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## INTRODUCTION TO DIFFERENCE METHODS OF SOLUTION OF THERMAL CONDUCTIVITY PROBLEMS WITH EXAMPLES

Part 1

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- Reduction method
- Iterative methods
- Relaxation method
- Method of lines
- Nonuniform mesh
- Algorithm with variable time step
- Problems with moving bondaries
- Integro-interpolation method
- Examples of problems with convective heat
- Factorization method for different problems
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- Equations with mixed derivatives
- Method of finite volumes
- Introduction to Fnite elemets method Введение в методы конечных элементов
- Examples of two- and three-dimensional problems