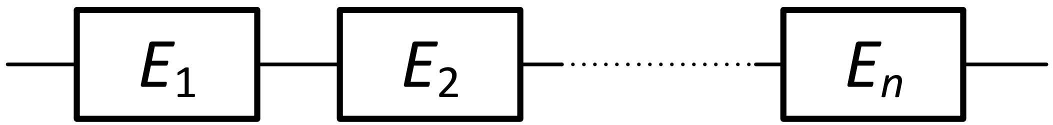
THE RELIABILITY OF SERIES SYSTEMS

The Reliability of Series Systems

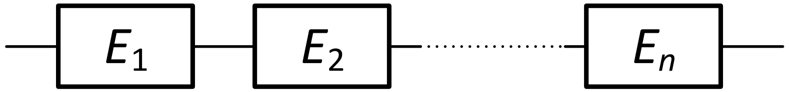
Components of a system are said to be connected in series if each one of them must be operational for the system to be operational,

i.e. *the failure of any one of its component causes the system to fail*.

The *reliability block diagram* (*RBD*) of a series system with *n* elements is as follows:



The Reliability of Series Systems



For calculation purposes it is in general tacitly assumed that for series systems, each element operates and fails independently from each other element.

Let $\{e_i\}$, $i = 1, 2,\ldots,n$ be the event

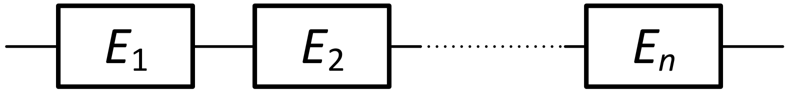
$$\{e_i\} = \{E_i \; new \; at \; t = 0 \; \cap \; E_i \; up \; in \; [0, t]\}$$

Assuming E_i is new at $t = 0$, the probability of $\{e_i\}$ is

$$Pr\{e_i\} = Pr\{E_i \; new \; at \; t = 0\} \cdot Pr\{E_i \; up \; in \; [0, t]\} = 1 \cdot R_i(t)$$

with $R_i(t)$ as reliability function of E_i.

The Reliability of Series Systems



The system does not fail in the interval [0, *t*] if and only if all elements $E_1, E_2, \ldots, E_n$ do not fail in that interval, thus

$$R_S(t) = Pr\{e_1 \cap e_2 \cap \cdots \cap e_n\}.$$

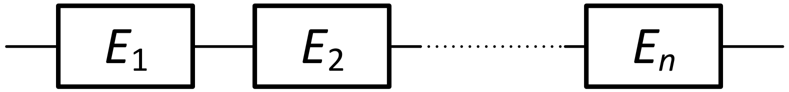
Here and in the following, *S* stands for system.

Due to the assumed independence among the elements $E_1, E_2, \ldots, E_n$, it follows for the reliability function $R_S(t)$:

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

Product Law of Reliabilities

The Reliability of Series Systems



Since the reliability of each component is a positive number less than one,

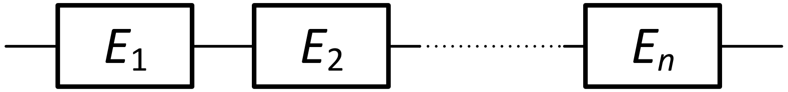
$$R_i(t) \in (0,1), t > 0$$

the product value is less then each term.

It follows that the reliability of a series system is less than the reliability of each constituent component and, hence, is less than the reliability of its least reliable component:

$$R_S(t) < \min_{i=1..n} R_i(t).$$

The Reliability of Series Systems



As we know

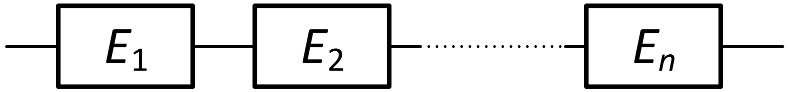
$$R(t) = e^{-\int_0^t h(\tau)d\tau}.$$

Given that, let $h_i(t)$ be the hazard rate of element E_i.

Hence, for series systems

$$R_S(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\int_0^t h_i(\tau)d\tau} = e^{-\int_0^t [\sum_{i=1}^{n} h_i(\tau)]d\tau}.$$

The Reliability of Series Systems



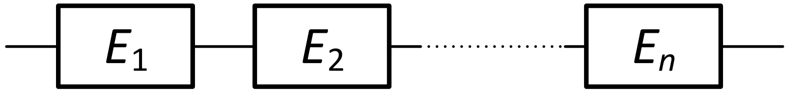
By taking

$$h_S(t) = \sum_{i=1}^{n} h_i(t)$$

we can infer:

The hazard rate of a series system, consisting of independent elements, is the sum of the hazard rates of its elements.

The Reliability of Series Systems



If $h_i(t) = \lambda_i = const.$, i.e. failure times of all elements of series system are exponentially distributed r.v., we have

$$h_S(t) = \sum_{i=1}^{n} \lambda_i = \lambda_S = const.$$

Thus, time to failure of the series systems itself is also an exponentially distributed r.v.

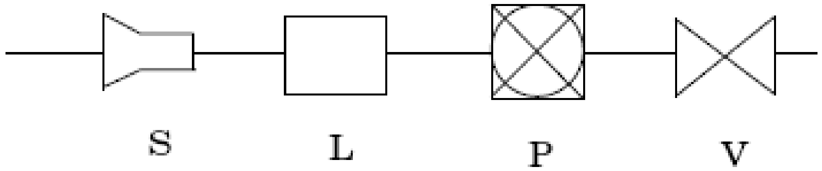
What other distributions exhibit the same property?

Rayleigh? Weibull?

The Reliability of Series Systems

Ex.: Consider a system used to maintain the fluid pressure in a tank at a constant value.

The system is composed of four components (or blocks): a pressure sensor (S), a control logic (L), a motor-pump group (P) and a valve (V).



The components are connected in series since the correct operation of each one of them is needed to guarantee the correct functioning of the system.

Often, the information that is possible to retrieve from a data bank is in the form of a (constant) hazard rate, thus implicitly assuming an exponential failure time distribution.

Assume that consulting a data bank the following values have been obtained (expressed in failure per hour = f/h):

S = Pressure sensor	$\lambda_S = 2 \cdot 10^{-6}$	f / h
L = Control logic	$\lambda_L = 5 \cdot 10^{-6}$	f / h
P = Motor-pump group	$\lambda_P = 2 \cdot 10^{-5}$	f / h
V = Valve	$\lambda_V = 1 \cdot 10^{-5}$	f / h

The Reliability of Series Systems

The reliability of each component after 1 year (t=8760 h) of continuous operation is given by

$$R_S\ (t = 8760\ h) = e^{-\lambda_S t} = 0.983$$
$$R_L\ (t = 8760\ h) = e^{-\lambda_L t} = 0.957$$
$$R_P\ (t = 8760\ h) = e^{-\lambda_P t} = 0.839$$
$$R_V\ (t = 8760\ h) = e^{-\lambda_V t} = 0.916$$

and for the series system

$$R_S(t = 8760\ h) = 0.983 \cdot 0.957 \cdot 0.839 \cdot 0.916 = 0.723$$

The Reliability of Series Systems

Alternatively, this result can be obtained by summing the hazard rates of the constituent components:

$$\lambda_S = \lambda_R + \lambda_L + \lambda_P + \lambda_V = 3.7 \cdot 10^{-5}\, f/h$$

$$R_S(\text{t} = 8760\text{ h}) = e^{-\lambda_S t} = 0.723$$

ACTIVE REDUNDANCY

Active Redundancy

When the reliability of a series system does not reach the design goal, it becomes necessary to act at the structure level and to resort to redundant configurations.

A system configuration is said to be redundant, when the occurrence of a component failure does not necessarily causes a system failure.

Various redundant architectures have been studied and applied in practice, and they will be illustrated in the following sections.

Active Redundancy

From the operating point of view, we can distinguish between:

- *Active Redundancy (parallel, hot)*: Redundant elements are subjected from the beginning to the same load as the operating elements;
- *Warm Redundancy (lightly loaded)*: Redundant elements are subjected to a lower load until they become operating;
- *Standby Redundancy (cold, unloaded)*: Redundant elements are subjected to no load until they become operating, and the hazard rate in reserve (standby) state is assumed to be zero.

Active Redundancy

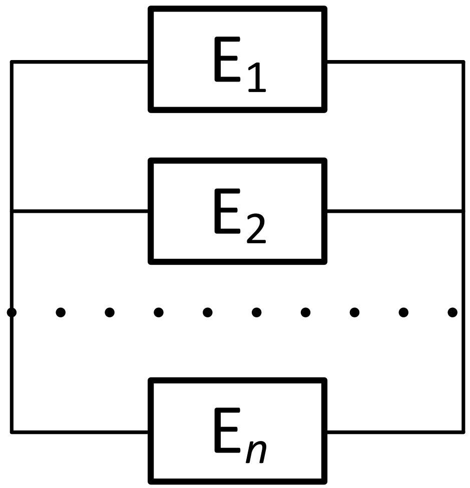
By default, we assume the components of the system with redundancy are statistically independent, i.e. the failure of one of them doesn’t affect the reliability of the rest.

However, if this is not the case, i.e. the component(s) that are still operating assume the failed unit's portion of the load, such type of redundancy is called *load sharing*.

The reliability of load sharing configurations is much harder to compute.

Active Redundancy

A *parallel model* consists of n (*often statistically identical*) elements in active redundancy, of which k ($1 \leq k < n$) are necessary to perform the required function and the remaining $(n - k)$ are in reserve.



Such a structure is designated as a *k-out-of-n* (or *k-out-of-n:G*) redundancy.

1-out-of-n redundancy is also called hot redundancy.

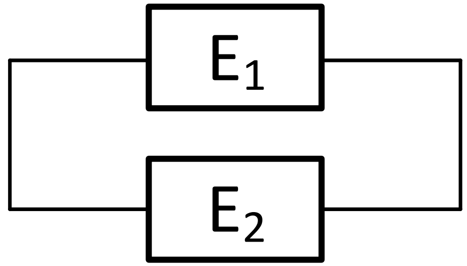
Active Redundancy

Let's consider at first the case of an active (hot) *1-out-of-2* redundancy.

The required function is fulfilled if at least one of the elements E_1 or E_2 works without failure in the interval (0, t]. In other words, the system fails if both elements failed in the interval (0, t].

Let $\{\overline{e_i}\}, i = 1,2$ be the event of ith element failing in (0, t], then $F_i(t) = Pr\{\overline{e_i}\}$ is the failure probability of E_i.

Active Redundancy



Then, if $F_S(t)$ denotes the failure probability of the entire system, it follows:

$$F_S(t) = Pr\{\overline{e_1} \cap \overline{e_2}\} = Pr\{\overline{e_1}\} \cdot Pr\{\overline{e_2}\} =$$
$$= F_1(t) \cdot F_2(t)$$

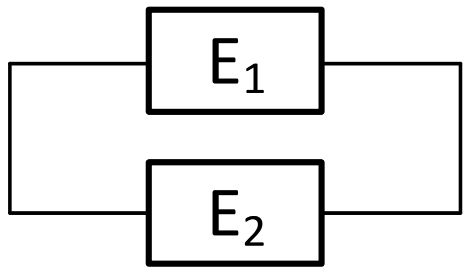
Generalizing for 1-out-of-*n* system, we obtain

$$F_S(t) = \prod_{i=1}^{n} F_i(t)$$

Product Law of Unreliabilities

Active Redundancy

Substituting $F_*(t)$ with $1 - R_*(t)$, we obtain



$$R_S(t) = R_1(t) + R_2(t) - R_1(t)R_2(t)$$

For *1-out-of-n* system, we get

$$R_S(t) = 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right)$$

Active Redundancy

As it is often that all elements in parallel system are statistically identical, i.e. $R_1(t) = R_2(t) = \cdots = R_n(t) = R(t)$, we can obtain for this particular case

$$R_S(t) = 2R(t) - R(t)^2$$ for *1-out-of-2* system

and

$$R_S(t) = 1 - \left(1 - R(t)\right)^n$$ for *1-out-of-n* system.

Thus, for parallel systems of independent components, we have a product law of "unreliabilities" analogous to the product law of reliabilities for series systems.

It follows that the reliability of a parallel system is greater than the reliability of each constituent component ...

... and, hence, a parallel system is more reliable than the most reliable of its components.

Active Redundancy

Let's assume the parallel system consists of 2 identical components, and

$$R_1(t) = R_2(t) = R(t) = e^{-\lambda t},$$

i.e. the failure time of each component is an exponentially distributed r.v.

Moreover, the hazard rate of each component is constant.

The reliability of the system then is

$$R_S(t) = 2R(t) - R(t)^2 = 2e^{-\lambda t} - e^{-2\lambda t}$$

Obviously, we can't find such $\lambda_S > 0$ so that

$$e^{-\lambda_S t} = 2e^{-\lambda t} - e^{-2\lambda t}$$

This means that the hazard rate of the entire system is not constant, and its failure time is *not an exponentially distributed r.v.*

To demonstrate this, let's find the expression for the system's hazard rate.

Active Redundancy

We know that

$$h(t) = -\frac{R'(t)}{R(t)}$$

Given that

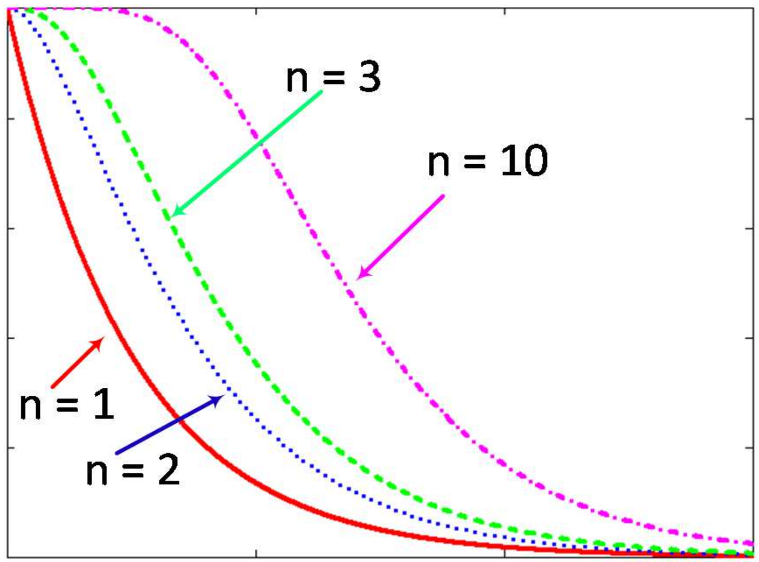
$$R_S(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

we obtain

$$h_S(t) = \frac{2\lambda e^{-\lambda t} - 2\lambda e^{-2\lambda t}}{2e^{-\lambda t} - e^{-2\lambda t}} = \frac{2\lambda e^{-\lambda t}\left(1 - e^{-\lambda t}\right)}{e^{-\lambda t}(2 - e^{-\lambda t})} =$$

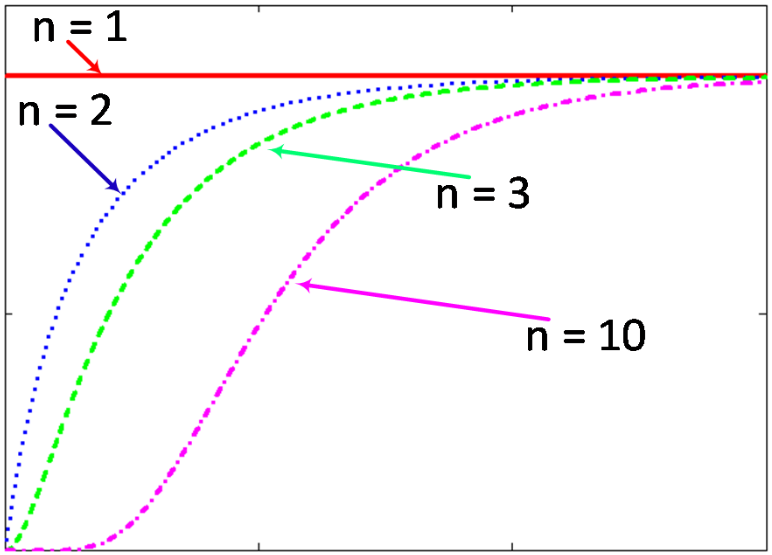
$$= \frac{2\lambda\left(1 - e^{-\lambda t}\right)}{2 - e^{-\lambda t}}.$$

Active Redundancy



Reliability

hazard rate



Active Redundancy

As previously mentioned, a hot redundancy system is the special case of *k-out-of-n* redundancy systems (with *k=1*).

Mind that for *k-out-of-n* systems correct operation of *k* components is sufficient for the system to be operable, hence the system can tolerate *(n-k)* failures of its components.

By default we assume that all items in *k-out-of-n* systems are statistically identical.

Consider a *k-out-of-n* system ($2 \leq k < n$) with reliability of each item equals *R*.

The system is operable if it has *n, n-1, n-2, ..., k+1, k* operable components.

The probability that exactly *n-1* components are operable is given by $n \cdot R^{n-1} \cdot (1-R)^1$. Here the multiplier *n* is stands for the number of all possible combinations of operating items, i.e.

$$\binom{n}{n-1} = \frac{n!}{(n-1)!\,(n-n+1)!} = n.$$

The probability that exactly *n-2* components are operable is given by $\binom{n}{n-2} \cdot R^{n-2} \cdot (1-R)^2$.

...

Finally, the probability that exactly *k* components are operable is given by $\binom{n}{k} \cdot R^k \cdot (1-R)^{n-k}$.

Since we have listed all possible conditions for the system to be operable, the reliability of the system is given by

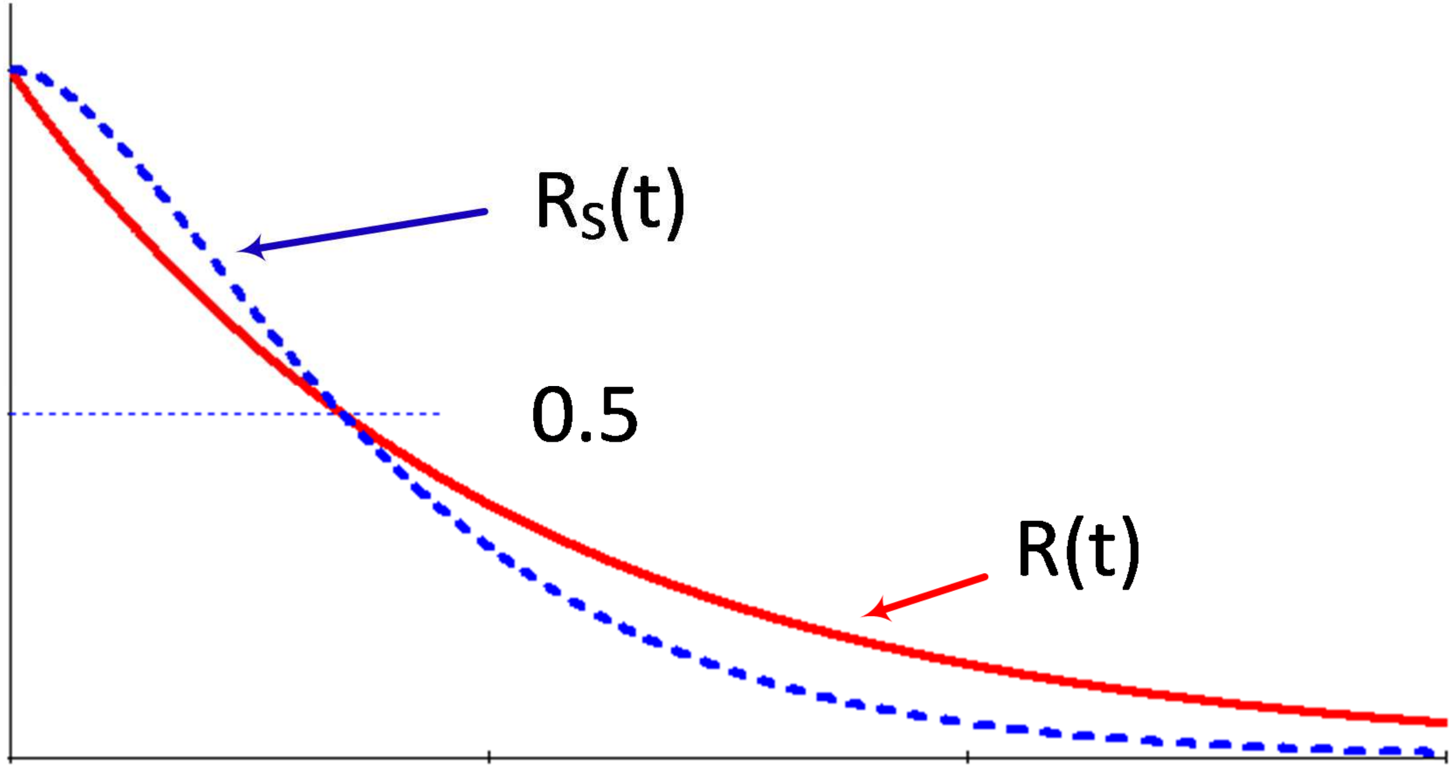
$$R_S = \sum_{i=k}^{n} \binom{n}{i} \cdot R^i \cdot (1-R)^{n-i}$$

Ex.: Consider a *2-out-of-3* system with reliability of each component given as $R(t) = e^{-\lambda t}$. Find the reliability function, MTTF and hazard rate function for the entire system.

First, the reliability of the system is obtained as

$$R_S(t) = \sum_{i=2}^{3} \binom{3}{i} \cdot R(t)^i \cdot \big(1 - R(t)\big)^{3-i} =$$
$$= 3R(t)^2\big(1 - R(t)\big) + R(t)^3 =$$
$$= 3R(t)^2 - 2R(t)^3 =$$
$$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

Active Redundancy



The system is more reliable if the reliability of each item is greater than 0,5.

Second, the MTTF is given by

$$MTTF = \int_0^\infty R_S(t)dt = \int_0^\infty \left(3e^{-2\lambda t} - 2e^{-3\lambda t}\right)dt =$$

$$= \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda}.$$

Note that MTTF of the *2-out-of-3* system is less than the MTTF of a single item!

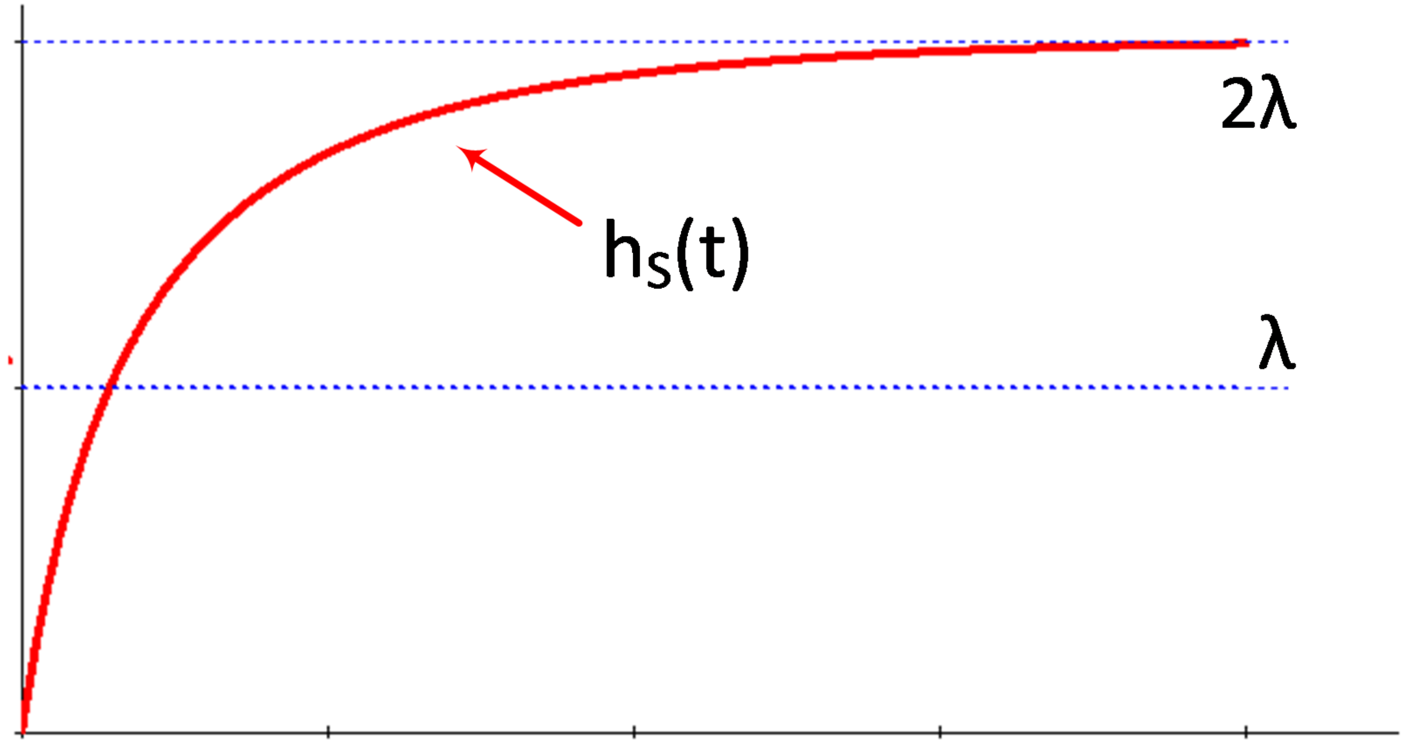
Finally, the hazard rate is given by

$$h_S(t) = -\frac{R_S{}'(t)}{R_S(t)} = \frac{6\lambda e^{-2\lambda t} - 6\lambda e^{-3\lambda t}}{3e^{-2\lambda t} - 2e^{-3\lambda t}} =$$

$$= \frac{6\lambda e^{-2\lambda t}\left(1 - e^{-\lambda t}\right)}{e^{-2\lambda t}\left(3 - 2e^{-\lambda t}\right)} =$$

$$= 6\lambda \cdot \frac{1 - e^{-\lambda t}}{3 - 2e^{-\lambda t}}.$$

Active Redundancy

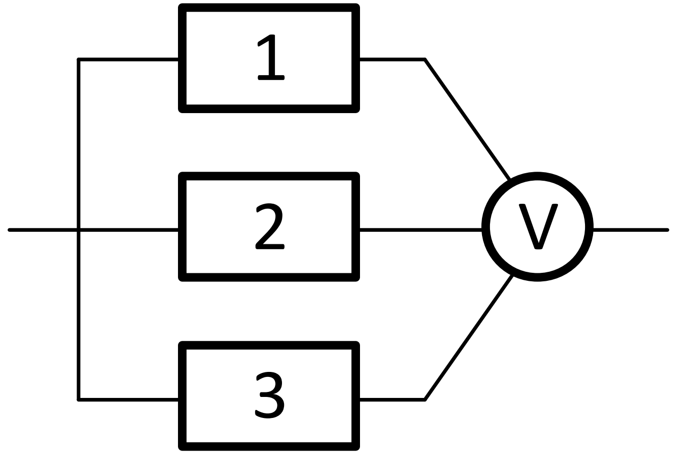


The *2-out-of-3* system is the most common case of *k-out-of-n* redundancy, since all other variants are more costly.

Often, *2-out-of-3* system is called TMR system, where TMR stands for triple modular redundancy.

Also, there is a subclass of *k-out-of-n* systems called majority voting systems. Here *n* is always odd number, and $k = \frac{n+1}{2}$.

The RBD for 2-out-of-3 system is as follows:



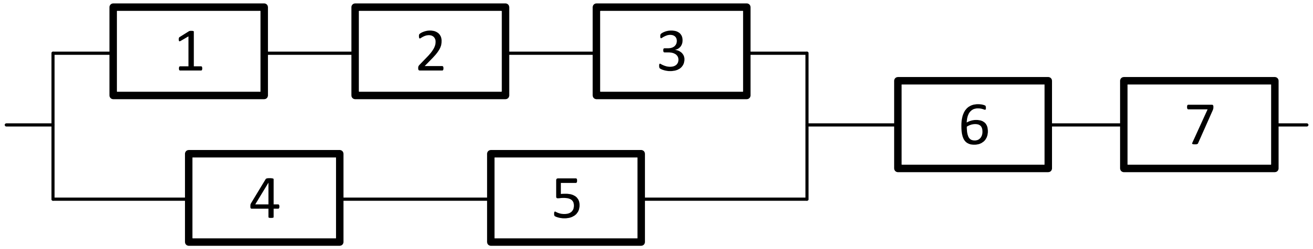
Often, the element *V* (*voter*) is assumed to be perfectly reliable, i.e. $R_V(t)=1$.

Active Redundancy

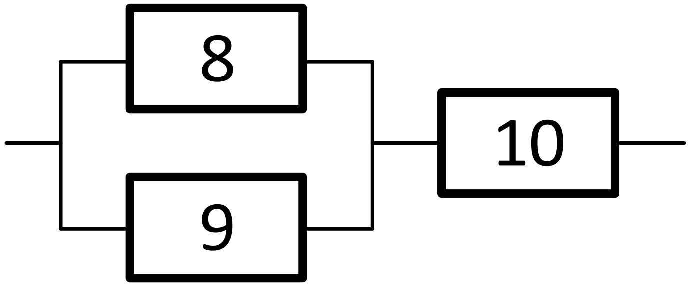
The formulae for the reliability computation of series and parallel systems can be used in combination to compute the reliability of a system having both series and parallel parts (*series-parallel systems*).

The computational procedure consists of a progressive reduction of the system complexity by substituting blocks of components in series/parallel with a single equivalent block.

Ex.: Consider a system with the following RBD:



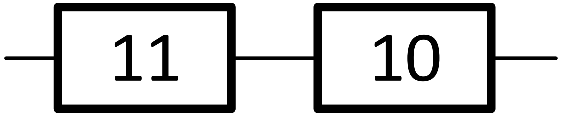
1st step: replace all series elements with equivalent ones:



$$R_8 = R_1 R_2 R_3$$
$$R_9 = R_4 R_5$$
$$R_{10} = R_6 R_7$$

Active Redundancy

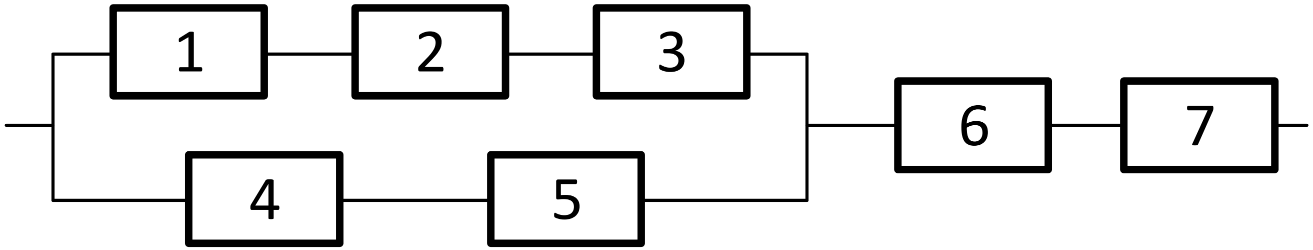
2^{nd} step: replace parallel elements 8 and 9 with equivalent one:



$R_{11} = R_8 + R_9 - R_8 R_9$

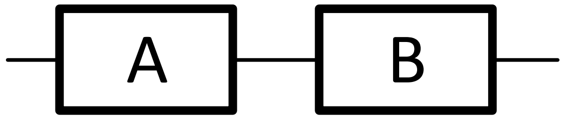
Finally, reliability of the entire system is given by

$$R_S = R_{10} R_{11} = (R_1 R_2 R_3 + R_4 R_5 - R_1 R_2 R_3 R_4 R_5) \cdot R_6 R_7$$



When discussing series-parallel systems, we should address another topic, namely, *system redundancy vs. component redundancy*.

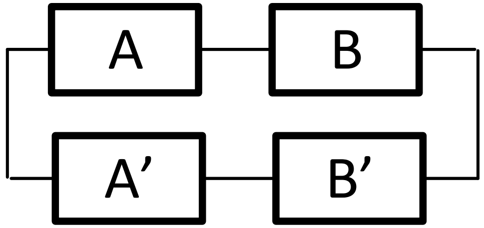
Consider a system composed of two series components A and B:



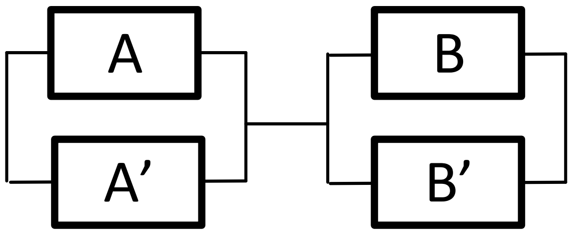
Let's denote its reliability as $R_1 = R_A R_B$.

Active Redundancy

If we decide to improve the reliability of the system by applying redundancy using one single replica for each component, two solutions are possible. Either we replicate the complete line (system redundancy):

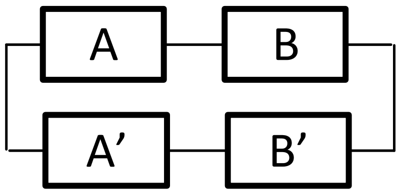


or we replicate each component individually (component redundancy):



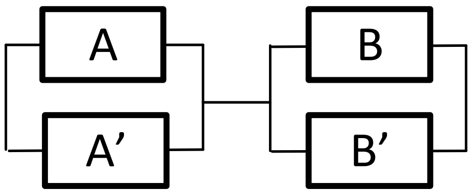
Active Redundancy

Reliability of the system with system redundancy is



$$R_2 = 2R_AR_B-(R_AR_B)^2 = R_AR_B(2 - R_AR_B)$$

Reliability of the system with component redundancy:



$$R_3 = (2R_A - R_A{}^2)(2R_B - R_B{}^2) =$$

$$= R_AR_B\,[4 – 2(R_A + R_B) + R_AR_B]$$

Active Redundancy

$R_1 = R_A R_B$

$R_2 = R_A R_B (2 - R_A R_B)$

$R_3 = R_A R_B [4 – 2(R_A + R_B) + R_A R_B]$

It is easy to see that $R_2 > R_1$ and $R_3 > R_1$; both redundant configurations are more reliable than the original system.

Next, we compare between the two redundant configurations

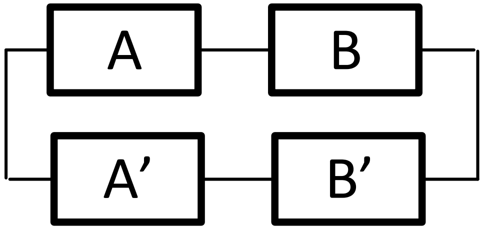
Active Redundancy

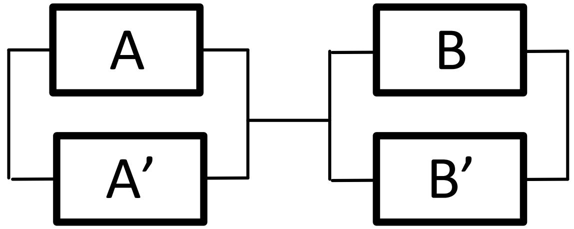
$$\frac{R_3}{R_2} = \frac{4 - 2(R_A + R_B) + R_A R_B}{2 - R_A R_B} = 1 + \frac{2(1 - R_A)(1 - R_B)}{2 - R_A R_B} > 1$$

It should, however, be noted that configuration 3 is more complex than configuration 2, since each type A component need to be possibly connected with any type B component.

This higher complexity requires an additional control logic (not considered in the formulae) that may reduce the benefits calculated from the equation.

The reason why configuration 3 is more reliable than configuration 2 can be also explained on a qualitative basis, noticing that there are failure combinations of basic blocks that cause failure of configuration 2, but not of configuration 3.





STANDBY REDUNDANCY

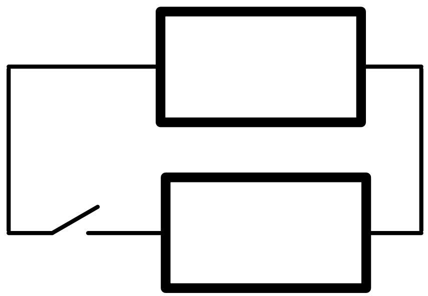
(cold, warm)

Standby Redundancy

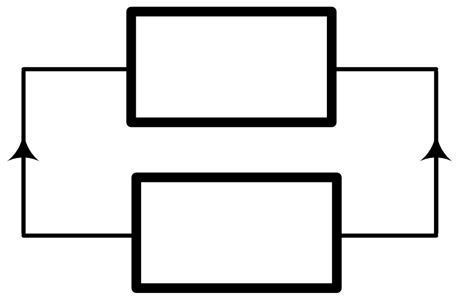
From now on, we will denote both cold and warm redundancy as a *standby redundancy*.

Also, we will consider statistically identical items with constant hazard rates, unless otherwise stated.

The RBDs for these configurations are as follows:



Cold Redundancy



Warm Redundancy

The most general way to compute the reliability of standby system (of 2 components) is to evaluate the integral:

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

where:

R_1 is the reliability of the active component;

f_1 is the pdf of the active component;

$R_{2;SB}$ is the reliability of the standby component when in standby mode (*quiescent* reliability);

$R_{2;A}$ is the reliability of the standby component when in active mode;

t_e is the equivalent operating time for the standby unit, if it had been operating at an active mode, such that:

$$R_{2;SB}(x) = R_{2;A}(t_e)$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Note that the formula above may involve different distributions of component failure time.

Furthermore, you can compute the reliability of active redundancy with this formula as well, though such an approach wouldn't be the most convenient.

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

To provide an example, we will consider the system of two statistically identical components with constant hazard rate.

$$R_1(t) = R_{2;A}(t) = e^{-\lambda t} \qquad f_1(t) = \lambda e^{-\lambda t}$$

The quiescent reliabilities are:

$R_{2;SB}(t) = 1$ for cold redundancy;

$R_{2;SB}(t) = e^{-\lambda_{SB} t}$ for warm redundancy ($\lambda_{SB} < \lambda$);

$R_{2;SB}(t) = R_{2;A}(t) = e^{-\lambda t}$ for hot (active) redundancy.

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

The equivalent operating time t_e is obtained from the equation $R_{2;SB}(x) = R_{2;A}(t_e)$, so for the case of cold redundancy:

$$1 = e^{-\lambda t_e} \;\Rightarrow\; t_e = 0.$$

For warm redundancy:

$$e^{-\lambda_{SB} x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = \frac{\lambda_{SB}}{\lambda} x.$$

For hot redundancy:

$$e^{-\lambda x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = x.$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

The equivalent operating time t_e is obtained from the equation $R_{2;SB}(x) = R_{2;A}(t_e)$, so for the case of cold redundancy:

$$1 = e^{-\lambda t_e} \;\Rightarrow\; t_e = 0.$$

For warm redundancy:

$$e^{-\lambda_{SB} x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = \frac{\lambda_{SB}}{\lambda} x.$$

For hot redundancy:

$$e^{-\lambda x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = x.$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Let’s start with hot redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda x} \cdot \frac{e^{-\lambda(x+t-x)}}{e^{-\lambda x}} dx =$$

$$= e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda x} \cdot \frac{e^{-\lambda t}}{e^{-\lambda x}} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{-\lambda x} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \left[-\frac{e^{-\lambda t}}{\lambda} + \frac{1}{\lambda} \right] = 2e^{-\lambda t} - e^{-2\lambda t}$$

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Next, for cold redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot 1 \cdot \frac{e^{-\lambda(0+t-x)}}{1} dx =$$

$$= e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} e^{-\lambda t} e^{\lambda x} \, dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t}[t - 0] = e^{-\lambda t}(1 + \lambda t)$$

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

And finally, for warm redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda_{SB} x} \cdot \frac{e^{-\lambda\left(\frac{\lambda_{SB}}{\lambda}x + t - x\right)}}{e^{-\lambda\left(\frac{\lambda_{SB}}{\lambda}x\right)}} dx =$$

$$= e^{-\lambda t} + \lambda \int_0^t e^{-\lambda x} e^{-\lambda_{SB} x} \frac{e^{-\lambda_{SB} x} e^{-\lambda t} e^{\lambda x}}{e^{-\lambda_{SB} x}} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{-\lambda_{SB} x} dx =$$

$$= e^{-\lambda t} \left[1 + \frac{\lambda}{\lambda_{SB}} \left(1 - e^{-\lambda_{SB} t}\right)\right]$$

These results can also be obtained via state-space method, which will be addressed later.

For now, we can have the formulae for certain special cases of *cold* and *warm* redundant configurations, namely, systems of *m+1* statistically identical components with constant hazard rate.

Standby Redundancy

For cold standby redundancy ($\lambda_{SB} = 0$):

$$R(t) = e^{-\lambda t} \sum_{i=0}^{m} \frac{(\lambda t)^i}{i!} \qquad MTTF = \frac{m+1}{\lambda}$$

For warm standby redundancy ($\lambda_{SB} < \lambda$):

$$R(t) = e^{-\lambda t} \left(1 + \sum_{i=1}^{m} \frac{a_i}{i!} \left(1 - e^{-\lambda_{SB} t}\right)^i \right) \qquad MTTF = \frac{1}{\lambda} \sum_{i=0}^{m} \frac{1}{1 + ik}$$

where

$$k = \frac{\lambda_{SB}}{\lambda} \qquad a_i = \prod_{j=0}^{i-1} \left(j + \frac{1}{k}\right)$$

Ex.: Consider a single-component system with constant hazard rate ($\lambda = 0.001\ h^{-1}$).

Providing that redundant components are statistically identical, determine the minimal number *m* of redundant elements for hot, warm and cold redundancy, sufficient for the system reliability at a mission time $t = 1000\ h$ be greater than *0.9*.

(for warm standby $\lambda_{SB} = \frac{1}{6}\lambda$)

To begin with, let's compute the reliability of a single component at *t = 1000 h*.

$$R(t = 1000) = e^{-0.001 \cdot 1000} = e^{-1} \approx 0.368$$

First, let's consider hot redundancy. Since all components are equally reliable, we have

$$R_H(t) = 1 - \left(1 - R(t)\right)^{m+1}$$

Rewriting, we get

$$1 - R_H(t) = \left(1 - R(t)\right)^{m+1}$$

By taking the logarithm, we obtain:

$$\ln\big(1 - R_H(t)\big) = (m+1)\ln\big(1 - R(t)\big)$$

Hence,

$$m = \frac{\ln\big(1 - R_H(t)\big)}{\ln\big(1 - R(t)\big)} - 1$$

Substituting $R_H(1000) = 0.9$ and $R(1000) = 0.368$, we get

$$m = \frac{\ln(1 - 0.9)}{\ln(1 - 0.368)} - 1 \approx 4.02$$

Since m must be integer, $m = 5$.

Since equations for warm and cold standby systems don't allow direct computing of *m*, we should use simple substitution.

So, for warm redundancy substituting *m = 1* yields

$$R_W(1000) = e^{-\lambda t}\left(1 + \sum_{i=1}^{1} \frac{a_i}{i!}\left(1 - e^{-\lambda_{SB} t}\right)^i\right) =$$

$$= e^{-1}\left(1 + 6\left(1 - e^{-1/6}\right)\right) \approx 0.707 < 0.9$$

$$k = \frac{1}{6} \qquad a_1 = \prod_{j=0}^{i-1}\left(j + \frac{1}{k}\right) = 6$$

Substituting *m = 2* yields

$$R_W(1000) = e^{-\lambda t}\left(1 + \sum_{i=1}^{2} \frac{a_i}{i!}\left(1 - e^{-\lambda_{SB} t}\right)^i\right) =$$

$$= e^{-1}\left(1 + 6\left(1 - e^{-1/6}\right) + \frac{42}{2}\left(1 - e^{-1/6}\right)^2\right) \approx$$

$$\approx 0.889 < 0.9$$

$$a_2 = \prod_{j=0}^{i-1}\left(j + \frac{1}{k}\right) = 6 \cdot 7 = 42$$

Substituting *m = 3* yields

$$R_W(1000) = e^{-\lambda t}\left(1 + \sum_{i=1}^{3} \frac{a_i}{i!}\left(1 - e^{-\lambda_{SB} t}\right)^i\right) =$$

$$= e^{-1}\left(1 + 6\left(1 - e^{-1/6}\right) + \frac{42}{2}\left(1 - e^{-1/6}\right)^2 + \frac{336}{6}\left(1 - e^{-1/6}\right)^3\right) \approx$$

$$\approx 0.963$$

$$a_3 = \prod_{j=0}^{i-1}\left(j + \frac{1}{k}\right) = 6 \cdot 7 \cdot 8 = 336$$

So, in the case of warm standby we need the main component and 3 redundant ones.

Standby Redundancy

For cold redundancy substituting *m = 1* yields

$$R_C(1000) = e^{-\lambda t} \sum_{i=0}^{1} \frac{(\lambda t)^i}{i!} = e^{-1}(1+1) \approx 0.736 < 0.9$$

Substituting *m = 2* yields

$$R_C(1000) = e^{-\lambda t} \sum_{i=0}^{2} \frac{(\lambda t)^i}{i!} = e^{-1}\left(1+1+\frac{1}{2}\right) \approx 0.92$$

Hence, in the case of cold standby we only need 2 extra components to meet the requirements.

As demonstrated by the previous example, cold standby redundancy is the most reliable configuration of the three from the above.

It can be shown that MTTF of the cold standby is also the largest.

So, why won't we always using cold redundancy?

Standby Redundancy

First, toggling the redundant components on requires switching devices.
Though their reliability is often considered to be perfect, it’s not!

Second, switching the redundant components from the standby mode can take considerable time, which negatively affect system’s fail-safety.