L e c t i o n 3
Convective heat exchange

Convection is the substance transfer due to macro volumes (particles) movement of fluids or gases. The motion of these particles can be observed with the naked eye. The mass transfer by convection is accompanied by the thermal conduction; the basic role belongs to the convection.

Combined heat transfer by the convection and thermal conduction is called as ***convection heat exchange***

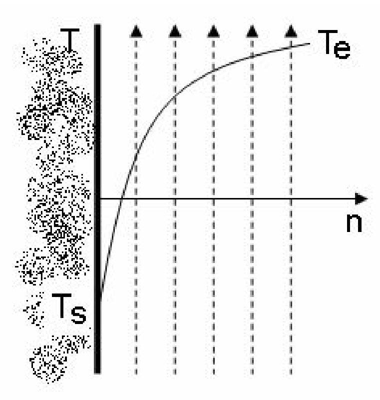
Ones distinct free and forced convection. Convection generated by forced way is called ***forced*** one. If the medium motion connects with the temperature difference and/or densities of medium elements, the convection is called ***free or natural***. Because the liquid velocity at the forced convection is more then at free one, in the fist case the more quantity of the heat can be transferred then in the second one.

If the liquid or gas interrelates with solid surface with other temperature, the process of heat energy exchange is called heat exchange (heat emission).

Reynolds O establishes in Year 884 that two liquid motion regimes exist: ***laminar and turbulent.*** At the laminar flow all fluid particles moves parallel to each other, no mixing. Therefore, the heat transfer in the surface normal direction is carried out by thermal conduction. (Fig.1,a). Due to low thermal conduction coefficients of liquids, the heat propagates in the volume of laminar flow very slowly. At the turbulent mode, the liquid particles move chaotically, with vortexes formation and velocity pulsations. Heat transfer is carried out in this case by the convection. But due to the liquid particles adherence to the surface ***the hydrodynamic boundary layer*** forms near by the body and moves parallel to the wall (Fig.1,b)

The hypothesis on the liquid adherence to the wall was developed by L.Prandtl in year 1904. The velocity in the boundary layer changes from zero to the velocity of basic flow. Within this layer the heat transfer is carried out by thermal conduction.

Hence, the liquid motion character determines the heat transfer mechanisms in the flow.



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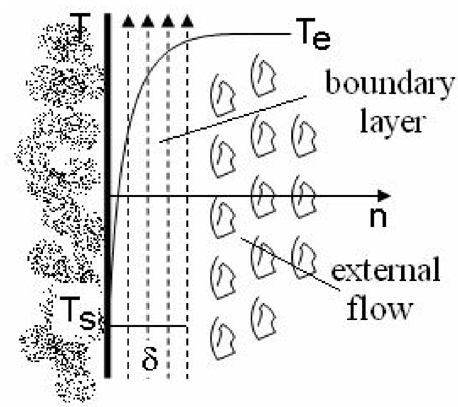

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Fig. 1. Heat emission at the various flow modes: a) at the laminar flow; b) at the turbulent flow

The thickness of the boundary layer δ is the conditional value.

At the heat exchange between the body surface and moving medium, the heat boundary layer appears. It is the wall liquid layer where the temperature changes from the wall temperature T_s to the external flow temperature T_e.

I. Newton notes that temperature difference is controlling factor in the heat exchange between the body and the medium. G. Richman gives firstly in XVIII century the analysis of the cooling processes of the heated body in the air and shows that the heat exchange depends not only on temperature drop, but on surface area and medium volume. The following investigations allow detecting extremely complicity of the heat exchange phenomena and their connection with the hydrodynamics. It was found that the heat quantity received or outputted by the body from environment is proportional to body surface area F, temperature difference T_e and T_s, process duration and depends on physical properties of medium, motion character, body form and size. For elementary area, the process is described by the equation

$$dQ_\tau = \alpha(T_e - T_s)dFd\tau, \tag{1}$$

called basic equation of convective heat exchange or ***Newton-Richman law***. Here $T_e - T_s$ is the temperature drop; α is proportionality coefficient called ***heat exchange coefficient***, W/(m^2·K).

For stationary heat exchange process at constant medium temperature and surface area, the heat flux follow from the equality

$$Q = \alpha(T_e - T_s)F, \tag{2}$$

and the density of the heat flux –

$$q = Q/F = \alpha(T_e - T_s) \tag{3}$$

We find from (3) and (1)

$$\alpha = \frac{dQ_\tau}{(T_e - T_s)dFd\tau} = \frac{q}{(T_e - T_s)}, \tag{4}$$

that is the heat exchange coefficient equals to the heat quantity received or outputted by surface unit during time unit at the temperature difference between the wall and moving medium equal to 1 K.

Heat exchange coefficient characterizes the intensity of the heat exchange between body surface and washed environment and take into account the partial conditions of the process. As opposed to the thermal conductivity coefficient, the heat exchange coefficient is the function of the process. Its values are not contained in reference books.

The convective heat exchange process can be stationary and no stationary. In the second case the temperature field of the liquid changes in time.

Heat transfer through plane wall

Heat transfer from one medium to other through divided them wall is called ***heat transmission.*** Example - the heat transfer from hot water to air through metal walls of radiator.

Let consider the heat transfer through one layer wall of thickness L and thermal conductivity coefficient λ (Fig. 2.). The wall divides two mediums with the temperatures T_{e1} and T_{e2}. As temperatures T_{e1} and T_{e2}, ones assume the media temperatures in enough distance from wall. Qualitative temperature distribution at the condition of stationary heat transfer is shown on this figure also.

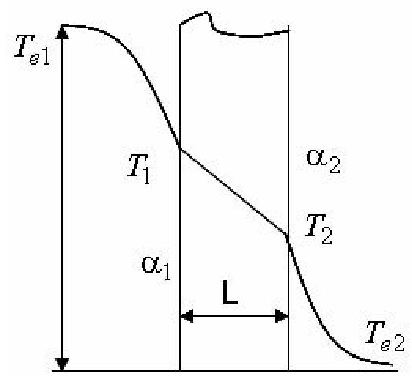

Fig. 2. Qualitative temperature distribution in the wall and in washed media.

Specific heat flux, received by wall, is determined by Newton-Richman law

$$q = \alpha_1(T_{e1} - T_1). \tag{5}$$

Due to continuity condition, it must equal to the heat rejected into the wall. This flux can be written in the form

$$q = \frac{\lambda}{L}(T_1 - T_2). \tag{6}$$

But, as opposed to previous problem, the temperatures T_1 and T_2 are not known. Corresponding to the same law, the heat flux emitted from wall, can be presented in the form

$$q = \alpha_2(T_2 - T_{e2}) \tag{7}$$

and equals also to the heat flux (6). The equation system (5) – (7) is solved very simple. It is follows from it

$$q\left(\frac{1}{\alpha_1}+\frac{L}{\lambda}+\frac{1}{\alpha_2}\right)=T_{e1}-T_{e2}$$

or

$$q=K\left(T_{e1}-T_{e2}\right), \tag{8}$$

where

$$K=\left(\frac{1}{\alpha_1}+\frac{L}{\lambda}+\frac{1}{\alpha_2}\right)^{-1} \text{ -} \tag{9}$$

is the heat transmission coefficient with Wt/(m$^2\cdot$K).

Reverse value

$$R_f=\frac{1}{\alpha_1}+\frac{L}{\lambda}+\frac{1}{\alpha_2} \text{ -} \tag{10}$$

is full thermal resistance, (m$^2\cdot$K)/Wt. Its value is determined by a sum of known resistance of thermal conduction and two resistances of heat emission $1/\alpha_1$ and $1/\alpha_2$.

Using (5) and (7), we shall find the wall surface temperatures

$$T_1=T_{e1}-\frac{q}{\alpha_1};\quad T_2=T_{e2}-\frac{q}{\alpha_2}. \tag{11}$$

If the thickness of metal wall is small $L/\lambda \to 0$, it is follows from (8)

$$K=\frac{1}{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}}.$$

Therefore, when the coefficients α_1 and α_2 are different essential, the transmission coefficient increases as the lesser heat transfer coefficient growths.

Thermal resistances of all layers are taken into account when multi layer wall has been studied.

The rigorous mathematical formulation of this problem includes stationary thermal conduction equation

$$\frac{d^2T}{dx^2}=0$$

and boundary conditions of third kind

$$x=0: -\lambda\frac{dT}{dx}=\alpha_1\left(T_{1e}-T\right);\; x=L:\; -\lambda\frac{dT}{dx}=\alpha_2\left(T_{2e}-T\right).$$

Similarity theory elements

The similarity conception occurs firstly in geometry in the school, when geometric figures. This conception can be propagating to any physical values, processes and phenomena.

Each variable has own similarity constant for complex processes characterizing by many physical values. If the phenomena are similar, then similarity constants correlate between each other by some way. For given physical process (or for the system) the choosing of the similarity constants is stipulated by the similarity conditions for physical phenomena. The similarity constants are the dimensionless complexes combined from physical values, typical for the process or phenomenon. They are called similarity criteria. Similarity criteria have the same values for all similar phenomena.

So, similarity criterion is the dimensionless complex compiled from the values important for given process.

All similarity criteria have certain physical since and their zero dimensions can serve the compilation correctness. Usually, the similarity criteria are called by the names of scientists making large contribution to the study of the processes of heat exchange, hydrodynamics etc.

Similarity theory is based on three theorems.

1. Similarity theory. *Similar phenomena have equal similarity criteria.*

This theorem allows deducting the equation fro the similarity criteria and indicates that it is necessary to measure in experiment only that values, which present in the similarity criteria of the phenomena under study.

2. Similarity theory. *Initial mathematical equation characterizing given physical phenomena can always be presented in the form of the dependencies between similarity criteria suitable for this phenomenon.*

These functional dependencies between similarity criteria are called similarity equations or criterion equations. It is follows from this theorem that the experimental data should be handled and presented in the form criterion equations.

3. Similarity theory. *The phenomena are similar when their single-valuedness conditions are similar and their similarity criteria compiled using single-valuedness conditions are equal numerically. .*

Third theorem establishes the features which allow determine what phenomena are similar; it shows that phenomena for which the experimental results obtained for model system could be propagate.

So, basic similarity criteria for heat, mechanical and hydrodynamical phenomena are obtained from the mathematical equations described corresponding process.

For example, the relation between inertia forces

$$F_i = ma = \frac{\rho l^3 w}{\tau}$$

and mass forces (gravitational force)

$$F_m = mg = \rho l^3 g$$

in the liquid flow is characterized by dimensionless complex

$$\frac{F_m}{F_i} = \frac{\rho l^3 g}{\left(\rho l^3 w\right)/\tau} = \frac{gl}{w(l/\tau)} = \frac{gl}{w^2},$$

which is called ***Froude's criterion***:

$$Fr = \frac{gl_0}{w^2}. \qquad (12)$$

In these equations g is gravitational acceleration; ρ is the liquid density; l_0 is specific linear size; w is the rate; τ is the time (или масштаб времени).

Froude's criterion characterizes the relation of mass forces and inertia forces at the forced liquid motion.

The connection between inertia forces F_i and pressure forces $F_p = \Delta p l^2$ (Δp is the pressure drop) at forced liquid motion

$$\frac{F_p}{F_i} = \frac{\Delta p l^2}{\left(\rho l^3 w\right)/\tau} = \frac{\Delta p}{(\rho l w)/\tau} = \frac{\Delta p}{\rho w^2}$$

is characterized by Euler criterion

$$Eu = \frac{\Delta p}{\rho w^2}. \qquad (13)$$

It is follows from this equation that *Euler number is relationship measure for static pressure drop in the flow (hydraulic resistance) and flow kinetic energy*.

Dimensionless complex showing the connection between inertia forces and viscosity forces $F_\mu = \dfrac{\mu l^2 \Delta w}{\Delta l}$ is very important for the problems of hydrodynamics and forced convection:

$$\frac{F_i}{F_\mu} = \frac{\rho l^3}{\mu l^2}\frac{\Delta w/\Delta\tau}{\Delta w/\Delta l} = \frac{\rho l\,\Delta l/\Delta\tau}{\mu} = \frac{\rho w\, l}{\mu}.$$

It is called ***Reynolds criterion***

$$\mathrm{Re} = \frac{\rho w l_0}{\mu} = \frac{w l_0}{\nu}, \tag{14}$$

where ν is kinematic viscosity.

Reynolds number characterizes the relation between inertia forces and forces of molecular friction and determines the hydrodynamic mode of forced medium motion.

At free medium motion (natural convection), when the motion is carried out only due to difference in densities connecting with temperature field nonuniformity, Grashof number is similarity criterion determining the heat propagation in the medium. It is found from the product of Reynolds number and relation of upward force $F_\rho = \rho g \beta \Delta T l^3$, where β is volume temperature expansion coefficient, to viscosity F_μ:

$$\mathrm{Re}\frac{F_\rho}{F_\mu} = \frac{\rho w l}{\mu}\frac{\rho g \beta \Delta T l^3}{\mu l^2\, w/l} = \frac{g l^3}{(\mu/\rho)^2}\beta\Delta T$$

or

$$Gr = \frac{g\, l_0^{\,3} \beta \Delta T}{\nu^2} \tag{15}$$

Grashof number characterizes the relation between upward force appearing in the medium due to difference in the densities and molecular friction force.

So, Fr (or Gr), Eu and Re are basic similarity criteria for hydrodynamic similarity of flows.

The basic heat similarity criteria can be obtained from the basic equations of heat transfer.

Corresponding to Fourier low, the heat quantity transferred by thermal conduction is

$$Q_\lambda = [(T_1 - T_2)/(\delta/\lambda)]F\tau = \lambda(\Delta T/l)l^2\tau. \tag{16}$$

The heat quantity transmitted due to heat transfer is

$$Q_\alpha = \alpha(T_1 - T_2)dFd\tau = \alpha\Delta T l^2 \tau. \tag{17}$$

The heat received by body of mass M is

$$Q = Mc(T_2 - T_1) = c\rho l^3 \Delta T \tag{18}$$

We obtain from (16) and (18)

$$\frac{Q_\lambda}{Q} = \frac{\lambda}{c\rho}\frac{\Delta T l^2 \tau (1/l)}{l^3 \Delta T} = \frac{a\tau}{l^2}.$$

It is one of basic similarity criteria called ***Fourier number:***

$$Fo = a\tau/l_0^{\,2}, \tag{19}$$

where a is the thermal diffusivity coefficient , $a = \lambda/(c\rho)$.

Fourier number is dimensionless time and characterizes the connection between change rate of temperature field, physical properties and body sizes.

Fourier criterion together with ***Bio criterion***

$$Bi = \alpha \mathrm{l}_0/\lambda_f, \tag{20}$$

where λ_f is thermal conductivity coefficient of the wall, characterizes no stationary processes of the heat propagation.

Physical since of Bio number consists in the establishment of the correlation between the intensities of heat transfer from body surface and the heat supply by thermal conduction from internal body layers to its surface.

From above we can find

$$\frac{Q}{Q_\lambda} = \frac{c\rho}{\lambda}\frac{l^3 \Delta T}{l^2 \Delta T (\tau / l)} = \frac{c\rho w \Delta T}{\lambda (\Delta T / l)} = \frac{wl}{a}.$$

This complex is called as ***Peclet number***

$$Pe = \frac{w\,l_0}{a}. \qquad (21)$$

Peclet criterion is the relation of heat flux density, transported by convection, to the heat flux density transferred by thermal conduction.

The ***Nusselt number***, important in the convective heat exchange theory, can be found from (16) and (17):

$$\frac{Q_\alpha}{Q_\lambda} = \frac{\alpha}{\lambda}\frac{l^2 \Delta T \tau}{l^2 \Delta T \tau (1/l)} = \frac{\alpha l}{\lambda}.$$

Nusselt number is dimensionless heat transfer coefficient

$$Nu = \frac{\alpha l_0}{\lambda} \qquad (22)$$

and characterizes *the heat exchange intensity in the interface «solid body – liquid».*

The heat exchange coefficient enters in Nusselt number and is usually determined value in heat exchange theory. Seemingly, Nu coincides with Bi. But there is significant difference between these criteria. Thermal conductivity coefficient of the wall enters in Bio criterion. It has been used for internal heat exchange description. The conductivity coefficient of washed medium enters in Nusselt criterion; it has been used for external heat exchange description.

One can compile the ***Prandtl criterion*** from (21) and (22)***:***

$$\frac{Pe}{\mathrm{Re}} = \frac{wl}{\mathrm{a}} : \frac{wl}{\nu} = \frac{\nu}{\mathrm{a}} = \mathrm{Pr} \qquad (23)$$

Prandtl criterion characterizes the thermal physical properties influence on the convective heat exchange and is the similarity measure of the temperature and velocity fields.

The similarity criteria Fo, Pe (или Re), Nu must have the same values at heat similarity.

Besides heat similarity, hydrodynamical and geometrical similarity should take a place.

Determined linear size l_0 enters in many criteria. It is the size from that the convective heat exchange development depends farthest.