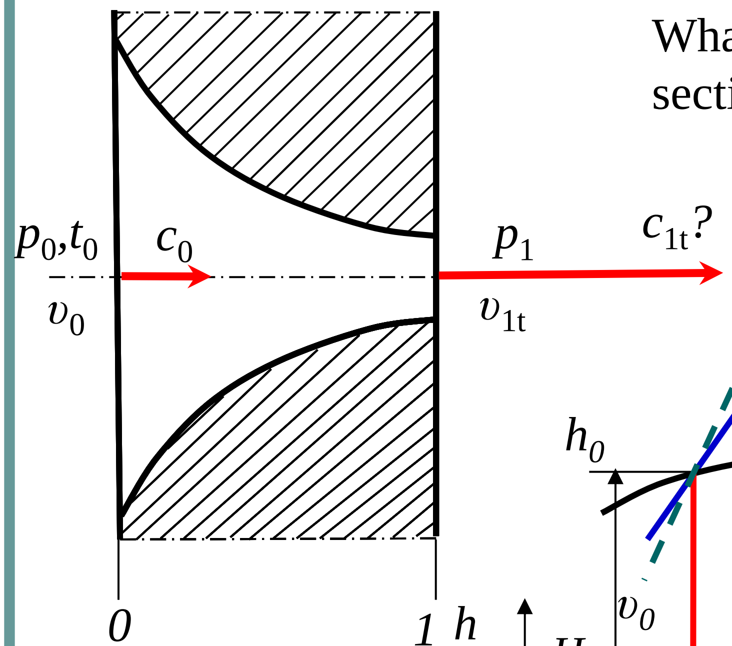
2.2. The flow characteristics during isentropic expansion of gas in the channels

The channel, in which the flow smoothly accelerates, is called a **nozzle one** or a **nozzle**.

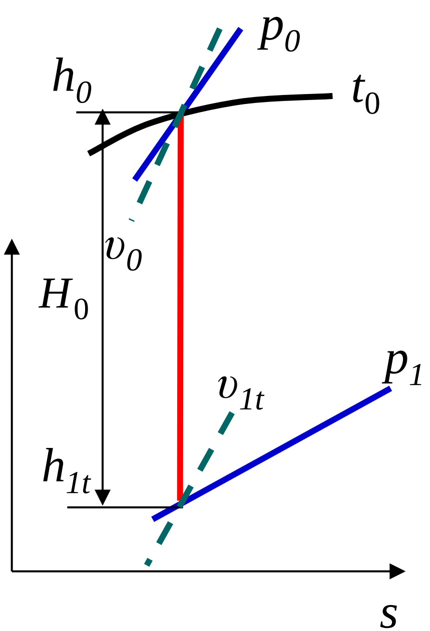
The channel, in which the flow smoothly slows down, is called a **diffusion one** or a **diffuser.**

2.2.1. Acceleration of the flow in the channel



What will be the flow rate at the channel outlet (in section 1)?

$$\frac{c_{1t}^2 - c_0^2}{2} = h_0 - h_{1t}$$



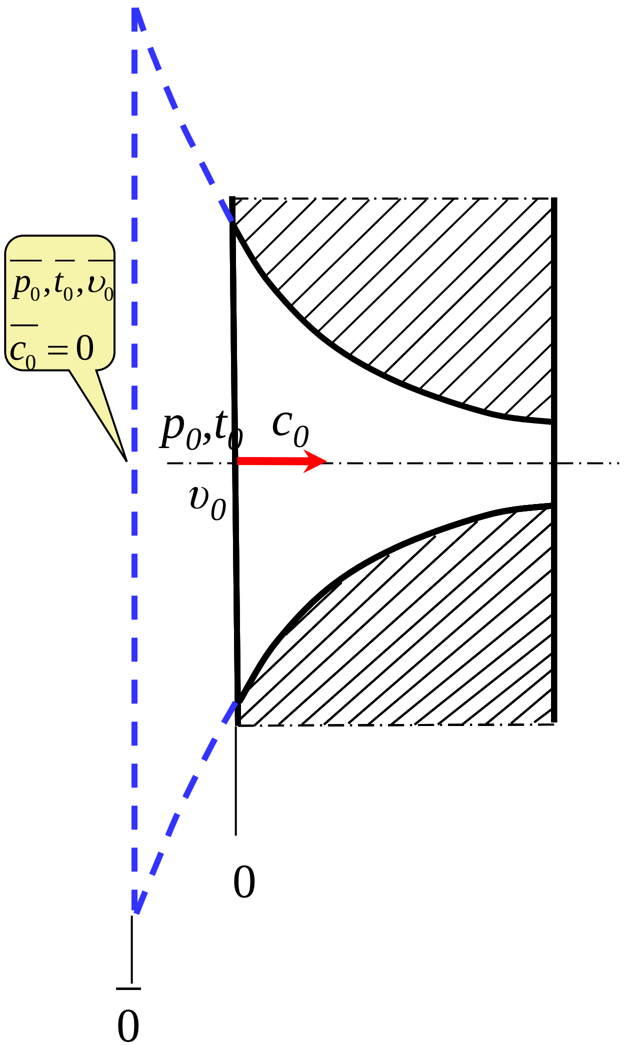
$H_0 = h_0 - h_{1t}$ is available heat drop per channel (by static parameters)

$$h = \frac{k}{k-1} p\upsilon + const$$

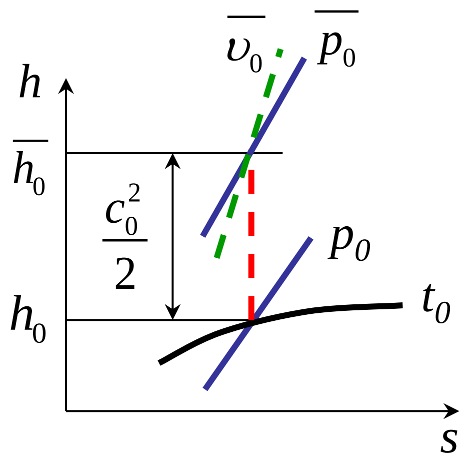
$$\frac{c_{1t}^2}{2} = \frac{k}{k-1}\left(p_0\upsilon_0 - p_1\upsilon_{1t}\right) + \frac{c_0^2}{2}$$

$$\frac{c_{1t}^2}{2} = \frac{k}{k-1}\left(p_0\upsilon_0 - p_1\upsilon_{1t}\right) + \frac{c_0^2}{2}$$

* Deceleration parameters. How did the velocity c_0 appeared?



$$\frac{c_0^2}{2} = \frac{k}{k-1}\left(\bar{p}_0\bar{\upsilon}_0 - p_0\upsilon_0\right) = \bar{h}_0 - h_0$$



$$\bar{p}_0 = p_0 + \frac{c_0^2}{2\upsilon_0};$$

$$\bar{\upsilon}_0 = \upsilon_0 + \frac{c_0^2}{2kp_0}$$

$$\frac{c_{1t}^2}{2} = \frac{k}{k-1}\left(\bar{p}_0\bar{\upsilon}_0 - p_1\upsilon_{1t}\right)$$

$$\frac{c_{1t}^2}{2}=\frac{k}{k-1}\left(\overline{p}_0\overline{\upsilon}_0-p_1\upsilon_{1t}\right)=\overline{h}_0-h_{1t}$$

$\overline{H}_0=\overline{h}_0-h_{1t}$ available heat drop for the channel (by the parameters of deceleration at the outlet)

$$\frac{c_{1t}^2}{2}=\frac{k}{k-1}\overline{p}_0\overline{\upsilon}_0\left(1-\frac{p_1\upsilon_{1t}}{\overline{p}_0\overline{\upsilon}_0}\right)$$

$$\overline{H}_0-H_0=\frac{c_0^2}{2}$$

$$\frac{p_1\upsilon_{1t}}{\overline{p}_0\overline{\upsilon}_0}=\frac{p_1}{\overline{p}_0}\cdot\left(\frac{\overline{p}_0}{p_1}\right)^{\frac{1}{k}}=\varepsilon\cdot\varepsilon^{-\frac{1}{k}}=\varepsilon^{\frac{k-1}{k}}$$

a) According to the isentrope equation $p\upsilon^k=const$ we have

$$\overline{p}_0\overline{\upsilon}_0^{\,k}=p_1\upsilon_{1t}^k \Longrightarrow \frac{\upsilon_{1t}}{\overline{\upsilon}_0}=\left(\frac{\overline{p}_0}{p_1}\right)^{\frac{1}{k}}$$

b) Denote

$$\varepsilon=\frac{p_1}{\overline{p}_0}$$

The relation of pressures on the channel (nozzle)

$$\frac{c_{1t}^2}{2}=\frac{k}{k-1}\overline{p}_0\overline{\upsilon}_0\left(1-\varepsilon^{\frac{k-1}{k}}\right)$$

2.2.2. Critical parameters of the flow

Critical parameters are determined by the critical flow velocity

Compare the flow velocity with the local speed of sound

The speed of sound is rate of propagation of small changes in pressure

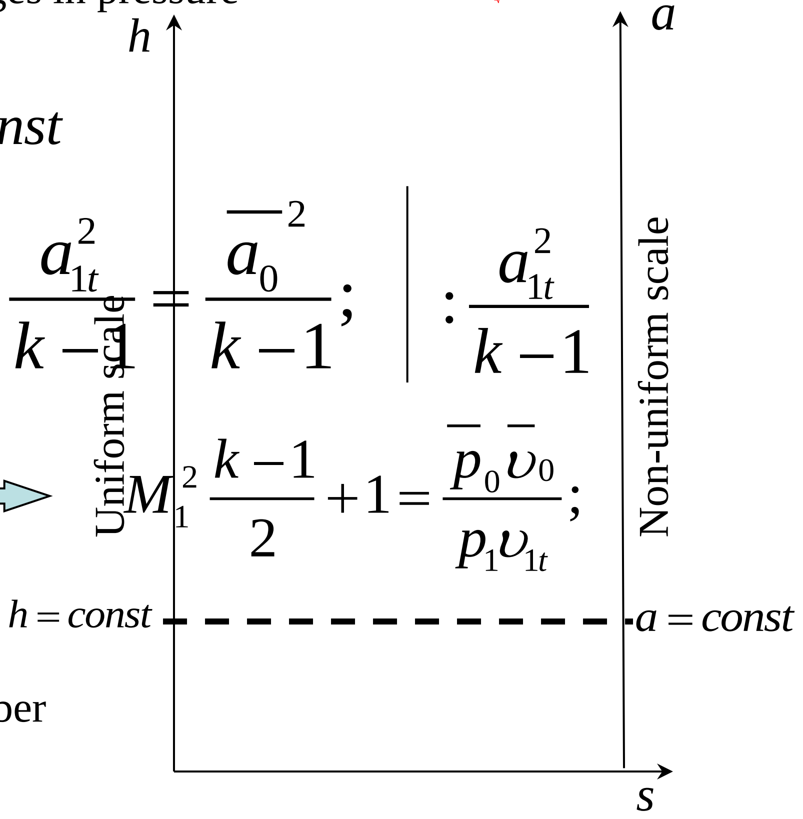
$$a = \sqrt{kp\upsilon}$$

$$\frac{c_{1t}^2}{2} + h_{1t} = \bar{h}_0; \qquad h = \frac{k}{k-1} p\upsilon + const$$

$$\frac{c_{1t}^2}{2} + \frac{kp_1\upsilon_{1t}}{k-1} = \frac{k\,\bar{p}_0\bar{\upsilon}_0}{k-1}; \quad \Rightarrow \quad \frac{c_{1t}^2}{2} + \frac{a_{1t}^2}{k-1} = \frac{\bar{a}_0^2}{k-1}; \quad \Big| \; : \frac{a_{1t}^2}{k-1}$$

$$\frac{c_{1t}^2}{a_{1t}^2}\,\frac{k-1}{2} + 1 = \frac{\bar{a}_0^2}{a_{1t}^2} \quad \Rightarrow \quad M_1^2\,\frac{k-1}{2} + 1 = \frac{\bar{p}_0\bar{\upsilon}_0}{p_1\upsilon_{1t}};$$

$$\frac{c_{1t}}{a_{1t}} = M_{1t}$$ is Mach number



$$M_1^2\frac{k-1}{2}+1=\frac{\overline{p}_0\overline{\upsilon}_0}{p_1\upsilon_{1t}};$$

$$\frac{\overline{p}_0\overline{\upsilon}_0}{p_1\upsilon_{1t}}=\varepsilon^{\frac{1-k}{k}}$$

$$M_{1t}=\sqrt{\frac{2}{k-1}\left(\varepsilon^{\frac{1-k}{k}}-1\right)}$$

$$M_{1t}=1: \qquad c_{1t}=a_{1t}=a_*$$

$$\frac{k-1}{2}+1=\varepsilon_{*t}^{\frac{1-k}{k}} \longrightarrow \frac{k+1}{2}=\varepsilon_{*t}^{\frac{1-k}{k}} \longrightarrow \varepsilon_{*t}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$\varepsilon_{*t}=\dfrac{p_{1*}}{\overline{\overline{p}}_0}$ is critical pressure ratio

$$\boxed{\varepsilon_{*t}=\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}$$

For:

Superheated steam $(k=1{,}3)$ $\varepsilon_{*t}=0{,}546$

Wet steam $(k=1{,}135)$ $\varepsilon_{*t}=0{,}577$

air $(k=1{,}4)$ $\varepsilon_{*t}=0{,}526$

What is special about the **critical parameters?**

1.At critical parameters the flow velocity is equal to the local speed of sound.

2. Analyze the following problem:

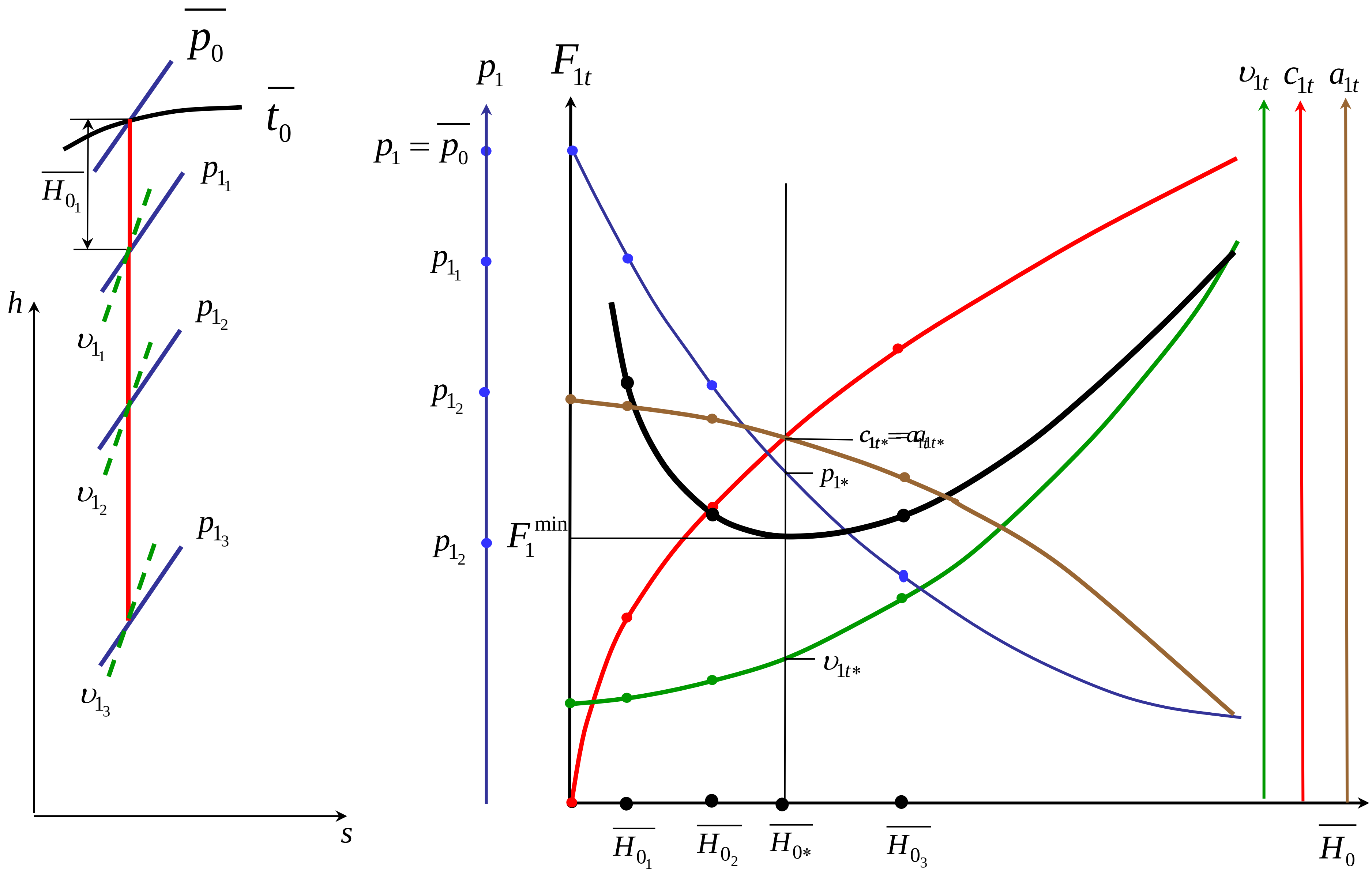
The initial parameters and steam flow through the nozzle are set.

Determine the change in the area of the nozzle outlet section when the outlet pressure is changed

$$F_{1t} = G\frac{\upsilon_{1t}}{c_{1t}}$$

$$c_{1t} = \sqrt{\frac{2k}{k-1}\bar{p}_0\bar{\upsilon}_0\left(1-\varepsilon^{\frac{k-1}{k}}\right)} = \overline{h_0} - h_{1t}$$

$$\varepsilon = \frac{p_1}{p_0}$$



$$\frac{dF}{F} = \frac{d\upsilon}{\upsilon} - \frac{dc}{c}$$