
RELIABILITY THEORY



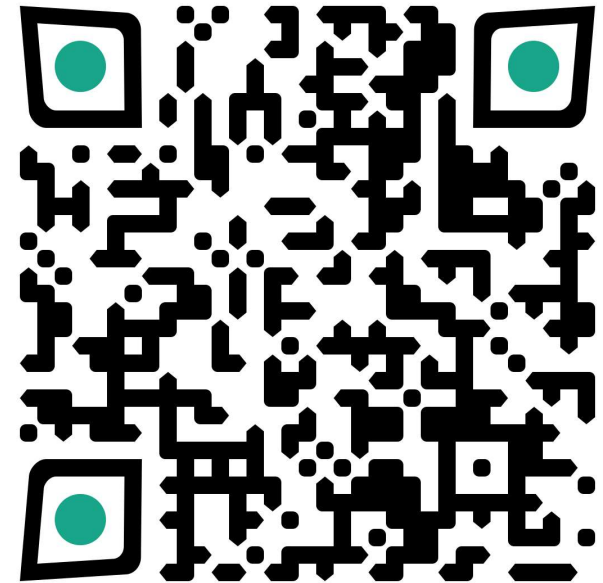
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INTRODUCTION

Introduction

As per IEEE standards, *reliability* is defined as the ability of a system or component to perform its required functions under stated conditions for a specified period of time.

The key elements of the definition are

- ability,
- required function,
- conditions,
- and specified period of time.

Introduction

Ability is expressed quantitatively with probability.

Required function relates to expected performance.

Stated conditions usually refer to environmental conditions of operation.

Specified period of time is also referred as mission time which provides expected duration of operation.

Introduction

We can distinguish between three main branches of reliability theory:

- Hardware reliability
- Software reliability
- Human reliability

This course is concerned with the first of these branches: the reliability of technical components and systems.

Introduction

Within hardware reliability we may use two different approaches:

- The physical approach
- The actuarial (probabilistic) approach

In the physical approach the strength of a technical item is modeled as a random variable S . The item is exposed to a load L that is also modeled as a random variable.

A failure will occur as soon as the load is higher than the strength.

Introduction

The physical approach is mainly used for reliability analyses of structural elements, like beams and bridges. The approach is therefore often called *structural reliability analysis*.

In the actuarial approach, we describe all our information about the operating loads and the strength in the probability distribution function $F(t)$ of the time to failure T .

Introduction

Various approaches can be used to model the reliability of systems of several components and to include maintenance and replacement of components.

When several components are combined into a system, the analysis is called a *system reliability analysis*.

Introduction

The concept of reliability as a probability means that any attempt to quantify it must involve the use of probabilistic and statistical methods.

An understanding of probability theory and statistics as applicable to reliability engineering is therefore a necessary basis for progress.

Introduction

The Institute of Electrical and
Electronics Engineers (IEEE) – *Eye-triple-E*

stated (*adj*) – *заданный, известный*

specified (*adj*) – *заданный заранее, конкретный,*
['spesə,faɪd] *точно определенный*

maintenance (*n*) – *техническое обслуживание и ремонт,*
['meɪnt(ə)nəns] *поддержание работоспособного*
состояния

to maintain (*v*) – *поддерживать, содержать*
[meɪn'teɪn]

Introduction

to define / definition / defined, predefined;
to perform / performance;
to rely / reliance, reliability;
to state / statement;
to specify / specification / specific;
to relate / relation / related;
to refer / reference;
to occur / occurrence;
to deduce / deduction;
to vary / variety, variance;
to note / notation;
to consider / consideration;

Introduction

to provide;
to distinguish;
to concern;
to involve;
to interact;
to expose;
to deteriorate;
to obtain;
to approach;
to propose;
to prefer.

BASIC CONCEPTS OF PROBABILITY THEORY

Basic Concepts of Probability Theory

Randomness is the lack of pattern or predictability in events.

Individual random events are by definition unpredictable, but in many cases the frequency of different outcomes over a large number of events (or "trials") is predictable.

Randomness is a measure of uncertainty of an outcome.

Basic Concepts of Probability Theory

Probability is the measure of the likelihood that an event will occur.

Probability quantifies as a number between 0 and 1, where, loosely speaking, 0 indicates impossibility and 1 indicates certainty. The higher the probability of an event, the more likely it is that the event will occur.

In any situation in which one of a number of possible outcomes may occur, the theory of probability provides methods for quantifying the chances, or likelihoods, associated with the various outcomes.

Basic Concepts of Probability Theory

In probability, an experiment refers to any action or activity whose outcome is subject to uncertainty.

Although the word “experiment” generally suggests a planned or carefully controlled laboratory testing situation, we use it here in a much wider sense.

Basic Concepts of Probability Theory

Thus experiments that may be of interest include tossing a coin once or several times, selecting a card or cards from a deck, weighing a loaf of bread, measuring time to failure of a particular device.

The sample space of an experiment, denoted by Ω , is the set of all possible outcomes of that experiment.

Basic Concepts of Probability Theory

When dealing with experiments that are random, probabilities can be numerically described by the number of desired outcomes divided by the total number of all outcomes.

$$P = \frac{n}{N}$$

The probability of an event A is written as

$$P(A), p(A), Pr\{A\}$$

Basic Concepts of Probability Theory

In probability and statistics, a random variable is a variable whose possible values are outcomes of a random phenomenon.

More specifically, a random variable X is defined as a function that maps the outcomes of an unpredictable process to numerical quantities, typically *real numbers*.

$$X: \Omega \rightarrow D$$

We can consider other sets D , such as Boolean values, complex numbers, matrices, categorical values, etc.

Basic Concepts of Probability Theory

Random variables can be classified into two categories, namely, discrete and continuous random variables.

A random variable is said to be discrete if its sample space is countable.

If the elements of the sample space are infinite in number and sample space is continuous, the random variable defined over such a sample space is known as continuous random variable.

Basic Concepts of Probability Theory

event [ɪ'vent] - *событие*

outcome ['aʊtkʌm] - *исход, результат, следствие*

trial ['trɪəl] - *испытание*

to occur [ə'kəː] - *происходить, случаться*

occurrence [ə'klɜ(ə)ns] - *возникновение, появление*

uncertainty [ʌn'səːt(ə)nti] - *неопределённость*

sample ['sɑːmp(ə)l] - *выборка, образец, экземпляр*

sample space - *пространство элементарных
событий (исходов)*

to map - *отображать*

Basic Concepts of Probability Theory

Try to explain the difference between these concepts:

- ☐ *possibility* (*possible, impossible*);
- ☐ *probability* (*probable, improbable*);
- ☐ *likelihood* (*likely, unlikely*).

DISTRIBUTION FUNCTIONS

Distribution Functions

In probability theory and statistics, a probability distribution is a mathematical function that provides the probabilities of occurrence of different possible outcomes in an experiment.

In more technical terms, the probability distribution is a description of a random phenomenon in terms of the probabilities of events.

To specify a random variable is to define a probability distribution function.

Distribution Functions

For a discrete random variable, the distribution is often specified by just a list of the possible values along with the probability of each.

In some cases, it is convenient to express the probability in terms of a formula.

A *probability mass function (pmf)* is a function that gives the probability that a discrete r.v. X is exactly equal to some value x .

$$f_X(x) = \Pr\{X = x\}$$

Distribution Functions

In other words, for every possible value x of the random variable, the *pmf* specifies the probability of observing that value when the experiment is performed.

Each *pmf* must satisfy two conditions:

$$\forall x \in D \ f_X(x) \geq 0$$

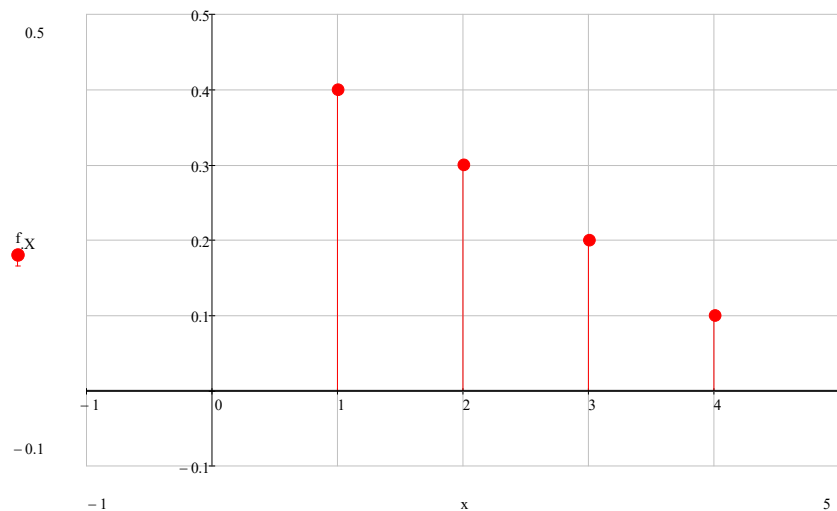
$$\sum_{x \in D} f_X(x) = 1$$

Distribution Functions

The *pmf* can be presented compactly in a tabular form:

x	1	2	3	4
$f_X(x)$	0.4	0.3	0.2	0.1

where any x value not listed receives zero probability.



Distribution Functions

The concept similar to *pmf*, but applied for a continuous random variable, is called *probability density function (pdf)*.

A *pdf*, or density of a continuous r.v., is a function whose value at any given sample (or point) in the sample space can be interpreted as providing a *relative likelihood* that the value of the random variable would equal that sample.

Distribution Functions

A *pmf* differs from a *pdf* in that the values of the *pdf* are not probabilities as such: a *pdf* must be integrated over an interval to yield a probability.

$$Pr\{a \leq x \leq b\} = \int_a^b f_X(x) dx$$

Similar to *pmf*, any *pdf* must satisfy two conditions:

$$\forall x \in D \ f_X(x) \geq 0 \qquad \int_{x \in D} f_X(x) dx = 1$$

Distribution Functions

For some fixed value x , we often wish to compute the probability that the observed value of X will be at most x .

For example, let X be the number of beds occupied in a hospital's emergency room at a certain time of day, and suppose the *pmf* of X is given by

x	0	1	2	3	4
$f_X(x)$	0.2	0.25	0.3	0.15	0.1

Then the probability that at most two beds are occupied is

$$Pr\{X \leq 2\} = f_X(0) + f_X(1) + f_X(2) = 0.75$$

Distribution Functions

Furthermore, we also have $Pr\{X \leq 2.7\} = 0.75$,
and similarly $Pr\{X \leq 2.999\} = 0.75$.

Since 0 is the smallest possible X value,

$$Pr\{X \leq -1.5\} = 0, \quad Pr\{X \leq -10\} = 0$$

and in fact for any negative number x , $Pr\{X \leq x\} = 0$.

And because 4 is the largest possible value of X ,

$$Pr\{X \leq 4\} = 1, \quad Pr\{X \leq 9.8\} = 1,$$

and so on.

Distribution Functions

In probability theory and statistics, the cumulative distribution function (cdf) of a real-valued random variable X , or just distribution function of X , evaluated at x , is the probability that X will take a value less than or equal to x .

$$F_X(x) = Pr\{X \leq x\}$$

In this definition, the "less than or equal to" sign is a convention, not a universally used one, but is important for discrete distributions.

Distribution Functions

Every *cdf* F_X is non-decreasing and right-continuous function. Furthermore,

$$\lim_{x \rightarrow -\infty} F_X(x) = 0, \quad \lim_{x \rightarrow +\infty} F_X(x) = 1$$

$$F_X(x) = \sum_{x_i \leq x} f_X(x_i) = \int_{-\infty}^x f_X(\theta) d\theta$$

$$Pr\{a \leq x \leq b\} = F_X(b) - F_X(a) = \int_a^b f_X(x) dx$$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Distribution Functions

