BASIC CONCEPTS
OF
RELIABILITY THEORY

Reliability is a characteristic of the item, expressed by the *probability* that it will perform its *required function* under *given conditions* for a *stated time interval*. It is generally designated by *R*.

From a *qualitative* point of view, reliability can be defined as the *ability of the item to remain functional*.

Quantitatively, reliability specifies the *probability that no operational interruptions* will occur during a stated time interval.

To make sense, a numerical statement of reliability (e. g. R = 0.9) must be accompanied by the definition of the *required function*, *the operating conditions*, and *the mission duration*.

In general, it is also important to know whether or not the item can be considered new when the mission starts.

An *item* is a functional or structural *unit* of arbitrary complexity (e.g. component, assembly, equipment, subsystem, system) that can be considered as an entity for investigations.

It may consist of *hardware*, *software*, or *both*, and may also include *human resources*.

Often, ideal human aspects and logistic support are assumed, even if (for simplicity) the term *system* is used instead of *technical system*.

Often the mission duration is considered as a parameter *t*, the *reliability function* is then defined by *R(t)*.

R(t) is the probability that no failure at item level will occur in the interval (0, *t*].

For the systems of *physical nature* reliability function is a *decreasing (non-increasing) function*.

Reliability function is sometimes referred to as a *survival function, S(t)*.

A distinction between *predicted* and *estimated* reliability is important:

- the first is calculated on the basis of the item's reliability structure and the reliability characteristics of its components;
- the second is obtained from a statistical evaluation of *reliability tests* or from *field data*.

The concept of reliability can be extended to processes and services, although human aspects can lead to modeling difficulties.

A *failure* occurs when the item stops performing its required function. As simple as this definition is, it can become difficult to apply it to complex items.

The *failure-free time* (hereafter used as a synonym for *failure-free operating time*) is generally a random variable.

A general assumption in investigating failure-free times is that at t = 0 the item is free of *defects* and *systematic failures*.

Failures can be classified as *sudden* or *gradual*.

We will consider time at which failure occurs (*failure time*) as a nonnegative continuous r.v. with particular *cdf* *F(t)* and *pdf* *f(t)*.

In reliability theory *F(t)* is referred to as probability of failure or *failure probability function*.

Similarly, *f(t)* is often (and not quite correctly) called *failure density function*.

The failure probability function is sometimes referred to as a *life distribution*.

Basic Concepts of Reliability Theory

We can infer the following formulae:

	$R(t)$	$F(t)$	$f(t)$
$R(t) =$		$1 - F(t)$	$\int_t^{\infty} f(\tau)\, d\tau$
$F(t) =$	$1 - R(t)$		$\int_0^{t} f(\tau)\, d\tau$
$f(t) =$	$-\frac{dR(t)}{dt}$	$\frac{dF(t)}{dt}$	

The (*instantaneous*) *failure rate* can be defined as the ratio of the items failed in the *infinitesimal* interval (t, $t+\delta t$] to the number of items still operating at time t.

Some authors refer to the failure rate as the *hazard rate* and consider these term synonyms (others – don't!).

Failure rate function is often denoted as $\lambda(t)$. To avoid confusion with the parameter of exponential distribution we will denote it as $h(t)$.

By definition

$$h(t) = \frac{f(t)}{R(t)}.$$

Omitting the derivation, it is possible to define failure rate in terms of the other reliability measures:

$$h(t) = -\frac{R'(t)}{R(t)} = \frac{F'(t)}{1 - F(t)} = \frac{f(t)}{\int_t^{\infty} f(\tau)d\tau}$$

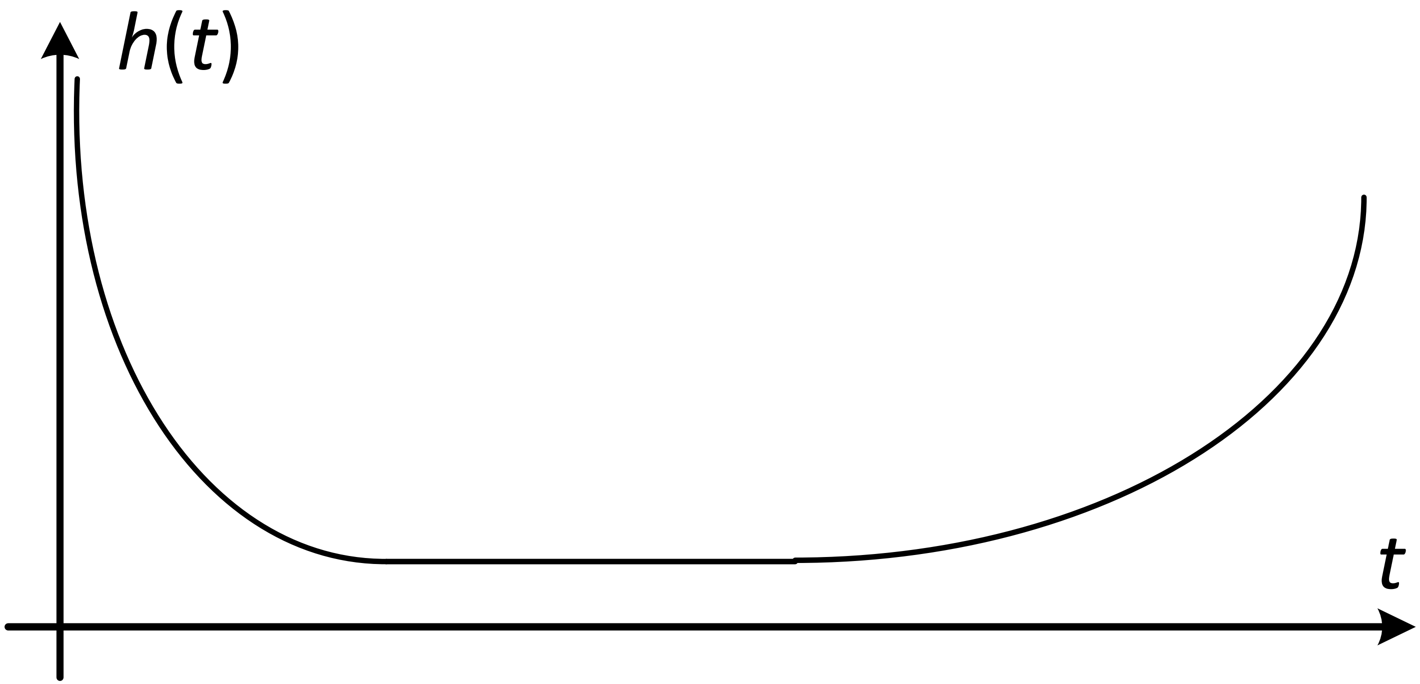
The reverse formulae could also be of use:

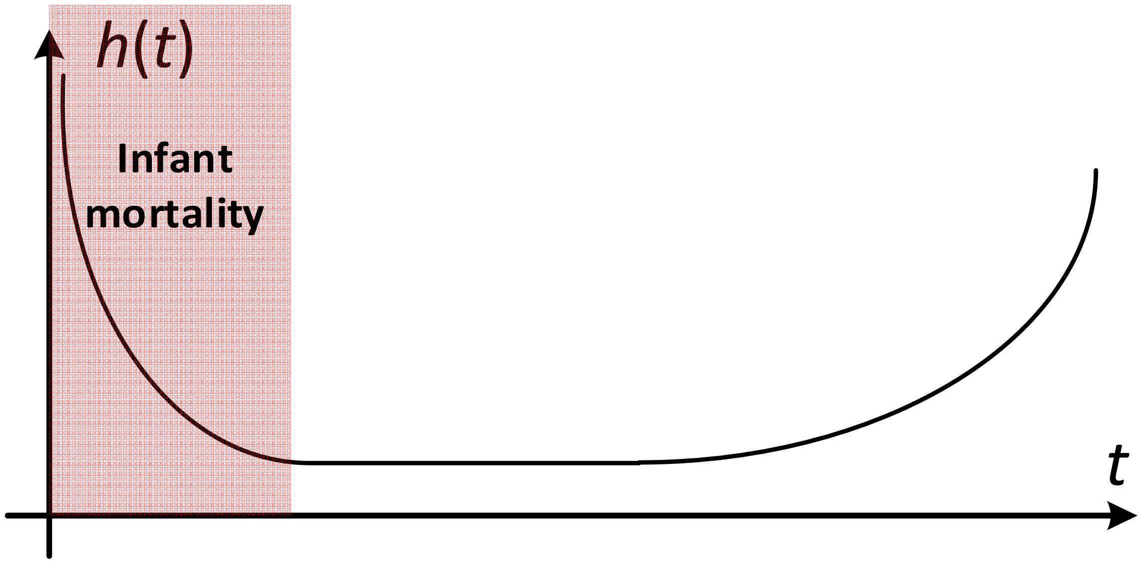
$$R(t) = e^{-\int_0^t h(\tau)d\tau} \quad F(t) = 1 - e^{-\int_0^t h(\tau)d\tau} \quad f(t) = -\left[e^{-\int_0^t h(\tau)d\tau}\right]'$$

Now we can update the table:

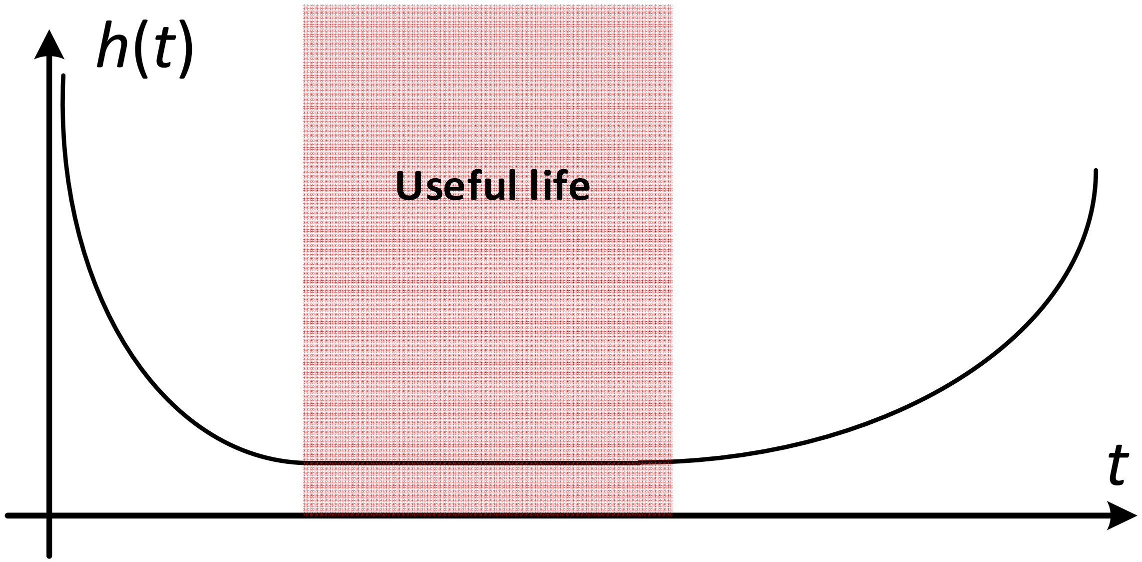
	$R(t)$	$F(t)$	$f(t)$	$h(t)$
$R(t) =$		$1 - F(t)$	$\int_t^{\infty} f(\tau)\, d\tau$	$e^{-\int_0^t h(\tau) d\tau}$
$F(t) =$	$1 - R(t)$		$\int_0^{t} f(\tau)\, d\tau$	$1 - e^{-\int_0^t h(\tau) d\tau}$
$f(t) =$	$-\frac{dR(t)}{dt}$	$\frac{dF(t)}{dt}$		$-\left[e^{-\int_0^t h(\tau) d\tau}\right]'$
$h(t) =$	$-\frac{R'(t)}{R(t)}$	$\frac{F'(t)}{1 - F(t)}$	$\frac{f(t)}{\int_t^{\infty} f(\tau) d\tau}$	

The failure rate of a large population of statistically identical and independent items often exhibits a typical *bathtub curve*:

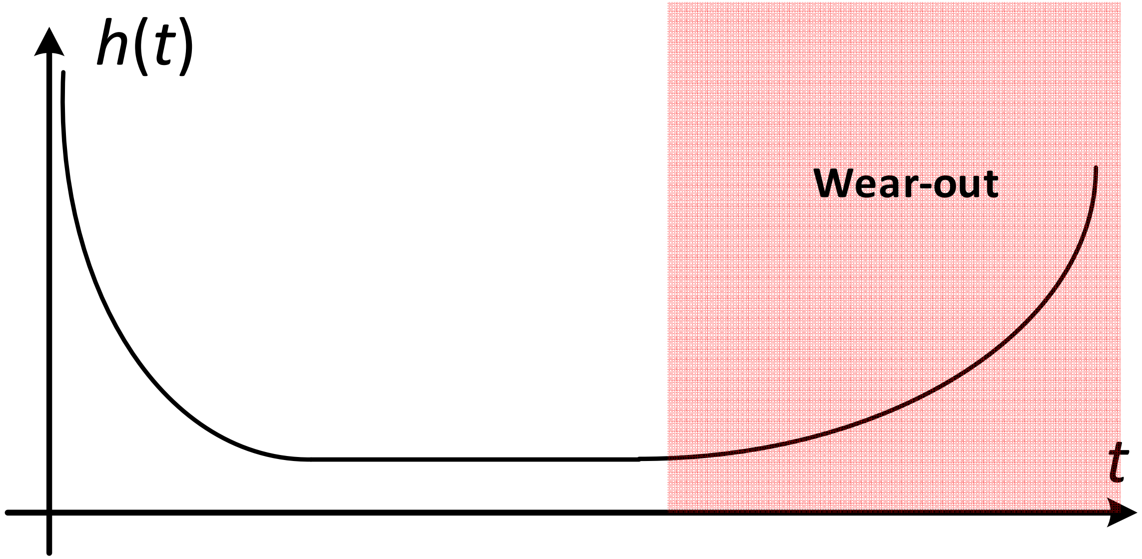




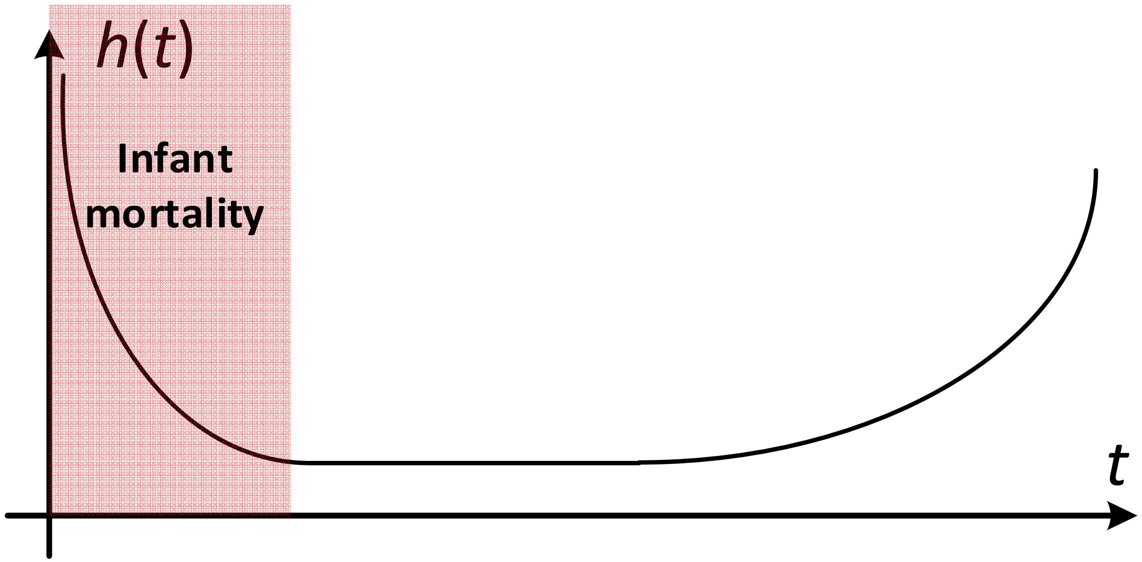
As can be seen from this plot, many products will begin their lives with a higher failure rate (which can be due to manufacturing defects, poor workmanship, poor quality control of incoming parts, etc.) and exhibit a *decreasing failure rate*.



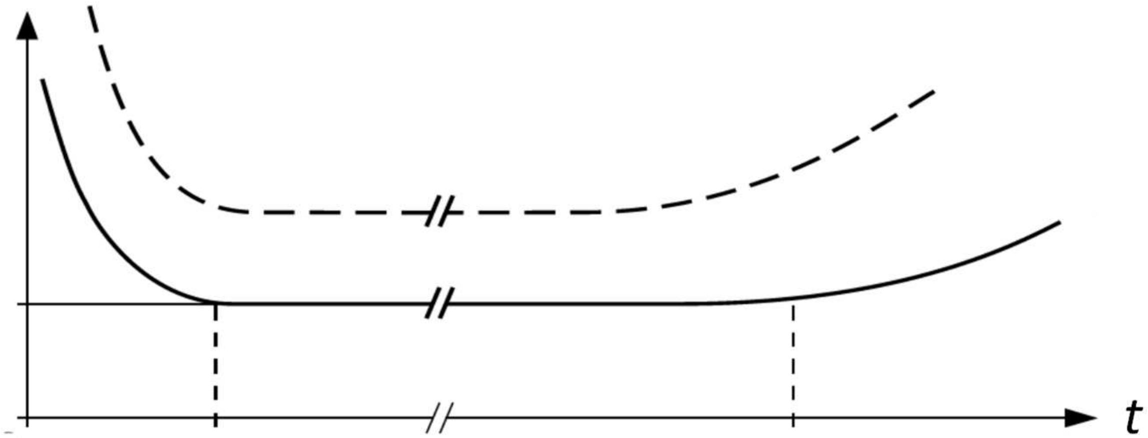
During the *useful life* region the item failure rate remains *nearly constant* with respect to time. Some of the main reasons for the occurrence of failures during this region are undetectable defects, higher random stress than expected, abuse, low safety factors, and human error.



As the products experience more use and wear, the *failure rate begins to rise* as the population begins to experience failures related to aging, wear-out, fatigue, etc.



It should be obvious that it would be best to ship a product at the beginning of the useful life region, rather than right off the production line; thus preventing the customer from experiencing early failures. This practice is what is commonly referred to as "*burn-in*", and is frequently performed for electronic components.



The failure rate strongly depends upon the item's operating conditions.

Typical figures for $h(t)$ are 10^{-10} to 10^{-7} h^{-1} for electronic components at 40°C, doubling for a temperature increase of 10 to 20°C.

Since time to failure, *T*, is a random variable, its mean is an important and the most obvious reliability measure.

In reliability theory it is called *“mean time to failure” (MTTF)*, and, by definition, it can be computed as

$$E[T] = MTTF = \int_0^\infty t f(t) dt \,.$$

Another useful formula for the MTTF is as follows:

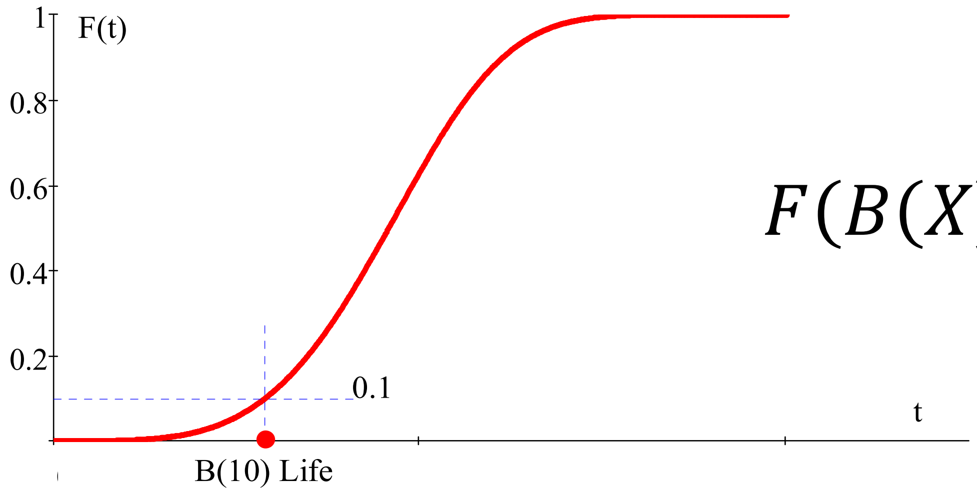
$$MTTF = \int_0^{\infty} R(t)dt\,.$$

Note that, although

$$MTTF = \int_0^{\infty} R(t)dt = \int_0^{\infty} tf(t)dt\,,$$

$$\boldsymbol{R(t) \neq tf(t)}\;\textbf{!}$$

B(X) Life is yet another reliability measure. It is defined as the time at which *X*% of the units in a population will have failed, or as the time when the probability of failure will reach a specified point (*X*%).



$$F(B(X)\ Life) = \frac{X}{100}$$

RELIABILITY MODELS (LIFE DISTRIBUTIONS)

Reliability Models (Life Distributions)

When specifying certain probability distribution as a reliability model for an element, a system or a subsystem, we assume that failure time of such items is a random variable distributed according to this particular distribution.

Generally, *any distribution* can be regarded as a life distribution.

Often, it is assumed that failure probability at $t = 0$ is equal to 0, so, failure time is considered to be a *non-negative continuous r.v.*

However, even a distribution $F(t)$ with an infinite support $(-\infty; +\infty)$ can be employed as a reliability model in cases when

- $F(t = 0)$ is a negligible quantity;
- $F(t = 0) > 0$ a priori, i.e. the assumption on $F(t = 0) = 0$ is not valid.

When failure occurs after random number of on-off cycles, $F(t)$ could be discrete.

Reliability Models (Life Distributions)

The most popular reliability models are exponential and Weibull distributions.

As most of characteristics of these distributions were addressed in preceding sections, we will consider only failure rate according to these two models.

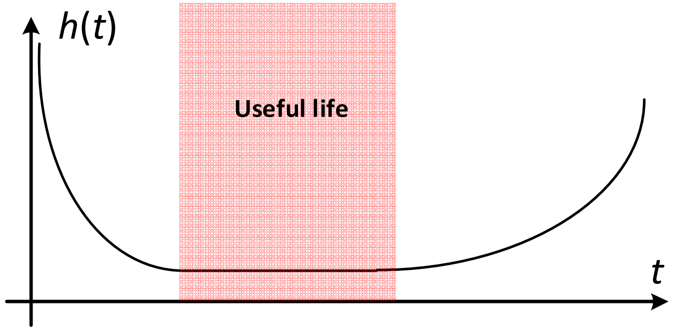
Exponential Reliability Model (ERM)

Given that $h(t) = \dfrac{f(t)}{R(t)}$ and

$$f(t) = \lambda e^{-\lambda t}$$
$$R(t) = 1 - F(t) = e^{-\lambda t}$$

we obtain $h(t) = \lambda = const.$

Therefore, ERM is valid for items in their useful life period.



By using ERM we assume that the manufacturer carried out complete burn-in and that the item will be in use for a time interval not extending into wear-out period.

Weibull Reliability Model (WRM)

Given that $h(t) = \dfrac{f(t)}{R(t)}$ and

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad R(t) = 1 - F(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

we get

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

The obtained equation offers various distinct shapes of the failure rate.

First, if $\beta=1$, Weibull distribution reduces to the exponential:

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\Bigg|_{\beta=1} = \frac{1}{\eta} = const.$$

Second, if $\beta<1$, the failure rate of Weibull distribution is a convex decreasing function.

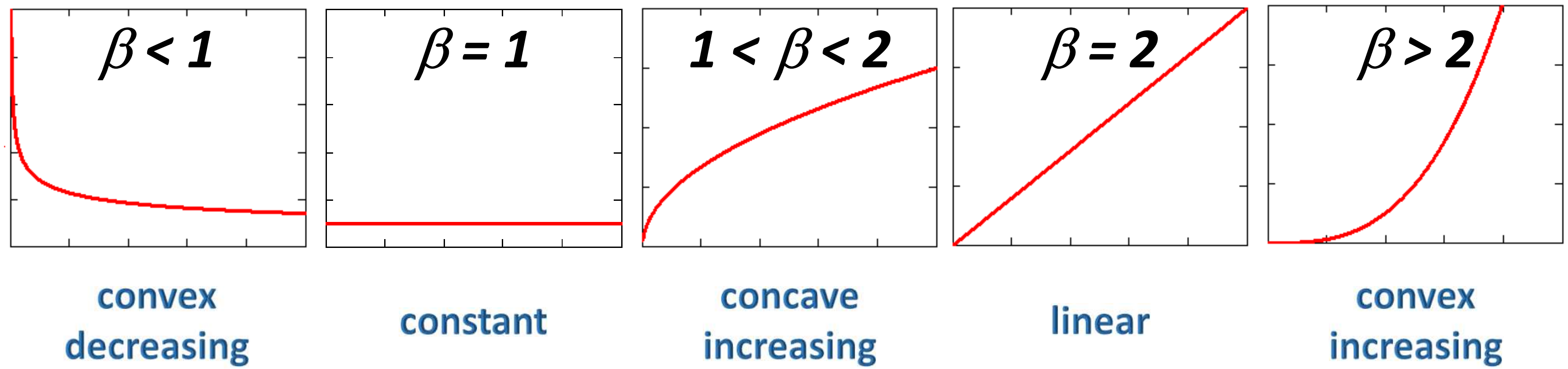
Third, if $1<\beta<2$, the failure rate of Weibull distribution is a concave increasing function.

Next, if β=2, Weibull distribution reduces to the Rayleigh distribution, and *h(t)* becomes linear:

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\Bigg|_{\beta = 2} = \frac{2}{\eta^2}t.$$

Finally, if β>2, the failure rate of Weibull distribution is a convex increasing function.

The following images illustrate the shapes of the Weibull failure rate:



Reliability Models (Life Distributions)

Though Weibull distribution can't provide bathtub shape for the failure rate, it is widely applied in reliability analysis due to its relative simplicity and flexibility.

There were suggested various compound distributions that demonstrate this particular shape of the failure rate.

Some of them are based on either exponential or Weibull distributions.

Consider so called Bi-Weibull distribution with a reliability function defined as

$$R(t) = e^{-\left(\frac{t}{\eta 1}\right)^{\beta 1}-\left(\frac{t}{\eta 2}\right)^{\beta 2}}.$$

Adjusting its parameters we can obtain the following shape of the failure rate function: