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Digital Signal Processing in Pulse Method for Measuring Frequency Response Function of High-Current Shunt

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Abstract – In the paper a digital signal processing method for measuring frequency response function of high-current shunt are described. This is based on a joint digital processing of the input short pulse signal and a corresponding output. In order to measure the frequency response function, a short current pulse is applied to the shunt input; input and output signals of the shunt are recorded in a digital oscilloscope memory; then spectra of these two signals are computed. Amplitude and phase responses are determined by the arithmetic ratio of spectral components of the input and output signals at appropriate frequencies. For this purpose, an algorithm has been proposed and investigated.

I. Introduction

Traditionally, current shunts are employed to measure high currents under significant electromagnetic interference conditions [1, 2]. The measured peak current can reach dozens of kiloampers, and the current signals themselves may have a complicate waveform and wide-band spectra. Investigation of dynamic behavior of the current shunt requires to measure small impedance of the order of dozens or hundreds of microohms [3]. Classical methods for measurement of small impedances employ current calibrators and precise voltmeters. This rather expensive metrological equipment is available to a limited number of specialized laboratories. There are also so called pulse methods characterize high current shunt dynamic behavior. In this approach, the shunt input is affected by a short current pulse [4]. The input and output shunt signals are measured by means of reference current to voltage transducer (for example, by current transformer) and a digital oscilloscope. Then a frequency response functions are calculated using a digital signal processing algorithm. The latter method provides reduced requirements to a source of the input test pulse signal of the shunt. The object of this paper is the description of digital signal processing method for measuring frequency response function of high-current shunt.

II. Determination of frequency response functions

In order to measure the frequency response function of high-current shunt, a short current pulse is applied to the shunt input; input and output signals of the shunt are recorded in a digital oscilloscope memory; then spectra of these two signals are computed. Amplitude and phase responses are determined by the arithmetic ratio of spectral components of the input and output signals at appropriate frequencies. In practice, the test input signal must be wideband and also satisfy the Dirichlet conditions. It means that the test signal must be piecewise continuous, and be characterized with finite number of extrema. These conditions can be hardly satisfied. For example, the signal piecewise continuity means that first kind discontinuities do not exist. However, the discontinuities can take place because of finite sampling time. Reducing a discretization step results in the signal noise pollution that, in turn, increases a number of local extrema.

The recorded input and output shunt signals are presented by N-element sequences readouts of input signal $x(t_k)$ and output signal $y(t_k)$, where t_k is an acquisition time for k-th sequence element ($1 \le k \le N$). Discrete Fourier transform, being applied to the signals, produces spectra of the signals as complex number vectors $X(f_k)$ and $Y(f_k)$ as follows:

$$X(f_k) = \sum_{j=1}^{N} x(t_j)\, f_N^{(j-1)(k-1)},\; Y(f_k) = \sum_{j=1}^{N} y(t_j)\, f_N^{(j-1)(k-1)},\; f_N = \frac{1}{2\pi} e^{-2\pi i/N}, \tag{1}$$

where f_k is a vector of frequencies corresponding to complex vectors $X(f_k)$ and $Y(f_k)$ obtained using the sampling length and Nyquist frequency; i is imaginary unit.

Then, the shunt frequency response $K(f_k)$ can be determined by means of the expression

$$K(f_k) = \frac{Y(f_k)}{X(f_k)}. \tag{2}$$

Then modulus of $K(f_k)$ is the frequency response and argument of $K(f_k)$ is the phase response.

In real physical experiment conditions the pulse signal is noised and characterized with amplitude and phase instabilities. Traditionally, the noise impact is minimized by repeated measurements and ensemble averaging. However, instability of the pulse length and oscilloscope triggering conditions restricts the applicability of this approach. At the same time, the moving average method allows to reduce the noise impact [5]. In order to determine the moving averaging time windows for the input W_X and output W_Y shunt signals, one should apply Fourier series expansion of the sequences $x(t_k)$ and $y(t_k)$. Next stage is determination of the spectral bands ΔF_X and ΔF_Y at level -10 dB. Averaging time windows are inversely proportional to the spectra bands. That is, we have the following expressions:

$$W_X = \frac{1}{\Delta F_X}, \; W_Y = \frac{1}{\Delta F_Y}. \tag{3}$$

Then the smoothed initial signals are calculated by the following formulae:

$$\bar{x}(t_j) = \frac{1}{W_X} \sum_{k=k-\frac{W_X}{2}}^{k+\frac{W_X}{2}} x(t_k), \; \bar{y}(t_j) = \frac{1}{W_Y} \sum_{k=k-\frac{W_Y}{2}}^{k+\frac{W_Y}{2}} y(t_k). \tag{4}$$

The discrete Fourier transform applied to the smoothed signals allows to have their spectral composition:

$$\hat{X}(f_k) = \sum_{j=1}^{N} \bar{x}(t_j) f_N^{(j-1)(k-1)}, \; \hat{Y}(f_k) = \sum_{j=1}^{N} \bar{y}(t_j) f_N^{(j-1)(k-1)}. \tag{5}$$

Under steady sample size N and sampling step of the oscilloscope, the discrete Fourier transform frequency components f_k remain invariable and also are not affected by the offset signal relatively to the sample beginning.

Consequently, the accuracy of the proposed method can be increased by means of uniform ensemble averaging over realizations of the smoothed signal spectral components $\hat{X}(f_k)$ and $\hat{Y}(f_k)$ at fixed frequencies f_k. When carrying out n measurements of the input and output shunt pulse signals, averaged spectral signals components $\bar{X}(f_k)$ and $\bar{Y}(f_k)$ can be found by using the following expressions:

$$\bar{X}(f_k) = \frac{1}{n} \sum_{j=1}^{n} \left| \hat{X}_j(f_k) \right| e^{i \frac{1}{n} \sum_{j=1}^{n} \arg\left(X_j(f_k)\right)}, \; \bar{Y}(f_k) = \frac{1}{n} \sum_{j=1}^{n} \left| \hat{Y}_j(f_k) \right| e^{i \frac{1}{n} \sum_{j=1}^{n} \arg\left(Y_j(f_k)\right)}. \tag{6}$$

Then, from expression (5), the shunt frequency response (1) can be rewritten as follows:

$$\bar{K}(f_k) = \frac{\bar{Y}(f_k)}{\bar{X}(f_k)}. \tag{7}$$

Uncertainty of the method for measuring shunt frequency response can be estimated by calculation of the coherence function γ_{xy} between the spectral densities output signal and the test signal as proposed in [4]:

$$\bar{\gamma}_{xy}^2(f_k) = \frac{\left| \sum_{j=1}^{n} \bar{X}_j^*(f_k) \bar{Y}_j(f_k) \right|^2}{\sum_{j=1}^{n} \left| \bar{X}_j(f_k) \right|^2 \sum_{j=1}^{n} \left| \bar{Y}_j(f_k) \right|^2}, \tag{8}$$

where $\bar{X}_j^*$ is a j-th conjugate complex value of the averaged signal spectral component.

Standard deviation σ for frequency response $K(f_k)$ determination can be obtained from expression (8) as follows:

$$\sigma\left(\bar{K}(f_k)\right) = \frac{1}{\sqrt{2n}} \frac{\sqrt{1 - \bar{\gamma}_{xy}^2(f_k)}}{\left| \bar{\gamma}_{xy}(f_k) \right|}. \tag{9}$$

III. Digital signal processing algorithm

The above reasoning allows to propose an algorithm of digital signal processing in the pulse method for measuring frequency and phase response functions of high-current shunt, first described in [6]. The algorithm is implemented in MATLAB [7].

In order to minimize an available memory, the algorithm uses consecutive loading of readout files. We use Goertzel algorithm [8] to determine the signal spectral components in the bandwidths ΔF_X and ΔF_Y at level -10 dB. This also allows you to save computer memory and improves the computational efficiency of the proposed algorithm. Spectral components were determined in the previously defined frequency range.

Algorithm: Determination of frequency and phase response functions of high-current shunts

1: **Input:**

n number of pulses in ensemble;

t_k time counts;

$x(t_k)$ sequence of input signals;

$y(t_k)$ sequence of output signals;

Goertzel algorithm parameters:

α alpha-parameter;

W_N rotary factor;

vx vector of length N to store interim results for input signal;

vy vector of length N to store interim results for output signal;

2: **for** j = 1, n **do**

3: Loading readouts of input <t_k, $x(t_k)$> and output <t_k, $y(t_k)$> signals;

4: Determination of the sampling length for input and output signals N;

5: Dividing of loaded samples into three sequences t_k, $x(t_k)$ and $y(t_k)$;

6: Determination of quantization step t_s as the average difference between adjacent elements in a sequence t_k;

7: Calculation of fast Fourier transform for sequences x and y and determination of signal spectral components X and Y;

8: Determination of frequencies of signal spectral components f_k $f_k \leftarrow k/2t_S$ for k = 0, …, N/2 + 1;

9: Determination of signal bandwidths and numbers kx and ky of spectral components corresponding to upper bandwidth limits;

10: Determination of averaging windows W_X and W_Y using expression (2) ;

11: Temporal signals averaging using the expression (4) ;

12: Applying Goertzel algorithm to averaged sequences of readouts $\bar{x}$ and $\bar{y}$ and determination of spectral components $\widehat{X}$ and $\widehat{Y}$:

for k = 0, max (kx, ky) **do**

13: $\alpha \leftarrow \cos\left(\frac{2\pi k}{N}\right)$;

14: $W_N \leftarrow \cos\left(\frac{2\pi k}{N}\right) + i\sin\left(\frac{2\pi k}{N}\right)$;

15: Calculation of initial values of vectors vx and vy:

$vx(1) \leftarrow \bar{x}(1)$, $vx(2) \leftarrow \bar{x}(1)(1+\alpha)$;

$vy(1) \leftarrow \bar{y}(1)$, $vy(2) \leftarrow \bar{y}(1)(1+\alpha)$;

16: Iterative calculation of vectors elements for vx and vy:

for j = 3, N **do**

17: $vx(j) \leftarrow \bar{x}(j) + \alpha \cdot vx(j-1) - vx(j-2)$;

18: $vy(j) \leftarrow \bar{y}(j) + \alpha \cdot vy(j-1) - vy(j-2)$;

19: **end for**

20: Calculation of signal spectral components:

$\widehat{X}(f_k) \leftarrow vx(N) \cdot reW_N - vx(N-1) + i \cdot vx(N) \cdot imW_N$;

$\widehat{Y}(f_k) \leftarrow vy(N) \cdot reW_N - vy(N-1) + i \cdot vy(N) \cdot imW_N$;

21: **end for**

22: Averaging signal spectral components using expression (5):

$$\bar{X}(f_k) \leftarrow \left(\left|\bar{X}(f_k)\right| + \frac{1}{n}\left|\widehat{X}_j(f_k)\right|\right) e^{i\left[\arg\left(\bar{X}(f_k)\right) + \frac{1}{n}\arg\left(\widehat{X}_j(f_k)\right)\right]};$$

$$\bar{Y}(f_k) \leftarrow \left(\left|\bar{Y}(f_k)\right| + \frac{1}{n}\left|\widehat{Y}_j(f_k)\right|\right) e^{i\left[\arg\left(\bar{Y}(f_k)\right) + \frac{1}{n}\arg\left(\widehat{Y}_j(f_k)\right)\right]};$$

23: **end for**

24: Determination of frequency response $|K(f_k)|$ and phase response $\arg(K(f_k))$ using expression (7) ;

25: Determination of coherence function $\gamma_{xy}(f_k)$ and standard deviation $\sigma(f_k)$ using expressions (8) and (9) ;

26: Saving and indication of calculation outcomes.

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IV. Experimental studies of high-current shunt frequency and phase responses

The proposed method was applied to experimentally determine frequency response for a coaxial high-current shunt that was designed for measurement of pulse current up to 20 кA [2]. The resistance of the shunt is about 170 μOhm.
For this experiment, a simple pulse current source was designed. Pulse current source yields a current pulse up to 20 kA with duration 150 μs. The experimental setup is shown in Figure 1.

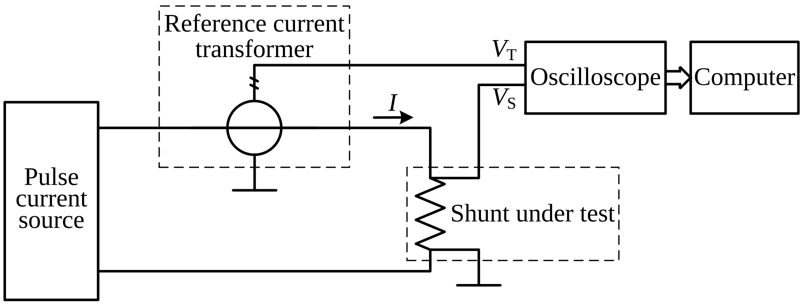

Figure 1. The experimental setup diagram to determine the shunt frequency response

As a reference current transformer the unit 13W0100 produced by Lilco Ltd has been used [9]. Its specifications are as follows: bandwidth 0.3 Hz – 25 MHz, accuracy 0,5 % and peak current 5 kA. Oscilloscope LeCroy WaveSurfer 62S has been also used.
Figure 2 shows typical waveforms obtained from outputs of the reference current transformer (curve 1) and shunt under test (curve 2); for the sake of clarity the signals in Figure 2 are reduced to the same scale.

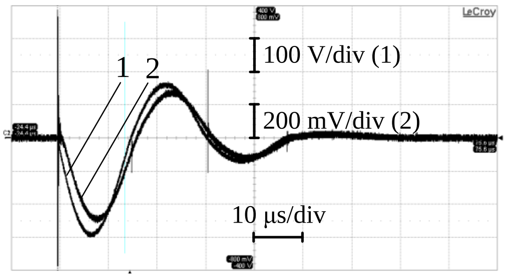

Figure 2. Output waveforms of reference current transformer (curve 1) and shunt under test (curve 2)

It can be seen from Figure 2 that amplitude of the pulse current via the shunt approaches to 3 kA.
Further, according to the proposed algorithm averaged spectra of signals from outputs of the reference current transformer (curve 1) and shunt under test (curve 2) were determined. These spectra are shown in Figure 3.

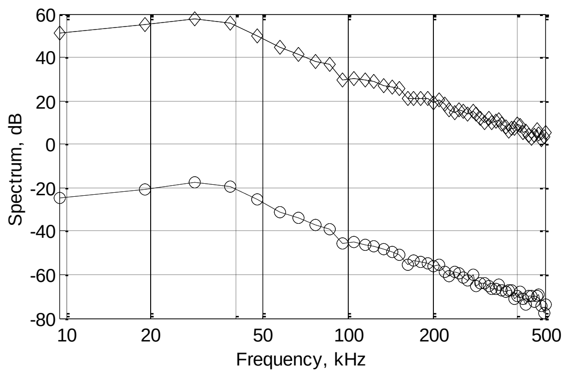

Figure 3. Spectra of the signals of reference current transformer (curve 1) and shunt under test (curve 2)

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It is clear from Figure 3 that after frequency approx. equal to 200 kHz spectral noises caused by quantization and inherent circuit properties take place. This confines the possibility to determine the upper frequency limit of broadband shunts.
Figure 4 shows final outcomes of application of the proposed algorithm to the experimental data; it can be seen that in the bandwidth up to approx. 200 kHz the shunt under test has a linear gain about -75.5 dB.

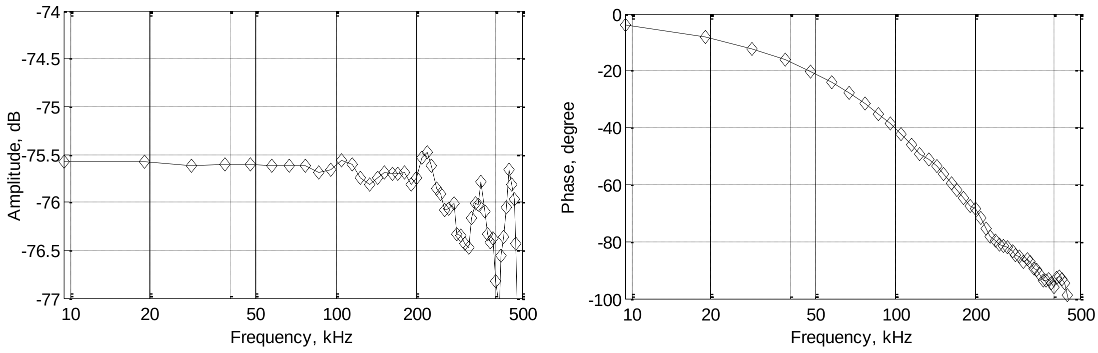

Figure 4. Frequency (a) and phase (b) response functions of the high-current shunt

V. Conclusions

Pulse method for analysis of high-current shunts dynamic behavior has been proposed and experimentally investigated. The method includes ensemble averaging of spectral components of the pulse signals that results in increase of shunt frequency response measurement accuracy. Future investigations on the topic aim at restoring current pulse waveform due to distortions induced by noise and quantization errors [5] in the framework of various ways for restoring signals in time and frequency domains [10].

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