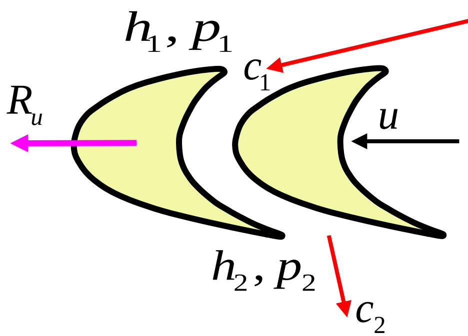
3.5. Steam expansion over rotor blades

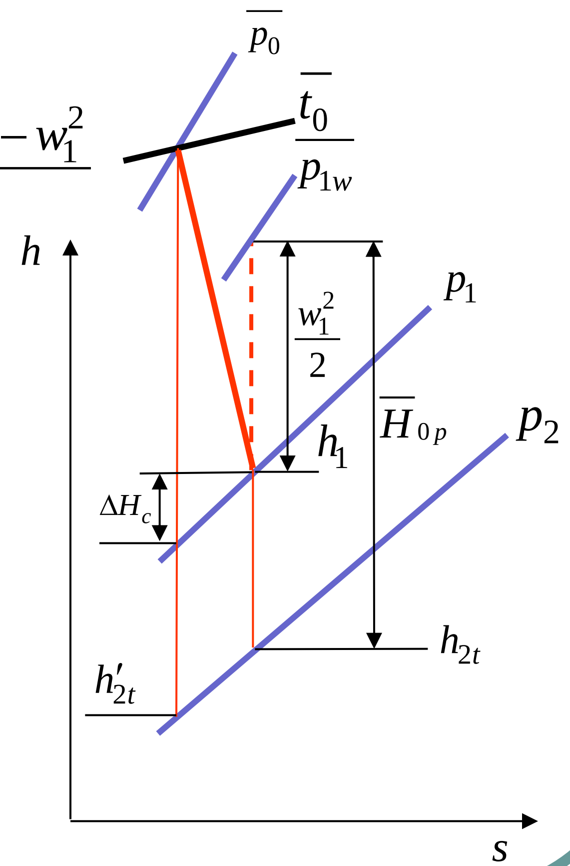
The law of conservation of energy for rotor blades (G=1 kg/s)



$$\frac{c_1^2}{2}+h_1=\frac{c_2^2}{2}+h_2+L_u$$

$$\frac{c_1^2}{2}+h_1=\frac{c_2^2}{2}+h_2+\frac{c_1^2-c_2^2+w_2^2-w_1^2}{2}$$

$$\frac{w_1^2}{2}+h_1=\frac{w_2^2}{2}+h_2$$



For ideal steam expansion

$$\frac{w_1^2}{2}+h_1=\frac{w_{2t}^2}{2}+h_{2t}$$

$$w_{2t}=\sqrt{2\left(h_1-h_{2t}\right)+w_1^2}=\sqrt{2\overline{H}_{0p}}$$

$$\varepsilon_2=\frac{p_2}{\overline{p}_{1w}}$$

For actual expansion:

$$\frac{w_1^2}{2}+h_1=\frac{w_2^2}{2}+h_2$$

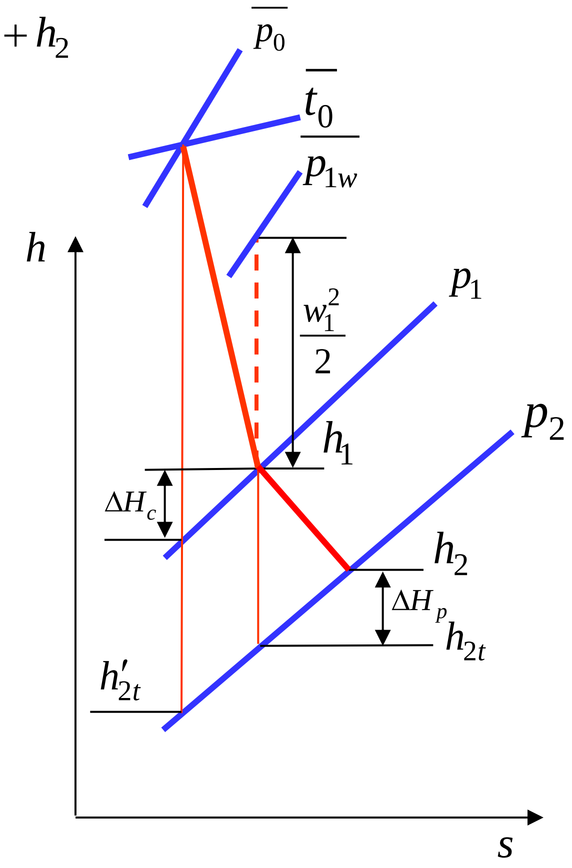
$$\psi=\frac{w_2}{w_{2t}}$$

$$\psi=\sqrt{1-\zeta_p}$$

$$\zeta_p=1-\psi^2$$

$$w_2=\psi w_{2t}$$

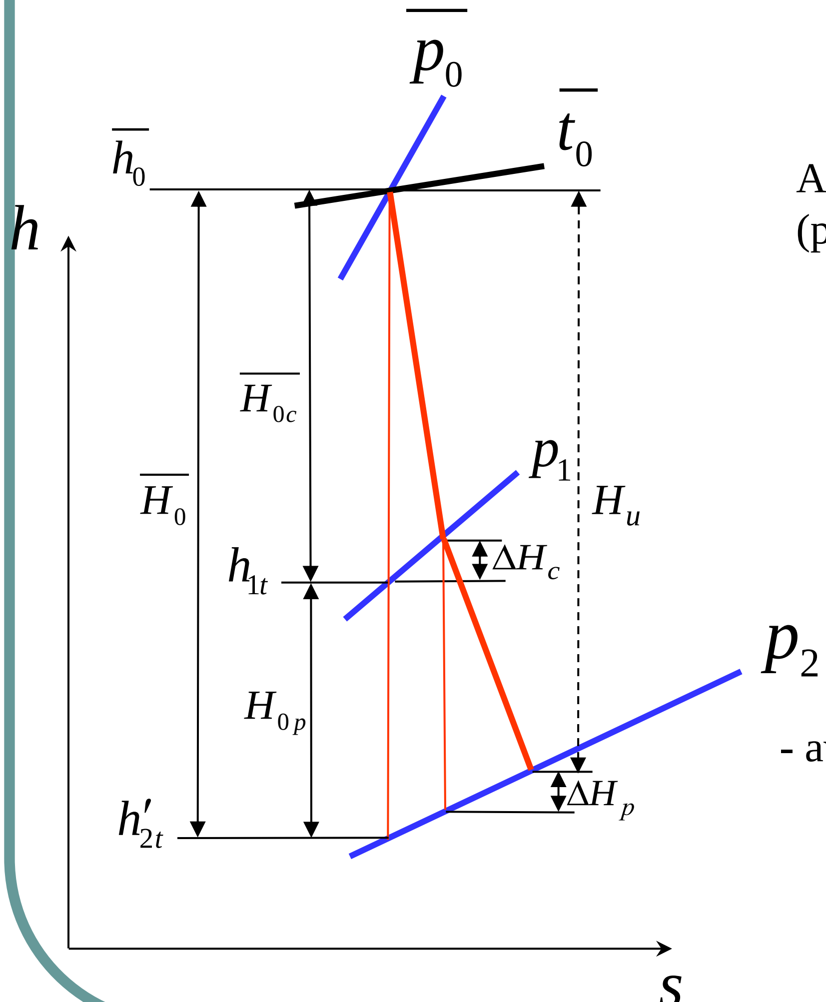
$$\Delta H_p=\frac{w_{2t}^2-w_2^2}{2}=\frac{w_{2t}^2}{2}\left(1-\frac{w_2^2}{w_{2t}^2}\right)=\frac{w_{2t}^2}{2}\left(1-\psi^2\right)=h_2-h_{2t}$$



Particular case: $\rho=0$

$$H_{0p}=0 \Rightarrow h_{2t}=h_1 \Rightarrow w_{2t}=w_1 \Rightarrow w_2=\psi w_1$$

3.6. Work (power) generated by 1 kg of gas in the stage (according to the equation of conservation of energy)



$\rho > 0: \quad H_{0p} > 0; \quad p_1 > p_2$

According to the equation of conservation of energy, (power) work transferred by 1 kg of gas to blades :

$$L_u = H_u = \overline{H}_0 - \Delta H_c - \Delta H_p \quad (?)$$

- available heat drop of the stage:

$$\overline{H}_0 = \overline{H}_{0c} + H_{0p}$$

- available heat drop in nozzles:

$$\overline{H}_{0c} = \frac{c_{1t}^2}{2}$$

- available heat drop in rotor blades:

$$H_{0p} = \frac{w_{2t}^2 - w_1^2}{2}$$

- available energy losses in nozzles:

$$\Delta H_c = \frac{c_{1t}^2 - c_1^2}{2}$$

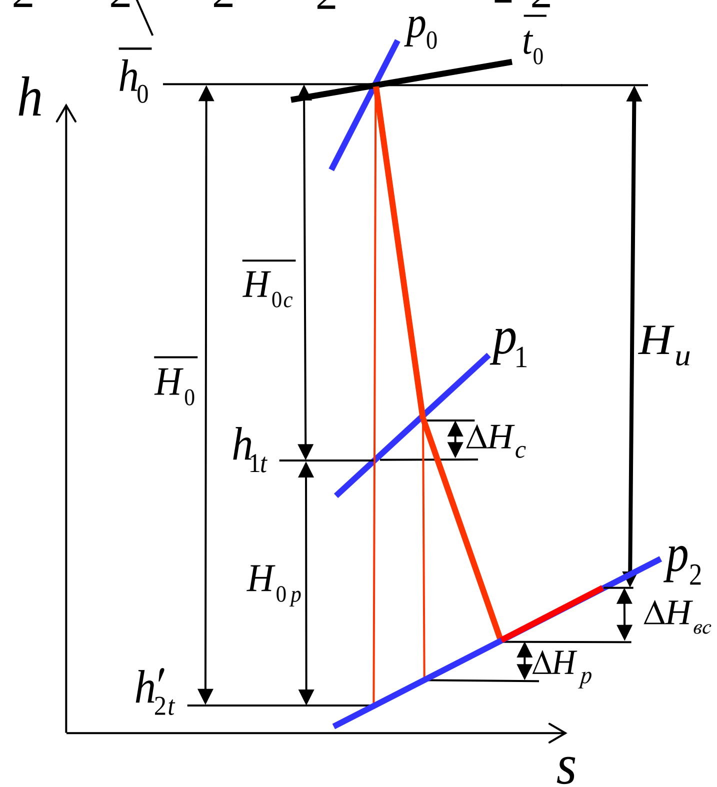
- available energy losses in rotor blades:

$$\Delta H_p = \frac{w_{2t}^2 - w_2^2}{2}$$

Assume

$$L_u = H_u = \overline{H}_0 - \Delta H_c - \Delta H_p - \Delta H_{вс} = \cancel{\frac{c_{1t}^2}{2}} + \cancel{\frac{w_{2t}^2}{2}} - \frac{w_1^2}{2} - \cancel{\frac{c_{1t}^2}{2}} + \frac{c_1^2}{2} - \cancel{\frac{w_{2t}^2}{2}} + \frac{w_2^2}{2} - \frac{c_2^2}{2} = \frac{c_1^2 - c_2^2 + w_2^2 - w_1^2}{2}$$

Compare this formula with the formula of the work produced in the blades according to the equation of momentum:



$$L_u = \frac{c_1^2 - c_2^2 + w_2^2 - w_1^2}{2}$$

$$\Delta H_{вс} = \frac{c_2^2}{2}$$ **- loss with outlet velocity**

4. Relative efficiency of the vane stage

It characterizes the perfection (efficiency) of energy conversion in the flow range of the stage:

By definition of EF

By equation of energy conservation

$$\eta_{ол} = \frac{L_u}{\overline{H_0}} = \frac{\overline{H_0} - \Delta H_c - \Delta H_p - \Delta H_{BC}}{\overline{H_0}} = 1 - \xi_c - \xi_p - \xi_{вс} =$$

$$= \frac{u\left(c_1 \cos\alpha_1 + c_2 \cos\alpha_2\right)}{\overline{H_0}} = \frac{2u\left(w_1 \cos\beta_1 + w_2 \cos\beta_2\right)}{c_ф^2} = \frac{c_1^2 - c_2^2 + w_2^2 - w_1^1}{c_ф^2}$$

By equation of equation of momentum

$\overline{H_0} = \frac{c_ф^2}{2}, c_ф -$ **fictitious velocity in the stage,** equivalent to the available energy in the stage

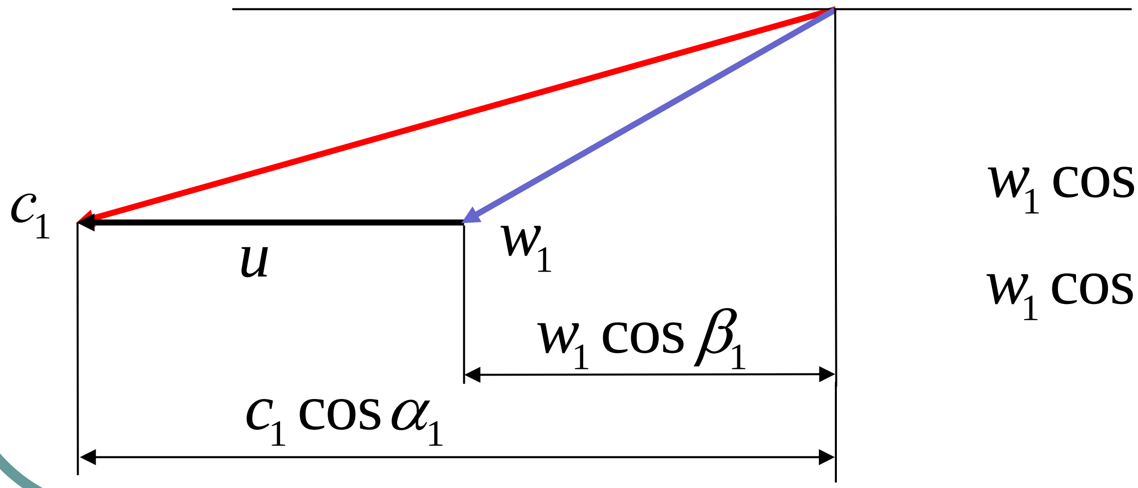
4.1. Dependence of the relative blade efficiency on dimensionless ratio of velocities

A. For "pure impulse" stage

$$\rho = 0 \quad \Longrightarrow \quad H_{0p} = 0; \quad \overline{H_{oc}} = \overline{H_0}; \quad \Longrightarrow \quad c_{1t} = c_{\phi}$$

I. We use the formula for determining the EF according to the equation of momentum:

$$\eta_{ол} = \frac{2u\left(w_1 \cos\beta_1 + w_2 \cos\beta_2\right)}{c_{\phi}^2} = \frac{2u}{c_{\phi}^2} w_1 \cos\beta_1 \left(1 + \frac{w_2 \cos\beta_2}{w_1 \cos\beta_1}\right)$$



$$w_1 \cos\beta_1 = c_1 \cos\alpha_1 - u$$

$$w_1 \cos\beta_1 = \varphi c_{\phi} \cos\alpha_1 - u$$

$$\eta_{ол} = \frac{2u}{c_{ф}^2}\left(\varphi c_{ф}\cos\alpha_1 - u\right)\left(1+\frac{w_2\cos\beta_2}{w_1\cos\beta_1}\right) = 2\left(\frac{u}{c_{ф}}\varphi\cos\alpha_1 - \frac{u^2}{c_{ф}^2}\right)\left(1+\frac{w_2\cos\beta_2}{w_1\cos\beta_1}\right)$$

$$x_{ф} = \frac{u}{c_{ф}}$$

- dimensionless ratio of velocities

as $w_2 = \psi w_{2t}$, and $\underset{\rho=0}{w_{2t}} = w_1$, $\frac{w_2}{w_1} = \psi$

$$\left(\eta_{ол}\right)_{\rho=0} = 2\left(x_{ф}\varphi\cos\alpha_1 - x_{ф}^2\right)\left(1+\psi\frac{\cos\beta_2}{\cos\beta_1}\right)$$

Function $\eta_{ол} = f\left(x_{ф}\right)$ parabolic (has its maximum)

$$\frac{d\eta_{ол}}{dx_{ф}} = \varphi\cos\alpha_1 - 2x_{ф} = 0 \quad \Longrightarrow \quad x_{ф}^{opt} = \frac{\varphi\cos\alpha_1}{2}$$

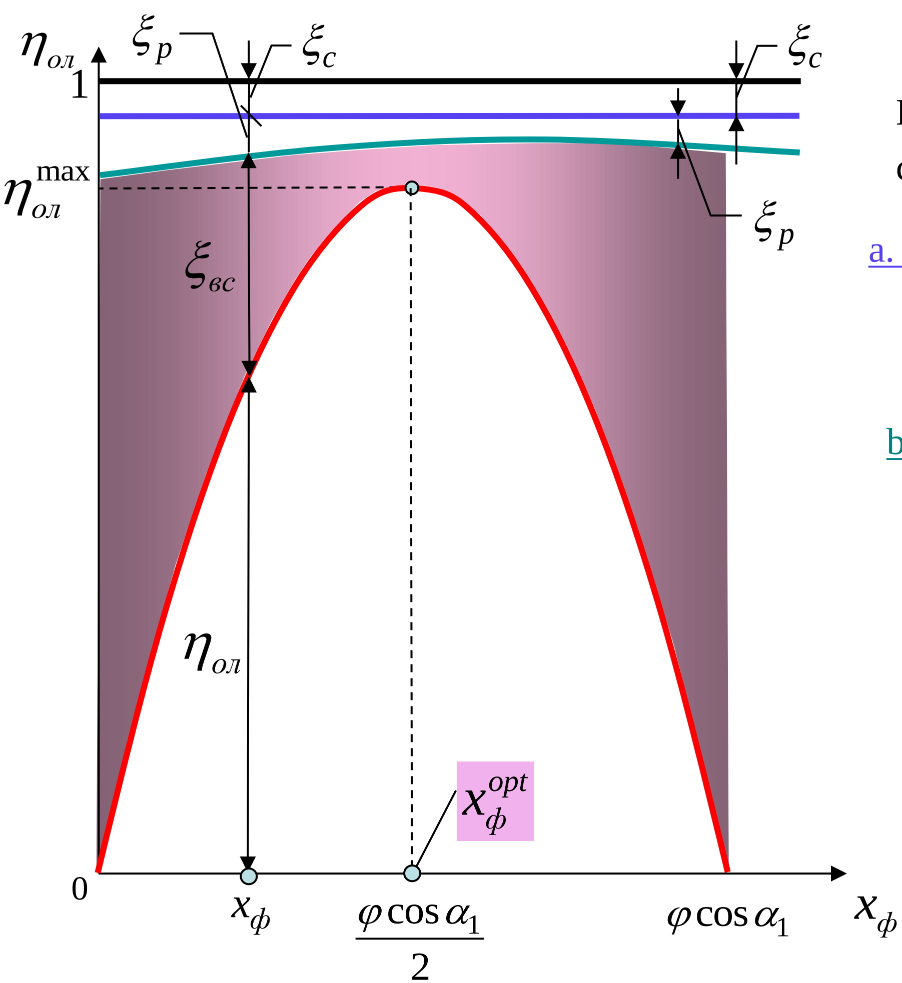
First dogmata: for maximum EF, the ratio of velocities should be **optimal**

$$\left(\eta_{ол}^{\max}\right)_{\rho=0} = 2\left(\frac{\varphi\cos\alpha_1}{2}\varphi\cos\alpha_1 - \frac{\varphi^2\cos^2\alpha_1}{4}\right)\left(1+\psi\frac{\cos\beta_2}{\cos\beta_1}\right)$$

$$\left(\eta_{ол}^{\max}\right)_{\rho=0} = \frac{\varphi^2\cos^2\alpha_1}{2}\left(1+\psi\frac{\cos\beta_2}{\cos\beta_1}\right) \approx \varphi^2\cos^2\alpha_1$$

Second dogmata: the smaller the angle nozzle ring outlet flow, the higher the EF

II. We use the formula for determining the EF by the equation of energy conservation:



$$\eta_{ол} = 1 - \xi_c - \xi_p - \xi_{вс}$$

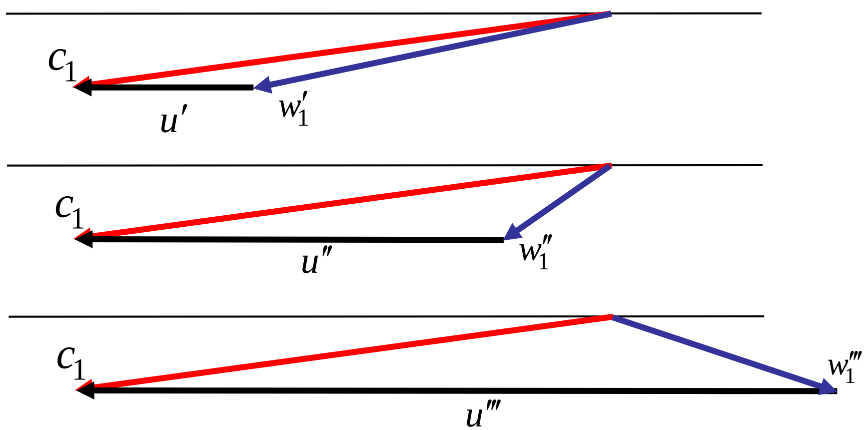
How do the individual components of available energy losses change relative to $x_ф$?

a. Nozzle loss

$$\xi_c = \frac{\Delta H_c}{\overline{\overline{H}}_0} = \frac{c_{1t}^2 - c_1^2}{c_ф^2} = 1 - \varphi^2$$

b. Rotor blade loss

$$\xi_p = \frac{\Delta H_p}{\overline{\overline{H}}_0} = \frac{w_{2t}^2 - w_2^2}{c_ф^2} = \frac{w_{2t}^2}{c_{1t}^2}\left(1-\psi^2\right) = \varphi^2\,\frac{w_1^2}{c_1^2}\left(1-\psi^2\right)$$



c. Loss in the velocity of exhaust

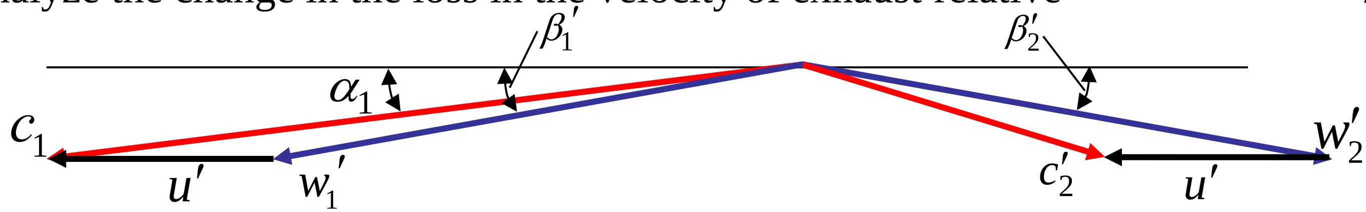
$$\xi_{вс} = \frac{\Delta H_{вс}}{\overline{\overline{H}}_0} = \frac{c_2^2}{c_ф^2} = 1 - \eta - \xi_c - \xi_p$$

Most strongly changing available energy loss relative to $x_ф$.

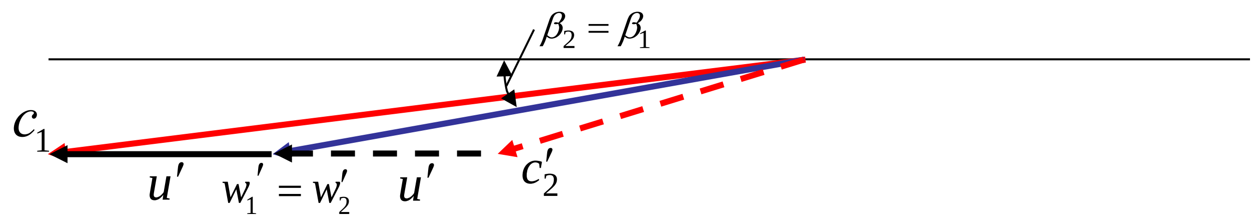
Analyze the change in the loss in the velocity of exhaust relative

$x_\phi : c_\phi = const; u = \text{var}$

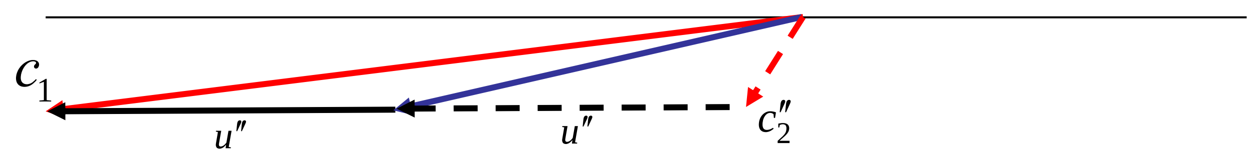
I.



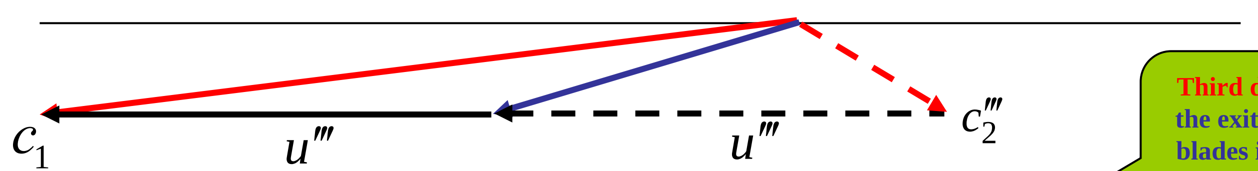
Assume $\psi = 1,$ then $w_2 = w_1.$ If $\rho = 0$ $\beta_2 = \beta_1$



II.

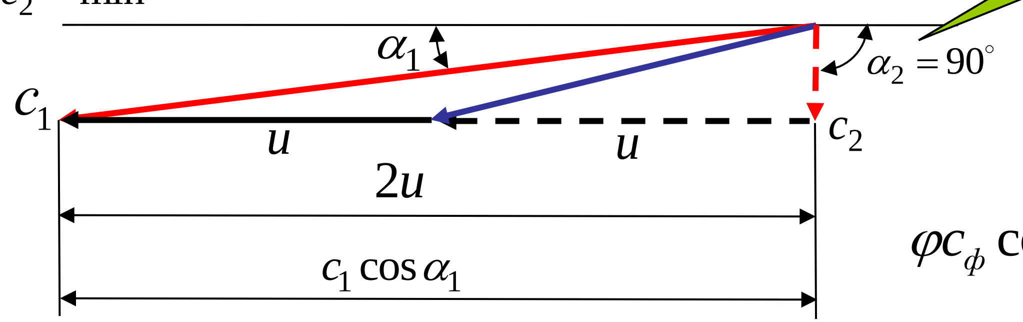


III.



Third dogmata: **EF will be of the highest level if the exit angle of the absolute velocity from rotor blades is 90°, i.e., the flow direction is parallel to the rotation axis**

IV. $c_2 = \min$



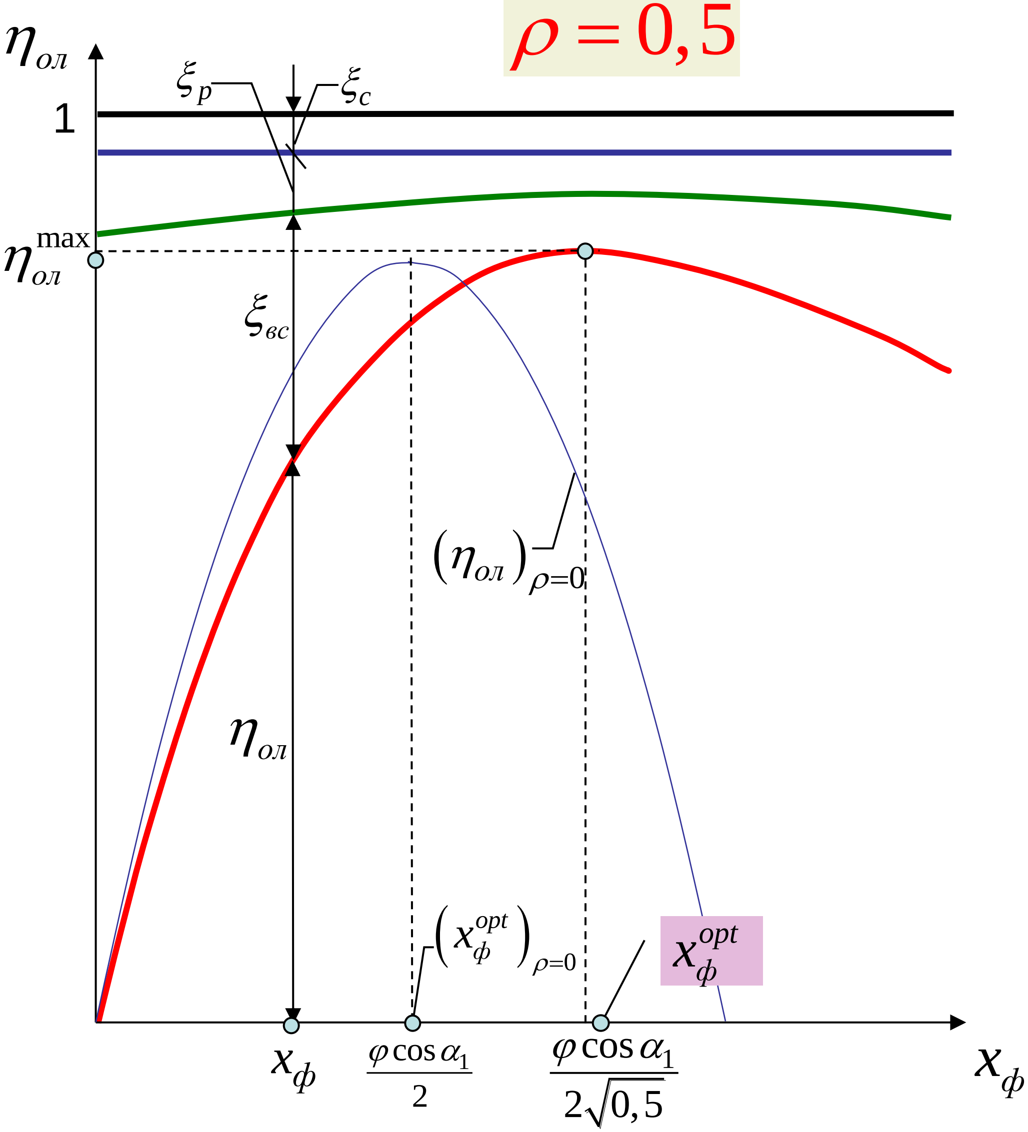
$$c_1 \cos \alpha_1 = 2u$$

$$\varphi c_\phi \cos \alpha_1 = 2u \Rightarrow x_\phi^{opt} = \frac{\varphi \cos \alpha_1}{2}$$

B. For the stage for any value of reaction degree :

$$\eta_{ол} = 2x_{ф}\left[\varphi\cos\alpha_1\sqrt{1-\rho} - x_{ф} + \psi\cos\beta_2\sqrt{\varphi^2(1-\rho) + x_{ф} - 2x_{ф}\varphi\cos\alpha_1\sqrt{1-\rho} + \rho}\right]$$

$$\rho = 0,5$$



$$x_{ф}^{opt} = \frac{\varphi\cos\alpha_1}{2\sqrt{1-\rho}}$$

Losses:

a. Nozzle loss

$$\xi_c = \frac{\Delta H_c}{\overline{H}_0} = \frac{c_{1t}^2 - c_1^2}{c_{ф}^2} = \frac{c_{1t}^2}{c_{ф}^2}\left(1-\varphi^2\right) = (1-\rho)\left(1-\varphi^2\right)$$

$$c_{1t}^2 = 2(1-\rho)\overline{H}_0; \quad c_{ф}^2 = 2\overline{H}_0$$

b. Rotor blade loss

$$\xi_p = \frac{\Delta H_p}{\overline{H}_0} = \frac{w_{2t}^2}{c_{ф}^2}\left(1-\psi^2\right) = \left(\rho + \frac{w_1^2}{c_{ф}^2}\right)\left(1-\psi^2\right)$$

$$w_{2t}^2 = 2\rho\overline{H}_0 + w_1^2$$

c. Loss in the velocity of exhaust

Minimal at $\alpha_2 \approx 90^\circ$

$$\frac{\left(x_{ф}^{opt}\right)_{\rho=0,5}}{\left(x_{ф}^{opt}\right)_{\rho=0}} = \sqrt{2}$$

4.2. Optimal available heat drop of the stage

Set: stage diameter and the rotor angular rate

Determine: **what heat drop provides the greatest EF of the stage?**

$$\overline{H_0} = \frac{c_\phi^2}{2}; \qquad x_\phi = \frac{u}{c_\phi}; \qquad \overline{H_0} = \frac{u^2}{2x_\phi^2}$$

$$\overline{H_0}^{opt} = \frac{u^2}{2\left(x_\phi^{opt}\right)^2} \qquad x_\phi^{opt} = \frac{\varphi\cos\alpha_1}{2\sqrt{1-\rho}} \qquad u = \pi dn$$

$$\overline{H_0}^{opt} = \frac{2\pi^2 d^2 n^2 (1-\rho)}{\varphi^2\cos^2\alpha_1}$$

Particular cases:

a) $$\frac{\left(\overline{H_0}^{opt}\right)_{\rho=0}}{\left(\overline{H_0}^{opt}\right)_{\rho=0,5}} \approx 2$$

b) ρ=0; if: n= 50 c^{-1}; α_1=13°; φ=0,97.

$$\left(\overline{H_0}^{opt}\right)_{\rho=0} = 52,5d^2$$