RELIABILITY THEORY

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INTRODUCTION

Introduction

As per IEEE standards, *reliability* is defined as the ability of a system or component to perform its required functions under stated conditions for a specified period of time.

The key elements of the definition are

- ability,
- required function,
- conditions,
- and specified period of time.

Ability is expressed quantitatively with probability.

Required function relates to expected performance.

Stated conditions usually refer to environmental conditions of operation.

Specified period of time is also referred as mission time which provides expected duration of operation.

We can distinguish between three main branches of reliability theory:

- Hardware reliability
- Software reliability
- Human reliability

This course is concerned with the first of these branches: the reliability of technical components and systems.

Within hardware reliability we may use two different approaches:

- The physical approach
- The actuarial (probabilistic) approach

In the physical approach the strength of a technical item is modeled as a random variable S. The item is exposed to a load L that is also modeled as a random variable.
A failure will occur as soon as the load is higher than the strength.

The physical approach is mainly used for reliability analyses of structural elements, like beams and bridges. The approach is therefore often called *structural reliability analysis*.

In the actuarial approach, we describe all our information about the operating loads and the strength in the probability distribution function $F(t)$ of the time to failure T.

Various approaches can be used to model the reliability of systems of several components and to include maintenance and replacement of components.

When several components are combined into a system, the analysis is called a *system reliability analysis*.

The concept of reliability as a probability means that any attempt to quantify it must involve the use of probabilistic and statistical methods.

An understanding of probability theory and statistics as applicable to reliability engineering is therefore a necessary basis for progress.

The Institute of Electrical and Electronics Engineers (IEEE) –

stated (*adj*) –

specified (*adj*) –
[ˈspesəˌfaɪd]

maintenance (*n*) –
[ˈmeɪnt(ə)nəns]

to maintain (*v*) –
[meɪnˈteɪn]

Introduction

to define / definition / defined, predefined;
to perform / performance;
to rely / reliance, reliability;
to state / statement;
to specify / specification / specific;
to relate / relation / related;
to refer / reference;
to occur / occurrence;
to deduce / deduction;
to vary / variety, variance;
to note / notation;
to consider / consideration;

to provide;
to distinguish;
to concern;
to involve;
to interact;
to expose;
to deteriorate;
to obtain;
to approach;
to propose;
to prefer.

BASIC CONCEPTS
OF
PROBABILITY THEORY

Randomness is the lack of pattern or predictability in events.

Individual random events are by definition unpredictable, but in many cases the frequency of different outcomes over a large number of events (or "trials") is predictable.

Randomness is a measure of uncertainty of an outcome.

Basic Concepts of Probability Theory

Probability is the measure of the likelihood that an event will occur.

Probability quantifies as a number between 0 and 1, where, loosely speaking, 0 indicates impossibility and 1 indicates certainty. The higher the probability of an event, the more likely it is that the event will occur.

In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances, or likelihoods, associated with the various outcomes.

In probability, an experiment refers to any action or activity whose outcome is subject to uncertainty.

Although the word experiment generally suggests a planned or carefully controlled laboratory testing situation, we use it here in a much wider sense.

Thus experiments that may be of interest include tossing a coin once or several times, selecting a card or cards from a deck, weighing a loaf of bread, measuring time to failure of a particular device.

The sample space of an experiment, denoted by $\boldsymbol{\Omega}$, is the set of all possible outcomes of that experiment.

Ex.: An experiment consists of examining a single device to see whether it is defective.

The sample space for this experiment can be abbreviated as $\boldsymbol{\Omega}=\{D,N\}$, where D represents defective, N represents not defective, and the braces are used to enclose the elements of a set.

Ex.: If we examine three devices in sequence and note the result of each examination, then an outcome for the entire experiment is any sequence of *N*s and *D*s of length 3, so

$$\boldsymbol{\Omega}=\{NNN, NND, NDN, NDD, DNN, DND, DDN, DDD\}$$

Basic Concepts of Probability Theory

When dealing with experiments that are random, probabilities can be numerically described by the number of desired outcomes divided by the total number of all outcomes.

$$P = \frac{n}{N}$$

The probability of an event *A* is written as

$$P(A), p(A), Pr\{A\}$$

In probability and statistics, a random variable is a variable whose possible values are outcomes of a random phenomenon.

More specifically, a random variable *X* is defined as a function that maps the outcomes of an unpredictable process to numerical quantities, typically *real numbers*.

$$X\colon \Omega \to D$$

We can consider other sets *D*, such as Boolean values, complex numbers, matrices, categorical values, etc.

Random variables can be classified into two categories, namely, *discrete* and *continuous* random variables.

A random variable is said to be *discrete* if its *sample space is countable*.

If the elements of the sample space are *infinite in number* and *sample space is continuous*, the random variable defined over such a sample space is known as *continuous* random variable.

Basic Concepts of Probability Theory

event [ɪˈvɛnt] -

outcome [ˈaʊtkʌm] -

trial [ˈtrʌɪəl] -

to occur [əˈkəː] -

occurrence [əˈkʌr(ə)ns] -

uncertainty [ʌnˈsəːt(ə)nti] -

sample [ˈsɑːmp(ə)l] -

sample space -

to map -

Try to explain the difference between these concepts:

- *possibility* (*possible, impossible*);
- *probability* (*probable, improbable*);
- *likelihood* (*likely, unlikely*).

DISTRIBUTION FUNCTIONS

Distribution Functions

In probability theory and statistics, a ***probability distribution*** is a mathematical function that provides the probabilities of occurrence of different possible outcomes in an experiment.

In more technical terms, the probability distribution is a description of a random phenomenon in terms of the probabilities of events.

To specify a random variable is to define a probability distribution function.

Distribution Functions

For a *discrete random variable*, the distribution is often specified by just a list of the possible values along with the probability of each.

In some cases, it is convenient to express the probability in terms of a formula.

A *probability mass function (pmf)* is a function that gives the probability that a discrete r.v. X is exactly equal to some value x.

$$f_X(x) = Pr\{X = x\}$$

In other words, for every possible value *x* of the random variable, the *pmf* specifies the probability of observing that value when the experiment is performed.

Each pmf must satisfy two conditions:

$$\forall x \in D \; f_X(x) \geq 0$$

$$\sum_{x \in D} f_X(x) = 1$$

Ex.: Consider a group of five potential blood donors—A, B, C, D, and E—of whom only A and B have type O+ blood.

Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified.

Let the r.v. X = the number of typings necessary to identify an O+ individual.

Then the *pmf* of X is

Distribution Functions

$$f_X(1) = Pr\{X = 1\} = Pr\{A \text{ or } B \text{ typed first}\} = \frac{2}{5} = 0.4$$

$$f_X(2) = Pr\{X = 2\} = Pr\{C, D, \text{or } E \text{ first, and then } A \text{ or } B\} = \frac{3}{5} \cdot \frac{2}{4} = 0.3$$

$$f_X(3) = Pr\{X = 3\} = Pr\{C, D, \text{or } E \text{ first and second, and then } A \text{ or } B\} =$$

$$= \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} = 0.2$$

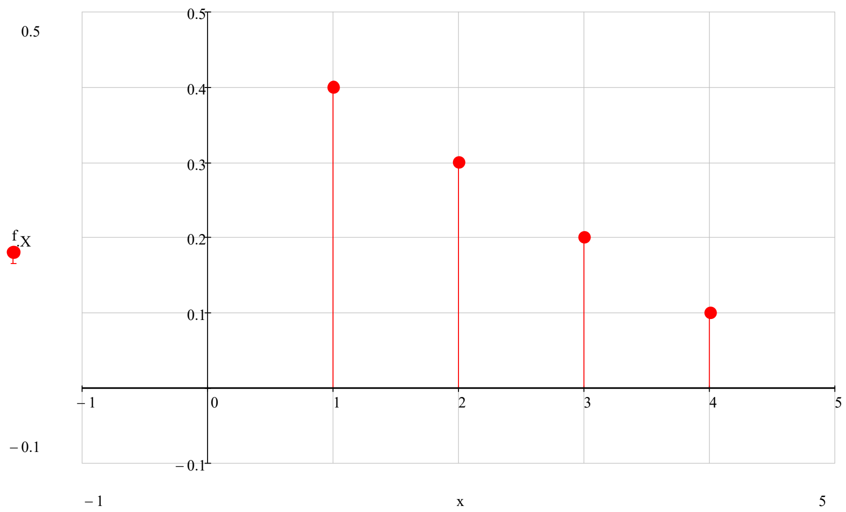
$$f_X(4) = Pr\{X = 4\} = Pr\{C, D, \text{or } E \text{ all done first}\} = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot 1 = 0.1$$

$$f_X(x) = 0 \quad \text{for } x \neq 1,2,3,4$$

The *pmf* can be presented compactly in a tabular form:

x	1	2	3	4
$f_X(x)$	0.4	0.3	0.2	0.1

where any x value not listed receives zero probability.



Distribution Functions

The concept similar to *pmf*, but applied for a continuous random variable, is called ***probability density function (pdf)***.

A *pdf*, or density of a continuous r.v., is a function whose value at any given sample (or point) in the sample space can be interpreted as providing a *relative likelihood* that the value of the random variable would equal that sample.

Distribution Functions

A *pmf* differs from a *pdf* in that the values of the *pdf* are not probabilities as such: a *pdf* must be integrated over an interval to yield a probability.

$$Pr\{a \le x \le b\} = \int_a^b f_X(x)dx$$

Similar to *pmf*, any *pdf* must satisfy two conditions:

$$\forall x \in D \ f_X(x) \ge 0 \qquad \int_{x \in D} f_X(x) = 1$$

For some fixed value *x*, we often wish to compute the probability that the observed value of *X* will be at most *x*.

For example, let *X* be the number of beds occupied in a hospital's emergency room at a certain time of day, and suppose the *pmf* of *X* is given by

x	0	1	2	3	4
$f_X(x)$	0.2	0.25	0.3	0.15	0.1

Then the probability that at most two beds are occupied is

$$Pr\{X \leq 2\} = f_X(0) + f_X(1) + f_X(2) = 0.75$$

Furthermore, we also have $Pr\{X \leq 2.7\} = 0.75$,
and similarly $Pr\{X \leq 2.999\} = 0.75$.

Since 0 is the smallest possible X value,

$$Pr\{X \leq -1.5\} = 0, \qquad Pr\{X \leq -10\} = 0$$

and in fact for any negative number x, $Pr\{X \leq x\} = 0$.

And because 4 is the largest possible value of X,

$$Pr\{X \leq 4\} = 1, \qquad Pr\{X \leq 9.8\} = 1,$$

and so on.

Distribution Functions

In probability theory and statistics, the *cumulative distribution function (cdf)* of a real-valued random variable X, or just distribution function of X, evaluated at x, is the probability that X will take a value less than or equal to x.

$$F_X(x) = Pr\{X \leq x\}$$

In this definition, the "less than or equal to" sign, is a convention, not a universally used one, but is important for discrete distributions.

Every *cdf* F_X is non-decreasing and right-continuous function. Furthermore,

$$\lim_{x \to -\infty} F_X(x) = 0, \qquad \lim_{x \to +\infty} F_X(x) = 1$$

$$F_X(x) = \sum_{x_i \le x} f_X(x_i) = \int_{-\infty}^{x} f_X(\theta)\, d\theta$$

$$Pr\{a \le x \le b\} = F_X(b) - F_X(a) = \int_{a}^{b} f_X(x) dx$$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Distribution Functions