The Reduction of the Multidimensional Model of the Nonlinear Heat Exchange System with Delay

Aleksandr Shilin and Viktor Bukreev

National Research Tomsk Polytechnic University, 634050, Russia, Tomsk
{shilin,bukreev}@tpu.ru

Abstract. A method of reducing of a multi-dimensional model of the complex non-linear heat exchange system (HES) with delay based on structural changes in equilibrium points and approximation of delay function by the end of inertial units is presented. Simulation results confirming the adequacy of the process of reduced and original models, as well as their compliance with the real data of the experimental control object are demonstrated.

Keywords: complex heat-exchange system, equilibrium state, structural transformation, reduction of multidimensional models.

1 Introduction

In modern systems of intelligent management of complex equipment there are often used optimal control algorithms providing real-time management of production processes. One of the well-approved approaches for the synthesis of such algorithms is the theory of linear systems, which assumes a description of nonlinear processes and objects in terms of linearized models [1]. The main obstacle to the use of these methods is a significant degree of differential equations describing the behavior of a complex multi-dimensional control object with the necessary range of accuracy. The most preferred variant of priori mathematical models allowing to record the control laws in an analytical form, and submit the results of the analysis of dynamic processes in a convenient form, is second order differential equations. For example, in the relay control systems in sliding mode control methods for nonlinear second-order systems are well-developed [2], [3].

There are several approaches to reduction of dimensional models of high dimension [4]: the use of linearized matrix properties when converting to block matrix according to division into independent blocks ; the use of the coefficients properties of low interconnection; aggregating the matrix elements; the models selection according to the frequency hierarchy of submatrices; separation in time or frequency.

1.1 Problem Formulation

We will consider the problem of reduction of non-linear mathematical model of a distributed heat-exchange system with maximum accounting its features. We

A. Dudin et al. (Eds.): ITMM 2014, CCIS 487, pp. 387–396, 2014.

assume interval stationary of HES element parameters and determinate nature of interrelated thermal processes. It is appropriate to present the approximation procedure in several stages [4]: decomposition of the original model [5]; formation of several reduced models [6]; structural transformation in equilibrium state points and leading to a simpler form; checking the approximate model for the adequacy of the complete model or the real process of high order.

2 The Original Non-linear Model in the Space of State Variables

As an illustrative example here is the heat exchange system (Fig. 1), characterized by significant non-linear properties [7]. Notation: 1 heat exchanger, 2 circulating pumps, 3 control valve with AC motor 4, 5 temperature sensors and a microprocessor controller (MC).

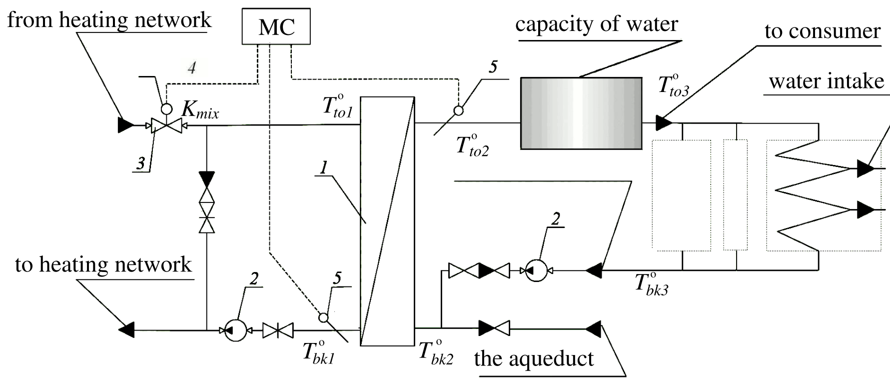

Fig. 1. The technological scheme structure of the heat exchange system

Salient features of HES with high-performance heat exchangers are not only the non-linearity of the three-point relay control, but also the delay in the formation channels of control actions and the flow of heat transfer in a distributed pipe network of the secondary circuit. In addition, significant disturbance on the characteristics and parameters of the heat exchange system is made by a periodic flow of cold water into the secondary loop of the HES to compensate for the irreplaceable HES discharge.

Arbitrary arrangement of risers-of couplers of the HES secondary circuit and their different distances from the heat exchanger causes a variable delay which makes a significant impact on the dynamics of thermal processes in the primary circuit.

Under certain admissions for thermal processes taking place in contours of HES, the original non-linear model of the system in the space of state variables can be represented by the following differential equations:

$$\begin{cases} \frac{dK_{mix}(t)}{dt} = (k_{mx} - K_{mix}(t)) \cdot \frac{k_h}{T_{vlv}} \cdot u(t) \\ \frac{dT^{\circ}_{to1}(t)}{dt} = \frac{(T^{\circ}_1 - T^{\circ}_{bk1}(t)) \cdot K_{mix}(t) + T^{\circ}_{bk1}(t) - T^{\circ}_{to1}(t)}{T_{mix}} \\ \frac{dT^{\circ}_{to2}(t)}{dt} = \frac{k_{exc} \cdot T^{\circ}_{to1}(t) + (1-k_{exc}) \cdot T^{\circ}_{bk2}(t) - T^{\circ}_{to2}(t)}{T_{exc}} \\ \frac{dT^{\circ}_{bk1}(t)}{dt} = \frac{k_{exc} \cdot T^{\circ}_{bk2}(t) + (1-k_{exc}) \cdot T^{\circ}_{to1}(t) - T^{\circ}_{bk1}(t)}{T_{exc}} \\ \frac{dT^{\circ}_{to3}(t)}{dt} = \frac{T^{\circ}_{to2}(t) - T^{\circ}_{to3}(t)}{T_{cp}} \\ \forall i = 1..n \rightarrow \left\{ \forall j = 1..m \rightarrow \left\{ \frac{dT^{\circ}_{bkz(i,j)}(t)}{dt} = \frac{T^{\circ}_{bkz(i,j-1)}(t) - T^{\circ}_{bkz(i,j)}(t)}{\tau_{z(i)}/m} \right\} \right\} \\ \frac{dT^{\circ}_{bk2}(t)}{dt} = \frac{k_{cw} \cdot T^{\circ}_{cw} + (1-k_{cw}) \cdot T^{\circ}_{bk3}(t) - T^{\circ}_{bk2}(t)}{T_{cw}} \end{cases} \quad (1)$$

$$\forall i = 1..n \rightarrow T^{\circ}_{bkz(i,0)}(t) = (1 - k_{cl}) \cdot T^{\circ}_{to3}(t) + k_{cl} \cdot T^{\circ}_{rm}$$

$$T^{\circ}_{bk3}(t) = \sum_{i=1}^{n} \left(k_{zi} \cdot T^{\circ}_{bkz(i,m)}(t) \right), \sum_{i=1}^{n} k_{zi} = 1$$

where $K_{mix}(t)$ - the coefficient of coolant mixing in the external circuit to the mixing unit; k_{mx} and k_h - coefficient characterizing the nonlinear properties of the mixing process; T_{vlv} - the time constant of the electric control valve; $u(t)$ - electric valve control action that takes one of discrete values $u \in (-1, 0, +1)$; T°_{to1} - the coolant temperature at the inlet HES in the external circuit; T°_1 - the coolant temperature coming out of the backbone network; $T_{mix}, T_{exc}, T_{cp}, T_{cw}$ - respectively, constants mixing time of the valve in the heat exchanger, in the intermediate storage device, in the input node of cold water; T°_{bk1} - the coolant temperature at the outlet of the external circuit of HES; T°_{to2} - the coolant temperature at the outlet of the internal circuit of HES; T°_{bk2} - the coolant temperature at the inlet of the internal circuit of HES; k_{exc} - the coefficient of heat exchange efficiency; T°_{to3} - the coolant temperature at the outlet of the intermediate storage, which is located in the internal circuit; T°_{bk3} - return temperature before unit mixing with cold water; $\tau_{z(i)}$ - the time delay of the transport carrier in the secondary circuit (i riser-branch); $T^{\circ}_{bkz(i,j)}$ - the equivalent temperature in inertial units to be used for the approximation of the transport delay; k_{cw} - the coefficient of cold water influence on the coolant in the internal circuit; k_{cl} - the coefficient of the coolant cooling in the internal circuit; n - the number of secondary circuits (riser-branch) in HES; m - the number of inertial units approximating transport delay.

Obviously, the order of the system of differential equations (1) will be determined by $(6 + n \cdot m)$. For example, for a system with two risers $n = 2$ and branches of inertial delay line units equal to $m = 5$ the number of equations of the system will be 16. The coefficient k_{zi} means relative proportion of the i-th heat flow throughout the entire volume of the coolant flow q_2, and characterizes the distribution of the i-th flow on risers-branches.

3 The Reduction of the Original Non-linear Multidimensional Model

The procedure of the original nonlinear model HES converting assumes finding the equilibrium points, which can be calculated by solving the system (1) with priori known parameters of the object and control $u(t) = 0$.

As a result of this calculation with the required accuracy we can find steady values of the following variables state of the heat exchange system:

$$\left[K^0_{mix}, T^{\circ 0}_{to1}, T^{\circ 0}_{to2}, T^{\circ 0}_{bk1}, T^{\circ 0}_{to3}, \forall i = 1..n \rightarrow \left\{\forall j = 1..m \rightarrow \left\{T^{\circ 0}_{bkz(i,j)}\right\}\right\}, T^{\circ 0}_{bk3}\right] \quad (2)$$

where the superscript "^{0}" denotes the state variables belong to the fields of steady state. Further, using the calculated values (2) we write equations that reflect dynamic processes in a neighborhood of steady state:

$$\begin{cases} x_1(t) = K_{mix}(t) - K^0_{mix} \\ x_2(t) = T^{\circ}_{to1}(t) - T^{\circ 0}_{to1} \\ x_3(t) = T^{\circ}_{to2}(t) - T^{\circ 0}_{to2} \\ x_4(t) = T^{\circ}_{bk1}(t) - T^{\circ 0}_{bk1} \\ x_5(t) = T^{\circ}_{to3}(t) - T^{\circ 0}_{to3} \\ \forall i = 1..n \rightarrow \left\{\forall j = 1..m \rightarrow \left\{x_{5+(i-1)\cdot m+j}(t) = T^{\circ}_{bkz(i,j)}(t) - T^{\circ 0}_{bkz(i,j)}\right\}\right\} \\ x_{6+n\cdot m}(t) = T^{\circ}_{bk3}(t) - T^{\circ 0}_{bk3} \end{cases} \quad (3)$$

Defining the $x(t) = [x_1(t), x_2(t), .., x_{6+n\cdot m}]^T$, the linearized model can be written in a vector-matrix form:

$$\dot{x} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdot & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 & a_{2,4} & 0 & 0 & \cdot & 0 & 0 \\ 0 & a_{3,2} & a_{3,3} & 0 & 0 & 0 & \cdot & 0 & a_{3,6+nm} \\ 0 & a_{4,2} & 0 & a_{4,4} & 0 & 0 & \cdot & 0 & a_{4,6+nm} \\ 0 & 0 & a_{5,3} & 0 & a_{5,5} & 0 & \cdot & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{in1} & A_1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & A_{inn} & 0 & \cdot & A_n & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{out1} & \cdot & A_{outn} & a_{6+nm,6+nm} \end{vmatrix} \cdot x + \begin{vmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdot \\ 0 \\ 0 \end{vmatrix} \cdot u \quad (4)$$

where the matrices $A_1, A_2, .., A_n$ represent an approximation of the transport delay of the coolant in riser-branches by inertial units:

$$\forall i = 1..n \rightarrow A_i = \begin{vmatrix} ai_{1,1} & 0 & \cdot & 0 \\ ai_{2,1} & ai_{2,2} & \cdot & 0 \\ 0 & 0 & \cdot & ai_{m,m} \end{vmatrix};$$

where $\forall j = 1..m \rightarrow ai_{j,j} = -m/\tau_{zi}, ai_{j,j-1} = m/\tau_{zi}$ the input vectors-columns A^T_{ini} of dimension m are calculated as follows:

$$\forall i = 1..n \rightarrow A^T_{ini} = [(1 - k_{cl}) \cdot m/\tau_{zi}, 0, .., 0]$$

the output vectors-lines A_{outi} of the dimension m are defined by the expression:

$$\forall i = 1..n \rightarrow A_{outi} = [0, .., 0, (1 - k_{cw}) \cdot k_{ci}/T_{cw},]$$

where $\sum_{i=1}^{n} k_{ci} = 1$; the coefficient $b_1 = \left(k_{mx} - K_{mix}^0\right) \cdot \frac{k_h}{T_{vlv}}$.

Let us consider in more details the peculiarities of a heat exchange system as a control object. As it is known from the description of the object its nonlinear properties are reflected in the coefficients $a_{2,1}, a_{2,4}, b_1$. The remaining elements of the matrix HES parameters are stationary coefficients, the components $A_1, A_2, .., A_n$ are determined by transport delay with different values within certain limits.

The traditional problem of regulating in heat exchange systems is to stabilize the temperature $T_{to3}^{\circ}(t)$ of the coolant at the outlet of the intermediate tank, which in terms of the taken denotation corresponds to a state variable $x_5(t)$. Further, assuming that the $a_{2,1}, a_{2,4}, b_1$ coefficients of the linearized model are stationary points in the equilibrium state, the transfer functions can be used for structural analysis of the object mathematical model. In addition, note the assumption that is made on the analysis of the functioning of the control object. This assumption is as following: the minimum time of transport delay in heat exchange systems is next larger than the mixing time constant, therefore the impact of the return coolant in the HES is considered as an external disturbance on the closed loop control. In the absence of the influence of cold water on the coolant in the inner-loop heating systems, i.e. $k_{cw} = 0$, this disturbance will be stationary and it can be used in the mathematical model (4) in the form of a fixed coefficient.

In case of significant effect of cold water on the heat exchange system, which leads to the inequality coefficient $k_{cw} > 0$, the disturbance takes the form of time-dependent function, which in the mathematical model (4) is appropriate to distinguish as a separate term. Denoting the disturbance as a symbol $v(t)$, we can reduce the dimension of the mathematical model to the fifth order:

$$\dot{x} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 & a_{2,4} & 0 \\ 0 & a_{3,2} & a_{3,3} & 0 & 0 \\ 0 & a_{4,2} & 0 & a_{4,4} & 0 \\ 0 & 0 & a_{5,3} & 0 & a_{5,5} \end{vmatrix} \cdot x + \begin{vmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \cdot u + \begin{vmatrix} 0 \\ q_3 \\ q_4 \\ 0 \end{vmatrix} \cdot v(t) \tag{5}$$

where $q_3 = a_{3,6+nm}, q_4 = a_{4,6+nm}$

Next, using the Laplace transformation in the point of the equilibrium state, we will write the linearized model:

$$\begin{cases} X_1(s) = \frac{b_1 \cdot U}{s} \\ X_2(s) = \frac{a_{2,1} \cdot X_1 + a_{2,4} \cdot X_4}{s - a_{2,2}} \\ X_3(s) = \frac{a_{3,2} \cdot X_2 + q_3 \cdot V_3}{s - a_{3,3}} \\ X_4(s) = \frac{a_{4,2} \cdot X_2 + q_4 \cdot V_4}{s - a_{4,4}} \\ X_5(s) = \frac{a_{5,3} \cdot X_3}{s - a_{5,5}} \end{cases} \tag{6}$$

With the notation of the functional blocks

$$W_1(s) = \frac{-a_{2,1} \cdot b_1 \cdot a_{2,2}^{-1}}{s}, W_2(s) = \frac{-a_{3,2} \cdot a_{3,3}^{-1}}{1 - a_{2,2}^{-1} \cdot s}, W_3(s) = \frac{1}{1 - a_{3,3}^{-1} \cdot s},$$
$$W_4(s) = \frac{-a_{2,4} \cdot a_{4,4}^{-1}}{1 - a_{4,4}^{-1} \cdot s}, W_5(s) = \frac{-a_{5,3} \cdot a_{5,5}^{-1}}{1 - a_{5,5}^{-1} \cdot s},$$
$$K_{v3} = -\frac{q_3}{a_{3,3}}, K_{v4} = -\frac{q_4}{a_{4,4}}, K_{2,4} = \frac{a_{4,2} \cdot a_{3,3}}{a_{3,2} \cdot a_{4,4}}; \tag{7}$$

the mathematical model (6) can be represented as a block diagram (Fig. 2): After conversion (dotted line marked shifts directions of adders) we obtain the

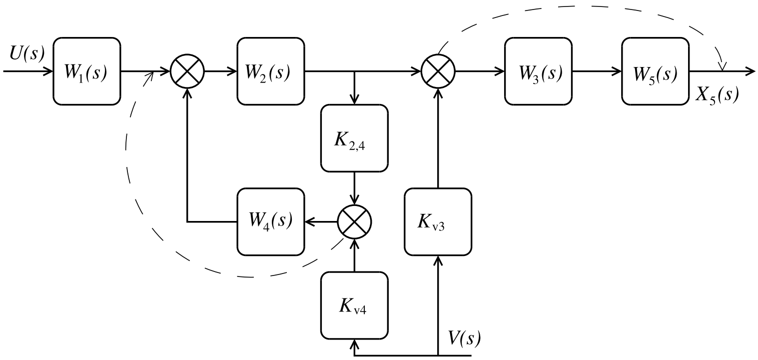

Fig. 2. The block diagram of the linearized model (7)

system shown in Figure 3.

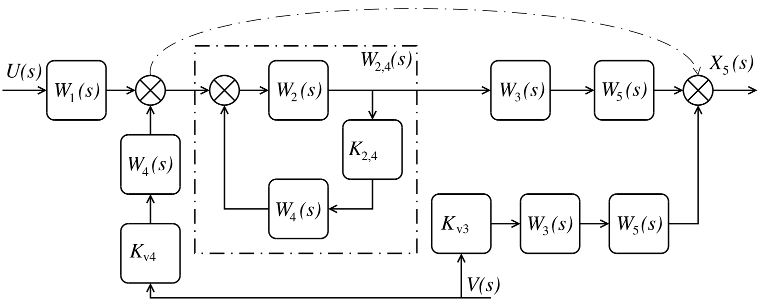

Fig. 3. The converted block diagram of the linearized model (7)

Under the conditions of HES functioning let us consider some assumptions that allow quite adequately to convert a block diagram (Fig. 3) in order to get an equivalent transfer function. Thus the equivalent element with transfer function $W_{2,4}(s)$ is assumed to be stable, because the static open-loop transfer coefficient is less than one:

$$W_2(0) \cdot k_{2,4} \cdot W_4(0) = \frac{a_{4,2} \cdot a_{2,4}}{a_{4,4} \cdot a_{2,2}} = (1 - K^0_{mix}) \cdot (1 - k_{exc}) < 0.2 \qquad (8)$$

where the coefficient $k_{exc} = 0.9$ (based on the practical experience of heat exchange systems maintenance). In addition, using Vieta theorem, we can write the following approximation:

$$\frac{(1 - T_2 \cdot s)}{(1 - T_2 \cdot s) \cdot (1 - T_1 \cdot s) - 0.2} \approx \frac{(1 - 0.2)^{-1}}{(1 - T_1 \cdot s)} \qquad (9)$$

This assumption is transformed to the ratio of the roots of the characteristic equation, which allows to write the conditions:

$$T_{min} < T_1 < T_{max}, \frac{T_{max}}{T_{min}} \approx \left(1 + 2\sqrt{1 - \frac{4 \cdot T_1 \cdot T_2 \cdot 0.8}{(T_1 + T_2)^2}}\right) < 2 \qquad (10)$$

Execution of the inequalities (10) presents the measure of inaccuracy of log-magnitude of the open loop not more than 6 dB.

Thus, the transfer function unit $W_{2,4}$ with a positive feedback, with the assumptions noted above, can be written as:

$$\begin{array}{l} W_{2,4}(s) = \frac{(-a_{3,2}/a_{3,3}) \cdot (1 - (a_{4,2} \cdot a_{2,4})/(a_{4,4} \cdot a_{2,2})^{-1})}{1 + (-a_{2,2} \cdot k_{tt}(K^0_{mix}) \cdot s)}, \\ \forall K^0_{mix} \in (0..1) \vee (k_{exc} > 0.9) \to k_{tt}(K^0_{mix}) \in \left((\sqrt{2})^{-1}..\sqrt{2}\right) \end{array} \qquad (11)$$

where $k_{tt}(K^0_{mix})$ - the function that characterizes the change in the time constant of the object.

Next, the object is divided into the following parts: the unit of control signal delay $W_z(s)$, the integrating part which is a unit of the equivalent transfer function $W_i(s)$ of the electric control valve, the inertial part which is an aperiodic link $W_o(s)$ of the first-order transfer function of the thermal object, the transfer function $W_v(s)$ of the disturbance signal $V(s)$. As a result of transformations, we obtain the final block diagram (Fig. 4), representing the linearized model graphically (6). The corresponding transfer functions are defined by the following equations:

$$\begin{array}{l} W_z(s) \cdot W_i(s) \cdot W_o(s) = W_1(s) \cdot W_{2,4}(s) \cdot W_3(s) \cdot W_5(s), \\ W_v(s) = (K_{v3} + K_{v4} \cdot W_4(s) \cdot W_{2,4}(s)) \cdot W_3(s) \cdot W_5(s) \end{array} \qquad (12)$$

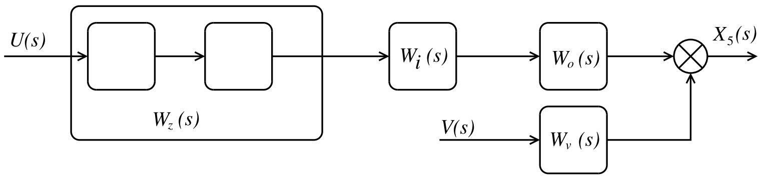

Fig. 4. The final block diagram

The blocks are distributed in such a way that the inertial link with a maximum time constant to be present in the unit $W_o(s)$, the other two units are replaced by the delay link (Fig. 3).

As a result of the transformation we obtain more convenient for the synthesis of closed-loop control transfer function of the electric control valve, including the main parameters of the heat exchanger system:

$$W_i(s) = \left(\frac{b_1 \cdot a_{3,2} \cdot a_{2,1} \cdot a_{5,3}}{a_{3,3} \cdot a_{2,2} \cdot a_{5,5}}\right) \cdot \left(1 - \frac{a_{4,2} \cdot a_{2,4}}{a_{4,4} \cdot a_{2,2}}\right)^{-1} \cdot s \tag{13}$$

After the substitution of physical quantities, which are the parameters of HES, this expression takes on the following form:

$$W_i(s) = \frac{k_h \cdot k_{exc} \cdot (k_{mx} - K^0_{mix}) \cdot (T_1^\circ - T_{bk1}^{\circ 0}(k_{cw}))}{(1 - (1 - k_{exc}) \cdot (1 - K^0_{mix})) \cdot (T_{vlv} \cdot s)} \tag{14}$$

In the research of complex objects with delay a very important requirement is a preliminary assessment of the main parameters that have a significant impact on the stability of the closed-loop control system. A valid transfer coefficient and time constants in the inner loop heat exchange systems that define the retarded reaction of control to the disturbance can be such parameters for the object in question.

For the specific HES performance with known interval values of the constituents parameters, in particular, the transfer coefficient k_g of the control valve, the coefficient $k_c w$ of cold water influence on the coolant in the internal contour of the heat exchange system we can rather accurately estimate the range of variation of the static coefficient of the actuator transfer function $W_i(s)$.

The determining factor in assessing of the delay in the HES control channel is the ratio of the time constant T_{cp} of the fluid mixing in the storage container (if it exists in the HES) and time constant T_{mix} of the fluid mixing at the valve of the system. It is clear that in the absence of the storage container the delay duration in the control channel will be determined only by the time constant T_{mix}. Let us write the transfer functions $W_o(s)$ and $W_z(s)$ the coefficients of which are largely determined by the ratio of the time constants data:

$$\begin{array}{l} T_{cp} > T_{mix} \begin{cases} W_o(s) = \frac{1}{1 - a_{5,5}^{-1} \cdot s} = \frac{1}{1 + T_{exc} \cdot s}, \\ W_z(s) = exp((k_{tt}(K^0_{mix}) \cdot a_{2,2}^{-1} + a_{3,3}^{-1}) \cdot s) \\ = exp((k_{tt}(K^0_{mix}) \cdot T_{mix} + T_{exc}) \cdot s) \end{cases} \\ T_{cp} < T_{mix} \begin{cases} W_o(s) = \frac{1}{1 - k_{tt}(K^0_{mix}) \cdot a_{2,2}^{-1} \cdot s} = \frac{1}{1 + k_{tt}(K^0_{mix}) \cdot T_{mix} \cdot s}, \\ W_z(s) = exp((a_{5,5}^{-1} + a_{3,3}^{-1}) \cdot s) = exp((T_{cp} + T_{exc}) \cdot s) \end{cases} \end{array} \tag{15}$$

To solve the tasks of HES research we can use the following fact: in the inequality $T_{cp} > T_{mix}$ there is nonstationary delay time for control, which varies not more than two fold and corresponds to $(k_{tt}(K^0_{mix}) \cdot T_{mix} + T_{exc})$. Similarly, when performing inequality $T_{cp} < T_{mix}$ nonstationary time constant of the object $(k_{tt}(K^0_{mix}) \cdot T_{mix})$ also changes not more than two fold.

4 The Nonlinear HES Model of the Second Order with Delay in Controlling

Using the equations (15) without disturbance, with the notation $y_1(t) = x_5(t)$, $y_2(t)$ - output unit with the transfer function $W_i(s)$

$$\begin{array}{l} k_g(s) = \frac{k_h \cdot k_{exc} \cdot (k_{mx} - K^0_{mix}) \cdot (T^\circ_1 - T^{\circ 0}_{bk1}(k_{cw}))}{\left(1-(1-k_{exc})\cdot(1-K^0_{mix})\right)\cdot T_{vlv}} \\ T_{cp} > T_{mix} \left\{ T_o = T_{cp}, \tau_z = (k_{tt}(K^0_{mix}) \cdot T_{mix} + T_{exc}) \right. \\ T_{cp} < T_{mix} \left\{ T_o = k_{tt}(K^0_{mix}) \cdot T_{mix}, \tau_z = (T_{cp} + T_{exc}) \right. \end{array} \tag{16}$$

we can write the system of differential equations of the second order

$$\begin{cases} \dot{y}_1(t) = \frac{y_2(t) - y_1(t)}{T_o(t,y)}, \\ \dot{y}_2(t) = k_g(t,y) \cdot u(t - \tau_z) \end{cases} \tag{17}$$

The canonical representation of the system of differential equations in Frobenius form after changing variables $z_1(t) = y_1(t), z_2(t) = \dot{y}_1(t)$ becomes:

$$\begin{cases} \dot{z}_1(t) = z_2(t), \\ \dot{z}_2(t) = (k_g(t,z)/T_o(t,z)) \cdot u(t - \tau_z) - T_o^{-1}(t,z) \cdot z_2(t) \end{cases} \tag{18}$$

4.1 The Results of the Simulation

The adequacy of the resulting model (18) is confirmed by comparing the s-shaped curves of the speed-up after the numerical simulation of the transition process which conforms to conditions of the experiment [8]. The experiment was performed in the same conditions for each model, the parameters of the model (18) were calculated by the formula (16) on the base of the parameters of the initial model (1). The simulation was performed in the C language; the source code is available on the web resource. [9]

Fig. (5a) illustrates transients: full red line T°_{to3} for the model (1); green dotted line $y_1(t) = z_1(t)$ for the model (18), and curve T°_{sto3} from the sensor output of the current heat exchange system - the blue dotted line. Some difference between

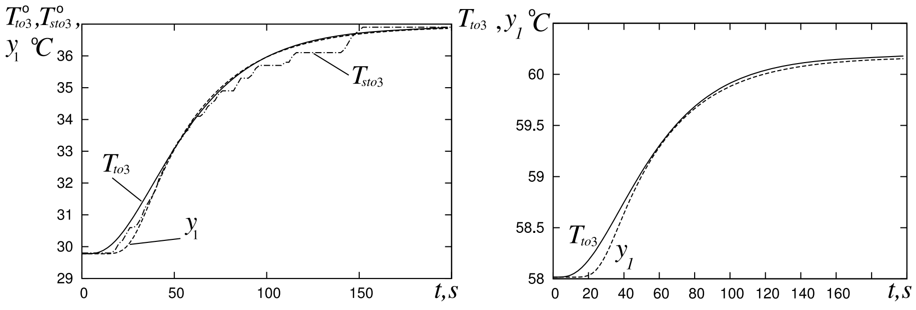

Fig. 5. Comparison of the modeling and experiment outcomes

the models at the beginning of the transition process can be explained by the fact that in the system of equations (18) several inertial links are replaced by a single delay element.

Fig. 5b shows the laws T°_{to3} and $y_1(t) = z_1(t)$ for both models. They correspond to the same conditions and values of the parameters in the control valve stem position $h = 0.7$. The results of modeling and experimental research imply a high degree of adequacy of the reduced model to the real object.

Conclusion

A solution to the problem of approximation of a complex nonlinear mathematical model of heat transfer delay system to a nonlinear system of differential equations of the second order allows us to use modern methods of relay control. For the dimensional model reduction we used the method of decomposition of the linearized model at the steady state point, where the basic coefficients are given in a general form which allows us to get the non-linear law of the reduced model coefficients at an arbitrary point of equilibrium. The results of numerical simulations confirm the adequacy of the reduced model and real heat exchange system.

References

1. Klyuev, A.S., Kolesnikov, A.A.: Optimization of automatic speed control systems, p. 240. Energoizdat, Moscow (1982)
2. Khalil Hassan, K.: Nonlinear Systems, 3rd edn., p. 750. Prentice Hall (2002) ISBN 0-13-067389-7
3. Shilin, A.A., Bukreev, V.G.: Investigation of the three-position relay temperature controller in a sliding mode. Reoprts of TUSUR, Russian Tomsk 2(1), 251–257 (2012)
4. Buslenko, N.P.: Modeling of complex systems, p. 360. Nauka, Moscow (1978)
5. Anderson, B.D.O., Liu, Y.: Controller reduction: concepts and approaches. IEEE Trans. Automat. Control. 34, 802–812 (1989)
6. Moore, B.C.: Principal component analysis in linear systems: controllability, observability and model reduction. IEEE Trans. Automat. Control. AC-26, 17–32 (1981)
7. Shilin, A.A., Bukreev, V.G., Koykov, K.I.: Mathematical model of a nonlinear system with delay. Devices and Systems. Management, Monitoring, Diagnostics (6), 3–10 (2013)
8. Pan'ko, M.A.: Selecting the mathematical models of a controlled piece of equipment from experimental data. Thermal Engineering 53(10), 782–786 (2006) ISSN 0040 6015
9. Shilin, A.A.: Materials for the article: Decomposition of nonlinear heat transfer model with delay, `http://portal.tpu.ru/SHARED/s/SHILIN/tethiss/Tab4`