PSEUDO-RANDOM NUMBER SAMPLING

Pseudo-Random Number Sampling

Quite often in statistics and simulation we need to obtain samples of random numbers distributed according to a given probability distribution.

Modern mathematical software is equipped with *pseudo-random number generator* producing *uniformally* distributed samples.

To generate samples drawn from other distributions we need to resort to *pseudo-random number sampling* techniques.

Pseudo-Random Number Sampling

The most common technique is *inverse transform sampling (Smirnov transform, inverse transformation method)*.

Inverse transform sampling takes uniform samples of a number *u* between 0 and 1, interpreted as a probability, and then returns the largest number *x* from the domain of the distribution $P(X)$ such that $P(-\infty < X < x) \leq u$.

Pseudo-Random Number Sampling

Computationally, this method involves computing the quantile function of the distribution — in other words, computing the cumulative distribution function (cdf) of the distribution and then inverting that function.

For a continuous distribution we need to integrate the probability density function (pdf) of the distribution or to obtain quantile function in an explicit form, which is impossible to do analytically for most distributions (including the normal distribution).

Pseudo-Random Number Sampling

Let X be a random variable whose distribution can be described by the *cdf* F_X. We want to generate values of X which are distributed according to this distribution.

The inverse transform sampling method works as follows:

- Generate a random number u from the standard uniform distribution in the interval $[0,1]$.
- Find the inverse of the desired *cdf*, $F_X^{-1}(x)$.
- Compute $X = F_X^{-1}(u)$. The computed random variable X has distribution $F_X(x)$.

Pseudo-Random Number Sampling

Ex.: Suppose we have a random variable $U \sim Unif(0,1)$ and a *cdf* of Weibull distribution

$$F(x) = 1 - exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right)$$

In order to perform an inversion we want to solve for

$$F\left(F^{-1}(u)\right) = u.$$

Pseudo-Random Number Sampling

$$F(x) = 1 - exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right) \qquad F\left(F^{-1}(u)\right) = u$$

$$1 - exp\left(-\left(\frac{F^{-1}(u)}{\eta}\right)^{\beta}\right) = u$$

$$exp\left(-\left(\frac{F^{-1}(u)}{\eta}\right)^{\beta}\right) = 1 - u$$

$$\left(\frac{F^{-1}(u)}{\eta}\right)^{\beta} = -\ln(1 - u)$$

$$F^{-1}(u) = \eta \cdot \sqrt[\beta]{-\ln(1 - u)}$$

Pseudo-Random Number Sampling

Once the samples of components' failure times are generated, we can obtain the sample of the system failures, providing that the system configuration is known.

For example, if the *series system* consists of *m* components and for each of them failure time samples $X^{<i>}$ $(i = 1..m)$ of size n were generated, then we can get the sample Y of system's failure times as follows:

$$Y_j = \min_{i \in (1,m)} X_j^{\langle i \rangle}, j = 1..n$$

Pseudo-Random Number Sampling

For the parallel hot redundancy system of *m* components we get:

$$Y_j = \max_{i \in (1,m)} X_j^{\langle i \rangle}$$

For the cold standby redundancy system of *m* components we get:

$$Y_j = \sum_{i=1}^{m} X_j^{\langle i \rangle}$$

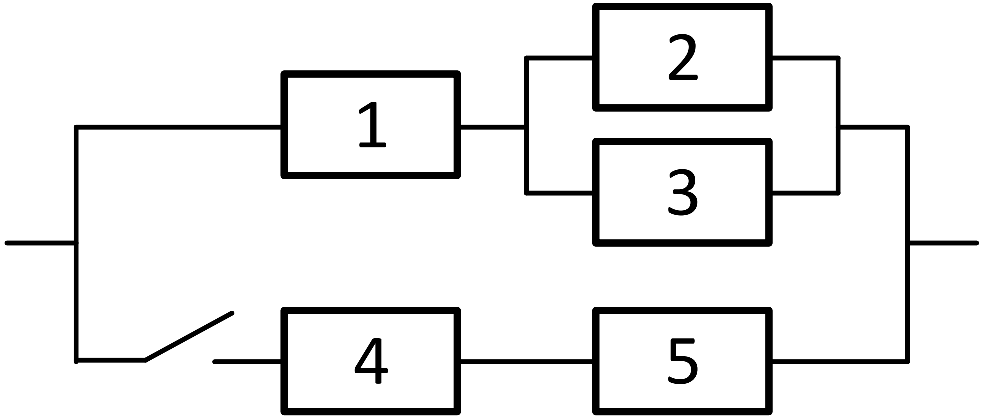
For the *k*-out-of-*n* system

$$Y_j = \vec{X}_j^{\langle k \rangle}$$

where $\vec{X}$ is the *ordered* set.

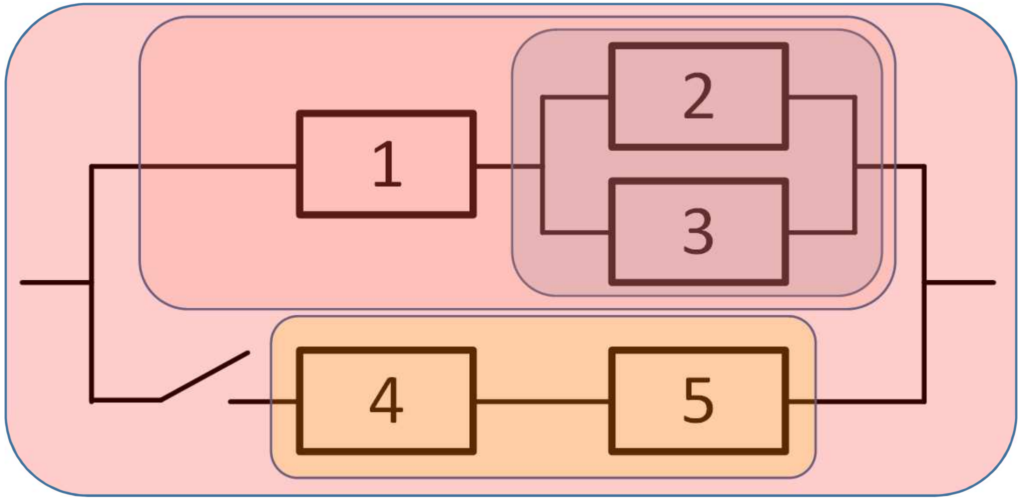
Pseudo-Random Number Sampling

Ex.: For the system with RBD shown below provide an algorithm of generating the sample of *n* system failures.



We assume the failure times of components 1 – 5 are collected into samples X1 – X5, respectively.

Pseudo-Random Number Sampling



$\max(X2_j, X3_j)$

$\min\left[X1_j, \max(X2_j, X3_j)\right]$

$\min(X4_j, X5_j)$

$$Y_j = \min\left[X1_j, \max(X2_j, X3_j)\right] + \min(X4_j, X5_j)$$

Homework Assignment 1

Consider the system with the RBD according to your assigned variant. Each RBD has at least one component of the Type 1 with complex distribution (F_1) of its time to failure; other types of components have exponentially distributed TTF.

1. Write a Mathcad program that generates n random numbers drawn from the F_1 distribution.
2. Select the values of F_1 parameters so that its mean would equals T (see the Table). The l-values of other components are provided in the Table.
3. Generate the samples of system failure times of size N and $4N$, where $N = 50 + 2V$, and V is the number of your variant.
4. Save these samples into separate Excel files.
5. Reopen the samples in new Mathcad file.

Homework Assignment 1

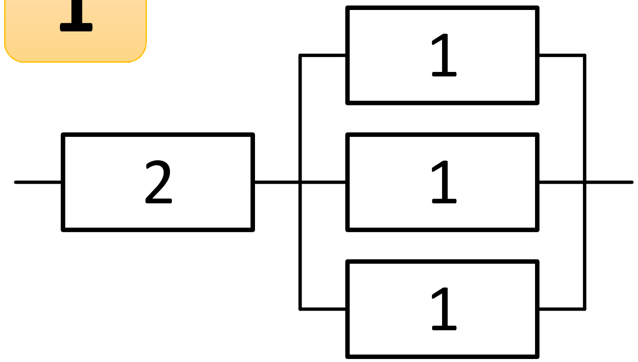
5. Define the reliability function and the failure probability function for your system, find the system's MTTF.
6. For each generated sample obtain the EDF and Median Rank values (either by Filliben's estimate or exact values).
7. Plot both EDFs together with theoretical failure probability function.
8. Plot both Median Ranks together with theoretical failure probability function.
9. For each generated sample find estimates of the MTTF.
10. Compare the obtained results with the theoretical reliability measures.

Homework Assignment 1

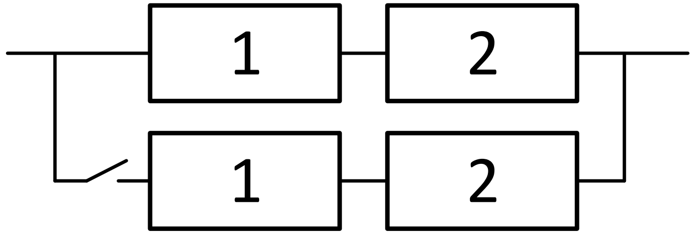
8T8A		F_1	T	Схема	2	3	4
					λ, 10^{-5}	λ, 10^{-5}	λ, 10^{-5}
1	Базаргуруева Валерия Лыгдыновна	GW	650	6	50	35	
2	Богданова Вероника Антоновна	GCEG	300	3	100		
3	Боровской Артём Романович	Kw-R	750	4	65		
4	Глебов Александр Глебович	EW	450	8	100	70	
5	Головченко Станислав Сергеевич	CWG	1650	2	30		
6	Горбачев Александр Сергеевич	Kw-E	1750	5	20	15	8
7	Ивашкин Евгений Александрович	CWG	800	8	60	40	
8	Коробов Максим Сергеевич	ECEG	1150	1	35		
9	Куцюк Тарас Викторович	CWG	1300	6	25	15	
10	Лаврентьев Виктор	GCRG	1450	4	30		
11	Левченко Кирилл Николаевич	Kw-E	100	7			
12	Махно Анна Сергеевна	Kw-R	1500	1	20		
13	Мусаев Назим	EW	550	2	90		
14	Новиков Павел	Kw-R	950	3	35		
15	Овдиенко Никита Александрович	ECEG	1100	7			
16	Рахманов Сардор Джамолиддинович	EW	700	3	45		
17	Цымжитов Тимур Булатович	Kw-E	850	5	40	30	15
18	Чащин Данил Александрович	GCEG	1850	1	30		

Homework Assignment 1

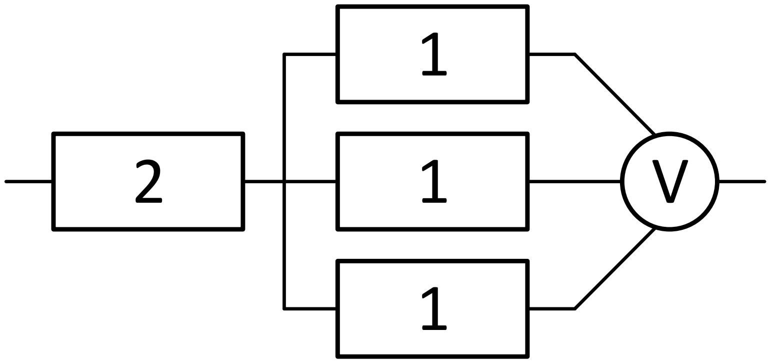
1



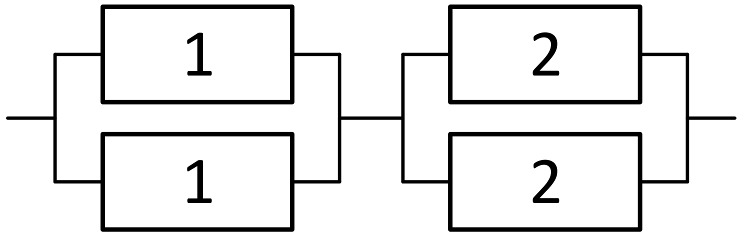
2



3

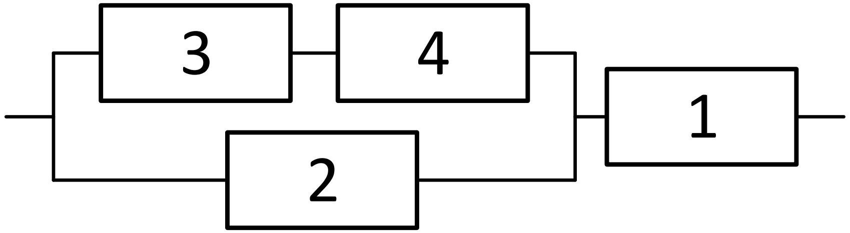


4

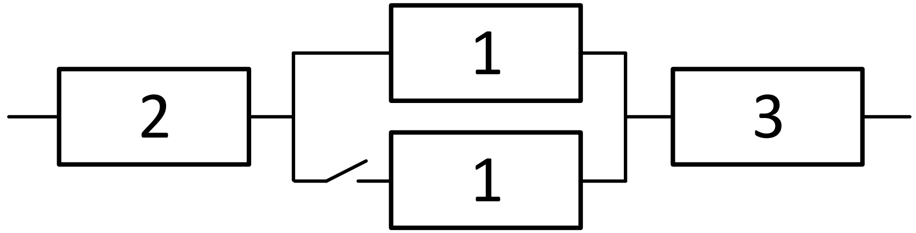


Homework Assignment 1

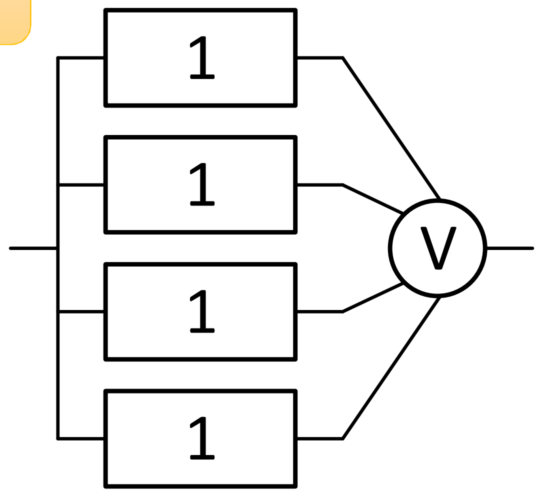
5



6



7



8