TOMSK POLYTECHNIC UNIVERSITY

D.S. Nikitin, A.S. Saigash,
A.A. Sivkov, I.A. Rakhmatullin

RELIABILITY OF POWER SUPPLY

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Nikitin D.S.

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This book provides state of the art tools for analyzing power supply system's reliability. This material will be useful for those who need to use these tools as well as those who want to do further research. They will be able to use this knowledge to make trade-offs between reliability, cost, environmental issues and other factors as needed.

The book is intended for Master in the direction of 13.04.02 "Electric Power and Electrical Engineering" (Program name: "Electric Power Generation and Transportation"), studying reliability of power supply. It may be of interest to master and postgraduate students of department of electric power and electrical engineering.

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INTRODUCTION

Institutional and individual customers have increasingly better and broader awareness of products (and services) and are increasingly making smarter choices in their purchases. In fact, because society as a whole continues to become more knowledgeable of product performance, quality, reliability, and cost, these attributes are considered to be market differentiators.

People are responsible for designing, manufacturing, testing, maintaining, and disposing of the products that we use in daily life. Perhaps you may agree with Neville Lewis (2003), who wrote, "*Systems do not fail, parts and materials do not fail – people fail!*" It is the responsibility of people to have the knowledge and skills to develop products that function in an acceptably reliable manner. These concepts highlight the purpose of this book: to provide the understanding and methodologies to efficiently and cost effectively develop reliable power supply systems and to assess and manage the operational availability of complex products, processes, and systems.

In general, reliability needs to be built, as far as possible, at the design or planning stage of a product or a system. Corrective actions to fix the reliability are generally more inconvenient and expensive. So far as the power grid is concerned, it is emerging as a highly complex system with heavy penetration of renewable energy sources, central and distributed energy storage and massive deployment of distributed communication and computational technologies allowing smarter utilization of resources. In addition, as the shape of the grid unfolds, there will be higher uncertainty in the planning and operation of these systems. As the complexity and uncertainty increase, the potential for possible failures with a significant effect on industrial complexes and society can increase drastically. In these circumstances maintaining the grid reliability and economy will be a very important objective and will be a challenge for those involved. Although many activities are involved in meeting these goals, educating the engineers in the discipline of reliability provides them with tools of analysis, trade-off and mental models for thinking. Reliability cannot be left to the goodwill of those designing or planning systems nor as a by-product of these processes but must be engineered into the grid and its subsystems in a systematic and deliberate manner. An important step in this process is to model, analyze and predict the effect of design, planning and operating decisions on the reliability of the system. So there is a need for educational tools covering the spectrum of reliability modeling and evaluation tools needed for this emerging complex cyber-physical system.

This book provides state of the art tools for analyzing power supply system's reliability. This material will be useful for those who need to use these tools as well as those who want to do further research. They will be able to use this knowledge to make trade-offs between reliability, cost, environmental issues and other factors as needed.

To achieve this objective, we provide a strong background in general reliability that cultivates a deep understanding that can be used to develop appropriate tools as needed. We then use this foundation to build the tools for analyzing the power systems. The book can thus be used both by those who want to understand the tools of reliability analysis and those who want expertise in power system reliability.

Chapter 1
PROBABILITY THEORY

Knowledge of probability concepts is essential for power systems reliability modeling and analysis. These serve as fundamental ideas for the understanding of random phenomenon in reliability engineering problems. Probability theory is used to describe or model random occurrences in systems that behave according to probabilistic laws. Basic probability theory is reviewed in this chapter, with emphasis on application to power systems.

1.1. State Space and Event

Sample space or state space, usually denoted by S, is a collection of all possible outcomes of a random phenomenon. Consider the following examples:

- Outcome of tossing a coin once: $S = \{\text{Head, Tail}\}$.
- Outcome of rolling a dice: $S = \{1, 2, 3, 4, 5, 6\}$.
- Status of a generator: $S = \{\text{Up, Down}\}$.
- Status of two transmission lines: $S = \{(1U, 2U), (1U, 2D), (1D, 2U), (1D, 2D)\}$, where U denotes that a transmission line is working (i.e., in the up state) and D denotes that the transmission line has failed (and is in the down state), as shown in Fig. 1.1.

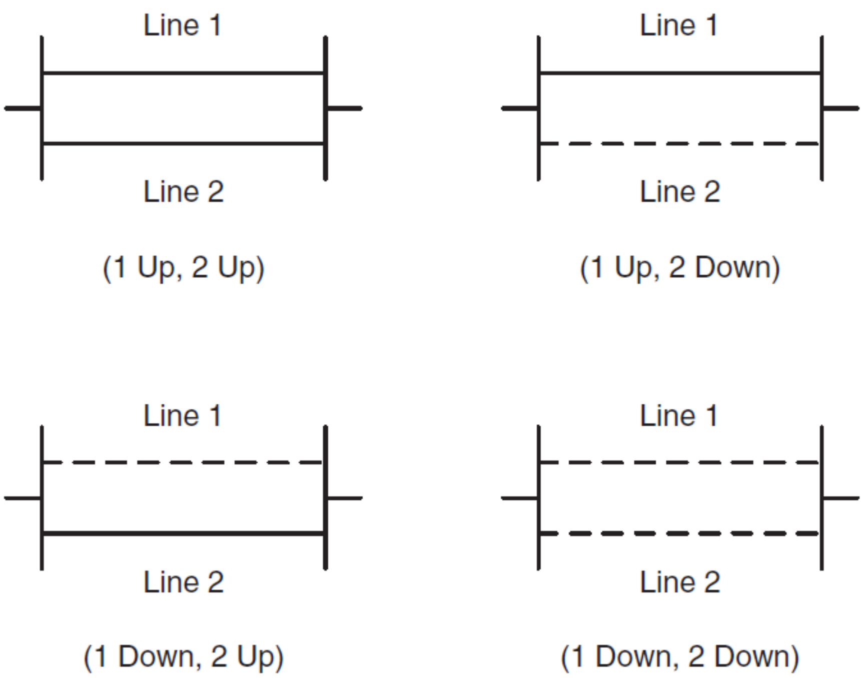

Fig. 1.1. Status of two transmission lines

In power system applications, we may want to focus our analysis on certain scenarios in the state space. For instance, in the example of the two transmission lines, we may be concerned only with the situation where at least one transmission line is working. This leads to what we call an **event**. An event is defined as a set of outcomes of a random phenomenon. It is a subset of a sample space.

For example,

- Rolling a dice yields a “1”: $E = \{1\}$;
- A generator has failed: $E = \{\text{Down}\}$;
- At least one transmission line is working, $E = \{(1U, 2U), (1U, 2D), (1D, 2U)\}$;
- Only one transmission line has failed, $E = \{(1U, 2D), (1D, 2U)\}$ as shown in Fig. 1.2.

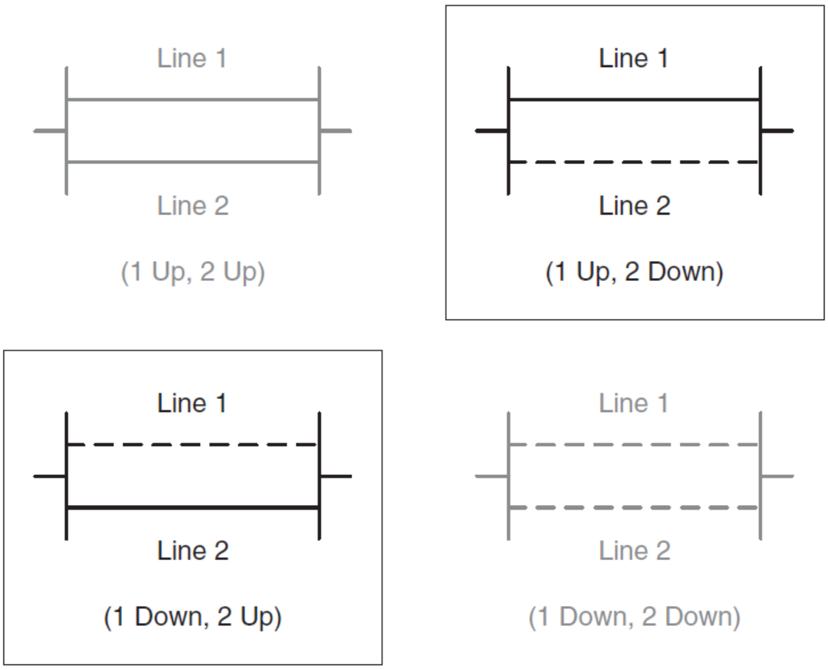

Fig. 1.2. The event that only one transmission line has failed

For any two events E_1 and E_2 in the state space S, the new event that contains outcomes from either E_1 or E_2 or both is called the **union of the events**, denoted by $E_1 \cup E_2$. For example, if E_1 is an event that at least one transmission line is up, $E_1 = \{(1U, 2U), (1U, 2D), (1D, 2U)\}$, and E_2 is an event that at least one transmission line is down, $E_2 = \{(1U, 2D), (1D, 2U), (1D, 2D)\}$, then, union of event E_1 and E_2 is

$$E_1 \cup E_2 = \{(1U, 2U), (1U, 2D), (1D, 2U), (1D, 2D)\}.$$

For any two events E_1 and E_2 in the state space S, the new event that contains outcomes from both E_1 and E_2 is called the **intersection of the**

events, denoted by $E_1 \cap E_2$. For example, the intersection of events E_1 and E_2 in the example of the two transmission lines is

$$E_1 \cap E_2 = \{(1U, 2D), (1D, 2U)\}.$$

There are cases where some events do not have any common outcome, i.e., the intersection of these events does not contain any outcome. Consider the event that both transmission lines are down, $E_3 = \{(1D, 2D)\}$; then the intersection of events E_1 and E_3 has no outcome. This null event is denoted by an empty set, $\varnothing$.

When the intersection of two events creates an empty set, the two events are said to be **mutually exclusive** or **disjoint events**. For example, if E_4 is an event that the two transmission lines are up, i.e., $E_4 = \{(1U, 2U)\}$, and E_5 is an event that two transmission lines are down, i.e., $E_5 = \{(1D, 2D)\}$, then it is impossible for E_4 and E_5 to happen together, and the intersection of E_4 and E_5 is a null set: $E_4 \cap E_5 = \varnothing$. We can conclude that E_4 and E_5 are mutually exclusive, and this is shown by the Venn diagram in Fig. 1.3.

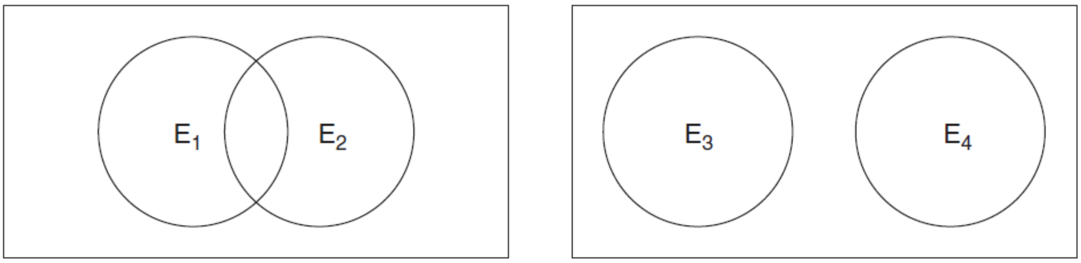

Fig. 1.3. Venn diagram between inclusive events and mutually exclusive events

The concept of union and intersection of events can be extended to include more than two events. If $E_1, E_2, \ldots, E_n$ are events in the state space S, then union of these events, denoted by $\bigcup_{i=1}^{n} E_i$, is the event that contains outcomes from any of the events $E_1, E_2, \ldots, E_n$. The intersection of these events, which is defined in a similar way, denoted by $\bigcap_{i=1}^{n} E_i$, is an event that selects only outcome(s) that is (are) common in all the events $E_1, E_2, \ldots, E_n$. The same concept also applies when n goes to infinity.

Example 1.1. Consider a system of three generators connected to a load, as shown in Fig. 1.4. A generator can assume two statuses, either working in the up state or failure in the down state.

Let us find the possible outcomes (state space) of the status of generators in this problem, the event that the one generator is working and the event that satisfies any of the following criteria:

- One generator is working;
- Three generators failed;
- The third generator failed.

And lastly, find the event that satisfies all the above criteria. The state space of this problem is shown in Fig. 1.5.

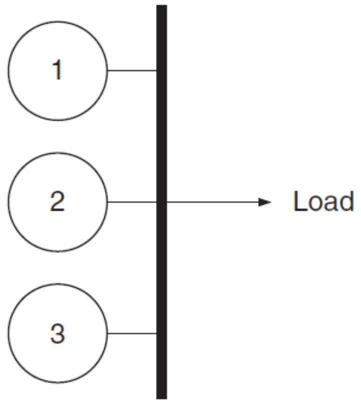

Fig. 1.4. Three-generator system representation in Example 1.1

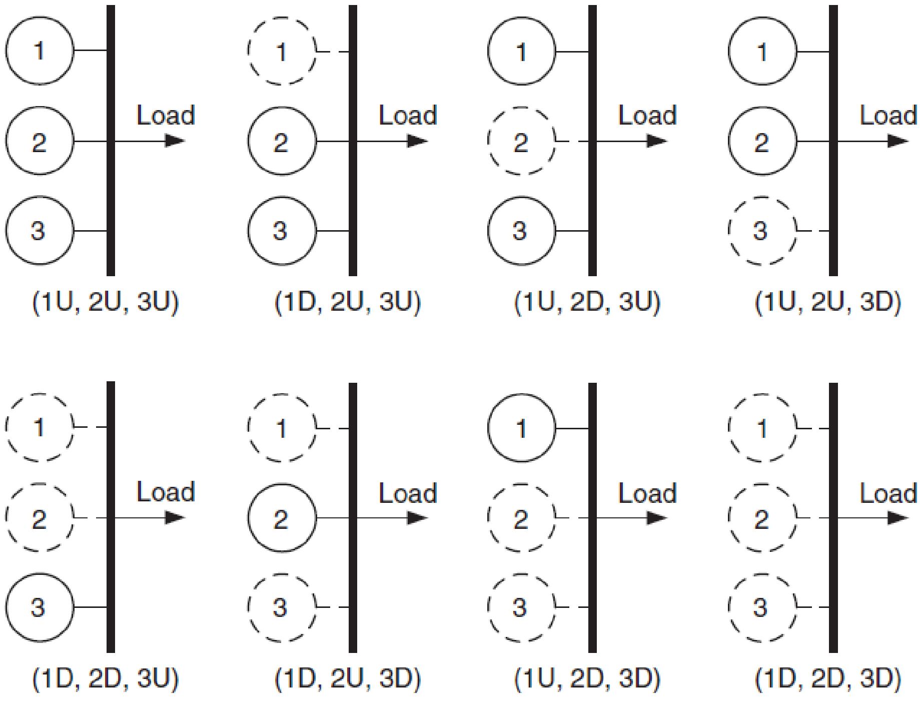

Fig. 1.5. State space of three-generator system in Example 1.1

Let S be a state space of a status of three generating units. Then

$$S = \{(1U, 2U, 3U), (1U, 2U, 3D), (1U, 2D, 3U), (1D, 2U, 3U), (1D, 2U, 3D), (1D, 2D, 3U), (1U, 2D, 3D), (1D, 2D, 3D)\},$$

where U denotes a unit that is working and D denote a unit that is failed. The state space, S, shows the possible outcomes of this problem.

Let E_1 be an event that one generating unit is up; then$E_1 = \{(1D, 2U, 3D), (1D, 2D, 3U), (1U, 2D, 3D)\}$.

Let E_2 be an event that three generating units are down then $E_2 = \{(1D, 2D, 3D)\}$, and E_3 be an event that the third unit is down, then

$$E_3 = \{(1U, 2U, 3D), (1D, 2U, 3D), (1U, 2D, 3D), (1D, 2D, 3D)\}.$$

The event that satisfies any one of the three criteria is given as the union of E_1, E_2, E_3 or $E_1 \cup E_2 \cup E_3 = \{(1U, 2U, 3D), (1D, 2U, 3D), (1D, 2D, 3U), (1U, 2D, 3D), (1D, 2D, 3D)\}$.

The event that satisfies all of the three criteria is given as the intersection of E_1, E_2, E_3 or, $E_1 \cap E_2 \cap E_3 = \{(1D, 2D, 3D)\}$.

Let us also find the event, denoted by E_3^c, that the third unit is up; then, $E_3^c = \{(1U, 2U, 3U), (1U, 2D, 3U), (1D, 2U, 3U), (1D, 2D, 3U)\}$.

Note that this event E_3^c contains all possible outcomes in the state space that are not in the event E_3, which describes the outcomes that the third unit is down.

We can now define a new event, denoted by E^c, a **complement of an event** E, to be the set of outcomes that are in the state space, S, but not included in an event E. This means that E^c will occur only when E does not occur. This also implies that E and E^c are mutually exclusive ($E \cap E^c = \varnothing$) and that the union of E and E^c yields the state space, $E \cup E^c = S$.

1.2. Probability Measure and Related Rules

Probability is defined as a quantitative measure of an event E in a state space S. This measure is denoted by $P(E)$, called probability of an event E and defined to satisfy the following properties.

1. $0 \le P(E) \le 1$.
2. $P(S) = 1$.
3. If E_1, E_2, …, E_i, are mutually exclusive events in S, then $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$.

For engineering applications, probability can be interpreted as a measure of how frequent an event will occur in a long-run experiment. Intuitively, this measure of an event should be proportional to a number of times that an outcome in the event occurs divided by the total number of experiments. For example,

- If a coin is tossed once, what is the probability of the outcome being a Head? If the coin is fair, i.e., there is equal chance to appear as Head or Tail,

$$P(\{\text{Head}\}) = P(\{\text{Tail}\}).$$

Since $S = \{\text{Head, Tail}\}$ and $P(S) = 1$, the probability of being Head is

$$P(\{\text{Head}\}) = 1/2.$$

- If a dice is rolled once, what is the probability of the outcome being "1"? If the dice is fair, then each number has the same chance to appear,

$$P(\{1\}) = P(\{2\}) = P(\{3\}) = P(\{4\}) = P(\{5\}) = P(\{6\}).$$

Using the 2nd and 3rd properties, we have

$$P(\{i\}) = 1\ 6,\ i \in \{1, 2,\ldots, 6\}.$$

The probability of rolling "1" is $P(\{1\}) = 1/6$.

- If a dice is rolled once, what is the probability of the outcome being an odd number?

The event of being odd number is $\{1, 3, 5\}$. Since these events are mutually exclusive, using the 3rd property, we have

$$P(\{1, 3, 5\}) = P(\{1\}) + P(\{2\}) + P(\{3\}) = 1/2.$$

For any set, E, the union of E and its complement, E^c, yields the state space, $E \cup E^c = S$. Since E and Ec are always mutually exclusive, by properties 2 and 3, we have $P(E \cup E^c) = P(E) + P(E^c) = P(S) = 1$. This implies that $P(E^c) = 1 - P(E)$. For some application problems, it is easier to calculate probability of a complement of an event than the probability of an event itself. We can use this property, called **complementation rule**, to help us find a probability of an event.

We can also derive another important rule, called **addition rule**, to find a probability of union of two events. If the two events are mutually exclusive, we arrive at the same result as the 3rd property. We now consider the case when the two events are not disjoint and introduce the concept by the following example.

Example 1.2. Consider a system of two transmission lines. If E_1 is an event that at least one transmission line is up, $E_1 = \{(1U, 2U), (1U, 2D), (1D, 2U)\}$, E_2 is an event that at least one transmission line is down, $E_2 = \{(1U, 2D), (1D, 2U), (1D, 2D)\}$; then, union of event E_1 and E_2 is

$$E_1 \cup E_2 = \{(1U, 2U), (1U, 2D), (1D, 2U), (1D, 2D)\},$$

and the intersection of event E_1 and E_2 is

$$E_1 \cap E_2 = \{(1U, 2D), (1D, 2U)\}.$$

If we add probability of E_1 to probability of E_2, we have $P(E_1) + P(E_2) = P(\{(1U, 2U), \underline{(1U, 2D), (1D, 2U), (1U, 2D), (1D, 2U)}, (1D, 2D)\})$. Note that the events $\{(1U, 2D), (1D, 2U)\}$ appear twice. Rearranging the events, we have

$$P(E_1) + P(E_2) = P(\{(1U, 2U), (1U, 2D), (1D, 2U), (1D, 2D)\}) + \\ + P(\{(1D, 2U), (1U, 2D)\}).$$

This means that $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$.

For any two events, we can calculate the probability of union of two events as follows.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

Note that if E_1 and E_2 are mutually exclusive, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ follows from the 3rd condition. This implies $P(E_1 \cap E_2) = 0$, and we can conclude that $P(\varnothing) = 0$ since $E_1 \cap E_2 = \varnothing$.

In general, the probability of union of n events can be found as shown below:

$$P\left(E_1 \cup E_2 \cup \ldots \cup E_n\right) = \sum_i P\left(E_1\right) - \sum_{i<j} P\left(E_i \cap E_j\right) +$$
$$+ \sum_{i<j<k} P\left(E_i \cap E_j \cap E_k\right) - \ldots + (-1)^{n-1} P\left(E_1 \cap E_2 \cap \ldots \cap E_n\right);$$
$$P(E_1 \cup E_2 \cup \ldots \cup E_n) = \sum \mathrm{i}\, P(E_i) - \sum \mathrm{i} < \mathrm{j}.$$

Consider again a system of two transmission lines. Suppose that the first transmission line fails, and we may wish to know the probability of the second line failing. Let E_1 be an event that the first transmission line fails, $E_1 = \{(1D, 2U), (1D, 2D)\}$, and $E_2|E_1$ be an event that the second transmission line fails, given that one transmission line has already failed. The event that the second transmission line fails is $E_2 = \{(1U, 2D), (1D, 2D)\}$. However, $(1U, 2D)$ cannot occur in this problem since the first transmission line has already failed. We are interested in calculating the **conditional probability** that the event E_2 occurs, given that the event E_1 has already occurred. We denote this probability as $P(E_2|E_1)$.

Intuitively, when E_1 has already occurred, we can only consider the states with occurrences E_1 in the state space. This means that the state space has shrunk to become the set E_1, and the events in E_2 will have to be in common with the events in E_1. This leads to the following formula for conditional probability:

$$P(E_2 \mid E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)}.$$

This formula is properly defined only when $P(E_1) > 0$.

Example 1.3. Consider the same system as in Example 1.2. The state space of this problem is $S = \{(1U, 2U), (1U, 2D), (1D, 2U), (1D, 2D)\}$. The probability of each event is given as shown in Table 1.1. Let us calculate the probability that the second transmission line fails given that the first transmission line has already failed.

Table 1.1

Probability of an Event in Example 1.3

Event	**Probability**
(1*U*, 2*U*)	0.81
(1*U*, 2*D*)	0.09
(1*D*, 2*U*)	0.09
(1*D*, 2*D*)	0.01

Let E_1 be an event that the first transmission line fails, $E_1 = \{(1D, 2U), (1D, 2D)\}$, and E_2 be an event that the second transmission line fails,

$E_2 = \{(1U, 2D), (1D, 2D)\}$. We need to calculate $P(E_2|E_1)$. Fig. 1.6 shows the state space of this example.

Since $E_2 \cap E_1 = \{(1D, 2D)\}$, $P(E_2 \cap E_1) = 0.01$ and $P(E_1) = 0.09 + 0.01 = 0.1$, then we have

$$P(E_2|E_1) = P(E_2 \cap E_1)\ P(E_1) = 0.01/0.1 = 0.1.$$

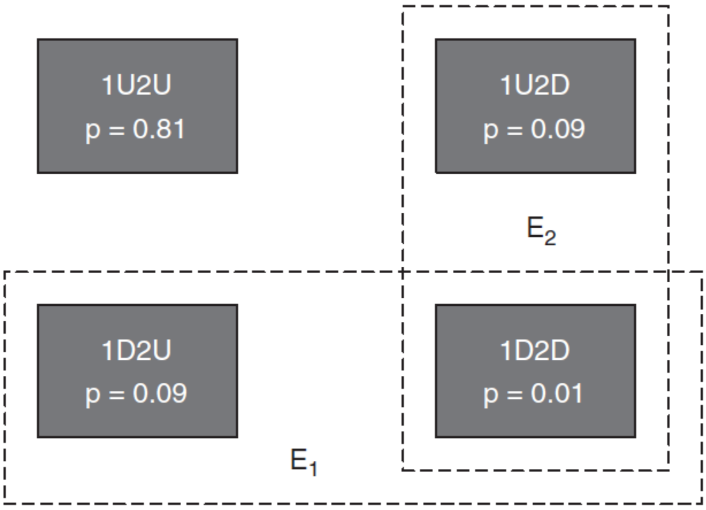

Fig. 1.6. State space of two-transmission line in Example 1.3

This conditional probability rule is in fact a very powerful technique to help us calculate the probability of an event in a state space. To illustrate how we can apply this technique, let us first divide the state space into two mutually exclusive sets, $S = B_1 \cup B_2$, $B_1 \cap B_2 = \varnothing$. An event E in the state space has to be inclusive with either event B_1 or B_2 and can be described as $E = (E \cap B_1) \cup (E \cap B_2)$.

Using addition rule,

$$P(E) = P(E \cap B_1) + P(E \cap B_2) - P(E \cap E \cap B_1 \cap B_2).$$

Since $B_1 \cap B_2 = \varnothing$, it follows that $P(E \cap E \cap B_1 \cap B_2) = 0$. From conditional probability rule

$$P(E \cap B_1) = P(E|B_1) \times P(B_1),$$
$$P(E \cap B_2) = P(E|B_2) \times P(B_2).$$

Then,

$$P(E) = P(E|B_1) \times P(B_1) + P(E|B_2) \times P(B_2).$$

This expression can be interpreted as a weighted average of the conditional probability of E on a given event when the weight is the probability of the event in which E is conditioned to occur.

For n mutually exclusive events, B_i that $\bigcup_{i=1}^{n} B_i = S$. We can find a probability of any event E from the following **Bayes' rule**

$$P(E) = \sum_{i=1}^{n} P(E \mid B_i) \times P(B_i).$$

For example, if the state space is divided into five mutually exclusive events, B_1, B_2,…, B_5, then the probability of an event E can be found from conditional probability of event E on each of the disjoint events and can be shown in graphical form in Fig. 1.7.

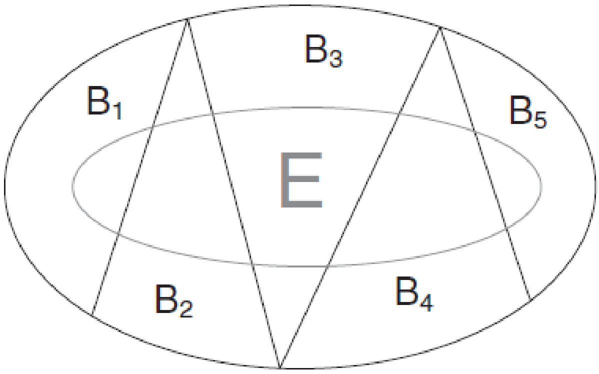

Fig. 1.7. Graphical representation of Bayes' rule

The conditional probability rule can also be used to calculate the probability of an intersection of events. Recall that for any two events,

$$P(E_2 \cap E_1) = P(E_2|E_1) \times P(E_1).$$

We now define another important property of two events called **independent**. The two events are independent, if and only if

$$P(E_2 \cap E_1) = P(E_2) \times P(E_1).$$

This also implies that $P(E_2|E_1) = P(E_2)$, which means that the probability that E_2 will occur does not depend on whether E_1 has already occurred or not. This property helps us to calculate the probability of the intersection of two events by simply multiplying the probabilities of the two events. We call this **multiplication rule**. It should be noted that the independence property is different from mutually exclusive property of two events and cannot be described using a Venn diagram.

Example 1.4. Consider the same system as in Example 1.3. Determine whether or not the event of failure of the first transmission line and the event of failure of the second transmission line are independent. Let E_1 and E_2 be events that the first transmission line and the second transmission line fail, then

$$E_1 = \{(1D, 2U), (1D, 2D)\},$$
$$E_2 = \{(1U, 2D), (1D, 2D)\}.$$

We can find $P(E_1) = 0.09 + 0.01 = 0.1$, and $P(E_2) = 0.09 + 0.01 = 0.1$.

Since $E_2 \cap E_1 = \{(1D, 2D)\}$, $P(E_2 \cap E_1) = 0.01$. Then, we have $P(E_2 \cap E_1) = $ $= P(E_2) \times P(E_1) = 0.01$.

This means that the event that the first transmission line fails and the event that the second transmission line fails in this problem are independent.

Now, if the probability of this state space has changed and is given by the Table 1.2, determine if the two events are still independent or not.

Table 1.2

Probability of an Event in Example 1.4

Event	Probability
$(1U, 2U)$	0.80
$(1U, 2D)$	0.10
$(1D, 2U)$	0.09
$(1D, 2D)$	0.01

In this case, $P(E_1) = 0.09 + 0.01 = 0.1$, $P(E_2) = 0.10 + 0.01 = 0.11$ and $P(E \cap E_1) = 0.01$. Then,

$$P(E_2 \cap E_1) = 0.01 \neq P(E_2) \times P(E_1) = 0.011.$$

This shows that the two events are not independent. This implies that the event that the first transmission line fails is dependent on the event that the second transmission line fails and vice versa.

From Example 1.4, since a transmission line can assume either up or down state, i.e., the state space of one transmission line is $S = \{U, D\}$. This means that if the failure probability of a transmission line is $P(D) = 0.1$, the probability that a transmission line will work is $P(U) = 1 - P(D) = 0.9$ according to the complementary rule

For a system of two identical transmission lines, let E be an event that the first transmission line is working and the second transmission line fails, $E = \{(1U, 2D)\}$. If the working or failure statuses of the two transmission lines are independent, then the probability of this event can be found from multiplication rule,

$$P(E) = P(1U \cap 2D) = P(1U) \times P(2D) = 0.09,$$

which is the same as shown in Example 1.3.

Example 1.5. Consider the same system of three generators connected to a load as shown in Example 1.1. Assume that each generator has 50 MW capacity with the probability of failure of 0.01, and each generator fails independently. Let us find the probability that the system will supply 0, 50, 100 and 150 MW to the load and the probability of loss of load when the load is 50, 100 or 150 MW with 0.20, 0.75 and 0.05 probability accordingly

Let us first define the events as follows:

E_1 Event that the system will supply 0 MW.

E_2 Event that the system will supply 50 MW.

E_3 Event that the system will supply 100 MW.

E_4 Event that the system will supply 150 MW.

These events are given in the following:

$E_1 = \{(1D, 2D, 3D)\}$.

$E_2 = \{(1U, 2D, 3D), (1D, 2U, 3D), (1D, 2D, 3U)\}$.

$E_3 = \{(1U, 2U, 3D), (1D, 2U, 3U), (1U, 2D, 3U)\}$.

$E_4 = \{(1U, 2U, 3U)\}$.

In order to calculate the probability of each event, it is important to note that each generator has failure probability of 0.01. Since all generators fail independently, we can use multiplication rule:

$$P(E_1) = P(1D \cap 2D \cap 3D) = P(1D) \times P(2D) \times P(3D) = 0.00001.$$

$$\begin{gathered} P(E_2) = P\{(1U, 2D, 3D) \cup (1D, 2U, 3D) \cup (1D, 2D, 3U)\} = \\ = P(1U \cap 2D \cap 3D) + P(1D \cap 2U \cap 3D) + P(1D \cap 2D \cap 3U) = \\ = \{P(1U) \times P(2D) \times P(3D)\} + \{P(1D) \times P(2U) \times P(3D)\} + \\ + \{P(1D) \times P(2D) \times P(3U)\} = 0.000297. \end{gathered}$$

$$\begin{gathered} P(E_3) = P\{(1U, 2U, 3D) \cup (1D, 2U, 3U) \cup (1U, 2D, 3U)\} = \\ = P(1U \cap 2U \cap 3D) + P(1D \cap 2U \cap 3U) + P(1U \cap 2D \cap 3U) = \\ = \{P(1U) \times P(2U) \times P(3D)\} + \{P(1D) \times P(2U) \times P(3U)\} + \\ + \{P(1U) \times P(2D) \times P(3U)\} = 0.029403. \end{gathered}$$

$$P(E_4) = P(1U \cap 2U \cap 3U) = P(1U) \times P(2U) \times P(3U) = 0.970299.$$

The loss of load can occur in three mutually exclusive load scenarios, i.e., when the load is 50, 100 or 150 MW. First, we define the following events.

F Event of loss of load.

B_1 Event that load is 50 MW.

B_2 Event that load is 100 MW.

B_3 Event that load is 150 MW.

Then, the probability of loss of load can be found using Bayes' rule as follows.

$$P(F) = P(F|B_1) \times P(B_1) + P(F|B_2) \times P(B_2) + P(F|B_3) \times P(B_3).$$

The loss of load event given that the load is 50 MW, described by Fig. 1.8, will occur when all units fail. Thus, $P(F|B_1) = P(1D \cap 2D \cap 3D) = P(E_1) = 0.00001$.

The loss of load event given that the load is 100 MW will occur when at least two units fail. This event is described by Fig. 1.9. Thus, $P(F|B_2) = P\{(1U, 2D, 3D) \cup (1D, 2U, 3D) (1D, 2D, 3U) \cup (1D, 2D, 3D)\} = = P(E_2) + P(E_1) = 0.000298$.

The loss of load event given that the load is 150 MW will occur when at least one unit fails. This event is described by Fig. 1.10. Equivalently, no loss of load will occur when all units are working. Using complementary rule, $P(F|B_3) = 1 - P(1U, 2U, 3U) = 1 - P(E_4) = 0.029701$.

From Bayes' rule, we can calculate the loss of load probability as follows.

$$P(F) = P(F|B_1) \times 0.20 + P(F|B_2) \times 0.75 + P(F|B_3) \times 0.05 = 0.00170875.$$

We can also use Bayes' rule to calculate loss of load probability by conditioning on the delivered capacity of the three generators instead of conditioning on the different load levels shown in this example.

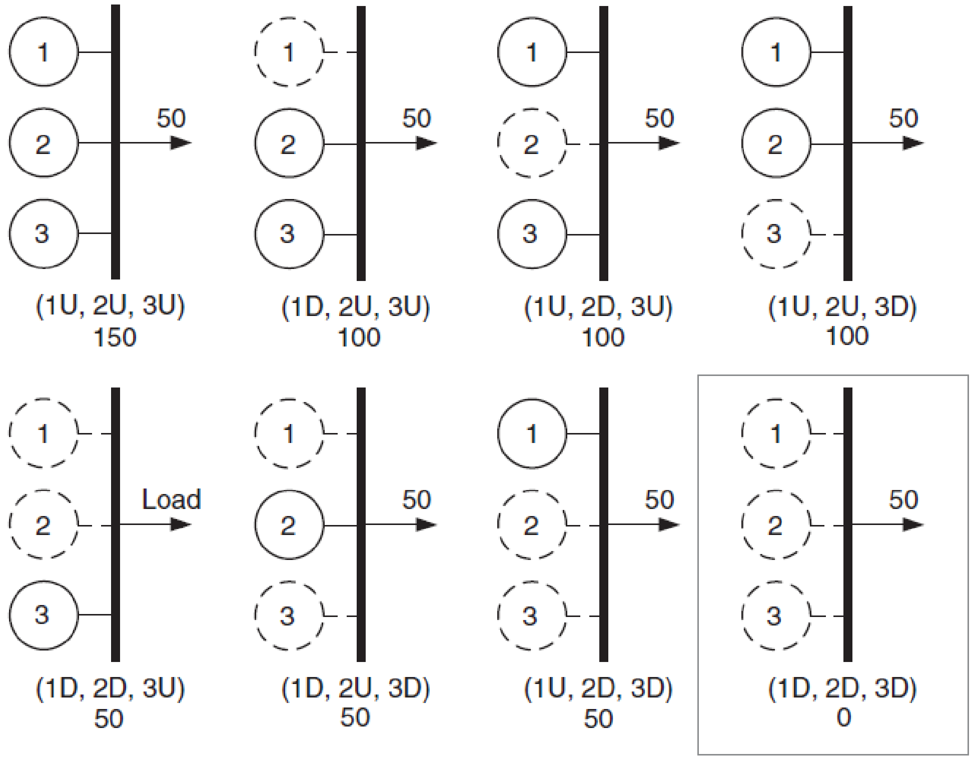

Fig. 1.8. State space representation for loss of load event when load is 50 MW

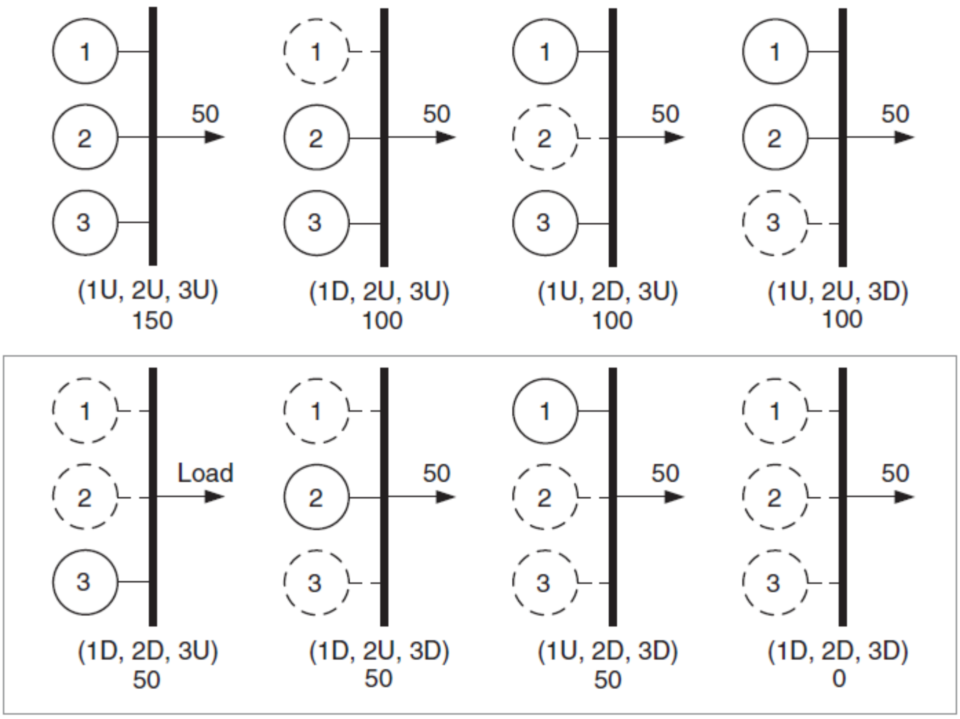

Fig. 1.9. State space representation for loss of load event when load is 100 MW

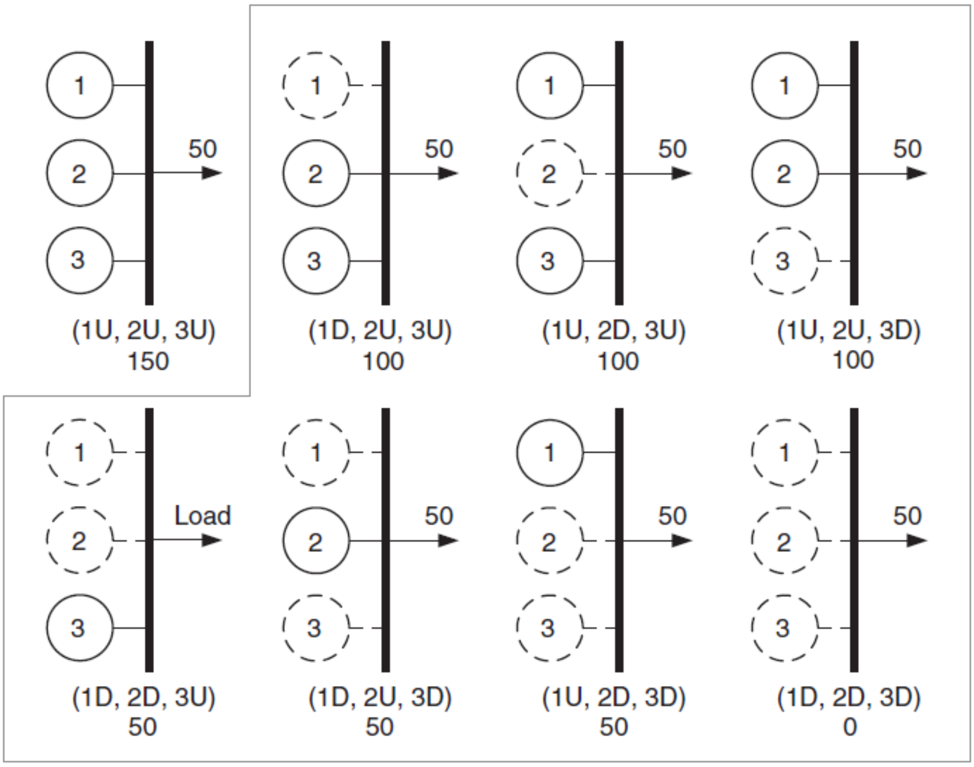

Fig. 1.10. State space representation for loss of load event when load is 150 MW

As seen in Example 1.5, we are interested in knowing the total generating capacity of the system rather than the status of each generator. This real-valued quantity is more important for our analysis since we are interested in a function of the outcome (generating capacity) of the event rather than the outcome (status of each generator) itself. This quantity is a real-valued function defined on the sample space and is called a **random variable**.

Chapter 2
RELIABILITY PRINCIPLES AND CHARACTERISTICS

2.1. Introduction into reliability theory

The twentieth century was characterized by rapid changes in technology, with many changes occurring at an exponential rate, and this is certainly continuing in the twenty-first century as well. **Technology** is, according to a standard dictionary definition, "*the totality of the means employed to provide objects necessary for human sustenance and comfort*." Electrical "objects" can vary from relatively simple products, such as light bulbs, to power generating systems. Items such as these are engineered and manufactured to perform in some specified manner when operated under normal operating conditions. By and large, engineered objects such as these perform satisfactorily, but occasionally they fail. A dictionary definition of **failure** is "*falling short in something expected, attempted, or desired, or in some way deficient or lacking*." From an engineering point of view, it is useful to define failure in a broader sense. Witherell (1994) elaborates as follows: "*It [failure] can be any incident or condition that causes an industrial plant, manufactured product, process, material, or service to degrade or become unsuitable or unable to perform its intended function or purpose safely, reliably, and cost-effectively*." Accordingly, "*the definition of failure should include operations, behavior, or product applications that lead to dissatisfaction, or undesirable, unexpected side effects*." When a failure occurs, no matter how benign, its impact is felt. Failure causes a certain degree of inconvenience or result in personal injury, damage to property, and a significant economic loss. When the failure is catastrophic, the total economic damage and loss of life can be very dramatic, affecting society as a whole. Failures occur in an uncertain manner and are influenced by factors such as design, manufacture or construction, maintenance, and operation. In addition, the human factor is important.

There is no way that failures can be totally eliminated. Every engineered object is unreliable in the sense that it will fail sooner or later, even with the best design, construction, maintenance, and operation. The reason for this is that there are limits to everything and as a result all objects, whether engineered and manufactured or natural (living organisms) must fail eventually. What can be done is reduce the chance of occurrence of failures within a limited time frame. This requires effective integration of good engineering with good management so that the failures and their consequences are minimized and the object can fulfill its intended purpose.

Engineered objects are becoming more and more complex. This, combined with the use of new materials and new construction methods, often increases the risk of failure and the possible damage that may result. Civilized society has always taken a dim view of the damage suffered by its members that is caused by someone or some activity and has demanded a remedy or retribution for offenses against it. Consequently, manufacturers are required to provide compensation for any damages resulting from failures of an object. This has serious implications for manufacturers of engineered objects (including for electricity suppliers). Product-liability laws and warranty legislation are signs of society's desire to ensure fitness of products for their intended use and compensation for failures. Similarly, the actions of user-owners (e. g., operations and maintenance) of engineered objects may have an impact on failure, and individuals and businesses need to understand the implications of this. For example, operating an engine at a higher load than that for which it is rated might lead to increased output but hasten its failure and hence lead to loss rather than gain. A generator that is not properly maintained or is allowed to be overloaded may collapse even though it was properly engineered. The study of various aspects related to failures of engineered objects, the consequence of such failures, and techniques for their avoidance requires that we begin with a good and clear conceptual understanding and have a framework that allows us to integrate the various issues involved in an effective manner. The systems approach provides the framework needed. An important feature of this approach is the use of mathematical models to obtain solutions to a variety of problems of interest to manufacturers and user-owners. We commence with a description of a few engineered objects ranging from a simple product to a complex system. We discuss characterization of a product or system in terms of its various parts. This is essential to the analysis, since the failure of a product or system is related to failure of one or more parts.

For complex products, the number of parts is large. Parts counts, in fact, can provide a crude notion of relative reliability; the more parts, the lower the reliability, all other things being equal, simply because there are more things that can go wrong. Systems of the type indicated above are very complex, indeed, requiring huge charts and schematics for design and analysis. We illustrate in detail the decomposition of products and systems into parts with one or more intermediate levels through a few examples, beginning with much simpler products and extending to complex systems. We shall see that even for quite simple products there are many possible causes of failure. Theoretically, any part, even some not explicitly shown, such as adhesives, could fail and lead to item failure.

Example 2.1 Incandescent Electric Bulb

The components of a typical incandescent light bulb are shown in Fig. 2.1. The light-emitting part is the filament. When heated to 2000 to 3000 °C, it emits light due to incandescence. The source of heating is the resistance of the filament to the electrical current flowing through it. The filament is made by first pressing tungsten ingots and sintering them. The ingot is shaped into round rods and drawn through a die to produce thin wire. The lead-in wires are usually made of nickel, copper, or molybdenum and the support wires are made of molybdenum. The base is made of aluminum. The bulb is filled with an inert gas, usually a mixture of nitrogen and argon.

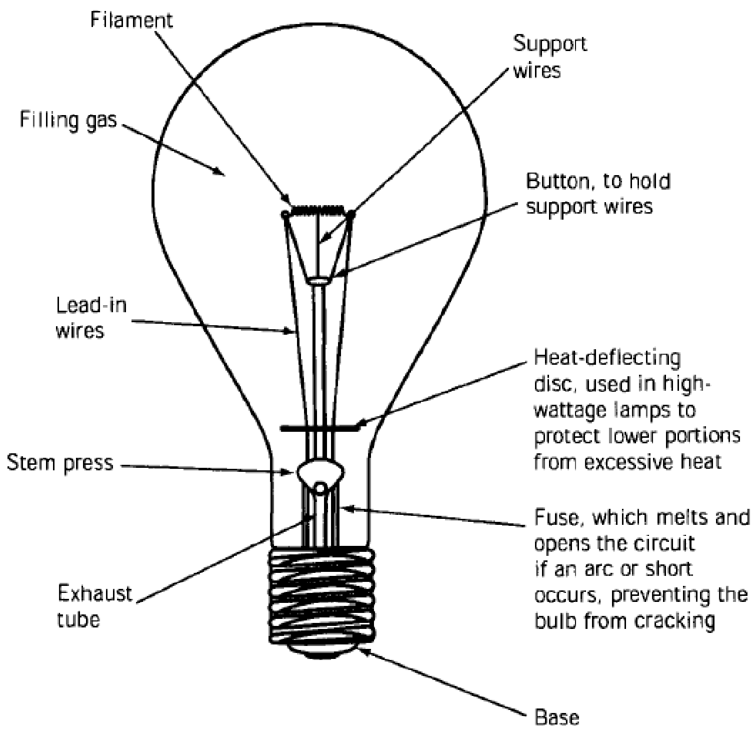

Fig. 2.1. Components of a typical light bulb

Example 2.2 Electric Power System

Industrial nations require electrical energy for use in homes as well as in commerce and industry. This electricity is generated by power plants and transmitted to demand centers (which include domestic and/or commercial and industrial consumers) using a network of high- and low-voltage transmission lines. In schematic form, an electric power system can be represented as a network, as shown in Fig. 2.2. The network consists of two types of nodes–square nodes representing power plants and round nodes representing demand centers–and connecting arcs representing transmission lines that transfer the power from power plants to demand centers.

Each power plant is a complex subsystem consisting of several elements. The main elements of a thermal power plant are shown schematically in Fig. 2.3. The basic process in a thermal power station is as follows. The chemical energy contained in the fuel is converted into heat energy through combustion in the boiler. This energy is used to generate steam from water. The steam is used to drive a turbine that converts the thermal energy into mechanical energy. Finally, the generator transforms the mechanical energy into electrical energy for transmission over high-voltage lines.

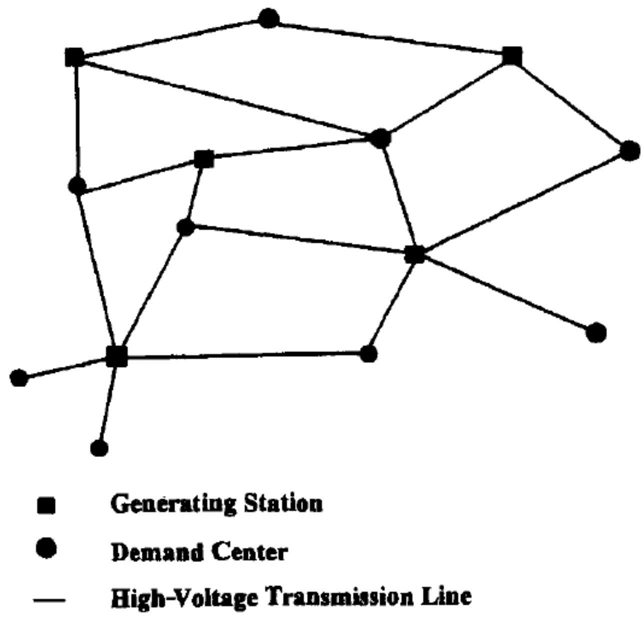

Fig. 2.2. Schematic representation of a power system network

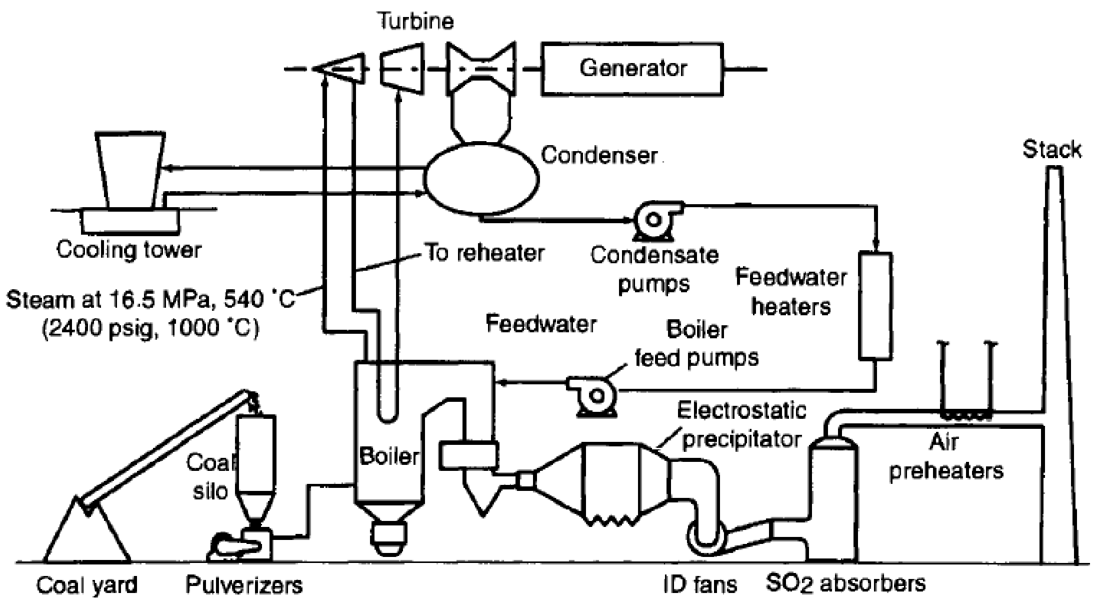

Fig. 2.3. Schematic of a Coal -Fired Steam Power Plant

Each element of a thermal power plant consists of several components and these in turn can be decomposed into various parts. A partial list of the components for the different elements are as follows:

- Boiler: water tubes, drum, headers, superheater/reheater tubes.
- Turbine: high-, intermediate-, and low-pressure units, rotor disk, bladings, inner casings, steam chests.
- Generator: stator, rotor, retaining rings, coils.

2.2. Failures and Faults

We have defined failure in an intuitive manner. In this section we refine this concept and discuss some related notions in order to define clearly what is meant by deterioration and failure in a system.

We begin with some definitions of **failure**.

- "*Failure is the termination of the ability of an item to perform a required function*." [International Electronic Commission, IEC 50(191)]
- "*Equipment fails, if it is no longer able to carry out its intended function under the specified operational conditions for which it was designed*." (Nieuwhof, 1984)
- "*Failure is an event when machinery/equipment is not available to produce parts at specified conditions when scheduled or is not capable of producing parts or perform scheduled operations to specification. For every failure, an action is required*." (Society of Automotive Engineers, "Reliability and Maintainability Guideline for Manufacturing Machinery and Equipment")
- "*Recent developments in products-liability law has given special emphasis to expectations of those who will ultimately come in direct contact with what we will do, make or say or be indirectly affected by it. Failure, then, is any missing of the mark or falling short of achieving these goals, meeting standards, satisfying specifications, fulfilling expectations, and hitting the target*." (Witherell, 1994)

As can be seen, the key term in the above definitions is the inability of the system or product to function as required. Rausand and Oien (1996) suggest a classification of functions for items of a complex system. The various functions in their classification are as follows:

1. **Essential functions**: This defines the intended or primary function. In Example 2.1, the primary function is to provide light. In Example 2.2, it is to provide electric power on demand to the consumers who are part of the network.
2. **Auxiliary functions**: These are required to support the primary function. In Example 2.2, transport of power from areas of low demand to areas of high demand, storage of excess capacity, and sale to other networks are examples of auxiliary functions.
3. **Protective functions**: The goal here is to protect people and the environment from damage and injury. In Example 2.2, relays in the network serve

the primary role of offering protection against current surges, and scrubbers on smokestacks remove particulate matter to protect the environment.

4. **Information functions**: These comprise condition monitoring, gauges, alarms, etc. In Example 2.2, the main control panel displays various bits of information about the different subsystems, e. g., voltage and current output of generators, pressure and temperature of steam in various parts of a power generating plant, and so on.

5. **Interface functions**: This deals with the interface between the item under consideration and other items.

6. **Superfluous functions**: These are superfluous to the system. They occur due to modifications to a system that make an item no longer necessary.

A **fault** is the state of the system characterized by its inability to perform its required function. (Note, this excludes situations arising from preventive maintenance or any other intentional shutdown period during which the system is unable to perform its required function.) A fault is hence a state resulting from a failure.

It is important to differentiate between failure (fault) and error. According to the International Electrotechnical Commission [IEC 50(191)], an error is a "*discrepancy between a computed, observed or measured value or condition and the true, specified or theoretically correct value or condition.*" As a result, an error is not a failure because it is within the acceptable limits of deviation from the desired performance (target value). An error is sometimes referred to as an incipient failure.

2.3. Failure Modes

A **failure mode** is a description of a fault. It is sometimes referred to as fault mode [for example, IEC 50(191)]. Failure modes are identified by studying the (performance) function of the item. Blache and Shrivastava (1994) suggest a classification scheme for failure modes; this is shown in Fig. 2.4.

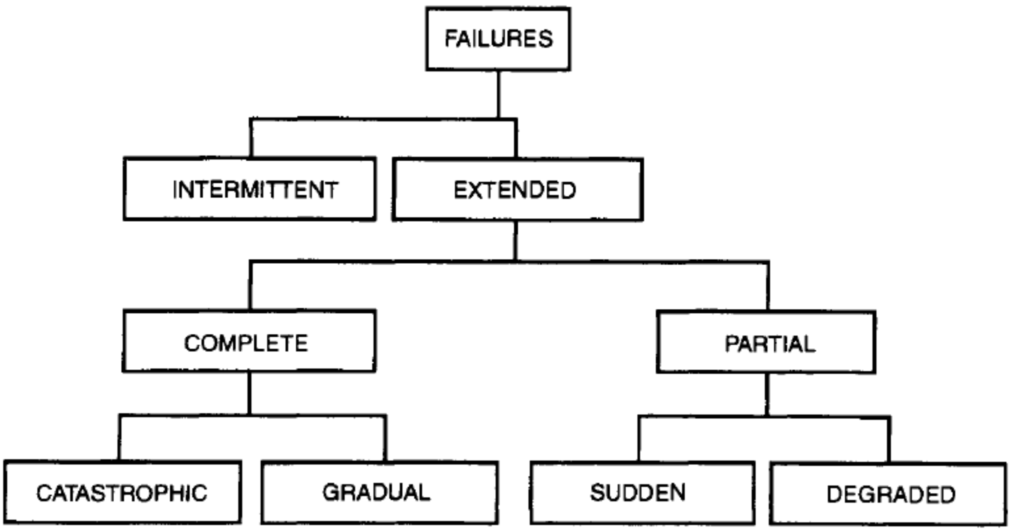

Fig. 2.4. Failure classification

A brief description of the different failure modes is as follows:

1. **Intermittent failures**: Failures that last only for a short time. A good example of this is software faults that occur only under certain conditions that occur intermittently.

2. **Extended failures**: Failures that continue until some corrective action rectifies the failure. They can be divided into the following two categories:

a) **Complete failures**, which result in total loss of function

b) **Partial failures**, which result in partial loss of function

Each of these can be further subdivided into the following:

a) **Sudden failures**: Failures that occur without any warning

b) **Gradual failures**: Failures that occur with signals to warn of the occurrence of a failure

A complete and sudden failure is called a **catastrophic failure** and a gradual and partial failure is designated a **degraded failure**.

In Example 2.1, failure of the item is sudden and catastrophic in the sense that failure is essentially instantaneous and after failure the bulb no longer emits light. In a power station (Example 2.2), the bearings (for turbine and generator) often fail due to wear, resulting in gradual deterioration. In contrast, failure due to a lightning strike is sudden and can lead to either partial or complete failure of the network.

2.4. Failure Causes and Severity

According to IEC 50(191), **failure cause** is "*the circumstances during design, manufacture or use which have led to a failure*." Failure cause is useful information in the prevention of failures or their reoccurrence. Failure causes may be classified (in relation to the life cycle of the system) as shown in Fig. 2.5.

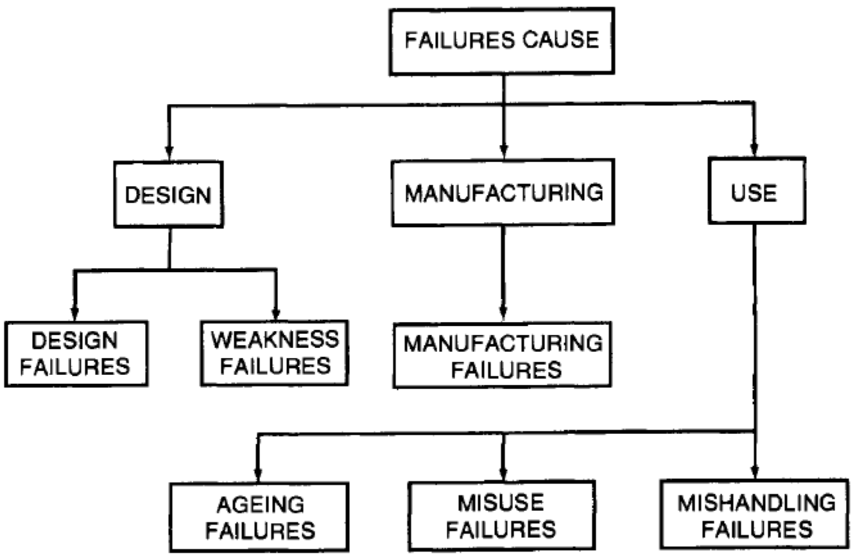

Fig. 2.5. Failure cause classification [from IEC 50(191)]

First, we briefly describe each of these failure causes.

1. **Design failure**: Due to inadequate design.
2. **Weakness failure**: Due to weakness (inherent or induced) in the system so that the system cannot stand the stress it encounters in its normal environment.
3. **Manufacturing failure**: Due to nonconformity during manufacturing.
4. **Aging failure**: Due to the effects of age and/or usage.
5. **Misuse failure**: Due to misuse of the system (operating in environments for which it was not designed).
6. **Mishandling failure**: Due to incorrect handling and/or lack of care and maintenance.

Note that the various failure causes shown in Fig. 2.5 are not necessarily disjoint. Also, one can differentiate between primary (or root) cause and secondary and other levels of failures that result from a primary failure.

In Example 2.2, blades in the steam turbine can fail due to excessive thermal stress resulting from poor design. Bearings of the turbine may fail due to improper lubrication (mishandling), aging, or, in fact, for any of the other reasons. A mishandling failure in the case of Example 2.1 occurs when the filament breaks loose under a mechanical impact.

Finally, the severity of a failure mode signifies the impact of the failure mode on the system as a whole and on the outside environment. A severity ranking classification scheme (MIL-STD 882) is as follows:

1. **Catastrophic**: Failures that result in death or total system loss.
2. **Critical**: Failures that result in severe injury or major system damage.
3. **Marginal**: Failures that result in minor injury or minor system damage.
4. **Negligible**: Failures that result in less than minor injury or system damage.

Another classification is given in the reliability centred maintenance (RCM) approach, where the following severity classes (in descending order of importance) are used:

1. Failures with safety consequences.
2. Failures with environmental consequences.
3. Failures with operational consequences.
4. Failures with non-operational consequences.

The specific causes of failures of components and equipment in a system can be many. Some are known and others are unknown due to the complexity of the system and its environment. A few of them including the causes mentioned earlier are listed below:

1. Poor Design, Production and Use

Poor design and incorrect manufacturing techniques are obvious reasons of the low reliability. Some manufacturers hesitate to invest more money on an improved design and modern techniques of manufacturing and testing. Improper selection of materials is another cause for poor design.

Components and equipment do not operate in the same manner in all conditions. A complete knowledge of their characteristics, applications, and limitations will avoid their misuse and minimize the occurrence of failures. All failures have a cause and the lack of understanding these causes is the primary cause of the unreliability of a given system.

2. System Complexity

In many cases a complex and sophisticated system is used to accomplish a task which could have been done by other simple schemes. The implications of complexity are costly. First it employs more components thereby decreasing overall reliability of the system. Second, a complex scheme presents problems in terms of users' understanding and maintenance.

On the other hand, simplicity costs less, causes less problems, and has more reliability. A basic rule of reliability with respect to complexity is: Keep the system as simple as is compatible with the performance requirements.

3. Poor Maintenance

The important period in the life cycle of a product or a system is its operating period. Since no product is perfect, it is likely to fail. However its life time can be increased if it can be repaired and put into operation again. In many cases preventive-measures are possible and a judiciously designed preventive-maintenance policy can help eliminate failures to a large extent. The adage Prevention is better than cure applies to products and equipment as well.

4. Communication and Coordination

Reliability is a concern of almost all departments of an organization. It is essentially a birth-to-death problem involving such areas as raw material and parts, conceptual and detailed engineering design, production, test and quality control, product shipment and storage, installation, operation and maintenance.

A well-organized management with an efficient system of communication is required to share the information and experiences about components. Sufficient opportunity should be available for the people concerned to discuss the causes of failures. In some organizations, rigidity of rules and procedures prohibits the creative-thinking and design.

5. Human Reliability

In spite of increased application of automation techniques in industries and other organisations, it is impossible to completely eliminate the human involvement in the operation and maintenance of systems. The contribution of human-errors to the unreliability may be at various stages of the product cycle. Failures due to the human- error can be due to:

- lack of understanding of the equipment,
- lack of understanding of the process,

- carelessness,
- forgetfulness,
- poor judgemental skills,
- absence of correct operating procedures and instructions,
- physical inability.

Although, it is not possible to eliminate all human-errors, it is possible to minimize some of them by the proper selection and training of personnel, standardization of procedures, simplification of control schemes and other incentive measures. The designer should ensure that the operation of the equipment is as simple as possible with practically minimum probability for error. The operator should be comfortable in his work and should be free from unnecessary stresses. The following checklist should prove useful to the design engineer:

- Is the operator position comfortable for operating the controls?
- Do any of the operations require excessive physical effort?
- Is lighting of the workplace and surrounding area satisfactory?
- Does the room temperature cause any discomfort to the operator?
- Are noise and vibration within the tolerable limits?
- Does the layout ensure the required minimum movement of operator?
- Can the operator's judgement be further minimized?

With all this care, human operators are still likely to make errors. A human error may or may not cause a failure. Consequently, the quantitative measurement of the human reliability is required in order to present a correct picture of the total system reliability.

2.5. Catastrophic failures and degradation failures

When the ability of an item to perform its required function is terminated the item is said to have failed. As failure is an ill-defined term, we have tried to cross-reference some of the more important kinds of failures by way of a contingency Table 2.1. A failure may be **complete** or **partial** depending upon how complete the lack of the required function is. If we follow a particular item in time as it functions and finally fails we will see that it may fail in one of two ways, by a **catastrophic failure** or by a **degradation failure**.

Catastrophic failures are characterized as being both complete and sudden. Complete in the sense that the change in output is so gross as to cause complete lack of the required function, and sudden in the sense that the failure could not be anticipated. For example, at the system level the event of the gain of an amplifier suddenly going to zero would be a catastrophic failure.

Degradation failures often called **drift failures**, require further categorization. We can distinguish between monotonic and non-monotonic drift. **Monotonic drift** is characterized by an output variable continuously varying in the same direction as illustrated in the Fig. 2.6. At some point in time the value of the output crosses one of the constraints, giving rise to failure. **Non-monotonic** drift is characterized by both positive and negative excursions of an output variable as shown in Fig. 2.7, *a*, the excursions being somewhat similar to Brownian movements. The definition of unsatisfactory performance (especially failure) in the case of non-monotonic drift is not quite so straightforward as for monotonic drift. Of course, violation of the constraints at any point must strictly speaking be classified as a failure.

Table 2.1

Failures

	Sudden failures: Failures that could not be anticipated by prior examination. (Sudden failures are similar to random failures. A random failure is any failure whose time of occurrence is unpredictable).	Gradual failures: Failures that could be anticipated by prior examination.
Complete failures: Failures resulting from deviations in characteristic (s) beyond specified limits.	Catastrophic failures: Failures that are both sudden and complete.	This state of affairs may be the end result when degradation failures are left unattended
Partial failures: Failures resulting from deviations in characteristic (s) beyond specified limits but not such as to cause complete lack of required function.	We define marginal failures as failures which are observed at time t = 0, when the item has just been finished. Sudden and partial failures are rarely seen later in life of an item.	Degradation failures: Failures that are both gradual and partial.

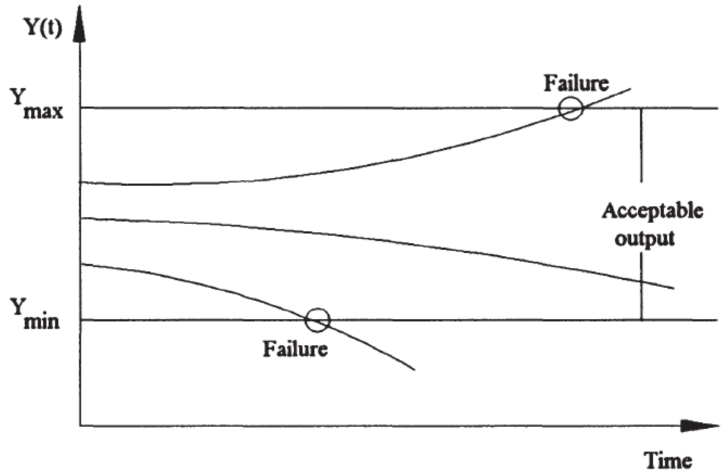

Fig. 2.6. Three examples or monotonic drift two or which give rise to failures

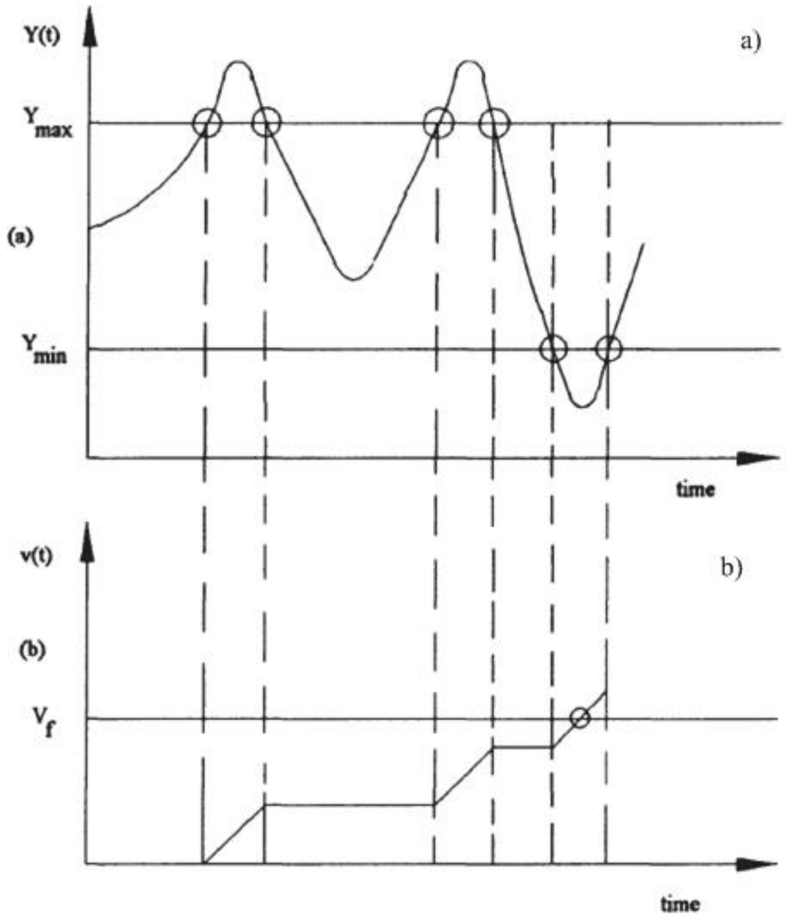

Fig. 2.7. a – Non- monotonic drift or a variable.
b – v(t) Is the total time Y(t) has spent in the region or degradation

However, in the case of non-monotonic drift, it may happen that the output drifts back into the acceptable region shortly afterwards if so the short-lasting excursion into the region of unsatisfactory performance may not have harmed the system performance appreciably. Depending on the system, this consequence of drift may more properly be defined in terms of the accumulated amount of resulting degradation.

As an example, consider the definition of a possible function $v(t)$ for measuring the accumulated degradation as shown in Fig. 2.7, *b*. Only when the accumulated amount of degradation defined by this function exceeds a specified level, V_f, is the system deemed to have performed unsatisfactorily. Other indications of unsatisfactory performance are also possible in the case of non- monotonic drift. We might for example use the area of $V(t)$ above or below the limits for acceptable performance as an indicator. Unsatisfactory performance would then be evidenced when the area exceeds a specified amount. A third possibility would be to use the number of crossings of the limits as an indicator of unsatisfactory performance.

2.6. Characteristic types of failures

Reliability Engineering distinguishes three characteristic types of failures (excluding damage caused by careless handling, storing, or improper operation by the users) which may be inherent in the equipment and occur without any fault on the part of the operator.

First, there are the failures which occur early in the life of a component. They are called **early failures**. Some examples of early failures are:

- Poor welds or seals.
- Poor solder joints.
- Poor connections.
- Dirt or contamination on surfaces or in materials.
- Chemical impurities in metal or insulation.
- Voids, cracks, thin spots in insulation or protective coatings.
- Incorrect positioning of parts.

Many of these early failures can be prevented by improving the control over the manufacturing process. Sometimes, improvements in design or materials are required to increase the tolerance for these manufacturing deviations, but fundamentally these failures reflect the **manufacturability** of the component or product and the control of the manufacturing processes. Consequently, these early failures would show up during:

- In-process and final tests.
- Process audits.
- Life tests.
- Environmental tests.

Early failures can be eliminated by the so-called **debugging** or **burn-in** process. The debugging process consists of operating equipment for a number of hours under conditions simulating actual use. The weak or substandard components fail in these early hours of the equipment’s operation and they are replaced by good components. Similarly poor solder connections or other assembly faults show up and they are corrected. Only then is the equipment released for service.

Secondly, there are failures which are caused by wearout of parts. These occur in equipment only if it is not properly maintained-or not maintained at all. **Wearout failures** are due primarily to deterioration of the design strength of the device as a consequence of operation and exposure to environmental fluctuations. Deterioration results from a number of familiar chemical and physical phenomena:

- Corrosion or oxidation.
- Insulation breakdown or leakage.
- Ionic migration of metals in vacuum or on surfaces.
- Frictional wear or fatigue.
- Shrinkage and cracking in plastics.

In most cases wearout failures can be prevented. For instance, in repeatedly operated equipment one method is to replace at regular intervals the accessible parts which are known to be subject to wearout, and to make the replacement intervals shorter than the mean wearout life of the parts. Or, when the parts are inaccessible, they are designed for a longer life than the intended life of the equipment. This second method is also applied to so-called **one-shot** equipment, such as missiles, which are used only once during their lifetime.

Third, there are so-called **chance failures** which neither good debugging techniques nor the best maintenance practices can eliminate. These failures are caused by sudden stress accumulations beyond the design strength of the component. Chance failures occur at random intervals, irregularly and unexpectedly. No one can predict when chance failures will occur. However, they obey certain rules of collective behaviour so that the frequency of their occurrence during sufficiently long periods is approximately constant. Chance failures are sometimes called catastrophic failures, which is inaccurate because early failures and wearout failures can be as catastrophic as chance failures. It is not normally easy to eliminate chance failures. However, reliability techniques have been developed which can reduce the chance of their occurrence and, therefore, reduce their number to a minimum within a given time interval.

Reliability engineering is concerned with eliminating early failures by observing their distribution and determining accordingly the length of the necessary debugging period and the debugging methods to be followed. Further, it is concerned with preventing wearout failures by observing the statistical distribution of wearout and determining the overhaul or preventive replacement periods for the various parts or their design life. Finally, its main attention is focused on chance failures and their prevention, reduction, or complete elimination because it is the chance failure phenomenon which most undesirably affects after the equipment has been debugged and before parts begin to wear out.

2.7. Useful life of components

If we take a large sample of components and operate them under constant conditions and replace the components as they fail, then approximately the same number of failures will occur in sufficiently long periods of equal length. The physical mechanism of such failures is a sudden accumulation of stresses acting on and in the component. These sudden stress accumulations occur at random and the randomness of the occurrence of chance failures is therefore an obvious consequence.

If we plot the curve of the failure rate against the lifetime T of a very large sample of a homogeneous component population, the resulting failure rate graph is shown in Fig. 2.8. At the time $T = 0$ we place in operation a very large number of new components of one kind. This population will initially exhibit a high failure rate if it contains some proportion of substandard, weak specimens. As these weak components fail one by one, the failure rate decreases comparatively rapidly during the so-called **burn-in** or **debugging** period, and stabilizes to an approximately constant value at the time T_b when the weak components have died out. The component population after having been burned in or debugged, reaches its lowest failure rate level which is approximately constant. This period of life is called the **useful life** period and it is in this period that the exponential law is a good approximation. When the components reach the life T_w wearout begins to make itself noticeable. From this time on, the failure rate increases rather rapidly. If upto the time T_w only a small percentage of the component population has failed of the many components which survived up to the time T_w, about one-half will fail in the time period from T_w to M. The time M is the mean wearout life of the population. We call it simply mean life, distinguished from the mean time between failures, $m = 1/\lambda$. in the useful life period.

If the chance failure rate is very small in the useful life period, the mean time between failures can reach hundreds of thousands or even millions of hours. Naturally, if a component is known to have a mean time between failures of say 100,000 hours (or a failure rate of 0.00001) that certainly does not mean that it can be used in operation for 100,000 hours.

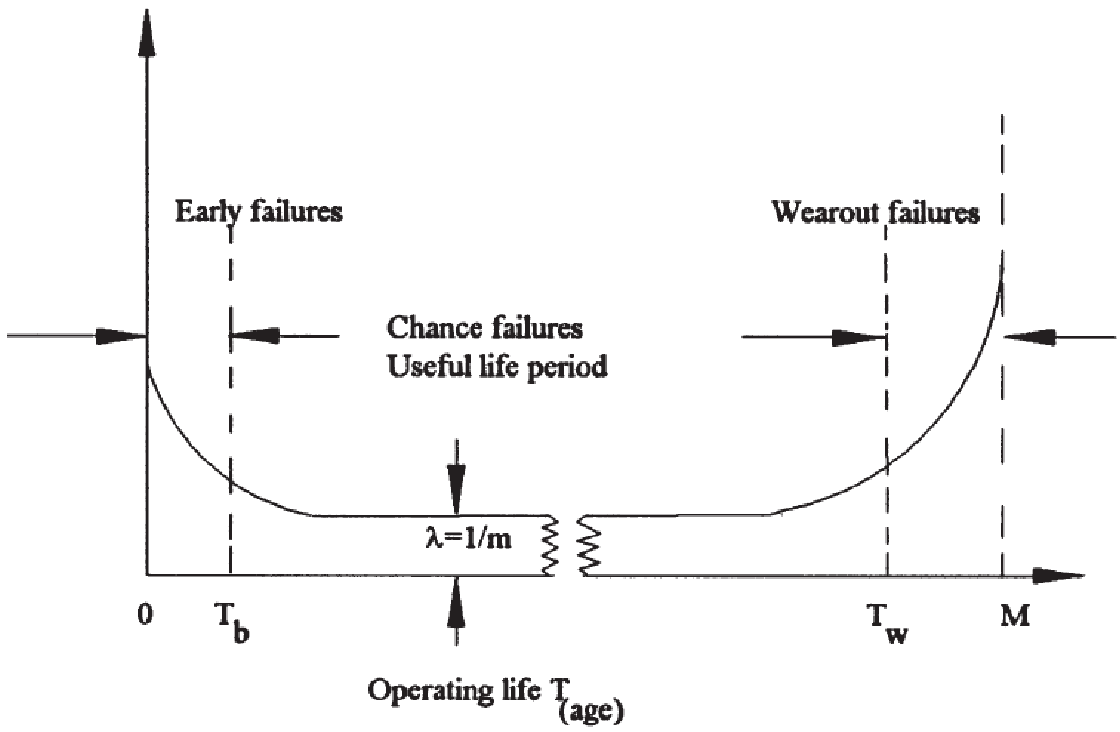

Fig. 2.8. Component failure rate as a function of age

The mean time between failures tells us how reliable the component IS In its useful life period, and such information is of utmost importance. A component with a mean time between failures of 100,000 hours will have a reliability of 0.9999 or 99.99 percent for any 10-hour operating period. Further if we operate 100,000 components of this quality for 1 hour, we would expect only one to fail. Equally, would we expect only one failure if we operate 10,000 components under the same conditions for 10 hours, or 1000 components for 100 hours, or 100 components for 1000 hours.

Chance failures cannot be prevented by any replacement policy because of the constant failure rate of the components within their useful life. If we try to replace good non-failed components during useful life, we would improve absolutely nothing. We would more likely do harm, as some of the components used for replacement may not have been properly burned in, and the presence of such components could only increase the failure rate. Therefore, the very best policy in the useful life period of components is to replace them only as they fail. However, we must stress again that no component must be allowed to remain in service beyond its wearout replacement time T w. Otherwise, the component probability of failure increases tremendously and the system probability of failure increases even more.

The **golden rule** of reliability is, therefore: *Replace components as they fail within the useful life of the components, and replace each component preventively, even if it has not failed, not later than when it has reached the end of its useful life*. The burn-in procedure is an absolute must for missiles, rockets, and space systems in which no component replacements are possible once the vehicle takes off and where the failure of any single component can cause the loss of the system. *Component burn-in before assembly followed by a debugging procedure of the system* is, therefore, **another golden rule** of reliability.

Failure is often a result of the effect of **deterioration**. The deterioration process leading to a failure is a complicated process, and this varies with the type of the system and the material used. Failure mechanisms may be divided into two broad categories (Dasgupta and Pecht, 1991): (i) overstress failures, and (ii) wear-out failures.

Overstress failures are those due to brittle fracture, ductile fracture, yield, buckling, large elastic deformation, and interfacial deadhesion. Wear-out failures are those due to wear, corrosion, dendritic growth, interdiffusion, fatigue crack propagation, diffusion, radiation, fatigue crack initiation, and creep.

As an illustrative example, consider fatigue failure. When cyclic stress is applied to a mechanical component, failure of the material occurs at stresses much below the ultimate tensile strength of the material because of the accumulation of damage. Fatigue failure begins with the initiation of a small, microscopic crack. The crack typically develops at a point of discontinuity or at a defect in the material that can cause local stress and plastic strain concentration. This is termed fatigue crack initiation. Once a fatigue crack has been initiated, the crack can propagate in a stable fashion under cyclic stress, until it becomes unstable under applied stress amplitude. The crack propagation rate is a material property. The rate at which the deterioration occurs is a function of time and/or usage intensity.

2.8. What Is Quality?

The word **quality** comes from the Latin qualis, meaning "how constituted." Dictionaries define quality as the essential character or nature of something, and as an inherent characteristic or attribute. Thus, a product has certain qualities or characteristics, and a product's overall performance, or its effectiveness, is a function of these qualities.

Juran and Gryna (1980) looked at multiple elements of fitness for use and evaluated various quality characteristics (or "qualities"), such as technological characteristics (strength, weight, and voltage), psychological characteristics (sensory characteristics, aesthetic appeal, and preference), and time-oriented characteristics (reliability and maintainability). Deming (1982) also investigated several facets of quality, focusing on quality from the viewpoint of the customer.

The American Society for Quality (ASQC Glossary and Tables for Statistical Quality Control 1983) defines quality as the "totality of features and characteristics of a product or service that bear on its ability to satisfy a user's given needs." Shewhart (1931) stated it this way:

The first step of the engineer in trying to satisfy these wants is, therefore, that of translating as nearly as possible these wants into the physical characteristics of the thing manufactured to satisfy these wants. In taking this step, intuition and judgment play an important role, as well as a broad knowledge of the human element involved in the wants of individuals. The second step of the engineer is to set up ways and means of obtaining a product which will differ from the arbitrary set standards for these quality characteristics by no more than may be left to chance.

One of the objectives of **quality function deployment** (**QFD**) is to achieve the first step proposed by Shewhart. QFD is a means of translating the "voice of the customer" into substitute quality characteristics, design configurations, design parameters, and technological characteristics that can be deployed (horizontally) through the whole organization: marketing, product planning, design, engineering, purchasing, manufacturing, assembly, sales, and service.

Products have several characteristics, and the "ideal" state or value of these characteristics is called the target value (Fig. 2.9). QFD (Fig. 2.10) is a methodology to develop target values for substitute quality characteristics that satisfy the requirements of the customer. Mizuno and Akao (1931) have developed the necessary philosophy, system, and methodology to achieve this step.

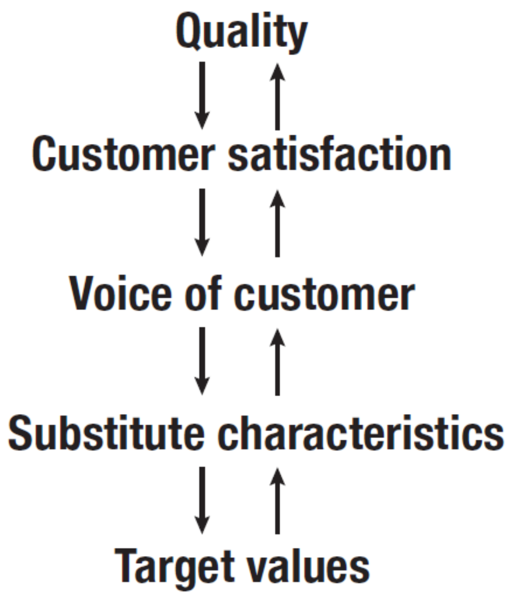

Fig. 2.9. The relationship of quality, customer satisfaction, and target values

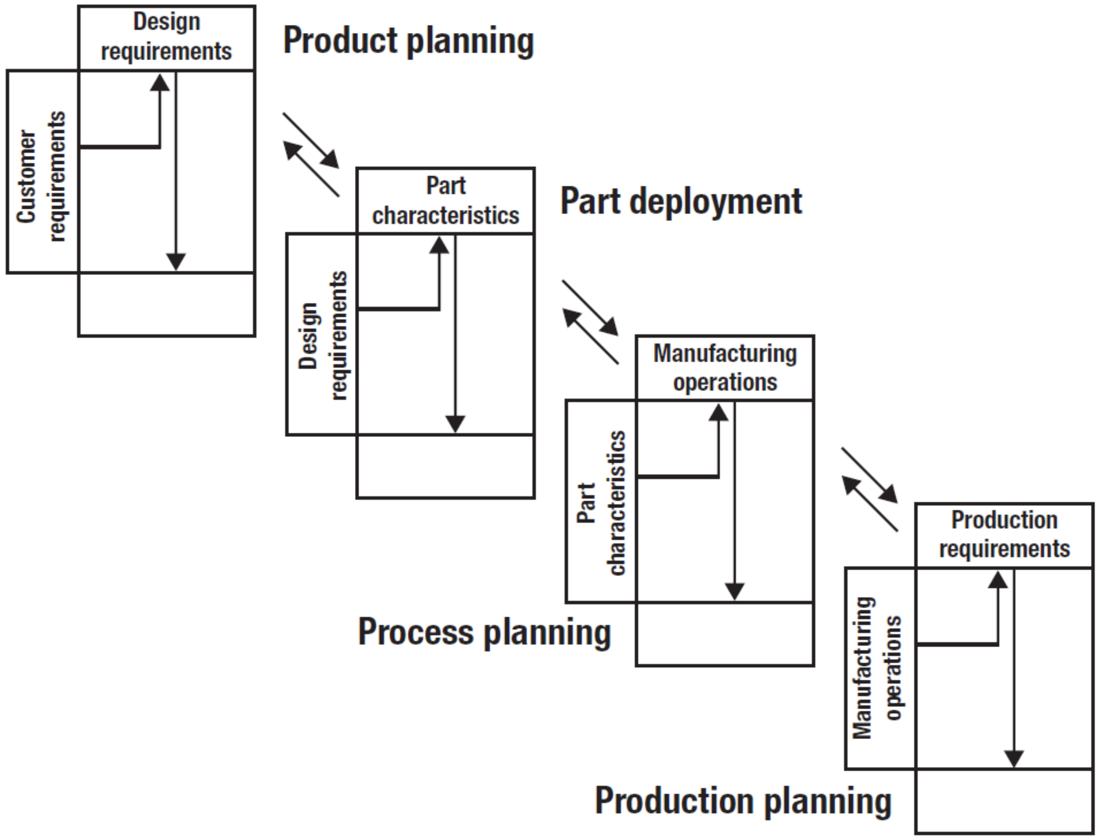

Fig. 2.10. Illustration of the steps in QFD

2.9. Basic concepts of reliability

Reliability of a system conveys the concept of dependability, successful operation or performance, and the absence of failures. Unreliability (or lack of reliability) conveys the opposite. Since the process of deterioration leading to failure occurs in an uncertain manner, the concept of reliability requires a dynamic and probabilistic framework.

The concept of reliability has been interpreted in many ways in numerous works. Since many of these do not agree in content, it is expedient to examine the main ones.

The following definitions of **reliability** are most often met with in the literature.

1. *Reliability is the integral of the distribution of probabilities of failure – free operation from the instant of switch- on to the first failure*.
2. *The reliability of a component (or a system) is the probability that the component (or a system) will not fail for a time t*.
3. *Reliability is the probability that a device will operate without failure for a given period of time under given operating conditions*.
4. *Reliability is the mean operating time of a given specimen between two failures*.
5. *The reliability of a system is called its capacity for failure -free operation for a definite period of time under given operating conditions, and for minimum time lost for repair and preventive maintenance*.
6. *The reliability of equipment is arbitrarily assumed to be the equipment's capacity to maintain given properties under specified operating conditions and for a given period of time*.

One of the definitions which has been accepted by most contemporary reliability authorities is given by the Electronics Industries Association, (EIA) USA (formerly known as RETMA) which states:

The reliability of an item (a component, a complex system, a computer program or a human being) is defined as the probability of performing its purpose adequately for the period of time intended under the operating and environmental conditions encountered.

This definition stresses four elements:

1. **Probability**.
2. **Adequate performance**.
3. **Time**.
4. **Operating and environmental conditions**.

We will also use the following definition:

The reliability of a system is the probability that the system will perform its intended function for a specified time period when operating under normal (or stated) environmental conditions.

The true reliability is never exactly known, but numerical estimates quite close to this value can be obtained by the use of statistical methods and probability calculations. How close the statistically estimated reliability comes to the true reliability depends on the amount of testing, the completeness of field service reporting all successes and failures, and other essential data. For the statistical evaluation of equipment, the equipment has to be operated and its performance observed for a specified time under actual operating conditions in the field or under well-simulated conditions in a Laboratory. Criteria of what is considered an **adequate performance** have to be exactly spelled out for each case, in advance.

Measurement of the adequate performance of a device requires measuring all important performance parameters. As long as these parameters remain within the specified limits, the equipment is judged as operating satisfactorily. When the performance parameters drift out of the specified tolerance limits, the equipment is judged as having malfunctioned or failed. For instance, if the gain of an electronic amplifier reduces to a value K_1 from the designed value K its performance may have to be considered unsuitable for a control system application but may still be quite acceptable for consumer electronics equipment.

In the probability context, satisfactory performance is directly connected to the concepts of failure or malfunction. The relation between these two is that of mutually exclusive events-which means the equipment when in operation, is either operating satisfactorily or has failed or malfunctioned. Sometimes, it may be simpler to specify first what is regarded as **failure** and **satisfactory performance** is then every other operating condition which is not a failure. The frequency at which failures occur is called the **failure rate** (λ). It is usually measured in number of failures per unit operating hour. Its reciprocal value is called the **mean time between failures** (m) and this is measured in hours.

It is true that only in some simple cases, where devices of the *go-no-go* type are involved, the distinction between adequate performance and failure is a very simple matter. For instance, a switch either works or does not work – it is good or bad. But there are many more cases where such a clear-cut decision can not be made so easily and a number of performance parameters and their limits must first be specified.

Since reliability is a yardstick of capability to perform within required limits when in operation, it normally involves a parameter which measures time. This may be any time unit which is preferable in cases where continuous operation is involved; it may be number of cycles when the equipment operates only sporadically, in regular or irregular periods, or a combination of both. It is meaningful to speak of the operating hours of an engine, generator, aircraft, etc.

But for a switch or relay it may be more meaningful to speak of the number of operations which such a device has to perform. The probability that no failure will occur in a number of operations (cycles) may in these cases tell much more than the probability of no failure in a number of hours. Thus, a switch measures its **time** in cycles of operation rather than in hours. Similarly, a vehicle may more meaningfully measure its **time** in miles or kilometers rather than in hours.

In addition to the conventional systems approach to reliability studies, we also frequently use **Failure mode and effects analysis** (**FMEA**), and **Fault tree analysis** (**FTA**) approaches. Failure mode and effects analysis is a preliminary design evaluation procedure used to identify design weakness that may result in safety hazards or reliability problems. The FMEA procedure may be termed a **what if** approach in that it starts at component level and asks **what if this component fails**. The effects are then traced on to system level. Any component failures that could have a critical effect on the system are identified and either eliminated or controlled, if possible. Fault tree analysis begins with the definition of an undesirable event and traces this event down through the system to identify basic causes. In systems parlance, the FMEA is a *bottom-up* procedure while the FTA is a top-down technique.

Thus, **reliability theory** deals with the interdisciplinary use of probability, statistics, and stochastic modeling, combined with engineering insights into the design and the scientific understanding of the failure mechanisms, to study the various aspects of reliability. As such, it encompasses issues such as (i) reliability modeling, (ii) reliability analysis and optimization, (iii) reliability engineering, (iv) reliability science, (v) reliability technology, and (vi) reliability management.

Reliability modeling deals with model building to obtain solutions to problems in predicting, estimating, and optimizing the survival or performance of an unreliable system, the impact of the unreliability, and actions to mitigate this impact.

Reliability analysis can be divided into two broad categories: (i) qualitative and (ii) quantitative. The former is intended to verify the various failure modes and causes that contribute to the unreliability of a product or system. The latter uses real failure data in conjunction with suitable mathematical models to produce quantitative estimates of product or system reliability.

Reliability engineering deals with the design and construction of systems and products, taking into account the unreliability of its parts and components. It also includes testing and programs to improve reliability. Good engineering results in a more reliable end product.

Reliability science is concerned with the properties of materials and the causes for deterioration leading to part and component failures. It also deals with the effect of manufacturing processes (e. g., casting, annealing) on the reliability of the part or component produced.

Reliability management deals with the various management issues in the context of managing the design, manufacture, and/or operation of reliable products and systems. Here the emphasis is on the business viewpoint, as unreliability has consequences in cost, time wasted, and, in certain cases, the welfare of an individual or even the security of a nation.

"The soundness of management is reflected in the quality of products produced and in customer satisfaction. Reliability is merely one quality of the product; others might be performance, style, convenience, economy and so on." (Lloyd and Lipow, 1962).

2.10. Product life cycle

A **product life cycle** (for a consumer durable or an industrial product, e. g. an electrical installation), from the point of view of the manufacturer, is the time from initial concept of the product to withdrawal of the product from the marketplace. It involves several stages, as indicated in Fig. 2.11.

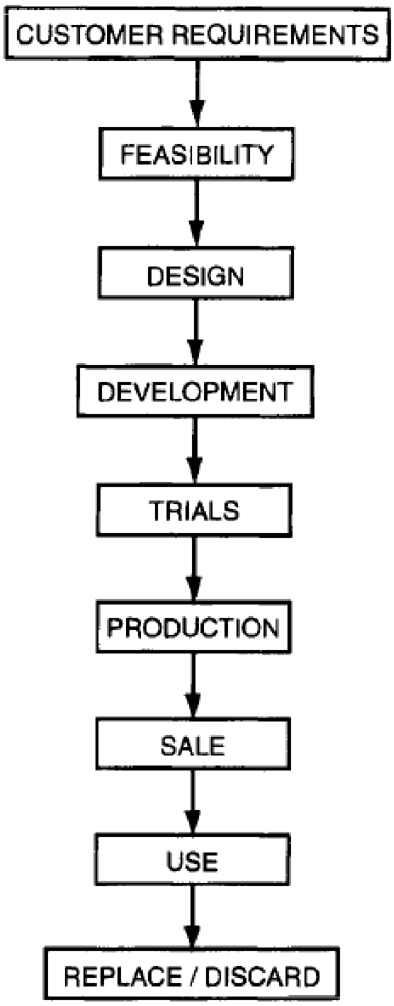

Fig. 2.11. Product life cycle

The process begins with an idea to build a product to meet some customer requirements, such as performance (including reliability) targets. This is usually based on a study of the market and potential demand for the product being planned. The next step is to carry out a feasibility study. This involves evaluating whether it is possible to achieve the targets within specified cost limits. If this analysis indicates that the project is feasible, an initial product design is undertaken. A prototype is then developed and tested. It is not unusual at this stage to find that achieved performance levels of the prototype product are below the target values. In this case, further product development is undertaken to overcome the problem. Once this is achieved, the next step is to carry out trials to determine performance of the product in the field and to start a preproduction run. This is required because the manufacturing process must be fine tuned and quality control procedures established to ensure that the items produced have the same performance characteristics as those of the final prototype. After this, the production and marketing efforts begin. The items are produced and sold. Production continues until the product is removed from the market because of obsolescence and/or the launch of a new product.

The life cycle for more specialized industrial products is similar. Here, the product requirements are supplied by the customer and the manufacturer builds the product to these specifications.

We focus our attention on the reliability of the product over its life cycle. Although this may vary considerably, a typical scenario is as shown in Fig. 2.12. A feasibility study is carried out using the specified target value for product reliability. During the design stage, product reliability is assessed in terms of part and component reliabilities. Product reliability increases as the design is improved. However, this improvement has an upper limit. If the target value is below this limit, then the design using available parts and components achieves the desired target value. If not, then a development program to improve the reliability through test-fix-test cycles is necessary. Here the prototype is tested until a failure occurs and the causes of the failure are analyzed. Based on this, design and/or manufacturing changes are introduced to overcome the identified failure causes. This process is continued until the reliability target is achieved.

The reliability of the items produced during the preproduction run is usually below that for the final prototype. This is caused by variations resulting from the manufacturing process. Through proper process and quality control, these variations are identified and reduced or eliminated and the reliability of items produced is increased until it reaches the target value. Once this is achieved, full-scale production commences and the items are released for sale.

The reliability of an item in use deteriorates with age. This deterioration is affected by several factors, including environment, operating conditions, and maintenance. The rate of deterioration can be controlled through preventive maintenance, as shown in Fig. 2.12.

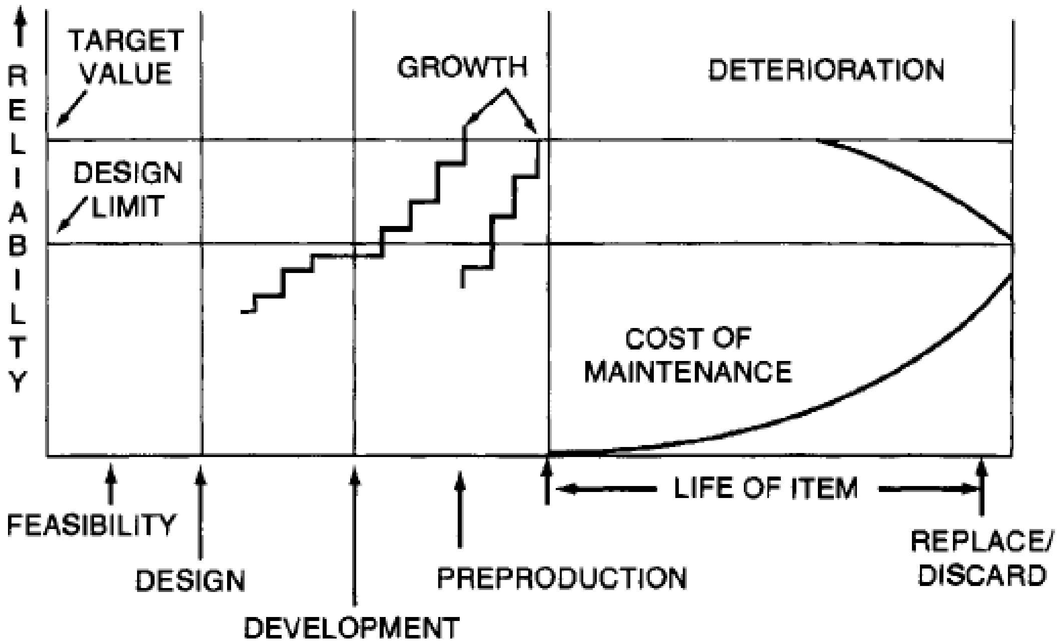

Fig. 2.12. Reliability over the product life cycle

It is worth noting that if the reliability target values are too high, they might not be achievable with development. In this case, the manufacturer must revise the target value and start with a new feasibility study before proceeding further.

The changing nature of reliability of a product over its life cycle has implications for both the manufacturer and the buyer (owner-user).

2.11. Reliability and the System Life Cycle

Reliability activities should span the entire life cycle of the system. Fig. 2.13 shows the major points of reliability practices and activities for the life cycle of a typical system. The activities presented in Fig. 1.6 are briefly explained in the following sections.

Step 1: Need. The need for reliability must be anticipated from the beginning. A reliability program can then be justified based on specific system requirements in terms of life-cycle costs and other operational requirements, including market competitiveness, customer needs, societal requirements in terms of safety and public health, liability, and statutory needs.

Step 2: Goals and Definitions. Requirements must be specified in terms of welldefined goals.

Step 3: Concept and Program Planning. Based on reliability and other operational requirements, reliability plans must be developed. Concept and program planning is a very important phase in the life cycle of the system. Fig. 2.14 illustrates that 60…70 % of the life cycle may be determined by the decisions made at the concept stage. Thus, the nature of the reliability programs will also determine the overall effectiveness of the total program.

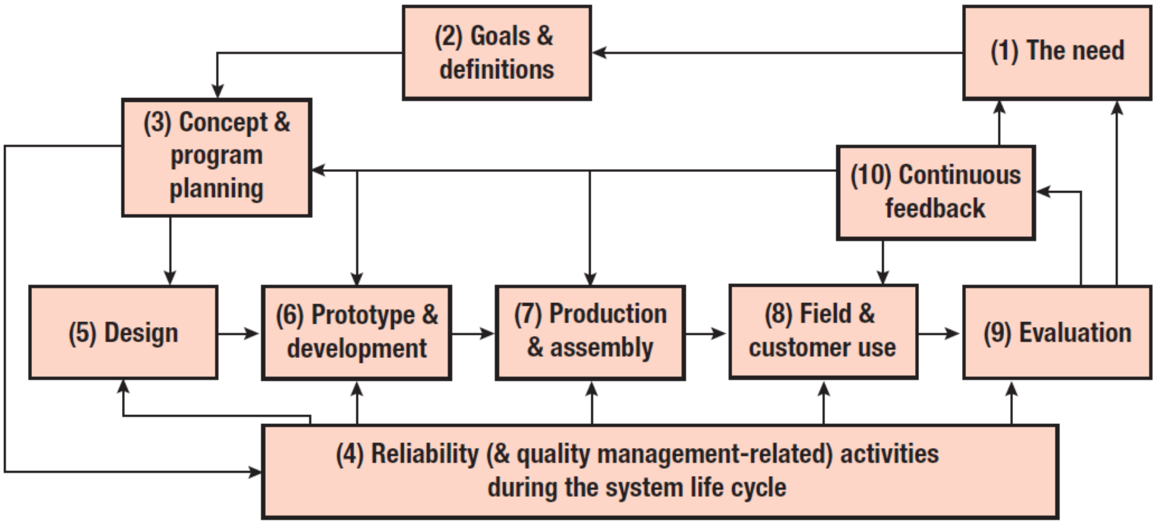

Fig. 2.13. Reliability (and quality management related activities) during system life cycle

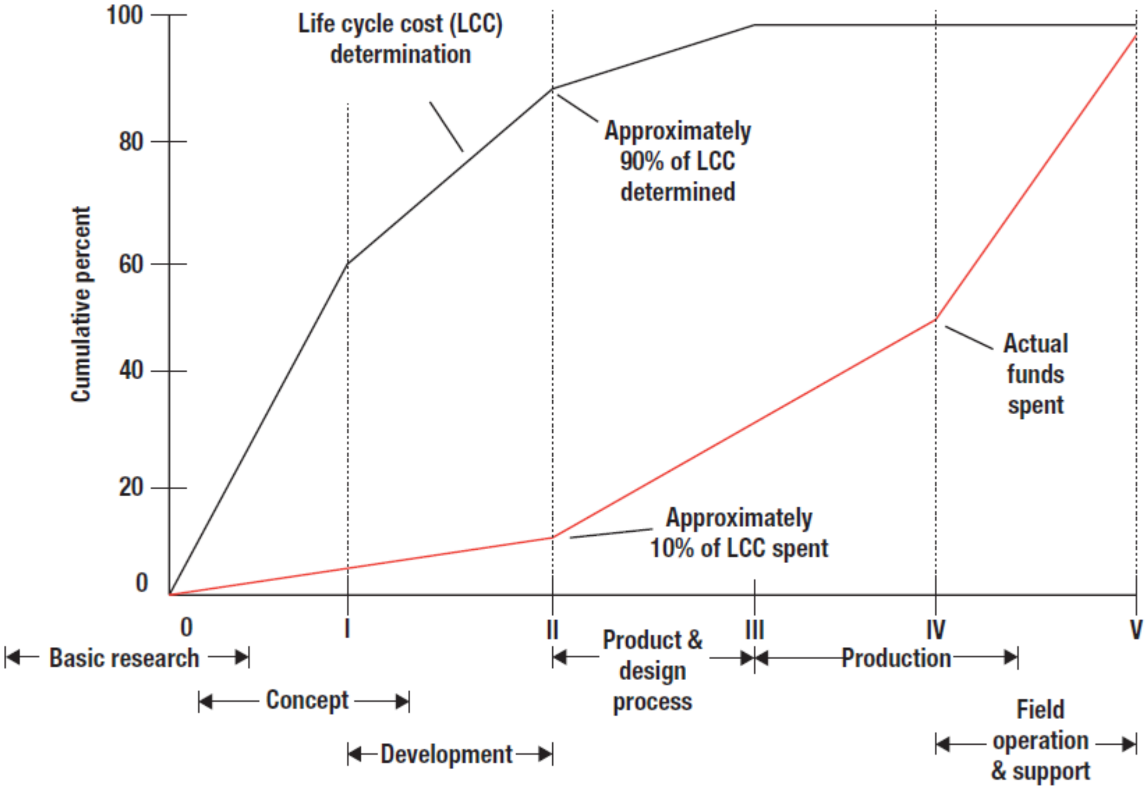

Fig. 2.14. Conceptual relationship of life-cycle cost and different phases of life cycle

Step 4: Reliability and Quality Management Activities. The plans developed in step 3 are implemented, and the total program is continuously monitored in the organization for the life-cycle phases. An organizational chart for the implementation of these plans must exist with well-defined responsibilities. Some guiding principles that can be used for any reliability program and its processes and management include:

- **Customer Focus**. Quality, and reliability as one of its qualities, is defined and evaluated by the customer, and the organization has a constancy of purpose to meet and/or exceed the needs and requirements of the customer (We use the word **customer** in a very broad sense. Anything the system affects is the customer. Thus, in addition to human beings and society, the environmental and future impacts of the product are considered in the program).
- **System Focus**. Emphasis is on system integration, synergy, and the interdependence and interactions of all the parts of the system (hardware, software, human, and other elements). All the tools and methodologies of systems engineering and some of the developments in **Design for Six Sigma** (**DFSS**) are an integral part of this focus.
- **Process Focus**. Design and management of reliability processes should be well developed and managed using cross-functional teams using the methodology of concurrent design and engineering (Fig. 2.15).

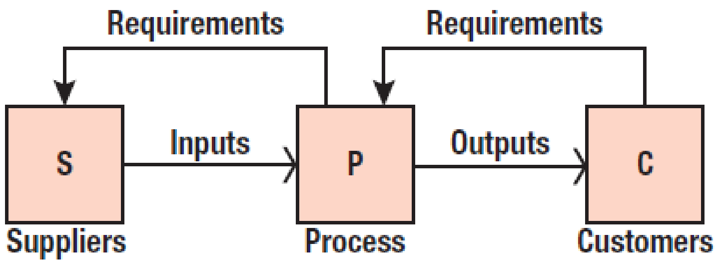

Fig. 2.15. Process development

- **Structure**. The reliability program must understand the relationships and interdependence of all the components, assemblies, and subsystems. High reliability is not an end in itself but is a means to achieve higher levels of customer satisfaction, market share, and profitability. Thus, we should be able to translate reliability metrics to financial metrics that management and customers can understand and use for decision-making processes.
- **Continuous Improvement and Future Focus**. Continuous, evolutionary, and breakthrough improvement is an integral part of any reliability process. The organization should have a philosophy of never-ending improvement and reliance on long-term thinking.
- **Preventive and Proactive Strategies**. The real purpose of reliability assurance processes is to prevent problems from happening. Throughout the

book, we will present many design philosophies and methodologies to achieve this objective.

- **Scientific Approach**. Reliability assurance sciences are based on mathematical and statistical approaches in addition to using all the other sciences (such as the physics, chemistry, and biology of failure). We must understand the causation (cause–effect and means–end relationships), and we should not depend on anecdotal approaches. Data-driven and empirical methods are used for the management of reliability programs.
- **Integration**. Systems thinking includes broader issues related to the culture of the organization. Thus, the reliability program must consider the integration of cultural issues, values, beliefs, and habits in any organization for a quality and productivity improvement framework.

Step 5: Design. Reliability is a design parameter, and it must be incorporated into product development at the design stage. Fig. 2.16 illustrates the importance of design in terms of cost to address or fix problems in the future of the life cycle of the product.

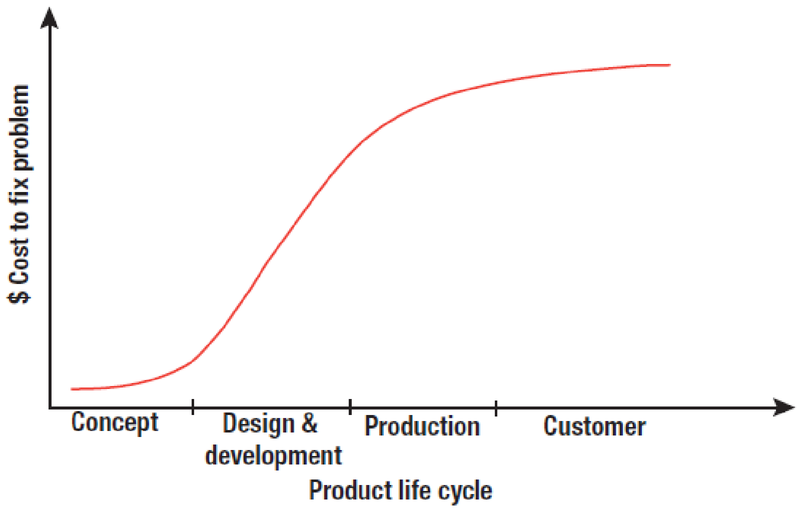

Fig. 2.16. Conceptual illustration of cost to fix problems versus product life cycle

Step 6: Prototype and Development. Prototypes are developed based on the design specifications and life-cycle requirements. The reliability of the design is verified through development testing. Concepts, such as the design and development of reliability test plans, including accelerated testing, are used in this step. If the design has deficiencies, they are corrected by understanding the root failure causes and their effect on the design. After the product has achieved the required levels of reliability, the design is released for production.

Step 7: Production and Assembly. The product is manufactured and assembled based on the design specifications. Quality control methodologies, such as **statistical process control** (**SPC**), are used. One of the objectives of quality assurance programs during this phase of the system is to make sure that the product reliability is not degraded and can be sustained in the field.

Step 8: Field and Customer Use. Before the product is actually shipped and used in the field by customers, it is important to develop handling, service, and, if needed, maintenance instructions. If high operational availability is needed, then a combination of reliability and maintainability will be necessary.

Step 9: Continuous System Evaluation. The product in the field is continuously evaluated to determine whether the required reliability goals are actually being sustained. For this purpose, a reliability monitoring program and field data collection program are established.

Step 10: Continuous Feedback. There must be continuous feedback among all the steps in the life cycle of the product. A comprehensive data gathering and information system is developed. A proper communication system is also developed and managed for all the groups responsible for the various steps. This way, all field deficiencies can be reported to the appropriate groups. This will result in continuous improvement of the product.

2.12. Framework for solving reliability related problems

As can be seen from the list of problems of interest to buyers and to manufacturers, the study of product reliability requires a framework that incorporates many interrelated technical, operational, commercial and management issues. We list some important issues in each of these areas.

Technical issues:

- Understanding of deterioration and failure (material science).
- Effect of design on product reliability (reliability engineering).
- Effect of manufacturing on product reliability (quality variations and control).
- Testing to obtain data for estimating part and component reliability (design of experiments).
- Estimation and prediction of reliability (statistical data analysis).

Operational issues:

- Operational strategies for unreliable systems (operations research).
- Effective maintenance (maintenance management).

Commercial issues:

- Cost and pricing issues (reliability economics).
- Marketing Implications (warranties, service contracts).

Management issues:

- Administration of reliability programs (engineering management).
- Impact of reliability decisions on business (business management).
- Risk to individuals and society resulting from product unreliability (risk theory).

- Effective management of risks from a business point of view (risk management).

The uncertain nature of deterioration and failure implies the need for a suitable framework in which to formulate and solve these problems. The systems approach provides an integrated framework for effectively addressing the issues raised. Fig. 2.17 shows many of the important issues and the disciplines involved in their analysis in a diagrammatic manner.

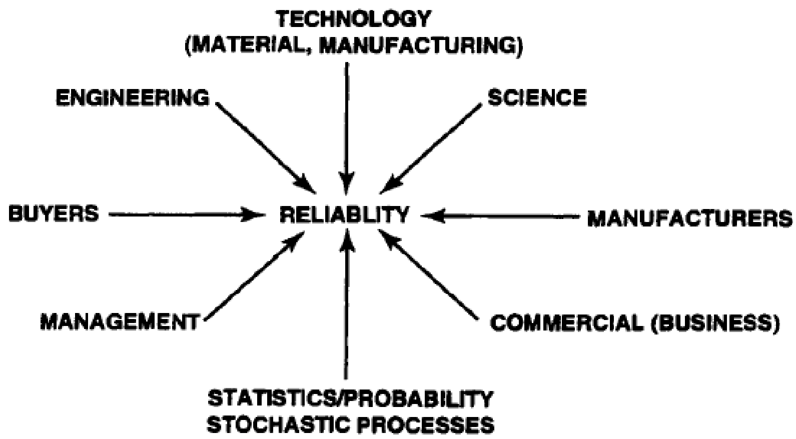

Fig. 2.17. Framework for the study of reliability

The Systems Approach

The **systems approach** to problem solving in the real world involves several stages. These are shown in Fig. 2.18. The execution of each stage requires a good understanding of concepts and techniques from many disciplines.

The key step is characterization of the system in such a way that the details of the system that are relevant to the problem being addressed are made apparent and appropriately modeled. The variables used in the system characterization and the relationships between them depend on the problem. If the problem is to understand system failures, then the variables of the system characterization are from the relevant engineering sciences; if the problem is to study the impact of reliability on sales, then one would use variables from the theory of marketing and economics in the system characterization; and so forth.

For reliability related problems, most of the variables used in the system characterization are dynamic (changing with time) and stochastic (changing in an uncertain manner). The mathematical formulations needed for modeling reliability are obtained from statistics, probability theory, and stochastic processes. It is important to ensure that the model used is adequate for solving the real problem and that adequate and relevant data can be obtained. If not, then the analysis will yield results that are of limited use for solving the problem. In general, obtaining an adequate model requires an iterative approach, wherein changes are made to the simplification and/or the mathematical formulation during each iteration. Adequate and relevant data are obtained by proper testing as well as from other sources. Statistical methods are used in test design and for both parameter estimation and model validation.

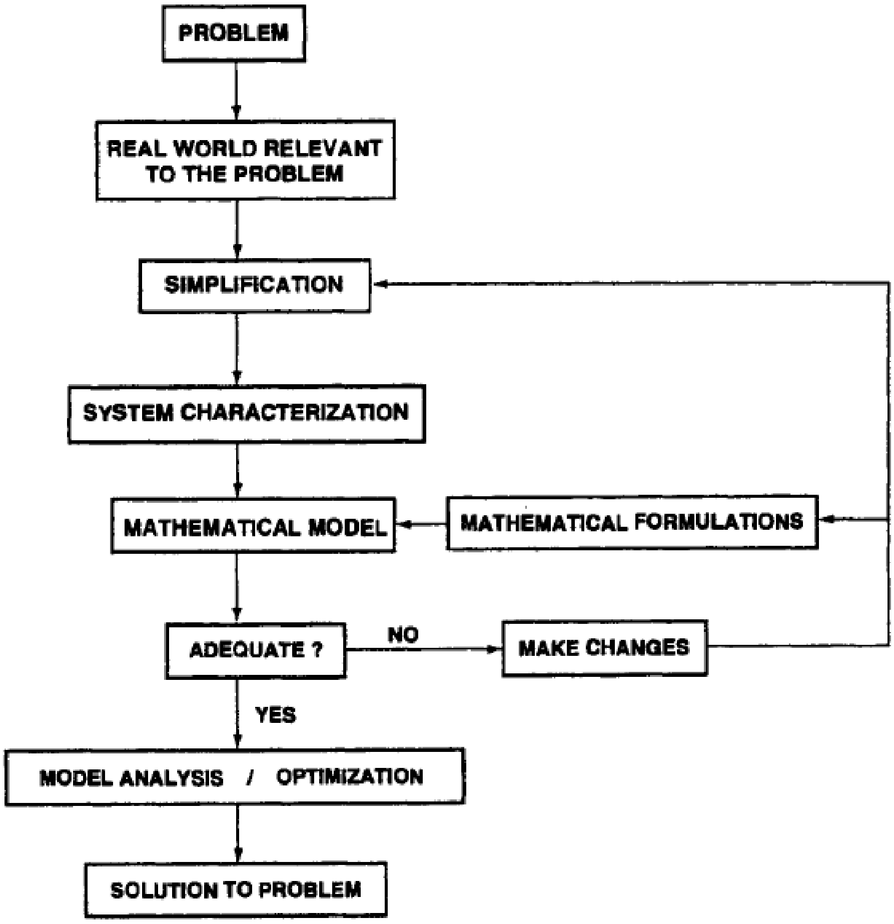

Fig. 2.18. Systems approach

Once an adequate model is developed, techniques from statistics, probability theory and stochastic processes and optimization theory are needed for analysis and optimization. Reliability theory provides the concepts and tools for this purpose.

2.13. Consequences of Failure

There is always a risk of a product failing in the field. For some products, the consequences of failure can be minor, while for others, it can be catastrophic. Possible consequences include financial loss, personal injury, and various intangible costs. Under U.S. law, consequences of product failure may also include civil financial penalties levied by the courts and penalties under statutes, such as the Consumer Product Safety Act, building codes, and state laws. These penalties can include personal sanctions such as removal of professional licenses, fines, and jail sentences.

Financial Loss

When a product fails, there is often a loss of service, a cost of repair or replacement, and a loss of goodwill with the customer, all of which either directly or indirectly involve some form of financial loss. Costs can come in the form of losses in market share due to damaged consumer confidence, increases in insurance rates, warranty claims, or claims for damages resulting from personal injury. If negative press follows a failure, a company's stock price or credit rating can also be affected.

Often, costs are not simple to predict. For example, a warranty claim may include not only the cost of replacement parts, but also the service infrastructure that must be maintained in order to handle failures (Dummer et al. 1997). Repair staff must be trained to respond to failures. Spare parts may be required, which increases inventory levels. Service stations must be maintained in order to handle product repairs.

The cost of failure also often includes financial losses for the customer incurred as a result of failed equipment not being in operation. For some products, this cost may greatly exceed the actual cost of replacing or repairing the equipment.

Breach of Public Trust

The National Society of Professional Engineers notes that "*Engineers, in the fulfillment of their professional duties, shall hold paramount the safety, health, and welfare of the public*" (National Society of Professional Engineers 1964). In many cases, public health, safety, and welfare are directly related to reliability.

Legal Liability

There are a number of legal risks associated with product reliability and failure. A company can be sued for damages resulting from failures. A company can also be sued if they did not warn users of defects or reliability problems. In extreme cases of negligence, criminal charges can be brought in addition to civil damages.

Most states in the United States operate on the theory of strict liability. Under this law, a company is liable for damages resulting from a defect for no reason other than that one exists, and a plaintiff does not need to prove any form of negligence to win their case. Companies have a duty to exercise "ordinary and reasonable care" to make their products safe and reliable. If a plaintiff can prove that a defect or risk existed with a product, that this defect or risk caused an injury, that this defect or risk was foreseeable, and that the company broke their duty of care, damages can be assessed. A defect, for legal purposes, can include manufacturing flaws, design oversights, or inadequacies in the documentation accompanying a product. Thus, almost every job performed by a designer or an engineer can be subjected to legal scrutiny.

Intangible Losses

Depending on the expectations that customers have for a product, relations with customers can be greatly damaged when they experience a product failure. Failures can also damage the general reputation of a company. A reputation for poor reliability can discourage repeat and potential future customers from buying a product, even if the causes of past failures have been corrected.

In some cases, the effects of a lack of reliability can hurt the national psyche, for example, failures in space, military, and transportation applications. The higher the profile of a failure event, the greater the effect is on society. Failures that affect public health and the environment can also create discontent with government and regulatory bodies.

Suppliers and Customers

The rapid pace of technological developments and the globalization of supply chains have made customers dependent upon worldwide suppliers who provide parts (materials), subassemblies, and final products. When customers have to wait until they receive their parts, subassemblies, or products to assess if they are reliable, this can be an expensive iterative process. An upfront evaluation of suppliers is a beneficial alternative. Measuring the reliability capability of a supplier yields important information about the likelihood that a reliable product can be produced (Tiku et al., 2007). Reliability capability can be defined as follows.

Reliability capability is a measure of the practices within an organization that contribute to the reliability of the final product, and the effectiveness of these practices in meeting the reliability requirements of customers.

To obtain optimal reliability and mutually beneficial results, suppliers and customers in the supply chain should cooperate. The IEEE Reliability Program Standard 1332 (IEEE Standards Project Editors 1998) identifies three reliability objectives between suppliers and customers:

- The supplier, working with the customer, should determine and understand the customer's requirements and product needs so that a comprehensive design specification can be generated.
- The supplier should structure and follow a series of engineering activities so that the resulting product satisfies the customer's requirements and product needs with regard to product reliability.
- The supplier should include activities that assure the customer that reliability requirements and product needs have been satisfied.

Summary

Reliability pertains to the ability of a product to perform without failure and within specified performance limits for a specified time in its life-cycle application conditions. Performance and quality are related to reliability. Reliability engineering deals with preventing, assessing, and managing failures. The tools of reliability engineers include statistics, probability theory, and many fields of engineering and the sciences related to the problem domain.

Chapter 3
QUANTITATIVE RELIABILITY

The term reliability of power supply system is generally used to relate to the ability of this system to perform its intended function which is to supply electricity to a customer. The term is also used in a more definite sense as one of the measures of reliability and indicates the probability of not failing by the end of a certain period of time, called the mission time. In this book, this term will be used in the former sense unless otherwise indicated. In a qualitative sense, planners and designers are always concerned with reliability, but the qualitative sense does not help us understand and make decisions while dealing with complex situations. However, when defined quantitatively it becomes a parameter that can be traded off with other parameters, such as cost and emissions.

There can be many reasons for quantifying reliability. In some situations, we want to know what the reliability level is in quantitative measures. In power system applications, we want to know what the reliability actually is, as we are risking lives. In commercial applications, reliability has a definite trade-off with cost. So we want to have a decision tool for which reliability needs to be quantified. The following example will illustrate this situation.

Example 3.1. A system has a total load of 500 MW. The following options are available for satisfying this load, which is assumed constant for simplicity:

5 generators, each with 100 MW;
6 generators, each with 100 MW;
12 generators, each with 50 MW.

The question we need to answer in terms of design and operation aspect is: *Which of these alternatives has the best reliability?*

A little thinking will show that there is no way to answer this question without some additional data on the stochastic behavior of these units, which are failure and repair characteristics. After we obtain this data, models can be built to quantify the reliability for these three cases, and then the question can be answered.

3.1. General characteristics of quantitative reliability

Most of the applications of reliability modeling are in the steady state domain or in the sense of an average behavior over a long period of time. If we describe the system behavior at any instance of time by its state, the collection of possible states that the system may assume is called the state space, denoted by S.

In reliability analysis, one can classify the system state into two main categories, success or failure states. In success states the system is able to do its intended function, whereas in the failed states it cannot. We are mostly concerned with how the system behaves in failure states. The basic indexes used to characterize this domain are as follows.

Probability of failure

Probability of failure, denoted by p_f, is the steady state probability of the system being in the failed state or unacceptable states. It is also defined as the long run fraction of the time that system spends in the failed state. The probability of system failure is easily found by summing up the probability of failure states as shown in (3.1):

$$p_f = \sum_{i \in Y} p_i \, , \tag{3.1}$$

where p_f system unavailability or probability of system failure; Y set of failure states, $Y \subset S$; S system state space.

Frequency of failure

Frequency of failure, denoted by f_f, is the expected number of failures per unit time, e. g., per year. This index is found from the expected number of times that the system transits from success states to failure states. This index can be easily obtained by finding the expected number of transitions across the boundary of subset Y of failure states.

Mean cycle time

Mean cycle time, denoted by T_f, is the average time that the system spends between successive failures and is given by (3.2). This index is simply the reciprocal of the frequency index:

$$T_f = \frac{1}{f_f} \, . \tag{3.2}$$

Mean down time

Mean down time, denoted by T_D, is the average time spent in the failed states during each system failure event. In other words, this is the expected time of stay in Y in one cycle of system up and down periods. This index can be found from (3.3):

$$T_D = \frac{p_f}{f_f} \, . \tag{3.3}$$

Mean up time

Mean up time, denoted by T_U, is the mean time that the system stays in the up states before system failure and is given by (3.4):

$$T_U = T_f - T_D \, . \tag{3.4}$$

There are several other indices that can be obtained as a function of the above indices.

There are also applications in the time domain, say [0, *T*]. For example, at time 0, we may be interested in knowing the probability of not having sufficient generation at time *T* in helping decide the start of additional generation. The following indices could be used in such situations:

1. **Probability of failure at time *T***. This indicates the probability of being in the failed state at time *T*. This does not mean that the system did not fail before time *T*. The system may have failed before *T* and repaired, so this only indicates the probability of the system being in a failed state at time *T*.
2. **Reliability for time *T***. This is the probability that the system has not failed by time *T*.
3. **Interval frequency over [0, *T*]**. This is the expected number of failures in the interval [0, *T*].
4. **Fractional duration**. This is the average probability of being in the failed state in interval [0, *T*].

The most commonly computed reliability measures can be categorized as three indices as follows.

1. **Expected value indexes**: These indices involve **Expected Power Not Supplied** (**EPNS**) or **Expected Unserved Energy** (**EUE**).
2. **Probability indices** such as **Loss of Load Probability** (**LOLP**) or **Loss of Load Expectation** (**LOLE**).
3. **Frequency and duration indices** such as **Loss of Load Frequency** (**LOLF**) or **Loss of Load Duration** (**LOLD**).

3.2. Basic Approaches for Considering Reliability in Decision-Making

Having quantified the attributes of reliability, the next step is to see how it can be included in the decision process. There are perhaps many ways of doing it, but the most commonly used are described in this section. It is important to remember that the purpose of reliability modeling and analysis is not always to achieve higher reliability but to attain the required or optimal reliability.

Reliability as a constraint

Reliability can be considered a constraint within which other parameters can be changed or optimized. Until now this is perhaps the most common manner in which reliability considerations are implemented. For example, in generation reliability there is a widely accepted criterion of loss of load of one day in 10 years.

Reliability as a component of overall cost optimization

The conceptual relationship between cost and reliability can be appreciated from Fig. 3.1. The overall cost is a combination of the investment cost and the cost of failures to the customers. The investment cost would tend to increase if we are interested in higher levels of reliability. The cost of failures to the customers, on the other hand, tends to decrease with increased level of reliability. If we combine these costs, the total cost is shown by the solid curve, which has a minimum value. The reliability at this minimum cost may be considered an optimal level; points to the left of this would be dominated by customer dissatisfaction, while points to the right may be dominated by investment cost considerations.

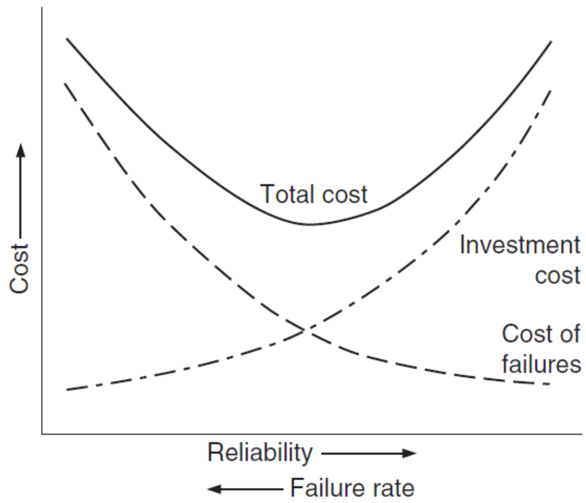

Fig. 3.1. Trade-off between reliability and cost

It can be appreciated that in this type of analysis we need to calculate the worth of reliability. In other words, how much do the customers think that interruptions of power cost them? One way of doing this is through customer damage function, like the one shown in Fig. 3.2.

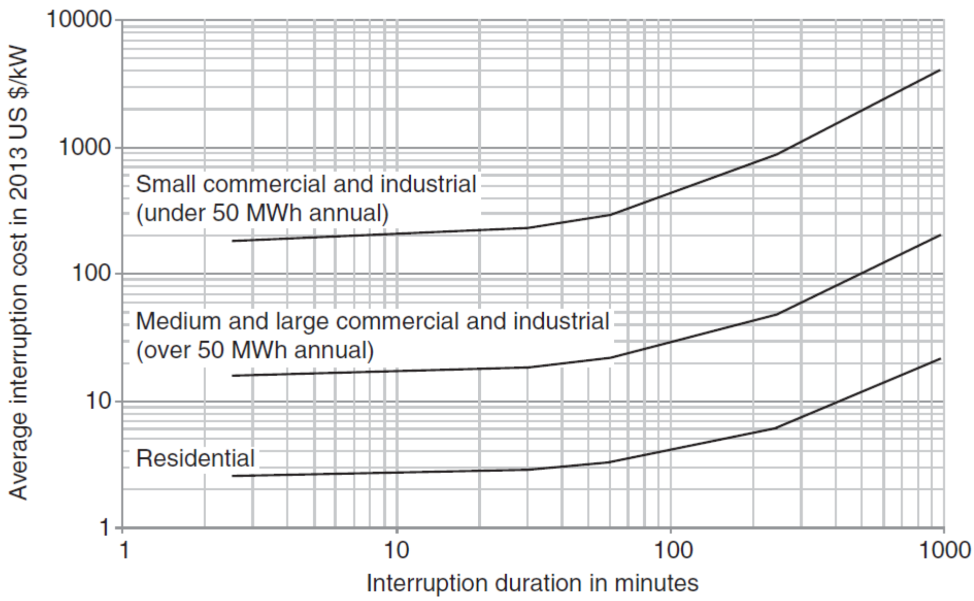

Fig. 3.2. Customer damage function

The customer damage function provides the relationship between the duration of outage and the interruption cost in \$/kW. The damage function is different depending on the type of customer. The damage function is clearly nonlinear with respect to the duration, increasing at much higher rates for longer outages. The frequency and duration indices defined earlier can be combined to yield the cost of interruptions using (3.5):

$$IC = \sum_{i=1}^{n} L_i f_i c_i(d_i), \tag{3.5}$$

where n – number of load points in the system; L_i – load requirement at load point i in kW; f_i – failure frequency at load point i in number of occurrence per year; $c_i(d_i)$ – customer damage function at load point i in \$ per kW in terms of outage duration d_i; d_i – outage duration at load point i in hours.

Multi objective optimization and pareto-optimality

Generally there are conflicting objectives to be satisfied or optimized. For example, cost and reliability are conflicting objectives. Multi objective-optimization, also known as multi criteria or multi attribute optimization, is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. In multi objective optimization, Pareto-optimal solutions are usually derived, where the improvement of an objective will inevitably deteriorate at least another one. An example can be seen in Fig. 3.3 – given that lower values are preferred to higher values, point C is not on the Pareto frontier because it is dominated by both point A and point B; and points A and B are non inferior.

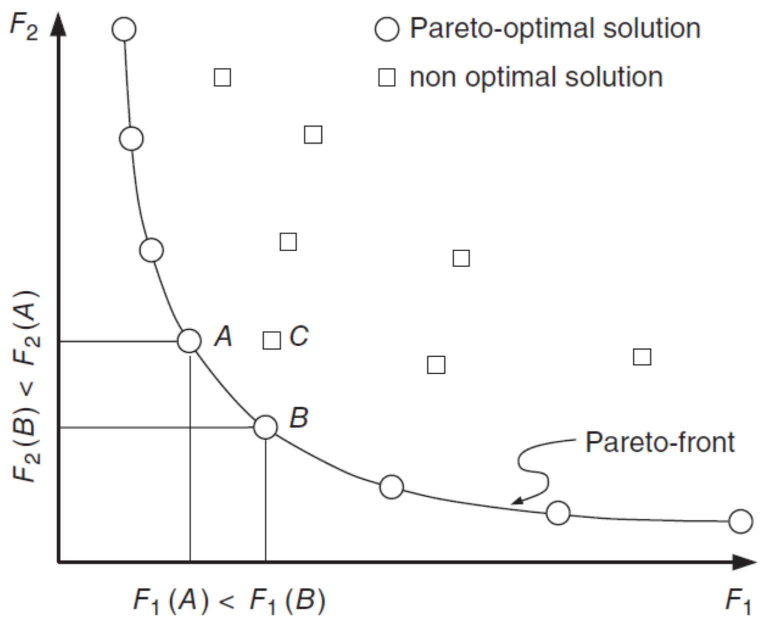

Fig. 3.3. Multi objective optimization

3.3. Random Variables

A **random variable** is a real-valued function that assigns numerical values to all outcomes in the state space. A random variable can take on either countable real values or continuous real values. We call the random variables with countable values discrete random variables, and those with continuous real values continuous random variables. For example:

Discrete random variable: Number of failed transmission lines in the system with the state space of two transmission lines shown in Fig. 3.4.

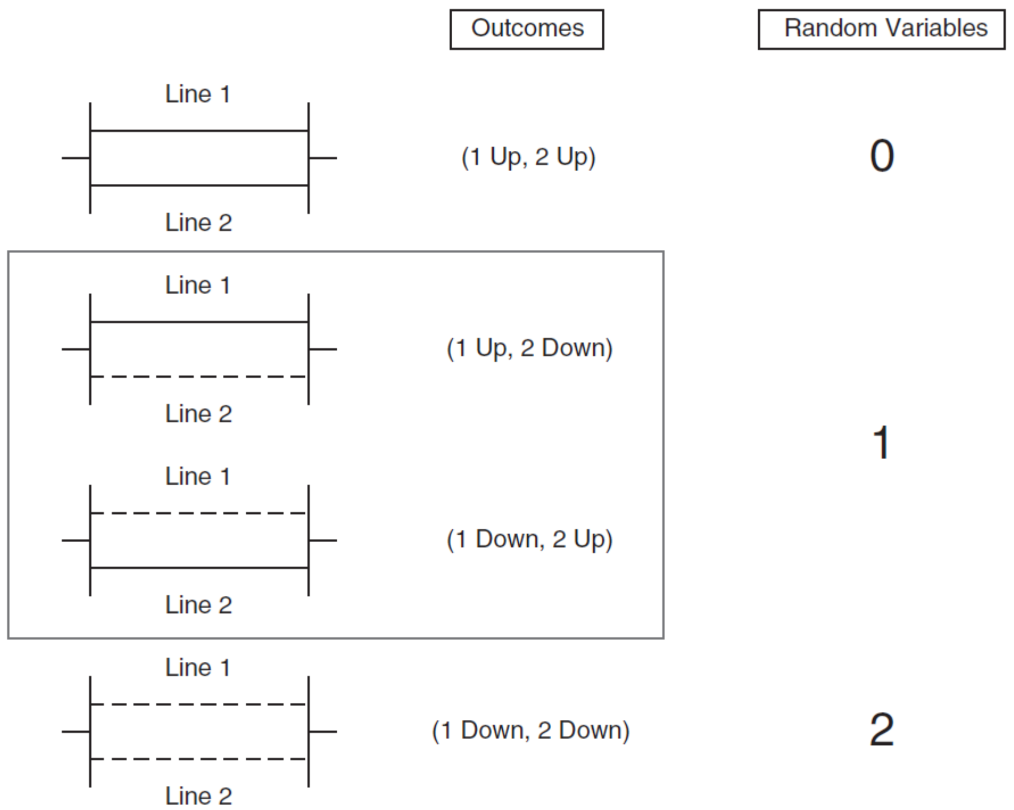

Fig. 3.4. Example of discrete random variable

Since each outcome is associated with a probability measure, we can assign probabilities to any possible value of the random variable. For example, consider the system of two transmission lines if the random variable is the number of failed transmission lines and the probability of the outcomes is the same as given in Example 1.3. Then

$P\{X = 0\} = P(\{(1U, 2U)\}) = 0.81,$

$P\{X = 1\} = P(\{(1U, 2D), (1D, 2U)\}) = 0.9 + 0.9 = 0.18,$

$P\{X = 2\} = P(\{(1D, 2D)\}) = 0.01.$

Note that $P\{X = 0\} + P\{X = 1\} + P\{X = 2\} = 1.$

Continuous random variable: Time to failure of a generator shown in Fig. 3.5.

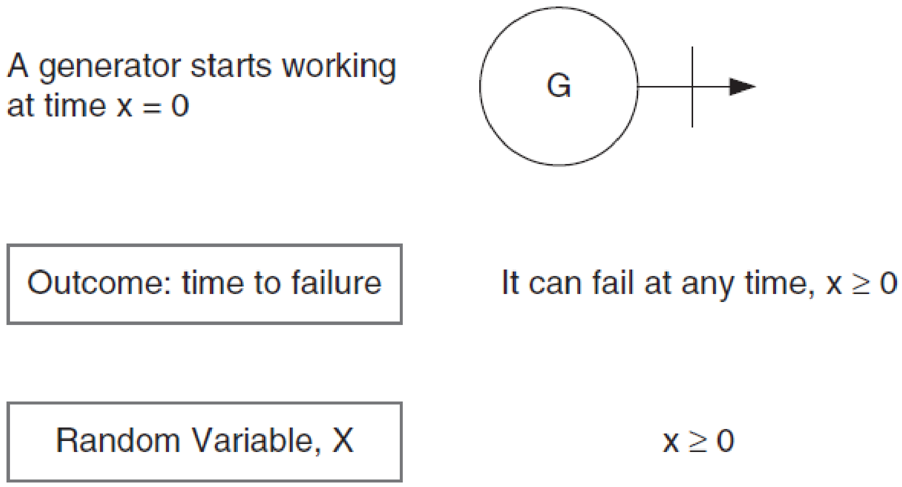

Fig. 3.5. Example of continuous random variable

3.4. Probability Density Function

A discrete random variable assumes only countable values of x_1, x_2, …, x_n from the set of real numbers. The function that gives probabilities associated with all possible values of a discrete random variable, X, is called **probability mass function**, denoted by

$$p(x) = P\{X = x\}.$$

The following are the properties of probability mass function of a discrete random variable:

1. $0 \le p(x_i) \le 1,\ i = 1, 2, \ldots, n$.
2. $p(x_i) = 0$ if $i \notin \{1, 2, \ldots, n\}$.
3. $\sum_{i=1}^{\infty} p(x_i) = \sum_{i=1}^{\infty} P\{X = x_i\} = 1.$

A graphical representation of a probability mass function is shown in Fig. 3.6. In this example, a discrete random variable can assume any of the values of 1, 2, 3, 4 or 5 with 0.1, 0.2, 0.3, 0.3 and 0.1 probabilities accordingly.

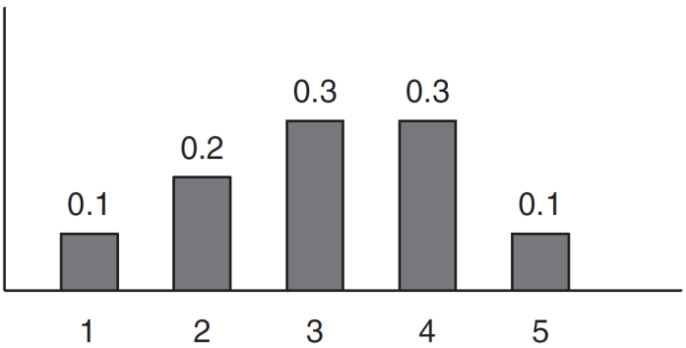

Fig. 3.6. Example of probability mass function of a discrete random variable

Similarly, for a continuous random variable X, we define a non negative function, $f(x)$, called a **probability density function** or **pdf**, for all real numbers $x \in (-\infty, \infty)$. A function $f(x)$ has to satisfy the criteria that for any set A of real numbers $x \in A$,

$$\int_A f(x)dx = P\{X \in A\}. \tag{3.6}$$

A continuous random variable assigns real values to all outcomes in the state space; thus the probability of all real values has to add up to one, which is given as

$$\int_{-\infty}^{\infty} f(x)dx = P\{X \in (-\infty, \infty)\} = 1. \tag{3.7}$$

Example 3.2. Let X denote a continuous random variable representing time to failure (in days) of a generator by probability density function $f(x)$; what is the probability that a generator will fail during the 2nd and 3rd days, $A = [2, 3]$?

This probability can be found from

$$P\{X \in [2,3]\} = P\{2 \le X \le 3\} = \int_2^3 f(x)dx.$$

Note that with this definition, the probability that a continuous random variable will assume any particular value a will be zero since $P\{X = a\} =$ $= \int_a^a f(x)dx = 0$.

3.5. Probability Distribution Function

A random variable can also be characterized by a **cumulative distribution function** or **distribution function** or **cdf**, denoted by $F(a)$, which gives the probability that a random variable takes on value less than or equal to a real number a.

$$F(a) = P\{X \le a\}. \tag{3.8}$$

For a discrete random variable,

$$F(a) = \sum_{x_i \le a} P\{X = x_i\} = \sum_{x_i \le a} p(x_i). \tag{3.9}$$

A graphical representation of a distribution function is shown in Fig. 3.7. In this example, a discrete random variable can assume values of 1, 2, 3, 4 or 5 with corresponding probabilities of 0.1, 0.2, 0.3, 0.3 and 0.1.

For a continuous random variable,

$$F(a) = P\{X \in (-\infty, a)\} = \int_{-\infty}^{a} f(x)dx. \tag{3.10}$$

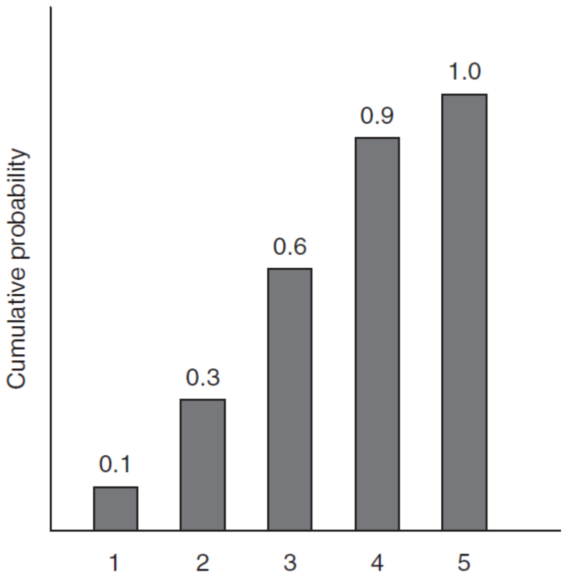

Fig. 3.7. Example of distribution function of a discrete random variable

From the above expression, a distribution function $F(a)$ is a non decreasing function of a, $\lim_{a\to\infty}F(a) = F(\infty) = P\{X \in (-\infty, \infty)\} = 1$ and $\lim_{a\to-\infty}F(a) = F(-\infty) = P\{X \in (-\infty, -\infty)\} = 0$.

Example 3.3. Let X denote a continuous random variable representing time to failure (in days) of a generator by a distribution function $F(x)$; what is the probability that a generator will fail during the 2nd and 3rd days, $A = [2,3]$.

This probability can be found from $P\{X \in [2, 3]\} = P\{2 \le X \le 3\} = P\{X \in (-\infty, 3)\} - P\{X \in (-\infty, 2)\} = F(3) - F(2)$.

From (3.10), if we differentiate both sides, we have

$$\frac{dF(a)}{da} = F'(a) = f(a). \tag{3.11}$$

This means that the probability density function can be found from differentiating the distribution function. Equivalently, we can also write

$$f(a) = \lim_{\Delta a \to 0} \frac{F(a + \Delta a) - F(a)}{\Delta a} = \lim_{\Delta a \to 0} \frac{P\{a \le X \le a + \Delta a\}}{\Delta a}. \tag{3.12}$$

3.6. Survival Function

Consider a continuous random variable representing, for example, time to failure of a component. The probability density function $f(x)$ of this random variable can give the probability of failure at a certain time. The probability distribution function $F(a)$ gives the probability that it will fail within time a. In reliability analysis, it is sometimes more interesting to know the probability that the component will fail beyond a specified time. In this case it is more convenient to work with the complementary of the distribution function called **survival function**.

Survival function, denoted by $R(a)$, gives the probability of a component surviving beyond time a.

$$R(a) = P\{X > a\}, \tag{3.13}$$

when X is a random variable of time to failure of the component.

Survival function can be determined from either the probability density function or the probability distribution function, as shown below:

$$R(a) = P\{X > a\} = 1 - P\{X \leq a\} = 1 - F(a); \tag{3.14}$$

$$R(a) = P\{X > a)\} = \int_a^{\infty} f(x)dx. \tag{3.15}$$

It follows from (3.11) that

$$\frac{d(1 - R(a))}{da} = -R'(a) = f(a). \tag{3.16}$$

Any one of the density function, distribution function or survival function can be determined from the others. In other words, we can denote a random variable with one of the three functions interchangeably.

3.7. Hazard Rate Function

The failure of a population of products (electrical equipment) can arise from inherent design weaknesses, manufacturing- and quality control-related problems, variability due to customer usage, the maintenance policies of the customer, and improper use or abuse of the product. The **hazard rate**, $h(t)$, is the number of failures per unit time per number of nonfailed products remaining at time t. An idealized (though rarely occurring) shape of the hazard rate of a product is the bathtub curve (Fig. 3.8). A brief description of each of the three regions is given in the following:

1. **Infant Mortality Period**. The product population exhibits a hazard rate that decreases during this first period (sometimes called "burn-in," "infant mortality," or the "debugging period"). This hazard rate stabilizes at some value at time t_1 when the weak products in the population have failed. Some manufacturers provide a burn-in period for their products, as a means to eliminate a high proportion of initial or early failures.

2. **Useful Life Period**. The product population reaches its lowest hazard rate level and is characterized by an approximately constant hazard rate, which is often referred to as the "constant failure rate." This period is usually considered in the design phase.

3. **Wear-Out Period**. Time t_2 indicates the end of useful life and the start of the wear-out phase. After this point, the hazard rate increases. When the hazard rate becomes too high, replacement or repair of the population of products should be conducted. Replacement schedules are based on the recognition of this hazard rate.

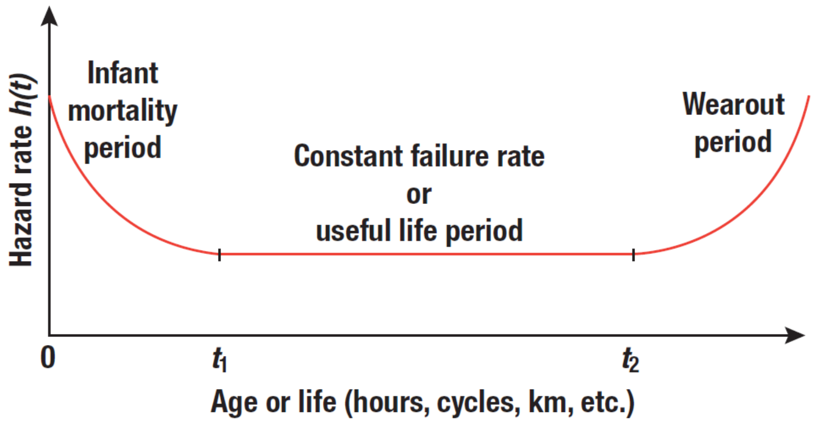

Fig. 3.8. Idealized bathtub hazard rate curve

Optimizing reliability must involve the consideration of the actual life-cycle periods. The actual hazard rate curve will be more complex in shape and may not even exhibit all of the three periods.

Thus, **hazard rate function**, denoted by $h(a)$ is widely used to describe a random variable X representing time to failure of a component.

A hazard rate function is a function that gives a rate at time a at which a component fails given that it has survived for time a. This function is the rate of a conditional probability of failure at time a and is given by (3.17):

$$h(a) = \lim_{\Delta a \to 0} \frac{P\{a \le X \le a + \Delta a \mid X > a\}}{\Delta a}. \tag{3.17}$$

As $\Delta a \to 0$, the hazard rate function can be written as $h(a)\Delta a = P\{a \le X \le a + \Delta a | X > a\}$. This gives a conditional probability of a random variable taking a value in the interval $[a, a + \Delta a]$ given that the value is greater than a. When X represents time to failure of a component, $h(a)\Delta a$ gives the probability that a component will fail during interval $[a, a + \Delta a]$ given that it has been working (not failed) up to time a.

Depending on the context of usage, a hazard rate function is known by a variety of names, such as age specific failure rate, failure rate, repair rate and force of mortality. We can find this function from the density and survival function as follows. Using conditional probability rule,

$$\begin{aligned} h(a) &= \lim_{\Delta a \to 0} \frac{P\{a \le X \le a + \Delta a \cap X > a\}}{\Delta a} \times \frac{1}{P\{X > a\}} = \\ &= \lim_{\Delta a \to 0} \frac{P\{a \le X \le a + \Delta a\}}{\Delta a} \times \frac{1}{P\{X > a\}} = \frac{f(a)}{R(a)}. \end{aligned} \tag{3.18}$$

Using (3.16), we have

$$h(a) = \frac{-R'(a)}{R(a)} = \frac{-d}{da}\left[\ln R(a)\right]. \tag{3.19}$$

Integrating (3.19) yields

$$R(a) = e^{-\int_0^a h(x)dx}. \tag{3.20}$$

In addition,

$$f(a) = h(a)e^{-\int_0^a h(x)dx}. \tag{3.21}$$

Equations (3.20) and (3.21) allow us to uniquely determine the probability density function and survival function from the hazard rate function. All three functions can be used to calculate one another. Their mathematical relationship can be derived and is shown in Fig. 3.9.

3.8. Jointly Distributed Random Variables

The previous sections only consider a single random variable. Sometimes we need to consider the joint probabilities behavior of two or more random variables. As an example, consider a power system consisting of generators and transmission lines. In this case, both generating capacity and transmission line capabilities exhibit probabilistic behavior. If we need to find out the total available capacity of the system, we therefore need to describe the two uncertainties using two random variables with some distribution functions. This leads us to consider the situation of two or more random variables.

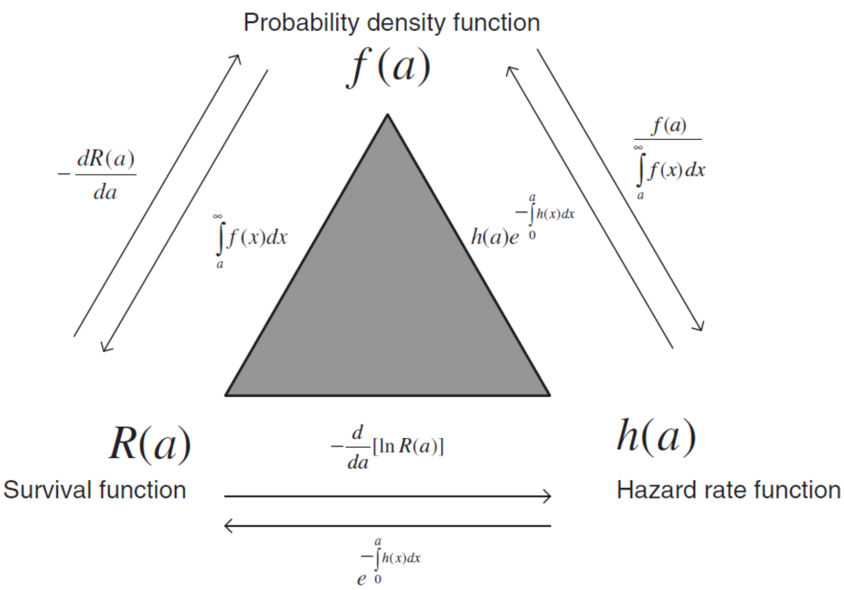

Fig. 3.9. Triangle defining relationship between density function, survival function and hazard rate function

Consider two random variables X and Y. We define a **joint probability distribution function** of these two random variables as follows:

$$F(a, b) = P\{X \le a,\ Y \le b\}, \tag{3.22}$$

where $a, b \in (-\infty, \infty)$.

Similar to the single random variable cases, we define the joint probability density function as follows. For discrete case random variables X and Y, the joint probability density function is given below:

$$p(x, y) = P\{X = x,\ Y = y\}. \tag{3.23}$$

For continuous random variables X and Y, we define a non negative joint probability density function, $f(x, y)$, for all real numbers $x, y \in (-\infty, \infty)$. A function $f(x, y)$ has to satisfy the criteria that for any set A, B of real number $x \in A$, $y \in B$,

$$\int_B \int_A f(x, y)dxdy = P\{X \in A, Y \in B\}. \tag{3.24}$$

If we know the joint probability density function of X and Y, we can find a probability density function of X since $y \in (-\infty, \infty)$, we have

$$f(x) = \int_{-\infty}^{\infty} f(x, y)dy. \tag{3.25}$$

Similarly, a probability density function of Y is

$$f(y) = \int_{-\infty}^{\infty} f(x, y)dx. \tag{3.26}$$

If the two random variables are independent, then from (3.27) we can write $F(a, b) = P\{X \le a,\ Y \le b\} = P\{X \le \mathrm{a} \cap\ Y \le b\} = P\{X \le a\}P\{Y \le b\}$, which also implies $f(x, y) = f(x)f(y)$.

3.9. Expectation, Variance, Covariance and Correlation. MTTF and MTBF

A random variable can be expressed by density function, survival function or hazard rate function. These functions yield the probability associated with a real valued variable representing different outcomes in the state space. It is sometimes of interest to represent a random variable X by a single value. This value is called **expectation** or **expected value**, denoted by $E[X]$ or μ.

An expected value is an average of real possible values that a random variable assumes randomly for a long run experiment. For a discrete random variable X, we have

$$E[X] = \sum_i x_i P\{X = x_i\}. \tag{3.27}$$

This expectation is therefore the weighted sum of all discrete real values x_i, each x_i weighted by the probability of X assuming the value x_i.

For a continuous random variable X having density function $f(x)$, we have

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx. \tag{3.28}$$

It can be verified from both (3.27) and (3.28) that for a random variable X, $E[aX + b] = aE[X] + b$. The expected value of a summation of random variable can be found as follows:

$$E\left[\sum_i X_i\right] = \sum_i E[X_i]. \tag{3.29}$$

Suppose that we want to calculate the expected value of a function $g(.)$ of a random variable X. Then this function is also a random variable, $g(X)$. We can find the expected value of $g(X)$ as follows.

For a discrete random variable,

$$E[g(X)] = \sum_i g(x_i)P\{X = x_i\}. \tag{3.30}$$

For a continuous random variable

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx. \tag{3.31}$$

The expected value only gives a single real value to describe a random variable, but the two random variables having the same expected value may exhibit different behavior of variability. **Variance**, denoted by $Var(X)$ or σ^2 of a random variable X, is measured as a squared distance of a real value from its expected value $E[X]$. A formula for $Var(X)$ is as follows.

$$Var(X) = E[(X - E[X])^2]. \tag{3.32}$$

For a discrete random variable,

$$Var[X] = \sum_i (x_i - E[X])^2 P\{X = x_i\}. \tag{3.33}$$

For a continuous random variable,

$$Var[X] = \int_{-\infty}^{\infty} (x - E[X])^2 f(x)dx. \tag{3.34}$$

From (3.32), we can also find the variance from the following

$$\begin{aligned} Var(X) &= E[X^2] - 2E[XE[X]] + E[(E[X])^2] = \\ &= E[X^2] - 2E[X]E[X] + (E[X])^2 = E[X^2] - (E[X])^2. \end{aligned} \tag{3.35}$$

Variance is a measure of weighted deviations from the average value of a random variable. If the random variable assumes real values that move away from μ with high probability, the variance will be large. On the other hand, the variance will be small if the random variable assumes real values that lie closer to the average value μ.

The square root of a variance is called standard deviation, denoted by σ. We can also find the expected value of the jointly distributed random variables. Consider two random variables X and Y with a joint probability density function $f(x, y)$. We can calculate the expected value of a function $g(X, Y)$ of the two random variables as follows.

For discrete X and Y,

$$E[g(X,Y)] = \sum_i \sum_j g(x_i, y_j) P\{X = x_i, Y = y_i\}. \qquad (3.36)$$

For continuous X and Y,

$$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) dxdy. \qquad (3.37)$$

If $g(X, Y) = aX + bY$, we can find the expectation from (3.39):

$$\begin{aligned} E[g(X,Y)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (ax+by) f(x,y) dxdy = \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} axf(x,y) dxdy + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} byf(x,y) dxdy = \\ &= a \int_{-\infty}^{\infty} xf(x) dx + b \int_{-\infty}^{\infty} yf(y) dy = aE\left[X\right] + bE\left[Y\right]. \end{aligned} \qquad (3.38)$$

In general, for n random variables we can write:

$$E[a_1X_1 + a_2X_2 + \ldots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \ldots + a_nE[X_n]. \qquad (3.39)$$

If $g(X, Y) = XY$, we have:

$$E[X,Y] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x,y) dxd. \qquad (3.40)$$

If X and Y are independent, then $f(x, y) = f(x)f(y)$, and the expectation will be as follows:

$$\begin{aligned} E[X,Y] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x) f(y) dxdy = \\ &= \left(\int_{-\infty}^{\infty} xf(x) dx \right) \left(\int_{-\infty}^{\infty} yf(y) dy \right) = E\left[X\right] E\left[Y\right]. \end{aligned} \qquad (3.41)$$

In the case of two random variables X and Y with a joint probability density function $f(x, y)$, we are also interested in seeing the deviations of the two random variables from their respective expected value. This measure is called **covariance**, denoted by $Cov(X, Y)$. We can find the covariance from the following expression:

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])]. \qquad (3.42)$$

Equivalently,

$$\begin{aligned} Cov(X, Y) &= E[(X - E[X])(Y - E[Y])] = \\ &= E[XY - YE[X] - XE[Y] + E[X]E[Y]] = \\ &= E[XY] - E[X]E[Y] - E[Y]E[X] + E[X]E[Y] = \\ &= E[XY] - E[X]E[Y]. \end{aligned} \tag{3.43}$$

Note that if X and Y are independent, $E[XY] = E[X]E[Y]$, and $Cov(X, Y) = 0$.

For jointly discrete random variables,

$$Cov(X,Y) = \sum_i \sum_j \left(x_i - E[X]\right)\left(y_j - E[Y]\right) P\{X = x_i, Y = y_i\}. \tag{3.44}$$

For jointly continuous random variables

$$Cov(X,Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(x - E[X]\right)\left(y - E[Y]\right) f(x,y)\,dxdy. \tag{3.45}$$

The covariance gives the tendency that the two variables will vary together. This means that if X and Y move in the same direction, both deviations will be in the same sign, either positive or negative, and the resulting covariance will be positive. If X and Y vary in the opposite direction, then the deviation of each variable will be in different sign, and the covariance will be negative. However, when X and Y are independent, the covariance will be zero. It should also be noted from (3.32) that $Var(X) = E[(X - E[X])^2]$; this means that $Var(X) = Cov(X, X)$.

For any random variables X, Y and Z we can also write

$$\begin{aligned} Cov(X, Y + Z) &= E[X(Y + Z)] - E[X]E[Y + Z] = \\ &= E[XY] + E[XZ] - E[X]E[Y] - E[X]E[Z] = \\ &= Cov(X, Y) + Cov(X, Z). \end{aligned} \tag{3.46}$$

We can use this property to calculate variance of sum of random variables X_i as follows:

$$\begin{aligned} Var\left(\sum_i X_i\right) &= Cov\left(\sum_i X_i, \sum_j Y_i\right) = \sum_i \sum_j Cov\left(X_i, Y_i\right) = \\ &= \sum_i Cov\left(X_i, X_i\right) + 2\sum_i \sum_{j<i} Cov\left(X_i, X_j\right) = \\ &= \sum_i Var\left(X_i\right) + 2\sum_i \sum_{j<i} Cov\left(X_i, X_j\right). \end{aligned} \tag{3.47}$$

Note that when all X_i are independent, then $Cov(X_i, X_j) = 0$, and $Var\left(\sum_i X_i\right) = \sum_i Var\left(X_i\right)$.

We can use another dimensionless quantity called **correlation coefficient** to measure the tendency of two random variables. The correlation coefficient, denoted $Corr(X, Y)$ or $\rho_{X,Y}$, is defined below:

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}. \tag{3.48}$$

It can be shown from Cauchy-Schwarz inequality that the correlation coefficient lies in the range [−1, 1]. The correlation coefficient can be thought of as the covariance of two random variables being normalized by the product of standard deviation of the two random variables. The correlation coefficient can only indicate linear dependence of two random variables. If the two random variables are independent, the covariance will be zero and the correlation coefficient will also be zero. However, the reverse is not true i.e. if the correlation coefficient is zero, it does not imply that the two random variables are independent.

For a given underlying probability density function, **the mean time to failure** (**MTTF**) is the expected value for the time to failure. It is defined as

$$E[T] = \text{MTTF} = \int_0^\infty t f(t) dt. \tag{3.49}$$

It can also be shown that MTTF is equivalent to

$$\text{MTTF} = \int_0^\infty R(t) dt. \tag{3.50}$$

Thus, $E[T]$ is the first moment or the center of gravity of the probability density function (like the fulcrum of a seesaw). $E[T]$ is also called the **mean time between failures** (**MTBF**), when the product exhibits a constant hazard rate; that is, the failure probability density function is an exponential.

The MTTF should be used only when the failure distribution function is specified, because the value of the reliability function at a given MTTF depends on the probability distribution function used to model the failure data. Furthermore, different failure distributions can have the same MTTF while having very different reliability functions.

The first few failures that occur in a product or system often have the biggest impact on safety, warranty, and supportability, and consequently on the profitability of the product. Thus, the beginning of the failure distribution is a much more important concern for reliability than the mean.

3.10. Moment Generating Function

It can be seen that the expectation $E[X]$ and variance, $Var(X) = E[X^2] - (E[X])^2$, of a random variable can be computed from the expected value of a simple function $g(X) = X^k$ of a random variable when $k = 1$ and 2. The expectation of this function $g(X)$ is called ***k*^*th*^ initial** or **raw moment** of X, denoted by μ_k and is given by

$$\mu_k = E[X_k]. \tag{3.51}$$

For a discrete random variable X, we have

$$\mu_k = \sum_i x_i^k P\{X = x_i\}. \tag{3.52}$$

For a continuous random variable X having density function $f(x)$, we have

$$\mu_k = \int_{-\infty}^{\infty} x^k f(x) dx. \tag{3.53}$$

The first initial moment, $\mu_1 = E[X]$, is the expected value of a random variable. Similarly, we define the **k^{th} central moment** of X, denoted μ'_k, as

$$\mu'_k = E[(X - E[X])^k]. \tag{3.54}$$

For a discrete random variable X, we have (2.54):

$$\mu'_k = \sum_i (x_i - E[X])^k P\{X = x_i\}. \tag{3.55}$$

For a continuous random variable X having a density function $f(x)$, we have:

$$\mu'_k = \int_{-\infty}^{\infty} (x - E[X])^k f(x) dx. \tag{3.56}$$

The second central moment, $\mu'_2 = E[(X - E[X])^2]$, is the variance of a random variable.

If a random variable is symmetrical around its expected value, the odd central moments will be zero. The effect of asymmetry of the distribution can be detected from the odd central moments and is assessed by the following expression, called **skewness**:

$$Skew = \frac{\mu'_3}{\sqrt{\mu'^3_2}}. \tag{3.57}$$

We can convert the raw moment to central moment and vice versa. We can also find a moment of two or more random variables X_i, $i = 1, 2, \ldots, n$ in a similar manner:

$$(k_1, k_2, \ldots, k_n) = E[X_1^{k_1}, X_2^{k_2}, \ldots, X_m^{k_n}]; \tag{3.58}$$

$$\mu'(k_1, k_2, \ldots, k_n) = E[(X_1 - E[X_1])^{k_1}, (X_2 - E[X_2])^{k_2}, \ldots, (X_n - E[X_n])^{k_n}].$$

The moment is a powerful tool used to match two distributions. It can also be used for fitting a distribution to the raw data or when approximating a discrete distribution with a continuous distribution. We can generate moments of a random variable X using a **moment generating function**, denoted by $\phi(t)$.

The moment generating function is defined as:

$$\phi(t) = E[e^{tX}]. \tag{3.59}$$

Let us differentiate this function one time with respect to t,

$$\phi'(t) = \frac{d}{dt}\phi(t) = \frac{d}{dt}E\left[e^{tX}\right] = E\left[\frac{d}{dt}e^{tX}\right] = E\left[Xe^{tX}\right] \tag{3.60}$$

and then differentiate this function one more time with respect to t,

$$\phi''(t) = \frac{d^2}{dt}\phi(t) = \frac{d}{dt}E\left[Xe^{tX}\right] = E\left[\frac{d}{dt}Xe^{tX}\right] = E\left[X^2e^{tX}\right]. \tag{3.61}$$

If we let $t=0$, we have $\phi'(0)=E[X]$, and $\phi''(0)=E[X^2]$. This means that the moment generating function allows us to simply calculate the successive moments by differentiating the moment generating function and substitute $t=0$.

Generally

$$E[X^k] = \phi^{(k)}(0). \tag{3.62}$$

We can use the moment generating function to calculate the moments of summation of two independent random variables, $X + Y$. The moment generating function of this summation is given by (3.63):

$$\phi_{X+Y}(t) = E[e^{t(X+Y)}] = E[e^{tX}e^{tY}] = E[e^{tX}]E[e^{tY}] = \phi_X(t)\phi_Y(t). \tag{3.63}$$

In general, the moment generating function of the summation of independent random variables, X_i, $i = 1, 2,\dots, n\dots, n$, can be found as follows. Let $Y = \sum_i X_i$; then

$$\phi_Y(t) = E\left[e^{t\sum_i X_i}\right] = E\left[\prod_i e^{tX_i}\right] = \prod_i E\left[e^{tX_i}\right] = \prod_i \phi_{X_i}(t). \tag{3.64}$$

It is important to note that the moment generating function uniquely determines the distribution function.

In reliability analysis, we often deal with a random variable X representing time. This means that the random variable will only assume value from zero to infinity. In this case, we can calculate its moments from its density function, $f(x)$, using Laplace transformation. The Laplace transformation of the density function is denoted $\overline{f}(s)$, where s is a complex variable:

$$L\left[f(x)\right] = \overline{f}(s) = \int_0^\infty f(x)e^{-sx}dx = E\left[e^{-sX}\right]. \tag{3.65}$$

This expression is similar to the definition of the moment generating function shown in (3.59). The only difference is the negative sign of the variable s.

We can use Laplace transformation of the density function to calculate for the kth initial moments from

$$E\left[X^k\right] = (-1)^k \overline{f}^{(k)}(0). \tag{3.66}$$

We can also use the moments to help us construct a distribution function. Using Taylor's expansion to (3.65) and $e = \sum_{k=0}^{\infty} \frac{a^k}{k!}$, we have:

$$\overline{f}(s) = E\left[\sum_{k=0}^{\infty} \frac{(-sX)^k}{k!}\right] = E\left[\sum_{k=0}^{\infty} \frac{(-1)^k s^k X^k}{k!}\right] = \\ = \left[\sum_{k=0}^{\infty} \frac{(-1)^k s^k E[X^k]}{k!}\right] = \sum_{k=0}^{\infty} (-1)^k \frac{s^k}{k!} \mu_k. \tag{3.67}$$

If we have information about the moments of a random variable, we can use (3.67) to construct a distribution function.

Chapter 4
FUNCTIONS OF RANDOM VARIABLES

We examine in this section some random variables, both discrete and continuous, that are commonly used in power system reliability applications.

4.1. Bernoulli Random Variable

A **Bernoulli random variable**, X, is a discrete random variable whose outcome can only be either success or failure. This is also called a Bernoulli trial. In power system reliability analysis we usually use this distribution to represent a status of a transmission line, which can be either up or down. This can be denoted by a discrete random variable assuming 0 if it is down and 1 if it is up. The following probability mass function is used to characterize a Bernoulli random variable:

$$P\{X=0\}=1-p; \tag{4.1}$$

$$P\{X=1\}=p, \tag{4.2}$$

where probability p is between 0 and 1, which denotes probability of success.

It can be seen that this distribution is only concerned with two possible outcomes of a component. The next distribution considers outcomes of multiple components.

4.2. Binomial Random Variable

Consider a generation system with n identical generators, and each generator is working independently and has probability of working (success) of p, thereby having a failure probability of $1-p$. We are interested in knowing the number of unit(s) that is (are) working. Let X be number of working (success) unit(s) among n generators, taking value of 0, 1, 2, …, n.

A random variable X is said to have **binomial distribution** with parameter (n, p). The probability mass function of this discrete random variable is given by:

$$P\{X=a\}=\binom{n}{a}p^{a}(1-p)^{n-a}, \tag{4.3}$$

where $\binom{n}{a}=\dfrac{n!}{a!(n-a)!}$.

The expected value of X, $E[X]=np$, and variance is $Var[X]=np(1-p)$.

Example 4.1. For a system of three identical and independent generators, each having probability of success of 0.9, let us find the probability that two generators are working.

In this example, X is a binomial random variable with parameter (3, 0.9). The probability that two generators are working can be found as follows:

$$P\{X=2\}=\binom{3}{2}0.9^{a}(1-0.9)=0.243.$$

In general, binomial distribution is used to describe n independent trials, each trail resulting in either success with probability p or failure with probability $1-p$. Then, a binomial random variable, X, denotes the number of success in n independent trials.

Consider a case when number of trials reaches a very large quantity, and the probability of success is small. Let Λ be number of successes in n independent trials. We can approximate the success probability by $p = \Lambda/n$. Then

$$P\{X=a\}=\binom{n}{a}p^{a}(1-p)^{n-a}=\frac{n!}{a!(n-a)!}\left(\frac{\Lambda}{n}\right)^{a}(1-\frac{\Lambda}{n})^{n-a}. \tag{4.4}$$

As $n \to \infty$, using binomial series, $(1+\gamma)^{\alpha}=\sum_{k=0}^{\infty}\binom{\alpha}{k}\gamma^{a}$, and Taylor's expansion, $e^{\alpha}=\sum_{k=0}^{\infty}\frac{\alpha^{k}}{k!}$, we have the following approximation:

$$\begin{aligned}\left(1-\frac{\Lambda}{n}\right)^{n-a}&\approx\left(1-\frac{\Lambda}{n}\right)^{n}=1+\binom{n}{1}\frac{-\Lambda}{n}+\binom{n}{2}\left(\frac{-\Lambda}{n}\right)^{2}+\ldots\\&\approx 1-\Lambda+\frac{\Lambda^{2}}{2!}-\frac{\Lambda^{3}}{3!}+\ldots=e^{-\Lambda}.\end{aligned} \tag{4.5}$$

Then, the probability is written as:

$$\begin{aligned}P\{X=a\}&=\frac{(n(n-1)\ldots(n-a+1)(n-a))}{a!(n-a)!}\frac{\Lambda^{a}}{n^{a}}e^{-\Lambda}=\\&=\frac{(n(n-1)\ldots(n-a+1))}{a!}\frac{\Lambda^{a}}{n^{a}}e^{-\Lambda}.\end{aligned} \tag{4.6}$$

As $n \to \infty$, $n(n-1)\ldots(n-\mathrm{a}+1)\approx n^{a}$, we have (4.7):

$$P\{X=a\}=\frac{\Lambda^{a}}{a!}e^{-\Lambda}. \tag{4.7}$$

Expression (4.7) shows that when a number of trials is very large, we can calculate the probability that the trial will be successful a times by using the average number of successes, Λ, over a long-run trial. This leads us to the next distribution, called Poisson distribution.

4.3. Poisson Random Variable

A **Poisson random variable** X is a discrete random variable taking value 0, 1, 2,… with a parameter Λ, for some Λ > 0. The probability mass function of this random variable is given in (2.74):

$$P\{X = a\} = \frac{\Lambda^a}{a!} e^{-\Lambda}. \quad (4.8)$$

The expected value of X, $E[X] = \Lambda$ and variance is $Var[X] = \Lambda$.

A Poisson random variable is widely used to describe the number of occurrences (either failures or successes) in a fixed time given that an average or expected number of occurrences is Λ. For example, in reliability analysis it is commonly used to describe number of failures of a component within a certain time period given that the number of failures on average, Λ, is known.

Example 4.2. A transmission line fails on an average two times per year. If the number of failures can be described by Poisson distribution, what is the probability of having two failures in 5 years? What is the probability of having three failures in 10 years?

We first let X_5 be a Poisson random variable representing the number of failures in 5 years; its expected or average number of occurrences is $\Lambda_5 = 2 \times 5 = 10$ failures in 5 years. Then, the probability mass function of this random variable is

$$P\{X_5 = a\} = \frac{10^a}{a!} e^{-10}.$$

The probability of having two failures in 5 years is $P\{X_5 = 2\} = \frac{10^2}{2!} e^{-10} = 0.00227$.

Similarly, let X_{10} be a Poisson random variable representing number of failures in 10 years; its average number of occurrences is $\Lambda_{10} = 2 \times 10 = 20$ failures in 10 years. Then, the probability mass function of this random variable is

$$P\{X_{10} = a\} = \frac{20^a}{a!} e^{-20}.$$

The probability of having three failures in 5 years is $P\{X_{10} = 3\} = = \frac{20^3}{3!} e^{-20} = 4.12 \cdot 10^{-7}$.

If, after a long experiment, we found out that within a time period, t, we have a number of failures equal to Λ; the average number of failures is $\lambda = \Lambda/t$; this number is the average number of occurrences per unit time interval. The probability mass function can be rewritten as:

$$P\{X = a\} = \frac{(\lambda t)^a}{a!} e^{-\lambda t}. \quad (4.9)$$

If number of failure in this time interval is zero, then

$$P\{X = 0\} = e^{-\lambda}. \tag{4.10}$$

We will see later in this chapter that (4.10) gives a widely known distribution function of a continuous random variable called exponential.

4.4. Uniform Random Variable

A continuous random variable is **uniformly distributed** on interval (α, β) if the probability density function is given as:

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{if } \beta < x < \alpha; \\ 0 & \text{otherwise.} \end{cases} \tag{4.11}$$

The probability distribution function is given as:

$$f(x) = \begin{cases} 0 & \text{if } a < \beta; \\ \dfrac{a - \alpha}{\beta - \alpha} & \text{if } \beta < a < \alpha; \\ 1 & \text{if } a \geq \beta. \end{cases} \tag{4.12}$$

This distribution is frequently used for generating random numbers for Monte Carlo simulation.

4.5. Exponential Random Variable

An **exponential random variable**, *X*, is a non negative random variable on [0, ∞) whose probability density function is given by (4.13) for some positive constant, $\lambda > 0$:

$$f(x) = \lambda e^{-\lambda x}. \tag{4.13}$$

The probability distribution function is given in:

$$F(a) = 1 - e^{-\lambda a}. \tag{4.14}$$

If we take Laplace transformation as follows, we can find the momentssj from the following moment generating function:

$$E\left[X^k\right] = (-1)^k \overline{f}^{(k)}(0), \tag{4.15}$$

where

$$L\left[f(x)\right] = \overline{f}(s) = \int_0^\infty f(x) e^{-sx} dx = \frac{\lambda}{\lambda + s}. \tag{4.16}$$

The expected value is the first moment given by:

$$E\left[X\right] = -\frac{d\overline{f}(s)}{ds}\Big|_{s=0} = \frac{\lambda}{(\lambda + s)^2}\Big|_{s=0} = \frac{1}{\lambda}. \tag{4.17}$$

Similarly, the second moment is given by:

$$E\left[X^2\right] = -\frac{d^2\overline{f}(s)}{ds}\Big|_{s=0} = \frac{2\lambda}{(\lambda+s)^3}\Big|_{s=0} = \frac{2}{\lambda^2}. \tag{4.18}$$

Then, the variance is found from (4.20):

$$Var(X) = E\left[X^2\right] - \left(E\left[X\right]\right)^2 = \frac{1}{\lambda^2}. \tag{4.19}$$

If X denotes time to failure of a component, the expected value of X gives the expected value of time to failure of the component, and λ is called failure rate of the component. The survival function is shown in:

$$R(a) = \int_a^\infty f(x)dx = e^{-\lambda a}. \tag{4.20}$$

The hazard rate function can be found using

$$h(a) = \frac{f(a)}{R(a)} = \lambda. \tag{4.21}$$

Recall that the hazard rate function gives the rate of a conditional probability of failure at time a given that the component has survived up to time a. As $\Delta a \to 0$, the hazard rate function can be written as $h(a)\Delta a = P\{a \le X \le a + \Delta a | X > a\}$. When the hazard rate function is constant, it implies that a component will fail at a constant rate irrespective of how long it has been in operation. This property is known as **memoryless property** and can be proven by finding the residual lifetime of a component.

When X is a random variable denoting time to failure of a component, assume that a component has operated up to time t without failure. Let $Y = X - t$ denote residual lifetime of this component. Then,

$$F_Y(a) = Pr\{Y \le a | X > t\}. \tag{4.22}$$

The equation (4.22) gives the distribution function $F_Y(a)$ of a random variable Y, representing a residual lifetime of this component. Using conditional probability rule, we can find

$$F_Y(a) = Pr\{Y \le a \mid X > t\} = Pr\{X - t \le a \mid X > t\} =$$

$$= \frac{Pr\{t < X \le a + t\}}{Pr\{X > t\}} = \frac{\int_t^{a+t} \lambda e^{-\lambda x} dx}{e^{-\lambda t}} = 1 - e^{-\lambda a}. \tag{4.23}$$

Since X is exponentially distributed, $F_X(a)$ is given by (4.14), which is exactly the same as above. This means that Y, the residual lifetime, is also exponentially distributed. The distribution of residual lifetime is independent of the time that a component has been in operation. It will not fail because of the gradual degradation of the component itself but from the random failure; in other words, this component does not age. Since the hazard rate function uniquely determines the distribution function, it follows from (4.18) that the exponential distribution is the only distribution that has memoryless property

There is a connection between Poisson distribution and exponential distribution. Recall that Poisson distribution is used to describe the number of occurrences (either failures or successes) in a fixed time t given that an average or expected number of occurrences is Λ. Let X denotes = number of failures in this time interval $(0, t)$, and X is a Poisson random variable. The average number of failures within this time interval is $\lambda = \Lambda/t$. We found out from (4.10) that when number of failures is zero during this time interval, the probability is $P\{X=0\}=e^{-\lambda t}$. In other words, a system has been operated in the time $(0, t)$ and no failure happens, which implies that a system will fail at a time greater than t.

Let Y denote time to failure of this system; then $Pr\{Y > t\} = e^{-\lambda t}$, and we can write

$$F_Y(t) = Pr\{Y \le t\} = 1 - Pr\{Y > t\} = 1 - e^{-\lambda t}.$$

This shows that the random variable Y is exponentially distributed. Thus, we can conclude that for a Poisson random variable X, if we focus on time between failures and denote this time by a continuous random variable Y, this time is exponentially distributed.

4.6. Normal Random Variable

A **normal random variable**, X, is a continuous random variable on $(-\infty, \infty)$ with parameter μ and σ^2, whose probability density function is given by (4.24). We can write $X \sim N(\mu, \sigma^2)$.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \quad x \in (-\infty, \infty). \qquad (4.24)$$

The normal distribution is bell-shaped and has symmetry around μ, which is its mean value. The variance of this random variable is $Var[X] = \sigma^2$.

The probability distribution function is given by:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx. \qquad (4.25)$$

Let $z = \frac{(x-\mu)}{\sigma}$, $dx = \sigma dz$. We can write:

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{z^2}{2}} dz. \qquad (4.26)$$

The integral in (4.26) cannot be expressed explicitly. Numerical integration is used to obtain the value of this integral in a tabular form.

We can also use the moment generating function to calculate for mean and variance as follows:

$$\phi(t) = E\left[e^{tX}\right] = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx - \frac{(x-\mu)^2}{2\sigma^2}} dx . \tag{4.27}$$

Let $z = \dfrac{(x-\mu)}{\sigma}$ and $dx = \sigma dz$. We can write

$$\phi(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\sigma z+\mu) - \frac{z^2}{2}} dz = \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(z-\sigma t)^2}{2} + \frac{\sigma^2 t^2}{2}} dz = e^{t\mu + \frac{\sigma^2 t^2}{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(z-\sigma t)^2}{2}} dz .$$

Let $w = z - \sigma t$ and $dw = dz$. We have

$$\phi(t) = e^{t\mu + \frac{\sigma^2 t^2}{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{w^2}{2}} dw = e^{t\mu + \frac{\sigma^2 t^2}{2}} . \tag{4.28}$$

The first and second moments are found from $\phi'^{(0)} = E[X] = \mu$ and $\phi''^{(0)} = E[X_2] = \mu^2 + \sigma^2$. The mean and variance are the same as previously stated.

This function leads to the special case of normal random variable when the mean value is zero and variance is one, $Z \sim N(0, \sigma^2)$. This random variable Z is called standard normal random variable, which has the following density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}, z \in (-\infty, \infty) . \tag{4.29}$$

It should be noted that normal random variable can take any value in the real axis. When we use this random variable, $X \sim N(\mu, \sigma^2)$, to represent operating time of a component, the function needs to be modified to reflect the fact that the operation time can only be positive. This is done by truncating the normal distribution by:

$$f(x) = \frac{1}{\alpha\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \; x \in [0, \infty), \tag{4.30}$$

where α is a normalizing parameter and $\alpha = \dfrac{1}{\sigma\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$.

The survival function and hazard rate function can be found from (3.15) and (3.19). It should be noted that the hazard rate function of the normal distribution is monotonically increasing, as shown in Fig. 4.1.

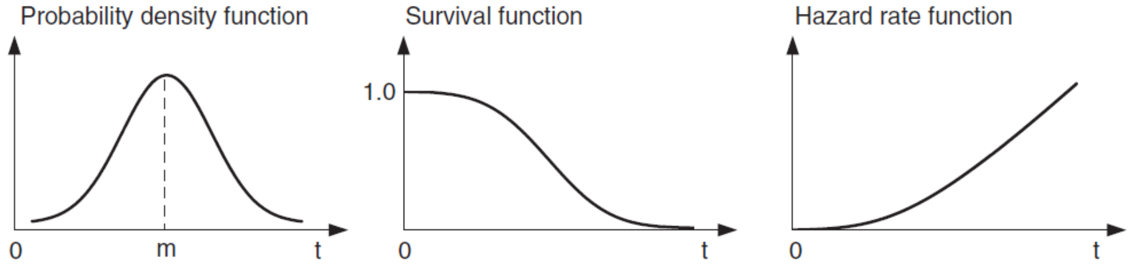

Fig. 4.1. Characteristics of a normal random variable

4.7. Log-Normal Random Variable

A **log-normal random variable**, X, is a continuous random variable on $(0, \infty)$ with parameter μ and σ^2, and if $Y = \log X$ is normally distributed with parameter μ and σ^2. Since a logarithm is a non-decreasing function, we can write

$$F_X(x) = Pr\{X \le x\} = Pr\{\log X \le \log x\} = Pr\{Y \le y\} = F_Y(y).$$

Using chain rule, we have

$$f_X(x) = \frac{d}{dx}F_X(x) = \frac{d}{dx}F_Y(y) = f_Y(y)\frac{dy}{dx} = f_Y(\log x)\frac{dy}{dx} = \frac{1}{x}f_Y(\log x).$$

The probability density function of X is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\log x - \mu)^2}{2\sigma^2}}, x \in (0, \infty). \tag{4.31}$$

The mean and variance of this random variable can be found from moment generating function. Since $Y = \log X$ is normally distributed with the moment generating function shown in (4.28), and $X = e^Y$, we can write:

$$E[X] = E[e^Y]. \tag{4.32}$$

From $\phi_Y(t) = E\left[e^{tY}\right] = e^{t\mu + \frac{\sigma^2 t^2}{2}}$, let $t = 1$, and we have $E\left[X\right] = e^{\frac{\mu+\sigma^2}{2}}$, the mean value of X. Similarly, $E[X^2] = E[e^{2Y}]$, let $t = 2$, and we have $E\left[X^2\right] = e^{2\mu + 2\sigma^2}$. The variance can be found from:

$$Var(X) = E\left[X^2\right] - \left(E\left[X\right]\right)^2 = e^{2(\mu+\sigma^2)} - e^{\mu+\sigma^2}. \tag{4.33}$$

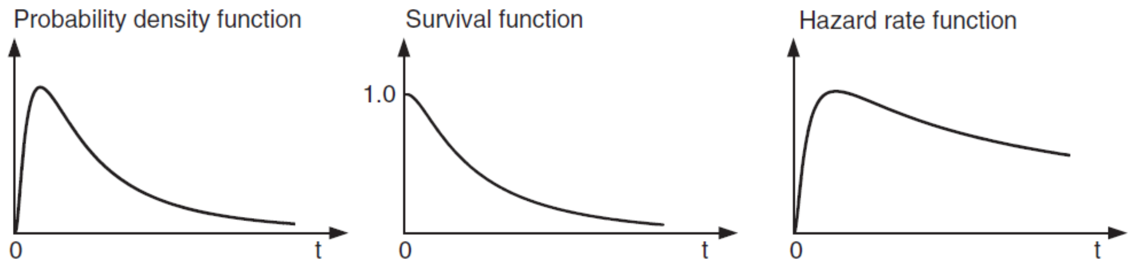

Fig. 4.2. Characteristics of a log-normal random variable

The hazard rate function of log-normal is shown in Fig. 4.2, which demonstrates that the function is not monotonically increasing and does not seem to model a lifetime of a physical component. However, in several cases, it shows reasonable fit for repair times.

4.8. Gamma Random Variable

A **gamma random variable**, X, is a continuous random variable on $[0, \infty)$ with parameter $\lambda > 0$ and $\alpha > 0$, whose probability density function is given by (4.34):

$$f(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, x \in [0, \infty), \tag{4.34}$$

where $\Gamma(\alpha)$ is a **gamma function** and is defined as:

$$\Gamma(\alpha) \equiv \int_0^\infty z^{\alpha-1} e^{-z} dz. \tag{4.35}$$

Note that when α is a positive integer, we can write $\Gamma(\alpha) = (\alpha - 1)!$, thus $\Gamma(1) = \Gamma(2) = 1$; using integration by parts, we can write $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$.

The probability distribution function is found from

$$F(a) = \frac{1}{\Gamma(\alpha)} \int_0^a \lambda e^{-\lambda x}(\lambda x)^{\alpha-1} dx.$$

Substitute $z = \lambda x$ and $dz = \lambda dx$ and we have:

$$F(a) = \frac{1}{\Gamma(\alpha)} \int_0^{\lambda a} e^{-z} z^{\alpha-1} dz. \tag{4.36}$$

When α is integer, we can use integration by parts to find the distribution function.

$$F(a) = 1 - \sum_{k=0}^{\alpha-1} \frac{\lambda e^{-\lambda a}(\lambda a)^k}{k!}, \text{ a is integer.} \tag{4.37}$$

We can compute the kth moment of X directly as follows:

$$E[X^k] = \int_0^\infty \frac{\lambda e^{-\lambda x} x^k (\lambda x)^{\alpha-1}}{\Gamma(\alpha)} dx.$$

Substitute $z = \lambda x$ and $dz = \lambda dx$ and we have

$$E[X^k] = \frac{1}{\lambda^k \Gamma(\alpha)} \int_0^\infty e^{-z} z^{k+\alpha-1} dz = \frac{\Gamma(k+\alpha)}{\lambda^k \Gamma(\alpha)}. \tag{4.38}$$

The first and second moment is found from the following:

$$E[X] = \frac{\Gamma(\alpha+1)}{\lambda\Gamma(\alpha)} = \frac{\Gamma(\alpha)}{\lambda\Gamma(\alpha)} = \frac{\alpha}{\lambda}; \tag{4.39}$$

$$E[X^2] = \frac{\Gamma(\alpha+2)}{\lambda^2\Gamma(\alpha)} = \frac{\alpha^2}{\lambda^2}. \tag{4.40}$$

The mean value is α/λ, and the variance of this random variable is $Var[X] = \alpha/\lambda^2$.

The survival function is shown in:

$$R(a) = P\{X > a\} = \int_a^\infty \frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)} dx. \tag{4.41}$$

The hazard rate unction of a gamma random variable is

$$h(a) = \frac{f(a)}{R(a)} = \frac{1}{\int_a^\infty e^{-\lambda(x-a)}\left(\frac{x}{a}\right)^{\alpha-1} dx}.$$

Let $z = x - a$ and $dz = dx$; then,

$$h(a) = \frac{1}{\int_0^\infty e^{-\lambda z}\left(1+\frac{z}{a}\right)^{\alpha-1} dz}. \tag{4.42}$$

Note that if $\alpha = 1$, then $\int_0^\infty e^{-\lambda z} dz = \lambda$, which is a constant value. If $\alpha > 1$, the hazard rate function will be increasing; if $0 < \alpha < 1$, the hazard rate function will be decreasing, as shown in Fig. 4.3.

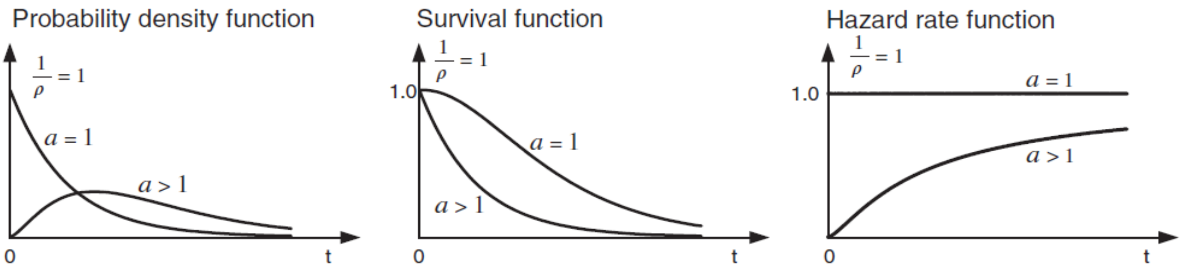

Fig. 4.3. Characteristics of a gamma random variable

4.9. Weibull Random Variable

A **Weibull random variable**, *X*, is a continuous random variable on (0, ∞) with parameter $\lambda > 0$ and $\alpha > 0$, and if *Y* is exponentially distributed with parameter λ, then $X = Y^{\frac{1}{\alpha}}$ or $Y = X^\alpha$. This means that an exponential random variable is a special case of a Weibull random variable with $\alpha = 1$. Since $\alpha > 0$, we can write

$$F_X(x) = Pr\{X \le x\} = Pr\{X^\alpha \le x^\alpha\} = Pr\{Y \le y\} = F_Y(y).$$

Thus, the probability distribution function is:

$$F_X(x) = 1 - e^{-\lambda y} = 1 - e^{-\lambda x^\alpha}, x \in (0, \infty). \quad (4.43)$$

Using chain rule, we have

$$f_X(x) = \frac{d}{dx}F_X(x) = \frac{d}{dx}F_Y(y) = f_Y(y)\frac{dy}{dx} = f_Y(x^\alpha)\frac{dy}{dx} = \alpha x^{\alpha-1} f_Y(x^\alpha).$$

The probability density function of X is (2.110), which can also be found from differentiating (4.43):

$$f(x) = \alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha}, x \in (0, \infty). \quad (4.44)$$

For a Weibull random variable, it is simpler to compute the kth moment of X directly as follows:

$$E[X^k] = \int_0^\infty x^k \alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha} dx.$$

Substitute $z = \lambda x^\alpha$ and $dz = \alpha\lambda x^{\alpha-1}dx$, and we have

$$E[X^k] = \frac{1}{\lambda^{\frac{k}{\alpha}}} \int_0^\infty z^{\frac{k}{\alpha}} e^{-z} dz.$$

From the previous section, $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} e^{-z} dz$; we can use gamma function to calculate the moment as follows:

$$E[X^k] = \frac{1}{\lambda^{\frac{k}{\alpha}}} \Gamma(1 + \frac{k}{\alpha}). \quad (4.45)$$

The expected value is found when $k = 1$,

$$E[X] = \frac{1}{\lambda^{\frac{1}{\alpha}}} \Gamma(1 + \frac{1}{\alpha}). \quad (4.46)$$

The second moment is when $k = 2$, $E[X^2] = \frac{1}{\lambda^{\frac{2}{\alpha}}} \Gamma(1 + \frac{2}{\alpha})$. The variance is calculated directly from the first and second moment as follows:

$$Var[X] = \frac{1}{\lambda^{\frac{2}{\alpha}}} \Gamma\left(1 + \frac{2}{\alpha}\right) - \left(\Gamma\left(1 + \frac{2}{\alpha}\right)\right)^2. \quad (4.47)$$

The survival function is shown in:

$$R(a) = P\{X > a\} = 1 - F(a) = e^{-\lambda a^\alpha}. \quad (4.48)$$

Thus, the hazard rate function is:

$$h(x) = \frac{f(x)}{R(x)} = \frac{\alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha}}{e^{-\lambda x^\alpha}} = \alpha\lambda x^{\alpha-1}. \quad (4.49)$$

Expression (4.49) shows that if $\alpha = 1$, the hazard rate function is constant. If $\alpha > 1$, the hazard rate function is increasing, and if $\alpha < 1$, the hazard rate function is decreasing. With this flexibility, a Weibull random variable is often used to model time to failure of a component. The characteristics of this distribution are given in Fig. 4.4.

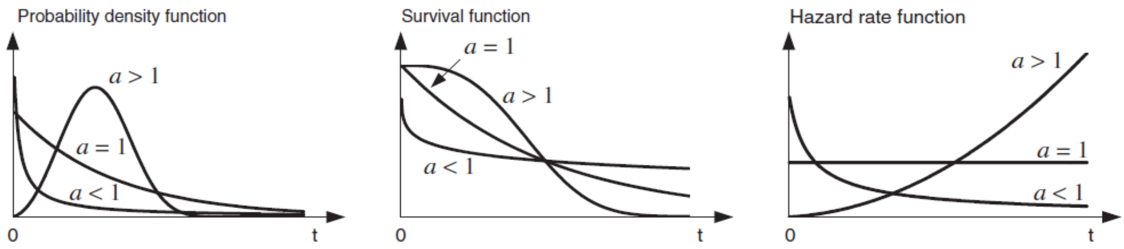

Fig. 4.4. Characteristics of a Weibull random variable

Chapter 5
METHODS OF ANALYSIS OF POWER SUPPLY SYSTEM RELIABILITY INDICES

The ultimate goal of reliability calculation of power supply system is the quantitative estimation (assessment) of reliability indicators (indices) concerning the certain load nodes (centers) and the development of measures based on the results of this estimation aimed at their changes.

Quantitative characteristics of complex reliability indicators (indices) depend on the system condition (state) at every instant, the power and energy demand in the load centers. The number of discrete states in a complex system is rather huge. It is impossible in practice to assess (estimate) the system reliability without development of an efficient method of the reduction of the number of considered states till acceptable level and the achievement of specific aims.

The most widespread and detailed methods of reliability indicators (indices) calculation of the system taking into consideration the specific of the solved problems (tasks) are considered in simplified version in this chapter. The notation conventions of reliability parameters are the same as in the references in order to make the understanding of this material easier.

5.1. Method of analysis of reliability indices using (applying) the models of random (stochastic) processes

The processes of the system state (condition) transitions, which are influenced by random failures of separate elements, are described by means of Poisson random processes. By exponential time distribution between failures and exponential distribution of failure states durability there is a chance to apply a well developed apparatus of waiting (queuing) theory, and particularly the apparatus of Markovian processes.

The process is called Markovian, if for every instant the probability of any state of the element or system in future depends only on the current state (condition) and does not depend on how the element has acquired this state (condition).

Let us consider more detailed the conditions of application of Markovian processes to describe the state transitions of the system consisting of some separate elements.

The occurrence of failures in an element at one time lag (slice) practically does not influence the probability of failure occurrence at another time lag (slice) (excluding the cascade development of failure in the system). Therefore, the failure flow (stream) of such an element (subsystem) can be viewed as Poisson one. This condition can be violated if the element (subsystem) includes not very reliable parts (elements). Then the failure flow (stream) will only consist of these elements and the conditions that this Poisson flow model corresponds (stationarity, ordinarity, absence of aftereffects) will not be satisfied. Moreover, the failure events in this case can not be considered as rare ones.

Within the run-in period of the element as well as in the period of intensive aging and wear-out, the failure flow (stream) of the elements also does not posses Markovian property (feature). In practice there is an attempt to reduce these periods (till commencement of operation) by carrying out trials (tests) on separate elements of power system and replacing of the outdated (out-of-date), worn equipment by modern one. Therefore, the mathematical models that satisfy the normal operating conditions of the elements (systems), being of interest in practice will be considered further.

The models of random failure process and power system recovery is applied for assessment (evaluation) of complex reliability indices at relatively short time lags (slices), that are comparable with recovery time after failure taking into account the initial state of some separate elements.

The application of the models of random processes in engineering calculations of power supply systems reliability allows to justify and find the areas of applications for more simplified algorithms. As more typical example we will consider the concepts of network analyzer mapping (building) for elementary circuits (diagrams) (redundant and non redundant).

5.1.1. Processes of failures and recoveries of one element circuit (scheme)

Let us assume that the process of failures and recoveries of the component (element) possesses the properties of Markovian random process. If the process that takes place in a physical system with countable collection (set) of conditions and continuous time is Markovian one, then it can be described with a simple differential equation, where unknown quantity is probabilities of states.

Let us consider an element (component) that can be in two states: **0** – is failure free operation, **1** – are states of failure (recovery). Let us define the corresponding probabilities of states of the element (component) $P_0(t)$, $P_1(t)$ at arbitrary time t by different initial conditions. This problem we will solve under the stipulation that the failure flow is elementary with the failure density (rate) $\lambda = \text{const}$ and recovery density $\mu = \text{const}$, the distribution time law between failures (the frequency of failures) is $a(t) = \lambda e^{-\lambda t}$, the recovery time is also described by exponential distribution law with parameters μ, e. g. $a_B(t) = \mu e^{-\mu t}$.

For any instant the probability sum is $P_0(t) + P_1(t) = 1$ – the probability of persistent (sure) event. Let us fix the time t and find the probability

$P_0(t+\Delta t)$ that at instant $t + \Delta t$ the element (component) will be in operation. This event will occur by fulfillment of two conditions.

1. At instant t the element (component) was in **0** state and over the period Δt there were no failures. The probability of the elements work (operation) is defined in accordance with the rule of multiplication of probability of independent events. The probability that at instant t the element was in **0** state is equal to $P_0(t)$. The probability that over the period Δt it does not fail is equal to $e^{-\lambda\Delta t}$. With the accuracy till the value of the higher infinitesimal order it can be presented as follows:

$$e^{-\lambda\Delta t} = 1 - \lambda\Delta t + \frac{\lambda^2\Delta t^2}{2} - \ldots \cong 1 - \lambda\Delta t . \tag{5.1}$$

Therefore, the probability of this hypothesis is equal to the product $P_0(t)(1-\lambda\Delta t)$.

2. The element (component) was in state 1 (recovery state) at instant t, over the period Δt the recovery was completed and the element went into **0** state. This probability we will also define in accordance with the rule of multiplication of probability of independent events. The probability that the element is in the state **1** at instant t is equal to $P_1(t)$. The probability that the recovery was completed we will define through the probability of opposite event, e. g. $1 - e^{-\mu\Delta t} \cong \mu\Delta t$. Hence, the probability of the second hypothesis is equal to $P_1(t)\,\mu\Delta t$.

The probability of operational state (operating condition) of the element (component) at instant $(t + \Delta t)$ is defined by the probability sum of independent events by the fulfillment of both hypotheses.

$$P_0(t+\Delta t) = P_0(t)(1-\lambda\Delta t) + P_1(t)\mu\Delta t . \tag{5.2}$$

or

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_1(t);$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = \frac{dP_0(t)}{dt}.$$

Hence, the first equation of this state is:

$$\frac{dP_0(t)}{dt} = -\lambda \mathrm{P}_0(t) + \mu \mathrm{P}_1(t) . \tag{5.3}$$

Analyzing in the same way the second state (condition) of the element – the failure state (recovery state), it is possible to write down the second equation of the state:

$$\frac{dP_1(t)}{dt} = -\mu \mathrm{P}_1(t) + \lambda \mathrm{P}_0(t) . \tag{5.4}$$

Thus, to describe the probability of the element (component) state (condition) two combined differential equations were obtained: (5.3) and (5.4).

It should be noted that λdt and μdt play a role of transition probabilities of the element in failed (failure) or operational state (condition). The process of state change of the considered element can be illustrated by means of the graph, which is presented in Fig. 5.1.The element states (**0**, **1**) correspond to the graph nodes and the possible transitions from one state into another one correspond to the ribs of graph.

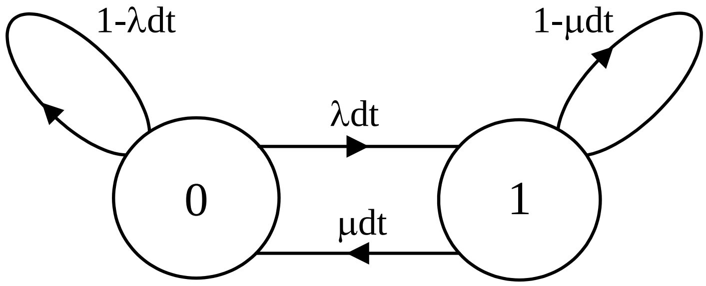

Fig. 5.1. Transition graph for one element scheme (circuit)

If there is a directed graph of an element or system state (condition), then the combined differential equations for the state probabilities P_k ($k = 0, 1, 2, \ldots$) can be written using the following simple rule. **The derivative *$dP_k(t)/dt$* is the first member of equation (in the left side of an equation) and there are so many members in the right side of an equation as many ribs are connected with the given state; if the rib finishes in the given state, then the member has plus sign, if it starts from this state it has minus sign. Every member is equal to the product of event flow rate, that transfers the element or system on the specified rib into another state (condition), into the probability of the state the rib originates.**

Combined differential equations can be used to define the probability of failure free operation of power supply system, the availability function and rate, the probability of repair (recovery), average time when system is in any state (condition), system failure density at very short time lags, when the initial state of the element is necessary to take into account.

Solution of the combined equations describing the state of the element by initial conditions [$P_0(0) = 1$; $P_1(0) = 0$], will be:

$$P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}. \tag{5.5}$$

The probability of failed state (failure mode) is:

$$P_1(t) = 1 - P_0(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}. \tag{5.6}$$

If at initial time the element was in failure sate (recovery state, e. g. $P_0(0) = 0$, $P_1(0) = 1$, then

$$P_0(t) = \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t}; \tag{5.7}$$

$$P_1(t) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda+\mu)t}. \tag{5.8}$$

For stationary sate ($t \to \infty$) the probability of the element operation is equal to stationary availability and the probability of failed state (failure mode) is equal to forced outage (breakdown) rate:

$$\lim_{t\to\infty} P_0(t) = K_{\text{g}} = \frac{\mu}{\mu + \lambda} = \frac{\bar{T}}{\bar{T} + \bar{t}_v}; \tag{5.9}$$

$$\lim_{t\to\infty} P_1(t) = K_p = \frac{\lambda}{\lambda + \mu} = \frac{\bar{t}_v}{\bar{T} + \bar{t}_v}, \tag{5.10}$$

where $\bar{T}$ is mean time of faultless operation, $\bar{t}_{\text{в}}$ is mean recovery time.

The time during which the probabilities $P_0(t)$ and $P_1(t)$ reach their steady-state value, depends on degree index, e. g. damping constant (coefficient) of the exponent.

If $\bar{T} >> \bar{t}_{\text{в}}$, then the damping constant (coefficient) of the exponent is:

$$\lambda + \mu = \frac{1}{\bar{T}} + \frac{1}{\bar{t}_{\text{в}}} = \frac{\bar{T} + \bar{t}_{\text{в}}}{\bar{t}_{\text{в}} \bar{T}} \approx \frac{1}{\bar{t}_{\text{в}}}. \tag{5.11}$$

The formulae (5.5)–(5.8) for practical calculations can be re-arranged as follows: $P_0(0) = 1$, $P_1(0) = 0$ (Fig. 5.2, *a*, *c*)

$$P_0(t) = K_{\text{g}} + K_p \exp(-t / \bar{t}_v); \tag{5.12}$$

$$P_1(t) = K_p - K_p \exp(-t / \bar{t}_v); \tag{5.13}$$

$P_0(0) = 0$, $P_1(0) = 1$ (Fig. 5.2, *c*, *d*);

$$P_0(t) = K_p - K_g \exp(-t / \bar{t}_v); \tag{5.14}$$

$$P_1(t) = K_p + K_g \exp(-t / \bar{t}_v). \tag{5.15}$$

The probabilistic system state by $t \to \infty$, e. g. by stationary conditions, does not depend on its initial state.

The availability and forced outage (downtime) coefficients can be interpreted as mean probability that the system is in operation sate and failure mode (Fig. 5.2, *a*–*d*).

From the analysis of the formulae (5.12)–(5.15) it is clear that the shorter (less) the mean recovery time of an element (component) (more $\mu = \bar{t}_{\text{в}}^{-1}$) is, the more the damping coefficient, and hence, the faster the process tends to the steady-state (final) probability value (in absolute time units), i.e. to stationary values K_g and K_p.

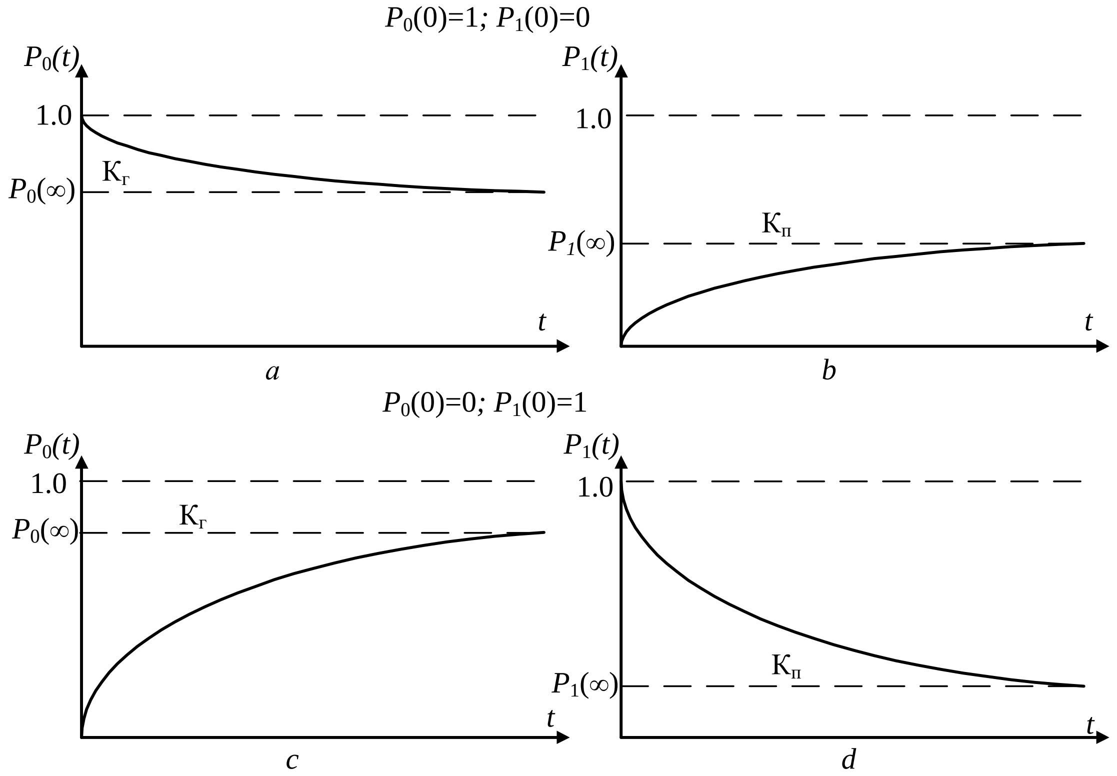

Fig. 5.2. Relations between the probability of faultless operation and the probability of failure of one-element scheme (circuit) by different initial conditions

The probabilities of the system state (condition) in the reliability indices calculations with high accuracy over relatively long periods ($t \geq (7-8)t_v$) can usually be defined by steady-state mean probabilities $P_0(\infty) = K_g = P_0$ и $P_1(\infty) = K_p = P_1$. This states from the point of view of reliability are called limiting (marginal) states. The probabilities of the steady (stationary) states ($t \to \infty$) can be found fairly simple by solving of combined algebraic equations, that were obtained from combined differential equations by equalization of derivatives (left parts) to zero, e. g. $dP_k(t)/dt = 0$, and by change of $P_k(t) = P_k = \text{const}$ and addition of normalizing condition $\sum_{k=0}^{n} P_k = 1$

Combined equations (set of equations) are of the following form:

$$\left.\begin{aligned} -\lambda P_0 + \mu P_1 = 0 \\ P_1 + P_0 = 1 \end{aligned}\right\}, \tag{5.16}$$

where

$$P_0 = K_g = \frac{\mu}{\lambda + \mu} = \frac{\frac{1}{\bar{t}_v}}{\frac{1}{\bar{T}} + \frac{1}{\bar{t}_v}} = \frac{\bar{T}}{\bar{T} + \bar{t}_v}; \tag{5.17}$$

$$P_1 = \frac{\lambda}{\lambda+\mu} = K_p = \frac{\bar{t}_v}{\bar{T}+\bar{t}_v}. \qquad (5.18)$$

Thus, the same result was obtained as by the analysis of limiting (marginal) state by means of differential equations.

It is to be emphasized, that by $\bar{T} >> \bar{t}_в$ the forced outage (downtime) is defined very simple:

$$K_p = \bar{t}_v / (\bar{t}_v + \bar{T}) \approx \bar{t}_v / \bar{T} = \lambda \bar{t}_v. \qquad (5.19)$$

In practical calculations (estimations) it is accepted that ω=λ, therefore the outage (breakdown) and operational states are defined in accordance with formulas:

$$P_1 = \lambda \bar{t}_в = \omega \bar{t}_в; \qquad (5.20)$$

$$P_0 = 1 - \omega \bar{t}_в. \qquad (5.21)$$

Hence, the forced outage (breakdown) coefficient (or mean probability of failure) is equal to product of failure flow parameter by mean recovery time of the element after one failure.

The same result can be obtained from general reasoning by absence of limits on the types of distribution laws of nonfailure operation and recovery time.

5.1.2. System, consisting of series connected repairable elements (renewable units)

The system which consists of n subsequent (series) repairable elements, fails in the cases when any element (component) fails (the probability of failure of few elements, by accepted assumption concerning the properties of failure flow is neglected). Therefore, compound (total) failure flow of all elements possesses practically the ordinary feature. That allows to neglect simultaneity (synchronism, coincidence) of two or more elements failure. The system consisting of *n* homogeneous elements connected in series have two states (conditions): **0** – all elements are in operational state; **1** – one element is in failed state (failure condition). Applying the mentioned above method of probability (frequency) definition by different initial conditions, we will get the following combined equations (set of equations):

$$\left.\begin{aligned} \frac{dP_0(t)}{dt} &= -n\lambda P_0(t) + \mu P_1(t) \\ \frac{dP_1(t)}{dt} &= -\mu P_1(t) + n\lambda P_0(t) \end{aligned}\right\} \qquad (5.22)$$

The probability of the n elements operation over the period *dt* is defined applying both-and rule (product rule) for compatible (joint) events – the operation of elements over the period *dt*:

$$\underbrace{e^{-\lambda dt} e^{-\lambda dt} e^{-\lambda dt} ... e^{-\lambda dt}}_{n} = e^{-n\lambda dt} \approx (1 - n\lambda dt). \qquad (5.23)$$

The recovering probability (probability of repair) of the failed (faulty) element μdt over the period dt is defined in the same way as for one-element (unicomponent) scheme (diagram). By solving the set of differential equations (combined equations) by initial conditions $P_0(0)=1$, $P_1(0)=0$, we will find:

$$P_0(t)=\frac{\mu}{n\lambda+\mu}+\frac{n\lambda}{n\lambda+\mu}e^{-(n\lambda+\mu)t}; \quad (5.24)$$

$$P_1(t)=\frac{n\lambda}{n\lambda+\mu}-\frac{n\lambda}{n\lambda+\mu}e^{-(n\lambda+\mu)t}. \quad (5.25)$$

By initial conditions $P_0(0)=0$, $P_1(0)=1$ (the circuit is in failed condition)

$$P_0(t)=\frac{\mu}{n\lambda+\mu}-\frac{\mu}{n\lambda+\mu}e^{-(n\lambda+\mu)t}; \quad (5.26)$$

$$P_1(t)=\frac{n\lambda}{n\lambda+\mu}+\frac{\mu}{n\lambda+\mu}e^{-(n\lambda+\mu)t}. \quad (5.27)$$

For stationary condition $t \to \infty$ the availability (in-commission rates) and breakdown rates are of the following form:

$$P_0=K_{\text{г}}=\frac{\mu}{n\lambda+\mu}=\frac{\overline{T}}{n\overline{t}_{\text{в}}+\overline{T}}; \quad (5.28)$$

$$P_0=K_{\text{п}}=\frac{n\lambda}{n\lambda+\mu}=\frac{n\overline{t}_{\text{в}}}{n\overline{t}_{\text{в}}+\overline{T}}. \quad (5.29)$$

If the elements (components) of series circuit are heterogeneous, e. g. $\lambda_1 \neq \lambda_2 \neq \lambda_n$, then

$$P_1=\sum_{i=1}^{n}\lambda_i\overline{t}_{\text{в}i}; \quad (5.30)$$

$$P_0=1-\sum_{i=1}^{n}\lambda_i\overline{t}_{\text{в}i}. \quad (5.31)$$

By calculation of elementary circuits (schemes) with low component count (low number of elements) and $\mu >> \lambda$, the error by application of these formulae is small (insignificant).

5.1.3. System consisting of parallel connected repairable elements (renewable units)

Parallel connected repairable elements from the standpoint of reliability mean, that in the case of failure of one of the elements (components) the system keeps functioning, i.e. automatic redundancy of each element with capacity (throughput performance) sufficient to satisfy consumers' power is assumed.

In a general case, when the system by such redundancy consists of n independent elements (components), the number of possible system states (conditions) will be 2^n assuming that each element (component) can be in two states (conditions): in operational and failed (failure condition) state. System failure (breakdown) occurs when all elements (components) of the system are in failed state (failure condition).

Let us consider more thoroughly the simplest case, which is commonly encountered in power (electrical) systems – parallel connection of two elements (two circuits of a transmission line, two transformer substations and etc.). Such system can be in four states (conditions): 1 – both elements are in operational state; 2 – the first element is in failed state (failure condition), and the second one is in operational state; 3 – the second element is in failed state (failure condition), and the first one is in operational state; 4 – both elements are in failed state (failure condition). The corresponding probabilities of these states (conditions) will be $P_1(t)$, $P_2(t)$, $P_3(t)$, $P_4(t)$.

The methodology of differential equations set-up (generation of equation) and solution of differential equations for this case will be based on the principles mentioned above. This methodology is given in [8, 9]. We will consider only the conclusions (developments) of their solution.

For stationary (steady) state (when $t \to \infty$) the mean probability of states (conditions) will be as follows:

$$P_1 = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\overline{T}_1}{(\overline{T}_1+\overline{t}_{v1})}\frac{\overline{T}_2}{(\overline{T}_2+\overline{t}_{v2})} = K_{g1}K_{g2}; \tag{5.32}$$

$$P_2 = \frac{\lambda_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\overline{t}_{v1}}{(\overline{T}_1+\overline{t}_{v1})}\frac{\overline{T}_2}{(\overline{T}_2+\overline{t}_{v2})} = K_{g1}K_{g2}; \tag{5.33}$$

$$P_3 = \frac{\lambda_2\mu_1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\overline{t}_{v2}}{(\overline{T}_2+\overline{t}_{v2})}\frac{\overline{T}_1}{(\overline{T}_1+\overline{t}_{v1})} = K_{p2}K_{g1}; \tag{5.34}$$

$$P_4 = \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\overline{t}_{v1}}{(\overline{T}_1+\overline{t}_{v1})}\frac{\overline{t}_{v2}}{(\overline{T}_2+\overline{t}_{v2})} = K_{p1}K_{p2}. \tag{5.35}$$

Stationary availability and steady-state unavailability of the system provided (given) $\overline{T}_i >> \overline{t}_{вi}$ is:

$$K_g = P_1 + P_2 + P_3; \tag{5.36}$$

$$K_p = P_4 = K_{p1}K_{p2} \approx \lambda_1\overline{t}_{v1}\lambda_2\overline{t}_{v2}. \tag{5.37}$$

This result can be also obtained applying both-and rule (product rule) of independent events and without overlapping of the states (conditions) neither distribution laws of failure free time nor distribution laws of recovery time.

In fact, the system consisting of two independent mutual redundant (backing up) elements will fail in case of intersection of two events when both elements fail. The probability of this phenomenon is equal to the product of mean probabilities of failed state (failure condition) of each of this element – q_1 and q_2.

As the mean probabilities of failed sate (failure condition) are approximately equal to the product of the number of failures $\lambda_i$by mean recovery time $\bar{t}_{Bi}$, then

$$q_1 \approx K_{p1} = \lambda_1 \bar{t}_{v1},\ \ q_2 \approx K_{p2} = \lambda_2 \bar{t}_{v2}.$$

Hence,

$$K_p = q_1 q_2 = K_{p1} K_{p2} = \lambda_1 \bar{t}_{v1} \lambda_2 \bar{t}_{v2}. \tag{5.38}$$

When considering one-element (unicomponent) system it was shown that the exponent damping constant (attenuation coefficient) is inversely proportional to mean recovery time of the element by $\bar{T} >> \bar{t}_{в} : (\lambda + \mu) \approx \bar{t}_{в}^{-1}$.

In the system under consideration the probabilities of all states (conditions) are described by superposition of exponents with constant components, that can be approximately replaced (substituted) by one exponent with equivalent damping constant (attenuation coefficient), that is inversely proportional to equivalent recovery time of the system from the failed state into operational state, e. g.

$$e^{-(\lambda_i + \mu_i)t} \approx e^{-\frac{t}{\bar{t}_{vi}}}; \tag{5.39}$$

$$e^{-(\lambda_1 + \mu_1 + \lambda_2 + \mu_2)t} \approx e^{-t\frac{\bar{t}_{v1} + \bar{t}_{v2}}{\bar{t}_{v1}\,\bar{t}_{v2}}}, \tag{5.40}$$

where

$$\frac{\bar{t}_{v1}\bar{t}_{v2}}{\bar{t}_{v1} + \bar{t}_{v2}} = \bar{t}_{vm}, \tag{5.41}$$

has the meaning of equivalent recovery time of two parallel connected elements.

Considering the multiplex (redundant) system as one equivalent component (element) we can represent it as follows

$$K_{п} = \lambda_{с} \bar{t}_{вс},$$

where

$$\lambda_c = \frac{K_p}{\bar{t}_{vm}} = \frac{\lambda_1 \bar{t}_{v1} \lambda_2 \bar{t}_{v2} (\bar{t}_{v1} + \bar{t}_{v2})}{\bar{t}_{v1} \bar{t}_{v2}} = \lambda_1 K_{p2} + \lambda_2 K_{p1}. \tag{5.42}$$

Thus, failure rate of a system consisting of two elements that back up (reserve) each other is equal to the product of failure rate of the first element multiplied by mean probability of failed state of the second element and failure rate of the second element multiplied by mean probability of the failed state of the first element.

The obtained algorithm of failure rate definition of the multiplex (redundant) system is of great practical importance due to its simplicity and clearness (visualization).

Failure rate of a system can be approximately assessed (estimated) without superposition of distribution function of failure free operation and elements renewal (recovery).

Let us study two independent hypotheses concerning possible system failures (breakdowns) analyzing its state (condition) over sufficiently long period (interval) T.

1. The number of systems failures over the interval T in the process of recovery of the first element is equal to the product of number of failures of the second element multiplied by mean probability of failed sate q_1 of the first element.

$$N_1 = \lambda_2 T q_1 = \lambda_2 T K_{p1}. \tag{5.43}$$

2. The number of systems failures over the interval T in the process of recovery of the second element is equal to the product of number of failures of the first element multiplied by mean probability of failed sate q_2 of the second element.

$$N_2 = \lambda_1 T q_2 = \lambda_1 T K_{p2}. \tag{5.44}$$

Total number of failures is equal to the sum of failures by two hypotheses:

$$N = N_1 + N_2 = (\lambda_2 K_{p1} + \lambda_1 K_{p2})T. \tag{5.45}$$

Average number of systems failures per time unit (failure rate) is defined as:

$$\lambda_c = N/T = \lambda_1 K_{p2} + \lambda_2 K_{p1}. \tag{5.46}$$

In other words, **the summands $\lambda_1 K_{p2}$ and $\lambda_2 K_{p1}$ have sense of average number of failures during the failed state period of the second and first elements correspondingly.**

In the systems where redundancy is carried out are called systems with reliability redundancy (excess) (redundancy on reliability). Two parallel connected elements, where each element is capable of functioning (to transmit required capacity (power)) form (comprise) an elementary (simplest) system with redundancy (excess) on reliability (reliability redundancy (excess)). It is necessary to mention the important feature of such systems. If failure flow and flow of renewals (recoveries) that are included in the system with reliability redundancy (excess) (redundancy on reliability) possess the properties (features) of a simplest (elementary) Markovian process, then the failure rate and flow rate of renewals (recoveries) considered as equivalent element with sufficient for practical purposes accuracy can be also regarded as possessing these properties (features), i.e. stationary and absence of aftereffects. This failure flow of systems with reliability redundancy (excess) (redundancy on reliability) will be Poisson one, as the probability of systems failure (breakdown) is considerably less than the probability of failure of separate (certain) elements.

The obtained practical algorithm for failure rate definition of the system with redundancy (renewals) can be applied for the case, when n elements back up each other (parallel connection in terms of reliability).

In order to define the failure rate of this system it is necessary to consider as many summands as many elements are there in the system, e. g.

$$\lambda_c = \sum_{i=1}^{n} \lambda_i \prod_{\substack{j=1 \\ j \neq i}} \lambda_j \bar{t}_{\text{в}j}\,; \tag{5.47}$$

$$\bar{t}_{\text{vm}} = \frac{K_{\text{п}}}{\lambda_c} = \prod_{i=1}^{n} \lambda_i \; \bar{t}_{vi} \Big(\sum_{i=1}^{n} \lambda i \prod_{\substack{j=1 \\ j \neq i}} \lambda_j \bar{t}_{\text{v}j} \Big)^{-1}. \tag{5.48}$$

In a particular case, when the elements (components) have the same reliability indices:

$$\lambda_c = n \lambda^n \bar{t}_{\text{в}}^{\,n-1} = n\lambda^n / \mu^{n-1}; \tag{5.49}$$

$$\bar{t}_{\text{vm}} = \frac{\lambda^n \bar{t}_{\text{v}}^{\,n}}{n\lambda^n \bar{t}_{\text{v}}^{\,n-1}} = \frac{1}{n\mu} = \frac{\bar{t}_{\text{v}}}{n}; \tag{5.50}$$

$$\bar{T}_c \approx \frac{1}{\lambda_c} = \frac{\mu^{n-1}}{n\lambda^n}. \tag{5.51}$$

For two similar (identical) mutual backing up elements

$$\lambda_c = 2\bar{t}_{\text{в}} / \bar{T}^2; \qquad \bar{t}_{\text{вс}} = \bar{t}_{\text{в}} / 2; \qquad \bar{T}_c = \bar{T}^2 / 2\bar{t}_{\text{в}}. \tag{5.52}$$

The obtained approximate algorithm of reliability indices definition can be used for systems of any complexity with free (arbitrary) (in terms of reliability) connection of elements, if the complete failure (breakdown) indices are defined for the system.

The failure rate of a system consisting of n independent elements (components) is equal to the sum of productions of failure rate of each element multiplied by mean probability of subsystems failure, which is left (remained) after this element was removed. If the failure of the element under consideration leads to systems failure, then the probability of remained (left) subsystem (part) failure is assumed as equal to 1 (unity) (for example series in terms of reliability connection of elements).

By increasing the number of elements (components) in the calculating system the number of its possible states (conditions) grows rapidly (for example, in the system with n elements without regard for deliberate elements cutoff (disconnection) the number of states (conditions) is $N = 2^n$). Therefore, the application of Markovian process theory for reliability evaluation (assessment) using the complete differential combined equations (set of differential equations), their analysis and solution presents some difficulties.

The solution of the problem using the values λ_i, μ_i, $P_k(0)$ makes the result acquisition (obtaining) considerably easier, but does not exclude (eliminate) symbolic manipulations (transformations) that are necessary by determinant calculation. Therefore, in the case, when it is necessary to research (study) the system reliability over short periods (intervals) the number of states N is limited, integrating the element group in one element with equivalent reliability indices ($\lambda_э$, μ and etc.), that can be approximately estimated (assessed) by the mentioned above methods.

5.1.4. Reliability indices calculation (estimation) taking into consideration repair state and deliberate cutoff (disconnection) of elements (components)

Timely and reasonable preventive equipment repair (maintenance work) allows not only to increase the technical capability (characteristics) of the operating system and improve their reliability indices, but also to reduce maintenance (operating) costs. However, preventive repair and complete overhaul of the equipment is connected with elements cutoff (disconnection), switching scheme (commutation scheme) changes, that leads to the change of power supply reliability level at this period.

Power equipment repair volume and schedule are determined by annual agenda (plan). As a rule preventive repair and overhaul schedule are planned to reduce the possible losses (damage) from undersupply of energy. Besides, the equipment and apparatuses repair that is directly linked (coupled) with technological apparatuses is carried out as far as possible together with the repair of the latest one.

Intentional (deliberate) cut off (disconnection) of elements (components) is carried out not only for preventive repairs and complete overhauls, but on the demand of different enterprises. Outage (cut off) frequency and duration of power supply elements (components) in general depend on random factors. Therefore, in reliability calculations the intentional cut off (outage) is reasonable to set the flow parameter of intentional (deliberate) outage (cut off, disconnection) λ_{dt} and their average time $\overline{t}_{pr}$.

Average time of preventive repair and complete overhaul is mainly defined by engineering instruction (operational regulations). Deviation from the average time is determined by the influence of weather conditions, repair base (service station) condition, availability of replacement components and etc.

Intentional element cutoff (disconnection) duration is usually compared with emergency repair (maintenance) duration. Therefore, the reliability indices calculations on the short intervals by taking into consideration intentional cutoffs (disconnection) are necessary to carry out taking into account initial states (conditions) of the elements. On the basis of these concepts the theory of Markovian processes for assessment (evaluation) of state probability can be applied with some assumptions (as the intentional cutoff (disconnection) duration is distributed not in accordance with exponential law and flow parameter of intentional cutoffs (disconnection) of the elements varies with time). Moreover, if the states of intentional cut off (disconnection) of the elements is described by differential equations, then their total number for a system consisting of *n* elements will increase by 3^n, that considerably complicates the solution obtaining.

In practical reliability calculations over sufficiently longer intervals $\left(t >> \overline{t}_{в}\right)$ mean probabilities are usually used, and the record (calculation) of initial states (conditions) is kept simplified, where such concepts (terms) as "element" and "system" are conditional. If the system is excessive (surplus) on reliability, then by intentional cut off (disconnection) of the element under consideration the remained (left) part of the system (within the solving problem) is considered as one equivalent element with equivalent reliability indices. The reliability indices are calculated by taking into account the initial conditions of operation, as it is assumed that at the moment (at instant) of intentional cut off (disconnection) of the given element *i* the equivalent element was in operational state.

The main point of this method is that the probability of emergency shutoff (disconnection) overlapping of one element on the intentional cut off (disconnection) of another element (but not vice versa) was defined taking into account the initial (starting) conditions. Hence, the probability of emergency (breakdown) of the equivalent element over the period of intentional cut off (disconnection) of the given element is lower (smaller) than the mean probability of its emergency cut off (disconnection). Thus, in accordance with formula (5.6) the probability of failure of the equivalent element during the intentional cut off (disconnection) of *i* element is:

$$P_{1e}(t) = \frac{\lambda_e}{\lambda_e + \mu_e} - \frac{\lambda_e}{\lambda_e + \mu_e} e^{-(\lambda_e + \mu_e)t} = K_{pe} - K_{pe} e^{-\frac{t}{t_{ve}}}. \qquad (5.53)$$

If we will assume that the intentional cut off (disconnection) duration is equal to $\overline{t}_{pri}$, then

$$P_{1e}(t) = K_{pe}\left(1 - e^{-\frac{\overline{t}_{pri}}{\overline{t}_{ve}}}\right) = K_{pe} K_{pri}. \qquad (5.54)$$

Where K_{npi} is coefficient that depends on the relation between recovery time of the back up (redundant) equivalent element and the period of intentional cut off (disconnection) of *i* element (Fig. 5.3). This coefficient takes into account the fact of decrease of probability coincidence of the intentional cut off (disconnection) of one element and emergency cut off (disconnection) of another back up (redundant) element.

For steady-state probability ($t \to \infty$), the coefficient that takes into account the probability of failure overlapping (superposition) of back up (redundant) element on intentional cut off (disconnection) of *i* element, can be accepted as equal to:

$$K_{pri} = \bar{t}_{pri} / \left(\bar{t}_{pri} + \bar{t}_{ve} \right). \qquad (5.55)$$

If the system consists of *n* elements with arbitrary switching scheme (setup sheet; commutation diagram), then to calculate (estimate) the reliability indices it is necessary to consider *n* hypotheses; in every of these hypotheses the intentional cut off (disconnection) of element is planned (supposed). The resulting index is defined on base of reliability indices of the whole system at each hypothesis.

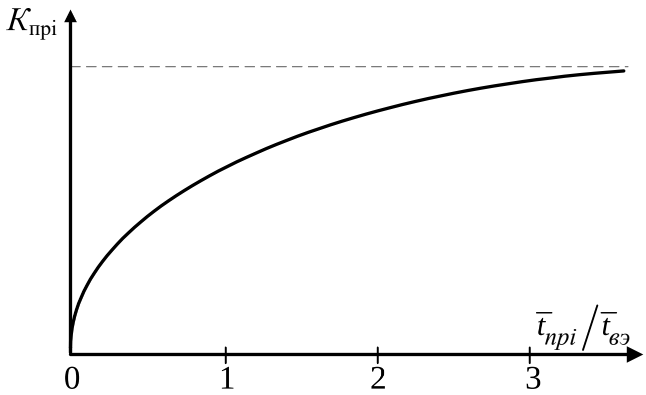

Fig. 5.3. Relation (Dependence) of K_{dt} from the ratio of intentional cut off time and recovery of the remained part

- ***The system with series connected elements (components)***. In order to reduce the number of probabilities of off state (open position) and power supply interruption (interruption of the mains supply) in the system with series connected elements there is a tendency to combine the intentional cut off (disconnection) of the elements for preventive repair and complete overhaul. For approximate calculations, particularly for design, the forced outage coefficient of such system is defined in accordance with the following formula:

$$K_{\text{ps}} = \sum_{i=1}^{n} \lambda_i \bar{t}_{vi} + \left(\lambda_{vi} \bar{t}_{pri} \right)_{\text{нб}}. \qquad (5.56)$$

Where $\left(\lambda_{\text{п}i} \bar{t}_{\text{пр}i} \right)$ is the largest probability of intentional cut off (disconnection) of one of the *n* system elements.

Net (total) failure rate (cut off rate) and equivalent recovery time are equal to:

$$\lambda_{\text{c}} = \sum_{i=1}^{n} \lambda_i + \lambda_{\text{п.нб}}; \qquad (5.57)$$

$$\bar{t}_{\text{vc}} = \frac{\sum_{i=1}^{n} \lambda_i \bar{t}_{vi} + \left(\lambda_{\text{п}i} t_{\text{пр}i}\right)_{\text{нб}}}{\sum_{i=1}^{n} \lambda_i + \lambda_{\text{п.нб}}} = \frac{K_{\text{пс}}}{\lambda_{\text{c}}}. \tag{5.58}$$

Where $\lambda_{\text{п.нб}}$ is the maximum cut off (outage) frequency of one element from *n* elements of the system.

The more precise (accurate) method of intentional (deliberate) cut off (disconnection) of such circuits (schemes) is used in analysis technique of system state probability.

- ***The system with parallel connected elements (components)***. Firstly, let us consider a scheme (diagram) with two back up (redundant) elements 1 and 2. The coefficient of forced outage (mean probability of off state) of this system in accordance with the mentioned above principles is:

$$K_{\text{пс}} = \lambda_1 \bar{t}_{\text{v}1} \lambda_2 \bar{t}_{\text{v}2} + \lambda_{\text{пр}1} \bar{t}_{\text{пр}1} K_{\text{пр}1} \lambda_2 \bar{t}_{\text{v}2} + \lambda_{\text{пр}2} \bar{t}_{\text{пр}2} K_{\text{пр}2} \lambda_1 \bar{t}_{\text{v}1}, \tag{5.59}$$

where

$$K_{\text{пр}1} = 1 - e^{-\frac{\bar{t}_{\text{пр}1}}{\bar{t}_{\text{в}2}}} \quad \text{or} \quad K_{\text{пр}1} = \frac{\bar{t}_{\text{пр}1}}{\bar{t}_{\text{пр}1} + \bar{t}_{\text{в}2}};$$

$$K_{\text{пр}2} = 1 - e^{-\frac{\bar{t}_{\text{пр}2}}{\bar{t}_{\text{в}1}}} \quad \text{or} \quad K_{\text{пр}2} = \frac{\bar{t}_{\text{пр}2}}{\bar{t}_{\text{пр}2} + \bar{t}_{\text{в}1}}.$$

Failure rate (failure flux parameter) of the system and equivalent recovery time are equal:

$$\lambda_{\text{c}} = \lambda_1 \left(\lambda_2 \bar{t}_{\text{v}2}\right) + \lambda_2 \left(\lambda_1 \bar{t}_{\text{v}1}\right) + \lambda_1 \lambda_{\text{пр}2} \bar{t}_{\text{пр}2} + \lambda_2 \lambda_{\text{пр}1} \bar{t}_{\text{пр}1}; \tag{5.60}$$

$$\bar{t}_{\text{вс}} = \frac{K_{\text{пс}}}{\lambda_{\text{c}}} = \frac{\lambda_1 \bar{t}_{\text{v}1} \lambda_2 \bar{t}_{\text{v}2} + \lambda_{\text{пр}1} \bar{t}_{\text{пр}1} K_{\text{пр}1} \lambda_2 \bar{t}_{\text{v}2} + \lambda_{\text{пр}2} \bar{t}_{\text{пр}2} K_{\text{пр}2} \lambda_1 \bar{t}_{\text{v}1}}{\lambda_1 \left(\lambda_2 \bar{t}_{\text{v}2}\right) + \lambda_2 \left(\lambda_1 \bar{t}_{\text{v}1}\right) + \lambda_1 \lambda_{\text{пр}2} \bar{t}_{\text{пр}2} + \lambda_2 \lambda_{\text{пр}1} \bar{t}_{\text{пр}1}}. \tag{5.61}$$

This method is easily applied to the system with *n* mutual back up (redundant) elements:

$$K_{\text{пс}} = \prod_{i=1}^{n} \lambda_i \bar{t}_{vi} + \sum_{i=1}^{n} \lambda_{\text{пр}i} \bar{t}_{\text{пр}i} K_{\text{пр}i} \prod_{\substack{j=1 \\ j \neq i}}^{n} \lambda_j \bar{t}_{vj}; \tag{5.62}$$

$$\lambda_{\text{c}} = \sum_{i=1}^{n} \lambda_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \left(\lambda_j \bar{t}_{\text{в}j} + \lambda_{\text{пр}j} \bar{t}_{\text{пр}j}\right); \tag{5.63}$$

$$\bar{t}_{\text{вс}} = K_{\text{пс}} / \lambda_{\text{c}}. \tag{5.64}$$

Example: The consumer (C) is supplied with power from two power sources P1 and P2 (Fig. 5.4). Each circuit can conduct all necessary power.

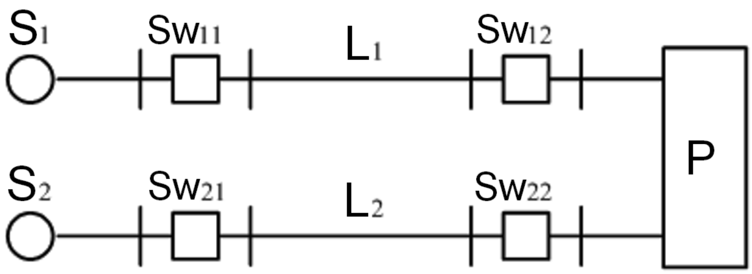

Fig. 5.4. Power supply scheme (circuit)

The failure rate (failure flux parameter) and intentional (deliberate) disconnection (cut off) of the elements of power supply system, mean recovery time and intentional (deliberate) cut off (disconnection) duration are presented in Table 5.1.

Table 5.1

Reliability parameters of elements

Parameter	Elements					
	Sw_{11}	L_1	Sw_{12}	Sw_{21}	L_2	Sw_{22}
λ_0, 1/(km)	0,099	0,023	0,048	0,137	0,019	0,137
L, km	–	80	–	–	30	–
$\bar{t}_B$, hours	10	30	10	15	30	15
λ_{pr}, 1/year	0,4	0,3	0,4	0,4	0,3	0,4
t_{pr}, hours	60	50	60	80	20	80

It is necessary to define failure rate of power supply system, average time of failure free operation, mean probability of failure, mean recovery time and also undersupply of energy per year, assuming that average annual consumer's power is $\overline{P} = 30\,\text{MW}$.

By calculation it is to assume that intentional (deliberate) cut off (disconnection) of series connected elements (components) of the circuits coincide (superpose) on time. The reliability of power sources is not taken into consideration.

The flow rate of the first and the second circuits (schemes), where each of these circuits consists of three series connected elements (components) in accordance with the formula (5.57) will be as follows:

$$\lambda_{\text{I}} = \lambda_{\text{ol1}} L_1 + \lambda_{\text{v11}} + \lambda_{\text{v12}} + \lambda_{\text{prv11}} = 0,023 \cdot 80 + 0,099 + 0,048 + 0,4 = 2,387\ 1/\text{year};$$

$$\lambda_{\text{II}} = \lambda_{\text{ol2}} L_2 + \lambda_{\text{v21}} + \lambda_{\text{v22}} + \lambda_{\text{prv21}} = 0,019 \cdot 30 + 2 \cdot 0,137 + 0,4 = 1,244\ 1/\text{year}.$$

The failure rate of a system is defined as for a system consisting of two parallel connected elements (components) in accordance with (5.60) and taking into account that superposition (overlapping) of emergency (accident) of the remained (left) part of the scheme (circuit) on the deliberate (intentional) cut off (disconnection) of the j element is possible to find:

$$\lambda_c = \lambda_I q_{II} + \lambda_{II} q_I + (\lambda_I - \lambda_{prv11}) q_{прII} + (\lambda_{II} - \lambda_{prv21}) q_{prI},$$

where

$$q_I = q_{B11} + q_{л1} + q_{B12} = \lambda_{B11} \bar{t}_{вB11} + \lambda_{л1} \bar{t}_{вл1} + \lambda_{B12} \bar{t}_{вB12} =$$

$$= (0,099 \cdot 10 + 1,84 \cdot 30 + 0,048 \cdot 10) / 8760 = 6,47 \cdot 10^{-3};$$

$$q_{II} = q_{B21} + q_{л2} + q_{B22} = (0,137 \cdot 15 + 0,57 \cdot 30 + 0,137 \cdot 15) / 8760 = 2,43 \cdot 10^{-3}.$$

Intentional (deliberate) circuit opening is taken into account by the element parameters B11 and B2, relatively q_{dtI} and q_{dtII}.

$$\lambda_c = 2,387 \cdot 2,43 \cdot 10^{-3} + 1,244 \cdot 6,47 \cdot 10^{-3} + (0,844 \cdot 0,4 \cdot 60 +$$
$$+ 1,987 \cdot 0,4 \cdot 80)/8760 = 23,41 \cdot 10^{-3} \text{ 1/year}.$$

Mean time of failure free operation is

$$\bar{T}_c = 1 / \lambda_c = 1 / 23,41 \cdot 10^{-3} = 42,7 \text{ years}.$$

where a = 0,1 [9] is estimated time of failure free operation

$$T_m = -ln\ (1-\alpha)\ \bar{T}_c = 0,105\ \bar{T}_c = 4,48 \text{ years}.$$

Mean time of emergency recovery (fall-back recovery) of the circuits is:

$$\bar{t}_{vI} = \frac{q_I}{\lambda_I - \lambda_{prB11}} = \frac{6,47 \cdot 10^{-3}}{1,987}\ 8760 = 28,5 \text{ hour};$$

$$\bar{t}_{vII} = \frac{q_{II}}{\lambda_{II} - \lambda_{prB21}} = \frac{2,43 \cdot 10^{-3}}{0,844}\ 8760 = 25,2 \text{ hour}.$$

The coefficients that take into account the factors of reducing of probability of intentional (deliberate) cut off (disconnection) of elements (components) Sw11 and Sw21,

$$K_{p1} = 1 - e^{-\frac{t_{pv11}}{\bar{t}_{vII}}} = 1 - e^{-\frac{60}{25,2}} = 0,9075;$$

$$K_{p2} = 1 - e^{-\frac{t_{pv21}}{\bar{t}_{vI}}} = 1 - e^{-\frac{80}{28,5}} = 0,939.$$

Mean probability of failed (failure) state is:

$$q_c = q_I q_{II} + K_{p1} \lambda_{prI} \cdot \bar{t}_{prI} \cdot q_{II} + K_{p2} \lambda_{prII} \cdot \bar{t}_{prII} \cdot q_I =$$

$$= 6,47 \cdot 10^{-3} \cdot 2,43 \cdot 10^{-3} + \Big(0,9075 \cdot 0,4 \cdot 60 \cdot 2,43 \cdot 10^{-3} +$$

$$+ 0,939 \cdot 0,4 \cdot 80 \cdot 6,47 \cdot 10^{-3}\Big) / 8760 = 43,93 \cdot 10^{-6}.$$

Mean recovery time of a system is:

$$\bar{t}_{vm} = \frac{q_c}{\lambda_c} = \frac{43,93 \cdot 10^{-6}}{23,41 \cdot 10^{-3}} 8760 = 16,43 \text{ h}.$$

Mathematical expectation of undersupplied energy to consumers is

$$\Delta E = \overline{\mathrm{E}} \cdot q_c = \bar{P} T q_c = 30 \cdot 10^{-3} \cdot 8760 \cdot 43{,}93 \cdot 10^{-6} = 11542 \ \text{KVh}.$$

If in the given example (equation) the deliberate (intentional) cut off (disconnection) is not taken into account, then we will get:

$$\lambda_c = 13{,}85 \cdot 10^{-3} \text{ 1/year}; \ q_c = 15{,}7 \cdot 10^{-6}.$$

Comparing the calculation data of reliability parameters taking into account and without taking into account the number of the intentional cut off (disconnection) it can be concluded that the intentional cut off has a considerable impact on the reliability parameters of power supply scheme (diagram).

The engineering-and-economical effects (consequences) from undersupply of energy and outages can be evaluated (assessed) by the obtained reliability indices.

From the mentioned above follows that the power supply system operation is sufficiently revealed (reproduced) in the model of Markovian processes. On basis of analyses of equation solution of this model, fairly simple algorithms for reliability indices calculation of typical power supply schemes (diagrams) from the point of view of reliability have been developed. With the increase of the number of elements of the system the number (quantity) of its possible states also increases sharply. Therefore, the application of Markovian processes for reliability evaluation (assessment) of repairable (restorable) system using the complete system of differential equations (set of differential equations), their analysis and solution presents some difficulties.

Simulation (modeling) of failure and renewal processes by means of Markovian processes is justified in the case when it is necessary to keep a record of initial states of some (separate) elements, e. g. the reliability indices are calculated over relatively short periods. To evaluate (assess) the probabilities of system states over sufficiently long period (season, year) much simpler asymptotic techniques (methods), based on mean probability values of the element state (condition) are mainly used.

5.2. Methods of reliability indices calculations (analyses) of power supply schemes (diagrams) on the mean probability values of element state (condition)

5.2.1. Mean probability values of an element

In reliability calculation of power supply systems, as well as in any other systems, there can be a contradiction: on the one hand the desire to have a precise model, that most sufficiently describes the failure and renewal processes, on the other hand the simplicity of calculations and the provision of calculated model by initial data.

The most widespread are the reliability calculation methods, which assume that a system consists of independent (separate), in terms of reliability, elements. In these methods the element is considered as failed (broken down) when its parameters spill over (run over) the tolerable practical standard limits. It is assumed, that when the element fails it is disconnected (disrupt) from the other part of the system by switching equipment. These calculation methods do not take into account functional relationship between the parameters of separate elements of power supply system, which can be viewed as one of their disadvantages. But taking into consideration the simplicity of calculations, the absence of necessary initial data and the possibility to obtain quantitative reliability evaluation (assessment) for modern complicated systems, the application of these methods at the current stage of theory development is justified.

The concepts (notions) "element" and "system" in reliability calculations are relative. The unit (object) that is considered as a system in one research can be viewed as an element in another research if the unit (object) of a larger scale is being studied. For example, if the reliability of power station is researched (studied), then the station is considered as a system, where generators, switches, bus bars, switch gears, turbines are regarded as separate elements. If the reliability of one generator is studied, then its parts such as a stator, rotor, exciter and etc. are considered as elements, and the generator is a system.

The system division into elements depends on the type of research (study) (functional, structural, circuit, operational element (component) and etc.), required accuracy of the conducted research, level of knowledge about the operation, the availability of statistical data and the scale of the unit (object) in whole. For example, when the reliability of a complicated system is analyzed (assessed) concerning (relatively to) the load center (node) the set of structural components (elements) of connection (joining) (disconnectors, switch with the set of relay protection and appropriate part of a bus bar) is regarded as one element with the same (integrated) reliability index, that includes the failures of these apparatuses under static and operating conditions. However, by assessment of chances of failure development in the system such agglomeration (enlargement) does not allow to solve the problem. In this case it is necessary to consider separately the switch failure under static and operating conditions and take into account the relay protection failures in them.

Convention (relativity) of the notions (concepts) "element" and "system" enables the wide application of step-by-step reliability calculation method (reliability analysis). The main idea of this method is that at the next stage of calculation (analysis) the element of the complicated system (station, substation, set of transmission lines) can be regarded as a separate system, where the reliability indices are specified in consecutive order. Thus, it is possible to make reliability calculations of more complicated systems.

In order to estimate (to assess) reliability indices on mean probability values of the element states (conditions) the following statistical data (information) is used:

1. Failure rate ω, e. g. average number of failures per time unit (usually per year) concerning one element (for simplest failure flow (simple failure stream) $\omega = \lambda$). For transmission line the failure rate usually concerns 1 km line [1/(km year.)].

2. Mean recovery time (replacement, breakdown maintenance) $\bar{t}_{в}$, hour/one renewal.

3. Failure rate of the intentional deliberate cut off (disconnection) of the element λ_n, 1/year.

4. Mean time of one intentional (deliberate) cut off (disconnection) of the element (mainly for preventive repair and complete overhaul of equipment) $\bar{t}_{пр}$, hour/one disconnection (cut off, outage).

The unreliability of an element (mean probability of failed state (failure condition) is defined by mean probability of its total downtime (idle) as a result of forced outage (cut off) due to damages and intentional (deliberate) cut off (disconnection) for preventive check (chapter 5.1).

The probability of forced outage (downtime)

$$q = \frac{\omega \bar{t}_v}{8760}. \tag{5.65}$$

The probability of intentional (deliberate) cut off (disconnection)

$$q_{\text{p}} = \frac{\lambda_p \bar{t}_{\text{pr}}}{8760}. \tag{5.66}$$

Mean probability of failed state (failure condition) (total)

$$q_{\Sigma} = q + q_p. \tag{5.67}$$

The probability of operation condition (availability, availability factor) is defined in accordance with the following formula

$$P = 1 - q_{\Sigma} = 1 - q - q_p = \bar{t}_{\text{p}} / 8760, \tag{5.68}$$

where $\bar{t}_{\text{p}}$ is failure-free time of the element.

If the time $\bar{t}_{\text{в}}$, $\bar{t}_{\text{pr}}$, $\bar{t}_{\text{p}}$ are measured in years, then

$$\left.\begin{aligned} q &= \lambda \bar{t}_{\text{в}} \\ q_p &= \lambda_{\text{n}} \bar{t}_{\text{pr}} \\ P &= \bar{t}_{\text{p}} \end{aligned}\right\} \tag{5.69}$$

The mentioned reliability indices can also characterize the entire system.

For the majority of the problem concerning technical and economic assessment of power supply system reliability there is no necessity to consider the reliability indices over (on) short intervals. Therefore, it is not necessary to take into account the initial conditions (states). Moreover the application of queuing analyses (techniques) (Markovian processes) for these purposes faces great difficulties with calculation, if the system has a great number of repairable elements (renewable units) and arbitrary commutation (switching) scheme. Therefore, by reliability indices calculation in time lags (slices) that are equal to a season, year***, and simpler probabilistic (chance) model, based on mean values of probabilities of element state (condition) can be applied.*** By the specified time lags (slices) the reliability calculation algorithms of its main indices (forced downtime rate, failure rate and mean recovery time) that were stated in chapter 5.1 provide sufficient accuracy, if the following conditions are carried out:

1) system element (component)failures are independent;
2) failure free time and recovery time are exponential distribution laws;
3) failure flow (stream) of the system element (component) is ordinary;
4) failure free time is much greater (longer) than the recovery time for all elements.

It is to be noticed, that to justify the possibility of application of algorithms on mean values of reliability indices two last described conditions are of great importance. These conditions are usually carried out practically for all system elements (components). Even if the distribution law of failure free time and recovery time significantly differ from exponential laws, the calculating error on mean values is small (insignificant).

Let us consider some dispositions of application of method of analysis (computing methods) on mean probabilities of element state (condition) with series and parallel connection.

5.2.2. The probabilities of failed (failure condition) and failure free states of a diagram (circuit) with series connected elements (components)

If the design (analytic) model (scheme) on reliability consists of *n* series connected elements (components), then it will be in operating condition when all *n* elements (components) are in operating condition. Complex (compound) event – the operation of all elements of a diagram (circuit) is the result of events overlapping – the operation of each element (component). Applying the probability multiplication theorem of independent events, we will get the operation condition of this diagram (circuit):

$$P_c = P_1 P_2 P_3 \dots P_n = \prod_{i=1}^{n} P_i . \tag{5.70}$$

The probability of failed state (failure condition) is defined as the probability of the event opposite to operation condition.

$$q_c = 1 - P_c. \tag{5.71}$$

In practical estimations (calculations) another method to define the probability of failed state (failure condition) of the elements is usually applied. The probability of diagram (circuit) failure in this method is defined as the probability of one element failure. The probability of this event is defined by applying the formula for the probabilities of the sum of compatible (joint) events:

$$q_c = \sum_{i=1}^{n} q_i - \sum_{i,j} q_i q_j + \sum_{i,j,k} q_i q_j q_k - ...(-1)^{n-1} q_1 q_2 q_3 ... q_n . \tag{5.72}$$

The relations, where q_i << 1 is typical for the elements (components) of electrical power systems. Therefore, by definition of probability of failed state (failure condition) of a system consisting of n series connected elements (components) the second, the third and etc. summands of the right side of equation of the last equation can be neglected as being the number of higher infinitesimal order (order of smallness) (formula (5.30)). Therefore, in practical calculation the following formula is used:

$$q_c \cong \sum_{i=1}^{n} q_i . \tag{5.73}$$

Calculating error does not exceed the value:

$$\left[\frac{1}{2}\sum_{i=1}^{n} q_i\right]^2 . \tag{5.74}$$

If the circuit (diagram) with series connected elements on reliability corresponds with electrical schematic diagram of elements connection, then if taking into consideration that in actual operating conditions the preventive repair of the elements of series circuit is carried out simultaneously, then the probability of circuit downtime (idle time) can be defined in accordance with formula (5.56):

$$q_{cp} \cong \sum_{i=1}^{n} q_i + q_p = q_c + q_p , \tag{5.75}$$

where q_p is the greatest probability from the probabilities of intentional (deliberate) circuit opening consisting of n elements.

5.2.3. The probabilities of failed (failure condition) and failure free states of a diagram (circuit) with parallel connected elements (components)

Let us consider a diagram (circuit) that consists of n parallel connected elements on the assumption of the independence of failure of each separate element and network capacity of each element sufficient to provide required power. This system will be in operating condition provided that only one element operates. The probability of operating condition (state) of the diagram (circuit) is defined by means of formula for probability sum of compatible (joint) independent events – each element operation:

$$P_c = \sum_{i=1}^{n} P_i - \sum_{i,j} P_i P_j + \sum_{i,j,k} P_i P_j P_k - \dots + (-1)^{n-1} P_1 P_2 \dots P_n . \tag{5.76}$$

The definition of probability of system operation applying this formula is very time-consuming as it is necessary to calculate and add (sum up) ($2^n - 1$) summands. All the summands should be taken into account as their values are close to one (unity). Therefore, the probability of reliable system operation is much easier to define on the probability of failed state (failure condition) of the elements. The system will be in failure condition under the assumption that all elements fail.

The probability of failure condition is defined by means of formula for product of independent events – failure of each system element (component):

$$q_c = q_1 q_2 \dots q_n = \prod_{i=1}^{n} q_i . \tag{5.77}$$

The probability of operating condition of such system is defined as probability of opposite event (systems failure (breakdown))

$$P_c = 1 - q_c = 1 - \prod_{i=1}^{n} q_i . \tag{5.78}$$

Let us consider the methodology of definition of the probability of failure condition of the system consisting of *n* parallels connected elements taking into account intentional (deliberate) cut off (disconnection) of separate elements. Moreover, not more than one element can be disconnected (cut off) simultaneously and intentionally and during the emergency recovery (fall-back recovery) the intentional cut off (disconnection) is not carried out. To define the probability of failure condition of this system it is reasonable to consider besides the probability of a complex (compound) event – the failure of all elements, also the probability of *n* hypotheses, where the systems failure is considered by intentional cut off (disconnection) of one element. As the hypotheses are independent due to independence of the elements, then the probability of systems failure condition (state) is defined as the sum of probabilities of failure conditions at each hypothesis (formula (5.62)).

$$\begin{aligned} q_{cp} &= \prod_{i=1}^{n} q_i + q_{p1} K_{p2} \prod_{\substack{i=2 \\ i \neq 1}}^{n} q_i + q_{p2} K_{p2} \prod_{\substack{i=1 \\ i \neq 2}}^{n} q_i + \dots + \\ &+ q_{pn} K_{pn} \prod_{\substack{i=1 \\ i \neq n}}^{n-1} q_i = \prod_{i=1}^{n} q_i + \sum_{j=1}^{n} q_{pj} K_{pj} \prod_{\substack{i=1 \\ i \neq j}}^{n} q_i . \end{aligned} \tag{5.79}$$

By the definition of probability of failure conditions at every hypothesis the reducing (decreasing) coefficient $K_{pj} < 1$ is introduced, that takes into account the reduction (decrease) of probability of damage (accident) superposition (overlapping) of the remained (left) part of the diagram (circuit) on intentional (deliberate) cut off (disconnection) of *j* element. The period of intentional (deliberate) cut off (disconnection) electric power systems elements is relatively not long, therefore, by the definition of the probability of emergency failure of the remained (left) part of the diagram (circuit) over this period it is necessary to take into consideration the initial states (conditions) of the element and simulate (design) the failure and renewal processes by means of Markovian processes. The remained (left) part of the diagram (circuit) is reasonable to present as one equivalent element with the properties (features) of the simple failure and renewal stream (simplest flow). Thus, the record of intentional (deliberate) cut off (disconnection) and the definition of reducing (decreasing) coefficient are necessary to keep according to the regulations stated in article (paragraph) 5.1.4.

When analyzing the algorithms of reliability indices calculation of arbitrary systems, it should be mentioned, that one of the most complicated and labor intensive and time-consuming task (problem) is repeated (multiple) definition of mean probability of failure of the remained (left) parts of the diagrams (circuits) after excluding each element alternately (in turn). In the general case, the complex diagrams (circuits) are practically not simplified after one element exclusion. By the calculation of forced outage coefficient concerning different load centers (nodes) the calculations are also quite laborious. Labor intensiveness and the number of calculations sharply increase by taking into consideration the intentional (deliberate) cutoff (disconnection) of the elements in complex (compound) systems. Therefore, one of the main tasks of the reliability analysis of electric power systems concerning the load centers (nodes) (or set of nodes) is the development of the methods intended to define the mean probabilities of their failure and failure free operation.

With the complication of interconnection (interrelation) between the elements it is impossible to bring the analytical model (design diagram) on reliability without applying some special techniques to the diagram (circuit) with parallel-series and series-parallel connected elements. For the diagram (circuit) of "bridge" type or diagram with a great number of transverse connections (cross-linkages) the conversion (transformation) rules of series parallel or parallel series circuits (diagrams) are unsuitable (inapplicable).

We will consider three basic methods from the analytic probabilistic methods of calculation of complex (compound) schemes (circuits) on the mean probability of the element (component) state (condition):

1) the method of analysis of the system state probability with the analysis of mode parameters in each state (the parameters by partial systems failures are determined by means of this method);

2) the method, that uses the formula of total probability and is based on it factorial expansion technique (method);

3) the method, that uses structural schematic representation, i.e. replacement of complex (compound) schemes (circuit) by equivalent ones with series-parallel or parallel –series connection of elements concerning the node points (centers).

5.2.4. Method of analysis of the system states probabilities

By means of this method the relation between modes of some separate elements and system and the system state probability can be taken into account, i.e. to estimate (evaluate) in terms of quantity the influence of capacity (throughput performance) limits of the elements on the system reliability indices (on heating current, voltage losses and etc.), particularly, on undersupply of energy.

To define the reliability indices of different system states (conditions) the calculating (estimated) elements are assigned taking into consideration the operation logic of system operation. The actual system elements (components) are combined (consolidated) in calculation (estimated) groups, the failure of these groups is not localized in them, but leads to cut off (disconnection) of related elements. This is as a rule a group of elements that are not separated in the scheme (circuit) by automatic switchboard (switching unit). In terms of reliability such elements are called series connected one. In accordance with the reliability indices of actual elements the reliability indices of calculation (estimated) elements are defined.

After that the modes of the system under different states (conditions) are analyzed – with one or two emergency disconnected (cut off) elements – and with superposition (overlapping) of the emergency condition of another element on each intentionally disconnected (curt off) element. The states (conditions) with three or more disconnected (cut off) elements are not considered as hardly probable.

The flow rate and intentional cut of (disconnection) $\lambda_{c,i,j}$ and its probability $Q_{i,j}$, are defined for each system state (condition).

Then the modes of element and system operation are estimated (calculated) and compared with admissible (allowable). After that the switched off capacity in nodes of the circuit (diagram) is evaluated (assessed) to ensure the mode of operation and minimum total damage from power limitations and undersupply of energy to consumers. Undersupplied energy is defined as the sum of undersupply at all system states (conditions).

At present, the method of analysis of system state probabilities is basic (common) for large power systems. It allows to show (to reflect) the features of different system states and modes in reliability evaluation (assessment). However, the calculations made by this method are very time-consuming and laborious, as almost for every state (condition) (and the quantity of states can be large) there is a necessity to estimate (calculate) flow distribution. More detailed information about the methodology (technique) and body of mathematics (mathematical apparatus) of this method you can find in.

By power supply system reliability evaluation (assessment) it is very often not necessary to take into account the throughput performance of the elements, but it is very important to evaluate (assess) structural reliability of the scheme (circuit) concerning each load node (center). In this case other types of methods are used, that are based on structural analysis of complex (compound) schemes (circuits) and total probability (overall probability) formula.

5.2.5. Methods with application of total probability (overall probability) formula

These methods allow by means of total probability formula to present complex (compound) scheme (circuit) in the form of equivalent series parallel one. Let us consider the main idea of this method by example of certain scheme (circuit) without taken into consideration intentional (deliberate) cut off (disconnection) of the elements.

Total (overall) probability formula intended to define reliable scheme (circuit) operation is interpreted in the following way. By this hypothesis the probability of any event (in the given case the system operation concerning the node) is calculated as the sum of product probabilities of incompatible hypotheses (as a hypothesis either the operation or failure of any element is considered) and probability of the event (event rate) (e. g. the operation of the remained part of the circuit).

Applying the total probability formula to the probability calculation of failure free operation of any scheme (circuit), the so called factorial expansion theorem can be formulated (defined). ***Circuit reliability with excess (redundancy) is equal to the product probability of failure free operation of i element of the circuit (multiplied) by (times) the probability of failure free operation of the remained (left) circuit (the termination points of i element are closed in a short circuit (are short-circuited) plus the product probability of the same i element failure (multiplied) by the probability of failure free operation of the remained (left) circuit (termination points of i element are open)***, i.e. for the distinguished element two hypotheses are considered.

Let us examine the expansion theorem, and hence, total probability formula intended to define reliability indices of complex (compound) scheme (circuits) by example of bridge circuit (connection) (Fig. 5.5). There can be considered two incompatible hypotheses concerning any circuit element: the operation with probability P and its failure with probability q.

Element 5 is chosen as such element. Then, applying the expansion theorem, it is not difficult to reduce the bridge circuit (Fig. 5.5) to the sum of two circuits: parallel-series and series-parallel (Fig. 5.6). The methods of calculation (analysis) of these circuits are well developed. The probability of failure free operation of this circuit (scheme) concerning the load node (center) IV is:

$$P_c = P_5\left[\left(1-q_1q_2\right)\left(1-q_3q_4\right)\right]+q_5\left[1-\left(q_1+q_3-q_1q_3\right)\left(q_2+q_4-q_2q_4\right)\right].$$

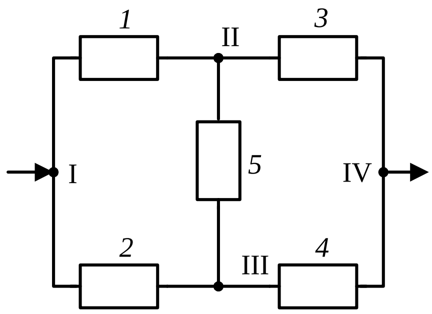

Fig. 5.5. Circuit of "bridge" type

There is probability of failure free operation of the circuit by the first hypothesis – failure free operation of the element 5 in this expression $\left(1-q_1q_2\right)\left(1-q_3q_4\right)$; $1-\left(q_1+q_3-q_1q_3\right)\left(q_2+q_4-q_2q_4\right)$ there is probability of failure free operation by the second hypothesis – failure of the element 5; P_5 is the probability of the first hypothesis; q_5 is the probability of the second hypothesis.

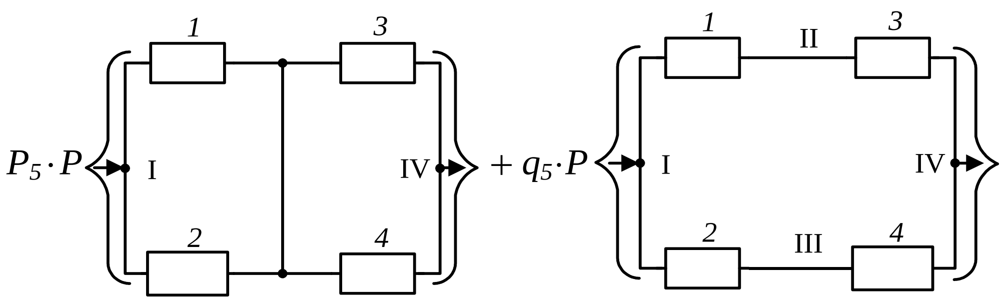

Fig. 5.6. Diagram, that illustrates the application of expansion theorem for the circuit of "bridge"-type

The total probability formula and factorial expansion theorem based on this formula play a significant role by the reliability analysis of complex (compound) diagram (circuit), as they enable to reduce any complex (compound) circuit to aggregate of elementary circuits (schemes). Moreover, this theorem is necessary to apply many times in a complex (compound) circuit (diagram).

The method of reliability evaluation (assessment), based on total probability formula is sufficiently convenient, simple and visual in the calculations even without using a computer of relatively small by volume schemes (circuits) with small number of nodes and paths, to which the in-plant power supply diagram refers. The application of this method for this irregular shape (complicated configuration) by means of a computer becomes complicated due to element choice, concerning which the expansion is carried out.

For pictorial (visual) presentation of repeated application expansion theorem the circuit (diagram) of "double bridge" type will be considered (Fig. 5.7). Let us define the probability of failure free operation of this scheme (circuit) concerning node IV without taken into consideration the intentional (deliberate) cut off (disconnection) of elements, if the mean probabilities of failed states (failure conditions) of the elements q_1, q_2, q_3, …, q_8 are known. The failures of node points (centers) are not taken into account. It is assumed that all circuit elements are independent in the terms of the probability of failures. Elements bandwidth on power is not limited.

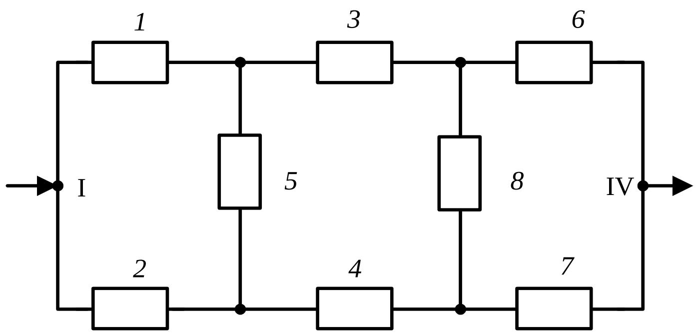

Fig. 5.7. Circuit of "double bridge) type

Applying sequentially the expansion theorem, firstly concerning the element 5 we will define the probability of failure free operation of the remained (left) part of the circuit, i.e. that includes the elements 1, 2, 3, 4, 6, 7, 8. To assess (estimate) the failure free operation of this remained (left) subcircuit (part of the circuit) the expansion theorem concerning the element 8 is applied. The circuit (diagram) that explains the consequence of these actions is represented in Fig. 5.8.

The probability of reliable operation of this system can be presented as follows:

$$P_c = P_5\,\{P_8\,(1-q_1q_2)(1-q_3q_4)(1-q_6q_7)+q_8(1-q_1q_2)[1-(q_3+q_6-\\ -q_3q_6)(q_4+q_7-q_4q_7)]\}+q_5\{P_8[1-(q_1+q_3-q_1q_3)\,(q_2+q_4-\\ -q_2q_4)](1-q_6q_7)+q_8[1-(q_1+q_3+q_6-q_1q_3-q_1q_6-q_3q_6+\\ +q_1q_3q_6)(q_2+q_4+q_7-q_2q_4-q_2q_7-q_4q_7+q_2q_4q_7)]\}.$$

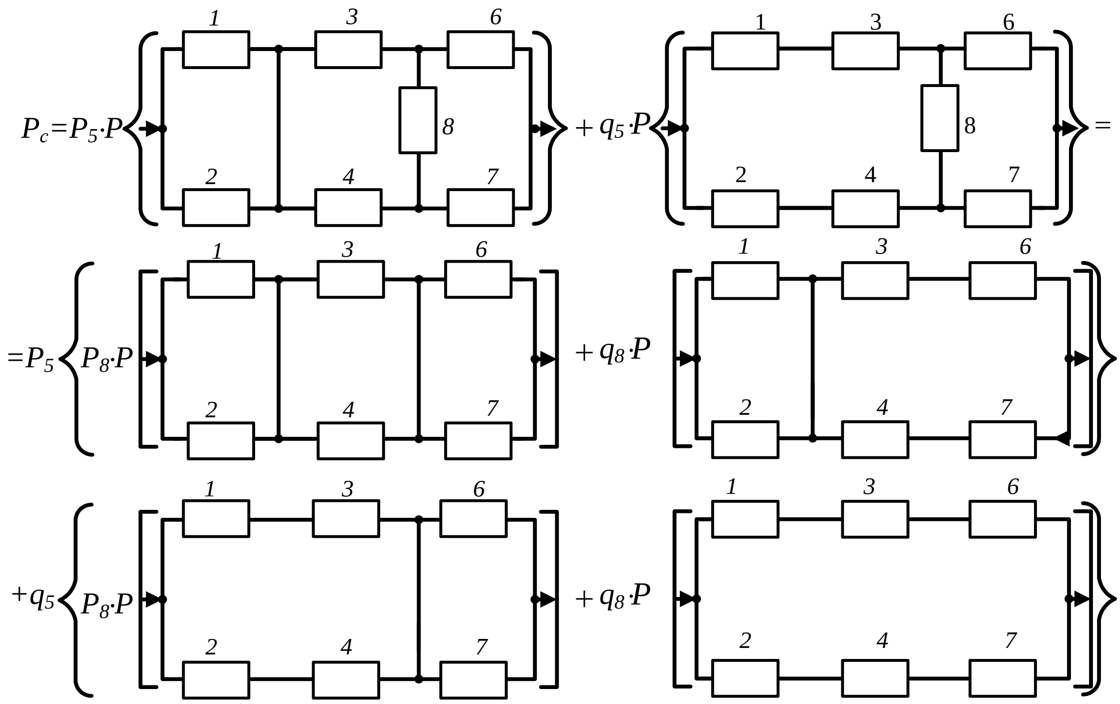

Fig. 5.8. Diagram, that illustrates double application of expansion theorem

5.2.6. The methods of structural analysis of complex (compound) schemes (circuits) and their application for reliability evaluation (assessment)

The application of the methods of structural analysis for power supply system diagram (circuit) study allows to study them in general. By reliability indices evaluation (assessment) by means of structural diagrams (circuits) not all possible states of the diagram (circuit) are analyzed, but only the failure free operation states of ***the minimum set of the elements, that ensure the normal operation of the diagram (power transmission) from the power source till the load node (center) (minimum path) or failure of this minimum set of the elements, which failure in any of these sets leads to systems failure concerning the node (minimal section) under consideration***.

From the definition of minimal paths and sections it follows that unlimited bandwidth of circuit elements concerning each given load node (center) is supposed. For example, for the circuit, illustrated in Fig. 5.5 without taken into account the reliability of node points (centers) the minimal paths concerning the node IV are {1,3}, {2,4}, {1,5,4},{2,5,3} (Fig. 5.9), and maximal sections – sets of elements are {1,2}, {3,4}, {1,5,4}, {2,5,3} (Fig. 5.10).

By means of minimal paths and sections, obtained as a result of structural analysis the probabilities of load node (center) blackout (zeroing) can be defined. Let us consider the fundamentals and definitions of graph theory, used in structural analysis, which purpose is to define minimal paths and maximal sections.

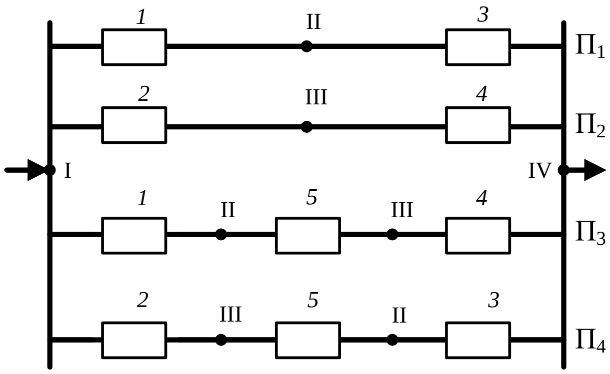

Fig. 5.9. Minimal paths for the circuit of "bridge" type

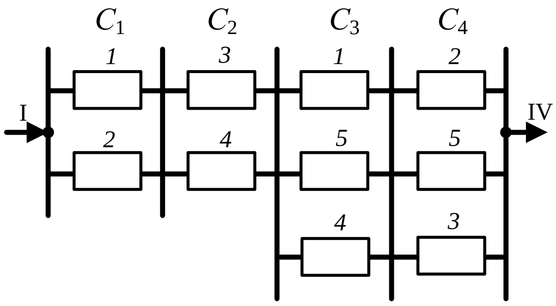

Fig. 5.10. Minimal sections for the circuit of "bridge" type

The graph is any set *A* and *B*, where each element from the set *A* corresponds to two elements from the set *B*. *A* and *B* are called respectively ribs and nodes (points) of graph. The nodes (points) that correspond to a rib are called ***the ends of a rib.*** The rib is called directed one, if one its end is considered as the tail of edge (rib) and the other one as the end. The directed rib (edge) is illustrated in the circuit (diagram) as a segment with an arrow. The ribs (edge), where separate ribs (edges) are directed are called ***partially-directed***. The graphs, where all ribs (edges) are directed are called ***directed***. The graphs without rib (edge) direction are called ***undirected edges*** (ribs).

The research of any diagram (circuit) is equivalent of the research of graph structure.

The graph is called ***planar (flat) graph*** if it can be illustrated (figured) on the plane without intersection of edges (ribs) in the points that are not the graph nodes (points), otherwise the graph is called ***nonplanar graph***. For power supply system the planar property as a rule, is carried out, as the line transition one over another one is a relatively rare phenomenon.

The relation analysis by reliability calculations consists first of all in finding and assessment (estimation) the paths between its nodes, e. g. between power source and load nodes (centers). Graph path is any edge (rib) sequence, where the end of any previous edge (rib) coincides with the tail of the next edge. One edge path is called ***direct (immediate)***, multi-edged paths is called ***transient***.

There are many ways (methods) to define the minimal graph paths. These methods are divided into analytical and logical-digital methods and are based on analytical presentation of the diagram (circuit) in the form of matrix of direct (immediate) path. The minimal section can be also defined on the graph paths. To plot a structural diagram (network graph) it is necessary in advance to convert electrical network system into design (analytic) reliability diagram, i.e. into network operation diagram (circuit). Series connected elements between two nodes are reasonable to replace by one equivalent, which parameters are defined in accordance with known formulae. The same technique is used for elements that are connected in-parallel between two nodes.

After that the graph edges are assigned the elements of design diagram, and the graph nodes are assigned the points of physical connections (bus bars, three winding transformers, the points of taps connection to mains (main feeds). The reliability of points of physical connection of the elements and switchgear (switching equipment) can be taken into account by introduction into the design diagram the elements accordingly to the logic of their operation into electrical system. Besides the mentioned graph nodes there will be one more singular vertex (node) in the network – vertex (top) of the sources. Power source, if the probability of its failure free operation distinguishes from 1, is introduced by designed (calculating) reliability element. All free ends of the elements edges of such power sources unite (join) in one node (vertex) – “source”.

The graph of a network subject to power flow (flux) direction in the elements is usually partially directional (oriented). The direction of network graph concerning different load nodes (centers) can be different. Therefore, to evaluate (assess) system reliability concerning different load nodes (centers), the orientation of edges of the initial (reference) graph should be checked every time. The construction and scheduling of reference diagram in the form of graph enables to simplify the research of system reliability by means of algebraic calculation procedure (computational technique).

The matrix of direct (immediate) path that is constructed in the following way is used as analytic image of the graph:

1. The node (vertex) of the reference graph is numbered. It is recommended to begin the numeration (numbering) with the node (top) of sources. The degree (order) of matrix is equal to the number of nodes (vertices) in the reference graph.
2. Matrix rows and columns are numbered by graph nodes (vertices).
3. The element belonging to i row and j column of matrix A is assigned certain number (unity or probability point (value) of reliable operation of the element), if there is a path from the node (vertex) i to the node (vertex) j. In the case when there is no path zero is set. If the specified element the value (number) 1 is given, then such matrix is called ***adjacency (connectivity) ma-***

trix. When reliability calculations of a very complicated configuration are made it is reasonable to divide the diagram (circuit) into some parts to reduce the degree (order) of matrix.

Applying the matrix of direct (immediate) paths A as an analytic image of analytical (design) model on reliability the minimal paths and sections in a complex (compound) diagram can be defined. There are some methods to define (estimate) minimal paths and relatively minimal sections. We will focus on how by means of minimal paths and sections the reliability indices of the reference diagram (circuit) can be calculated (estimated).

After the definition (estimation) of minimal paths and sections the reference complex design (calculating) model (diagram) is replaced by the equivalent diagram concerning the node: parallel series one when it concerns section and series parallel one when it concerns paths

This replacement enables to use known calculation techniques, particularly to apply formulae for the sum of probabilities of compatible events- the failure free operation or failure of section. It is necessary to take into account that the paths and sections in the general case are dependent, as they can be made up of the same elements. This relation is necessary to take into consideration by the definition (estimation) of the probability of reliable operation of some paths or the probability of failure of some sections in the formula for the sum of probability of compatible events on the assumption that each path can pass the required power in the load node (center).

Applying the formula for the sum of probabilities of compatible events (the path operation) to the equivalent series-parallel diagram (circuit) we will obtain for the probability of failure free operation of the diagram (circuit) concerning some node n the following result:

$$P_c = P\left(\sum_{i=1}^{k} \Pi_i\right) = \sum_{i=1}^{k} P(\Pi_i) - \sum_{i,j} P(\Pi_i \Pi_j) + \sum_{i,j,l} P(\Pi_i \Pi_j \Pi_l) - \qquad (5.80)$$

$$- \ldots + (-1)^{k-1} P(\Pi_1 \Pi_2 \ldots \Pi_k),$$

where k is the number of paths; Π_i is the event of operation of i-path; $P(\Pi_i)$ is the probability of failure free operation of i path:

$$P(\Pi_i) = \prod_{j=1}^{m_i} P_{i,j}, \qquad (5.81)$$

$P_{i,j}$ is the probability of failure free operation of j-element in the i-path(en route); m_i is the number of elements in the i path (en route);

$$P(\Pi_1 \Pi_2 \ldots \Pi_k) = P(\Pi_1) P(\Pi_2 / \Pi_1) \ldots P(\Pi_k / \Pi_1 \Pi_2 \ldots \Pi_{k-1}) \qquad (5.82)$$

is the probability of failure free operation of k paths (routs); $P(\Pi_2/\Pi_1)$ is the conditional chance (probability) of failure free operation of the second path (route) provided that the first path (route) will operate failure free. This probability can be obtained if in the sequence of the second path (route) the points of the elements connection that have been already included in the composition of the first path (route) are bridged (shorted), e. g. the probability of failure free operation by the calculation of conditional chance (probability) is accepted as being equal to one (unity). For example, for the diagram (circuit) illustrated in Fig. 5.9, $P(\Pi_1) = P_1 P_3$; $P(\Pi_3/\Pi_1) = P_4 P_5$.

By the definition (estimation) of any next conditional chance (probability) it is necessary to take into consideration the probability of failure free operation only of those elements that were not included into the previous paths (routes). The probability of failure free operation of the elements that were included in the in the previous paths (routes) is equal to 1. For example, the probability that the paths (routes) Π_1 and Π_3 operate without faults is equal to $P(\Pi_1 \Pi_3) = P(\Pi_1)\, P(\Pi_3/\Pi_1) = P_1 P_3 P_4 P_5$. Applying this concept to k path (route), it can be shown that:

$$P = \left(\Pi_1 \Pi_2 \Pi_3 \ldots \Pi_k\right) = \prod_{i=1}^{r} P_i . \tag{5.83}$$

Where r is the number of elements entering the k path, e. g. this probability is equal to the product probability of failure free operation of all elements entering this path, ***where every element is taken into account in the product only once***, although it can take part in several paths (routes). For the diagram presented in Fig. 5.9:

$$P\,(\Pi_1 \Pi_2 \Pi_3 \Pi_4) = P_1 P_2 P_3 P_4 P_5$$

by the definition (estimation) of the probability of circuit (diagram) malfunction concerning the load node (center), when it is replaced by equivalent parallel-series one (minimal sections), the formula of the sum of probabilities of compatible events – the section malfunction (failure) is also used (applied). The probability of circuit (diagram) malfunction concerning some load node (center) is:

$$\begin{aligned} Q_c = Q\left(\sum_{i=1}^{k} C_i\right) = \sum_{i=1}^{k} Q\left(C_i\right) - \sum_{i,j} Q\left(C_i C_j\right) + \\ + \sum_{i,j,e} Q\left(C_i C_j C_e\right) - \ldots (-1)^{k-1} Q\left(C_1 C_2 \ldots C_k\right), \end{aligned} \tag{5.84}$$

where C_i is the event of failure (malfunction) of i-section; k is the number of sections; $Q(C_i)$ is the probability of malfunction (failure) of i-section:

$$Q\left(C_i\right) = \prod_{j=1}^{m_i} q_{i,j} , \tag{5.85}$$

where $q_{i,j}$ is the probability of failure (malfunction) of j-element of i-section; m_i is the number of the elements in i-section;

$$Q\left(C_1C_2...C_k\right)=Q\left(C_1\right)Q\left(C_2/C_1\right)...Q\left(C_k/C_1C_2...C_{k-1}\right) \tag{5.86}$$

the probability of failure of k sections; $Q\left(C_2/C_1\right)$ is the conditional probability of the second section failure (malfunction) by the failure of the first section.

This probability can be obtained, if in the consequence of the second section the points of elements connection (switching), that were included (entered) in the composition of the first section are opened (are disconnected), e. g. the probability of their failure by the calculation of the conditional probability (chance) is accepted as equal to one (unity), for example for the diagram (circuit) illustrated in Fig. 5.10:

$$Q(C_1)=q_1\,q_2;\ Q(C_3/C_1)=q_4\,q_5.$$

By the definition (estimation) of every following (next) probability the probability of failure of only the elements that did not enter (were not included into) the previous section should be considered. The probability of elements failure, included into the previous sections is equal to 1. For example the probability of failure (malfunction) of the sections C_1 and C_3 is equal to:

$$Q(C_3\,C_1)=q_1\,q_2\,q_4\,q_5$$

and correspondingly the probability of failure (malfunction) of all k sections is:

$$Q\left(C_1C_2...C_k\right)=\prod_{i=1}^{r}q_i, \tag{5.87}$$

where r is the number of elements, included (entered) into k sections, e. g. this probability is equal to the product of probabilities of failure of all elements entering (included) into these sections, where ***every element is taken into account in the product only once***. For the circuit (diagram) in Fig. 5.10:

$$Q(C_1\,C_2\,C_3\,C_4)=q_1\,q_2\,q_3\,q_4\,q_5.$$

This method provides identity of the results, obtained by calculation (estimation) of complex (compound) reference (initial) diagrams (circuits) and equivalent structural diagrams (circuits). The neglect of this rule, particularly, in the definition (estimation) of failure free operation on the paths (en routes) leads to intolerably low (poor) accuracy (error).

Let us consider the features of these methods of determining (estimation) of reliability indices on the paths (routes) and sections. The number of summands in the formula of definition (estimation) of failure free operation with the application of paths (routes) is equal to (2^k-1) and not a single summand should be neglected (formula 5.80), as all summands are the product of actors (factors, multipliers), close to one (unity). By small (minor) complication of the diagram (circuit), in particular if the diagram (circuit) is multiply (multivariable) connected with a great number of cross-links, the number of paths (routes) increases and the calculations are getting very laborious.

The method, that applies (utilizes) the representation of reference (basic) diagram (circuit) in the form of minimal sections concerning the load nodes (centers) does not have this disadvantage, as in the majority of cases it is possible to take into account the summands, that consists of not more than three actors (multipliers). Approximately it can be assumed, that the probability of section failure (malfunction) is equal to the sum of the probabilities of their failure (malfunction):

$$Q\left(\sum_{i=1}^{k} C_i\right) \approx \sum_{i=1}^{k} Q(C_i) \tag{5.88}$$

and it is necessary to introduce into the calculation (estimation) the sections with the number of elements not more than two or three depending on special object and the required computational accuracy.

The final (ultimate) result – the calculation of the probability of systems failure concerning the specified load node (center) or the probability of failure free operation by the representation of the diagram (circuit) in the form of minimal sections – can be achieved faster and simpler, than the method of minimal paths (routes), but the process of section definition (estimation) is more laborious. At the current stage of development and application of these methods in electric power engineering it is not reasonable to oppose one method to another one, as for the diagrams (circuits) with extended (long distance) structure and low number of cross links the methods of paths will have some advantages, and for the diagrams (circuits) with concentrated structure and large number of cross links the method of sections (cutest method) is preferable.

Design methods (calculation methods) of reliability indices of complex (compound) diagrams (circuits) with minimum paths and sections enables to take into account the intentional (deliberate) elements disconnection (cut off) sufficiently easy, as by the representation of reference (basic) diagram in the form of minimum paths the probability of the failure of $Q_{\text{с.п}}$ diagram (circuit) concerning the n load node (center) is composed (formed) of the sum of the probabilities of two hypotheses: the failure (malfunction) of all paths $Q\left(\sum_{i=1}^{k} \Pi_i\right)$ and superposition (overlapping) of failure of the left part of the system $Q_{\text{п}}\left(\sum_{i=1}^{k-r_i} \Pi_i\right)$ on intentional (deliberate) cut off (disconnection) of i element. Let us assume, that deliberate (intentional) cut off (disconnection) elements are not supposed (do not coincide).

$$Q_{\text{с.п}} = Q\left(\sum_{i=1}^{k} \Pi_i\right) + Q_{\text{п}}\left(\sum_{i=1}^{k-r_i} \Pi_i\right), \tag{5.89}$$

where

$$Q\left(\sum_{i=1}^{k}\Pi_i\right)=1-P\left(\sum_{i=1}^{k}\Pi_i\right);\tag{5.90}$$

$$Q_p\left(\sum_{i=1}^{k-r_i}\Pi_i\right)=\sum_{i=1}^{m}K_{p,i}q_{p,i}Q\left(\sum_{i=1}^{k-r_i}\Pi_i\right);\tag{5.91}$$

$P=\left(\sum_{i=1}^{k}\Pi_i\right)$ – the probability of operation of all paths of the diagram (circuit) is defined (estimated) by the formula of the sum of the probabilities of compatible events (5.80); $q_{p,i}$ is the probability of intentional (deliberate) disconnection (cut off) of *i*-element of the diagram (circuit); r_i is the number of paths (routes) where the *i* element of the diagram (circuit) is included; $(k - r_i)$ is the number of paths (routes), remained (left) after *i* element of the diagram (circuit) disconnection; $K_{p,i} < 1$ – is the coefficient, that takes into account the decrease of failures probability due to the probability of superposition (overlapping) of the failure of the left (remained) of subcircuit (part of the diagram) on the intentional (deliberate) disconnection (cut off) of *I* element and not vice versa; *m* – is the number of the elements in a complex (compound) scheme (circuit).

If the diagram is represented in the form of minimum sections, then the failure (fault) probability concerning the *n* node (center) is:

$$\begin{aligned}Q_{cp}&=Q\left(\sum_{i=1}^{k}C_i\right)+\sum_{i=1}^{m}K_{p,i}q_{p,i}Q\left(\sum_{j=1}^{k-r_i}C_j\right)\approx\\&\approx\sum_{i=1}^{k}Q(C_i)+\sum_{i=1}^{m}K_{p,i}q_{p,i}\sum_{j=1}^{k-r_i}Q(C_j),\end{aligned}\tag{5.92}$$

where $(k - r_i)$ is the number of the sections, remained (left) after the *i* element exclusion (disconnection). The minimum sections in the remained (left) subcircuit (part of the diagram) after the *i* element exclusion is made up of the sections of reference (basic) complete diagram (circuit) after the exclusion of formed non-minimal sections.

Similar methods can be used (applied) for reliability calculation of those complex (compound) diagrams (circuits) where the combination (superposition) of intentional (deliberate) disconnection of different elements is possible. In this case the hypotheses of circuit (diagram) malfunction without taking into account the deliberate disconnection of elements are considered.

Example. It is necessary to define (estimate) the probability of systems failure for the load node (center) II (Fig. 5.11) taking into account the intentional (deliberate) disconnection of elements, if the following parameters are given: the probabilities of each element malfunction q_{p1}, q_{p2}, q_{p3}, q_{p4}, q_{p5}, the probabilities of their intentional (deliberate) disconnection q_{p1}, q_{p2}, q_{p3}, q_{p4}, $q_{п5}$, as well as the coefficients, that take into account the decrease of failure probability due to the failure superposition (overlapping) of the left (remained) subcircuit (part of the diagram) on the intentional (deliberate) disconnection of the element and not vice versa K_{p1}, K_{p2}, K_{p3}, K_{p4}, K_{p5}. The intentional disconnection of the elements is not supposed.

Let us use the diagram (circuit) representation in the form of minimum sections. The failures of node points are not taken into account.

Solution. The minimum sections of the reference (basic) circuit (diagram) will be $C_1 \approx \{1, 2\}$; $C_2 \approx \{3, 4\}$; $C_3 \approx \{1, 5, 4\}$; $C_4 \approx \{2, 5, 3\}$ (see. Fig. 5.11, *b*). The minimum sections after the disconnection (cut off) of the first (1) element are obtained from minimum sections of reference (basic) circuit (diagram) excluding the first element and then by the exclusion the no minimum sections of the obtained ones (Fig. 5.11, *c*). As a result the following sections are obtained: $C_{11} \approx \{2\}$; $C_{12} \approx \{3, 4\}$; $C_{13} \approx \{5, 4\}$. The sections by the disconnection of the second, the third, the fourth and the fifth element are defined (estimated) in the same way: 1 – the second element) $C_{21} \approx \{1\}$; $C_{22} \approx \{3, 4\}$; $C_{23} \approx \{5, 3\}$; 3 – the third element) $C_{31} \approx \{1, 2\}$; $C_{32} \approx \{4\}$; $C_{33} \approx \{2, 5\}$; 4 – the fourth element) $C_{41} \approx \{1, 2\}$; $C_{42} \approx \{3\}$; $C_{43} \approx \{1, 5\}$; 5 – the fifth element) $C_{51} \approx \{1, 2\}$; $C_{52} \approx \{3, 4\}$; $C_{53} \approx \{1, 4\}$; $C_{54} \approx \{2, 3\}$ (Fig. 5.11, *d*, *e*, *f*, *g*). Neglecting the probability of failure of more than three elements in the circuit (diagram) and applying the mentioned above algorithm for determination (estimation) of diagram (circuit) failure probability concerning the load node (center) taking into account the intentional (deliberate) elements disconnection (cut off) we will get the circuit (diagram) failure probability concerning the node II.

$$Q_{cp} \cong \sum_{i=1}^{k} q\left(C_i\right) + \sum_{i=1}^{m} q_{pi} K_{pi}\, q\left(\sum_{j=1}^{k-r_i} C_j\right) = q\left(C_1 + C_2 + C_3 + C_4\right) +$$

$$+ q_{p1} K_{p1} q\left(C_{11} + C_{12} + C_{13}\right) + q_{p2} K_{p2}\, q\left(C_{21} + C_{22} + C_{23}\right) +$$

$$+ q_{p3} K_{p3}\, q\left(C_{31} + C_{32} + C_{33}\right) + q_{p4} K_{p4}\, q\left(C_{41} + C_{42} + C_{43}\right) +$$

$$+ q_{p5} K_{p5}\, q\left(C_{51} + C_{52} + C_{53} + C_{54}\right) = q_1 q_2 + q_3 q_4 + q_1 q_5 q_4 + q_2 q_3 q_5 +$$

$$+ q_{p1} K_{p1}\left(q_2 + q_3 q_4 + q_4 q_5 - q_2 q_3 q_4 - q_2 q_4 q_5\right) +$$

$$+ q_{p2} K_{p2}\left(q_1 + q_3 q_4 + q_3 q_5 - q_1 q_3 q_4 - q_1 q_3 q_5\right) +$$

$$+ q_{p3} K_{p3}\left(q_4 + q_1 q_2 + q_2 q_5 - q_4 q_1 q_2 - q_4 q_2 q_5\right) +$$

$$+ q_{p4} K_{p4}\left(q_3 + q_1 q_2 + q_1 q_5 - q_3 q_1 q_2 - q_3 q_1 q_5\right) +$$

$$+ q_{p5} K_{p5}\left(q_1 q_2 + q_3 q_4 + q_1 q_4 + q_2 q_3\right).$$

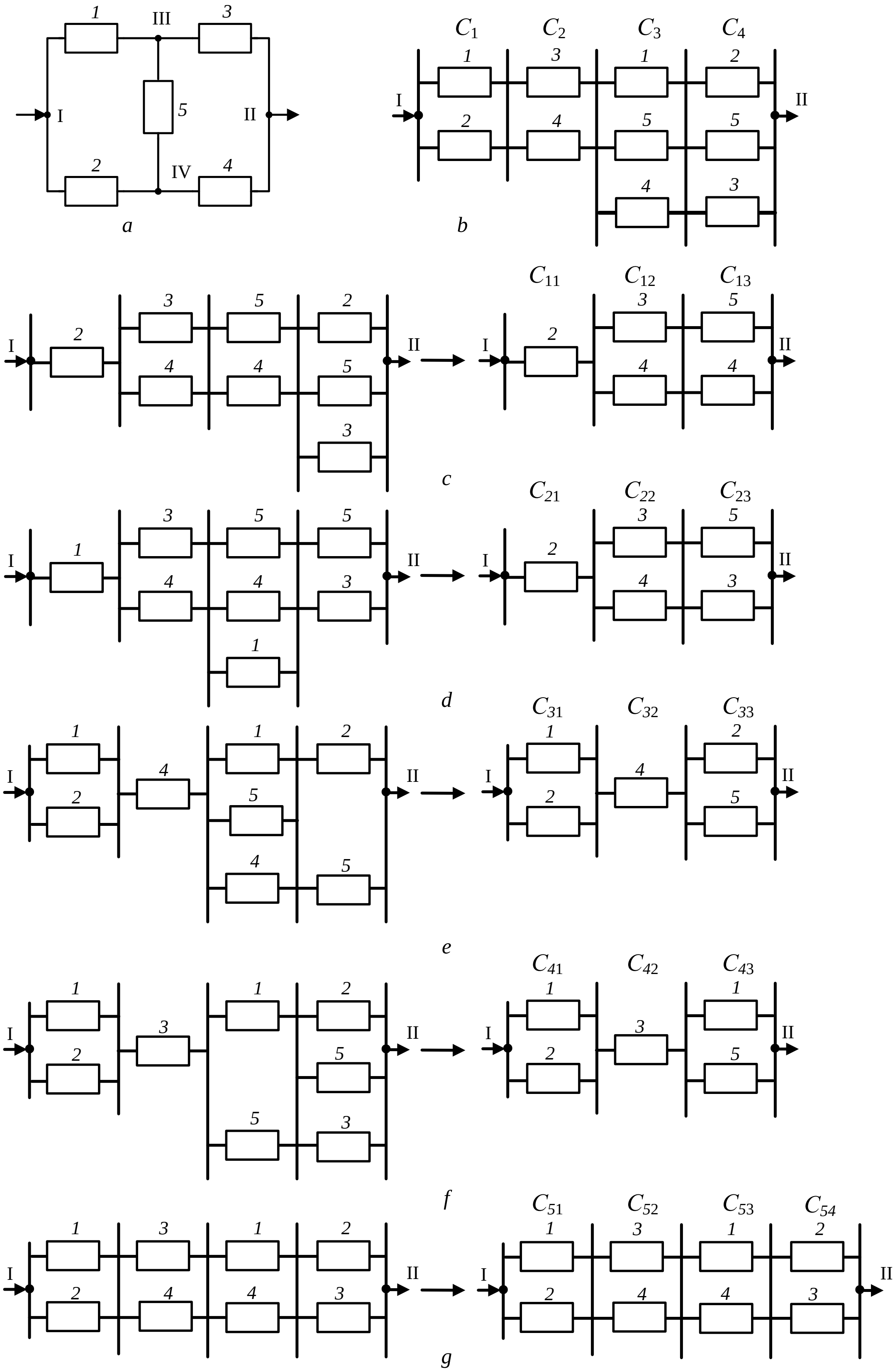

Fig. 5.11. Circuit (diagram) transformation based on the minimum section method

5.2.7. Drawing up of analytical diagrams and the features of reliability calculations of complex (compound) electric circuit diagrams

Modern power supply systems refer to the complex compound systems. The complicity of the system is determined not only by the number of its components, but by the complicity of functional and logical connections (relations) between separate parts and components of the system. Moreover, unlike some other engineering systems these systems are systems with a great deal of entrances (inputs) and outlets (outputs) (e. g. they have a lot of power sources and consumers). These factors determine the high requirements to analytic diagram drawing up on reliability intended for one or another system. The full accounting of all factors that have an impact on system reliability is obviously impossible due to their extreme variety. Therefore, the statistical approach to basic reliability indices calculation (estimation) of power system components has recently gained in popularity.

Before the system reliability calculation the logical diagram is made up. The logical diagram can differ from schematic circuit (diagram). For example, the parallel connection (shunt connection) of generators at power stations corresponds to the series connection in reliability design diagram (analytic model), if the question about generation reliability calculation (assessment) of the whole power plant output (capacity) arises.

Except the components of series electric circuit (lines, switch, transformer and etc.) the adjacent (related) switches are also introduced into series circuits of design diagram on reliability. The failure of the adjacent (related) switches can cause the failures of the given circuit (diagram) (for example, the switches of all connections, sectionalizing circuit breaker (section switch) of the bus bar to which the analyzed (considered) circuit (diagram) is connected. If the main feed with taps, which is not equipped with an automatic circuit breaker is analyzed, then the reliability indices of the taps from this line are also included into series circuit, but the probability of these indices failure is defined (estimated) as the product of failure rate of the taps times average switching period (it is assumed that the taps are equipped with disconnectors). This concept is fair for those taps, which are electrically switched in parallel concerning the analyzed load node (center). If the reliability indices of the load node (center) that are connected to the tap of the line are assessed (calculated), then this tap is introduced into calculation by mean recovery time.

The analytic diagram on reliability concerning the load nodes (centers) should reflect the operation logic of the reference (basic) of electric circuit. The variety of the methods intended for making up the analytic diagrams is justified by the variety of the used power supply diagrams and it is impossible to give common recommendations for solution of all possible problems. It is necessary to emphasize that power supply circuits (diagrams), where the automatic shutdown (disconnection) of some separate parts is combined with manual one by the redundancy (back up) commissioning (activation) (for example, in the loop design (circuit) in networks till 1000 Volts) claim some special attention. In these cases the mean failure probabilities of the design components are defined (estimated) not only by recovery time of the line section, but also mean switching time (period) to power backup (standby power supply).

Usually the aim of the calculations is to determine (estimate) the basic reliability indices concerning the load nodes (centers) or certain consumers. Therefore, the system is divided into some separate elements, which reliability behavior (characteristics) is relatively easy to assess (specify).

The next stage in calculation is to define the concept (notion) "failure" for the whole system and its separate elements (components). The failure of the system with limited bandwidth (capacity; output) can be considered, for example, as one or another consumers' power value limit. For some consumers, for example, the fact of no voltage at bus bars even for some split second (if the fallback (standby) circuit is switched on by means of reserve switching device) can be considered as a system failure.

Method of reliability analysis (calculation) is chosen depending on the certain task (aim) and the time period within which the reliability characteristics (performance) are determined (assessed). The calculations are made on mean (average) indices or with taking into consideration the reference (initial) elements (components) conditions over short periods (in the last case the model of random (stochastic) processes is used).

Drawing up of analytic diagram on reliability for complex (compound) power supply systems is very time-consuming and laborious task and can be compared by its labor intensiveness with reliability indices calculations. If the task is to estimate (evaluate) the reliability indices concerning the load nodes (centers), then this process can be formalized on computers, applying the method of structural analysis, notably, the method of power transmission ways (paths) creation.

Let us consider more detailed the logic of diagram (circuit) operation. Electric diagram (circuit) consists of nodes and branches (taps). As a rule, a node is a three winding transformer, bus bars or section of bus bars. A tap (branch) can consist of several elements (components): a line, a transformer, a switch and etc.

The failure of element, which is included into the branch (tap) can differently influence the operability of the whole branch (tap) and the adjacent nodes. The branch (tap) that contains this failed element (component) losses its ability to transmit power over (during) the recovery time of this element $\bar{t}_в$. The nodes that adjoin (border with) this branch (tap) lose their operability over the following periods: a) over the automatic disconnection (shutdown) time $t_а$ of the failed component (element) from the node, if there is a switch device (switching unit) that is influenced by relay protection between the node and this component (element) (the probability of failure of the switch device (switching unit) is not taken into consideration); b) over the period of manual commutation (switching) $\bar{t}_{op}$ that is required to cut off (disconnect) the failed component (element) from the node, if there is a disconnector or switch device (switching unit) between the failed component and the node that is not equipped by relay protection; c) over recovery time $\bar{t}_в$ of the failed component (element) if it is directly connected with the node.

It is not difficult to understand, that in the first case the node (center) will remain in operation (will function) and in the second and the third cases it will be in failed condition (state) over the prorated time. Therefore, the elements (components) that adjoin (border with) the node of branches (taps) that comply with conditions "*b*" and "*c*" should be introduced into analytic diagram on reliability sequentially with this node. These components should be introduced into all paths passing through this node.

The next procedure of calculations can be offered. All the paths (ways) for a certain (specified) consumer are formed on electric circuit (diagram) of power supply system. The paths (ways) are necessary to complete with other elements (components), that lead to the disconnection (cut off) of the node over the period $\bar{t}_v$ or $\bar{t}_{op}$. The minimum paths (ways) are obtained, that are constructed by analytic diagram on reliability. It should be noted that the same element (component) can be included into analytic diagram with the probabilities $\lambda\bar{t}_v$ и $\lambda\bar{t}_{op}$, where λ is failure rate (failure flux parameter).

By the evaluation (assessment) of reliability indices in the complex (compound) circuits (diagrams, network) two main approaches with application of different methods can be singled out.

1. The estimation (definition) of probability of different conditions of a complex (compound) system and the probability of power undersupply to consumers or complete (total) power supply loss (power supply disruption) of some separate consumers. The solution of this problem is concerned with the operating conditions (modes) analysis of some separate elements (components) in a complex (compound) system, with the estimation of probabilistic characteristic of the load in elements (components) and determination of the most loaded elements or groups in the circuit (diagram). The estimation of the circuit (diagram) state probability and reliability indices by different combinations of switched on and off elements (components) is carried out by

the method of analysis of basic states (conditions) probability. The state (condition) with the number of failed elements (components) more than three can be neglected. It is necessary to take into account the superposition (overlapping) of emergency outages on intentional (deliberate) disconnection (opening) of some separate circuits.

This method is considered as the basic one by the reliability indices calculation (estimation), consumers' power and energy limiting for the system with many inputs (inlets) (power sources) and outputs (outlets) (load nodes) and the limitations in elements (components) throughput performance (capacity) in post-fault conditions (states). The drawbacks of this method are the inconvenience in calculation, the necessity to analyze a great number of circuit (network; diagram) states (conditions), the difficulty with algorithmization by the application for computer calculations. For example, if the number of the calculating (design) elements (components) in a complex (compound) circuit (diagram) is n, it is necessary to analyze and calculate the modes for

$$N = \frac{1}{6}\left(n^3 + 5n\right) \tag{5.93}$$

circuit conditions even without taken into consideration the superposition (overlapping) of emergency outages on intentional ones (neglecting the failure (fault) probability of more than three elements (components)). ***The feature (peculiarity) of this method is that the reliability indices calculation is carried out applying the system approach.***

2. Reliability indices assessment sequentially concerning the load node (center) by means of formula of total probability or with representation of circuits (diagrams) in the form of structural series parallel (path diagram) or parallel-series (section diagram) circuits. The most appropriate for algorithmization is the method when the initial (reference) complex (compound) circuit (diagram) is presented in equivalent structural form. Moreover, as it was mentioned earlier the simpler algorithms of failed (failure condition) and failure-free state (condition) probability concerning the load node (center) is obtained when the circuit (diagram) is presented in the equivalent parallel-series (section circuit), although the algorithms for obtaining these sections is more complicated, than the algorithms intended to obtain the paths (ways).

With the increase of the number of elements (components) in a complex (compound) system the number of sections and paths (ways) concerning every node grows (goes up) very fast. Particularly, the record (accounting) of the node point failures leads to a sharp increase of the number of sections without causing the growth of the number of paths (ways) concerning the nodes. With the increase of the number of transverse connections (cross-linkages) the number of paths (ways) increases as well.

The basic feature of reliability indices calculation using the structural analysis concerning the nodes is the limit of possibilities of these methods due to difficulty of their application (implementation) by bandwidth (throughput performance) limitation of some separate elements (components), e. g. the difficulty to define (evaluate) partial power limiting and consumers' power.

The implementation of all these methods as well as the implementation of the methods of analysis of probabilities of circuit states (conditions) becomes significantly complicated with the growth of the number of calculated components of the analytic diagram. One of the ways to simplify the problem solution is to divide the initial (reference) circuit (diagram) into subcircuits (subdiagrams) on the node points of the network (tie-stations, bus bars, the point of power transformation etc.). The reliability indices calculation for the first circuit (diagram) is carried out concerning the points of circuit (diagram) division from the power supply (source) side. The points of division are the power sources for another circuit (diagram) and are introduced into analytic diagram by means of estimated (designed) components with the characteristics obtained in the calculations of the first sub-circuit (sub-diagram).

The division into sub-circuits (sub-diagrams) is reasonable to carry out thus, that the number of estimated (designed) elements (components) in every circuit (diagram) will not exceed 130–450. If the bandwidth (throughput performance) of the elements (components) of any part of the circuit (diagram) is limited, then the reliability indices concerning the division points are reasonable to assess (define) using the methods of structural analysis. The calculation of sub- circuits (sub-diagram) is carried out depending on the certain conditions: either by means of the methods of the analysis of state probabilities or applying the structural representation of the circuit (diagram).

5.2.8. Power supply system operating efficiency evaluation and the reliability decision criteria

Operating efficiency (reliability index (reliability criterion)) is one of the main criteria of power supply systems reliability and in quantitative form is characterized by relative magnitude (number) called mathematical expectation of supplied power that is determined (defined) by the analysis techniques (methods) of structural and functional reliability diagrams.

On the basis of the design and maintenance of power supply systems some practical criteria in the form of normative standards (regulations) to power supply reliability of power collectors are developed. All power collectors are divided into three categories, where the first category is marked out as the one that requires higher reliability level.

The decision-making criterion is the efficiency (objective) expenditure function, that includes as the most important component the damage (loss) from disrupt service (outage) and undersupply of energy to consumers.

The choice of means and measures of reliability level change provides for the necessity of quantitative estimation (evaluation) of reliability indices (indicators) in the variety of possible options determined by the number of collectors connected to the load centers that in general case are different in terms of materials costs.

As the reliability index (indicator) of power system operation is used the concept (the term) operating efficiency (reliability index).

Operating efficiency is the quality measure of intrinsic (self) functioning of a unit (facility) or the suitability (appropriateness) of its use to carry out the specified functions. In other words, the efficiency is the degree of the system suitability (adjustment) to carry out the corresponding functions in certain conditions.

As the efficiency index of an electric system is often accepted relative magnitude (number) of mathematical expectation of output effect, i.e. the mathematical expectation of supplied power:

$$\Phi(t)=\frac{E(t)}{E(t)+\Delta E(t)}=1-\frac{\Delta E(t)}{E(t)+\Delta E(t)}, \tag{5.94}$$

where *Э*(t) – is mathematical expectation of power of desired quality, that is obtained by consumers within the period (0, t); Δ *Э(t)* – is mathematical expectation of undersupplied power and power with quality index (factor) lower than required that is supplied to consumers within the period (0, t).

The modern level of scientific and engineering development allows creating systems, particularly electric power systems, practically with any reliability level. However, the development of such systems is connected with the increase of material and labor expenditures, as well as the increase of backing-up (redundancy) volume and maintenance cost and the application of more expensive and state-of-the-art technology and materials for equipment production and etc.

The experience of design and maintenance (service) of power systems based on experience generalization has developed some criteria in the form of normative requirements to provide reliability of power collectors, which are formulated (stated) in state standard specification. Apart from the power receivers (collectors) whose normal operation is connected with people's safety (security), the malfunction of other components leads to additional expenditure to compensate them. Therefore, the compulsory component in the objective function of expenditure on consumers' power supply must be also expenditure on compensations that deals with negative consequences connected with disrupt service (power outage). This expenditure is called damage from power undersupply, the cost of undersupplied energy can 20…100 times exceed the cost on its generation.

The decision criterion concerned with power supply reliability is as follows:

$$Z(H) = p_{norm} K(H) + I(H) + Y(H) \to \min, \tag{5.95}$$

where $Z(H)$ – current expenditure; $K(H)$, $I(H)$, $Y(H)$ are respectively (capital) investments, expenditures, damage depending on the reliability level; p_{norm} is normative efficiency factor (index).

As the decision criterion for power receivers (collectors) whose operation is connected with people's safety are accepted natural reliability indices (factors) (safety (security) is as a rule not estimated in monetary units).

Damages from disrupt service (power outage) and undersupply of energy

Quantitative characteristic of operating efficiency decrease of national economy that is caused by disruption of interconnection of power system with other subsystems expressed in cost estimation model (form) is called damage from electric power system reliability. The damage to national economy can be expressed in two components (constituents): fundamental (basic) component that is concerned with underproduction, idle time and downtime; additional component, which is concerned with sudden outage, when death of equipment, raw materials damage, degradation of finished commodity and uneconomical power system mode of operation can occur.

The increase in national economy efficiency is expressed in national income decrease, which is determined (estimated) by the value (cost) of produced output aggregate. Therefore, on the basis of relation (ratio) of consumed power per year and annual cost (value) of produced goods it is relatively simple to determine (to estimate) the mean value of specific damage from undersupply of unit of energy. By the ratio of current annual expenditure on the creation of enterprise production capacity and consumed power the lower bound (limit) of specific damage is estimated (determined). The specific damage reflects the minimum expenditure on backup (redundancy) creation only at one plant (without regard for connection (relation) with other ones, such as store, freezing (blocking) of backup (reserved) goods (output) and etc.

The technique (procedure) of damage determination, based on the analysis of the ratio between the energy required and the cost of goods (production) manufactured by the plants estimates (evaluates) the limits (bounds) of the first constituent (component), i.e. the direct damages (proximate damages). The second constituent (component), e. g. the additional (consequential) damage or the damage caused by abrupt disrupt service (outage) is connected with backup creation of certain (separate) parts of equipment, materials, raw materials, tools, workforce, that are necessary to compensate the consequences of disrupt service (outage) and possible operating procedure irregularity. In general, the additional (extra) damage depends nonlinearly on the time of disrupt service (outage). For complex load (different types of power receivers (collectors), that obtain power from the load center) whose disconnection (cutoff) is carried out partly (partially) the additional (extra) damage also depends on the depth of limits on capacity.

This component (constituent) is determined (estimated) by specified (specific) technological (engineering) characteristics of the plant and its calculation provides for thorough analysis of operating procedure (manufacturing process). The total damage (mathematical expectation) from disrupt service (outage) of load centers (nodes) with different classes of consumers is calculated by formula

$$Y(H)_{\Sigma} = \sum_{i=1}^{m} y_{0i} \Delta \overline{E}_{\text{н}i} + \sum_{j=1}^{n} y_{v.n}^{(0)} (t_{\text{v}}) \Delta P_j^{\beta_j}, \tag{5.96}$$

where y_{0i} is specific direct (proximate) damage; $\Delta \overline{E}_{ni}$ is mathematical expectation of undersupplied energy to *i*-consumer; $y_{v.n}^{(0)}$ is specific damage from suddenness, that depends on the duration of disconnection (cutoff) t_V; ΔP_j is relative value of disconnected (switched off) power (capacity); β_j is power exponent (index), that is determined by the kind (nature) of *j* consumer’s production; *m* and *n* are respectively the number of consumers where the suddenness of disrupt service (outage) does not or does influence the value of damage.

If the consumers’ cutoff (disconnection) is carried out with warning, then there is a possibility in many cases to organize the production (manufacturing) process in such manner in order to reduce the damage from disrupt service (outage). Therefore, the specific damages in the conditions where a warning was given are lower then in the cases where disrupt service (outage) took place.

The development of production all over the world recently causes the increase of environmental pollution. The damages from environmental pollution (ecological damage) leads to environmental deterioration and increase of harmful substances concentration in air, water and soil, the disturbance of thermal balance, landscapes, radiation pollution, the rise of noise level and vibration and etc.

Ecological damage caused by the equipment failure as well as power systems failures is an additional (extra) value concerned with ecological damage that takes place under normal operation of electrical installations. The total value of damage is determined by the sum of two components (constituents).

To the damages from ecological disturbances, which are caused by power supply interruption of some certain plants, belong also expenditures on environmental restoration (reclaiming) till the maximum permissible level of harmful substances concentration in air, water and soil.

By such power supply systems failures, as a rule, there is maximum (peak) concentration of harmful substances in the environment as a result of gas, discharged waters (waste water), dust and other types of emissions.

There is also a sudden increase of social factors, caused by environmental pollution. But the determination (calculation) of social damage from ecological disturbances in cost criterion (form) is one of the most complicated problems, which have not been still solved.

Ecological damage from failures (faults) is reasonable to determine as a sum from: extra (additional) expenditure on measures that are aimed at environmental protection in the condition of system failure; extra (additional) expenditure that are concerned with the environmental restoration (reclaiming) in the case of environmental pollution caused by system failures; extra (additional) expenditure on the protection of users natural resources and compensation from negative consequences, caused by ecological disturbances as the result of power supply system failure (fault).

The damage from expenditure increase in non-production sector due to ecological disturbances caused by power supply system failures (faults) is determined by the expenditure increase in municipal sector of national economy on sanitary purification and cleaning of the polluted area, the renovation of residential buildings and etc.

From the mentioned above it can be concluded, that quantitative estimation (assessment) of reliability is to make economically sound decision at different design, construction and maintenance (service) stages of power supply system. In general case there are the following goal settings of engineering decision making:

1. To find such engineering tools and technology for reliability growth (enhancement), that will correspond with the minimum of modified expenditure (costs).
2. To select the volume (amount) and sequence of engineering tools application for reliability growth (enhancement) as their efficiency will decrease.
3. To choose required engineering tools and technology that ensure maximum reliability level by specified limited resources (additional investment, volume and combination of engineering tools and technology for reliability enhancement, the limit of maintenance personnel).
4. To ensure (to provide) specified (normative) reliability level for power supply system (part of power supply system) or separate consumers by minimum of additional expenditure.

CONCLUSION

The main objective of this book is to address the subject of power supply reliability. Structurally, generating plants supply power to the grid, and the distribution system transports power to the end users. Although the reliability principles and models described in this book are universally applicable, each segment of the electric industry has developed its own specific and sometimes unique methods and metrics. Many professional organizations and regulatory entities have recommended and mandated industry practices, and continue to do so. The Institute of Electrical and Electronics Engineers has contributed to the development of the standards and recommended practices mentioned above. These standards and other documents have been developed collaboratively by experts from industry and academia and integrate mathematical principles and industry practices. These have been widely adopted by industry and by regulatory bodies.

The late 1990s witnessed a worldwide movement toward restructuring the electric utility industry. The restructuring process and policies produced several unintended consequences, most notably the following: there was no clear allocation amongst market participants of operating practices that contributed to system reliability; new operating practices caused unprecedented stresses on an already-aging infrastructure; and system operators lacked experience with markets and new regulatory and corporate policies. Regulatory bodies and Independent System Operators have responded by developing new policies and products (ancillary services). There were many other factors that conspired to bring about the August 2003 blackout, but one of the most significant actions taken in response to this event was the formation of the Reliability Functional Model, a document that addressed the first of the three issues enumerated above by clearly defining "functional entities," encompassing standards developers, reliability service providers, and system planners and operators, and assigning specific functions to each functional entity to ensure system reliability. Penalties for noncompliance can range from large fines to suspension of an entity's ability to perform the function.

Regardless of how the grid evolves, now one thing is clear: grid reliability methods and models will be necessary to understand the impact of adopting new technologies, operating strategies and planning and operating decisions. We believe that in the future there will be increased complexities and interdependencies between the various segments of the grid.

Some of the models described here will continue to be used, whereas others may need to be modified or enhanced to accommodate the changing scenarios. We have placed strong emphasis on the fundamentals so that the reader can acquire the capability to perform these modifications and enhancements.

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