In a number of problems in probability theory there's a need to determine the expected value and/or the variance of a certain random variables.

If the distribution of a r.v. is known and all of its parameters are defined, one always can consult a textbook for the appropriate formulae.

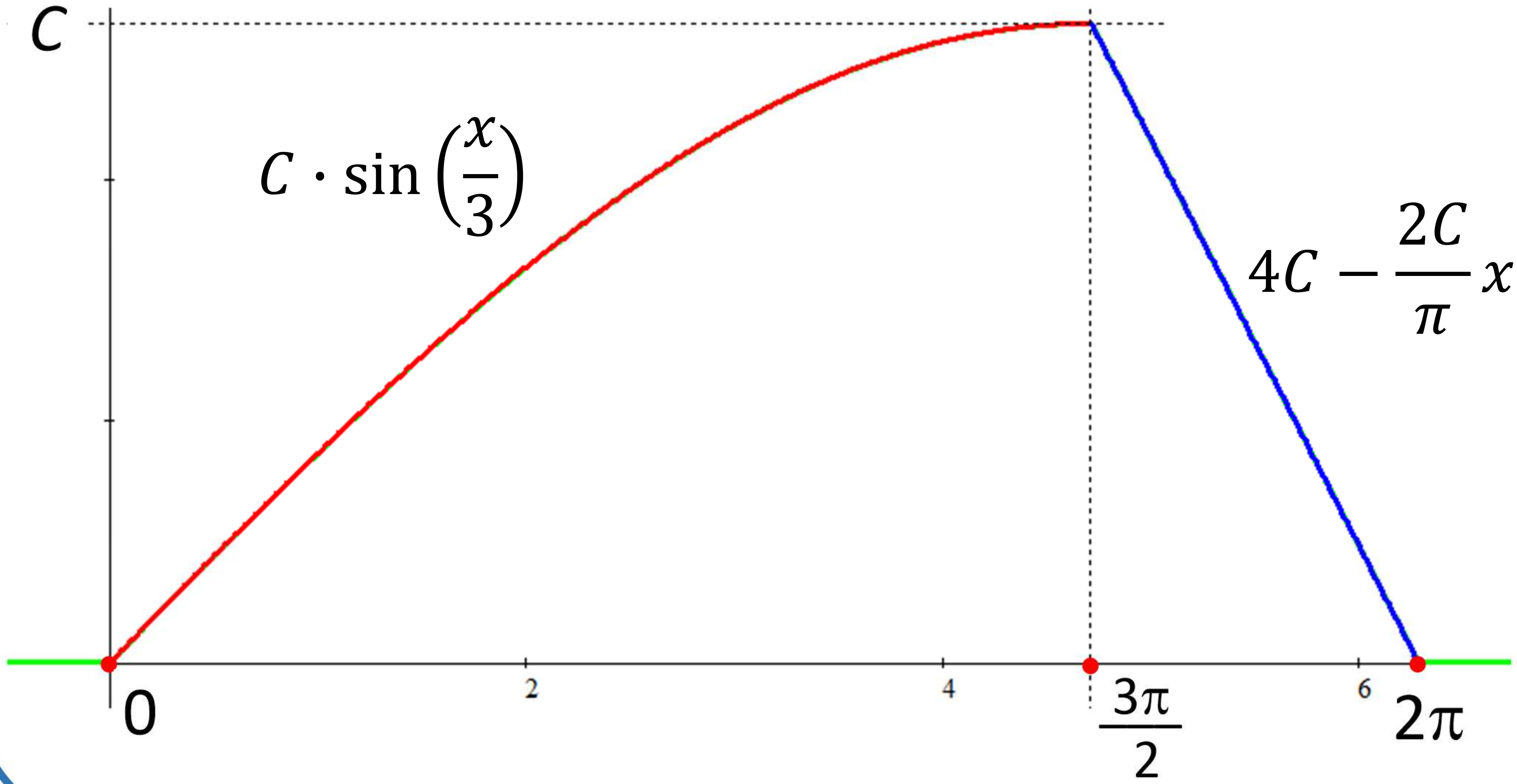
However, that is not always the case.

Example 1

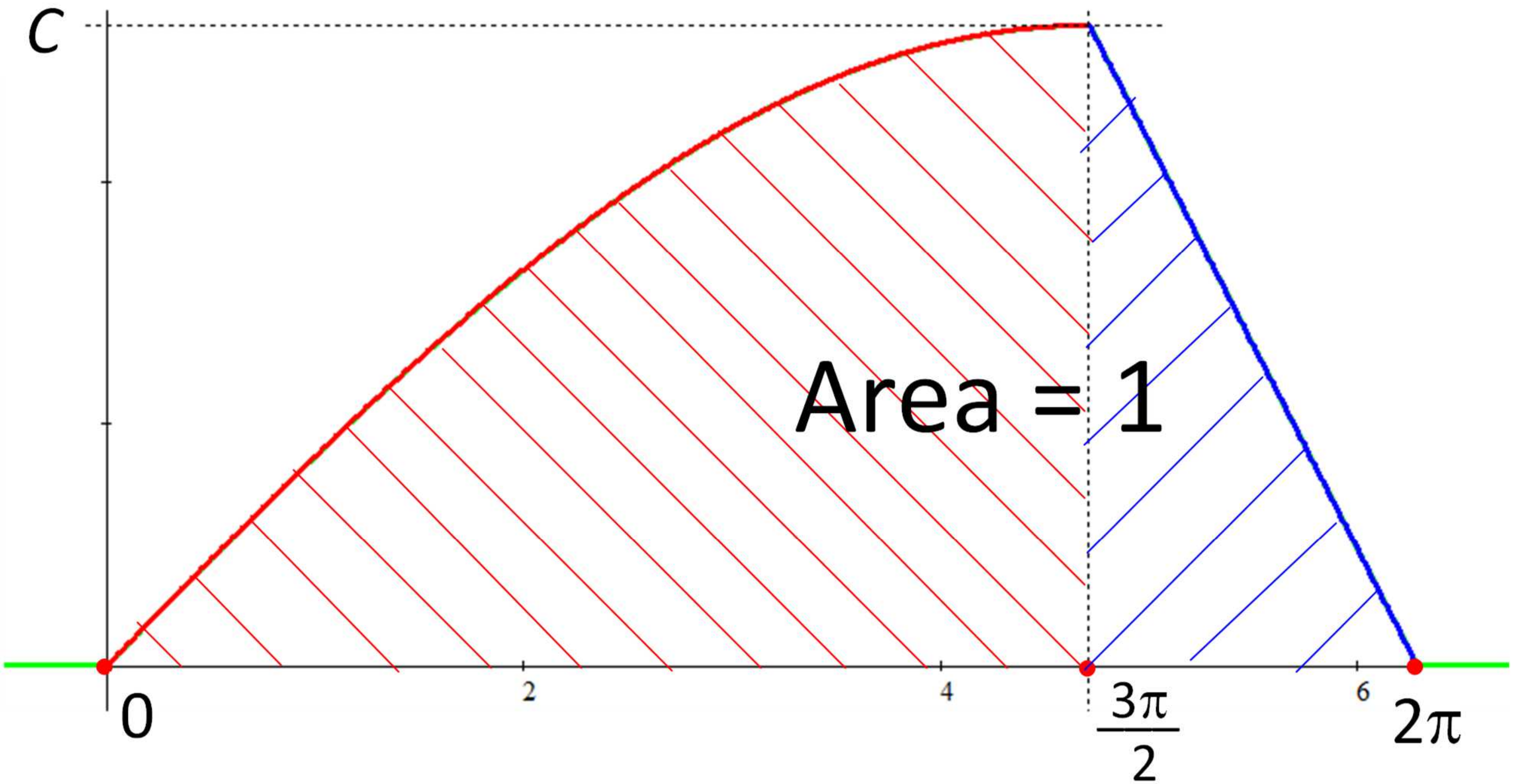
Consider a random variable X with the *pdf* given by the piecewise continuous function

$$f_X(x) = \begin{cases} C \cdot \sin\left(\frac{x}{3}\right), & 0 \le x < \frac{3\pi}{2}; \\ 4C - \frac{2C}{\pi}x, & \frac{3\pi}{2} \le x < 2\pi; \\ 0, & otherwise. \end{cases}$$

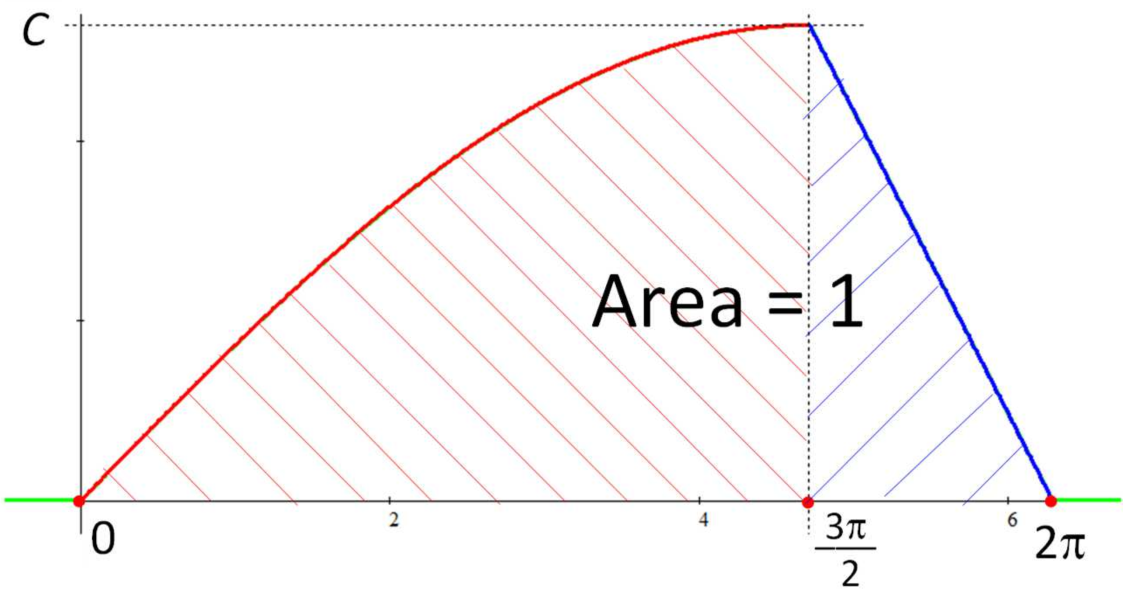
Here C is some unknown constant.



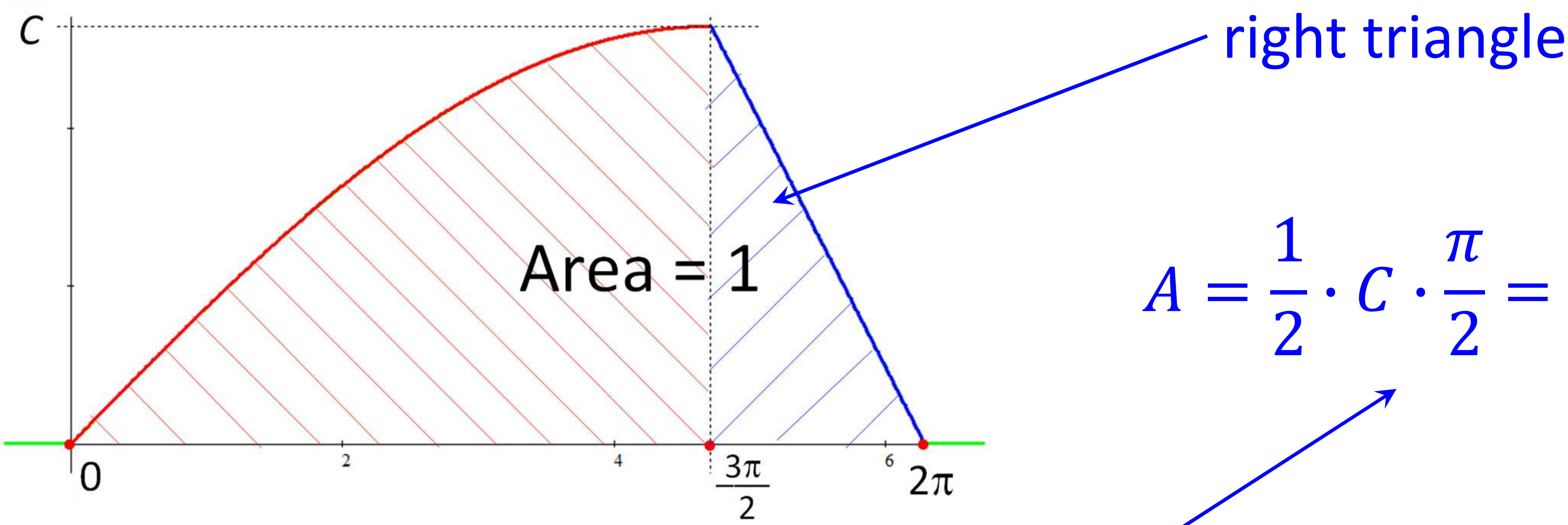
C
$C \cdot \sin\left(\frac{x}{3}\right)$
$4C - \frac{2C}{\pi}x$
2
4
6
0
$\frac{3\pi}{2}$
2π



C
Area = 1
0
2
4
6
3π/2
2π



$$\int_0^{3\pi/2} C \cdot \sin\left(\frac{x}{3}\right) dx + \int_{3\pi/2}^{2\pi} \left(4C - \frac{2C}{\pi} x\right) dx = 1$$



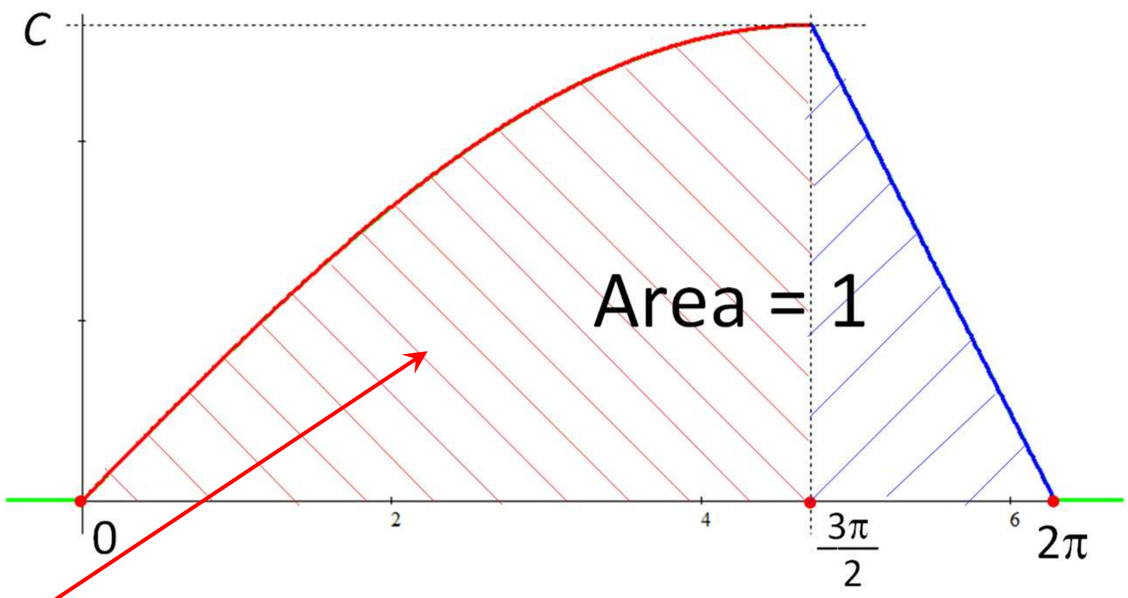
$$A = \frac{1}{2} \cdot C \cdot \frac{\pi}{2} = \frac{C\pi}{4}$$

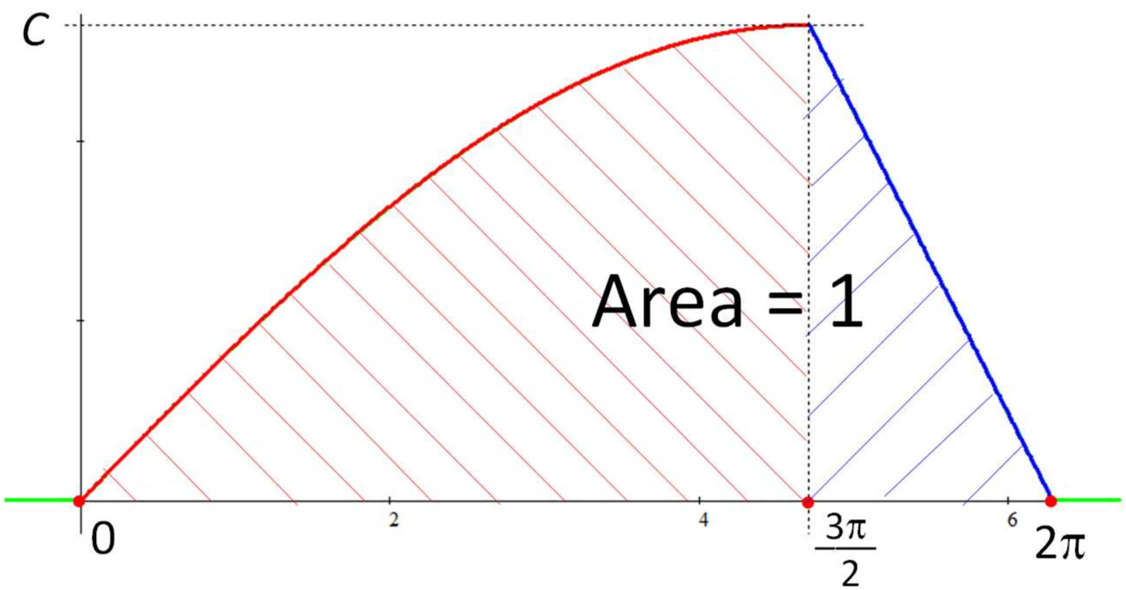
$$\int_{0}^{3\pi/2} C \cdot \sin\left(\frac{x}{3}\right) dx + \int_{3\pi/2}^{2\pi} \left(4C - \frac{2C}{\pi} x\right) dx = 1$$

$$\int_0^{3\pi/2} C \cdot \sin\left(\frac{x}{3}\right) dx =$$

$$= C \cdot \int_0^{3\pi/2} \sin\left(\frac{x}{3}\right) dx =$$

$$= \left[-3C\cos\left(\frac{x}{3}\right)\right]\Bigg|_0^{3\pi/2} = 3C$$





$$3C + \frac{C\pi}{4} = 1$$

$$C\left(3 + \frac{\pi}{4}\right) = 1$$

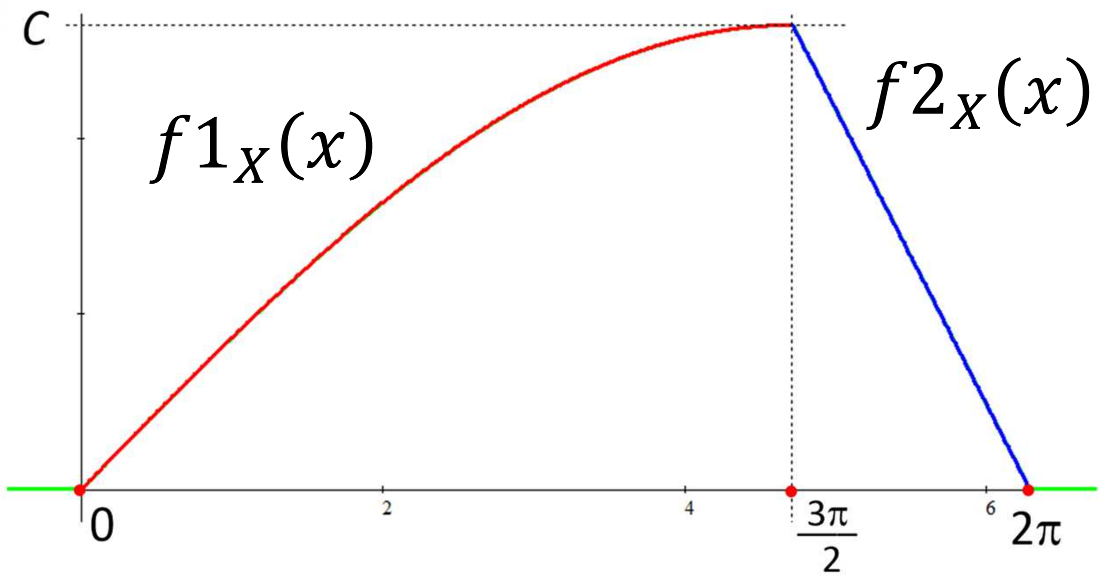
$$C = \frac{4}{12 + \pi}$$

$$f_X(x) = \begin{cases} C \cdot \sin\left(\frac{x}{3}\right), & 0 \leq x < \frac{3\pi}{2}; \\ 4C - \frac{2C}{\pi}x, & \frac{3\pi}{2} \leq x < 2\pi; \\ 0, & otherwise. \end{cases}$$

$$C = \frac{4}{12 + \pi}$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)dx$$

$$Var(X) = \int_{-\infty}^{\infty} (x - E[X])^2 \cdot f_X(x)dx$$



$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)dx =$$

$$= \int_{0}^{2\pi} x \cdot f_X(x)dx =$$

$$= \int_{0}^{3\pi/2} x \cdot f1_X(x)dx + \int_{3\pi/2}^{2\pi} x \cdot f2_X(x)dx\,.$$

For the variance we can either find

$$Var(X) = \int_{-\infty}^{\infty} (x - E[X])^2 \cdot f_X(x) dx\,,$$

or

$$Var(X) = E[X^2] - E[X]^2.$$

The latter expression is sometimes easier to compute.

Example 2

Consider a discrete random variable X defined by its tabulated *pmf*:

x	0	1	4
$f_X(x)$	0.1	p_1	p_2

Find $Var(X)$ assuming that $E[X] = 2.8$.

Example 3

A continuous r.v. X is defined by its *cdf*:

$$F_X(x) = \begin{cases} 1 - \dfrac{C^3}{x^3}, & x \geq C \ (C > 0); \\ 0, & x < C. \end{cases}$$

Find $f_X(x), E[X], Var(X)$.