Lection 2

Mechanisms of heat transfer: thermal conductivity, convection, Irradiation

There are three basic mechanisms of heat transfer:

Thermal conductivity is the heat transfer due to energy transfer by micro particles Molecules, atoms, electrons and other micro particles contained in substance move with rates proportional to their temperature and transfer the energy from zone with high temperature to zone with low temperature.

Heat transfer together with macroscopic volumes of substance is called ***convective heat transfer*** of convection. It is possible to pass the heat to large distances, from example from heat electro power station.

It is necessary often to calculate the convective heat transfer between liquid and solid surface. Such process has special name – *convective heat exchange.* (The heat goes from liquid to surface or inversely).

Irradiation is third way of heat transfer. Due to irradiation the heat could be propagate in all ray-transparent media including in vacuum, outer space, where the irradiation is unique way of heat exchange between bodies. *The photons* emitted and absorbed by bodies participating in heat exchange are energy carrier in this case.

Mass transfer from one space point to other appears at the presence of the difference in the concentration of this substance in chosen points. For example, the gas mixture components no uniform disturbed in volume can transfer from the region where their concentration is higher into regions where the concentration is low.

The ways of mass transfer analogously to heat ones can be various. When the mass is transferred due to atoms or molecules motion, the mass transfer is called ***diffusion***. Diffusion runs the most quickly in gases, where the particles are more mobile then in liquids and in solid bodies. ***The convective mass transfer*** is possible in gases and liquids due to macroscopic volumes translation. The analogy for irradiation is absent for mass transfer.

It have calculate the convective mass transfer from body surface to liquid or gas phase during sublimation, drying, chemical reactions. This process is called as mass exchange.

Analogous process of the heat and mass transfer are described by the same differential equations (with other variables and coefficients). Therefore, many formula and particular problems of the heat transfer theory will similar to the formulae and problems for theory of mass transfer.

The ***more complex mechanisms*** of heat and mass transfer exist and are observed in technology processes of material treatment (for example, (thermal diffusion, multi component diffusion etc.).

Temperature field characteristics

In any case the heat transfer is accompanied by body temperature change in space and in time. Analytical investigation of thermal conductivity process comes to the study of space-time distribution of the temperature that is to the equation finding

$$T = T(x, y, z, t). \quad (1)$$

That is mathematical expression of temperature field.

One's recognize the stationary and no stationary temperature fields.

If the temperature does not change in any point, temperature field is called as stationary. In this case we can write down

$$T = f_1(x, y, z);\ \partial T/\partial t = 0\,. \tag{2}$$

Temperature field can be three dimensional (1); two-dimensional

$$T = f_2(x, y, t);\ \partial T/\partial z = 0 \tag{3}$$

and one-dimensional

$$T = f_3(x, t);\ \partial T/\partial z = 0\,;\ \partial T/\partial y = 0\,. \tag{4}$$

That depends on physical situation.

We choose in solid body the surface so that all their points had the same temperature T_i in some time moment (Fig.1). That surface is called as ***isothermal surface*** of temperature T_i. These isothermal surfaces can locate any way. But two such surfaces can not intersect because none part has two temperature simultaneously.

The intersection of isothermal surfaces by plane gives the ***isotherm family.*** The isotherms on the fig.1 differ on ΔT

Temperature in the body changes only in the directions crossing isothermal surfaces. Thai with the largest temperature drop per unit length happens in the direction of the normal to isothermal surface.

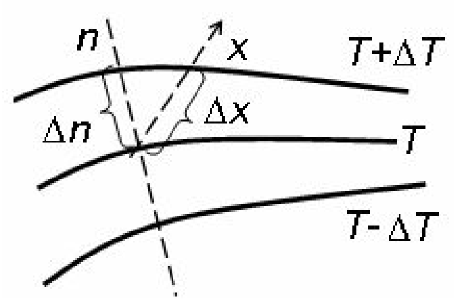


Fig. 1. Isotherms

Temperature growth in the direction of the normal to the isothermal surface is characterized by ***temperature gradient***.

Temperature gradient is the vector directed along the normal to the isothermal surface in the direction of temperature growth and equal to temperature derivative in this direction

$$\nabla T \equiv gradT = \vec{n}_0 \frac{\partial T}{\partial n}\,, \tag{5}$$

where $\vec{n}_0$ is the normal to isothermal surface directed to temperature increase; $\partial T/\partial n$ is normal-derivative of the temperature. производная температуры по нормали.

Projections of vector-gradient ∇T to coordinate axis's of Cartesian coordinate system Ox, Oy, Oz are

$$\begin{aligned}(\nabla T)_x &= \frac{\partial T}{\partial n}\cdot\cos(n, x) = \frac{\partial T}{\partial x};\\ (\nabla T)_y &= \frac{\partial T}{\partial n}\cdot\cos(n, y) = \frac{\partial T}{\partial y};\\ (\nabla T)_z &= \frac{\partial T}{\partial n}\cdot\cos(n, z) = \frac{\partial T}{\partial z}.\end{aligned} \tag{6}$$

Thermal conduction

The condition for heat propagation in a space consists in the temperature difference between differ points. It is correct for any heat exchange mechanisms.

The ***Fourier low*** is basic law of heat transfer by thermal conductivity. That hypothesis (1768-1830) is formulated by following way: elementary quantity of the heat

dQ (J), passing through element of isothermal surface dF during time dt is proportional to temperature gradient

$$dQ = -\lambda \frac{\partial T}{\partial n} dF dt\,. \tag{7}$$

The proportionality coefficient λ is the material constant called ***thermal conductivity coefficient.***

Heat flux density is determined by relationship

$$\mathbf{q} = -\lambda \vec{n}_0 \frac{\partial T}{\partial n} \tag{8}$$

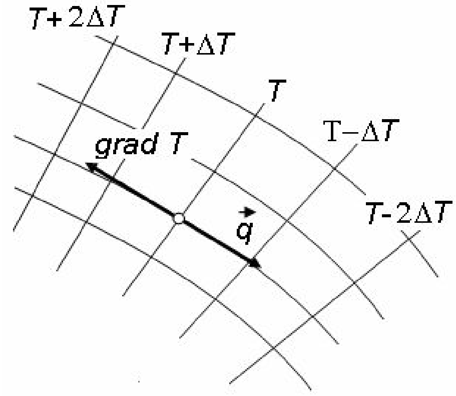


Fig. 2. Isotherms and streamlines

Heat flux density directs to isothermal surfaces. Its positive direction coincides with the direction of temperature decrease, that is the heat is transferred always from hot points to cold ones.

The lines, the tangents to which coincide with the direction of the vector of heat flux density $\mathbf{q}$, J/(s m^2), are called as the ***lines of heat flux***. The lines of the heat flux are orthogonal to isothermal surfaces (Fig. 2). Scalar value of the heat flux density is determined by

$$q = -\lambda \frac{\partial T}{\partial n}\,, \tag{9}$$

The Fourier hypothesis was confirmed experimentally.

Heat quantity through all isothermal surface during time unity, J/s, is

$$Q = \int_F q dF = -\int_F \lambda \frac{\partial T}{\partial n} dF\,,. \tag{10}$$

Full heat during the time τ through F, J, follows from

$$Q_\tau = -\int_0^\tau \int_F \lambda \frac{\partial T}{\partial n} dF dt \tag{11}$$

Components of the vector of the heat flux density are determined by formulae

$$q_x = -\lambda \frac{\partial T}{\partial x}\,;\; q_y = -\lambda \frac{\partial T}{\partial y}\,;\; q_z = -\lambda \frac{\partial T}{\partial z}\,. \tag{12}$$

Thermal conduction coefficient

Thermal conductivity coefficient equals to heat quantity which passes during time unity through isothermal surface unity at the temperature gradient equal to unity:

$$\lambda = \frac{|\mathbf{q}|}{|\nabla T|}\,.$$

The concrete mechanism of the heat transfer by thermal conductivity depends on physical properties of medium

Thermal conductivity coefficient of gases change from 0.006 to 0.6 Vt/(m.K); increases with the temperature, but does not practically with the pressure.

The thermal conductivity coefficient of dropping liquids_ lies in the limits from 0,07 to 0, 7 Vt/(m.K); for the most liquids decreases with the temperature excluding the water and glycerol and grows with the pressure.

Free electrons are basic transmitter for the heat in metals and alloys and are similar to ideal monoatomic gas. In metals, λ decreases with the temperature. Unlike pure metals, thermal conductivity coefficients of the alloys rise with temperature.

In solid bodies-dielectrics, thermal conductivity coefficient rises with the temperature growth. As a rule, the materials with greater density have a more high value. This coefficient depends on material structure.

The simplest problem on plane wall

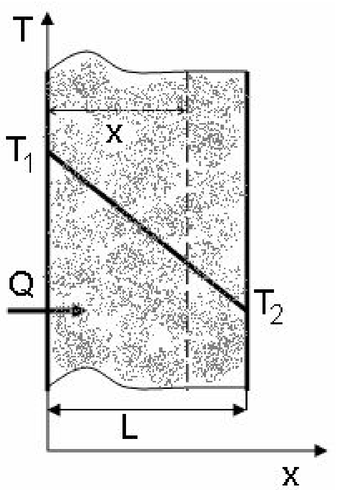


Fig. 3. Heat transfer by thermal conduction through plan wall

This problem is the simplest example of Fourier low employment. Let L (Fig. 3) is the thickness of the wall. The surface temperatures are constant and differ from each other ($T_1 > T_2$). Heat is transferred in one direction - along the normal to the wall surfaces.

Corresponding to Fourier law we have

$$dQ = -\lambda F \frac{dT}{dx} dx .$$

Integrating this equation for constant λ, we shall find

$$Q = \frac{\lambda F}{L}(T_1 - T_2) \text{ or } q = \frac{(T_1 - T_2)}{L/\lambda} \quad (13)$$

The value in denominator is called as ***thermal resistance or the resistance by thermal conduction*** for plane wall.

This and other (more complex) problems can be solved enough rigorous with the help of modern methods of mathematical physics/

Thermal conductivity equation

To solve the problems connecting with temperature fields determination it is necessary to have the differential thermal conductivity equation. There are many ways for this equation derivation

In the simplest case the following assumptions are taken. The body is uniform and isotropic; its physical parameters are constant, deformation connecting with temperature change is negligible small; internal heat sources can be given in the form

$$q_V = q_V(x, y, z, t), \quad (14)$$

and are even.

To deduct the differential equation the conservation energy law is used in the form:

The sum of heat quantity dQ_1, *entered in elementary volume during time* dt *due to thermal conduction and the heat from internal sources* dQ_2 *are equal to internal energy or enthalpy change* dQ *depending on process type*:

$$dQ_1 + dQ_2 = dQ\,. \tag{15}$$

Using the Fig. 4, we shall find the heat quantity transferred in three directions. For example

$$dQ_{x1} = dQ_x - dQ_{x+dx} = \\ = q_x dydzdt - q_{x+dx} dydzdt\,.$$

Expanding the heat flux in a series

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx + \frac{\partial^2 q_x}{\partial x^2}dxdx\frac{1}{2!} + \ldots .$$

and restricting by items of fist step of accuracy, we find

$$dQ_{x1} = -\frac{\partial q_x}{\partial x}dxdydzdt\,.$$

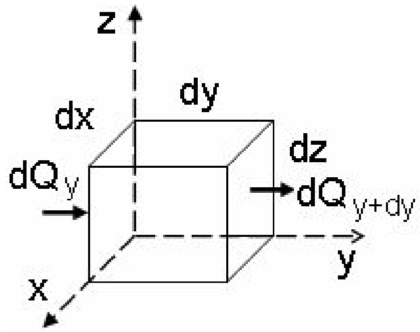


Fig. 4. Illustration to thermal conductivity equation derivation

As a result we obtain the heat entered by thermal conduction

$$dQ_1 = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)dxdydz\,. \tag{16}$$

We have for the heat release in the volume

$$dQ_2 = q_V dVdt\,, \tag{17}$$

where $dV = dxdydz$.

For isochoric process we had in the Lection 1

$$\left(dQ_3\right)_V = dU = \rho\frac{\partial u}{\partial t}dtdV = c_v\rho\frac{\partial T}{\partial t}dtdV\,. \tag{18}$$

We find from (15)-(18)

$$c_v\rho\frac{\partial T}{\partial t} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) + q_V$$

или

$$c_v\rho\frac{\partial T}{\partial t} = -\nabla\cdot\mathbf{q} + q_V\,. \tag{19}$$

For isobaric process, we obtain

$$c_p\rho\frac{\partial T}{\partial t} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) + q_V$$

or

$$c_p \rho \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + q_V \,. \tag{20}$$

For solid bodies the heat capacity c_p differs on c_v, as a rule, small. Using the Fourier law, we write

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + q_V \,. \tag{21}$$

The equation (19)-(20) are correct for any coordinate systems and for $\lambda = \lambda(T)$.

Problem formulation in thermal conductivity theory

We stop on one-dimensional problem, when thermal conductivity equation which for the Fig.3 has the form

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) \tag{22}$$

For one-dimensional stationary temperature field we come to the equation

$$\frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) = 0 \text{ or } \frac{d^2 T}{dx^2} = 0 \,. \tag{23}$$

Additionally we had two boundary conditions

$$x = 0: \; T = T_1 \text{ and } x = L: \; T = T_2 \,. \tag{24}$$

Without (24), the equation (23) has infinite many solutions.

Giving the boundary conditions (24) **we formulate the boundary problem**.

To solve arbitrary practical problem it is necessary to give certain data. The part of the conditions allows to go to more simple mathematical formulation (they are physical and geometrical conditions). Other part of the data allows choosing the unique solution corresponding to given boundary conditions. They are the single-valuedness conditions initial and boundary). ***The boundary conditions are divided on the conditions of first, second and third kind*** The conditions ***of fourth kind*** are used in the problems when the heat exchange between materials or various media is studied.

Electrical analogy for thermal conduction

It is expedient in some situations to use another approach to heat transfer. This approach is called as an ***analogy between transfer of the heat and the electricity***. If we assume that heat flux is similar to currant, the complex $L/(\lambda F)$ will been considered as resistance and temperature difference - as an analogy of potential drop, then the relation Если считать, что тепловой поток аналогичен электрическому току, комплекс рассматривать как сопротивление, а разность температур как аналог разности потенциалов, то соотношение (13) could be written by analogy Ohm law

$$Q = \frac{\Delta T}{R} \,,$$

where $\Delta T = T_1 - T_2$ is temperature drop (thermal potential), is thermal resistance. Reverse value of thermal resistance is thermal conductivity, and the relation λ/L is specific heat conductivity for conductive heat flux.

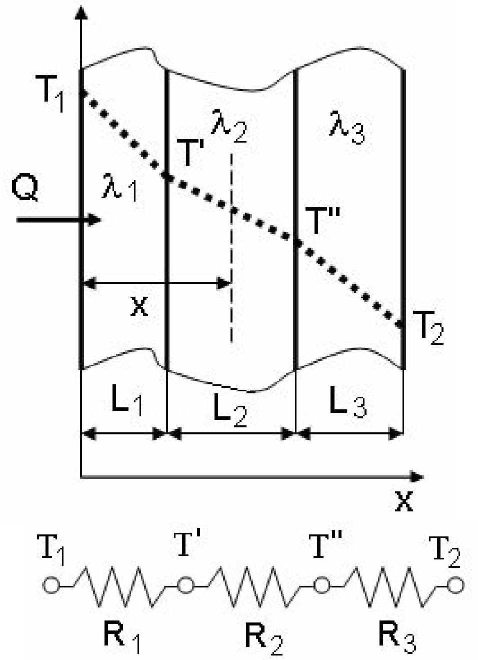


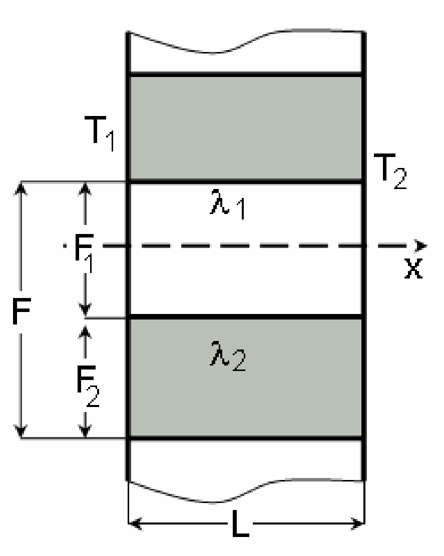
Fig. 5. Heat transfer by thermal conduction through three layer wall and electrical analogy

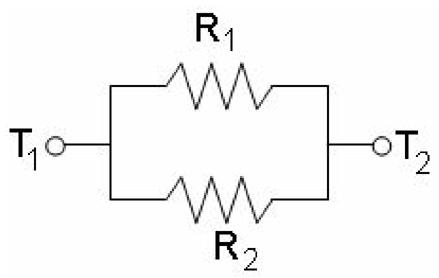
By analogous way ones can present the formulae for the heat flux through three layer wall (Fig.5):

$$Q = \frac{\Delta T}{R_1 + R_2 + R_3},$$

where $\Delta T = T_1 - T_2$, $R_i = L_i/(\lambda_i F)$, $i = 1,2,3$.

Electrical analogy can use for the solution of more complex problems. For example the thermal conduction process occurs in the material with the layers parallel to heat flux direction (Fig.6). The plate on this figure consist of two materials with sections F_1 and F_2. Corresponding electric chain is presented on the right hand. It is assumed here that the heat transfer process is one dimensional in each material. This approach is correct, when the temperature difference between contacted materials is small. In this case, the heat flux along the layers will larger then the heat flux in perpendicular direction.





$$R_1 = \frac{L}{\lambda_1 F_1}$$

$$R_2 = \frac{L}{\lambda_2 F_2}$$

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Fig 6. Thermal conduction through the wall with parallel sections