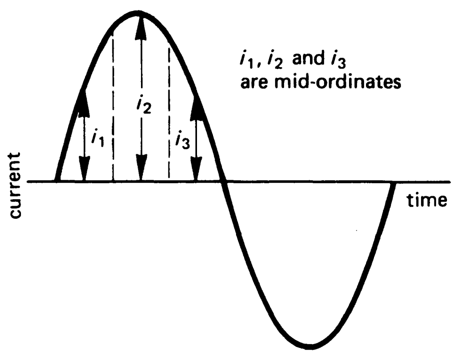
$$\text{mean value} = \frac{\text{area under one half of the waveform}}{\text{one-half of the periodic time}} \qquad (10.4)$$

To determine the area under the half cycle, we shall use the *mid-ordinate rule* as follows. The half cycle (a current waveform is chosen in this case - see Figure 10.2) is divided into an equal number of parts by lines known as *ordinates* (shown as dotted lines in the figure). *Mid-ordinates* i_1, i_2, i_3, etc, are drawn and measured, and the average value of the current in that half cycle is calculated as follows:

$$\text{average current, } I_{av} = \frac{\text{sum of the mid-ordinates}}{\text{number of mid-ordinates}} \qquad (10.5)$$

In Figure 10.2 only three mid-ordinates are shown, which is far too few to give a reliable answer; many more values are usually needed to give an accurate result.

fig 10.2 *the 'mean' value or 'average' value of an a.c. wave*



Suppose that the current waveform has a maximum value of 1 A and that we can divide the waveform up to give ten 18° mid-ordinates (the first being at 9° and the last at 171°). If the values of the mid-ordinates are as listed in Table 10.1, the average value of the current is

$$\text{average current, } I_{av} = \frac{\text{sum of mid-ordinate values}}{\text{number of mid-ordinates}}$$

$$= \frac{6.3922}{10} = 0.63922 \text{ A}$$

The above average value was, in fact, calculated for a sine wave using only ten mid-ordinates. A more accurate value is determined using more mid-ordinates, the most accurate value being obtained using the **calculus** which can be thought of as taking an infinite number of mid-ordinates. It can be shown that using the calculus the **mean value of a sinewave of current** is

$$\text{mean current, } I_{av} = 0.637 I_m \qquad (10.6)$$

where I_m is the maximum value of the current. In our case, the maximum value of the current is 1 A, so that the calculus gives an average current of 0.637 A (compared with our value of 0.639 A).

The same relationship holds for a voltage sine wave; that is **the average value of a sinusoidal voltage wave** is

$$\text{mean voltage, } V_{av} = 0.637\ V_m \tag{10.7}$$

Table 10.1 *Calculation of average value of current*

Angle	*Mid-ordinate current*
9°	0.1564
27°	0.4539
45°	0.7071
63°	0.891
81°	0.9877
99°	0.9877
117°	0.981
135°	0.7071
153°	0.4539
171°	0.1564

sum of mid-ordinates = 6.3922 A

Example

Calculate the mean value of a sinusoidal voltage wave whose maximum value is 100 V. Determine also the maximum value of a voltage wave whose average value is 90 V.

Solution

(i) $V_m = 100$ V

$$\text{mean voltage, } V_{av} = 0.637 V_m = 0.637 \times 100$$
$$= 63.7 \text{ V (Ans.)}$$

(ii) $V_{av} = 90$ V

$$\text{maximum value, } V_m = \frac{V_{av}}{0.637} = \frac{90}{0.637} = 141.3 \text{ V (Ans)}$$

10.3 THE EFFECTIVE VALUE OR ROOT-MEAN-SQUARE (r.m.s.) VALUE OF A SINE WAVE

The **effective value** of an alternating current (or an alternating voltage for that matter) is expressed in terms of its heating effect; that is, it is given in terms of its I^2R effect. The effective value is known as the **root-mean-square** (r.m.s.) value of the wave and is calculated as follows.

r.m.s. value = **square root** of the **mean** of the sum of **squares** (r.m.s.) of the mid-ordinate values of the wave.

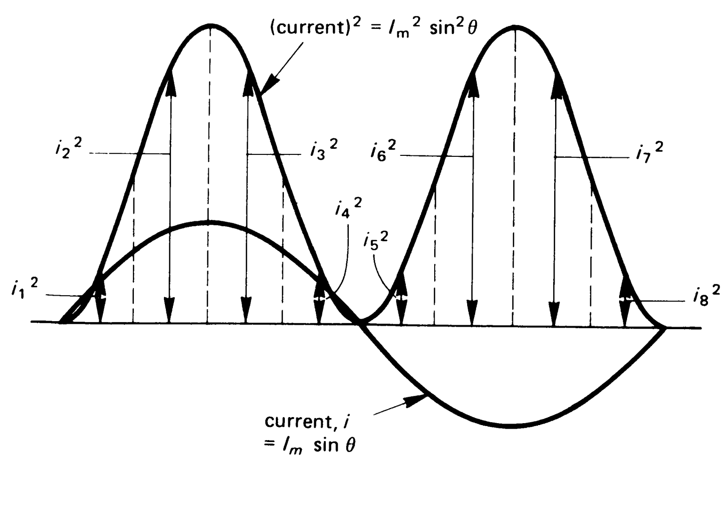
This is illustrated in Figure 10.3. Each value on the waveform (the current waveform in this case) is multiplied by itself (that is, it is 'squared'); if the wave is a sinewave, the equation of the wave is $I_m \sin\theta$, and the equation of the (current)^2 graph is $I_m^2 \sin^2\theta$. You will see that the (current)^2 graph has a positive value in the second half cycle even though the current is negative (this is because the product of two negative values is a positive value). The r.m.s. value of a sinusoidal current wave is given by

$$\text{r.m.s. current, } I = \sqrt{\frac{\text{(sum of the mid-ordinate (current)}^2\text{ values)}}{\text{number of mid-ordinates}}} \qquad (10.8)$$

or, alternatively

$$\text{r.m.s. current, } I = \sqrt{\frac{\text{(area under (current)}^2\text{ curve)}}{\text{(length of base of the waveform)}}} \qquad (10.9)$$

fig 10.3 *the r.m.s. value of an alternating wave*



In the following we will determine the r.m.s. value of a sinusoidal current waveform whose values are given in Table 10.2; mid-ordinates are taken at 36° intervals (the first at 18°). The results in the table give

$$\text{sum of (current)}^2 = 5.0 \text{ (amperes)}^2$$

hence

$$\text{r.m.s. value of current} = \sqrt{\frac{\text{sum of (current)}^2}{\text{(number of mid-ordinates)}}}$$

$$= \sqrt{\frac{5}{10}} = 0.7071 \text{ A}$$

That is to say, for a *current sine wave* the r.m.s. value is given by

$$\text{r.m.s. current, } I = 0.7071 I_m = \frac{I_m}{\sqrt{2}} \qquad (10.10)$$

Similarly, for a *sinusoidal voltage* the r.m.s. value is given by

$$\text{r.m.s. voltage, } V = 0.7071\ V_m = \frac{V_m}{\sqrt{2}} \qquad (10.11)$$

Example

(i) Calculate the r.m.s. value of a sinusoidal current wave of maximum value 20 A. Determine (ii) the maximum value of a voltage wave whose r.m.s. value is 240 V.

Table 10.2 *Calculation of r.m.s. value of current*

Angle	*Value of current*	*Value of (current)²*
18°	0.3090	0.0955
54°	0.8090	0.6545
90°	1.0	1.0
126°	0.8090	0.6545
162°	0.3090	0.0955
198°	−0.3090	0.0955
234°	−0.8090	0.6545
270°	−1.0	1.0
306°	−0.8090	0.6545
342°	−0.3090	0.0955
	sum of (current)^2 =	5.0

Solution

(i) $I_m = 20$ A

$$\text{r.m.s. current, } I = 0.7071\, I_m = 0.7071 \times 20$$
$$= 14.14 \text{ A (Ans.)}$$

(ii) $V_S = 240$ V r.m.s.

From eqn (10.7), $V_S = \frac{V_m}{\sqrt{2}}$, hence

$$\text{maximum voltage, } V_m = V_S \times \sqrt{2} = 240 \times 1.414$$
$$= 339.36 \text{ V (Ans.)}$$

10.4 AVERAGE VALUE AND r.m.s. VALUE OF A WAVE OF ANY SHAPE

So far we have discussed the mean and r.m.s. values of a sinewave. Every waveform has its own average and r.m.s. value, and can be calculated for current waves from eqns (10.4) and (10.5) and for voltage waveforms from eqns (10.8) and (10.9). Illustrative calculations are performed for the triangular wave in Figure 10.4.

Since the waveform has equal positive and negative areas, the mean value must be calculated over one half cycle; for convenience, the positive half-cycle is used. In this case the values are for a voltage wave (but it could well be a current wave!) and, for the wave in Figure 10.4:

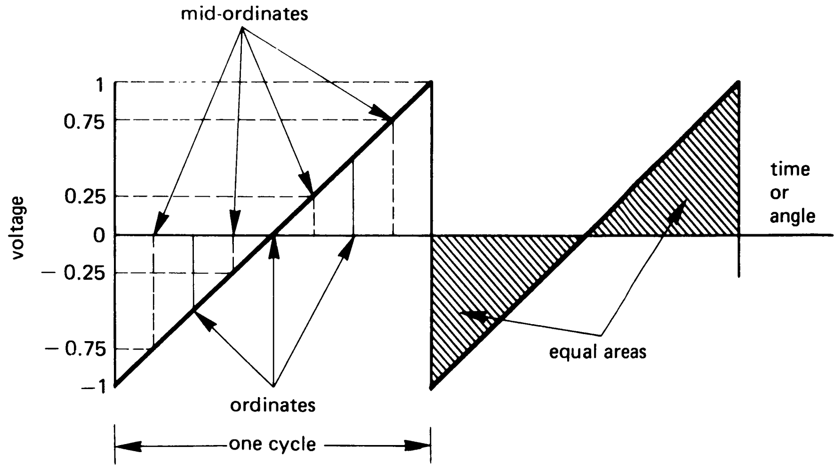
$$\text{mean voltage, } V_{av} = \frac{\text{sum of mid-ordinate voltages}}{\text{number of mid-ordinates}}$$
$$= \frac{(0.25 + 0.75)}{2} = 0.5 \text{ V (Ans.)}$$

The r.m.s. value of the wave in Figure 10.4 is calculated over a complete cycle using the equation

$$\text{r.m.s. voltage, } V = \sqrt{\frac{(\text{sum of mid-ordinate (current)}^2 \text{ values})}{(\text{number of mid-ordinates})}}$$
$$= \sqrt{\left(\frac{([-0.75]^2 + [-0.25]^2 + 0.25^2 + 0.75^2)}{4}\right)}$$
$$= 0.559 \text{ V (Ans.)}$$

Note

You should be aware of the fact that taking only a few mid-ordinates may give a result of low accuracy. Whilst the mean voltage calculated above is fig 10.4 *mean and r.m.s. values of a non-sinusoidal wave*



perfectly correct, the r.m.s. value obtained above is of limited accuracy (the value using many mid-ordinates is 0.5774 V.)

10.5 FORM FACTOR AND PEAK FACTOR OF A WAVEFORM

Information relating to the 'shape' of an a.c. waveform is often useful to electrical engineers. The factors known as the **form factor** and the **peak factor** (the latter also being known as the **crest factor**) act as electrical 'fingerprints' of the wave. Two differing waveforms may have the same value for one of the factors, but the other factor will differ between the two waves, indicating that the waveforms are different in shape. The two factors are defined below

$$\text{form factor} = \frac{\text{r.m.s. value of wave}}{\text{mean value of wave}} = \frac{I}{I_{av}} \text{ or } \frac{V}{V_{av}} \tag{10.12}$$

and

$$\text{peak factor} = \frac{\text{peak value of wave}}{\text{r.m.s. value of wave}} = \frac{I_m}{I} \text{ or } \frac{V_m}{V} \tag{10.13}$$

The **form factor and peak factor for a sinewave** (taking a current wave in this instance) are calculated as follows:

$$\text{form factor} = \frac{I}{I_{av}} = \frac{0.7071\, I_m}{0.637\, I_m}$$

$$= 1.11$$

$$\text{peak factor} = \frac{I_m}{I} = \frac{I_m}{0.7071\, I_m}$$

$$= 1.414$$

Example

Calculate the form factor of the triangular voltage wave in Figure 10.4.

Solution

The r.m.s. and mean values estimated for the wave are

r.m.s. value = 0.559 V
mean value = 0.5 V

hence

$$\text{form factor} = \frac{\text{r.m.s. value}}{\text{mean value}} = \frac{0.559\,V}{0.5\,V} = 1.118$$

Note

The above figure differs only slightly from that of the sinewave, the reason being that the r.m.s. value determined in the calculation is not sufficiently accurate. If the true r.m.s. value of 0.5774 V is used, the form factor of the wave is seen to be

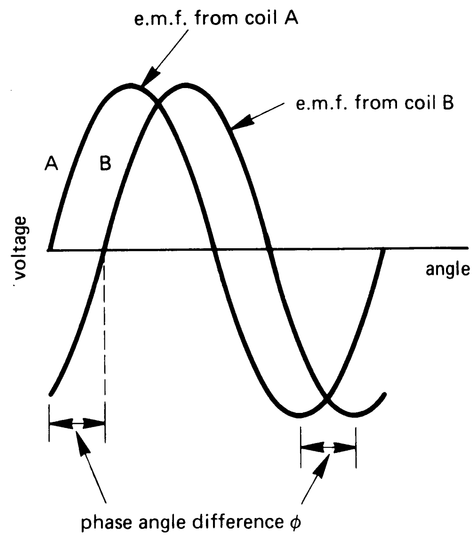
$$\frac{0.5774}{0.5} = 1.155$$

10.6 PHASE ANGLE DIFFERENCE BETWEEN TWO SINEWAVES

Suppose that we have a two-coil alternator on which both coils have the same number of turns but are in different physical positions on the rotor. Because of the difference in the position of the coils, the e.m.f. in each coil will differ, as illustrated in the waveforms in Figure 10.5 for coils A and B on the rotor of the alternator.

As each coil rotates, a sinewave of voltage is induced in it, but each wave differs from the other by an angle ϕ which is known as the *phase angle* difference (which can be expressed either in degrees or radians). The phase angle difference (often simply referred to as the *phase angle*) is the *angular difference between the two waves when they are at the same point on their waveform*. For example, the phase angle difference can be measured as the angular difference between the waves when they both pass through zero and are increasing in a positive direction (see Figure 10.5); alternatively it is the angular difference between the two waves when they pass through their peak negative voltage (also see Figure 10.5).

fig 10.5 *phase angle difference*



Lag or lead?

When describing phase angle difference, it is frequently necessary to know which wave 'lags' or which wave 'leads' the other, that is, which wave is 'last'and which is 'first'? The answer to this is revealed from a study of Figure 10.5. Clearly, waveform A passes through zero *before* waveform B; from this we can say that

waveform A leads waveform By by angle ϕ

Alternatively, since waveform B passes through zero *after* waveform A, we may say that

waveform B lags behind waveform A by angle ϕ

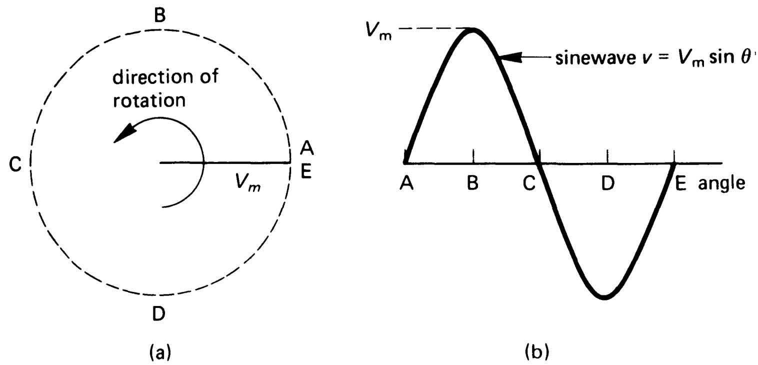
Both statements are equally correct.

10.7 PHASOR DIAGRAMS

Waveform diagrams are difficult to visualise, and engineers have devised a diagrammatic method known as a **phasor diagram** to simplify the problem.

Imagine a line of length V_m rotating in an anti-clockwise direction (see Figure 10.6(a)). If you plot the *vertical displacement* of the tip of the line at various angular intervals, the curve traced out is a sinewave (see Figure 10.6(b)).

fig 10.6 *production of a sinewave*

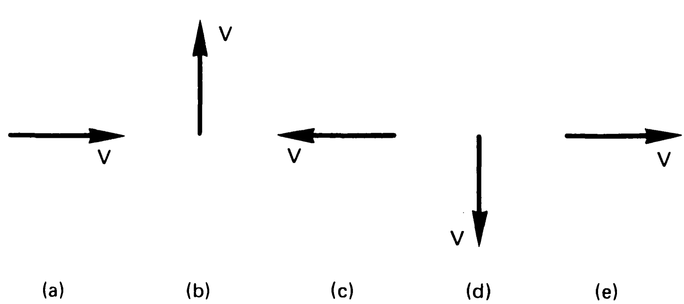


When the line is horizontal, the vertical displacement of the tip of the line is zero, corresponding to the start of the sinewave at point A. After the line has rotated through 90° in an anti-clockwise direction, the line points vertically upwards (corresponding to point B on the waveform diagram). After 180° rotation the line points to the left of the page, and the vertical displacement is zero once more (corresponding to point C on the waveform diagram). After a further 180° (360° in all) the rotating line reaches its starting position once more (corresponding to point E on the waveform diagram).

A **phasor** is a line representing the rotating line V_m, but is **scaled to represent the r.m.s. voltage, V, which is 'frozen' at some point in time.**

For example, if the rotating line is 'frozen' at point A in Figure 10.6(b) the corresponding voltage phasor which represents this is shown in Figure 10.7(a). If the rotating line is 'frozen' at point B, the voltage phasor is as shown in Figure 10.7(b). Voltage phasors representing the 'freezing' of the rotating line at points C, D and E in Figure 10.6 are represented by the

fig 10.7 *several phasor diagrams for Figure 10.5(b)*



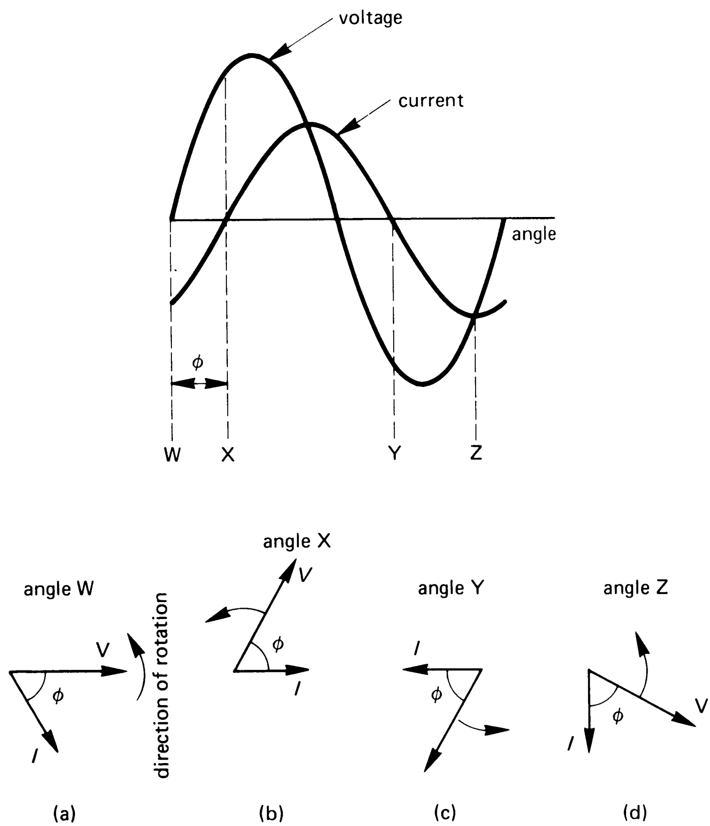
voltage phasors in diagrams (c), (d) and (e) in Figure 10.7. You will see that the 'length' of the phasor in Figure 10.7 is 0.7071 that of the rotating line in Figure 10.5 (remember, r.m.s. voltage = 0.7071 x maximum voltage).

Phasor representation of two waveforms

Consider the case of the two waveforms in Figure 10.8, one being a voltage wave and the other a current wave, which are out of phase with one another (in this case the current waveform lags behind the voltage waveform by angle ϕ).

If the phasor diagram for the two waves is drawn corresponding to point W on the wave, the corresponding phasor diagram is shown in diagram (a) in Figure 10.8 (**note**: the current phasor lags behind the voltage phasor in the direction of 'rotation' of the phasor).

fig 10.8 *phasor diagrams for a voltage wave and a current wave which are out of phase with one another*



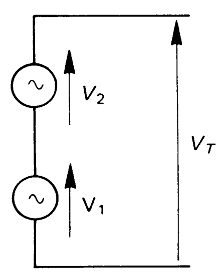
If, on the other hand, the wave is 'frozen' at point X, the corresponding phasor diagram is shown in diagram (b). Once again, the current phasor 'lags' behind the voltage phasor.

The phasor diagrams corresponding to points Y and Z on the waveform diagram are drawn in diagrams (c) and (d), respectively; you will note in each case that the current phasor always lags behind the voltage phasor.

10.8 ADDITION OF PHASORS

When two alternating voltages are connected in series in an a.c. circuit as shown in Figure 10.9, the two voltage waveforms may not be in phase with one another. That is, **you need to take account of the fact that there is a phase angle difference between the voltages.** It is rather like the case of a tug-of-war team in which one of the team pulls in the wrong direction; the total pull is not simply the sum of the pulls of the individual team members; account must be taken of the 'direction' of the pulls. In the a.c. circuit you must determine the **phasor sum** of the voltages.

fig 10.9 *phasor addition of voltages*



Suppose that voltage V_1 has an r.m.s. voltage of 15 V and V_2 has an r.m.s. voltage of 10 V, but V_2 leads V_1 by 60° (see Figure 10.10). The total voltage, V_T, in the circuit is given by

total voltage, V_T = **phasor sum** of V_1 and V_2

The solution can be obtained in one of two ways, namely:

1. the phasor diagram can be drawn to scale and the magnitude and phase angle of the resultant voltage, V_T, can be determined from the scale diagram;
2. you can 'add' the voltages together by resolving the components of the two voltages in the 'vertical' and 'horizontal' directions, and from this determine the 'vertical' and 'horizontal' components of the voltage V_T;

the magnitude and phase angle of V_T can be determined from the resolved components of V_T.

Method 1 needs only simple drawing instruments and, provided that the scale selected for the drawing is large enough, the method gives an answer which is accurate enough for most purposes. Figure 10.10 shows how this is done for the voltages given above. You will find it an interesting exercise to draw the phasor diagram to scale and to compare your results with the answer calculated below.

The solution using method 2 is described below for the voltages given above (see also the phasor diagram in Figure 10.10). First, the voltages V_1 and V_2 are resolved into their horizontal and vertical components.

Voltage V_1

Since we are starting with this voltage, we can assume that it points in the horizontal or 'reference' direction. That is to say, the voltage has a horizontal component and no vertical component, as follows:

Horizontal component = 15 V
Vertical component = 0 V

Voltage V_2

This voltage leads V_1 by an angle which is less than 90°, so that it has both horizontal and vertical components, which are calcualted as shown:

Horizontal component = 10 cos 60° = 10 × 0.5 = 5 V

Vertical component = 10 sin 60° = 10 × 0.866 = 8.66 V

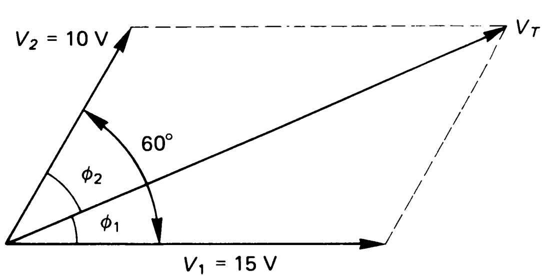
Resultant voltage, V_T

The horizontal and vertical components are calculated as follows:

horizontal component = sum of horizontal components of V_1 and V_2

$$= 15 + 5 = 20 \text{ V}$$

fig 10.10 *the phasor sum of two voltages*



vertical component = sum of the vertical components of V_1 and V_2

= 0 + 8.66 = 8.66 V

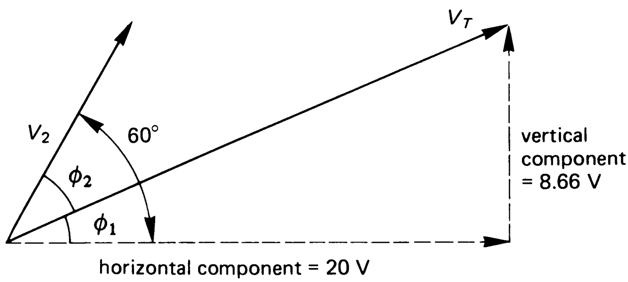
The magnitude of voltage V_T is determined by Pythagorus's theorem as follows (see also Figure 10.11)

$$V_T = \sqrt{[(\text{horizontal component of } V_T)^2 + (\text{vertical component of } V_T)^2]}$$

$$= \sqrt{(20^2 + 8.66^2)} = \sqrt{475}$$

= 24.8 V (Ans.)

fig 10.11 *determination of the voltage V_T*



The value of the phase angle ϕ between V_T and V_1 is calculated from the equation

$$\tan \phi = \frac{8.66}{24.8} = 0.349$$

hence

$$\phi_1 = \tan^{-1} 0.347 = 19.25°$$

where '$\tan^{-1}$' means 'the angle whose tangent is'. You may therefore say that V_T leads V_1 by 19.25°. However

$$\phi_1 + \phi_2 = 60°$$

or

$$\phi_2 = 60° - \phi_1 = 60° - 19.25° = 40.75°$$

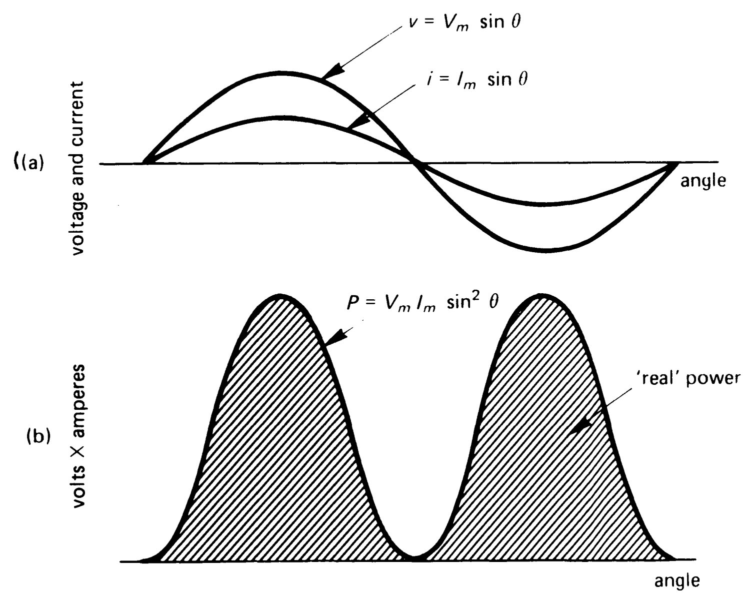
where ϕ_2 is the angle between V_2 and V_T. It follows that V_T lags behind V_2 by 40.75°.

10.9 VOLT-AMPERES, WATTS AND VOLT-AMPERES REACTIVE

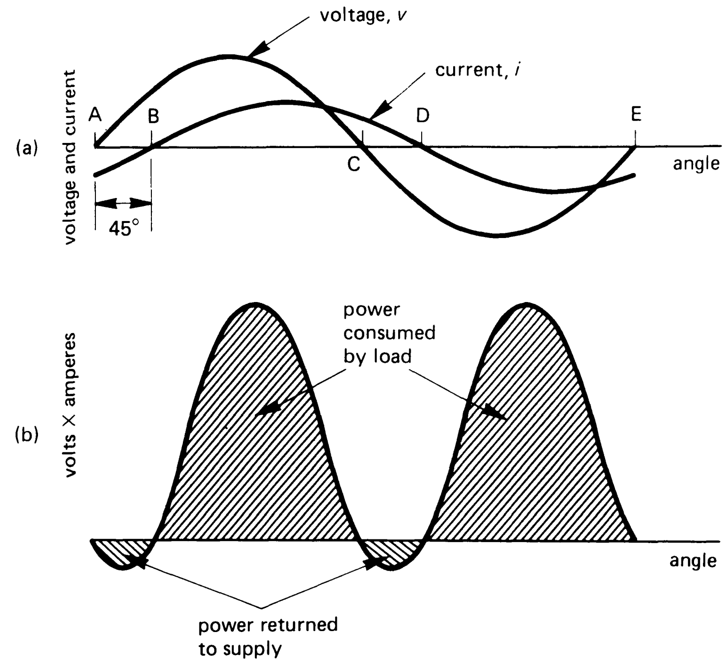
In a d.c. circuit, the product 'volts × amperes' gives the power consumed by the circuit. The situation is slightly more complex in an a.c. circuit, and we will study this situation in this chapter.

Let us look at a circuit in which the voltage and current waveforms are in phase with one another, that is, a circuit containing only a pure resistor, as shown in Figure 10.12(a). When the two waves are multiplied together to give the volt-ampere product, as shown in Figure 10.12(b), the power consumed by the circuit is given by the *average value of the area which is between the zero line and the volt-ampere graph*. You can see that, in this case, power is consumed all the time that current flows in the circuit.

fig 10.12 *(a) waveform diagram for a sinusoidal voltage wave and a current wave which are in phase with one another and (b) the corresponding volt-ampere product wave*



Let us now consider a circuit in which the current lags behind the voltage by an angle of 45° (see Figure 10.13(a)). When the voltage and current waveforms are multiplied together (Figure 10.13(b)) we see that in the time interval between A and B the voltage is positive and the current is negative; the volt-ampere product is therefore negative. Also in the time fig 10.13 *waveforms of voltage, current and volt-ampere product for two waves which are 45° out of phase with one another*



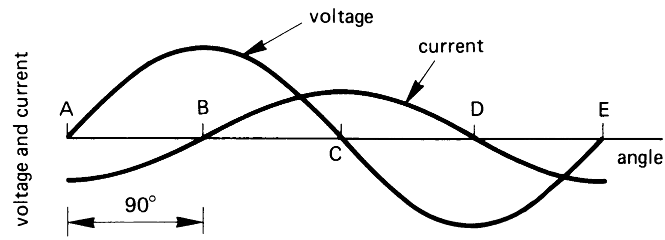
interval C-D, the voltage is negative and the current is negative, so that the volt-ampere product is negative once more.

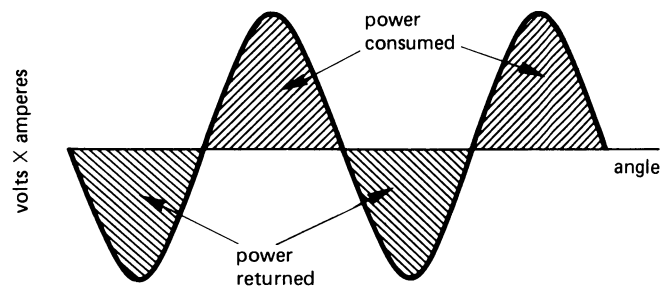
During the time interval B-C, both the voltage and the current are positive, giving a positive volt-amp product (as is also the case in the time interval D-E when both voltage and current are negative).

What is happening in this case is that when the volt-ampere product is positive, power is consumed by the load. When the volt-ampere product is negative, the power 'consumes' a negative power! That is, the load returns power to the supply; this occurs because the load (in this case) is inductive, and some of the energy stored in the magnetic field is returned to the supply when the magnetic field collapses.

You can see from Figure 10.13(b) that more energy is consumed by the load than is returned to the supply and, on average over the complete cycle, power is consumed by the load.

We turn our attention now to the case where the phase angle between the voltage and current is 90° (see Figure 10.14). In this case, the volt-ampere product graph has equal positive and negative areas, which tells us that **the load returns as much power to the supply as it consumes**. That is, fig 10.14 *waveforms for a phase angle of 90°; the average value of the power consumed is zero*





when the phase angle between the voltage and the current is 90°, the average power consumed by the load is zero.

It follows from the above that *the volt-ampere product in an a.c. circuit does not necessarily give the power consumed.* In order **to determine the power consumed by an a.c. circuit, you need to account not only for the volt-ampere product but also for the phase angle between the voltage and current.**

The product of the voltage applied to an a.c. circuit and the current in the circuit is simply known as the **volt-ampere product** (shortened to VA by engineers), and is given the symbol S. If the supply voltage is V_S and the current taken by the circuit, the VA consumed is

$$\text{volt-amperes, } S = V_S I \text{ VA} \qquad (10.14)$$

The volt-ampere product is also known as the **apparent power** consumed by the circuit or device.

The 'real' **power** or useful power in watts used to produce either work or heat is given by the equation

$$\text{power, } P = V_S I \cos \phi \text{ W} \qquad (10.15)$$

where ϕ is the phase angle between V_S and I.

There is also another element to the power 'triangle' in a.c. circuits which is the **reactive power** or **volt-amperes reactive** (VAr) symbol Q, and is given by the equation

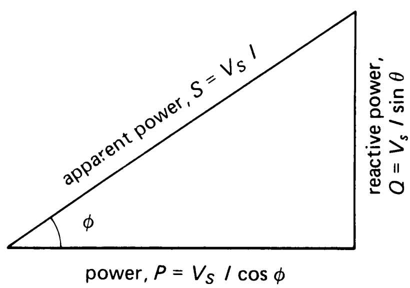
$$\text{reactive volt-amperes, } Q = V_S I \sin\phi \; VAr \qquad (10.16)$$

The three elements of 'power' in an a.c. circuit can be represented by the three sides of the **power triangle** in Figure 10.15 (for further details see Chapter 11).

Since the three 'sides' representing the apparent power (S), the real power (P), and the reactive power (Q) are related in a right-angled triangle, then

$$S^2 = P^2 + Q^2 \qquad (10.17)$$

fig 10.15 *the power triangle of an a.c. circuit*



Example
An a.c. circuit supplied at 11000 V r.m.s. draws a current of 50 A. The phase angle between the applied voltage and the current is 36°. Calculate for the circuit (i) the apparent power, (ii) the 'real' power and (iii) the reactive power consumed.

Solution

$V_S = 11\,000$ V; $I = 50$ A; $\phi = 36°$

(i) apparent power, $S = V_S I = 11\,000 \times 50 = 550\,000$ VA (Ans.)

(ii) power, $P = V_S I \cos\phi = 11\,000 \times 50 \times \cos 36°$

$= 444\,959$ W or 444.959 kW (Ans.)

(iii) reactive power, $Q = V_S I \sin\phi = 11\,000 \times 50 \times \sin 36°$

$= 323\,282$ VAr or 323.282 kVAr

You should note that whilst the circuit consumes 550 kVA (volts × amps), the useful power output from the circuit (which may be, for example, heat) is about 445 kW. The relationship between these two quantities is discussed further in section 10.10.

10.10 POWER FACTOR

The **power factor** of an a.c. circuit is the **ratio of the useful power (in watts [W]) consumed by a circuit to the apparent power (VA) consumed,** and is given by the equation

$$\text{power factor} = \frac{\text{'real' power in watts}}{\text{apparent power in volt-amperes}}$$

$$= \frac{V_S I \cos \phi}{V_S I} = \cos \phi \qquad (10.18)$$

where ϕ is the phase angle between V_S and I.

If for example, the phase angle between the current and the voltage is 0°, the power factor of the circuit is

$$\text{power factor} = \cos 0° = 1.0$$

That is, the 'real' power is equal to the apparent power, that is the power in watts consumed is equal to the number of volt-amperes consumed. Waveforms for this condition are shown in Figure 10.12, and correspond to the example of an a.c. circuit containing a pure resistive load.

If the phase angle between the voltage and the current is 45°, the power factor of the circuit is

$$\text{power factor} = \cos 45° = 0.7071$$

That is, for each 100 VA consumed by the circuit, 70.71 W are consumed. The reason why not all the VA is converted into watts is shown in the waveform in Figure 10.13.

If the phase angle of the circuit is 90°, the circuit power factor is

$$\text{power factor} = \cos 90° = 0$$

In this case the power consumed by the circuit is zero even though the current may be very large. The reason for this can be seen from the waveforms in Figure 10.14, where it was shown that the circuit returns as much power to the supply as it takes from it. This situation arises in any circuit where the phase angle between the voltage and the current is 90°, that is, in either a circuit containing a pure inductor or a pure capacitor.

Equation (10.18) may be re-written in the following form:

$$\text{Power} = \text{apparent power} \times \text{power factor}$$
$$= V_S I \cos\phi \text{ W} \quad (10.19)$$

Example
If the supply voltage to a circuit of power factor 0.8 is 11 000 V, and the power consumed in 200 kW, calculate the current in the circuit.

Solution
$V_S = 11\,000$ V; $P = 200\,000$ W; power factor = 0.8
From eqn (10.19) $P = V_S I \cos\phi$, hence

$$I = \frac{P}{(V_S \cos\phi)} = \frac{200\,000}{(11\,000 \times 0.8)}$$
$$= 22.73 \text{ A (Ans.)}$$

10.11 HARMONICS IN a.c. SYSTEMS

A device such as a resitor has a characteristic in which the current through it is proportional to the voltage applied to it (see Figure 10.16(a)). This type of characteristic is known as a **linear characteristic** or *straight line characteristics*; if the voltage applied to it is a sinewave, the current through it is also a sinewave.

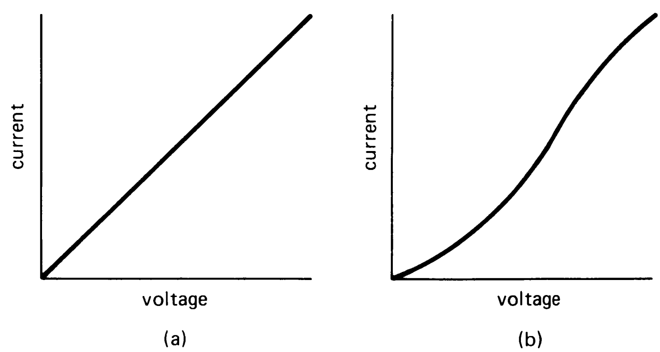
However, in a device which has a voltage–current characteristic which is not a straight line such as that in Figure 10.16(b), the current flowing through it is not proportional to the applied voltage. That is, if a sinusoidal voltage is applied, the current flowing through it is non-sinusoidal. This type of characteristic is said to be **non-linear**, that is, it is not a straight line which passes through zero.

Many common electrical devices have a non-linear characteristic, including fluorescent lights and iron-cored inductors. This results in the current waveshape being non-sinusoidal even though the voltage waveshape is sinusoidal; these current waveshapes are said to be **complex**.

Complex waveshapes which arise in circuits can be constructed or **synthesised** by adding together a series of sinewaves known as **harmonics** as follows:

Fundamental frequency
The fundamental frequency is a sinewave whish is the 'base' frequency on which the complex wave is built. The periodic time of the complex wave is equal to the periodic time of the fundamental frequency.

fig 10.16 *(a) a linear characteristic and (b) a non-linear characteristic*



Second harmonic
This is a sinewave whose frequency is twice that of the fundamental frequency;

Third harmonic
This is a sinewave whose frequency is three times that of the fundamental frequency;

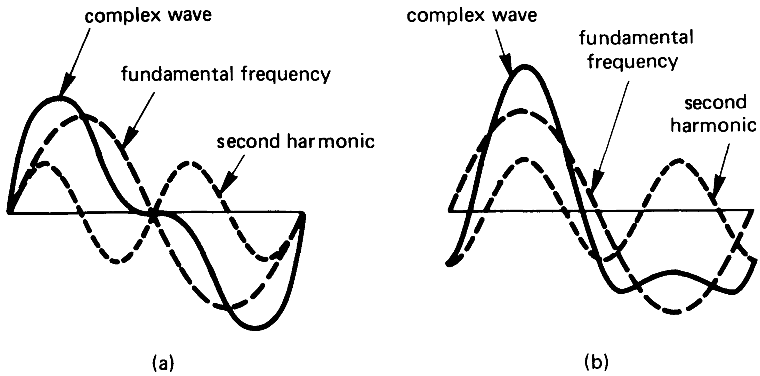
Fourth harmonic
This is a sinewave whose frequency is four times that of the fundamental frequency;

100th harmonic
This is a sinewave whose frequency is 100 times that of the fundamental etc.

The 'shape' of the resulting complex wave depends not only on the number and amplitude of the harmonic frequencies involved, but also on the phase relationship between the fundamental and the harmonics, as illustrated in Figure 10.17 which shows a fundamental frequency together with a second harmonic.

In Figure 10.17(a), the second harmonic has one-half the amplitude of the fundamental, and commences in phase with the fundamental. The complex wave produced by the addition of the two waves is symmetrical about the 180° point of the wave, the wave being 'peaky' in both half cycles.

In Figure 10.17(b), the same second harmonic is added to the fundamental but, in this case, the second harmonic lags behind the fundamental fig 10.17 *complex wave formed from (a) a fundamental and a second harmonic which are in phase with one another, (b) a fundamental and a second harmonic which lags by 90°*



frequency by 90° of the harmonic wave; the resulting complex wave is peaky in the first half cycle but has a flattened second half cycle.

If high-frequency harmonics are present in a wave, then the sound of the resultant complex wave is 'sharp' to the ear, that is it has a treble sound. However, the phase angle between the fundamental frequency and the harmonics has very little effect so far as hearing the sound is concerned since the human ear is insensitive to phase shift. Unfortunately, if the harmonic is produced in a TV video circuit, the phase shift of the harmonic produces quite a marked effect on the TV screen since it gives rise to a change in colour (we are all familiar with coloured 'interference' band patterns on the TV tube produced under certain conditions which result from harmonic effects).

Power electronic devices such as thyristors also give rise to harmonics in the power supply system. In industry, these harmonic currents are frequently carried by overhead power lines which can act as an aerial, giving rise to radiated electromagnetic energy; the radiated energy can produce interference in radios, TVs and other electronic equipment. It is for this reason that electricity supply authorities limit the amount of harmonic current that may be produced by industry.

SELF-TEST QUESTIONS

1. Calculate the periodic time of an alternating wave of frequency 152 Hz. Determine also the frequency of an alternating wave whose periodic time is 0.25 μs.

2. A sinusoidal current wave has a value of 10 A when the angle of rotation is 58°. Calculate (i) the peak-value of the current and (ii) the value of the current at an angle of 10°, 120°, 200°, 350°, 1 rad, 2 rad, and 4 rads.
3. Calculate the mean value and the r.m.s. value of a voltage sinewave of peak value 120 V.
4. What is meant by the 'form factor' and 'peak factor' of an alternating wave? Why do different types of wave have different combinations of form factor and peak factor?
5. Two series-connected sinusoidal voltages of r.m.s. value 200 V and 150 V, respectively, the 200-V wave leading the 100-V wave by 45°. Determine (i) the sum of, and (ii) the difference between, the waveforms.
6. An a.c. circuit consumes 10 kW of power when the applied voltage is 250 V. If the current is 50 A, determine (i) the VA consumed, (ii) the phase angle and the power factor of the circuit and (iii) the VAr consumed.

SUMMARY OF IMPORTANT FACTS

An **alternating quantity** is one which periodically reverses its direction or polarity. The **frequency**, f, of the wave is the number of alternations per second (Hz), and the **periodic time**, T, is the time taken for one complete cycle of the wave ($f = \frac{1}{T}$).

The **mean value** of a wave is the **average value** taken over a given period of the wave (in the case of an alternating quantity, the period is *one-half* of the periodic time of the wave). The **effective value** or **root-mean-square** (r.m.s.) value of the wave is the *effective heating value* of the wave. For a *sinewave*

mean value = 0.637 × maximum value

r.m.s. value = 0.7071 × maximum value

The factors which give some indication of the 'shape' of the waveform are the **form factor** and the **peak factor**. For a *sinewave* these factors are

form factor = 1.11
peak factor = 1.414

The angular difference between two sinewaves is known as the **phase difference**. A **phasor diagram** represents waveforms which are *frozen in time*, and are scaled to represent the r.m.s. values of the waves concerned. Phasors can be added to or subtracted from one another to give resultant values.

The product (volts × amperes) in an a.c. circuit is known as the **volt-ampere product** (VA) or **apparent power**. The **real power** or heating effect is measured in watts. The **reactive volt-amperes** (VAr) or **reactive power** is the volt-ampere product which does not produce any 'real' power consumption in the circuit.

The **power factor** of a circuit is given by the ratio of watts: volt-amperes.

Non-sinusoidal waveforms in a circuit can be thought of as consisting of a series of sinewaves known as **harmonics**, which are added together to form the complex wave in the circuit.

CHAPTER 11

INTRODUCTION TO SINGLE-PHASE a.c. CIRCUITS

11.1 RESISTANCE IN AN a.c. CIRCUIT

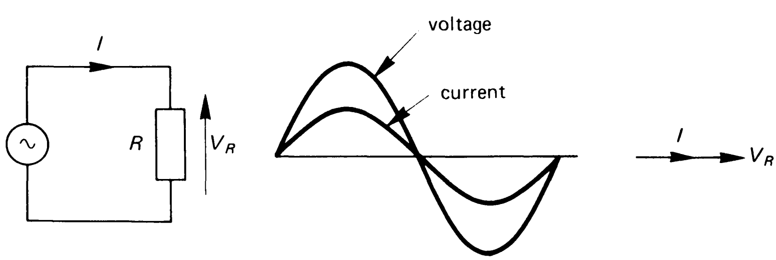
In an a.c. circuit at normal power frequency, a resistance behaves in the same way as it does in a d.c. circuit. That is, *any change in voltage across the resistor produces a proportional change in current through the resistor* (Ohm's law).

Suppose that a pure resistor is connected to a sinusoidal voltage as shown in Figure 11.1(a). The current through the resistor varies in proportion to the voltage, that is **the current follows a sinewave which is in phase with the voltage** (see Figure 11.1(b)). The corresponding phasor diagram for the circuit is shown in Figure 11.1(c).

Since *the phase angle between the voltage and the current is zero*, then the *power factor* of the resistive circuit is cos 0° = 1.0. The r.m.s. value of the current, I, in the circuit is calculated from Ohm's law as follows:

$$\text{current, } I = \frac{V_R}{R} \tag{11.1}$$

fig 11.1 *a pure resistor in an a.c. circuit*



circuit diagram (a) waveform diagram (b) phasor diagram (c) where V_R is the r.m.s. value of the voltage across the resistor. The power, P, consumed in the circuit is given by

$$\text{power}, P = V_R I \cos \phi \text{ W}$$

but, since $\cos \phi = 1$ in a resistive circuit, then

$$P = V_R I \text{ W}$$

However, by Ohm's law, $V_R = IR$, then

$$\text{power}, P = V_R I = (IR) \times I = I^2 R \text{ W} \qquad (11.2)$$

Example
Calculate the power consumed in a single phase a.c. circuit which is energised by a 240-V, 50-Hz sinusoidal supply, the load being a resistance of value 100 Ω.

Solution

$$V_S = 240 \text{ V}; R = 100 \ \Omega$$

Since the only element in the circuit is the resistor, then

$$V_R = V_S = 100 \text{ V}$$

From eqn (11.1)

$$\text{current}, I = \frac{V_R}{R} = \frac{240}{100} = 2.4 \text{ A}$$

and from eqn. (11.2)

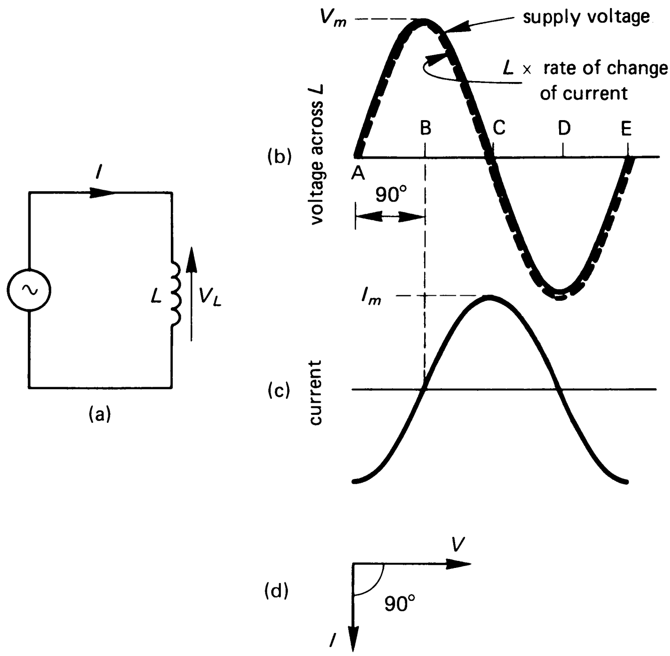
$$\text{power consumed}, P = I^2 R = 2.4^2 \times 100 = 576 \text{ W (Ans.)}$$

11.2 PURE INDUCTANCE IN AN a.c. CIRCUIT

A pure inductance is resistanceless, and its only property is that it produces a magnetic flux when a current flows through it. If it were connected to a d.c. supply, the current would be infinitely large because its resistance is zero! However, when a pure inductor is connected in an a.c. circuit the current in the circuit is limited in value; in the following we consider why this is the case.

When an alternating voltage is connected to a resistanceless inductor as shown in Figure 11.2(a), a current begins to flow in the circuit. This current produces a magnetic flux in the coil which, by Lenz's law, induces a 'back' e.m.f. in the coil which opposes the current producing the flux. This back e.m.f. therefore restricts the current in the coil and limits it to a safe value.

fig 11.2 *a pure inductor in an a.c. circuit*



This phenomenon gives rise to the property known as **inductive reactance**, X_L, of the inductor. Since the inductive reactance restricts the current in the circuit, *it has the dimensions of resistance*. The current, I, in a pure inductive reactance X_L is calculated from the equation

$$\text{current}, I = \frac{V_L}{X_L} \text{ A} \tag{11.3}$$

where V_L is the voltage across the inductor in volts, and X_L is the inductive reactance of the inductor in ohms. For example, if a resistanceless coil of 10 ohms inductive reactance is connected to a 240-V supply, then the current in the coil is

$$\text{current}, I = \frac{V_L}{X_L} = \frac{240}{10} = 24 \text{ A}$$

Turning now to the phase relationship between the voltage across the inductor and the current through it, your attention is directed to Figure 11.2. Since the supply voltage is connected to the resistanceless inductor, the voltage across the coil must be opposed by the back e.m.f. in the coil (this is clearly the case, since the coil has no resistance). You will recall from the earlier work on inductors that

'back' e.m.f. in an inductor = inductance × rate of change of current in the inductor

$$= L \frac{\Delta I}{\Delta t} \qquad (11.4)$$

where $\frac{\Delta I}{\Delta t}$ is the 'rate of change of current'; that is the current changes by ΔI amperes in Δt seconds. To determine what the current waveshape itself looks like, consider the voltage waveforem in Figure 11.2(b). Eqn (11.4) relates the voltage across the inductor to the rate of change of current through it; we therefore need to deduce a method of working back from eqn (11.4) to the current waveshape. Since we may assume that the inductance of the coil has a constant value, eqn (11.4) reduces to

voltage across the inductor α rate of change (the 'slope') of the current waveform

where α means 'is proportional to', that is, doubling the voltage across the inductor doubles the rate of change of current in the inductor. Looking at the points A to E in Figure 11.2(b) we deduce the results in Table 11.1.

Bearing in mind the fact that the right-hand column in Table 11.1 is the *slope of the current waveform*, we deduce that the actual *shape of the waveform of current through L is a sinewave which lags by 90° behind the current waveform* (see Figure 11.2(c)).

Table 11.1 *Relationship between voltage and current waveforms in a pure inductor*

Point in figure 11.2(b)	*Value of voltage*	*Slope of current waveform*
A	zero	zero
B	positive (large)	positive (large)
C	zero	zero
D	negative (large)	negative (large)
E	zero	zero

The same solution can be found using **the calculus** which is included for the reader with a mathematical turn of mind. If the instantaneous supply voltage v_S is given by the expression

$$v_S = V_{Sm} \sin \omega t$$

where V_{Sm} is the maximum value of the supply voltage and ω is the angular frequency of the supply in rad/s, and the instantaneous value of the e.m.f., e, induced in the inductor is equal to v_S. Also $e = \frac{Ldi}{dt}$, where L is the inductance of the inductor in henrys and $\frac{di}{dt}$ is the rate of change of current through the coil in amperes per second and, since $e = v_S$, then

$$\frac{Ldi}{dt} = V_{Sm} \sin \omega t$$

$$di = \frac{V_{Sm}}{L} \sin \omega t \, dt$$

Integrating the equation with respect to time gives the following result for the instantaneous current, i, in the circuit

$$i = -\frac{V_{Sm}}{\omega L} \cos \omega t = \frac{V_{Sm}}{\omega L} \sin (\omega t - 90^\circ)$$

$$= I_m \sin (\omega t - 90^\circ) \qquad (11.5)$$

where $(-\cos \omega t) = \sin (\omega t - 90^\circ)$ and $I_m = \frac{V_{Sm}}{\omega L}$.

Equn (11.5) says that the current has a sinusoidal waveshape of maximum value I_m and **lags behind the voltage across the inductor by 90°**.

11.3 CALCULATION OF INDUCTIVE REACTANCE, X_L

The value of the **inductive reactance, X_L**, can be determined from the following relationship:

induced e.m.f. = inductance × rate of change of current in L

which can be rewritten in the form

average supply voltage = inductance L × average rate of change of current in the inductor

You will recall from Chapter 10 that the average value of a sinewave is $0.637 V_m$, where V_m is the maximum value of the voltage. The average rate of change of current is calculated as follows. Since the maximum current I_m is reached after the first quarter cycle of the sinewave, the average rate of change of the current in this quarter cycle is $\frac{I_m}{T/4}$, where $\frac{T}{4}$ is one quarter of the periodic time of the cycle. Now, the periodic time of each cycle is $T = \frac{1}{f}$, where f is the supply frequency in Hz. That is,

$$\frac{1}{T/4} = \frac{1}{1/4f} = 4f$$

The equation for the average supply voltage therefore becomes

$$0.637V_m = L \times 4fI_m$$

or

$$\frac{V_m}{I_m} = \frac{4fL}{0.637} = 6.284fL = 2\pi fL = \omega L \qquad (11.6)$$

where $\omega = 2\pi fL$ rad/sec. Now $X_L = \frac{V_L}{I}$, where V_L and I are the respective r.m.s. values of the voltage across L and the current in it. Also $V_L = 0.7071V_m$ and $I = 0.7071I_m$, then

$$XL = \frac{V_L}{I} = \frac{0.7071V_m}{0.7071\,I_m} = \frac{V_m}{I_m}$$

and since $\frac{V_m}{I_m} = \omega L$, then

$$\textbf{inductive reactance, } X_L = \omega L = 2\pi fL\ \Omega \qquad (11.7)$$

Example
Calculate the current in a coil of 200 mH inductance if the supply voltage and frequency are 100 V and 500 Hz respectively.

Solution

$L = 200$ mH $= 0.2$ H; $V_S = 100$ V; $f = 500$ Hz

inductive reactance, $X_L = 2\pi fL = 2\pi \times 500 \times 0.2$

$= 628.3\ \Omega$

From eqn (11.3)

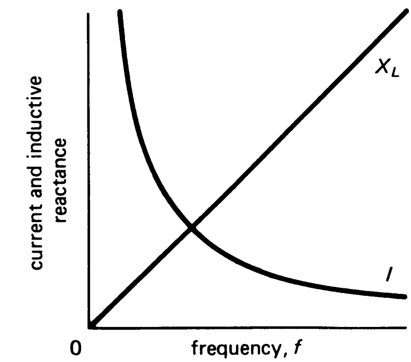
$$\text{current, } I = \frac{V_S}{X_L} = \frac{100}{628.3} = 0.159 \text{ A (Ans.)}$$

11.4 X_L, I AND FREQUENCY

The equation of X_L is $2\pi fL$, which implies that *the reactance of a fixed inductance increases with frequency*, that is

inductive reactance, $X_L \propto$ frequency, f

This relationship is shown as a graph in Figure 11.3. At *zero frequency*, that is, at direct current, the value of X_L is zero. At an infinitely high frequency, the inductive reactance is infinitely large. This means that the inductor 'looks' like a short circuit to a d.c. supply, and at infinite frequency it looks like an open-circuit. This is illustrated by calculating the reactance of a 200 mH inductor as follows fig 11.3 *inductive reactance, current and frequency*



frequency	*inductive reactance*
zero	zero
500 Hz	628.3 Ω
5 kHz	6.283 kΩ
5 MHz	6.283 MΩ
5 GHz	6283 MΩ

The current, I, in a circuit containing only a pure inductor of reactance X_L is

$$I = \frac{V_S}{X_L}$$

Hence, as *the frequency increases, so the inductive reactance increases and the current reduces in value*. This is illustrated in Figure 11.3. At zero frequency $X_L = 0$ and $I = \frac{V_S}{0} = \infty$, and at infinite frequency $X_L = \infty$ and $I = \frac{V_S}{\infty} = 0$.

Example

An inductor has an inductive reactance of 10 Ω at a frequency of 100 Hz. Calculate the reactance of the inductor at a frequency of (i) 50 Hz, (ii) 400 Hz, (iii) 1500 Hz. If the supply voltage is 100 V r.m.s., determine also the current in the inductor for each frequency.

Solution

$X_L = 10\ \Omega$; $f = 100$ Hz

If the inductor has a reactance of X_{L1} at frequency f_1 and X_{L2} at frequency f_2, then

$$X_{L1} = 2\pi f_1 L \text{ and } X_{L2} = 2\pi f_2 L$$

dividing the equation X_{L2} by X_{L1} gives

$$\frac{X_{L2}}{X_{L1}} = \frac{2\pi f_2 L}{2\pi f_1 L} = \frac{f_2}{f_1}$$

or

$$X_{L2} = X_{L1} \times \frac{f_2}{f_1}$$

If we let $X_{L1} = 10\ \Omega$ and $f_1 = 100$ Hz, then the reactances are calculated as follows

(i) $f_2 = 50$ Hz: $X_{L2} = X_{L1} \times \frac{f_2}{f_1} = 100 \times \frac{50}{100}$

$= 50\ \Omega$ (Ans.)

(ii) $f_2 = 400$ Hz: $X_{L2} = 100 \times \frac{400}{100} = 400\ \Omega$ (Ans.)

(iii) $f_2 = 1500$ Hz: $X_{L2} = 100 \times \frac{1500}{100} = 1500\ \Omega$ (Ans.)

The current in the inductor is calculated as follows.

(i) $I = \frac{V_S}{X_L} = \frac{100}{50} = 2$ A (Ans.)

(ii) $I = \frac{V_S}{X_L} = \frac{100}{400} = 0.25$ A (Ans.)

(iii) $I = \frac{V_S}{X_L} = \frac{100}{1500} = 0.0667$ A (Ans.)

11.5 CAPACITANCE IN AN a.c. CIRCUIT

We saw in Chapter 6 that when the voltage applied to a capacitor is increased, the capacitor draws a charging current (and when the voltage is reduced, the capacitor discharges). In an a.c. circuit the applied voltage is continually changing, so that the capacitor is either being charged or discharged on a more-or-less continuous basis.

Capacitive reactance

Since a capacitor in an a.c. circuit allows current to flow, it has the property of **capacitive reactance** which acts to restrict the magnitude of the current in the circuit. As with inductive reactance, *capacitive reactance has the dimensions of resistance*, that is, ohms. If V_C is the r.m.s. voltage across the capacitance and X_C is the capacitive reactance of the capacitor, then the equation for the r.m.s. current in the capacitor is

$$\text{current, } I = \frac{V_C \text{ (volts)}}{X_C \text{ (ohms)}} \tag{11.8}$$

For example, if a voltage of 20 V r.m.s. is applied to a capacitor and the current is 0.01 A, the capacitive reactance is

$$X_C = \frac{V_C}{I} = \frac{20}{0.01} = 2000\ \Omega$$

Suppose that the capacitor current is changing in a sinusoidal manner as shown in Figure 11.4(b). It was shown in Chapter 6 that

capacitor charging current = capacitance, C × rate of change of voltage across the capacitor

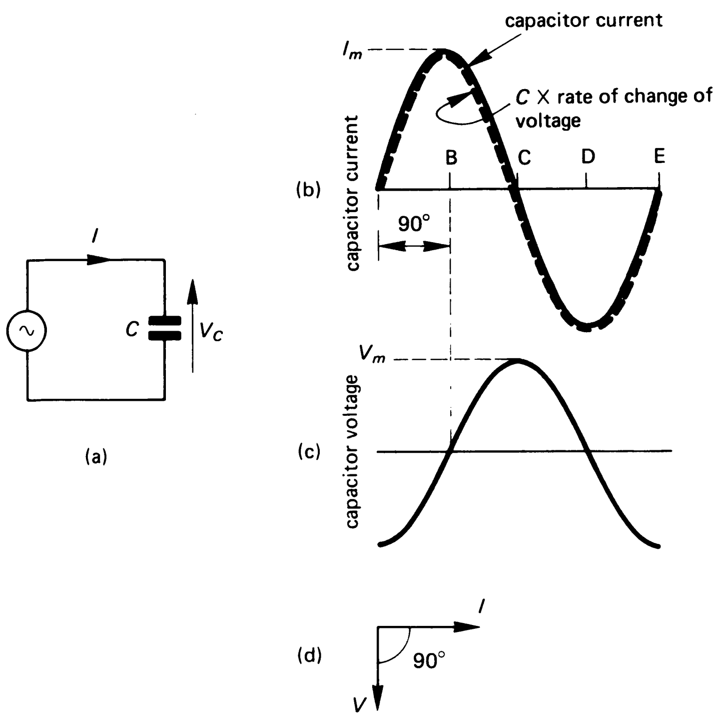
$$= C \times \frac{\Delta V_C}{\Delta t}$$

where C is the capacitance of the capacitor in farads, and $\frac{\Delta V_C}{\Delta t}$ is the rate of change of the capacitor voltage in V/s. Since the capacitor is the only element in the circuit in Figure 11.4(a), the capacitor current is equal to the supply current. We can determine the waveshape of the capacitor voltage from the above relationship as follows. Since the capacitance, C, in the foregoing equation is constant, then

capacitor current α rate of change (slope) of the capacitor voltage

An inspection of the points A–E on the capacitor current wave in Figure 11.4(b) provides the data in Table 11.2. Bearing in mind that the right-hand column of the table is the *slope of the voltage waveform*, it can be deduced that the *shape of the waveform of the voltage across C is a sine-wave which lags 90° behind the current through C* (see Figure 11.4(c)). The corresponding phasor diagram is shown in Figure 11.4(d).

The following alternative solution for the phase angle of the capacitor voltage is obtained using the **calculus**. If the instantaneous current taken by the capacitor is given by $i = I_m \sin \omega t$, where I_m is the maximum value of the current through the capacitor, and ω is the angular frequency of the fig 11.4 *pure capacitance in an a.c. circuit*



supply in rad/s, then the instantaneous voltage v_C across the capacitor is given by

$$i = C \times \frac{dv_C}{dt}$$

Table 11.2 ***Relationship between current and voltage waveforms in a pure capacitor***

Point in figure 11.4(b)	*Value of current*	*Slope of voltage waveform*
A	zero	zero
B	positive (large)	positive (large)
C	zero	zero
D	negative (large)	negative (large)
E	zero	zero

where C is the capacitance of the capacitor in farads, and $\frac{dv_C}{dt}$ is the rate of change of the voltage across the capcitor in V/s. Since the current is sinusoidal, then

$$\frac{C\,dv_C}{dt} = I_m \sin \omega t$$

hence

$$dv_C = \frac{I_m}{C} \sin \omega t \, dt$$

Integrating the equation with respect to time results in the following equation for the instantaneous voltage, v_C, across the capacitor:

$$v_c = \frac{-I_m}{\omega_C} \cos \omega t = \frac{I_m}{\omega_C} \sin (\omega t - 90°)$$

$$= V_m \sin (\omega t - 90°) \qquad (11.9)$$

where $-\cos \omega t = \sin (\omega t - 90°)$ and $V_m = \frac{I_m}{\omega C}$. Eqn (11.9) says that the voltage across the capacitor is sinusoidal and has a maximum voltage V_m, and that *the voltage lags behind the current by 90°.*

11.6 CALCULATION OF CAPACITIVE REACTANCE, X_C

The value of the **capacitive reactance,** X_C, can be deduced from the following equation for the charging current:

capacitor current = capacitance x rate of change of voltage across C

which can be rewritten in the form

average current through capacitor = C x average rate of change of voltage across capacitor

From Chapter 10, you will recall that the average value of a sinusoidal current is $0.637I_m$, where I_m is the maximum value of the current; this value will be used in the above equation.

The average rate of change of voltage across the capacitor is calculated as follows. Since the maximum voltage across the capacitor is V_m, and the time taken to reach this value is $\frac{T}{4}$ where T is the periodic time of the wave, the average rate of change of voltage during the first quarter cycle is $\frac{V_m}{T/4}$ or $V_m \times \frac{4}{T}$. Now the periodic time for each cycle is $T = \frac{1}{f}$, hence $\frac{T}{4} = \frac{1}{4f}$ therefore $\frac{V_m}{T/4} = \frac{V_m}{1/4f} = 4fV_m$. The above ewuation for the average current through the capacitor becomes

$$0.6371I_m = C \times 4fV_m$$

therefore

$$\frac{V_m}{I_m} = \frac{0.637}{4fC} = \frac{1}{6.28fC} = \frac{1}{2\pi fc} = \frac{1}{\omega C}$$

where $\omega = 2\pi f$. Now $X_C = \frac{V_C}{I}$, where V_C and I are the respective r.m.s. values of the voltage across and the current through the capacitor; also the r.m.s. voltage is given by $V_C = 0.7071 V_m$, and the r.m.s. current by $I = 0.7071 I_m$, hence

$$\textbf{capacitive reactance}, X_C = \frac{V}{I} = \frac{0.7071 V_m}{0.7071 I_m} = \frac{V_m}{I_m}$$

$$= \frac{1}{2\pi fC} = \frac{1}{\omega C}\ \Omega \qquad (11.10)$$

Example

A sinusoidal current of 0.1 A r.m.s. value flows through a capacitor connected to a 10-V, 1-kHz supply. Calculate the capacitance of the capacitor.

Solution

$$I = 0.1\text{ A};\ V_C = 10\text{ V};\ f = 1\text{ kHz} = 1000\text{ Hz}$$

From eqn (11.8), $I = \frac{V_C}{X_C}$, or

$$X_C = \frac{V_C}{I} = \frac{10}{0.1} = 100\ \Omega$$

Now $X_C = \frac{1}{2\pi fC}$, hence

$$\text{capacitance, } C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi \times 1000 \times 100)}$$

$$= 1.592 \times 10^{-6} F \text{ or } 1.592\ \mu\text{F (Ans.)}$$

11.7 X_C, I AND FREQUENCY

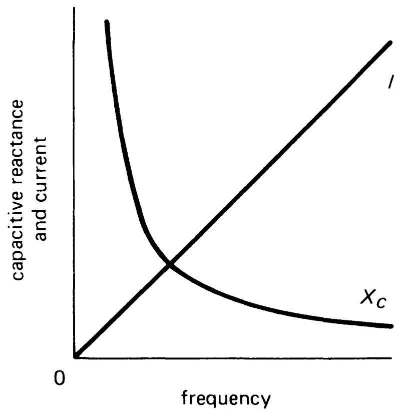
Since the equation for X_C is $\frac{1}{2\pi fC}$, then for a fixed value of C *the capacitive reactance decreases with frequency*, or

$$\text{capacitive reactance, } X_C \propto \frac{1}{\text{frequency}}$$

The relationship is shown in graphical form in Figure 11.5. At zero frequency, that is, direct current, the value of X_C is given by

$$X_C = \frac{1}{0} = \infty\ \Omega$$

fig 11.5 *capacitive reactance, current and frequency*



That is, the capacitor has infinite reactance to the flow of direct current.

As the frequency increases, X_C decreases and, at infinite frequency (which is beyond the scale of the graph in Figure 11.5), the capacitive reactance is

$$X_C = \frac{1}{\infty} = 0\ \Omega$$

That is the capacitor presents no opposition to the flow of very high frequency current. This can be summarised in the following.

At zero frequency (d.c.) the capacitor 'looks' like an open-circuit, and *at infinite frequency it 'looks' like a short-circuit.* For example a capacitor with a reactance of 100 Ω at a frequency of 1000 Hz has a reactance of 200 Ω at 500 Hz, and a reactance of 50 Ω at 2000 Hz.

The current in a circuit containing only a pure capacitance of reactance X_C is

$$I = \frac{V_C}{X_C} = \frac{V_C}{(1/2\pi fC)} = 2\pi C V_C \times f$$

That is, *as the frequency increases so the current through the capacitor increases*. At zero frequency the capacitive reactance is infinity, so that the current is

$$I = \frac{V_C}{\infty} = 0\ \text{A}$$

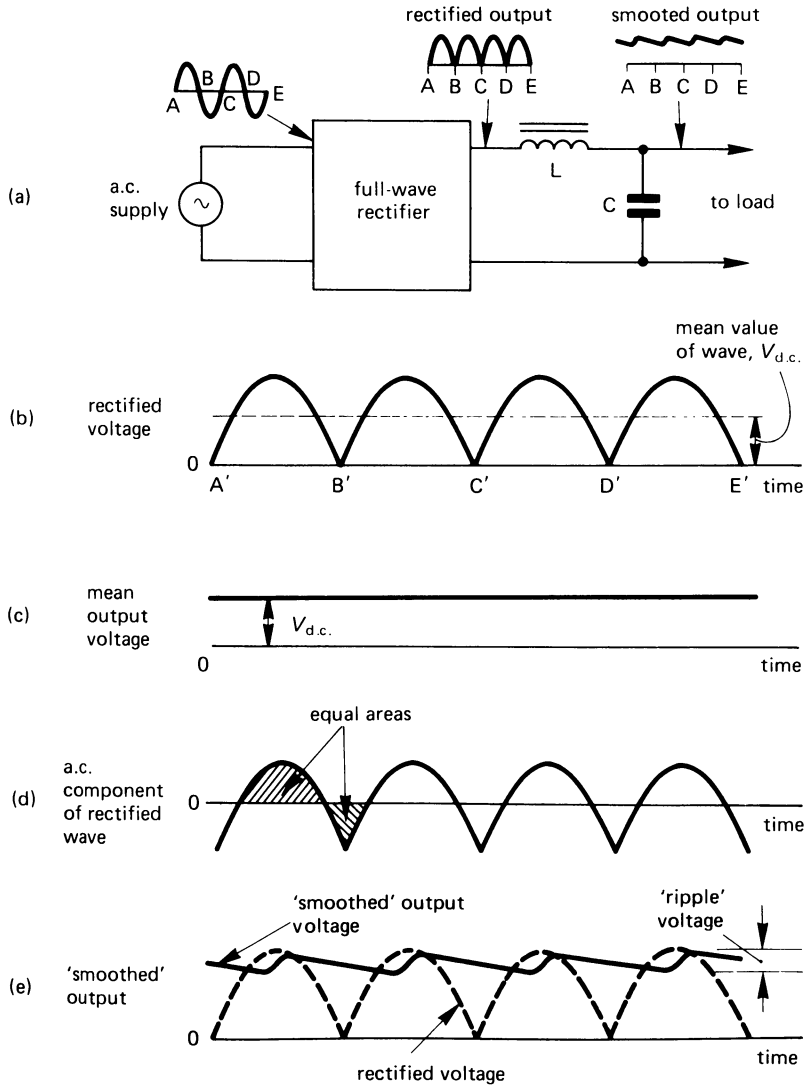


fig 11.6 *a full-wave rectifier and smoothing circuit with waveforms*

and at infinite freqùency the capacitive reactance is zero, and the current is

$$I = \frac{V_C}{0} = \infty \text{ A}$$

11.8 SUMMARY OF CURRENT AND VOLTAGE RELATIONSHIPS

In an **inductor** we may say that EITHER

the current through *L* lags behind the voltage across it by 90°,

OR

the voltage across *L* leads the current through it by 90°,

In a **capacitor** we may say that EITHER

the voltage across *C* lags behind the current through it by 90°,

OR

the current through *C* leads the voltage across it by 90°.

These are neatly summarised by the mnemonic **CIVIL** as follows:

In C, I leads V

C I V I L

V leads I in L

11.9 APPLICATIONS OF INDUCTIVE AND CAPACITIVE REACTANCE

Whenever an inductor or capacitor is used in an a.c. circuit, its effect is felt in terms of its reactance. A popular application of both *L* and *C* are in a 'smoothing' circuit which acts to 'smooth out' the ripples in the output voltage from a rectifier circuit (a rectifier is a circuit which converts an alternating supply into a d.c. supply; the operation of rectifiers is fully described in Chapter 16 and need not concern us here).

The circuit in Figure 11.6 shows a **full-wave** rectifier which converts both positive and negative half-cycles of the alternating supply (hence the name full-wave) into a unidirectional or d.c. supply. During the positive half-cycles A–B and C–D of the a.c. wave, the rectifier produces a positive output voltage (shown in Figure 11.6(b) as A'–B' and C'–D', respectively). During the negative half cycles B–C and D–E of the a.c. wave, the rectifier once more produces a positive output voltage (shown as B'–C' and D'–E'). The resulting 'd.c.' output waveform is shown in Figure 11.6(b).

The latter waveform can be considered to consist of two part which can be added together to form the composite waveform in Figure 11.6(b). These parts are the 'steady' d.c. output voltage $V_{d.c.}$ (see Figure 11.6(c)), and an 'a.c.' component (Figure 11.6(d)). The composite rectified wave (Figure 11.6(b)) is applied to an ***L–C*** **ripple filter**; you will see that the output current from the rectifier must flow through L, and that C is connected between the output terminals.

It is important to note that L has little resistance and therefore does not impede the flow of d.c. through it.

The frequency of the 'ripple component' of the wave (which is the 'a.c.' component of the composite output) in Figure 11.6(d) has the following effect on the smoothing circuit:

1. the reactance of the inductor is fairly high (remember, inductive reactance increases with frequency);
2. the reactance of the capacitor is fairly low (remember, capacitive reactance reduces with increasing frequency).

The two reactance effects combine to reduce the 'ripple' voltage at the output terminals as follows. The high inductive reactance impedes the flow of ripple current through the reactor (this current can be regarded as 'a.c.' current) and the low reactance of the capacitor applies an 'a.c. short-circuit' to the output terminals. The former restricts the flow of ripple current to the d.c. load, and the latter by-passes the ripple current from the load. In this way, the L–C circuit **smooths out** the ripple at the output terminals of the circuit.

When an inductor is used in a smoothing circuit in the manner described above, it is known as a **choke** since it 'chokes' the flow of ripple current. The capacitor is sometimes described as a **reservoir capacitor** since it acts as a reservoir of energy during periods of time when the output voltage of the rectifier approaches zero; that is at A′, B′, C′, D′, etc. in Figure 11.6(b).

Another application of inductive and capacitive reactance is in the **tuning** of radios and televisions. A 'tuning' circuit consists of an inductor connected in parallel with a capacitor, one of the two components having a variable value. The tuning control has the effect of altering either the capacitance of the capacitor or the inductance of the inductor. When the circuit is 'tuned' to the desired frequency, the impedance to a.c. current flow is at its highest, so that for a given current flow the voltage across it is at its highest.

The impedance of the parallel circuit is lower to frequencies other than that to which it is tuned, so that it is less sensitive to these frequencies. In this way the tuning circuit rejects other frequencies than the one selected by the L and C of the circuit.

SELF-TEST QUESTIONS

1. The power consumed in an a.c. circuit containing a pure resistance is 200 W. If the supply voltage is 240 V, determine the current in the circuit and the resistance of the circuit.
2. What is meant by (i) inductive reactance, (ii) capacitive reactance? Circuit A contains an inductance of reactance 150 Ω and circuit B contains a capacitance of reactance 80 Ω. If the supply voltage is 100 V, calculate for each circuit (i) the current in the circuit and (ii) the phase angle of the circuit.
3. An inductance of 0.5 H is connected to a 100-V a.c. supply of frequency (i) 50 Hz, (ii) 60 Hz, (iii) 1 kHz. Calculate in each case the inductive reactance and the current in the circuit.
4. An a.c. supply of 10 V, 1 kHz is connected to a capacitor. If the current in the circuit is 0.628 A, calculate the capacitance of the capacitor.
5. The reactance of (i) an inductor, (ii) a capacitor is 100 Ω at a frequency of 850 Hz. Determine the reactance of each of them at a frequency of 125 Hz.

SUMMARY OF IMPORTANT FACTS

In a **pure resistive** circuit, the **current is in phase with the voltage across the resistor**. In a **pure inductive** circuit, the **current lags behind the voltage across L by 90°**. In a **pure capacitive** circuit the **current in the capacitor leads the voltage across it by 90°**. The mnemonic CIVIL is useful to remember these relationships (**in C, I leads V; V leads I in L**).

The **current** in a resistive circuit is given by $\frac{V_R}{R}$, and the **power consumed** is I^2R.

Inductive reactance, X_L, is given by the equation

$$X_L = 2\pi fL \text{ (} f \text{ in Hz, } L \text{ in henrys)}$$

and the current in L is given by $I = \frac{V_L}{X_L}$. The **inductive reactance increases in proportion to the frequency**, that is $X_L \propto f$.

Capacitive reactance, X_C, is given by the equation

$$X_C = \frac{1}{2\pi fC} \text{ (} f \text{ in Hz, } C \text{ in farads)}$$

and the current in C is given by $I = \frac{V_C}{X_C}$. The **capacitive reactance is inversely proportional to frequency**, that is $X_C \propto \frac{1}{f}$ (X_C reduces as f increases).

CHAPTER 12

SINGLE-PHASE a.c. CALCULATIONS

12.1 A SERIES *R-L-C* CIRCUIT

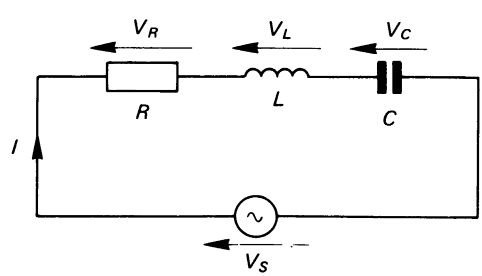
A single-phase circuit containing a resistor, an inductor and a capacitor is shown in Figure 12.1. You will recall that the phase relationship between the voltage and the current in a circuit element depends on the nature of the element, in other words, is it an R or an L or a C? This means that in an a.c. circuit you cannot simply add the *numerical* values of V_R, V_L and V_C together to get the value of the supply voltage V_S; the reason for this is that the *voltage phasors* representing V_R, V_L and V_C 'point' in different directions relative to the current on the phasor diagram. To account for the differing 'directions' of the phasors, you have to calculate V_S as the **phasor sum** of the three component voltages in Figure 12.1. That is

supply voltage, V_S = **phasor sum** of V_R, V_L and V_C

To illustrate how this is applied to the circuit in Figure 12.1, consider the case where the current, I, is 1.5 A, and the three voltages are

$V_R = 150$ V, $V_L = 200$ V, $V_C = 100$ V

fig 12.1 *an R-L-C series circuit*

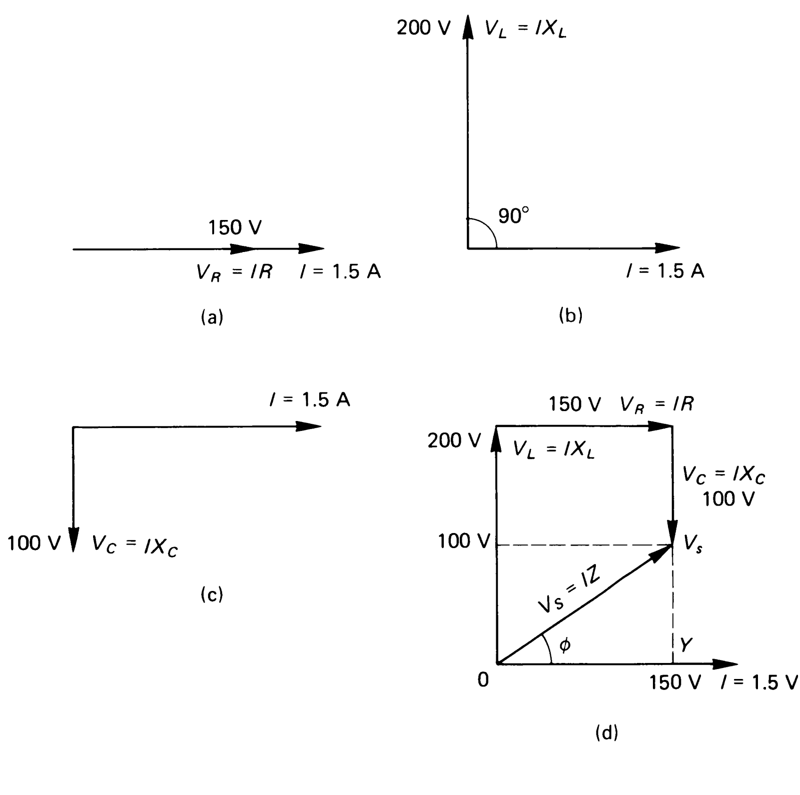


We shall consider in turn the phasor diagram for each element, after which we shall combine them to form· the phasor diagram for the complete circuit.

The phasor diagram for the *resistor* (Figure 12.2(a)) shows that the voltage V_R across the resistor is in phase with the current I. Phasor diagram (b) for the *inductor* shows that the voltage V_L across the inductor leads the current I (*remember* the mnemonic CIVIL). The phasor diagram for the *capacitor* (Figure 12.2(c)) shows the current I to lead the voltage V_C across the capacitor by 90°.

The **phasor diagram for the complete circuit** is obtained by combining the individual phasor diagrams. Since the current, I, is common to all three elements in the series circuit, it is drawn in the horizontal or **reference direction**. The combined phasor diagram is shown in Figure 12.2(d). First, V_L is shown leading I by 90°; next, V_R is added to V_L (remember, V_R is in phase with I, and therefore 'points' in the horizontal direction); next V_C is added to the sume of V_L and V_R to give the **phasor sum** of V_L, V_R

***fig 12.2** phasor diagrams for (a) R, (b) L and (c) C in Figure 12.1. The phasor diagram for the complete circuit is shown in (d)*



and V_C, which is equal to the supply voltage V_S. In engineering terms, the phasors V_L and V_C are in **quadrature** with I, meaning that they are at an angle of 90° to I or to the reference 'direction'. The supply voltage V_S has two components, namely its horizontal component and its quadrature component. These can be calculated from the phasor diagram as follows.

Since V_L and V_C do not have a horizontal component, then

$$\text{horizontal component of } V_S = V_R = 150 \text{ V}$$

Since V_R does not have a quadrature component, the upward (quadrature) component of V_S is

$$\begin{aligned}\text{quadrature component of } V_S &= V_L - V_C \\ &= 200 - 100 = 100 \text{ V}\end{aligned}$$

The triangle OV_SY in Figure 12.2(d) is a right-angled triangle, and from Pythagoras's theorem

$$\begin{aligned}V_S &= \sqrt{[(\text{horizontal component})^2 + (\text{vertical component})^2]} \\ &= \sqrt{[150^2 + 100^2]} = \sqrt{32500} \\ &= 180.3 \text{ V}\end{aligned}$$

The **phase angle**, ϕ, for the complete circuit (see Figure 12.2(d)) is calculated from the trigonometrical relationship

$$\begin{aligned}\tan\phi &= \frac{\text{vertical component of } V_S}{\text{horizontal component of } V_S} \\ &= \frac{100}{150} = 0.6666\end{aligned}$$

hence

$$\phi = \tan^{-1} 0.6666 = 33.69°$$

where $\tan^{-1}$ 0.6666 means 'the angle whose tangent is 0.6666'. The expression $\tan^{-1}$ is also known as '*arc tan*'.

In the circuit for which the phasor diagram in Figure 12.2(d) is drawn, the *current I lags behind the supply voltage V_S by angle ϕ* (alternatively, you may say that the supply voltage leads the current).

12.2 VOLTAGE DROP IN CIRCUIT ELEMENTS

In a *pure resistor*, the voltage drop across the resistor is given by the equation

$$V_R = IR \tag{12.1}$$

where I is the r.m.s. value of the current. This is shown in the phasor diagram in Figure 12.2(a). For the values given in that case ($I = 1.5$ A, $V_R = 150$ V), the resistance is

$$R = \frac{V_R}{I} = \frac{150\,(\mathrm{V})}{1.5\,(\mathrm{A})} = 100\ \Omega$$

In a *pure inductive reactance*, the voltage drop across it is given by the equation

$$V_L = IX_L \tag{12.2}$$

and is illustrated in Figure 12.2(b). For the values given in the problem ($I = 1.5$ A, $V_L = 200$ V), the value of X_L is

$$X_L = \frac{V_L}{I} = \frac{200}{1.5} = 133.3\ \Omega$$

Finally, in a pure *capacitive reactance*, the voltage drop across the capacitor is

$$V_C = IX_C \tag{12.3}$$

This is shown in Figure 12.2(c); for the values given ($I = 1.5$ A, $V_C = 100$ V) the value of X_C is

$$X_C = \frac{V_C}{I} = \frac{100}{1.5} = 66.6\ \Omega$$

12.3 CIRCUIT IMPEDANCE, Z

The *total opposition* to the flow of alternating current of the circuit is known as the **impedance** of the circuit, symbol Z. That is, Z is the effective opposition to alternating current flow of all the components (R, L and C) in the circuit. The *magnitude* of the r.m.s. current flow, I, in an a.c. circuit connected to a sinusoidal voltage V_S is given by

$$I = \frac{V_S}{Z} \tag{12.4}$$

where Z is the impedance of the circuit.

Taking the values given in the example in Figure 12.2, the impedance of the circuit is

$$Z = \frac{V_S}{I} = \frac{180.3}{1.5} = 120.2\ \Omega$$

This value should be compared with the ohmic values of the elements in the circuit, namely

$$R = 100\ \Omega$$

$$X_L = 133.3\ \Omega$$

$$X_C = 66.6\ \Omega$$

You should carefully note that the value of the impedance **is not equal to the sum of the ohmic values of the circuit components**; the impedance Z is calculated from R, X_L and X_C as follows.

Figure 12.3(a) shows the voltage components for the series circuit in Figure 12.1 (see also the phasor diagram in Figure 12.2(d)). The **voltage triangle** for the circuit comprises V_S, V_R and V_Q, where V_Q is the effective *quadrature voltage* in the circuit (that is $V_Q = V_L - V_C$). The value of each side of the voltage triangle is expressed in terms of

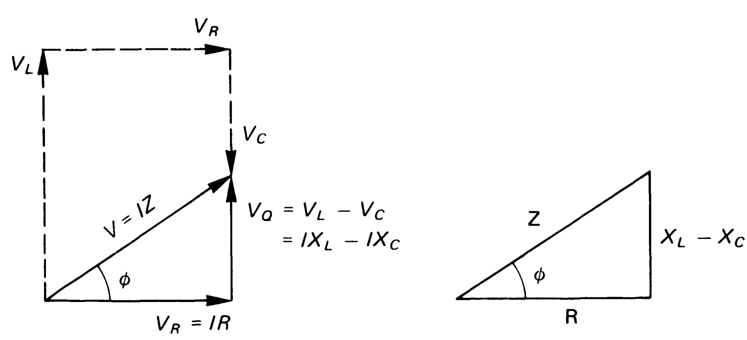
current (I) x ohmic value (R, X or Z)

so that if you divide each side of the voltage triangle by I, you will be left with the **impedance triangle** of the circuit (see Figure 12.3(b)) showing the respective ohmic values of the circuit, namely R, $(X_L - X_C)$ and Z. You will also note that the **phase angle**, ϕ, of the circuit between voltage V_S and I is the same as the angle between Z and R in the impedance triangle. That is to say, you can calculate the phase angle of the circuit from a knowledge of the ohmic values in the circuit!

Since the impedance triangle is a right-angled triangle, the value of the impedance Z can be determined by Pythagoras's theorem as follows:

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]} \qquad (12.5)$$

fig 12.3 *(a) the voltage triangle for a series circuit and (b) the impedance triangle*



(a) (b)

Using the values calculated for the series circuit in Figures 12.1 and 12.2 ($R = 100\ \Omega$, $X_L = 133.3\ \Omega$, $X_C = 66.6\ \Omega$), the impedance Z of the circuit is calculated as follows

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$
$$= \sqrt{[100^2 + (133.3 - 66.6)^2]}$$
$$= \sqrt{[10\,000 + 4\,449.9]} = 120.2\ \Omega$$

You will note that the above value is the same as that calculated from the equation $Z = \frac{V_S}{I}$.

12.4 POWER IN AN a.c. CIRCUIT

It was shown in Chapter 10 that when *the current and voltage are 90° out of phase with one another, the power consumed by the circuit is zero*. You have already seen that the phase angle not only for a *pure inductor* but also for a *pure capacitor* is 90°; that is

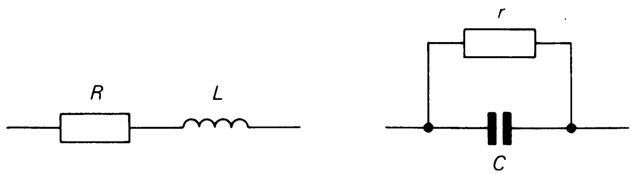
> *the power consumed by a* ***pure inductor*** *and a* ***pure capacitor*** *is zero.*

However, a *practical circuit* consists of a mixture of inductors, capacitors and resistors, and the *power loss* in the circuit occurs **in the resistive parts** of the circuit in the form of I^2R loss.

Moreover, since a *coil* is wound with wire which has resistance, *every coil has some resistance*; consequently, there is some power loss in the *coil* (there is, in fact, no power loss in the purely inductive part of the coil). Also, as mentioned earlier, there is also some power loss in the coil if it has an iron core attributed to eddy-current loss (due to current induced in the iron core) and to hysteresis loss (due to the continued reversal of the magnetic domains as the alternating current reverses the direction of magnetisation of the magnetic field). The two latter types of power loss have the effect of increasing the effective resistance of the coil. You must therefore remember that a practical coil is far from being a perfect inductor.

Although a *practical capacitor* is a more-nearly 'perfect' element, it too has some imperfections. Its dielectric is, in fact, not a perfect insulator, but allows 'leakage' current to flow through it; that is, it has some 'leakage resistance'.

A practical coil and a practical capacitor can be represented in the forms shown in Figure 12.4. The coil is represented by inductor L in series with resistor R which represents the combined effects of the resistance of the coil, the eddy-current power-loss, and the hysteresis power-loss. A fig 12.4 *the equivalent circuit of (a) a practical coil, (b) a practical capacitor*



(a) (b)

practical capacitor can be regarded as a pure capacitor C in parallel with resistor r, the resistance representing the 'leakage resistance'.

Consider now the series circuit in Figure 12.5 consisting of a resistor in series with a pure inductor (alternatively, you may think of the complete circuit as a single practical coil), connected to an a.c. supply. The circuit has an impedance Z of

$$Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 15^2)} = 18.03\ \Omega$$

The current I in the circuit is given by

$$I = \frac{V_S}{Z} = \frac{240}{18.03} = 13.31\ \text{A}$$

Now, *the pure inductive element does not itself consume power*, so that the **power** consumed by the circuit is

$$\text{power}, P = I^2R = 13.31^2 \times 10 = 1771.6\ \text{W}$$

The **apparent power** consumed by the circuit is

$$\text{apparent power}, S = V_S I = 240 \times 13.31 = 3194.4\ \text{VA}$$

and the **quadrature power** or VAr consumed is

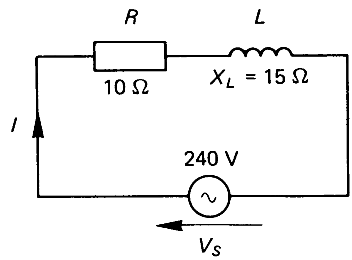
$$\text{quadrature power}, Q = \sqrt{(S^2 - P^2)}$$
$$= \sqrt{(3194.4^2 - 1771.6^2)} = 2658.1\ \text{VAr}$$

The above data allow us to calculate the **power factor** of the circuit as follows:

$$\text{power factor} = \frac{\text{'real' power}}{\text{apparent power}} = \frac{P}{S}$$
$$= \frac{1771.6}{3194.4} = 0.555$$

That is, 55.5 per cent of the volt-amperes consumed by the circuit are converted into watts.

fig 12.5 *a series a.c. circuit calculation*



Now it was shown in Chapter 10 that the power factor of the circuit is equal to cos ϕ for the circuit, where ϕ is the phase angle between the supply voltage and the current drawn by the circuit. If we draw the phasor diagram for the circuit as shown in Figure 12.6, we see that

$$\cos \phi = \frac{V_R}{V_S} = \frac{IR}{IZ} = \frac{R}{Z}$$

that is for the circuit in Figure 12.5, the power factor is

$$\text{power factor} = \cos \phi = \frac{R}{Z} = \frac{10}{18.03} = 0.555$$

This shows that the power factor can be calculated either from a knowledge of the watts and volt-amperes, or from a knowledge of the resistance and reactance of the circuit elements.

12.5 PARALLEL a.c. CIRCUITS

A parallel circuit is one which has the same voltage in common with all its circuit elements. A typical parallel *R-L-C* circuit is shown in Figure 12.7. The phasor diagram for the complete circuit is deduced from the phasors

fig 12.6 *phasor diagram for an R-L series circuit*

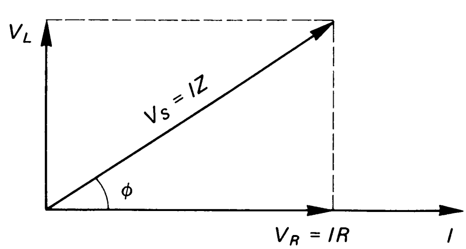
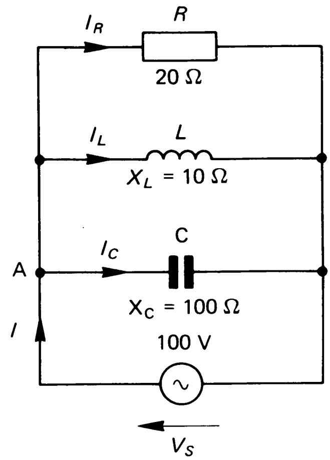


fig 12.7 *a parallel a.c. circuit*



of the individual parallel branches. The calculation is kept as simple as possible by using 'pure' C and L elements in the bottom and centre branches of the circuit respectively.

The phasor diagram for the resistive, the inductive and the capacitive branches of the circuit are shown in diagrams (a), (b) and (c), respectively of Figure 12.8. In the case of a parallel circuit V_S is common to all branches, so that *the supply voltage is used as the reference phasor*, and is shown in the horizontal (reference) direction.

Applying Kirchhoff's first law to junction A (or to junction B) of Figure 12.7, and *bearing in mind that we are dealing with an a.c. circuit*, the supply current is given by the equation

$$\text{supply current, } I = \text{phasor sum } (I_R + I_L + I_C) \qquad (12.6)$$

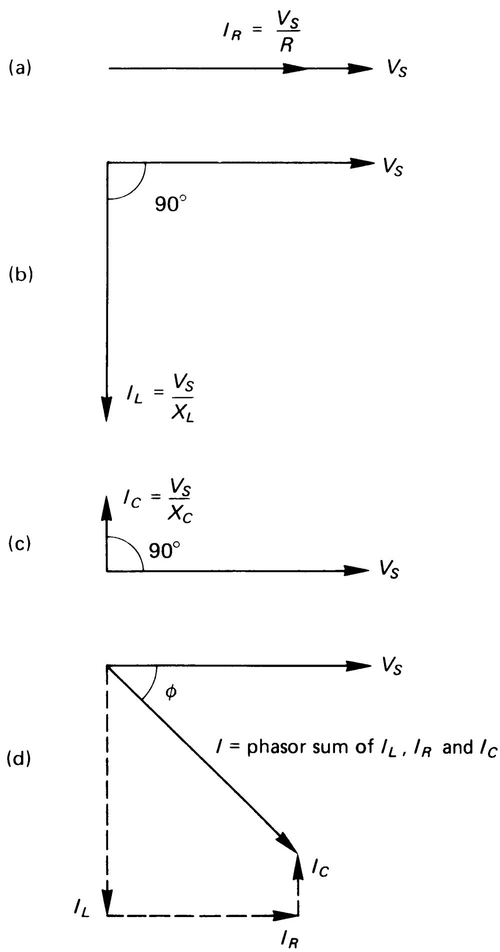
The complete phasor diagrams is obtained by adding the current phasors according to eqn (12.6) as shown in diagram (d) of Figure 12.8. The magnitude of the individual currents are calculated below:

$$I_R = \frac{V_S}{R} = \frac{100}{20} = 5 \text{ A (in phase with } V_S)$$

$$I_L = \frac{V_S}{L} = \frac{100}{10} = 10 \text{ A (lagging } V_S \text{ by } 90°)$$

$$I_C = \frac{V_S}{X_C} = \frac{100}{100} = 1 \text{ A (leading } V_S \text{ by } 90°)$$

fig 12.8 *phasor diagrams for (a) the resistive branch, (b) the inductive . branch, (c) the capacitive branch and (d) the complete circuit*



The *horizontal component*, I_h, of the total current is given by

horizontal component, $I_h = I_R$ p.245

The *vertically downwards component*, I_v, of the total current is

$I_v = I_L - I_C = 10 - 1 = 9$ A

The magnitude of the total current, I, drawn from the supply is determined using Pythagoras's theorem as follows:

$$\text{total current, } I = \surd(I_v^2 + I_v^2) = \surd(5^2 + 9^2)$$
$$= 10.3 \text{ A}$$

You will see from Figure 12.8(d) that *I lags behind V_S by angle ϕ*, where

$$\tan \phi = \frac{\text{vertically downwards component of } I}{\text{horizontal component of } I}$$

$$= \frac{(I_L - I_C)}{I_R} = \frac{9}{5} = 1.8$$

hence

$$\text{phase angle, } \phi = \tan^{-1} 1.8 = 60.95^\circ$$

The *power factor of the circuit* is given by

$$\text{power factor} = \cos \phi = \cos 60.95^\circ = 0.4856$$

and the*power consumed by the complete circuit* is

$$\text{power, } P = V_S I \cos \phi = 100 \times 10.3 \times 0.4856$$
$$= 500 \text{ W}$$

Since the inductor and the capacitor are 'pure' elements they do not consume any power, and *all the power* is consumed in resistor R. To verify this fact we will calculate the power consumed in R as follows:

$$\text{power consumed in } R = I_R^2 R = 5^2 \times 20 = 500 \text{ W}$$

12.6 RESONANCE IN a.c. CIRCUITS

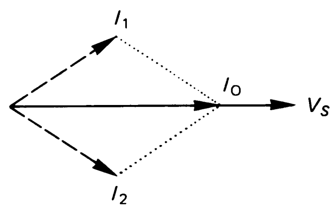
The word resonance means 'to reinforce a vibration or oscillation'. When resonance occurs in an electrical circuit, the circuit acts either to reinforce the current taken from the supply (**series resonance** or **acceptor resonance**) or to reinforce the current circulating between the branches of a parallel circuit (**parallel resonance** or **rejector resonance**).

The resonant condition occurs when the reactive elements (L and C) have values which cause the circuit to 'vibrate' electrically in sympathy with the electrical supply; this produces the reinforcement of either the voltage or current within the circuit.

The reason for resonance in an electrical circuit is, in fact, the interchange of the stored energy between the electromagnetic field of the inductor and the electrostatic field of the capacitor.

The electrical circuit'resonates' at some frequency f_O Hz (or ω_O rad/s), and the current I_O is **in phase with the supply voltage** at the resonant frequency. That is, a circuit containing L and C appears at the resonant frequency as though it were a pure resistive circuit (see I_O in Figure 12.9). At any other frequency the current either leads the supply voltage (see I_1 in Figure 12.9). or it lags behind it (see I_2).

fig 12.9 *resonance in an a.c. circuit*



12.7 SERIES RESONANCE

From eqn (12.5) the impedance of a series circuit *at any frequency*, is given by

$$\text{impedance}, Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

If both sides of the equation are 'squared' it becomes

$$Z^2 = R^2 + (X_L - X_C)^2$$

At the resonant frequency (f_O or ω_O), the 'impedance' is purely resistive (it must be resistive since the current and voltage are in phase with one another!); that is, the 'reactive' part $(X_L - X_C)^2$ must be zero. This part of the equation can only be zero if $X_L = X_C$, that is, if

$$\omega_O L = \frac{1}{\omega_O C}$$

Hence the resonant frequency of the circuit is given by

$$\omega_O = \frac{1}{(LC)} \text{ rad/s} \qquad (12.7)$$

where L is in henrys and C in farads. However, since $\omega_O = 2\pi f_O C$, the resonant frequency f_O is

$$f_O = \frac{1}{[2\pi\sqrt{(LC)}]} \qquad (12.8)$$

For the circuit in Figure 12.10, $L = 0.1$ H and $C = 10\ \mu F$, the resonant frequency is

$$\omega_O = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{(0.1 \times [10 \times 10^{-6}])}} = 1000 \text{ rad/s}$$

and

$$f_O = \frac{\omega_O}{2\pi} = \frac{1000}{2\pi} = 159.2 \text{ Hz}$$

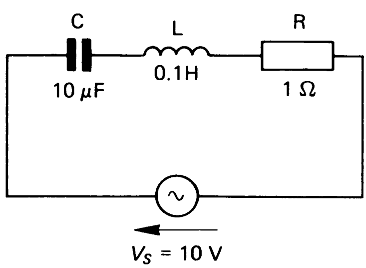
You will note that *the resistance of the circuit does not appear in any of the foregoing calculations since it does not affect the resonant frequency of the series circuit.*

As a matter of interest we will calculate the value of X_L and of X_C at resonance as follows:

$$X_L = \omega_O L = 1000 \times 0.1 = 100\ \Omega$$

$$X_C = \frac{1}{\omega_O C} = \frac{1}{(1000 \times [10 \times 10^{-6}])} = 100\ \Omega$$

fig 12.10 *series resonance*



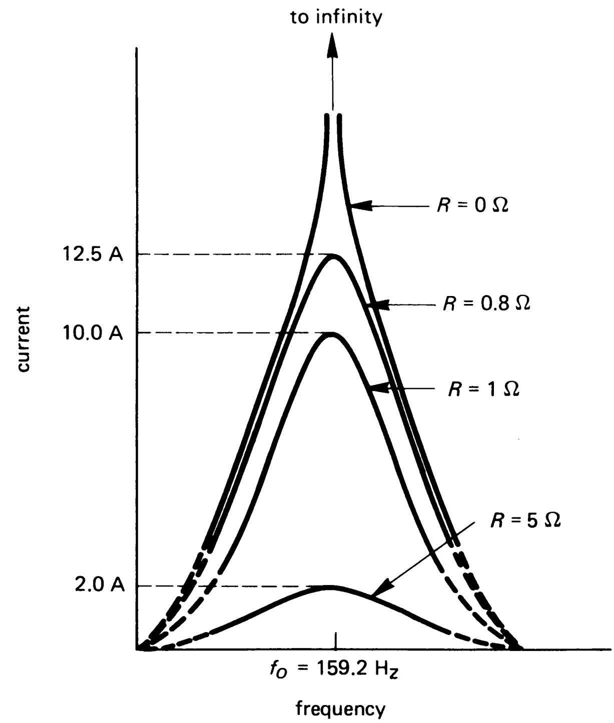
Resonance curves for a series circuit

At very low frequency, the capacitive reactance of the capacitor is very high (remember, $X_C = \frac{1}{2\pi fC}$), with the result that impedance of the series circuit is very high at low frequency. Consequently, the current flow in the circuit is small at low frequency (see Figure 12.11).

As the frequency of the supply increases, the capacitive reactance reduces in value, and with it the circuit impedance reduces in value. This results in an increase in current as the frequency increases in value (it being assumed for the moment that the supply voltage remains constant).

The current in a series circuit reaches its maximum value at the resonant frequency, when $Z = R$ and $I = \frac{V_S}{R}$.

fig 12.11 *frequency response of an R-L-C series circuit*



Beyond the resonant frequency there is a reduction in the circuit current (see Figure 12.11) for the following reason. As the frequency increases, the inductive reactance increases (remember, $X_L = 2\pi fL$) at a rate which is greater than the reduction in capacitive reactance. This results in a net increase in circuit impedance beyond the resonant frequency. Consequently, the current diminishes when the frequency passes through resonance.

At infinite frequency the inductive reactance X_L is infinitely large, so that the impedance of the circuit becomes infinity. That is, at infinite frequency the current diminishes to zero.

The curves in Figure 12.11 (known as the **frequency response curves**) show the variation in current in a series circuit for various values of frequency (the supply voltage being constant). You will see that the maximum r.m.s. value of current in the circuit is given by $I = \frac{V_S}{R}$; for a supply voltage of 10 V r.m.s., the resonant current in the circuit for a circuit resistance of 5 Ω is $\frac{10}{5} = 2$ A, for a resistance of 1 Ω is $\frac{10}{1} = 10$ A etc, and for a resistance of zero ohms is $\frac{10}{0} = \infty$ A.

12.8 *Q*-FACTOR OR QUALITY FACTOR OF A SERIES RESONANT CIRCUIT

We will illustrate the **Q-factor** or **voltage magnification factor** of a series circuit at resonance by means of the circuit in Figure 12.10. The impedance of the circuit *at its resonant frequency* of 1000 rad/s is

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{R^2 + (100 - 100)^2]}$$

$$= R = 1\ \Omega$$

The current drawn from the 10-V supply at resonance is

$$I = \frac{V_S}{Z} = \frac{10}{1} = 10\ \text{A}$$

Since this current flows through R, L and C, the voltage across each of these elements is

$$V_R = IR = 10 \times 1 = 10\ \text{V}$$

$$V_L = IX_L = 100 \times 10 = 1000\ \text{V}$$

$$V_C = IX_C = 100 \times 10 = 1000\ \text{V}$$

Although the supply voltage is only 10 V, the voltage across L and across C is 1000 V! That is, the circuit has 'magnified' the supply voltage 100 times! This is a measure of the 'quality' of the circuit at resonance; the value of this magnification is given the name **Q-factor**.

Let us determine how its value can be calculated. As you can see from the above calculations, the voltage across R is equal to the supply voltage, that is

$$V_S = V_R = IR$$

The voltage V_C across the capacitor is equal to IX_C, hence the Q-factor is

$$Q\text{-factor} = \frac{\text{voltage across } C \text{ at resonance}}{\text{voltage across } R \text{ at resonance}}$$

$$= \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1/\omega_O C}{R}$$

$$= \frac{1}{\omega_O CR} \qquad (12.9)$$

Similarly

$$Q\text{-factor} = \frac{\text{voltage across } L \text{ at resonance}}{\text{voltage across } R \text{ at resonance}}$$

$$= \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega_0 L}{R} \qquad (12.10)$$

Referring to the circuit in Figure 12.10 and using eqn (12.9)

$$Q\text{-factor} = \frac{1}{\omega_0 CR}$$

$$= \frac{1}{[1000 \times (10 \times 10^{-6}) \times 1]} = 100$$

It is left as an exercise for the reader to verify that eqn (12.10) gives the same result.

12.9 FEATURES OF SERIES RESONANCE

You can observe from the foregoing that **the total impedance of a series circuit is at its lowest value at the resonant frequency**, hence *the largest value of current flows in the circuit at resonance*. In the case of an *electronic circuit*, the resistance of the circuit is usually fairly high so that even at series resonance the current does not have a very high value.

On the other hand, in an electrical *power circuit* the resistance values are low (often only a fraction of an ohm), so that the current at resonance can have a dangerously high value. The high value of current produces not only damaging heating effects but also very high values of voltage across the inductors and capacitors in the circuit (remember, the voltage across these is Q times the supply voltage!) Both these effects can damage the circuit elements, and it is for this reason that **series resonance is avoided in power circuits.**

12.10 PARALLEL RESONANCE

Resonance occurs in the parallel circuit in Figure 12.12 when the total current I is in phase with the supply voltage V_S; it occurs at the resonant frequency ω_O rad/s or f_O Hz. The total current I is determined from the expression

total current, I = **phasor sum** of I_L and I_C

Now, the current in the branch containing the pure inductor is

$$I_L = \frac{V_S}{X_L}$$

and the current in the capacitive branch is

$$I_C = \frac{V_S}{X_C}$$

At the resonant frequency, ω_O, the reactive component of the total current is zero, that is $I_L = I_C$ or

$$\frac{V_S}{X_L} = \frac{V_S}{X_C}$$

that is

$$X_L = X_C$$

or

$$\omega_O L = \frac{1}{\omega_O C}$$

Cross-multiplying and taking the square root of both sides of the equation gives the resonant frequency ω_O as

$$\omega_O = \frac{1}{\sqrt{(LC)}} \text{ rad/s} \tag{12.11}$$

or

$$f_O = \frac{1}{2\pi\sqrt{(LC)}} \tag{12.12}$$

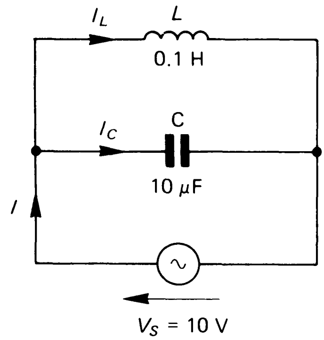
For the circuit in Figure 12.10

$$\omega_O = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{(0.1 \times [10 \times 10^{-6}])}}$$
$$= 1000 \text{ rad/s}$$

or

$$f_O = \frac{\omega_O}{2\pi} = \frac{1000}{2\pi} = 159.2 \text{ Hz}$$

Comparing the equations for the resonant frequency for the series circuit (eqns (12.7) and (12.8)) and those for the simplified parallel circuit above, you will see that they are the same!. However, whilst it is true that the resistance of the series circuit does not affect the calculation for the resonant frequency of the series circuit, it does in fact affect the parallel circuit. This fact did not arise in the case of Figure 12.12 since it was assumed that the coil was a 'perfect' inductor. It can be shown that if the fig 12.12 *parallel resonance*



coil contains some resistance R, the equation for the resonant frequency of the parallel circuit is

$$\omega_O = \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)} \text{ rad/s}$$

and

$$f_O = \frac{\omega_O}{2\pi} \text{ Hz}$$

If the resistance of the coil in Figure 12.12 is 1 Ω, the resonant frequency is calculated to be 999.95 rad/s or 159.15 Hz (compared with 1000 rad/s and 159.2 Hz, respectively), but if the resistance of the coil is 500 Ω, the resonant frequency is 866 rad/s or 137.8 Hz! Clearly, only if R is much less in value than X_L at the resonant frequency can its value be ignored.

12.11 CURRENT DRAWN BY A PARALLEL RESONANT CIRCUIT

The total current, I, drawn by a parallel resonant circuit at resonance is best determined by means of an example. Take the case of the circuit in Figure 12.12. The impedance of the inductive branch at the resonant frequency of 1000 rad/s is

$$X_L = \omega_O L = 1000 \times 0.1 = 100 \ \Omega$$

The impedance of the capacitive branch at the resonant frequency is

$$X_C = \frac{1}{\omega_O C} = \frac{1}{(1000 \times [10 \times 10^{-6}])} = 100 \ \Omega$$

The current, I_L, in the inductive branch is

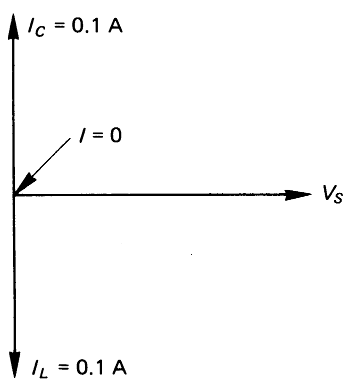
$$I_L = \frac{V_S}{X_L} = \frac{10}{100} = 0.1 \text{ A}$$

and the current I_C in the capacitive branch is

$$I_C = \frac{V_S}{X_C} = \frac{10}{100} = 0.1 \text{ A}$$

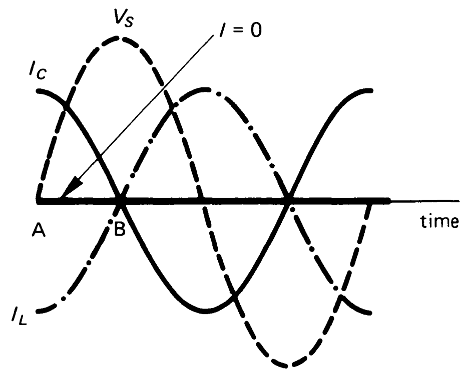
Let us now look at the phasor diagram for the circuit at the resonant frequency in Figure 12.13. The current in the inductive branch lags 90° behind V_S, and the current I_C in the capacitive branch leads V_S by 90°. Now, the total current I drawn by the circuit is the *phasor sum* of I_L and I_C; *this is clearly seen to be zero in Figure 12.13.* We now have to solve the apparent dilemma of the circuit, namely that whilst a current flows in both L and C, no current flows in the main circuit!

fig 12.13 *current at resonance in a parallel circuit*



A reason for this is offered by the waveform diagram for the circuit in Figure 12.14. Under resonant conditions, the value of the current in each branch has the same value (see the calculation above) but, whilst I_C leads V_S by 90°, I_L lags behind V_S by 90°; the corresponding waveform diagrams are as illustrated in Figure 12.14. If we add I_L and I_C together at every instant of time, we see that **the total current I is zero.**

The value of the total current can also be arrived at by considering the energy flow in the circuit. During the period A–B in Figure 12.14, the capacitor is absorbing energy from the supply (I_C is positive!) and, at the same time, the inductor is returning energy to the supply (I_L is negative!). In effect, during the period A–B, the energy which is given up by L is fig 12.14 *waveforms in a parallel resonant circuit*



absorbed by C! Hence, during that period of time there is no need for the power supply to provide energy (current) to the circuit.

That is, **a parallel resonant circuit containing a pure inductor and a pure capacitor does not draw any current from the supply.**

Current drawn when the circuit contains resistance

In a practical circuit, the coil in the circuit contains some resistance (as does the wiring and the capacitor for that matter), which consumes electrical energy. This energy must be supplied by the power source in the form of a current; that is, a 'practical' parallel resonant circuit takes some current from the supply at the resonant frequency.

The resistance, R_D (known as the **dynamic resistance**), of a practical resonant circuit at resonance is given by the equation

$$R_D = \frac{L}{CR}\ \Omega$$

For example, if $L = 0.1$ H, $C = 10\ \mu$F and $R = 1\ \Omega$, then

$$R_D = \frac{L}{CR} = \frac{0.1}{([10 \times 10^{-6}] \times 1)} = 10\,000\ \Omega$$

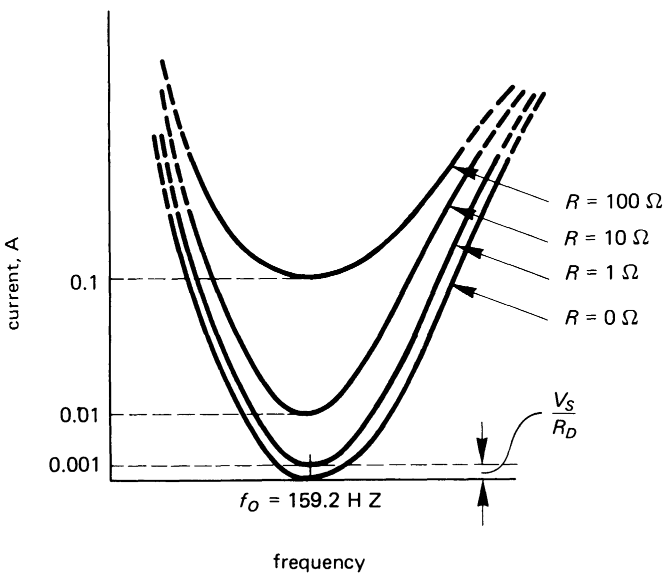
You may like to check that if $R = 10\ \Omega$ then $R_D = 1000\ \Omega$, and if $R = 100\ \Omega$ then $R_D = 100\ \Omega$. With a supply voltage of 10 V, the total current, I, drawn from the supply at the resonant frequency (calculated from the equation $I = \frac{V_S}{R_D}$) for various values of R is as listed in Table 12.1.

The current-frequency curves are shown in Figure 12.15. The interesting point to note about the curves is that the current drawn by the circuit at resonance *rises* as the *resistance of the inductive branch of the circuit*

Table 12.1 *Dynamic resistance and current for a parallel resonant circuit having resistance R ohms in its inductive branch*

Resistance, R ohms	*Dynamic resistance, R_D ohms*	*Current at resonance, I amperes*
0	∞	0
1	10 000	0.001
10	1 000	0.01
100	100	0.1

fig 12.15 *current-frequency graph for a parallel circuit having various resistance values*



rises. This is explained by the fact that more energy is dissipated in the resistive branch when it has a higher resistance.

12.12 *Q*-FACTOR OF A PARALLEL RESONANT CIRCUIT

The quality factor or *Q-factor* of a parallel resonant circuit is equal to the *current magnification* that the circuit provides at resonance, as follows:

$$Q\text{= factor} = \frac{\text{current in } C \text{ (or in } L\text{) at resonance}}{\text{current drawn from the supply at resonance}}$$

$$= \frac{I_C}{I} \text{ or } \frac{I_1}{I}$$

where I_1 is the current in the coil, that is, in the inductive branch. The Q-factors for the parallel circuit having the dynamic resistances listed in Table 12.1 are given in Table 12.2. You can also use the following equation to calculate the Q-factor for a practical parallel circuit containing some resistance in the inductive branch.

$$Q = \frac{\omega_O L}{R} \tag{12.13}$$

$$Q = \frac{1}{\omega_O CR} \tag{12.14}$$

Table 12.2 *Q-factor of a parallel resonant circuit*

$R(\Omega)$	$I(A)$	$IC(A)$	Q
0	0	0.1	∞
1	0.001	0.1	100
10	0.01	0.1	10
100	0.1	0.1	1

SELF-TEST QUESTIONS

1. A series R-L-C circuit contains a 100-Ω resistor, a 0.5-H inductor and a 10-μF capacitor. If the supply frequency is 200 Hz, calculate the reactance of the inductor and of the capacitor, and the impedance of the circuit.
2. In question 1, if the supply voltage is 10 V r.m.s., calculate the current in the circuit and the voltage across each element in the circuit. Draw the phasor diagram of the circuit to scale. What is the power consumed and the power factor of the circuit?
3. The three elements in question 1 are reconnected in parallel with one another in the manner shown in Figure 12.7. If the supply voltage is 10 V, 200 Hz, calculate (i) the current in each branch of the circuit, (ii) the total current drawn by the circuit, (iii) the phase angle of the

total current with respect to the supply voltage and the circuit power factor, (iv) the VA, the power and the VAr consumed by the circuit. Draw the phasor diagram of the circuit to scale.

4. What is meant by resonance in an a.c. circuit? Explain under what conditions (i) series resonance, (ii) parallel resonance occurs.
5. A series circuit containing a resistor of 10 Ω resistance, an inductor of 0.05 H inductance and a capacitor resonates at a frequency of 1 kHz. Calculate the capacitance of the capacitor. If the supply voltage is 15 V, determine the current in the circuit at resonance.
6. Determine the *Q*-factor of the circuit in question 5.

SUMMARY OF IMPORTANT FACTS

The **impedance** of a circuit is its **total opposition** to flow of current.

The **voltage triangle** of a series circuit shows the voltage across the resistive elements (IR), and across the reactive elements ($IX = IX_L - IX_C$), and across the complete circuit (IZ). The **phase angle** of the circuit is the angle between the supply voltage and the current drawn from the supply, and can be determined from the angle in the voltage triangle. The **impedance triangle** for a circuit is obtained by dividing each side of the voltage triangle by the current, I, and shows the total resistance, the effective reactance, and the impedance of the circuit.

The **volt-amperes** (VA) or **apparent power** consumed by an a.c. circuit is equal to the product of the supply voltage and the supply current. The **power consumed** is either equal to the sum of the I^2R products in the circuit (the power loss in all or the resistors), or is equal to $V_S I \cos\phi$, where V_S is the supply voltage, I is the current drawn from the supply and ϕ is the phase angle between V_S and I.

In a **series circuit**, the **phasor sum** of the voltages across the circuit elements is equal to the supply voltage. In a **parallel circuit**, the **phasor sum** of the current in each of the branches is equal to the supply current.

Resonance occurs in an *R-L-C* circuit when the current drawn from the supply is in phase with the supply voltage. A *series resonant circuit* is known as an *acceptor circuit*; a *parallel resonant circuit* is known as a *rejector circuit*.

The **impedance** of a *series resonant circuit* is equal to the resistance of the circuit. The **impedance** of a *parallel resonant circuit* is known as the **dynamic impedance** of the circuit and is equal to $\frac{L}{CR}$ ohms, where R is the resistance in the inductive branch of the circuit; the dynamic impedance of a parallel circuit usually has a very high value.

The **Q-factor** of a resonant circuit is a measure of the *quality* of the circuit in terms of magnifying either the voltage (series resonance) or the current (parallel resonance).

CHAPTER 13

POLY-PHASE a.c. CIRCUITS

13.1 FEATURES OF A POLY-PHASE SUPPLY

As its name implies, a poly-phase power supply or multi-phase supply provides the user with several power supply 'phases'. The way in which these 'phases' are generated is described in sections 13.2 and 13.3, and we concentrate here on the advantages of the use of a poly-phase supply which are:

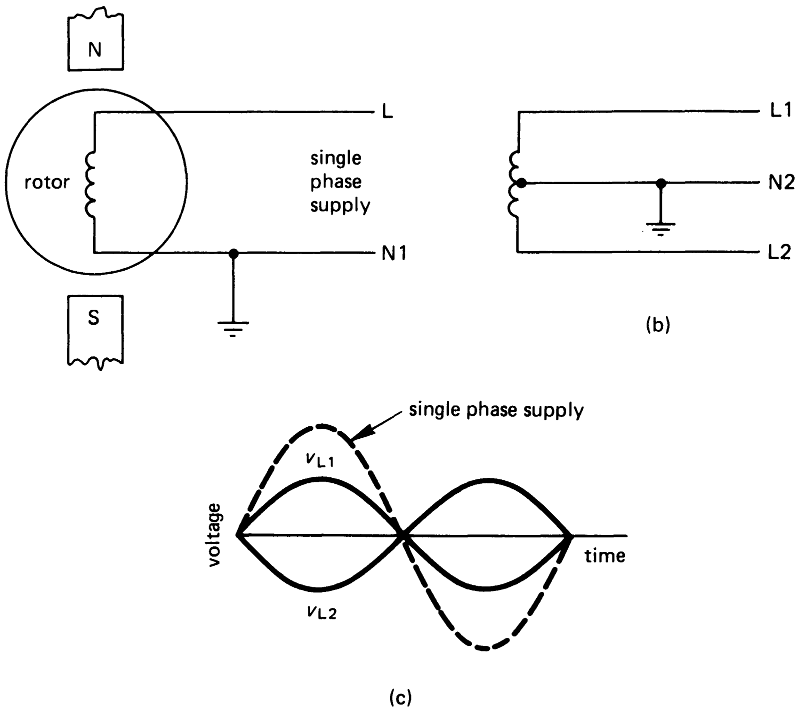
1. For a given amount of power transmitted to the user, the volume of conductor material needed in the supply cable is less than in a single-phase system to supply the same amount of power. A poly-phase transmission system is therefore more economical than a single-phase supply system.
2. Poly-phase motors and other electrical equipment are generally smaller and simpler than single-phase motors and equipment. For industry, poly-phase equipment is cheaper and easier to maintain.

A poly-phase supply system may have two, three, four, six, twelve or even twenty-four phases, with the **three-phase system** being the most popular. The National Grid distribution network is a three-phase system. An introduction to electrical power distribution systems was given in Chapter 8, where it was shown that power is distributed to industry using a three-phase system, a single-phase system being used for domestic power distribution.

13.2 A SIMPLE TWO-PHASE GENERATOR

Consider the simple single-phase generator or **alternator** in Figure 13.1(a). If line N1 is earthed, the waveform of the alternating voltage on line L is shown dotted in Figure 13.1(c).

fig 13.1 *a simple two-phase generator*



Suppose now that we disconnect the earth from line N1 and move it to the centre-point of the alternator winding (shown as N2 in diagram (b)). In effect, the two 'ends' of the winding are 'live' with respect to point N2, but they each have one-half the r.m.s. voltage on them when compared with the single-phase case (Figure 13.1(a)). Moreover, when line L1 is positive with respect to point N2, then L2 is negative with respect to it. The corresponding waveform diagrams for the voltages on lines L1 and L2 are shown in Figure 13.1(c).

Clearly, **by earthing the centre point of the alternator winding, we have produced two sets of 'phase' voltages which can be used independently and have (in this case) a phase angle difference of 180° between them.** This type of power supply is used in many bi-phase rectifier circuits (see Chatper 16).

13.3 A THREE-PHASE GENERATOR

A three-phase supply can be thought of as being generated by three windings on the rotor of an alternator connected as shown in Figure

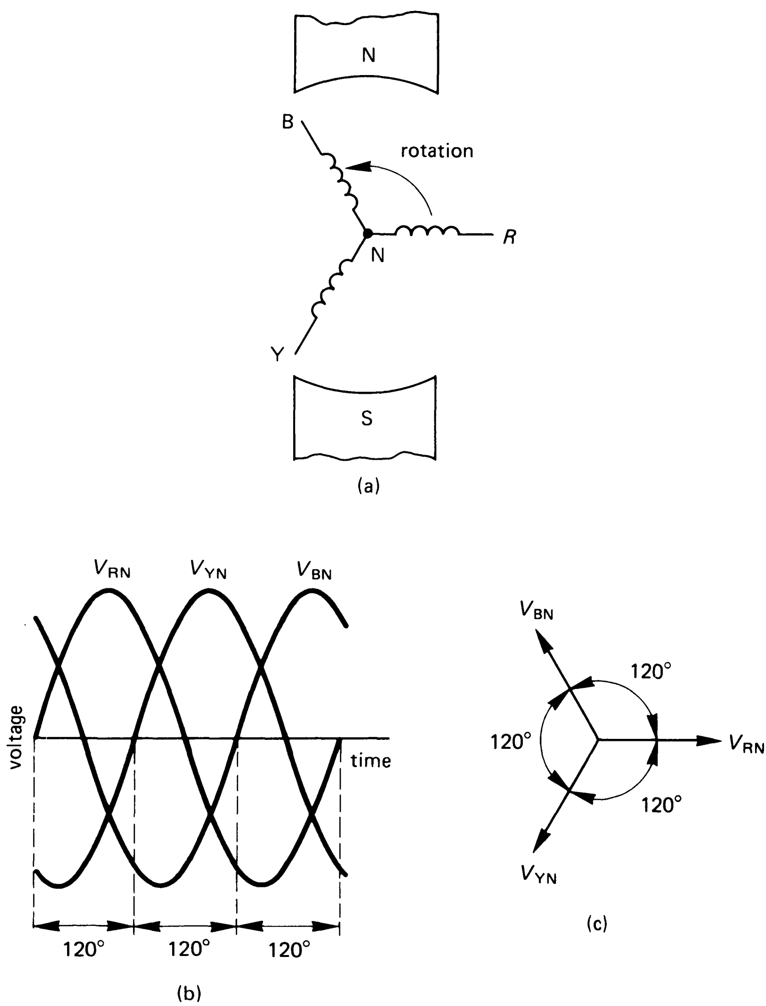
13.2(a) [you should note that this is only one of several possible methods of connection]. The voltage induced in each winding or **phase** is known as the **phase voltage**, and the three voltages are described as follows:

V_{RN} is the **red phase** voltage

V_{YN} is the **yellow phase** voltage

V_{BN} is the **blue phase** voltage

fig 13.2 *a three-phase generator*



The N-point in Figure 13.2(a) is known as the 'neutral' point since it is often connected to earth, that is, to a 'neutral' potential.

Since each winding on the alternator rotor is displaced from each of the other windings by 120°, the phase angle between each of the generated waveforms (Figure 13.2(b)) is 120°; the corresponding phasor diagrams are shown in Figure 13.2(c). With the conventional direction of rotation in Figure 13.2(a), the voltages become positive in the sequence red, yellow, blue; the **phase sequence *R, Y, B*** is known in electrical engineering as the **positive phase sequence**.

The phase sequence of a poly-phase electrical supply can be deduced as follows. Imagine that you are standing on the right-hand side of the alternator rotor in Figure 13.2(a) and looking towards point N. With the direction of rotation shown for the rotor, the windings pass you in the sequence R, Y, B; this means that the voltages become positive in the sequence R, Y, B.

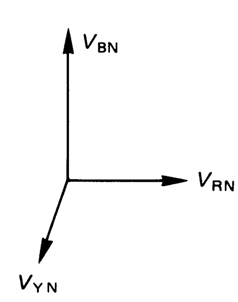
If the direction of the alternator rotor is reversed, the phase sequence is R, B, Y; this means that the phase voltages become positive in the sequence R, B, Y. The latter sequence is known as **negative phase sequence**. You would find it an interesting exercise to draw the waveform diagrams for an alternator having the phase sequence R, B, Y.

13.4 'BALANCED' AND 'UNBALANCED' THREE-PHASE SYSTEMS

A **balanced three-phase supply system** is one which has *equal values of phase voltage which are displaced from one another by 120°*. The phasor diagram in Figure 13.2(c) is for a balanced three-phase supply.

An **unbalanced three-phase supply** either has *unequal phase voltages, or has three voltages which are displaced from one another by an angle which is not 120°, or both*. The phasor diagram in Figure 13.3 is one for an unbalanced three-phase system; certain types of electrical loading and fault conditions produce unbalanced operation.

fig 13.3 *an unbalanced three-phase supply*



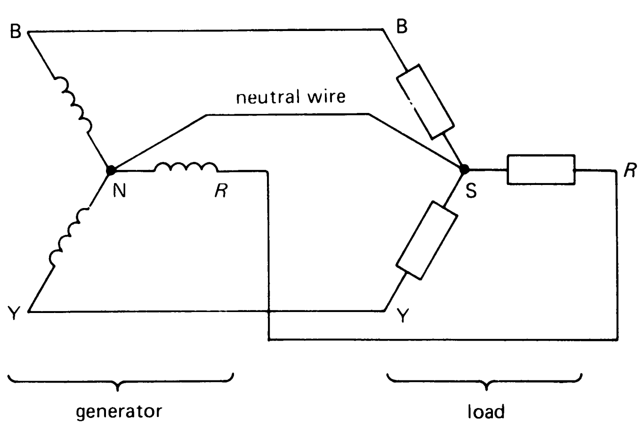
13.5 THE THREE-PHASE, FOUR-WIRE STAR CONNECTION

The **star connection**, as its name implies, means that the three windings of the alternator (or of the connected load) have a common point or **star point**. In Figure 13.4 both the generator and the load are 'star' connected but, for clarity, we have described the 'star' point of the generator as the neutral point (N) since it is usually connected to earth. The star point of the load is given the symbol S.

Figure 13.4 shows a **three-phase, four-wire** supply system since, as its name implies, four wires are used to connect the alternator to the load. The fourth wire is the **neutral wire**, which is used to carry current between the S-point of the load and the N-point of the generator. The three-phase, four-wire supply system is used to supply electricity to houses within a town or village. One group of houses would be supplied by power from the 'red' phase (strictly speaking, the power is supplied from the red-to-neutral (R-N) phase), a second group of houses would be supplied from the 'yellow' phase, and the remaining houses from the 'blue' phase. In the United Kingdom the 'phase voltage' supplied to houses is 240 V, 50 Hz; in the United States of America it is 110 V, 60 Hz.

As in any walk of life, anyone in a house is an individual and each has his own demand on the electricity system. It follows that each group of houses requires a different current at a different power factor (the latter depending on the type of load that is switched on) than the other groups. For example, the group of houses connected to the R-phase may need a current of 100 A, the group connected to the Y-phase may need 20 A, and the group connected to the B-phase may need 70 A. This load is *unbalanced*, and it is the *phasor sum* of these currents which returns to the N-point of

fig 13.4 *the three-phase, four-wire star connection*



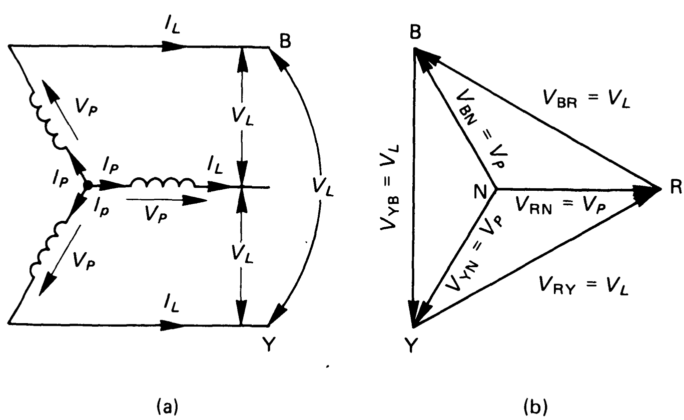
the generator along the neutral wire. This is one reason why the domestic power supply system needs the neutral wire or 'fourth' wire between the load and the supply.

Line voltage of a star connected system

When specifying a three-phase supply voltage, it is usual to quote the voltage between a pair of lines in the supply system; the **line-to-line** voltage is shortened to **line voltage**. The line voltages are specified as follows (see also Figure 13.5):

$$V_{RY} = \text{voltage of line } R \text{ relative to line } Y$$
$$= V_{RN} - V_{YN}$$
$$V_{BR} = \text{voltage of line } B \text{ relative to line } R$$
$$= V_{BN} - V_{RN}$$
$$V_{YB} = \text{voltage of line } Y \text{ relative to line } B$$
$$= V_{YN} - V_{BN}$$

fig 13.5 *phase and line voltage and currents in a star-connected system*



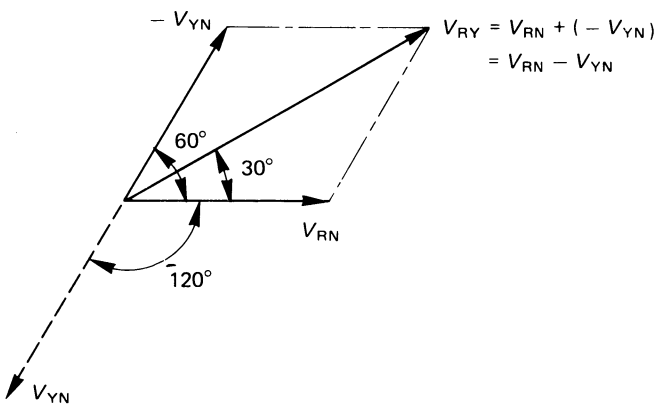
We will now determine the value of the line voltage V_{RY}, which is given by

$$V_{RY} = V_{RN} - V_{YN} = V_{RN} + (-V_{YN})$$

That is, we simply add the 'negative' of V_{YN} to V_{RN}; the voltage $(-V_{YN})$ is simply obtained by reversing the direction of the phasor V_{YN} as shown in Figure 13.6. By drawing the diagram accurately to scale you will find that the value of V_{RY} is:

$$V_{RY} = 1.732 V_{RN} = \sqrt{3} \times V_{RN}$$

fig 13.6 *determining the line voltage V_{RY}*



In general, we may say that

line voltage = $\sqrt{3}$ x phase voltage

or

$$V_L = \sqrt{3} \times V_P \tag{13.1}$$

For a phase voltage of 240 V (which is the nominal UK single-phase domestic voltage), the line voltage is

$$V_L = \sqrt{3} \times 240 = 415.7 \text{ V}$$

As mentioned ealier, three-phase supply systems are specified in terms of their line voltage, and we would describe the above supply as a 415.7 V, three-phase supply. The UK 400-kV grid system therefore operates with a line voltage of 400 kV and a line-to-neutral voltage of $\frac{400}{\sqrt{3}} = 230.9$ kV.

Line current of a star connected system

Since the current in one phase of the load flows in the supply line which is connected to it (see Figure 13.5), **the phase current, I_P, is equal to the line current, I_L**. That is

phase current = line current

or

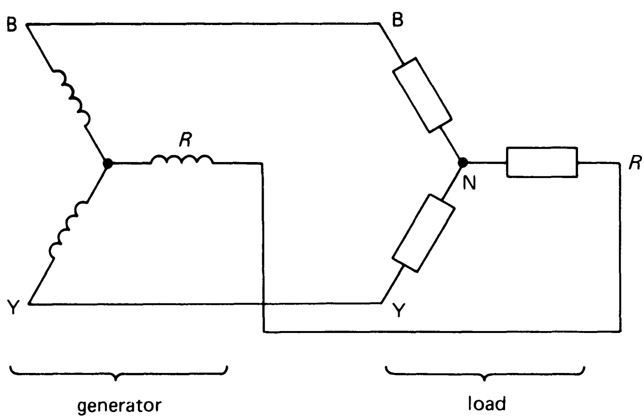
$$I_P = I_L \qquad (13.2)$$

13.6 THREE-PHASE, THREE-WIRE STAR CONNECTED SYSTEM

If the load connected to a three-phase system is *balanced*, that is the load has *the same impedance in each of its three branches and each branch has the same phase angle*, then the magnitude and phase angle of the current in each phase of the load is the same. In this case, *the phasor sum of the three phase currents at the star point of the load (or the neutral point of the supply) is zero*. That is, **the neutral wire current is zero**, so that **we can dispense with the neutral wire**! This is shown in Figure 13.7.

Practically all industrial installations have a balanced load, and the three-phase, three-wire supply system is universally adopted in industry. Reducing the number of supply conductors in this way does not have any technical drawbacks, and it significantly reduces the overall cost of the supply system.

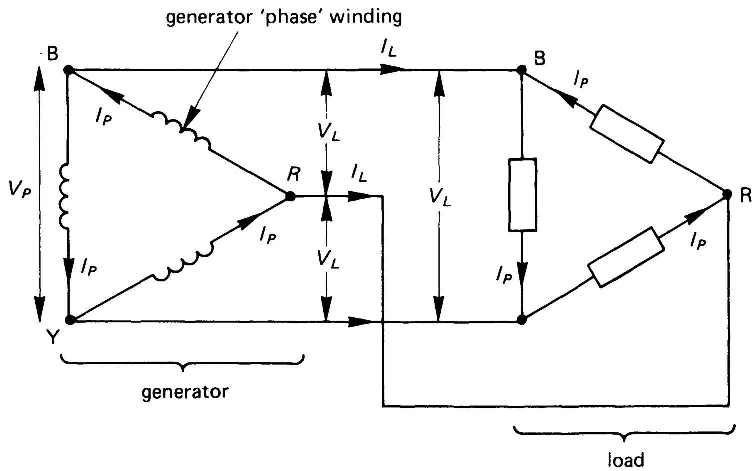
fig 13.7 *the three-phase, three-wire star connection*



13.7 THREE-PHASE DELTA OR MESH CONNECTION

If the three-phase generator described in section 13.3 has three separate windings which can be connected together in any way, and if we connect the 'end' of one winding to the 'start' of the next winding, we get the **delta connection** or **mesh connection** shown in Figure 13.8.

Whilst it may appear in this case that we are 'short-circuiting' the windings together, you must remember that the line voltages are at diffig 13.8 *the delta or mesh connection of a three-phase generator and load*



ferent phase angles to one another (120° in the case of a three-phase system), so that the voltage between the 'end' of the final voltage in the mesh and the 'beginning' of the first voltage is zero! Hence, when the mesh is connected as shown, no current circulates around the close mesh.

You will see that each winding of the generator (the 'phase' winding in which the phase voltage V_P is induced) is connected to two of the supply lines. That is, in a mesh or delta system

phase voltage = line voltage

or

$$V_P = V_L \tag{13.3}$$

That is, if the *line voltage* of a distribution system is 6.6 kV, it can be supplied from a delta-connected generator with a 'phase' voltage of 6.6 kV (if the generator was star-connected, the phase voltage of the generator would be $\frac{6.6}{\sqrt{3}}$ = 3.81 kV).

For a *balanced delta-connected load*, the relationship between the *phase current* I_P in the phase winding (or in one phase of the load), and the *line current* I_L in the supply line is

line current = $\sqrt{3}$ x phase current

or

$$I_L = \sqrt{3} \times I_P$$

For example, if the line current supplied to a delta-connected machine is 100 A, the phase current is

$$I_P = \frac{I_L}{3} = \frac{100}{\sqrt{3}} = 57.74 \text{ A}$$

13.8 SUMMARY OF STAR AND DELTA EQUATIONS

The relationship between the voltages and the currents in a balanced three-phase system are listed in Table 13.1.

Table 13.1 *Summary of three-phase line and phase quantities*

	Phase voltage	*Line voltage*	*Phase current*	*Line current*
Star	$V_P = V_L\sqrt{3}$	$V_L = \sqrt{3}V_P$	$I_P = I_L$	$I_L = I_P$
Delta	$V_P = V_L$	$V_L = V_P$	$I_P = I_L/\sqrt{3}$	$I_L = \sqrt{3}I_P$

13.9 POWER CONSUMED BY A THREE-PHASE LOAD

In *any three-phase load, either balanced or unbalanced*, **the total power consumed is equal to the sum of the power consumed in each of the three phases.**

Example

Calculate the total power consumed in a three-phase system having a phase voltage of 1000 V and the following phase current and power factor:

Red phase	10 A at 0.8 power factor
Yellow phase	20 A at 0.2 power factor
Blue phase	15 A at 0.6 power factor

Solution

The power consumed by each phase is given by the expression $V_P I_P \cos\phi_P$, where V_P is the phase voltage, I_P is the phase current, and $\cos\phi_P$ is the phase power factor. The power consumed in each phase is calculated as follows:

power in R phase = 1000 × 10 × 0.8 = 8000 W

power in Y phase = 1000 × 20 × 0.2 = 4000 W

power in B phase = 1000 × 15 × 0.6 = 9000 W and

$$\text{total power consumed} = 8000 + 4000 + 9000$$
$$= 21\,000 \text{ W (Ans.)}$$

13.10 VA, POWER AND VAr CONSUMED BY A BALANCED THREE-PHASE LOAD

In the case of a *balanced load*, the current and the power factor associated with each phase of the load is the same. This allows us to deduce some general equations as follows. In the following

V_P = phase voltage

V_L = line voltage

I_P = phase current

I_L = line current

$\cos\phi$ = power factor of the load

The reader may find it helpful to consult Table 13.1 when reading the following.

VA consumed by a balanced load

The volt-amperes consumed *by one phase* of the load is $V_P I_P$, and the *total VA* consumed by the load is $3V_P I_P$.

Star connected system

$$\text{total VA}, S = 3V_P I_P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L = \sqrt{3}V_L I_L \text{ VA}$$

Delta connected system

$$\text{total VA}, S = 3V_P I_P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} = \sqrt{3}V_L I_L \text{ VA}$$

Power consumed by a balanced load

The power consumed by *one phase* is $V_P I_P \cos\phi$, and the total power consumed is $3V_P I_P \cos\phi$.

Star connected load

$$\text{total power}, P = 3V_P I_P \cos\phi = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$

$$= \sqrt{3}\, V_L I_L \cos\phi = S \cos\phi \text{ W}$$

Delta connected system

$$\text{total power, } P = 3 V_P I_P \cos\phi = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$$
$$= \sqrt{3}\, V_L I_L \cos\phi = S \cos\phi \text{ W}$$

Volt-amperes reactive consumed by a balanced load

The VAr consumed by *one phase* is $V_P I_P \sin\phi$, and the total VAr consumed is $3 V_P I_P \sin\phi$.

Star connected load

$$\text{total VAr, } Q = 3 V_P I_P \sin\phi = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \sin\phi$$

$$= \sqrt{3}\, V_L I_L \sin\phi = S \sin\phi \text{ Var}$$

Delta connected load

$$\text{total VAr, } Q = 3 V_P I_P \sin\phi = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \sin\phi$$

$$= \sqrt{3}\, V_L I_L \sin\phi = S \sin\phi \text{ VAr}$$

Summary

In any three-phase balanced load

$$\text{total VA consumed, } S = \sqrt{3}\, V_L I_L \text{ VA} \qquad (13.5)$$

$$\text{total power consumed, } P = \sqrt{3}\, V_L I_L \cos\phi \text{ W} \qquad (13.6)$$

$$\text{total VAr consumed, } Q = \sqrt{3}\, V_L I_L \sin\phi \text{ VAr} \qquad (13.7)$$

Example

A three-phase motor has a mechanical output power of 10 kW and an efficiency of 85 per cent. If the motor is supplied at a line voltage of 416 V and the power factor of the motor is 0.8 lagging, calculate the line current drawn by the motor. Calculate the phase current and voltage of the motor if it is (a) star connected, (b) delta connected.

Solution

Output power = 10 kW = 10 000 W; efficiency = 0.85; $V_L = 416$ V; $\cos\phi = 0.8$.

The power input to the motor is

$$\text{power input} = \frac{\text{power output}}{\text{efficiency}} = \frac{10\,000}{0.85}$$

$$= 11\,765 \text{ W}$$

An electrical motor is a balanced load, so that we may use eqn (13.6) as follows

$$\text{electrical input power to the motor} = \sqrt{3}\, V_L I_L \cos\phi$$

hence

$$\text{line current, } I_L = \frac{P}{(\sqrt{3}\, V_L \cos\phi)}$$

$$= \frac{11\,765}{(\sqrt{3} \times 416 \times 0.8)}$$

$$= 20.41 \text{ A (Ans.)}$$

(a) For a *star connected* motor

$$I_P = I_L = 20.41 \text{ A (Ans.)}$$

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{416}{\sqrt{3}} = 240 \text{ V (Ans.)}$$

(b) For a *delta connected* motor

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{20.41}{\sqrt{3}} = 11.78 \text{ A (Ans.)}$$

$$V_P = V_L = 416 \text{ V (Ans.)}$$

SELF-TEST QUESTIONS

1. Explain why a poly-phase power distribution is used for the National Grid distribution system.
2. Describe the operation of a three-phase generator.
3. What is meant by (i) a balanced supply voltage and (ii) an unbalanced supply.
4. A three-phase supply has a line voltage of 33 kV; calculate the phase voltage if it is star connected. What is the line voltage of a star connected system which has a phase voltage of 100 V?
5. Under what circumstances are the following used? (i) A three-phase, four-wire system, (ii) a three-phase, three-wire system.

6. A factory draws a balanced three-phase load of 100 kW at a power factor of 0.8, the line voltage being 3.3 kV. Calculate the line current and the phase current if the load is (i) star connected, (ii) delta connected.
7. For the factory in question 6, calculate the phase angle of the load, the apparent power consumed, and the volt–amperes reactive consumed.

SUMMARY OF IMPORTANT FACTS

A **poly-phase supply** provides the user with several (usually three) power supply 'phases', the phase voltages being at a fixed angle to one another (which is 120° in a three-phase system). A poly-phase system can transmit more power for a given amount of conductor material than can an otherwise equivalent single-phase system. Poly-phase motors are generally smaller and more efficient than equivalent single-phase motors; polyphase switchgear and control gear is also smaller and cheaper than equivalent rated single-phase equipment.

A **balanced** or **symmetrical** three-phase supply has three equal phase voltages which are displaced from one another by 120°. The National Grid distribution network and local area distribution systems employ a three-phase distribution system.

Factories and other installations with three-phase **balanced loads** use a **three-phase, three-wire** distribution system. Housing estates and other consumers with an **unbalanced load** use a **three-phase, four-wire** distribution system; the fourth or **neutral wire** is used to return the out-of-balance current in the phases to the generator.

Generators and loads can be connected in one of several ways including **delta (mesh)** or **star**; star connected systems can use a three-phase, four-wire supply. In **star**, the line and phase currents are equal to one another; in **delta** or **mesh**, the line voltage is equal to the phase voltage.

The **power consumed** by a three-phase load (either balanced or unbalanced) is equal to the sum of the power consumed in each of the three loads. The *power consumed in a balanced load* is equal to $\sqrt{3}\,V_L I_L \cos\phi$.

CHAPTER 14

THE TRANSFORMER

14.1 MUTUAL INDUCTANCE

In Chapter 7 (Electromagnetism) it was shown that

1. when a current flows in a coil, a magnetic flux is established;
2. when a magnetic flux cuts a coil of wire, an e.m.f. is induced in the coil.

The above effects are involved in the process of **mutual induction**, illustrated in Figure 14.1, in which a changing alternating current in one coil of wire (the **primary winding**) induces an e.m.f. in the second coil (the **secondary winding**). The general principle of operation is described below.

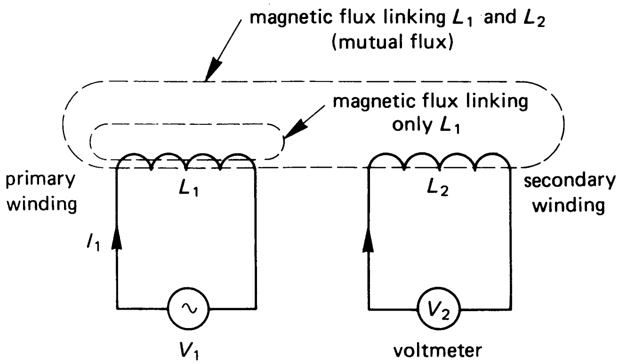
When a current flows in inductor L_1, it produces magnetic flux Φ. For practical reasons, not all this flux manages to link with the secondary winding; let us say that a proportion $k\Phi$ of this flux, where k is a number in the range zero to unity, links with the secondary winding. The flux which is common to the two windings is known as the **mutual flux**. This flux acts to induce mutually an e.m.f. V_2 in inductor L_2. The magnitude of V_2 is given by the formula

$$V_2 = M \frac{\Delta I_1}{\Delta t} \qquad (14.1)$$

where M is the **mutual inductance** in henrys between the two coils, and $\frac{\Delta I_1}{\Delta t}$ is the rate of change of current I_1 with respect to time (in A/s). For example, if the primary winding current changes at the rate of 80 A/s (corresponding approximately to a sinewave of peak current value 1 A at a frequency of 50 Hz), and if the mutual inductance is 100 mH, the induced e.m.f. is

$$V_2 = M \Delta I_1/\Delta t$$
$$= (100 \times 10^{-3}) \times 80 = 8 \text{ V}$$

fig 14.1 *mutual inductance: the basis of the transformer*



This is the principle of the **electrical transformer** in which the primary winding carries alternating current from the power supply, which is 'transformed' to a different voltage and current level by the mechanics of mutual inductance.

14.2 COUPLING COEFFICIENT

We saw in Figure 14.1 that not all the magnetic flux produced by the primary winding links with the secondary winding. The amount of magnetic flux linking with the secondary winding is $k\Phi$, where k is known as the **magnetic coupling coefficient** and is defined by the equation

$$k = \frac{\text{amount of flux linking the secondary winding}}{\text{amount of flux produced by the primary winding}}$$

$$= \frac{k\Phi}{\Phi}$$

The coupling coefficient has a value of zero when none of the flux produced by the primary winding links with the secondary winding, and has a value of unity when all the flux links with the secondary winding; the coupling coefficient is a dimensionless number.

Depending on the application, k may have a low value (a low value of k is used in certain radio and television 'tuning' circuits when k may be as low as 0.05) or it may have a high value (as occurs in electrical power transformers when k may be as high as 0.95, where we need a high electrical efficiency).

The mutual inductance, M henrys, between two coils of self inductance L_1 and L_2, respectively, is

$$M = k\sqrt{(L_1 L_2)} \tag{14.2}$$

Example

Determine the coupling coefficient between two coils of inductance 10 mH and 40 mH, respectively, if the mutual inductance is 5 mH.

Solution

From eqn (14.2)

$$k = \frac{M}{\sqrt{(L_1 L_2)}}$$

$$= \frac{5 \times 10^{-3}}{\sqrt{(10 \times 10^{-3} \times 40 \times 10^{-3})}} = 0.25 \text{ (Ans.)}$$

14.3 BASIC PRINCIPLE OF THE TRANSFORMER

The operating principle of the transformer can be described in terms of the diagram in Figure 14.2. We have seen earlier that an e.m.f. is induced in a coil whenever the magnetic flux linking with it changes, the equation for the induced e.m.f. being

$$E = N\frac{\Delta\Phi}{\Delta t}$$

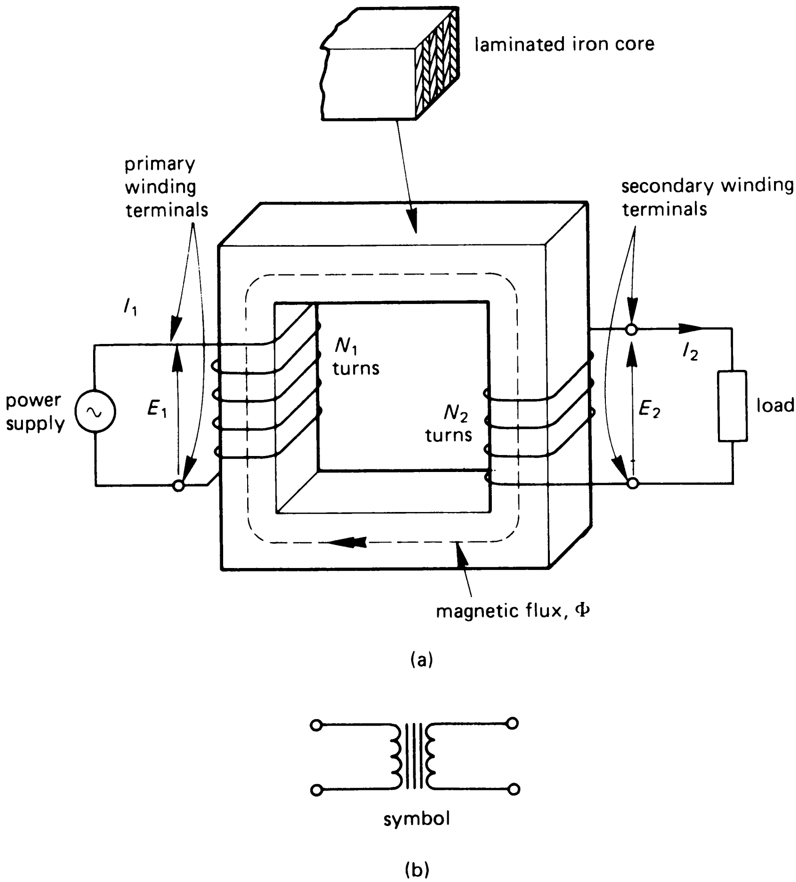
where N is the number of turns on the coil and $\frac{\Delta\Phi}{\Delta t}$ is the rate at which the magnetic flux changes.

However, in the transformer in Figure 14.2, the flux links with both the primary and secondary coils, so that the rate of change of magnetic flux is the same for both coils. That is, there is an induced e.m.f. E_2 in the secondary winding, and a 'back' e.m.f. E_1 in the primary winding. Hence

$$\frac{\Delta\Phi}{\Delta t} = \frac{E_1}{N_1} = \frac{E_2}{N_2} \tag{14.3}$$

That is, **each winding supports the same number of volts per turn.**

Now, if we can assume that the transformer has no power loss (this is not correct in practice since each winding has some resistance and therefore gives rise to an I^2R loss [often known as a **copper loss**], and the iron circuit of the transformer has both *eddy-current loss* and *hysteresis loss* [collectively known as the **iron loss**, the **core loss** or as the **no-load loss**]), then we may assume that the power consumed by the load connected to fig 14.2 *(a) a simple transformer, (b) circuit symbol*



the secondary terminals is equal to the power supplied by the primary winding. That is

$$E_1 I_1 \cos \phi = E_2 I_2 \cos \phi$$

where

E_1 = primary winding voltage
E_2 = secondary winding voltage
I_1 = primary winding current
I_2 = secondary winding current
$\cos \phi$ = power factor of the load

The cos ϕ term cancels on both sides of the equation, giving

$$E_1 I_1 = E_2 I_2 \tag{14.4}$$

Eqns (14.3) and (14.4) enable us to spell out two important design considerations, namely

1. **each winding supports the same number of volts per turn** (eqn (14.3));
2. **the volt–ampere product in each winding is the same** (eqn (14.4)).

We can combine the two equations into a composite equation as follows

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \tag{14.5}$$

The ratio $E_2 : E_1$ is described as the **voltage transformation ratio** of the transformer (this is the reciprocal of the ratio. $E_1 : E_2$ in eqn (14.5)). If this ratio is greater than unity then E_2 is greater than E_1, and the transformer has a **step-up voltage ratio**; if the ratio is less than unity, the transformer has a **step-down voltage transformation ratio.**

The ratio $I_2 : I_1$ is the **current transformation ratio.** If $I_2 : I_1$ is greater than unity, then I_2 is greater than I_1 and the transformer has a **step-up current ratio**; if $I_2 : I_1$ is less than unity, the transformer has a **step-down current ratio.**

A transformer which has a step-down voltage ratio has a step-up current ratio and vice versa. For example, if $E_1 = 400$ V and $E_2 = 20$ V, the transformer has a voltage ratio of $E_2 : E_1 = 20:400 = 1:20 = 0.05$ (which is usually described as a *step-down voltage ratio* of 20 to 1 or 20:1). The same transformer has a *step-up current ratio* of 20:1, so that if the current in the primary winding is 0.5 A, the secondary winding current is 10 A.

If we cross-multiply the centre term with the right-hand term in eqn (14.5) we get the equation

$$I_1 N_1 = I_2 N_2$$

which gives another cardinal design rule which is

3. **Ampere–turn balance is maintained between the windings.**

Example

A single-phase transformer supplied from a 1100-V supply has a step-down voltage ratio of 5 : 1. If the secondary load is a non-inductive resistor of 10 ohms resistance calculate (a) the secondary voltage, (b) the secondary current and the primary current, (c) the power consumed by the load.

Solution

$$E_1 = 1100 \text{ V}; \frac{E_2}{E_1} = \frac{1}{5}; R = 10\ \Omega$$

(a) $\frac{E_2}{E_1} = \frac{1}{5}$, hence

$$E_2 = \frac{E_1}{5} = \frac{1100}{5} = 220 \text{ V (Ans.)}$$

(b) $$I_2 = \frac{E_2}{R} = \frac{220}{10} = 22 \text{ A (Ans.)}$$

Also, since $\frac{E_2}{E_1} = \frac{1}{5}$, then $\frac{I_2}{I_1} = 5$

therefore

$$I_1 = \frac{I_2}{5} = \frac{22}{5} = 4.4 \text{ A (Ans.)}$$

(c) power consumed by the load $= I_2^2 R = 22^2 \times 10$

$$= 4840 \text{ W (Ans.)}$$

14.4 CONSTRUCTION OF A PRACTICAL TRANSFORMER

The operation of a *two-winding transformer* has been based on the construction in Figure 14.2, in which each winding is on a separate *limb* of the iron circuit. However, this arrangement is not efficient since the two windings are well separated from one another, resulting in a large magnetic leakage with an associated low value of magnetic coupling coefficient.

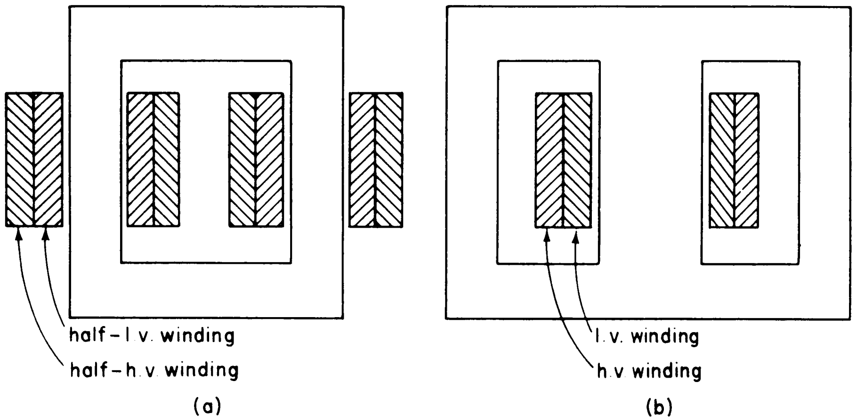
Efficient power transformer design brings the two windings into intimate contact with one another so that there is a close magnetic coupling between the two. The principal types of magnetic circuit and winding arrangements used in power transformers are described below.

Magnetic circuit design

A practical iron circuit has a certain amount of power loss because of induced eddy current power loss in the iron circuit. Using good engineering design, it is possible to minimise this power loss. The way in which this is done in power transformers is to construct the iron circuit using **thin iron laminations** as shown in Figure 14.2.

The laminations increase the resistance of the iron core to the flow of eddy current, so that the induced eddy-current power-loss in the core is reduced. The laminations are isolated from one another by a very thin coat of varnish (or oxide) on one side of each lamination. At the same time, the total cross-sectional area of the iron circuit is not reduced, so that the magnetic flux density is unchanged.

In the **core-type** magnetic circuit (see Figure 14.3(a)), one half of each winding is associated with each limb of the transformer, the magnetic fig 14.3 *(a) core-type magnetic circuit construction, (b) shell-type construction*



circuit having a uniform cross-sectional area throughout. In the **shell-type** magnetic circuit (see Figure 14.3(b)) both the primary and the secondary winding are on the centre limb which has twice the cross-sectional area of the outer limbs.

Winding design

In the **concentric construction** of windings (Figure 14.4(a)), the low-voltage and high-voltage windings are wound concentrically around the iron core. In the **sandwich construction** of windings (Figure 14.4(b)), the high-voltage winding is sandwiched between the two halves of the low voltage winding.

Other general features of transformer construction

In large power transformers, ventilation spaces are left in the windings to allow space for a coolant to circulate, which may be either air or oil. Oil is used as a coolant in large equipment because it is an efficient insulating medium at normal operating temperature.

14.5 AUTOTRANSFORMERS OR SINGLE-WINDING TRANSFORMERS

In some applications, a two-winding transformer of the type described above can be replaced by a simpler single-winding transformer known as an **autotransformer**. Two simplified versions of the autotransformer are shown in Figure 14.5. An autotransformer can either have a step-down voltage ratio (Figure 14.5(a)) or a step-up voltage ratio (Figure 14.5 (b)).

fig 14.4 *(a) concentric winding construction and (b) sandwich winding construction*

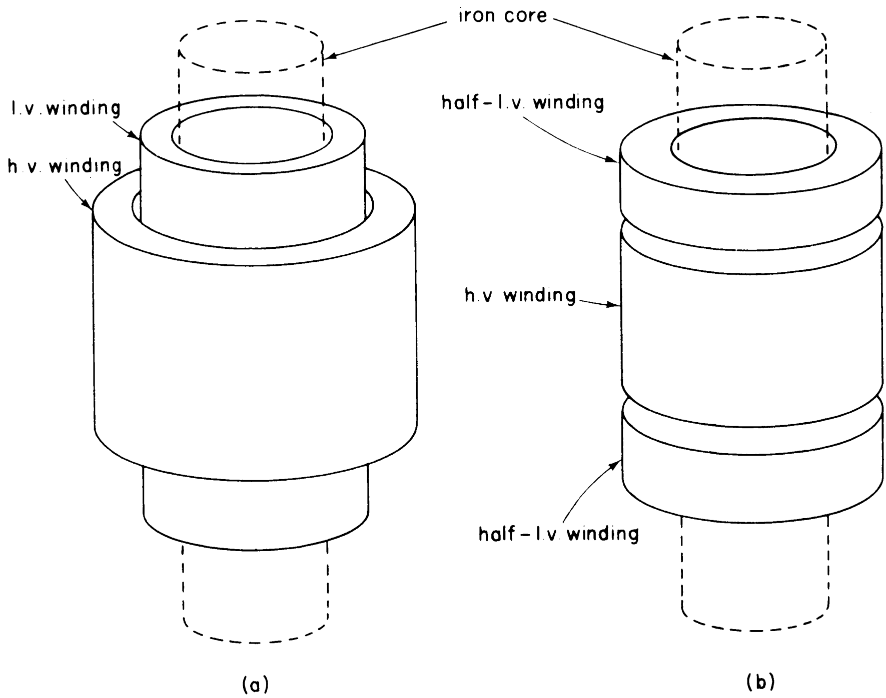
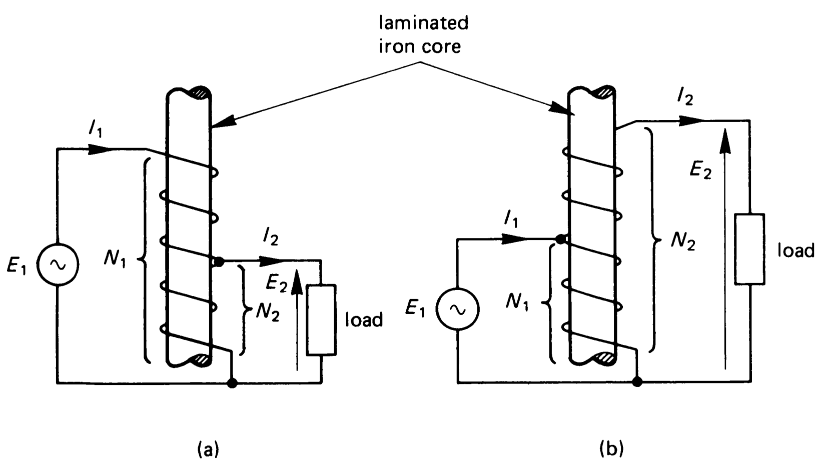


fig 14.5 *autotransformer with (a) a step-down voltage ratio, (b) a step-up voltage ratio*



The general principles of the transformer apply to the autotransformer, that is

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \quad \text{and} \quad I_1N_1 = I_2N_2$$

For some applications the autotransformer has the drawback that the primary and secondary windings are not electrically isolated from one another (as they are in the two-winding transformer); this introduces the hazard that the low-voltage winding can be accidentally charged to the high voltage potential under certain fault conditions.

14.6 TRANSFORMER EFFICIENCY

The efficiency of a transformer for a particular electrical load is given by the following:

$$\text{efficiency}, \eta = \frac{\text{output power (watts)}}{\text{input power (watts)}}$$

$$= \frac{\text{output power}}{\text{output power + power losses}}$$

$$= \frac{\text{input power} - \text{losses}}{\text{input power}}$$

A well-designed transformer has relatively low power-losses (low, that is, when compared with the power transmitted to the load), and the efficiency is quite high (95 per cent or better at normal load). The power loss occurring within a transformer can be divided into two types, namely

1. **I^2R loss or 'copper' loss** due to flow of load current in the transformer windings (this power loss varies with the load current).
2. **Power loss in the magnetic circuit** consisting of the hysteresis loss and the eddy-current loss (this loss is fairly constant and is independent of load current, and occurs even under no-load conditions).

Example

A 6600/550 V transformer has an iron loss of 350 W and a full-load copper loss of 415 W. If the full-load secondary current is 45 A at a power factor of 0.6 lagging, calculate the full-load efficiency of the transformer.

Solution

E_1 = 6600 V; E_2 = 550 V; iron loss = 350 W; full-load copper loss = 415 W; full-load secondary current = 45 A; power factor = 0.6.

Power output from transformer $= E_2 I_2 \cos \phi$

$= 550 \times 45 \times 0.6 = 14\,850$ W

The power loss in the transformer at full load is given by

copper loss + iron loss = 415 + 350 = 765 W

Hence

$$\text{full-load efficiency, } \eta = \frac{\text{output power}}{(\text{output power} + \text{losses})}$$

$$= \frac{14\,850}{(14850 + 765)}$$

= 0.951 per unit or 95.1 per cent (Ans).

14.7 INSTRUMENT TRANSFORMERS

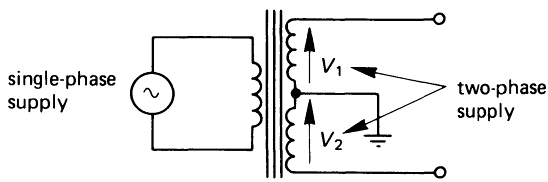
In industrial applications, many items of equipment require either a very high voltage to operate them, for example, several kilovolts, or a very high current, for example, hundreds or thousands of amperes. It is impractical in many cases to produce instruments which can be used to measure these values directly and, for this reason, 'instrument' transformers are used to transform the voltage or current level to a practical value which can be measured by conventional instruments (the normal range used is 1 A or 5 A for current and 110 V for voltage).

A **voltage transformer (VT)** or **potential transformer (PT)** is designed to operate with a high resistance voltmenter connected to its secondary terminals. That is, it is designed to operate with a very small load current.

A **current transformer (CT)** is designed to operate with an ammeter connected to its secondary terminals; that is to say, in normal operation the secondary terminals are practically short-circuited (note: since the ammeter has very little resistance, the CT transmits very little power to the ammeter). If the ammeter is disconnected for any reason (such as for maintenance purposes), **the secondary terminals of the CT should be short-circuited**. If this is not done, the primary current can endanger the transformer by causing (i) overheating of the iron core due to the high level of magnetic flux in it, and (ii) damage to the secondary winding by inducing a very high voltage in it.

14.8 USING A TRANSFORMER TO PRODUCE A TWO-PHASE SUPPLY

A two-phase supply is produced at the secondary of the transformer in Figure 14.6. The transformer has a centre-tapped secondary winding and, fig 14.6 *use of a centre-tapped secondary winding to produce a two-phase (bi-phase) supply*



in effect, each half of the secondary winding has the same value of voltage induced in it. However, the voltages V_1 and V_2 are 180° out of phase with one another (see also section 13.2). This type of circuit is used in certain types of rectifier circuit (see Chapter 16), and the same general principle is used to produce a six-phase supply from a three-phase supply (see section 14.10).

14.9 THREE-PHASE TRANSFORMERS

A three-phase transformer can be thought of as a magnetic circuit which has three separate primary windings and three secondary windings on it, either of which may be connected in star or in delta (some three-phase transformers use quite complex connections).

The basis of the three-phase transformer is illustrated in Figure 14.7, in which one phase of a three-phase transformer is shown. The general

fig 14.7 *windings on one limb of a three-phase transformer*

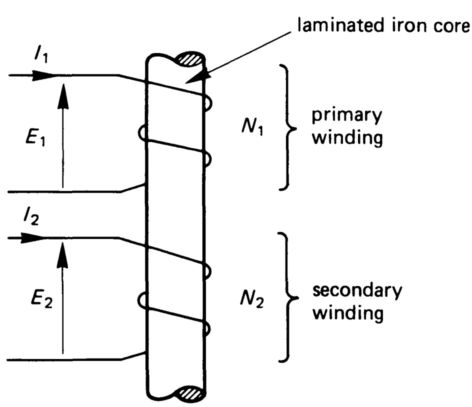
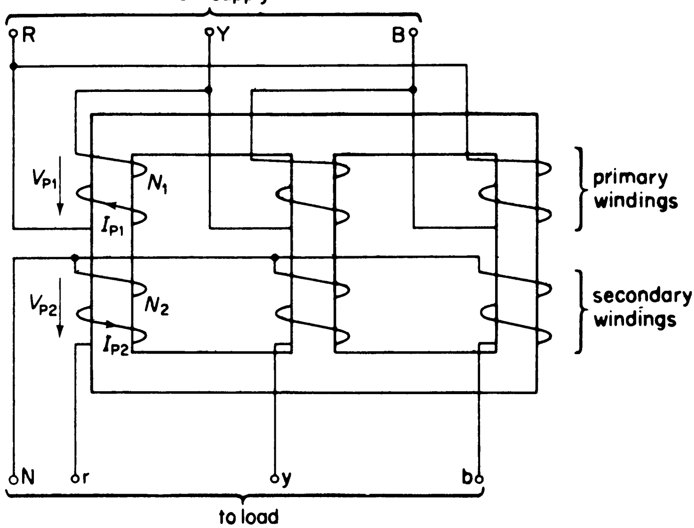
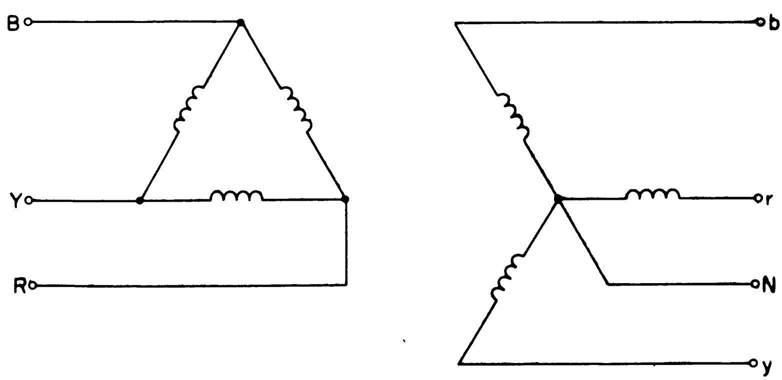


fig 14.8 *(a) a three-phase delta-star transformer and (b) connection diagram*



(a)



(b)

equation of the single-phase transformer holds good **for each phase of the three-phase transformer**; that is

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

and

$$I_1N_1 = I_2N_2$$

the values of E, N and I referred to above are shown in Figure 14.7.

Figure 14.8 shows a three-phase **delta–star** transformer, that is the primary winding is connected in delta and the secondary winding is connected in star. The physical arrangement of the transformer windings are illustrated in Figure 14.8(a) and the circuit diagram in Figure 14.8(b). This type of transformer enables the three-phase, three-wire supply connected to the primary winding to energise a three-phase, four-wire load. This type of transformer is used to connect, say, a local medium-voltage distribution system to a housing estate.

14.10 A THREE-PHASE TO SIX-PHASE TRANSFORMER

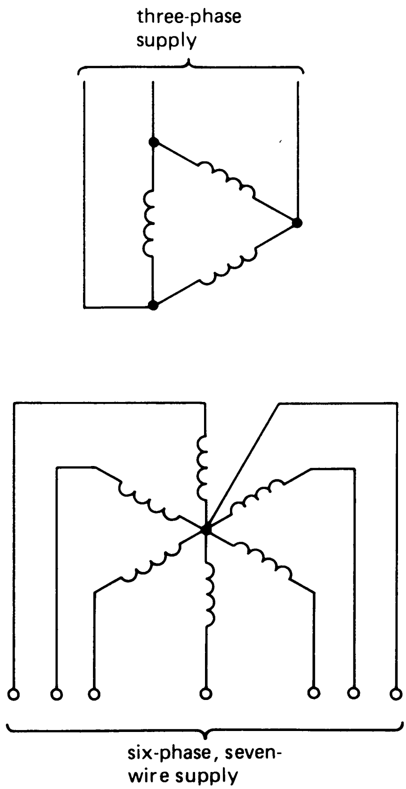
Many industrial power electronics circuits need a six-phase power supply. In this section we look at one method of producing a six-phase supply from a three-phase supply.

The transformer described here (see Figure 14.9) uses the method described in section 14.8 for the production of a two-phase supply from a single-phase supply. The basic transformer consists of a delta–star transformer of the type in Figure 14.8(a) with the difference that each secondary winding is centre-tapped. In this way, each of the secondary windings produces two supplies which are 180° out of phase with one another. All the centre-taps of the secondary windings are connected together to form a common neutral point (or star point) for the secondary winding. The net result is a three-phase to six-phase transformer, the secondary 'phase' voltages being displaced 60° from one another.

SELF-TEST QUESTIONS

1. Explain what is meant by the expression 'mutual inductance'. How is mutual inductance utilised in a transformer?
2. What is meant by the 'coupling coefficient' of a magnetic circuit? State the limiting values of the coupling coefficient; explain why its value cannot fall outside these limits.

fig 14.9 *a three-phase to six-phase transformer*



3. Two coils of equal inductance are magnetically coupled together. If the coupling coefficient is 0.5 and the mutual inductance is 0.5 H, determine the inductance of each coil.
4. A transformer has 50 turns of wire on its primary winding and 10 turns on its secondary winding. If the supply voltage is 100 V, calculate the secondary voltage. A 10-watt load is connected to the secondary terminals; determine the primary and secondary current.
5. Describe the construction and operation of a two-winding transformer. Include in your description details of the magnetic circuit construction and the winding construction.
6. How does an auto-transformer differ from a two-winding transformer?
7. A transformer has a 10:1 step-up voltage ratio. If the efficiency of the transformer is 95 per cent, calculate the power delivered to the load if the primary voltage and current are 100 V and 0.1 A, respectively.
8. Discuss applications of instrument transformers.

SUMMARY OF IMPORTANT FACTS

The **transformer** depends for its operation on the principle of **mutual induction**. The **primary winding** of the transformer is connected to the power source (which must be a.c.), and the load is connected to the **secondary winding**.

The transformer may have either a *single winding* (when it is known as an **autotransformer**) or *more than one winding* (**two-winding** transformers are the most common single-phase transformers). The **iron circuit** of the transformer is **laminated** to reduce the *eddy-current power-loss.* Important rules relating to transformer design are

1. **Each winding supports the same number of volts per turn.**
2. **Ampere-turn balance is maintained between the windings.**

If a transformer has a *step-down voltage ratio* it has a *step-up current ratio*, and vice versa.

The **efficiency** of a transformer is the ratio of the power it delivers to the load to the power absorbed by the primary winding.

Instrument transformers (*potential transformers* or *voltage transformers* and *current transformers*) are used to extend the range of a.c. measuring instruments.

A **three-phase transformer** may be thought of as three single-phase transformers using a common iron circuit. Special winding connections allow a six-phase supply (or a twelve-phase supply for that matter) to be obtained from a three-phase supply.

CHAPTER 15

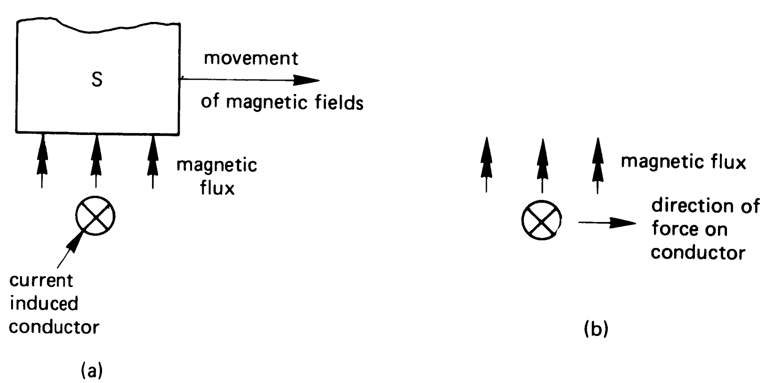
a.c. MOTORS

15.1 PRINCIPLE OF THE a.c. MOTOR

Imagine that you are looking at the end of the conductor in Figure 15.1(a) when the S-pole of a permanent magnet is suddenly moved from left to right across the conductor. By applying *Fleming's right-hand rule*, you can determine the direction of the induced e.m.f. and current in the conductor. You need to be careful when applying Fleming's rule in this case, because the rule assumes that the *conductor moves relatively to the magnetic flux* (in this case it is the *flux that moves relatively to the conductor*, so the direction of the induced e.m.f. is determined by saying that the flux is stationary and that the conductor effectively *moves to the right*). You will find that the induced current flows away from you.

You now have a current-carrying conductor situated in a magnetic field, as shown in Figure 15.1(b). There is therefore a force acting on the

fig 15.1 *(a) how current is induced in a rotor conductor and (b) the direction of the force on the induced current*



conductor, and you can determine the direction of the force by applying Fleming's *left-hand rule*. Application of this rule shows that **there is a force acting on the conductor in the direction of movement of the magnetic field.**

That is, the conductor is accelerated in the direction of the moving magnetic field.

This is the basic principle of the a.c. motor. An a.c. motor therefore provides a means for producing a 'moving' or 'rotating' magnetic field which 'cuts' conductors on the **rotor** or rotating part of the motor. The rotor conductors have a current induced in them by the rotating field, and are subjected to a force which causes the rotor to rotate in the direction of movement of the magnetic field.

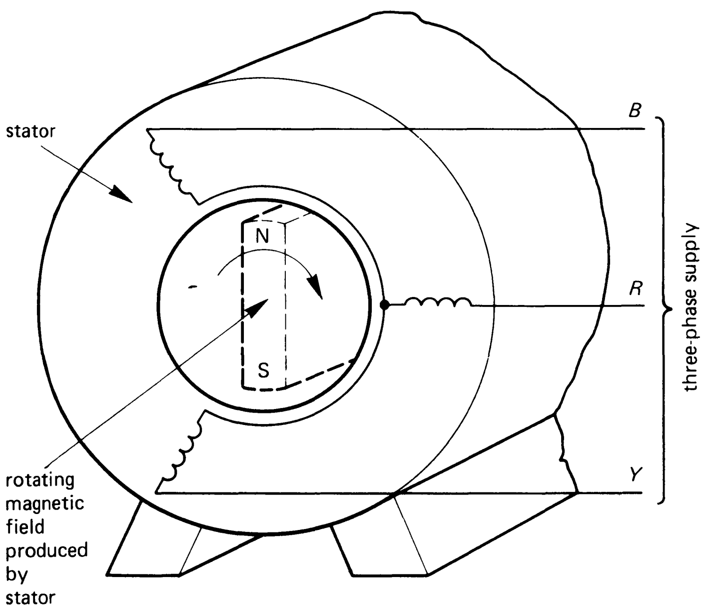
15.2 ROTATING AND 'LINEAR' a.c. MOTORS

Most electrical motors have a *cylindrical rotor*, that is, the rotor rotates around the axis of the motor shaft. This type of motor generally runs at high speed and drives its load through a speed-reduction gearbox. Applications of this type of motor include electric clocks, machines in factories, electric traction drives, steel rolling mills, etc.

Another type of motor known as a **linear motor** produces motion in a straight line (known as **rectilinear motion**); in this case the mechanical output from the motor is a linear movement rather than a rotary movement. An application of this type of motor is found in railway trains. If you imagine the train to be 'sitting' above a single metal track (which is equivalent to the 'conductor' in Figure 15.1) and the 'moving magnetic field' is produced by an electromagnetic system in the train then, when the 'magnet' is made to 'move' by electrical means, it causes the system to produce a mechanical force between the electromagnet and the track. Since the track is fixed to the ground, the train is 'pulled' along the conductor.

15.3 PRODUCING A ROTATING OR MOVING MAGNETIC FIELD

A simplified form of a.c. motor is shown in Figure 15.2. We will describe here a very simplified theory of the rotating field that will satisfy our needs. Imagine the three-phase supply to be a sequence of current 'pulses' which are applied to the red, yellow and blue lines in turn. Each current pulse causes one of the coils to produce a magnetic field, and when one electromagnet is excited, the others are not. As the current pulse changes fig 15.2 *production of a rotating magnetic field*



from the red line to the yellow line and then to the blue line, so the magnetic effect 'rotates' within the machine.

The net result is that we can regard this field as the rotating permanent magnet shown dotted in Figure 15.2. The speed at which the magnet rotates is known as the **synchronous speed** of the machine. Various symbols are used to describe the synchronous speed as follows

N_S is the synchronous speed in rev/min

n_S is the synchronous speed in rev/s

ω_S is the synchronous speed in rad/s

The relationship between them is

$$N_S = 60\, n_S$$

and

$$\omega_S = 2\pi n_S = 120\pi N_S \qquad (15.1)$$

In practice the synchronous speed of the rotating field is dependent not only on the number of pole-pairs produced by the motor (that is, the number of N- and S-pole combinations – the motor in Figure 15.2 has *one* *pole-pair*), but also on the frequency, f Hz, of the a.c. supply, the equation relating them is

$$\text{frequency}, f = n_S p \text{ Hz} \tag{15.2}$$

where

f = supply frequency in Hz

n_S = synchronous speed in rev/s

p = number of *pole-pairs* produced by the motor

Example
Calculate the speed of the rotating field of a two-pole motor in rev/min if the supply frequency is (i) 50 Hz, (ii) 60 Hz. What is the synchronous speed if the motor has two pole-pairs, that is, two N-poles and two S-poles?

Solution
(i) $p = 1$; $f = 50$ Hz. From eqn (11.2)

$$n_S = \frac{f}{p} = \frac{50}{1} = 50 \text{ rev/s}$$

and

$$N_S = 60n_S = 60 \times 50 = 3000 \text{ rev/min (Ans.)}$$

(ii) $p = 1$; $f = 60$ Hz.

$$n_S = \frac{f}{p} = \frac{60}{1} = 60 \text{ rev/s}$$

and

$$N_S = 60n_S = 60 \times 60 = 3600 \text{ rev/min (Ans.)}$$

Note: Since the US supply frequency is 60 Hz, then a motor with the same number of pole-pairs runs $\frac{60}{50} = 1.2$ times faster than the same motor in the UK.

You can see from eqn (15.2) that if the number of pole-pairs is doubled, then the speed is halved. The respective synchronous speeds for four-pole motors, that is, two pole-pair motors are

$$50 \text{ Hz:} \quad \frac{3000}{2} = 1500 \text{ rev/min (Ans.)}$$

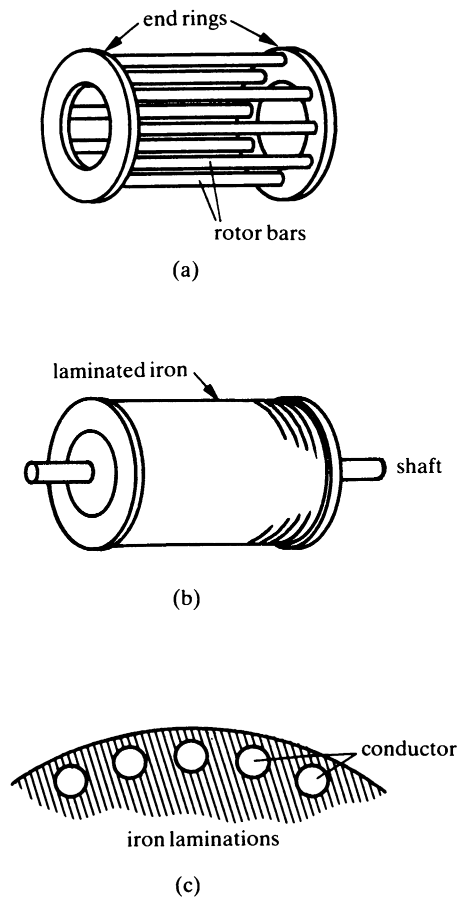
$$60 \text{ Hz:} \quad \frac{3600}{2} = 1800 \text{ rev/min (Ans.)}$$

15.4 A PRACTICAL a.c. MOTOR - THE INDUCTION MOTOR

The simplest form of motor is the **cage rotor induction motor** which has the rotor structure shown in Figure 15.3. The rotor conductors consist of a series of stout aluminium or copper **bars** which are short-circuited at the ends by substantial metal **end rings**. The conductor assembly resembles a 'squirrel cage' and, for this reason, the motor is sometimes described as a *squirrel cage induction motor.*

The rotor conductors are embedded in a *laminated iron core* (see Figure 15.3(b) and (c)) which provides the iron path for the magnetic flux produced by the windings on the **stator** or stationary part of the machine; the

fig 15.3 *construction of the rotor of a cage induction motor*



rotor is laminated in order to reduce the rotor iron loss. No electrical connections are made to the rotor of the machine.

The stator carries a three-phase winding which produces a rotating magnetic field when it is energised by a three-phase supply.

15.5 'FRACTIONAL SLIP' OF AN INDUCTION MOTOR

When a three-phase supply is connected to the stator winding of the induction motor, a magnetic field is produced which rotates at the synchronous speed of the motor, this flux cutting the rotor conductors. The net result is that a current is induced in the conductors in the manner described in section 15.1, and the rotor experiences a **torque** or *turning moment* which accelerates it in the direction of the rotating field.

Let us examine what happens next. As the rotor speeds up, the *relative speed* between the rotating field (which is the constant synchronous speed) and the rotor reduces. Consequently, the rotating magnetic field does not cut the rotor at the same speed; the net result is that the current is still induced in the rotor, the rotor continues to accelerate.

For the moment, let us assume that the rotor can continue acclerating until its speed is equal to that of the rotating field. In this event, the rotor conductors no longer 'cut' the magnetic field produced by the stator (since they both rotate at the same speed!). Since current is only induced in a conductor when it 'cuts' magnetic flux, it follows in this case that the rotor current falls to zero; that is, the rotor no longer produces any torque and it will begin to slow down. We therefore conclude that, under normal operating conditions, **the rotor of an induction motor will never rotate at the synchronous speed of the rotating magnetic field**.

There are, of course, special conditions under which the rotor rotates at a speed which is equal to or even higher than the synchronous speed, but a discussion of these is outside the scope of this book.

The actual difference in speed (in rev/min, or in rev/s or in rad/s) between the rotor and the rotating field is known as the **slip** of the rotor. It is more usual for engineers to refer to the **fractional slip**, s, which has no dimensions and is calculated as follows:

$$\text{fractional slip, } s = \frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}} \tag{15.3}$$

If the speed is in rev/min, the fractional slip is

$$s = \frac{(N_S - N)}{N_s}$$

If the speed is in rev/s, the fractional slip is

$$s = \frac{(n_S - n)}{n_S}$$

If the speed is in rad/s, the fractional slip is

$$s = \frac{(\omega_S - \omega)}{\omega_S}$$

where N, n and ω is the speed of the rotor in rev/min, rev/s and rad/s, respectively.

Example
Calculate the fractional slip of a three-phase, four-pole machine which has a supply frequency of 50 Hs if the rotor speed is 1440 rev/min.

Solution
$p = 2$ (2 pole-pairs); $f = 50$ Hz; $N = 1440$ rev/min.
From eqn (15.2)

$$n_S = \frac{f}{p} = \frac{50}{2} = 25 \text{ rev/s}$$

hence

$$N_S = 60n_S = 60 \times 25 = 1500 \text{ rev/min}$$

From eqn (15.3), the fractional slip is

$$s = \frac{(N_S - N)}{N_S} = \frac{(1500 - 1440)}{1500}$$

$$= 0.04 \text{ per unit or 4 per cent (Ans.)}$$

That is to say, the rotor runs at a speed which is 4 per cent below the synchronous speed.

15.6 THE SYNCHRONOUS MOTOR

A number of applications call for a motor whose rotor runs at precisely the same speed as that of the rotating magnetic field. This type of motor is known as a **synchronous motor**.

Quite simply, the principle is to replace the cage rotor with what is equivalent to a permanent magnet. The N-pole of this magnet is attracted to the S-pole of the rotating field, and the S-pole on the permanent magnet is attracted to the N-pole of the rotating field. Provided that the magnetic pull between the permanent magnet and the rotating field is large enough, the rotor runs at the same speed as the rotating field.

In practice, the 'permanent magnet' effect is produced by an electromagnet which is excited by a d.c. power supply which may be either a d.c. generator (known as an **exciter**) or from a rectifier.

Synchronous motors are generally more expensive than induction motors which (apart from providing an absolutely constant speed) have the ability to provide a certain amount of power factor 'correction' to an industrial plant. The latter feature means that the synchronous motor can 'look' like a capacitor so far as the a.c. power supply is concerned. It is this feature which makes the synchronous motor financially attractive to industry. Detailed treatment of this aspect of the operation of the synchronous motor is beyond the scope of this book.

15.7 SINGLE-PHASE INDUCTION MOTORS

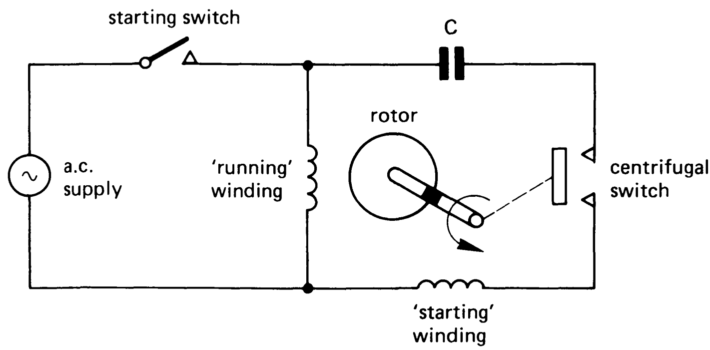
As explained earlier, in order to develop a torque in an induction motor, it is necessary to produce a rotating magnetic field. Unfortunately, a coil connected to a single-phase supply can only produce a *pulsating magnetic field*; that is to say, the magnetic field merely pulsates in and out of the coil and does not 'rotate' or sweep past the coil.

A rotating magnetic field can only be produced by using a supply with more than one phase; that is, at least two coils are needed, each of which is excited by a current which has a phase difference with the current in the other coil (in the three-phase case, three coils are used, the currents in the three coils being displaced from one another by 120°). It is therefore necessary to produce a second 'phase' from the single-phase supply in order to start a single-phase motor. One way in which this can be done is described below.

Once a rotating field has been produced, the rotor of the motor begins to rotate in the direction of the rotating field; at this point it is possible to 'switch off' the second phase, and the rotor will continue running so long as one coil is connected to the single-phase supply. This principle is utilised in the **capacitor-start motor** in Figure 15.4.

The single phase supply is connected directly (via the starting switch) to the **running winding**. The 'second phase' is obtained by means of capacitor C; the capacitor has the effect of supplying a current to the **starting winding** which has a phase angle which differs from the current in the running winding. The net result is that the starting and running windings can be thought of as being supplied from a two-phase supply; these produce a rotating magnetic field in the motor (its effect is not as good as a three-phase motor, but is adequate to start the motor).

fig 15.4 *a single-phase, capacitor-start induction motor*



The rotor begins to accelerate and, when it reaches a pre-determined speed set by a centrifugal switch, this switch cuts the starting winding out of circuit. The motor continues to run under the influence of the single-phase supply connected to the running winding.

Single-phase motors are well-suited to small drives used in domestic equipment, but are both uneconomic and inefficient for large industrial drives.

SELF-TEST QUESTIONS

1. Explain how a 'rotating' magnetic field is produced by a polyphase a.c. motor. How can this idea be adopted to produce a 'linear' motor?
2. A 10-pole a.c. motor is connected to a 50-Hz supply. Calculate the synchronous speed of the rotating field.
3. Explain the principle of operation of (i) an induction motor, (ii) a synchronous motor.
4. A 50-Hz, four-pole induction motor has a fractional slip of 5 per cent. Calculate the speed of the motor.
5. With the aid of a circuit diagram, explain the operation of one form of single-phase a.c. motor.

SUMMARY OF IMPORTANT FACTS

a.c. motor action depends on the production of a **rotating magnetic field** (or a *linearly moving magnetic field* in the case of a 'linear' motor).

A **cage rotor induction motor** has a rotor consisting of a cylindrical cage of conductors in laminated iron. The stator (which is also laminated) carries a poly-phase winding which produces a rotating magnetic field when excited by a poly-phase supply. The current induced in the rotor acts on the rotating field to produce a torque which acts **in the same direction as the rotating field**.

The supply frequency, f, is related to the synchronous speed of the rotating field, n_S rev/s, and the number of *pole-pairs*, p, by the equation

$$f = n_S p \text{ Hz}$$

The rotor of an induction motor runs at a speed, n rev/s, which is slightly less than the synchronous speed. The **fractional slip**, s, of an induction motor is given by

$$s = \frac{(n_S - n)}{n_S}$$

The rotor of a **synchronous motor** runs at the same speed as the synchronous speed of the rotating field. The primary advantage of the synchronous motor is that it can draw a current at a leading power factor (the current drawn by an induction motor lags behind the supply voltage), but it has the disadvantage of higher cost and complexity than an induction motor.

Single-phase induction motors are generally more complex than three-phase types because they need both a 'starting' and a 'running' winding, together with some means of making the transition from starting to running conditions.

CHAPTER 16

POWER ELECTRONICS

16.1 SEMICONDUCTORS

A **semiconductor** is a material which not only has a resistivity which is mid-way between that of a good conductor and an insulator, but has properties which make it very useful in the field of electronics and power electronics. There are two principal categories of semiconductor, namely *n*-type semiconductors and *p*-type semiconductors.

An ***n*-type semiconductor** is one whose physical make-up causes it to have **mobile negative charge carriers** in its structure, hence the name *n-type* semiconductor (these negative charge carriers are, of course, *electrons*). When current flows in an *n*-type semiconductor, the current flow is largely due to the movement of negative charge carriers, that is, electrons. Hence, in an *n*-type semiconductor, electrons are known as **majority charge carriers** (since they carry the majority of the current). A small amount of current flow in *n*-type material is due to the movement of **holes** (see Chapter 1 for a discussion on electronic 'holes'); in *n*-type material, holes are described as **minority charge carriers**.

A ***p*-type semiconductor** is one whose physical structure causes it to have **mobile positive charge carriers**, hence the name *p*-type semiconductor (the positive charge carriers are *holes*). The **majority charge carriers** in *p*-type material are holes, and current flow in *p*-type material can be thought of as being largely due to movement of holes. *Electrons* are the *minority charge carriers* in *p*-type material.

A range of semiconductor materials are used by the power electronics industry, the most popular being silicon, closely followed by germanium. A range of specialised items of equipment are manufactured using such materials as cadmium sulphide, indium antimonide, gallium arsenide, bismuth telluride, etc.

16.2 SEMICONDUCTOR DIODES (THE *p-n* JUNCTION DIODE)

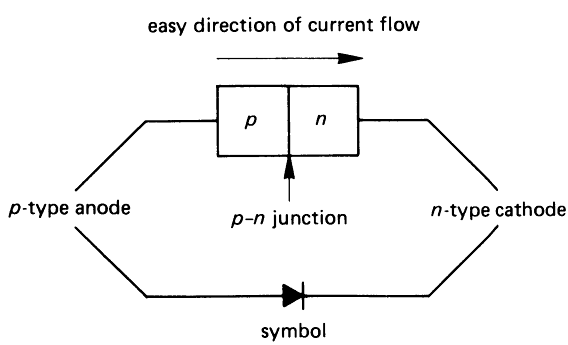
A **diode** is an electronic device which allows current to flow through it with little opposition for one direction of flow, but prevents current flow in the reverse direction. That is, it is the electrical equivalent of a 'one-way' valve.

It has *two electrodes* (see Figure 16.1), known respectively as the **anode** and the **cathode**; the anode is a *p*-type semiconductor and the cathode is *n*-type (a diode is a *single piece of semiconductor* with an *n*-region and a *p*-region; it *is not* simply a piece of *p*-type material which has been brought into contact with a piece of *n*-type material). The symbol representing a diode is shown in the lower part of Figure 16.1; the 'arrow' shape which represents the anode *points in the 'easy' direction of current flow*. That is

> **the diode offers little resistance to current flow when the anode is positive with respect to the cathode; the diode offers nearly infinite resistance to current flow when the anode is negative with respect to the cathode.**

You may therefore regard a diode as a 'voltage-sensitive' switch which is switched to the 'on' or is closed when the anode is positive with respect to the cathode, and is 'off' or open when the anode is negative.

fig 16.1 *a p-n junction diode*



The diode in Figure 16.1 is known as a *p-n* **junction diode**, and a practical diode of this type is physically very small (even a 'power' diode which can handle several hundred amperes is only a few millimetres in diameter!). However, each diode is housed in a container or canister of manageable size for two reasons:

1. it must be large enough to handle;
2. it must have sufficient surface area to dissipate any heat generated in it.

A photograph of a 'signal diode' (a small-current diode) is shown in Figure 16.2(a), its diameter being 2-3 mm. A 'power diode' is shown in Figure 16.2(b); this is seen to be physically larger than the signal diode, and is suitable for bolting onto a **heat sink** which is used to dissipate the heat produced by the diode in normal use.

fig 16.2 *photograph of (a) a signal diode and (b) a power diode*

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A heat sink is a metal (often aluminium) finned structure to which the diode is fastened. It may be painted black to increase its heat radiation capacity and, in some cases, may be fan-coiled. A selection of heat sinks are shown in Figure 16.3.

fig 16.3 *a selection of heat sinks*

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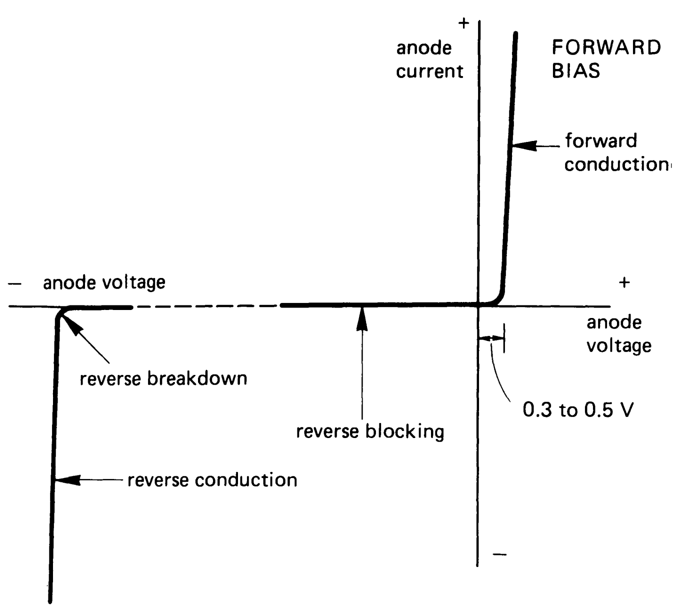
16.3 DIODE CHARACTERISTIC CURVES

The characteristic of a diode (technically known as the **static anode characteristic**) is shown in Figure 16.4. It has two operating areas corresponding respectively to the anode being positive with respect to the cathode and to the anode being negative.

In the *first quadrant* of the graph in figure 16.4, the *anode is positive with respect to the cathode*, and this is known as **forward bias** operation. In this mode, current flows easily through the diode, and the p.d. across the diode is more or less constant between about 0.3 V and 0.5 V (the value of this p.d. depends to a great extent on the type of semiconductor material used in the construction of the diode). When the diode is forward-biassed it is said to operate in its **forward conduction mode**.

In the *Third quadrant* of the characteristic curve, the *anode is negative with respect to the cathode*, and the diode is said to be **reverse biassed.** At normal values of reverse-bias voltage practically no current flows through the diode and it can be thought of as a switch is 'open' or off. In fact, a small value of **leakage current** of a few microamperes does flow through the diode; in most cases in electrical engineering this current can be ignored (this current is sometimes known as the **reverse saturation current**).

fig 16.4 *static anode characteristic of a diode*



If the reverse bias voltage is increased to a sufficiently high value, **reverse conduction** commences at a voltage known as the **reverse breakdown voltage**. Any attempt to increase the reverse bias voltage further results in a rapid increase in the current through the diode. If the current through the diode is not restricted in value (by, say, a fuse), the diode will rapidly overheat and will fail catastrophically.

If must be pointed out that diodes used in power supplies are operated well below their rated reverse breakdown voltage, so that the possibility of failure of the above reason is unlikely.

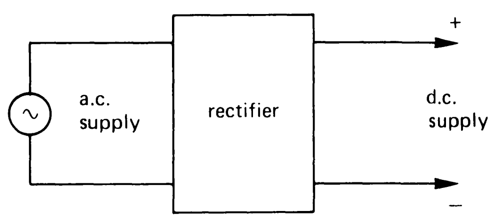
A branch of the diode family known as **Zener diode** is operated in the reverse breakdown mode, and details are given in *Mastering Electronics* by J. Watson.

16.4 RECTIFIER CIRCUITS

A **rectifier** is a circuit containing diodes which *convert alternating current into direct current (unidirectional current).* The basic arrangement is shown in Figure 16.5; alternating current enters the rectifier from the a.c. supply and leaves as d.c.

The a.c. supply may be either a single-phase supply, or a three phase supply, etc. The rectifier circuit can be any one of a number of possible types of circuit including half-wave or full-wave; these terms are described in the following sections

fig 16.5 *block diagram of a rectifier*

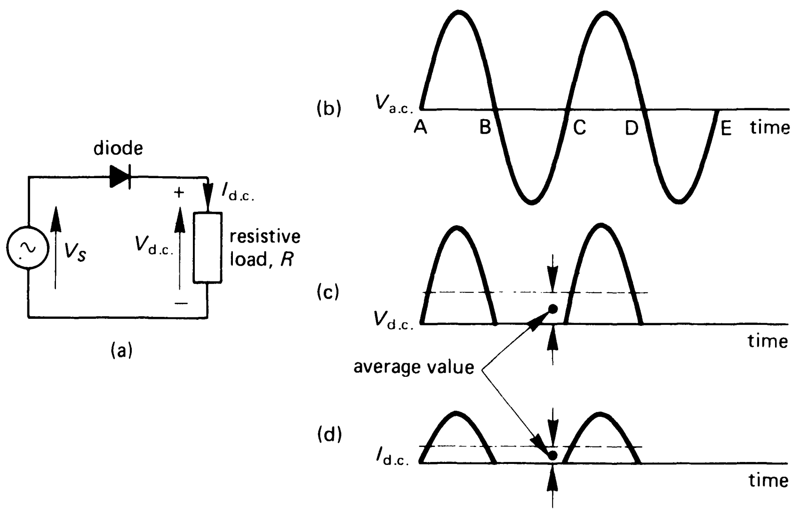


16.5 SINGLE-PHASE, HALF-WAVE RECTIFIER CIRCUIT

A **half-wave rectifier circuit** is one which utilises one-half of each complete wave or cycle of the a.c. supply in order to send current to the 'd.c.' load. A typical single-phase, half-wave rectifier circuit is shown in Figure 16.6(a).

The operation of the circuit is described below, and the corresponding waveform diagrams are shown in diagram (b), (c), and (d) of Figure 16.6.

During each *positive half-cycle* of the a.c. wave (that is, in the time intervals *A–B* and *C–D* of Figure 16.6(b)), the diode is *forward biassed* fig 16.6 *single-phase, half-wave rectifier circuit*



(that is, its anode is positive with respect to its cathode) and current flows through it. Since the 'd.c.' load is resistive, the current in the load is proportional to the voltage (Ohm's law), and the voltage across the load is the same as that of the alternating wave, that is, the 'd.c.' voltage is sinusoidal for the first half-cycle (see Figure 16.6(c)).

During each *negative half-cycle* of the a.c. wave (that is, during the time intervals *B-C* and *D-E* in Figure 16.6(b)), the diode is *reverse biassed* (that is, its anode is negative with respect to its cathode) and no current flows through it. Consequently, in the negative half-cycles of the supply, no current flows in load resistor, *R*, and no voltage appears across it. Hence the current on the d.c. side of the rectifier flows in one direction only, that is, it is **unidirectional.**

The value of the d.c. voltage, $V_{d.c.}$, across the load is calculated from the equation

$$V_{d.c.} = \frac{V_m}{\pi} = 0.318V_m = 0.45V_s \qquad (16.1)$$

where V_m is the maximum value of the a.c. supply voltage, and V_s is the r.m.s. value of the a.c. voltage. The d.c. current in the load is calculated from

$$I_{d.c.} = \frac{V_{d.c.}}{R} \qquad (16.2)$$

where *R* is the resistance of the load in ohms.

Example

A 240-V sinusoidal supply is connected to a single-phase, half-wave rectifier circuit of the type in Figure 16.6(a). Calculate (i) the direct voltage across the load and (ii) the current in a load resistance of 100 ohms. Determine also (iii) the power consumed by the load.

Solution

$V_s = 240$ V (r.m.s.); $R = 100$ ohms.

(i) $V_{d.c.} = 0.45 V_s = 0.45 \times 240 = 108$ V (Ans.)

(ii) $I_{d.c.} = \dfrac{V_{d.c.}}{R} = \dfrac{108}{100} = 1.08$ A (Ans.)

(iii) d.c. power $= 1.08^2 \times 100 = 116.6$ W (Ans.)

16.6 SINGLE-PHASE, FULL-WAVE RECTIFIER CIRCUITS

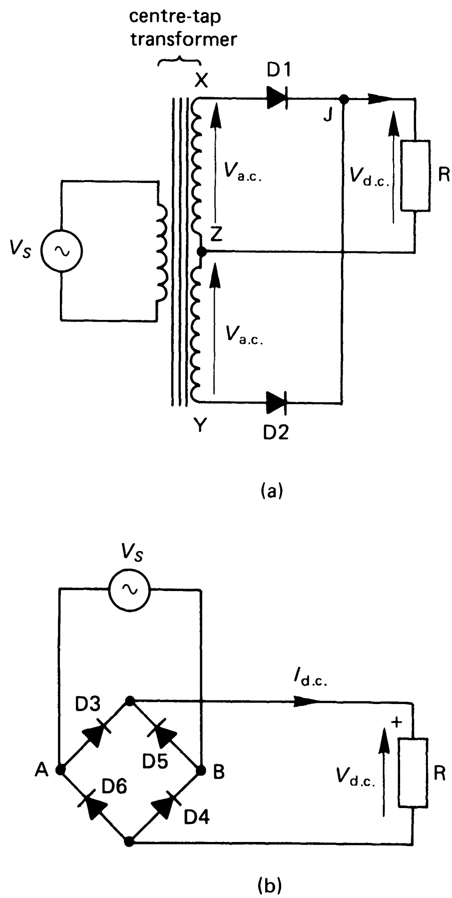
A **full-wave rectifier circuit** utilises both half-cycles of the a.c. supply wave, so that current flows in the d.c. side of the circuit during both half-cycles of the a.c. wave. With the same supply voltage and load resistance, this has the effect of doubling the d.c. voltage when compared with the half-waves case and that the d.c. current is doubled.

The popular forms of single-phase, full-wave rectifier circuit are the *centre-tap (or bi-phase)* circuit and the *bridge* circuit, and are described in the following paragraphs.

Single-phase centre-tap (or bi-phase) rectifier circuit)

The circuit is shown in Figure 16.7(a). It contains two diodes *D*1 and *D*2 which are energised by a transformer with a **centre-tapped secondary winding.** You will recall that it was shown in Chapter 14 that this type of winding provides a two-phase output voltage, the voltage at points X and Y being of opposite phase to one another (centre point Z being the 'common' zero volts line).

In one half-cycle of the supply, the voltage at X is positive with respect to Z on the secondary winding, and Y is negative. This causes D1 to be forward biassed at this time and D2 to be reverse biassed; the net result is that D1 conducts and D2 is cut off. Current therefore flows out of the positive terminal and into the load. In the other half-cycle of the supply, the polarity at X and Y are reversed so that D1 is reverse biassed and D2 is forward biassed. During these half-cycles, current flows through D2 and fig 16.7 *single-phase, full-wave (a) centre-tap circuit and (b) brige circuit*



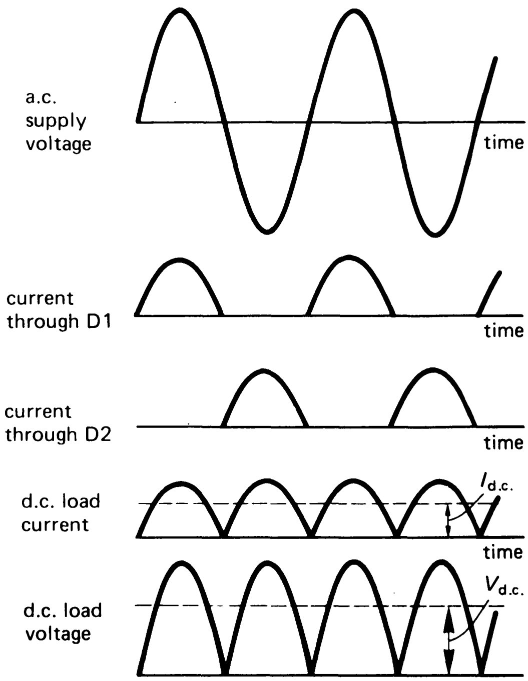
into the load via the positive terminal. That is, current always flows through the load in the same direction during both half-cycles of the a.c. waveform.

The corresponding voltage and current waveforms are shown in Figure 16.8. The current through the diodes is 'summed' at junction J in Figure 16.7 to give a pulsating but unidirectional (d.c.) load current.

The d.c. (average) load voltage, $V_{d.c.}$, is calculated from the equation

$$V_{d.c.} = 2V_m/\pi = 0.636V_m = 0.9V_{a.c.} \qquad (16.3)$$

fig 16.8 *voltage and current waveforms for a single-phase, full-wave rectifier circuit*



where V_m is the maximum value of the a.c. voltage induced in one-half of the secondary winding (see Figure 16.7(a)), and $V_{a.c.}$ as its r.m.s. value. The d.c. load current, $I_{d.c.}$ is given by

$$I_{d.c.} = V_{d.c.}/R \qquad (16.4)$$

where R is the resistance of the load in ohms.

A feature of this circuit is that you can select a direct voltage of your choice simply by obtaining a transformer with the correct voltage transformation ration. Another feature of the circuit is that the transformer enables the secondary circuit to be electrically 'isolated' from the mains supply; this feature is particularly useful in special cases such as medical installations.

A possible disadvantage of this circuit over a number of other rectifier circuits is that it needs a bulky and relatively expensive transformer. This disadvantage usually means that the centre-tap circuit is only used where other lighter and less expensive circuits cannot be used.

Single-phase bridge rectifier circuit

The circuit is shown in Figure 16.7(b); in this case the rectifier circuit contains four diodes D3–D6 which are connected in a 'bridge' formation.

When the a.c. supply causes point A to be positive with respect to point B, diodes D3 and D4 are forward biassed and diodes D5 and D6 are reverse biassed. Current therefore leaves point A and enters the load via diode D3 (D6 cannot conduct since it is reverse biassed at this time). The current returns to point B via the forward-biassed diode D4. That is, the upper terminal of the load resistor R is positive with respect to its lower terminal.

When the a.c. supply makes B positive with respect to A, diodes D5 and D6 are forward biassed and D3 and D4 are reverse biassed. Current enters the load via diode D5 and leaves it via D6. That is, the upper terminal of the load is once more positive.

In this way, current flows through the load resistor, R, *in one direction only* in both half-cycles of the a.c. supply.

The equations for the d.c. voltage and current in the bridge circuit are given by eqns (16.3) and (16.4), respectively, with the exception that $V_{a.c.}$ should be replaced by the r.m.s. value of the supply voltage, V_s.

Advantage of the bridge circuit include its simplicity and the fact that it does not need a transformer. **Disadvantages of the circuit** include the fact that the d.c. voltage is directly related to the a.c. supply voltage, that is the d.c. voltage cannot be altered without connecting a transformer between the a.c. supply and the rectifier. Another disadvantage is that the a.c. and d.c. voltages are not 'isolated' from one another; that is to say, if one of the a.c. supply lines is earthed, then you cannot earth one of the d.c. lines.

16.7 SMOOTHING CIRCUITS

The 'd.c.' voltage waveform from simple rectifier circuits of the type described in section 16.6 is in the form of 'pulses.; whilst this may be satisfactory for a number of applications, it is unsuited to applications which need a 'smooth' d.c. voltage. A number of simple low-cost **smoothing circuits** or **ripple filter circuits** using combinations of L and C are widely used to reduce the current and voltage ripple in the d.c. output of rectifier circuits.

Since the single-phase, half-wave rectifier conducts for only one half of each cycle, *it is not a practical proposition to try to obtain a 'smooth' output from it.* We therefore concentrate on circuits primarily used to smooth the output from a full-wave rectifier circuit.

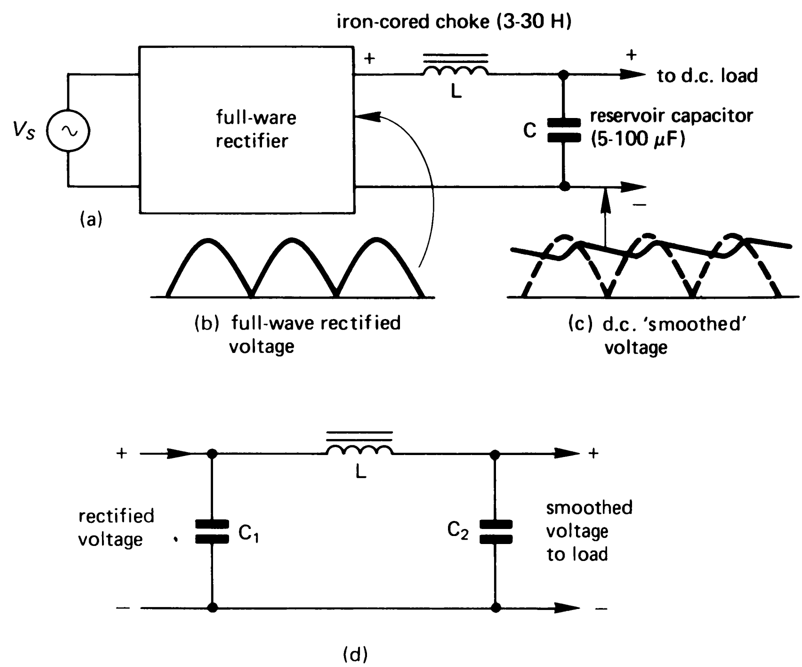
A simple **choke-input filter** is shown in Figure 16.9(a) and you will no doubt recall (see section 11.9 for details) that the function of the inductor or 'choke' is to introduce a high impedance to the 'alternating' ripple current component, and that the function of the reservoir capacitor is to short-circuit the tipple current from the d.c., load resistor. The net result is a significant reduction in the ripple component of the current in the d.c. load (see Figure 16.9(c)).

An improved filter circuit known as a **π-filter** is shown in Figure 16.9(d). This circuit uses two capacitors which are often equal in value and, for a given application, each has about twice the capacitance of C in Figure 16.9(a) (for a given application, the same value of L can be used in both circuits).

16.8 THE THYRISTOR

A **thyristor** is a *multi-layer semiconductor device*. There are two broad categories of thyristor, namely the *reverse blocking thyristor* and the *bidirectional thyristor* or *triac* (the latter being a trade name). Each type is described in this section.

fig 16.9 *(a) choke-input filter circuit and (b) a π-filter*



The reverse blocking thyristor

This is the type of device to which engineers generally refer when they mention the 'thyristor'; it is a four-layer semiconductor device which has the construction shown in Figure 16.10(a).

It has three electrodes, namely an **anode**, a **cathode** and a **gate**. The anode and cathode can be thought of in the same way as the anode and cathode of a diode; that is to say, the 'easy' direction of current flow is from the anode to the cathode. It is the four-layer construction and the 'gate' electrode which account for the difference between the thyristor and the diode.

The operation of the thyristor can be explained in terms of the simplified equivalent circuit in Figure 16.11. The thyristor can be thought of as a diode in series with an electronic switch, *S*, which is controlled by the signal applied to the gate electrode as follows:

> **When the gate current is zero, switch S is open and no current can flow through the thyristor. When gate current flows, switch S closes and current can flow through the thyristor (but only from the anode to the cathode).**

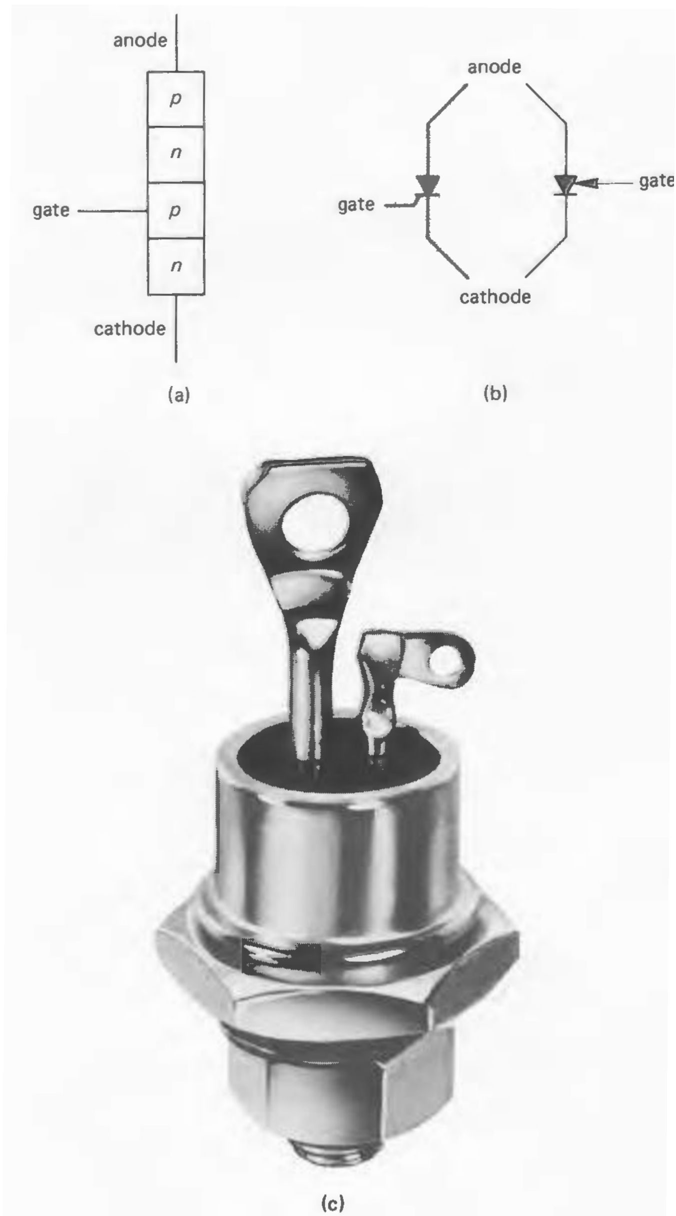
However, once switch S has been closed by the flow of gate current, *it will remain closed (even if the gate current falls to zero) so long as the anode is positive with respect to the cathode.*

The source of the gate current which 'closes' switch S can either be d.c. or a.c. (it must be the positive half-cycle of an a.c. wave!). In fact, the majority of gate 'driver' circuits are special pulse generator which supply a short pulse of current (say about 1 A or so for a few microseconds duration).

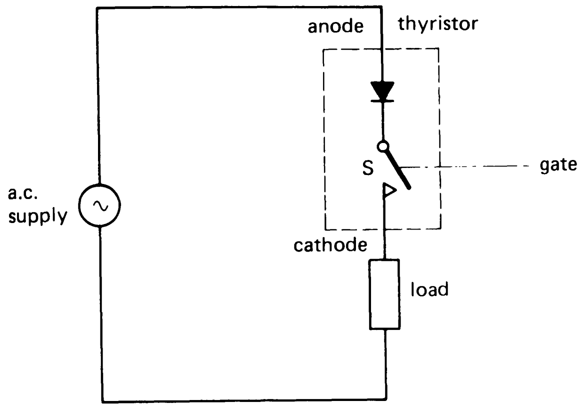
When the reverse blocking thyristor is used in an a.c. circuit, the current flowing into the anode of the thyristor falls to zero once during each cycle, so that the thyristor current automatically becomes zero at this time. When this occurs, switch *S* automatically opens, and current cannot flow again until current is applied to the gate electrode once more at some point in the next positive half-cycle.

That is to say, the point at which the current starts flowing in the load in each *positive half cycle* of the a.c. supply is controlled by the instant of time that the pulse of current is applied to the gate electrode. It is by controlling the point in the positive half cycle at which the gate is energised that you control the phase angle at which the load current is turned on; for this reason, this method of load current control is known as **phase control**.

When the thyristor is used in a d.c. circuit, the anode current does not fall to zero (assuming, that is, that the supply voltage is not switched off) fig 16.10 *(a) sectional diagram of a p-n-p-n reverse blocking thyristor, (b) circuit symbols and (c) a practical thyristor*



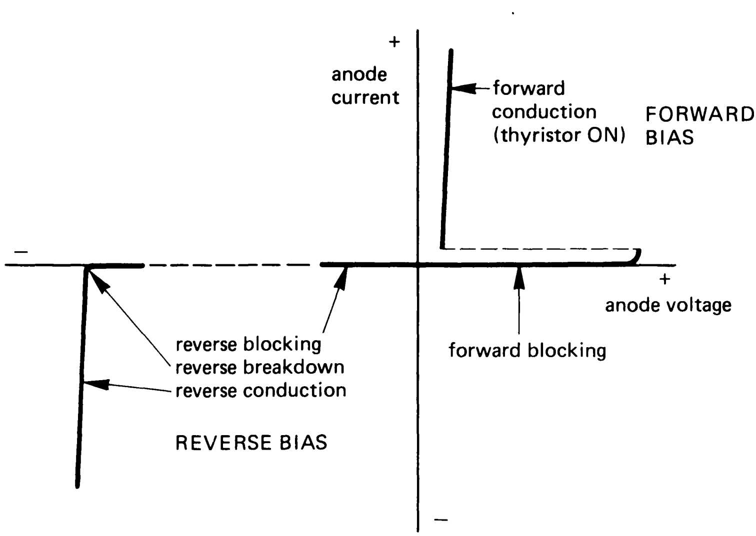
Published by kind permission of RS Components fig 16.11 *simplified operation of the thyristor*



and special current 'commutating' circuits have to be used in order to turn the thyristor off.

The characteristic of a reverse blocking thyristor is shown in Figure 16.12. In the **forward-biassed mode** (when the anode is positive with respect to the cathode) the thyristor has one of two operating conditions, namely

fig 16.12 *characteristic of a reverse blocking thyristor*



1. when the gate current is zero: in this case the thyristor 'blocks' the flow of current, that is, switch S in Figure 16.11 is open. This is known as the **forward blocking mode**;
2. when the thyristor is triggered: in this case the gate current is either flowing or it has just stopped flowing. This causes switch S in Figure 16.11 to close, allowing current to flow through the thyristor. This is known as the **forward conducting mode**.

In the **reverse biassed mode** (when the anode is negative with respect to the cathode) the thyristor blocks the flow of current (and is also known as the **reverse blocking mode**). At a high value of reverse voltage which is well in excess of the voltage rating of the thyristor, reverse breakdown occurs; the thyristor is usually catastrophically damaged if this happens.

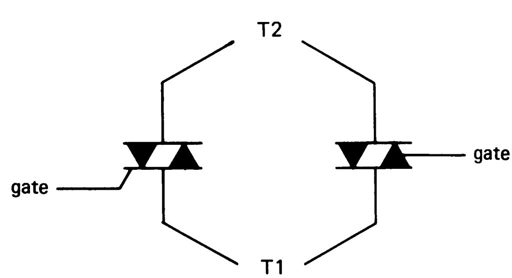
A typical application of a reverse blocking thyristor would be to an industrial speed control system, such as a steel rolling mill or an electric train. The operators control lever is connected to a potentiometer which is in a pulse generator in the gate circuit of the thyristor. Altering the position of the control lever has the effect of altering the 'pulse angle' at which the pulses are produced; in turn, this has the effect of controlling the current in (and therefore the speed of) the motor being controlled.

On the whole, the reverse blocking thyristor is more 'robust' than the triac (see next paragraph) and the thyristor can be used in all electric drives up to the largest that are manufactured.

The triac or bidirectional thyristor

This device is one that can conduct in two 'directions' and, although it has a more complex physical structure and operating mechanism than the reverse blocking thyristor, it still has three electrodes. These are known respectively as T1, T2 and the **gate**; circuit symbols for the triac are shown in Figure 16.13.

fig 16.13 *circuit symbols for a triac or bidirectional thyristor*

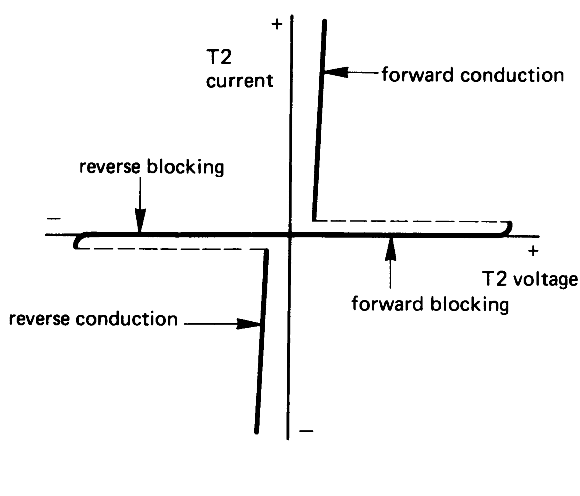


Since the triac can conduct in either direction, terminal T2 may be either positive or negative with respect to terminal T1 when current flow takes place. However, the triac must be triggered in to conduction by the application of a pulse to the gate electrode; the trigger pulse in this case may have either a positive or negative potential with respect to the 'common' electrode T1.

Applications of the triac are limited to power levels up to about a few hundred kilowatts, but this is a situation which is continually changing as technology advances. A popular application of the triac is to a domestic lighting control; the triac and its gate pulse circuitry is small emough to be housed in a standard plaster-depth switch, the knob on the front of the switch controlling the gate pulse circuitry. The knob is connected to a potentiometer which applies 'phase control' to the pulse generator. At switch-on, the triac gate pulses are 'phased back' to between about 150° and 170° so that current only flows for the final 30° to 10° of **each half-cycle** of the supply (remember, the triac conducts for both polarities of *T2*); this results in the lamp being dimly illuminated at switch-on. As the control knob is turned, the gate pulses are gradually 'phased forward' so that the triac fires at an earlier point in each half-cycle of the supply waveform, resulting in increased illumination. Finally, when the gate pulses are phased forward to 0°, the triac conducts continuously and the lamp reaches its full brilliance.

The characteristic of a triac is shown in Figure 16.14. Before gate current is applied (for either polarity of voltage between T2 and T1) the triac blocks the flow of current (shown as **forward blocking** and

fig 16.14 *characteristic of a triac*



reverse blocking in Figure 16.14). After the application of a gate pulse, the triac conducts for either polarity of potential across it (shown as **forward conduction** and **reverse conduction** in Figure 16.14); the triac continues conducting (even when the gate pulse is removed) so long as a p.d. is maintained across it. When the supply current falls to zero (as it does in an a.c. system), the triac reverts to one of its blocking modes until it is triggered again.

16.9 A 'CONTROLLED' THREE-PHASE POWER RECTIFIER

A **three-phase 'controlled' bridge rectifier** circuit is shown in Figure 16.15, and consists of three thyristors and three diodes, the pulses applied to the gates of the thyristors are 'separated' by the equivalent of 120°, so that only one thyristor conducts at a time.

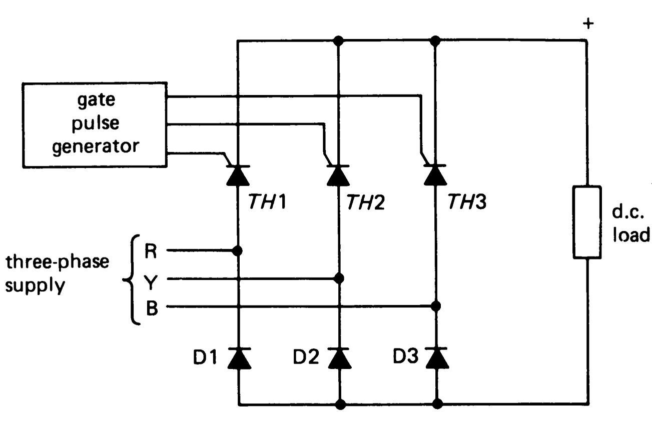
When TH1 is triggered, current passes through it to the positive terminal of the load, the returns to the supply via either D2 or D3. In fact, only one of the diodes conducts at any one time, which one it is depends on which of the Y or B supply lines is at the most negative potential. That is, current will return via D2 for one period of time, and via D3 for another period of time.

When TH2 is triggered into operation, current returns either via D1 or D3; when TH3 conducts, current returns either via D1 or D2.

Once again, the output current and voltage are controlled by *phase control*, that is the phase angle of the gate pulses is altered via the gate pulse generator.

The circuit in Figure 16.13 is known as a **half-controlled rectifier** because one-half of the devices in the circuit are thyristors.

fig 16.15 *a controlled three-phase bridge rectifier*



16.10 INVERTORS

An **invertor** is a circuit which **converts d.c. into a.c.**; for example, the circuit which provides the power from the battery of a bus to its fluorescent lights is an invertor. This circuit takes its d.c. supply from a 12-V battery and converts it into a higher voltage a.c. supply for the fluorescent lights.

For a circuit to be able to 'invert'. all the semiconductor elements in the invertor must be thyristors. For example, if all six devices in the half-controlled 'rectifier' in Figure 16.15 were thyristors, then it could act as a three-phase invertor. In this case the 'd.c. load' would be replaced by a battery or d.c. generator, and the 'three-phase supply' would be replaced by a three-phase load such as a motor or heating element.

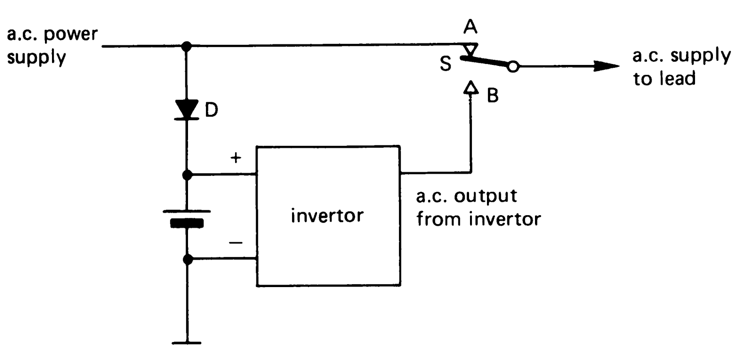
16.11 A STANDBY POWER SUPPLY

A number of installations need a power supply which is 100 per cent reliable. One method of providing such a power supply is shown in Figure 16.16.

Under normal operating conditions, the load is supplied directly from the mains via contact A of the electronic switch S (which would probably by a thyristor circuit). Whilst the main circuit is working normally, diode D trickle-charges the 'standby' battery.

When the power supply fails, the contact of switch S changes to position B, connecting the load to the output of the invertor circuit. Since the invertor is energised by the standby battery, the a.c. power supply to the load is maintained at all times.

fig 16.16 *one form of standby power supply*



SELF-TEST QUESTIONS

1. What is meant by a p-type semiconductor and an n-type semiconductor? Also explain the meaning of the expressions 'majority charge carrier' and 'minority charge carrier'.
2. Draw and explain the characteristic of a p–n junction diode. What is meant by 'forward bias' and 'reverse bias' in connection with a p–n diode?
3. In what respect do 'half-wave' and 'full-wave' rectifiers differ from one another?
4. A rectifier circuit is supplied from a 100-V r.m.s. supply. For (i) a half-wave and (ii) a full-wave rectifier circuit, calculate the no-load d.c. output voltage. Determine also the load current in each case for a 100-ohm load.
5. Explain the purpose of a 'smoothing' circuit or 'ripple' filter. Draw a circuit diagram for each of two such circuits and explain how they work.
6. What is a thyristor? Draw and explain the shape of the characteristic for (i) a reverse blocking thyristor and (ii) a bidirectional thyristor. Discuss applications of the two types of device.
7. Explain what is meant by an 'invertor'. Where might an invertor be used?

SUMMARY OF IMPORTANT FACTS

A **semiconductor** is a material whose resistivity is mid-way between that of a conductor and that of an insulator; popular semiconductor materials include silicon and germanium. The two main types of semiconductor are **n-type** and **p-type**; n-type has **mobile electrons** in its structure whilst p-type has **mobile holes.** In an n-type material, *electrons* are the **majority charge carriers**, and p-type *holes* are the majority charge carriers.

A **diode** permits current to flow without much resistance when the p-type anode is positive with respect to the n-type **cathode.** In this mode it is said to be **forward biassed.** The diode is **reverse biassed** when the anode is negative with respect to the cathode; in this mode the diode **blocks** the flow of current through it. **Reverse breakdown** occurs if the reverse bias voltage exceeds the reverse breakdown voltage of the diode; the diode can be damaged if the current is not limited in value when reverse breakdown occurs. Diodes designed to work in the reverse breakdown mode are known as **Zener diodes.**

A **rectifier circuit** converts an a.c. supply into d.c. The circuit may either be *single-phase* or *poly-phase*, and may either be *half-wave* or *full-*

wave. The 'ripple' in the output voltage or current from a rectifier can be reduced by means of a **smoothing circuit** or a **ripple filter**.

An **invertor** is the opposite of a rectifier, and converts d.c. into a.c.

A **thyristor** is a multi-layer semiconductor device. A **reverse-blocking thyristor** (which is the type referred to when thyristos are discussed) allows you to control the flow of current from the anode to the cathode by means of a signal applied to its *gate electrode*. A **bidirectional thyristor** (often called a **triac**) allows you to control the flow of current through it in either direction by means of a signal applied to its gate electrode. The gate signal of both types of thyristor may be either d.c. or a.c., or it may be a pulse.

SOLUTIONS TO NUMERICAL PROBLEMS

Chapter 1

4. 0.2 A
5. 100 S
6. non-linear
7. 1 kHz ; 10 μ s
8. 400 C; 400 W; 8000 J

Chapter 2

5. 0.1 Ω

Chapter 3

3. 5.51 Ω
4. 0.181 S
5. 86.4 Ω
6. (i) 40 Ω, (ii) 4 Ω
   (iii) series = 400 V, parallel = 40 V
   (iv) series = 4kW, parallel = 400 W
7. I_1 = 0.136A; I_2 = 0.318A; I + 1_2 = 0.454A

Chapter 4

6. 33.33 cm from one end and 66.67 cm from the other end.

Chapter 5

1. 144 MJ; 40 kWh

Chapter 6

2. 1 kV
4. (i) 10 V, (ii) 100 kV
6. 3.54 nF
7. (i) 12 μF ; (ii) 1.091 μF
8. (i) 120 μC ; 10.91 μC ; (ii) 600 μJ; 54.55 μJ
9. (i) 0.01 s ; (ii) 0.01 A ; (iii) 0.007 s ; (iv) 0.05 s ;
   (v) 10 V , 0.0 A

Chapter 7

2. 25 000 At; 53 052 At/m ; 0.209 mWb ; 0.209 T
5. 1293 At/m ; 2328 At
8. 125 J
9. (i) 0.5 s ; (ii) zero; 1.0 A ; (iii) 0.35 s (iv) 2.5 s ;
   (v) 2.5 J

Chapter 8

1. 200 Hz
2. (i) 1.047 rad ; (ii) 2.094 rad ; (iii) 91.67°
   (iv) 263.6°

Chapter 9

3. 0.372 T
4. 960 Nm

Chapter 10

1. 6.58 ms ; 4 MHz
2. (i) 11.79 A ; (ii) 0.174 A, 10.21 A , −4.03 A,
   −2.05 A, 9.92 A, 10.72 A , −8.92 A
3. 76.44 V ; 84.84 V
5. (i) 323.9 V leading the 150-V wave by 25.88° ;
   (ii) either 141.7 V lagging the 150-V wave by 86.5°
   (if the 200-V wave is subtracted), or 141.7 V
   leading the 200-V wave by 93.48° if the 150-V
   wave is subtracted)
6. (i) 12 500 VA ; (ii) 36.87°, 0.8 ; (iii) 7500 VAr

Chapter 11

1. 0.833 A ; 288Ω
2. Circuit A: (i) 0.667 A ; (ii) I lags behind V_s by 90° ;
   Circuit B: (i) 1.25 A ; (ii) I leads V_s by 90°
3. (i) 157.1 Ω, 0.636 A ; (ii) 188.5 Ω , 0.531 A;
   (iii) 314.2 Ω, 0.0318 A
4. 10 μF
5. (i) 14.71 Ω ; (ii) 680 Ω

Chapter 12

1. 628.3 Ω ; 79.6 Ω ; 557.7 Ω
2. 17.93 mA ; V_R = 1.793 V , V_L = 11.27 V ,
   V_C = 1.43 V ; P = 32.15 mW, power factor = 0.179
   (I lagging V_s)
3. (i) I_R = 0.1 A , I_L = 15.9 mA, I_C = 0.126 A ;
   (ii) 0.149 A ; (iii) 47.75° (I leading V_s), 0.672;
   (iv) 1.49 VA, 1W, 1.105 VAr
5. 0.507 μF; 1.5 A
6. 31.42

Chapter 13

4. 19.05 kV, 173.2V
6. (i) 21.87 A, 21.87 A; (ii) 21.87 A, 12.63 A
7. 36.87° ; 125 kVA ; 75 VAr

Chapter 14

3. 1 H
4. 20 V ; 0.5 A ; 0.1 A
7. 9.5 W

Chapter 15

2. 5 rev/s or 300 rev/min
4. 1425 rev/min

Chapter 16

4. (i) 45 V, 0.45 A; (ii) 90 V, 0.9 A

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