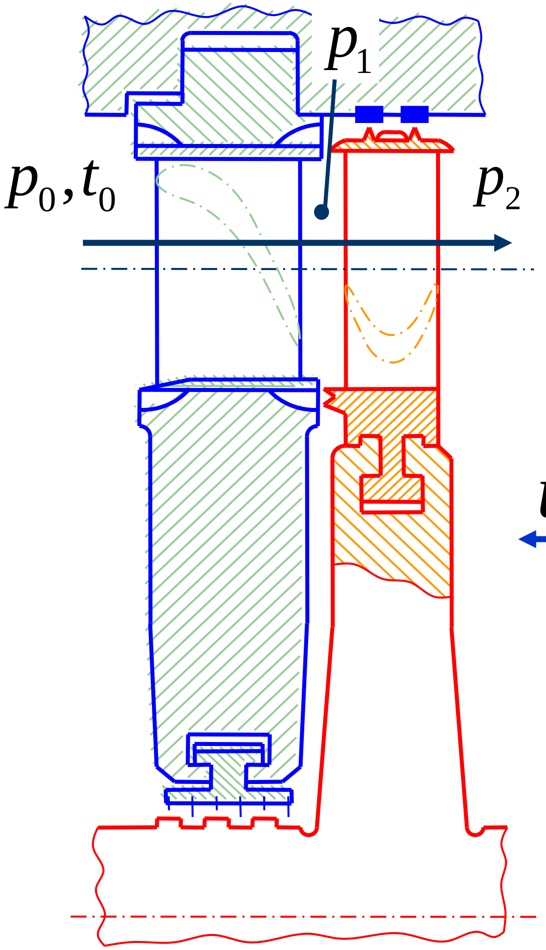
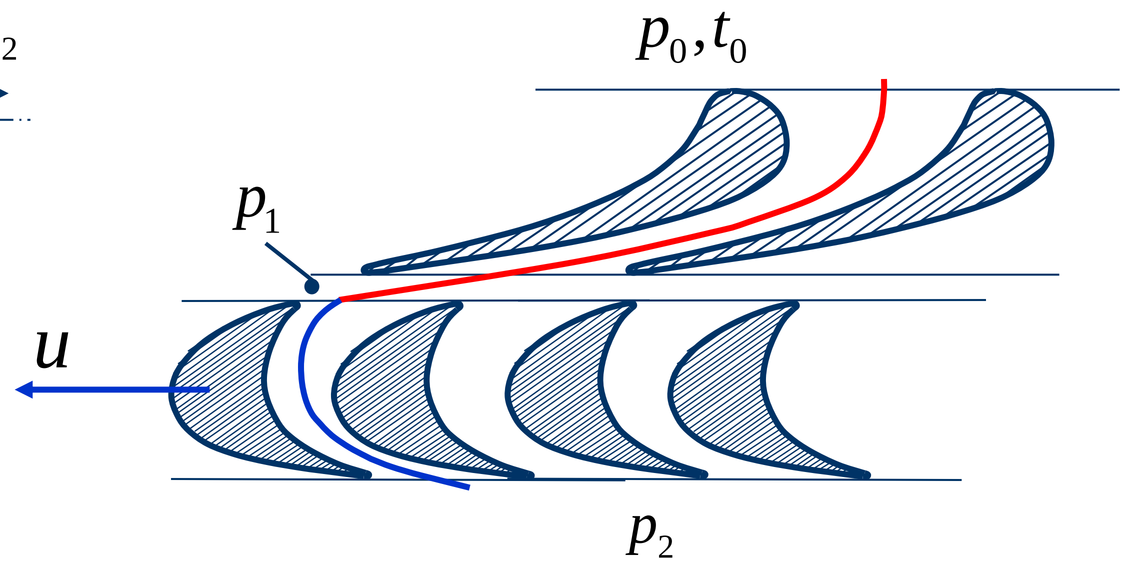
2. GAS FLOW IN THE TURBINE STAGE CHANNELS



$p_0 > p_2$



2.1. Basic equations of compressible fluid motion

System of equations **(!!!)**, ***adequately*** describes gas flow through channels:

- Equation of state
- Continuity equation
- Momentum equation
- Energy-conservation equation

2.1.1. Equation of state

The state of matter can be uniquely determined if we know two independent parameters.

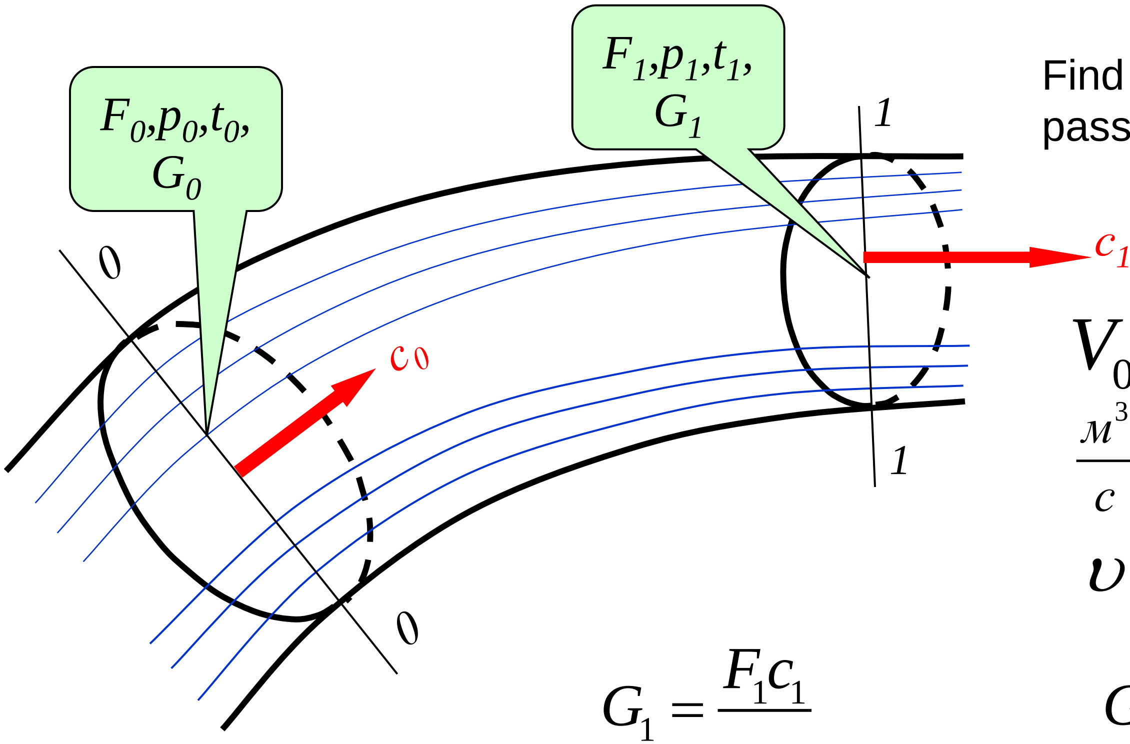
See: Lecture 1

The state changes as a result of the thermodynamic process.

We consider the adiabatic process - without external heat supply.
An ideal adiabatic (isentropic) process is described by the equation:

$$p\upsilon_t^{\,k} = const$$

2.1.2. Continuity equation



Find the volume flow rate [m^3/s], passing through section 0-0:

$$V_0 = F_0 c_0 = G_0 \upsilon_0$$

$$\frac{м^3}{с} = м^2 \frac{м}{с} = \frac{кг}{с} \frac{м^3}{кг}$$

$$\upsilon = f\left(p_0, t_0\right)$$

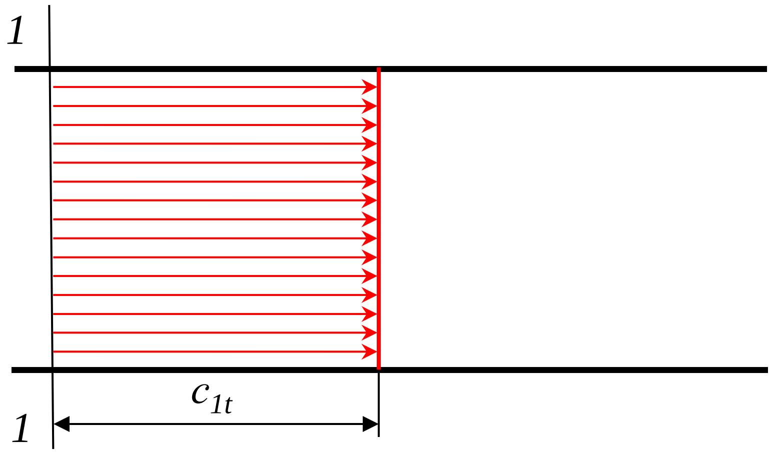
$$G_1 = \frac{F_1 c_1}{\upsilon_1}$$

$$G_0 = \frac{F_0 c_0}{\upsilon_0}$$

$$G_0 = G_1$$

$$\frac{F_0 c_0}{\upsilon_0} = \frac{F_1 c_1}{\upsilon_1}$$

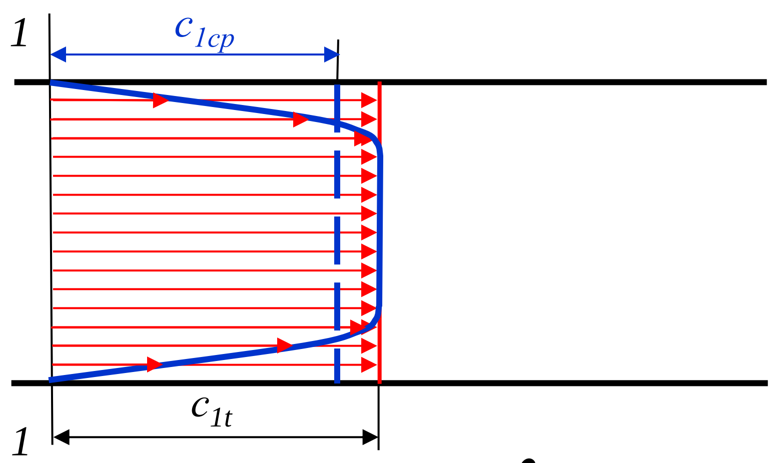
$$G = F \frac{c}{\upsilon} = const$$



a) Ideal flow is the one without friction

$$G_{1t} = \frac{F_1 c_{1t}}{\upsilon_{1t}}$$

$$\upsilon_{1t} = f\left(p_1, t_{1t}\right) = \varphi\left(p_1, s_0\right)$$



b) Real flow is the one where viscous forces (friction) are taken into account

$$G_{1\partial} = \int\limits_{(F_1)} \frac{c_1}{\upsilon_1} dF_1 = F_1 \frac{c_{1\text{cp}}}{\upsilon_{1\text{cp}}}$$

when $c_1 = \gamma\left(F_1\right)$ and $\upsilon_1 = \varphi\left(F_1\right)$

$$c_{1cp} = \frac{\int\limits_{(F_1)} c_1 dF_1}{F_1}$$

$$\upsilon_{1cp} = f\left(p_1, t_1\right)$$

$$G_1 = F_1 \frac{c_1}{\upsilon_1}$$

In the general case:

$$G_i = F_i \frac{c_i}{\upsilon_i}$$

Continuity equation in a differential form

$$G = F\frac{c}{\upsilon} = const$$

Take the logarithm of the continuity equation

$$\ln G = \ln F + \ln c - \ln \upsilon$$

Differentiate this expression (with *G* = *const* in mind)

$$0 = \frac{dF}{F} + \frac{dc}{c} - \frac{d\upsilon}{\upsilon}$$

$$\boxed{\frac{dF}{F} = \frac{d\upsilon}{\upsilon} - \frac{dc}{c}}$$

A) If $\frac{dc}{c} > \frac{d\upsilon}{\upsilon},\quad dF < 0$

The channel is to get narrowed.

B)) If $\frac{dc}{c} < \frac{d\upsilon}{\upsilon},\quad dF > 0$

The channel is to get expanded.

2.1.3. Equations of momentum

A) <u>The notion of momentum and energy in dynamic systems</u>

Newton's second law:

- for uniformly accelerated motion

$$R = ma$$

where ***R*** is force, *N*; ***m*** is mass, *kg*; ***a*** is acceleration, m/s^2.

- for motion with variable acceleration (differential form of the equation):

$$R = m\frac{\partial^2 s}{\partial \tau^2}$$

where ***s*** is distance; $\boldsymbol{\tau}$ is time.

$$\frac{\partial^2 s}{\partial \tau^2} = \frac{d}{d\tau}\left(\frac{ds}{d\tau}\right) = \frac{dc}{d\tau}$$

$$\frac{ds}{d\tau} = c$$ - velocity, *m/s.*

$$R = m\frac{dc}{d\tau} \xrightarrow[\text{variables}]{\text{devide}} Rd\tau = mdc$$

$$\int_0^\tau Rd\tau = \int_{c_0}^{c} mdc$$

$$R\tau = mc - mc_0$$ Equation of momentum

Force impulse is equal to the change in the amount of motion

the amount of motion is changed by the force impulse

$$\frac{\partial^2 s}{\partial \tau^2} = \frac{dc}{d\tau}\frac{ds}{ds} = \frac{ds}{d\tau}\frac{dc}{ds} = c\frac{dc}{ds}$$

$$R = mc\frac{dc}{ds} \longrightarrow Rds = mcdc$$

$$\int_0^s Rds = \int_{c_0}^{c} mcdc$$

$$Rs = \underbrace{\frac{mc^2}{2}} - \underbrace{\frac{mc_0^2}{2}}$$

Force multiplied by the distance is mechanical work

Kinetic energy

B) Applying the momentum equation for liquid (gas) circulation through the channel

I. Phenomenological approach

If the second mass flow ***G*** [kg/s] and flow velocities at the channel input $\boldsymbol{c_1}$ [*m/s*] and output $\boldsymbol{c_2}$ [*m/s*] are known, we can determine the force ***R*** [*N*], which changed the amount of motion.

$$R' = G\left(\overrightarrow{c_2} - \overrightarrow{c_1}\right)$$

As according to Newton's third law, to every action there is always opposed and equal reaction, we can determine the force with which the flow acts on the channel walls.

$$R = -R' = G\left(\overrightarrow{c_1} - \overrightarrow{c_2}\right)$$

II. The problem of determining the actual velocity