STANDBY REDUNDANCY

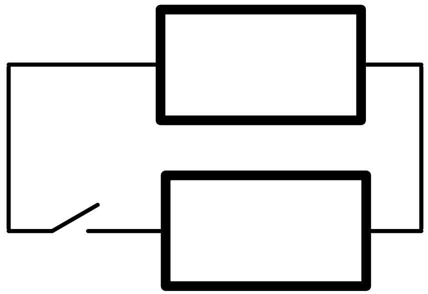
(cold, warm)

Standby Redundancy

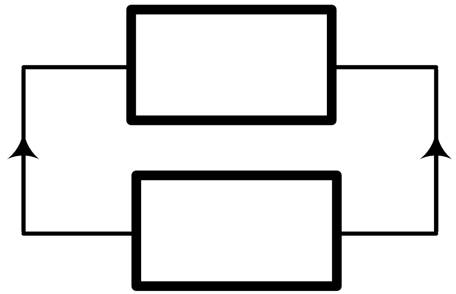
From now on, we will denote both cold and warm redundancy as a *standby redundancy*.

Also, we will consider statistically identical items with constant failure rates, unless otherwise stated.

The RBDs for these configurations are as follows:



Cold Redundancy



Warm Redundancy

Standby Redundancy

The most general way to compute the reliability of standby system (of 2 components) is to evaluate the integral:

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

where:

R_1 is the reliability of the active component;

f_1 is the pdf of the active component;

$R_{2;SB}$ is the reliability of the standby component when in standby mode (*quiescent* reliability);

$R_{2;A}$ is the reliability of the standby component when in active mode;

t_e is the equivalent operating time for the standby unit, if it had been operating at an active mode, such that:

$$R_{2;SB}(x) = R_{2;A}(t_e)$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Note that the formula above may involve different distributions of component failure time.

Furthermore, you can compute the reliability of active redundancy with this formula as well, though such an approach wouldn't be the most convenient.

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

To provide an example, we will consider the system of two statistically identical components with constant failure rate.

$$R_1(t) = R_{2;A}(t) = e^{-\lambda t} \qquad f_1(t) = \lambda e^{-\lambda t}$$

The quiescent reliabilities are:

$R_{2;SB}(t) = 1$ for cold redundancy;

$R_{2;SB}(t) = e^{-\lambda_{SB} t}$ for warm redundancy ($\lambda_{SB} < \lambda$);

$R_{2;SB}(t) = R_{2;A}(t) = e^{-\lambda t}$ for hot (active) redundancy.

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

The equivalent operating time t_e is obtained from the equation $R_{2;SB}(x) = R_{2;A}(t_e)$, so for the case of cold redundancy:

$$1 = e^{-\lambda t_e} \;\Rightarrow\; t_e = 0.$$

For warm redundancy:

$$e^{-\lambda_{SB} x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = \frac{\lambda_{SB}}{\lambda} x.$$

For hot redundancy:

$$e^{-\lambda x} = e^{-\lambda t_e} \;\Rightarrow\; t_e = x.$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

The equivalent operating time t_e is obtained from the equation $R_{2;SB}(x) = R_{2;A}(t_e)$, so for the case of cold redundancy:

$$1 = e^{-\lambda t_e} \Rightarrow t_e = 0.$$

For warm redundancy:

$$e^{-\lambda_{SB} x} = e^{-\lambda t_e} \Rightarrow t_e = \frac{\lambda_{SB}}{\lambda} x.$$

For hot redundancy:

$$e^{-\lambda x} = e^{-\lambda t_e} \Rightarrow t_e = x.$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Let's start with hot redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda x} \cdot \frac{e^{-\lambda(x+t-x)}}{e^{-\lambda x}} dx =$$

$$= e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda x} \cdot \frac{e^{-\lambda t}}{e^{-\lambda x}} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{-\lambda x} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \left[-\frac{e^{-\lambda t}}{\lambda} + \frac{1}{\lambda} \right] = 2e^{-\lambda t} - e^{-2\lambda t}$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

Next, for cold redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot 1 \cdot \frac{e^{-\lambda(0+t-x)}}{1} dx =$$

$$= e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} e^{-\lambda t} e^{\lambda x} \, dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t}[t - 0] = \boxed{e^{-\lambda t}(1 + \lambda t)}$$

Standby Redundancy

$$R(t) = R_1(t) + \int_0^t f_1(x) \cdot R_{2;SB}(x) \cdot \frac{R_{2;A}(t_e + t - x)}{R_{2;A}(t_e)} dx$$

And finally, for warm redundancy:

$$R(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda x} \cdot e^{-\lambda_{SB} x} \cdot \frac{e^{-\lambda\left(\frac{\lambda_{SB}}{\lambda}x + t - x\right)}}{e^{-\lambda\left(\frac{\lambda_{SB}}{\lambda}x\right)}} dx =$$

$$= e^{-\lambda t} + \lambda \int_0^t e^{-\lambda x} e^{-\lambda_{SB} x} \frac{e^{-\lambda_{SB} x} e^{-\lambda t} e^{\lambda x}}{e^{-\lambda_{SB} x}} dx =$$

$$= e^{-\lambda t} + \lambda e^{-\lambda t} \int_0^t e^{-\lambda_{SB} x} \, dx =$$

$$= e^{-\lambda t}\left[1 + \frac{\lambda}{\lambda_{SB}}\left(1 - e^{-\lambda_{SB} t}\right)\right]$$

Standby Redundancy

These results can also be obtained via state-space method, which will be addressed later.

For now, we can have the formulae for certain special cases of *cold* and *warm* redundant configurations, namely, systems of *m+1* statistically identical components with constant failure rate.

Standby Redundancy

For cold standby redundancy ($\lambda_{SB} = 0$):

$$R(t) = e^{-\lambda t} \sum_{i=0}^{m} \frac{(\lambda t)^i}{i!} \qquad MTTF = \frac{m+1}{\lambda}$$

For warm standby redundancy ($\lambda_{SB} < \lambda$):

$$R(t) = e^{-\lambda t} \left(1 + \sum_{i=1}^{m} \frac{a_i}{i!} \left(1 - e^{-\lambda_{SB} t}\right)^i \right) \qquad MTTF = \frac{1}{\lambda} \sum_{i=0}^{m} \frac{1}{1+ik}$$

where

$$k = \frac{\lambda_{SB}}{\lambda} \qquad a_i = \prod_{j=0}^{i-1} \left(j + \frac{1}{k} \right)$$

Ex.: Consider a single-component system with constant failure rate ($\lambda = 0.001\ h^{-1}$).

Providing that redundant components are statistically identical, determine the minimal number m of redundant elements for hot, warm and cold redundancy, sufficient for the system reliability at a mission time $t = 1000\ h$ be greater than 0.9.

(for warm standby $\lambda_{SB} = \frac{1}{6}\lambda$)

To begin with, let's compute the reliability of a single component at $t = 1000\ h$.

$$R(t = 1000) = e^{-0.001 \cdot 1000} = e^{-1} \approx 0.368$$

First, let's consider hot redundancy. Since all components are equally reliable, we have

$$R_H(t) = 1 - \left(1 - R(t)\right)^{m+1}$$

Rewriting, we get

$$1 - R_H(t) = \left(1 - R(t)\right)^{m+1}$$

By taking the logarithm, we obtain:

$$\ln\big(1 - R_H(t)\big) = (m + 1)\ln\big(1 - R(t)\big)$$

Hence,

$$m = \frac{\ln\big(1 - R_H(t)\big)}{\ln\big(1 - R(t)\big)} - 1$$

Substituting $R_H(1000) = 0.9$ and $R(1000) = 0.368$, we get

$$m = \frac{\ln(1 - 0.9)}{\ln(1 - 0.368)} - 1 \approx 4.02$$

Since m must be integer, $m = 5$.

Since equations for warm and cold standby systems don't allow direct computing of *m*, we should use simple substitution.

So, for warm redundancy substituting *m = 1* yields

$$R_W(1000) = e^{-\lambda t}\left(1 + \sum_{i=1}^{1} \frac{a_i}{i!}\left(1 - e^{-\lambda_{SB} t}\right)^i\right) =$$

$$= e^{-1}\left(1 + 6\left(1 - e^{-1/6}\right)\right) \approx 0.707 < 0.9$$

$$k = \frac{1}{6} \qquad a_1 = \prod_{j=0}^{i-1}\left(j + \frac{1}{k}\right) = 6$$

Substituting *m = 2* yields

$$R_W(1000) = e^{-\lambda t}\left(1+\sum_{i=1}^{2}\frac{a_i}{i!}\left(1-e^{-\lambda_{SB}t}\right)^i\right) =$$

$$= e^{-1}\left(1+6\left(1-e^{-1/6}\right)+\frac{42}{2}\left(1-e^{-1/6}\right)^2\right) \approx$$

$$\approx 0.889 < 0.9$$

$$a_2 = \prod_{j=0}^{i-1}\left(j+\frac{1}{k}\right) = 6\cdot 7 = 42$$

Substituting *m = 3* yields

$$R_W(1000) = e^{-\lambda t}\left(1 + \sum_{i=1}^{3} \frac{a_i}{i!}\left(1 - e^{-\lambda_{SB} t}\right)^i\right) =$$

$$= e^{-1}\left(1 + 6\left(1 - e^{-1/6}\right) + \frac{42}{2}\left(1 - e^{-1/6}\right)^2 + \frac{336}{6}\left(1 - e^{-1/6}\right)^3\right) \approx$$

$$\approx 0.963$$

$$a_3 = \prod_{j=0}^{i-1}\left(j + \frac{1}{k}\right) = 6 \cdot 7 \cdot 8 = 336$$

So, in the case of warm standby we need the <u>main component and 3 redundant ones</u>.

For cold redundancy substituting *m = 1* yields

$$R_C(1000) = e^{-\lambda t} \sum_{i=0}^{1} \frac{(\lambda t)^i}{i!} = e^{-1}(1 + 1) \approx 0.736 < 0.9$$

Substituting *m = 2* yields

$$R_C(1000) = e^{-\lambda t} \sum_{i=0}^{2} \frac{(\lambda t)^i}{i!} = e^{-1}\left(1 + 1 + \frac{1}{2}\right) \approx 0.92$$

Hence, in the case of cold standby we only need 2 extra components to meet the requirements.

Standby Redundancy

As demonstrated by the previous example, cold standby redundancy is the most reliable configuration of the three from the above.

It can be shown that MTTF of the cold standby is also the largest.

So, why won't we always using cold redundancy?

Standby Redundancy

First, toggling the redundant components on requires switching devices.
Though their reliability is often considered to be perfect, it’s not!

Second, switching the redundant components from the standby mode can take considerable time, which negatively affect system’s fail-safety.

Homework Assignment 1

1. For the system with the RBD, according to your variant,
   a) define the reliability function, failure density function and failure rate function;
   b) plot the graphs of these functions;
   c) find the value of MTTF for the system.
2. Repeat 1a) three more times, implementing 20% increase of parameters η or σ (20% decrease of parameter λ) separately for each component.
3. Plot the graphs of reliability functions and failure rate functions for the systems, defined in 2), together with the same graphs for the original system.
4. Find the values of MTTFs for the systems, defined in 2).
5. Make an observation on the influence of components' parameters on the system reliability measures.

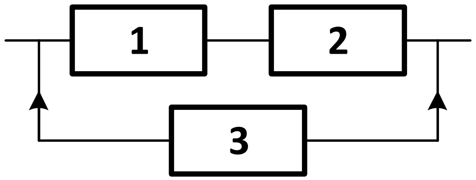
Homework Assignment 1

	8T91	№
1	Балахнин Илья Александрович	6.1
2	Балухта Алексей Игоревич	8.2
3	Долгих Владимир Алексеевич	4.5
4	Кирсанов Никита Артурович	5.1
5	Колотихин Евгений Иванович	5.2
6	Ларина Анастасия Валерьевна	8.5
7	Лесных Глеб Игоревич	3.1
8	Макев Адиль Нурланович	3.2
9	Оздоев Рафаэль Залимханович	5.5
10	Пешков Никита Сергеевич	1.4
11	Рыбин Владислав Евгеньевич	2.4
12	Сандул Андрей Анатольевич	5.6
13	Стрельникова Виктория Анатольевна	4.2
14	Суворов Данил Владиславович	4.1
15	Сурков Данила Сергеевич	1.5
16	Тятюшкин Данил Максимович	2.1
17	Черкасов Данил Дмитриевич	2.6

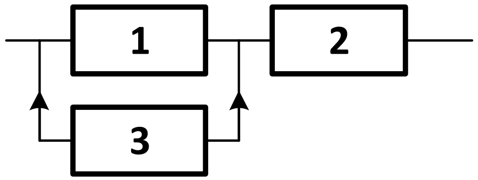
	8T92	№
1	Акулов Игорь Романович	1.2
2	Ароян Эдвард Варданович	4.4
3	Белоусова Анастасия Александровна	7.2
4	Бузмаков Илья Дмитриевич	1.1
5	Гурбанов Мекан	2.5
6	Данилюк Егор Дмитриевич	8.4
7	Долгушин Егор Евгеньевич	5.3
8	Жгута Вадим	6.2
9	Киргефнер Михаил Сергеевич	7.1
10	Курманов Станислав Анатольевич	5.7
11	Мамонтов Илья Сергеевич	3.3
12	Маракин Данил Денисович	2.3
13	Медведева Елизавета Васильевна	4.6
14	Мирошников Данил Александрович	7.5
15	Назыров Роман Павлович	4.3
16	Неволина Елена Сергеевна	7.3
17	Парипко Виктор Олегович	1.3
18	Рачковский Святослав Максимович	8.1
19	Салпыков Дамир Миржанович	5.8
20	Сладков Максим	7.4
21	Солодкин Алексей Дмитриевич	2.2
22	Уточкин Сергей Вячеславович	5.4
23	Щербашин Никита Геннадьевич	8.3

Homework Assignment 1

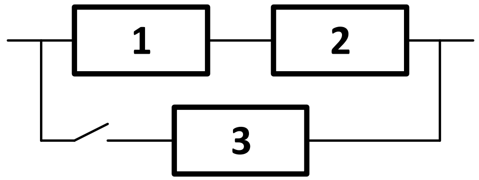
1



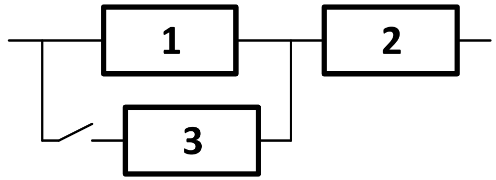
2



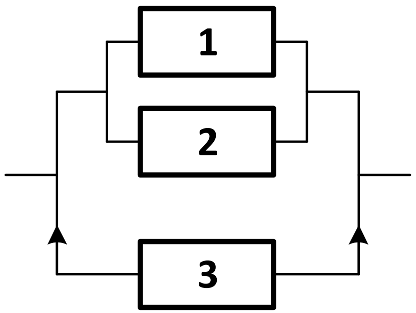
3



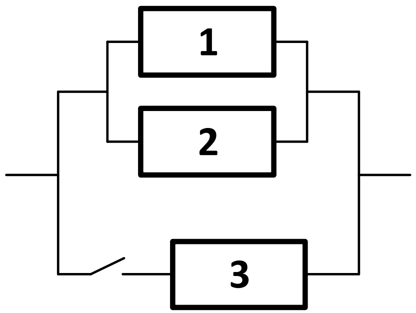
4



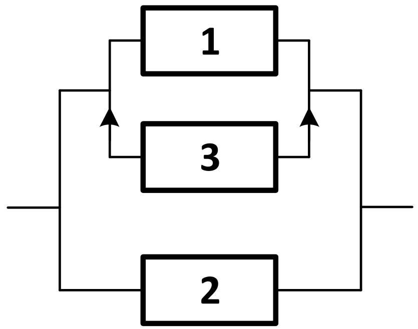
5



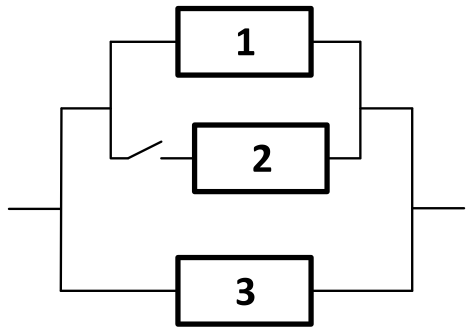
6



7



8



Homework Assignment 1

Схема	Вариант	Блок 1			Блок 2			Блок 3				
								в рабочем режиме			в теплом резерве	
		F1	p1	p2	F2	p1	p2	F_{3A}	p1	p2	p1	p2
1	1	R	3000		W	5000	1,4	E	0,0004		0,00005	
	2	R	2000		E	0,0005		W	1200	1,3	6000	1,3
	3	W	500	0,7	R	1000		E	0,0025		0,00025	
	4	W	800	0,6	W	6000	2,5	E	0,001		0,00015	
	5	W	3000	1,2	E	0,00035		W	1400	0,9	8400	0,9
2	1	R	300		W	2500	1,4	E	0,004		0,0005	
	2	E	0,001		R	2200		W	1000	1,15	7000	1,15
	3	W	2400		R	4000		E	0,0004		0,00006	
	4	R	2000		E	0,0003		W	2000	1,3	18000	1,3
	5	W	3500	2,6	R	3000		W	2500	1,2	15000	1,2
	6	E	0,0007		W	2700	0,65	E	0,0008		0,0001	
3	1	R	1050		W	2000	2,2	E	0,0009			
	2	E	0,0001		W	2500	1,75	R	1500			
	3	E	0,0004		R	1800		W	1250	0,75		
	4	W	2500	0,35	W	4000	1,4	R	1100			
	5	R	3000		R	5000		W	3000	0,85		
	6	R	900		W	500	0,25	W	500	1,1		
	7	E	0,0003		W	2500	1,25	E	0,0007			
4	1	R	850		W	4000	3,2	E	0,001			
	2	E	0,0005		W	3300	1,9	R	2000			
	3	R	1300		E	0,0003		W	1000	0,65		
	4	W	1600	1,65	E	0,00025		R	1350			
	5	R	400		W	4000	0,55	R	370			
	6	W	2500	1,5	E	0,00035		W	2400	1,75		

Homework Assignment 1

Схема	Вариант	Блок 1			Блок 2			Блок 3				
								в рабочем режиме			в теплом резерве	
		F1	p1	p2	F2	p1	p2	F_{3A}	p1	p2	p1	p2
5	1	R	1600		W	2000	1.2	E	0,0008		0,0003	
	2	R	500		E	0,0025		W	800	0,9	5600	0,9
	3	R	2000		R	1800		E	0,0004		0,0001	
	4	R	1000		R	1400		W	2400	1,15	29400	1,15
	5	E	0,001		E	0,0012		W	1000	0,65	3000	0,65
	6	E	0,0006		R	2500		W	3000	1,35	15000	1,35
	7	E	0,002		W	650	1,8	W	700	0,85	14000	0,85
	8	W	800	1,5	W	800	1,4	E	0,001		0,00014	
6	1	R	1200		W	1500	1,25	E	0,00065			
	2	W	750	0,9	E	0,0009		R	1100			
	3	R	1500		E	0,0005		W	1500	1,9		
	4	R	1000		R	1200		W	2000	3		
	5	W	1000	1,15	W	1000	1,3	E	0,00075			
	6	E	0,0005		E	0,00065		R	1600			
7	1	R	1350		W	2500	1,2	E	0,0008		0,0003	
	2	W	400	0,7	R	750		E	0,002		0,0001	
	3	R	1600		E	0,0004		W	1900	1,1	10000	1,1
	4	W	500	0,45	E	0,0005		W	1300	1,15	12000	1,15
	5	E	0,0007		R	2000		W	1250	0,8	2500	0,8
8	1	R	1400		W	1400	0,85	E	0,00075			
	2	E	0,002		W	650	2,65	R	800			
	3	E	0,00045		R	1800		W	2400	1,5		
	4	W	1200	1,35	W	1200	1,75	E	0,00055			
	5	R	300		R	270		W	850	2,3		