Digital Signal and Image Processing

Overview

- This course presents the fundamentals of digital signal processing with particular emphasis on problems in signal detection and image recognition.
- It covers principles and algorithms for processing both deterministic and random signals.
- Topics include data acquisition, imaging, filtering, coding, feature extraction, and modeling.
- The focus of the course is a series of labs that provide practical experience in noised data processing.
- The labs are done in MATLAB® during weekly lab sessions that take place in an electronic classroom.
- Lectures cover signal processing topics relevant to the lab exercises.

Motivation of the DSIP course study

- Integration role of signal and image processing in the frame of information engineering.
- Interdisciplinary area connecting mathematics and engineering: control, measuring engineering, vision, speech processing, biomedicine, environmental engineering, etc.
- Fundament for data acquisition, system identification and modelling, signal de-noising, feature extraction, segmentation, classification, compression, recognition.
- Similar mathematical background based on methods of time-frequency and time-scale analysis in different areas.

Similar Courses in the Top World Universities

- The basis of proposed course is the similar course created and delivered at the Massachusetts Institute of Technology (the first place in the QS World University Rankings® 2015/16). MIT Course Number HST.582J / 6.555J / 16.456J "Biomedical Signal and Image Processing“:

  http://web.mit.edu/6.555/www/

- There are other similar courses. See, e.g., DIGITAL SIGNAL AND IMAGE PROCESSING at the University of Central Lancashire (UCLan) in Preston, UK. It is the whole MSc and PGDip Programs containing a number of specific courses. School of computing, engineering and physical sciences provides programme: http://www.uclan.ac.uk/courses/msc_pgdip_digital_signal_and_image_processing.php

Welcome to the **QS World University Rankings® 2015/16**. Use the interactive ranking table to explore the world's top universities, with options to sort the results by country, region and subject area. You can also sort the ranking results based on the six individual indicators used (see the full methodology here). For alerts about the latest rankings releases, sign up to our newsletter.

Filter by region | Filter by location | reset

Filter by faculty | Note: Filtering by subject area will also resort the list by subject-area scores. | reset

RANK	UNIVERSITY	LOCATION	QS STARS ?
Overall Score	Search for universities...		Show only
1 100.0	Massachusetts Institute of Technology (MIT)		
2 98.7	Harvard University		
3 98.6	University of Cambridge		
3 98.6	Stanford University		
5 97.9	California Institute of Technology (Caltech)		

Course Features

- MIT course is based on Biomedical Signal and Image examples only. It is very narrow band for a wide engineer education.
- Proposed course has been modified by examples in technical areas: a problem weak signal detection in a strong noise, a problem of functional diagnostics of turn-to-turn short circuits in synchronous generator, a problem of a wildland fire recognition, etc.
- Almost all practical examples had been taken from an author's experience of performed and published research before. Hence, students will receive a first-hand information, an up-to-day knowledge including know-how.

DSIP Fundamentals Literature

- There are many tutorials, handbooks and articles dedicated DSIP Fundamentals. See, e.g.:
- J.-C. Pinoli, “Mathematical Foundations of Image Processing and Analysis,” Wiley, 2014.
- T. Bose, “Digital Signal and Image Processing,” Wiley, 2011.
- Up-to-day scientific articles have been publishing in specific journals. See, e.g.,
- "Signal Processing" Journal (http://www.journals.elsevier.com/signal-processing/) and
- "Digital Signal Processing" Journal (http://www.journals.elsevier.com/digital-signal-processing/) published by Elsevier.

Author's publications in DSIP area

- Device to calculate discretised continuous wavelet transform / European Patent Office, 2011.
- Parallel computation of the continuous wavelet transform for detection of narrow-band sonar signals / Electronic Journal «Technical Acoustics», 2012, 5.
- Apparatus for detecting narrow-band hydroacoustic noise signals based on continuous wavelet transformation /European Patent Office, 2013.
- Preprocessing of Coefficients for Reusable Continuous Wavelet Transform / Advanced Materials Research, vol. 1040, 2014.
- Numerical simulation of hydroacoustic noise signals detecting by a two stages wavelet transform / IEEE conference, 2014.
- And many other publications. See, e.g.: http://portal.tpu.ru/SHARED/a/AAXTPU

DSIP course troubles

- The similar MIT-course have 72 hours (26 lectures and 5 labs, see next slide). TPU-course curriculum allocated us 32 hours only practices without lectures.
- Trouble 1: How to deliver so much knowledge in such short period in the schedule?
- The details lab manuals and programs of MIT-course open resource are not available for all. It is the trouble number two. So we should create own labs.
- Trouble 3: Students should study many topics by himself. However, there are very complex topics in this course. It demands good mathematical knowledge.

Massachusetts Institute of Technology

HST.582J/6.555J/16.456J Course Calendar v3 Spring 2015

http://web.mit.edu/6.555/www/calendar.html

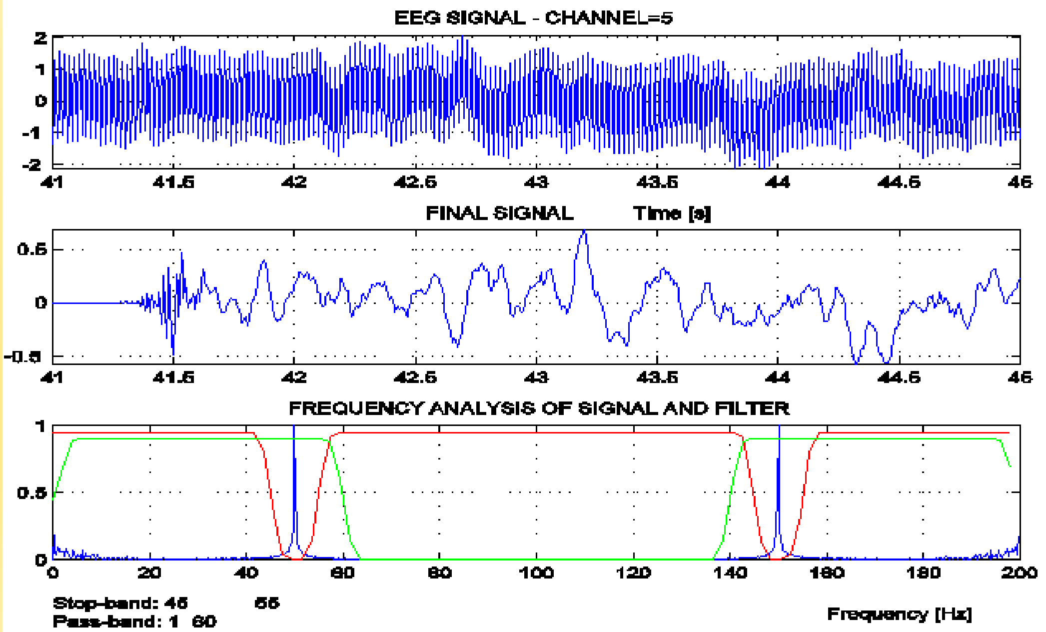
Monday	Tuesday	Wednesday	Thursday	Friday
2/2 REG DAY	**2/3** Lecture 1: JG Data Acquisition	**2/4** No Lab	**2/5** Lecture 2: guest ECG PS1 out	**2/6** Lab 0: TA Fundamentals of MATLAB®
2/9	**2/10** Lecture 3: JG Digital Filtering Lab 1 out	**2/11** Lab 1A: ECG	**2/12** Lecture 4: JG DTFT PS1 due / PS2 out	**2/13** Lab 1A: ECG
2/16 PRESIDENTS' DAY HOLIDAY	**2/17** MONDAY CLASS SCHEDULE: NO CLASS	**2/18** Lab 1B: ECG	**2/19** Lecture 5: JG DFT PS2 due / PS3 out	**2/20** Lab 1B: ECG
2/23	**2/24** Lecture 6: JG Sampling Revisited	**2/25** Lab 1C: ECG	**2/26** Lecture 7: JG Speech Signals PS3 due	**2/27** Lab 1C: ECG
3/2	**3/3** Lecture 8: JG Speech Coding Lab 2 out	**3/4** Lab 2A: Speech Coding	**3/5** Lecture 9: JG Image Processing I Lab 1 due / PSX out	**3/6** ADD DATE Lab 2A: Speech Coding
3/9	**3/10** Lecture 10: SW Image Processing II	**3/11** Lab 2B: Speech Coding	**3/12** Lecture 11: SW Image Registration I PSX Solutions out	**3/13** Lab 2B: Speech Coding
3/16	**3/17** not Lecture 12: Quiz I	**3/18** Lab 2C: Speech Coding	**3/19** Lecture 13: SW Image Registration II	**3/20** Lab 2C: Speech Coding
3/23 SPRING VACATION	**3/24** SPRING VACATION	**3/25** SPRING VACATION	**3/26** SPRING VACATION	**3/27** SPRING VACATION
3/30	**3/31** Lecture 14: JG Probability Lab 3 out	**4/1** Lab 3A: Image Registration	**4/2** Lecture 15:JG Random Signals I Lab 2 due / PS4 out	**4/3** Lab 3A: Image Registration
4/6	**4/7** Lecture 16: JG Random Signals II	**4/8** Lab 3B: Image Registration	**4/9** Lecture 17: guest Blind Source Separation	**4/10** Lab 3B: Image Registration
4/13	**4/14** Lecture 18: SW Imaging Modalities PS4 due / Lab 4 out	**4/15** Lab 4A: Blind Source Separation	**4/16** Lecture 19:JG Hypothesis Testing I Lab 3 due / PS5 out	**4/17** Lab 4A: Blind Source Separation
4/20 PATRIOTS DAY HOLIDAY	**4/21** PATRIOTS DAY HOLIDAY	**4/22** Lab 4B: Blind Source Separation	**4/23** DROP DATE Lecture 20: JG Hypothesis Testing II	**4/24** Lab 4B: Blind Source Separation
4/27	**4/28** Lecture 21: SW Image Segmentation PS5 due / Lab 5 out	**4/29** Lab 5A: Image Segmentation	**4/30** Lecture 22: guest MR Physics Lab 4 due / PSY out	**5/1** Lab 5A: Image Segmentation
5/4	**5/5** Lecture 23: guest Image Guided Therapy PSY Solutions out	**5/6** Lab 5B: Image Segmentation	**5/7** Not Lecture 24: Quiz 2	**5/8** Lab 5B: Image Segmentation
5/11	**5/12** Lecture 25: guest Diffusion Imaging Tractography	**5/13**	**5/14** Lecture 26: JG End-of-term wrap up Lab 5 due	**5/15**

DSIP course troubleshooting

- Trouble 1: How to learn so much knowledge in such a short syllabus time in the schedule?
- Answer: Schedule time are used only for practical works. Theory and fundamentals of Digital Signal and Image Processing are studied on one's own at home or library.
- Trouble 2. The details lab manuals and programs of MIT-course open resource are not available for all.
- Answer: Creating the unique lab works and manuals using author's research experience.
- Trouble 3. Mathematical knowledge. It is own problem of each student entirely.

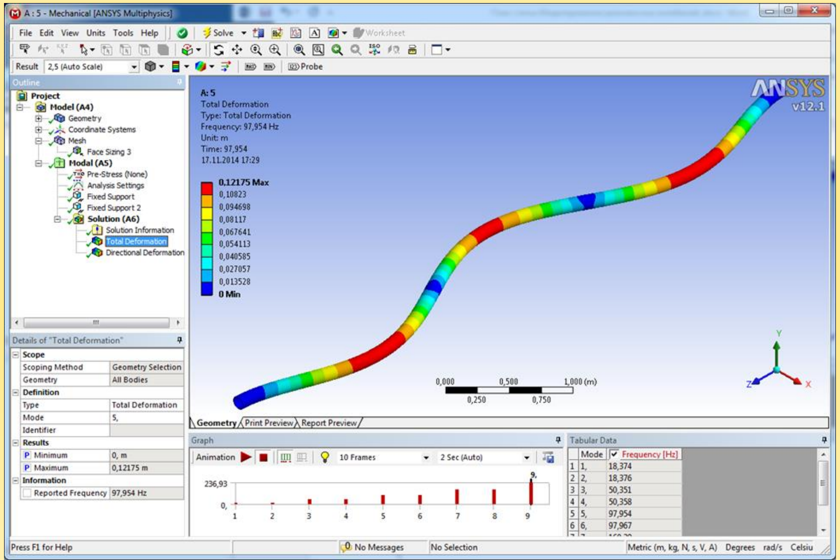
DSIP Syllabus: Lectures

- Part 1. Data Acquisition
- Part 2. Digital Filtering
- Part 3. Digital Processing and Analysis
- Part 4. Digital Signal and Image Processing Tools



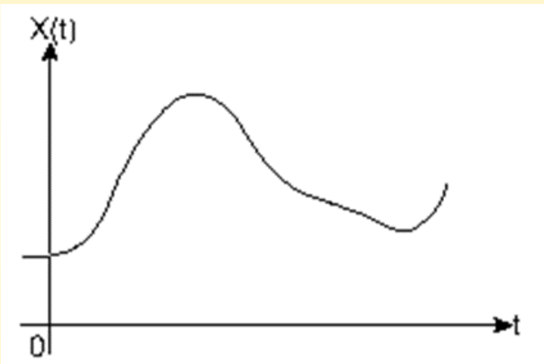
DSIP Syllabus: Practical Works

1. Identifying fault of rotor generators using wavelet analysis of magnetic field.
2. Detection of hydroacoustic noise using the integral wavelet spectrum.
3. Determining the forest fire type on the noise spectrum using Fourier analysis
4. Estimation of the resonance frequencies of pipelines using modal analysis.

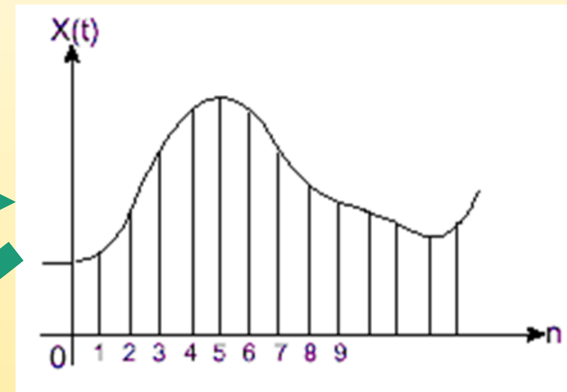


Part 1. Data Acquisition (DA)

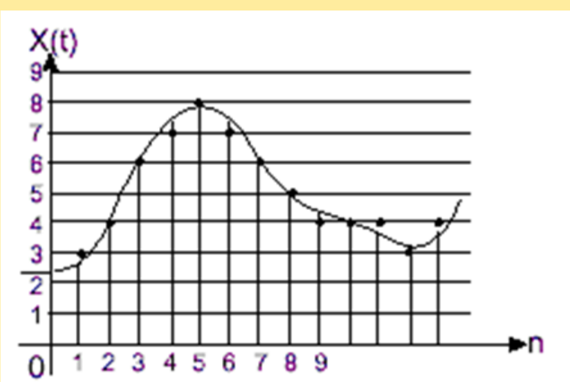
An analog-to-digital conversion stages



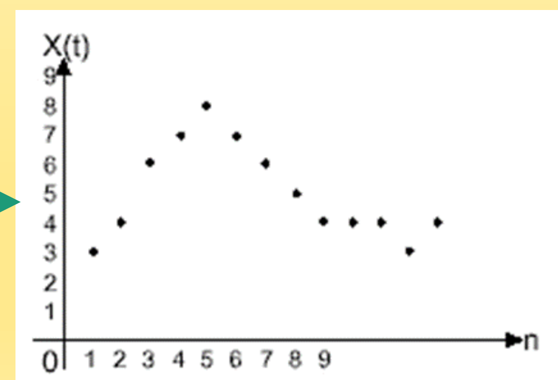
Analog signal



Sampling



Quantization



Digital signal

Part 1. DA: A/D Conversion

- An analog-to-digital converter is a device that transforms a continuous-time signal measured with a transducer into a digital signal that can be represented in a computer.
- Conceptually, it can be divided into a series of two operations that realized simultaneously in actual devices. First, sampling that the continuous-time analog signal is converted into one that is only defined for discrete times, but whose amplitude can take arbitrary values. Second, quantization that a continuous-amplitude signal is converted into a digital signal that can take a finite set of values only.
- The sampling and quantization operations are particularly critical if we want to avoid loss of information in the conversion.

Part 1. DA: Nyquist–Shannon Theorem

Sampling is the process of converting a signal (for example, a function of continuous time or space) into a numeric sequence (a function of discrete time or space). Shannon's version of the theorem states:

If a function $x(t)$ contains no frequencies higher than B cps, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart.

A sufficient sample-rate is therefore 2B samples/second, or anything larger. Conversely, for a given sample rate f_s the bandlimit for perfect reconstruction is $B \le f_s/2$. When the bandlimit is too high (or there is no bandlimit), the reconstruction exhibits imperfections known as aliasing.

- In Russia it called as Kotelnikov theorem.

P 1. N–S Theorem: mathematic form

Poisson shows that the Fourier series produces the periodic summation of X(f), regardless of fs and B. Shannon, however, only derives the series coefficients for the case fs = 2B. Virtually quoting Shannon's original paper:

Let $X(\omega)$ be the spectrum of $x(t)$. Then

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}\,\mathrm{d}\omega = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} X(\omega)e^{i\omega t}\,\mathrm{d}\omega$$

since $X(\omega)$ is assumed to be zero outside the band $|\frac{\omega}{2\pi}| < B$. If we let

$$t = \frac{n}{2B}$$

where n is any positive or negative integer, we obtain

$$x\left(\tfrac{n}{2B}\right) = \frac{1}{2\pi}\int_{-2\pi B}^{2\pi B} X(\omega)e^{i\omega \frac{n}{2B}}\,\mathrm{d}\omega.$$

On the left are values of $x(t)$ at the sampling points. The integral on the right will be recognized as essentially

P1. The N-S Theorem: Consequence 1

- Let x(n) be the n^{th} sample. Then the function x(t) is represented by:

$$x(t) = \sum_{n=-\infty}^{\infty} x_n \frac{\sin \pi(2Bt - n)}{\pi(2Bt - n)}.$$

- As in the other proof, the existence of the Fourier transform of the original signal is assumed, so the proof does not say whether the sampling theorem extends to bandlimited stationary random processes.

- In practice, we can not have an infinite sample, so for a limited n expression is written approximately and should be assessed accuracy.

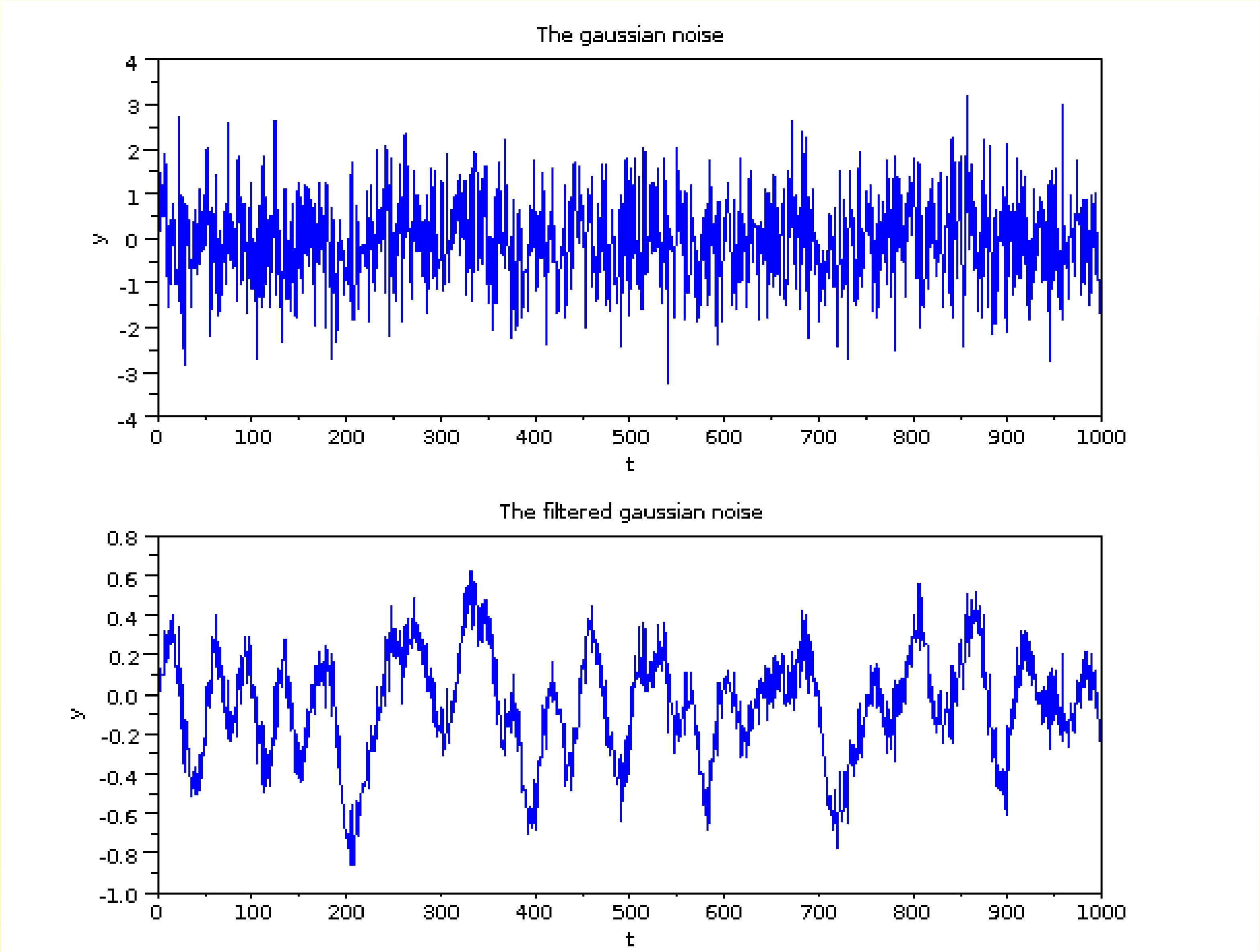
P1. The N-S Theorem: Consequence 2

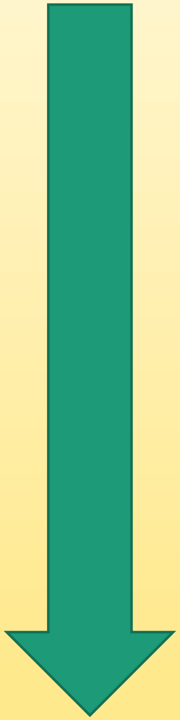
- An important consequence of the Nyquist-Shannon theorem is that a signal can be reconstructed when the sampling rate is more than twice the maximum frequency of the signal being sampled.
- This rule tells us that to reproduce sounds as high as human hearing 20,000 hertz we must take at least 40,001 samples per second.
- The world standard sampling rate for digital music in the music industry is 44,1 kHz and each sample is assigned 16 bits.
- Some theorem definitions describe this process as making a perfect recreation of the signal.

P 1. N–S Theorem: modern statements

- Modern statements of the theorem are sometimes careful to explicitly state that x(t) must contain no sinusoidal component at exactly frequency B, or that B must be strictly less than ½ the sample rate.
- The two thresholds, 2B and fs/2 are respectively called the Nyquist rate and Nyquist frequency. And respectively, they are attributes of x(t) and of the sampling equipment.
- The condition described by these inequalities is called the Nyquist criterion, or sometimes the Raabe condition.
- The theorem is also applicable to functions of other domains, such as space, in the case of a digitized image. The only change, in the case of other domains, is the units of measure applied to t, fs, and B.

Part 2. Digital Filtering





Part 2. Digital Filtering

A discrete-time system is any mathematical transformation that maps a discrete-time input signal x[n] into an output signal y[n]. We will begin by considering discrete-time systems defined by a linear, constant-coefficient difference equation (LCCDE), which constitute an important class of digital filters:

$$y[n] = \sum_{k=1}^{K} a_k y[n-k] + \sum_{m=0}^{M} b_m x[n-m]$$

Eq. can be used to compute the output y[n] at time n from a finite number of previous values of the input (x[n]) and the output.

Part 2. Digital Filtering

- Eq. can be used to compute the output $y[n]$ at time n from a finite number of previous values of the input ($x[n]$) and the output.
- The maximum of the numbers M and K is called the **Order of the Filter.**
- If the input signal is only defined after a certain time, say for $n \geq n0$, then values of both the input and output for a short time prior to $n0$ must be known in order to initialize the difference equation. Specifically, $y[n]$ must be known for $n0-K \leq n \leq n0-1$, and $x[n]$ for $n0-M \leq n \leq n0-1$.
- In many applications, it is justified to assume that these values are zero if the system has not received any input for a long time before $n0$, so that its response to any previous inputs has decayed to zero.

Part 2. DF: Examples of digital filters

1. Simple gain, or amplifier: $$y[n] = Gx[n]$$
2. Delay of n0 samples: $$y[n] = x[n - n_0]$$
3. Two-point moving average: $$y[n] = \frac{1}{2}(x[n] + x[n-1])$$
4. Euler's formula for approximating the derivative of a continuous-time function (where Ts is the sampling interval): $$y[n] = \frac{x[n] - x[n-1]}{T_s}$$
5. Averaging over N consecutive epochs of duration L: $$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n - kL]$$

Part 2. DF: Examples of digital filters 2

6. Trapezoidal integration formula:

$$y[n] = \frac{y[n-1] + (x[n] + x[n-1])T_s}{2}$$

7. Digital “leaky integrator”, or first-order low pass filter:

$$y[n] = ay[n-1] + x[n] \qquad \text{with} 0 < a < 1$$

This filter is the digital equivalent of an RC analog circuit. It is a useful “building block” for designing complex filters.

8. Digital resonator:

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + bx[n] \qquad \text{with} a_1^2 + 4a_2 < 0$$

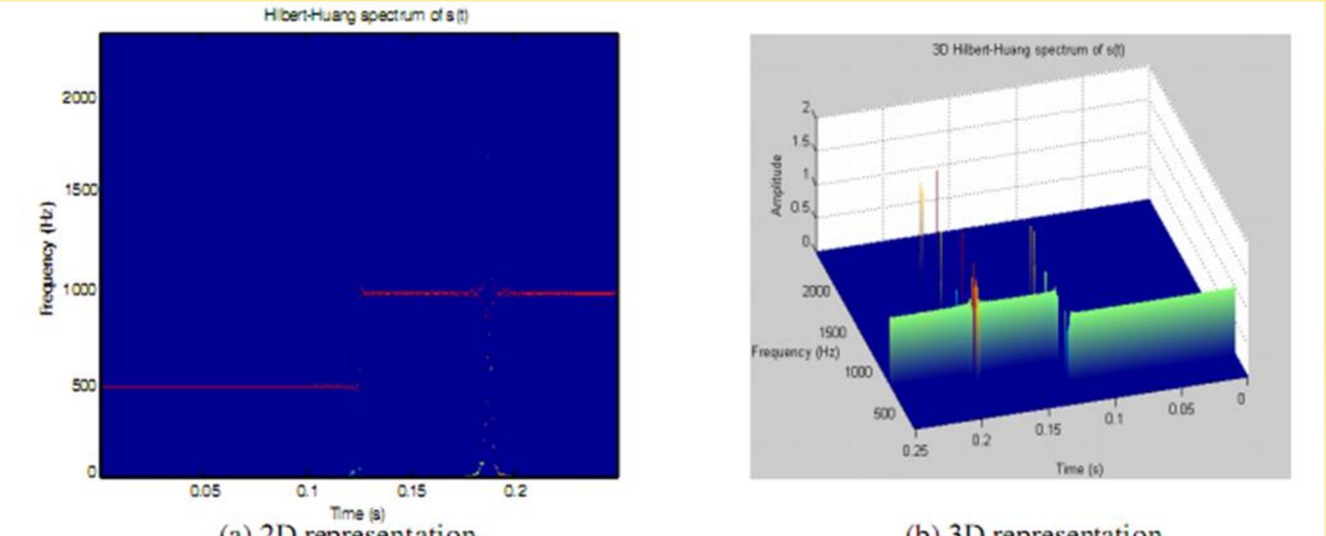
This is the digital equivalent of the harmonic oscillator.

Part 3. Digital Processing and Analysis

- Digital Processing and Analysis are based on Fourier Transform, Wavelet Transform, Hilbert-Huang Transform, and on other mathematical tools.
- The Fourier Transform expresses a function of time (or signal) in terms of the amplitude (and phase) of each of the frequencies that make it up. The resulting function, a complex amplitude that depends on frequency, is called the **frequency domain representation of the function**.
- The Wavelet Transform is a representation of a square-integrable (real- or complex-valued) function by a certain orthonormal series generated by a **wavelet**. The resulting function is called the **time-frequency domain representation of the function.** A complex amplitude this function depends on time and scale (frequency).

Part 3. Digital Processing and Analysis

- The Hilbert-Huang Transform (HHT) is a way to decompose a signal into so-called intrinsic mode functions (IMF), and obtain instantaneous frequency data. It is designed to work well for data that is nonstationary and nonlinear. In contrast to other common transforms like the Fourier Transform, the HHT is more like an algorithm (an empirical approach) that can be applied to a data set, rather than a theoretical tool.



(a) 2D representation (b) 3D representation

Part 3. DPA: Fourier Transform

- Continuous Fourier Transform (CFT):

The Fourier Transform .com

$$\mathcal{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t) e^{-i2\pi ft}\, dt$$

$$\mathcal{F}^{-1}\{G(f)\} = g(t) = \int_{-\infty}^{\infty} G(f) e^{i2\pi ft}\, df$$

- Discrete Fourier transform (DFT):

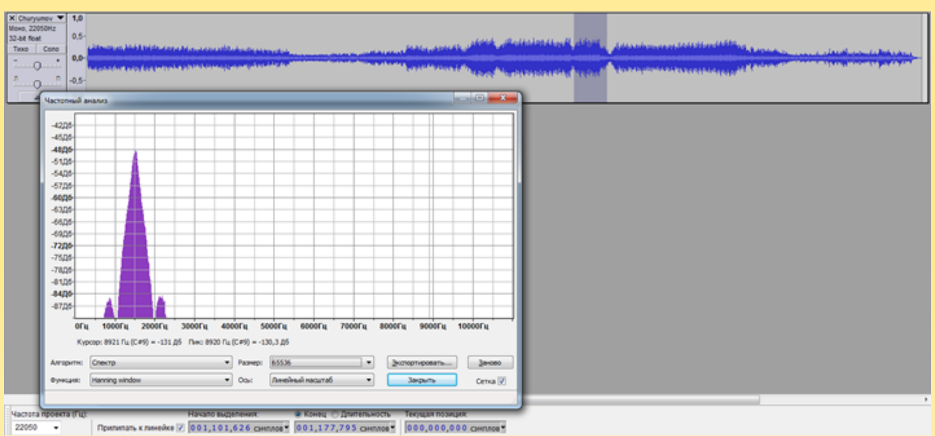
$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi kn/N}, \quad k \in \mathbb{Z} \text{ (integers)}$$

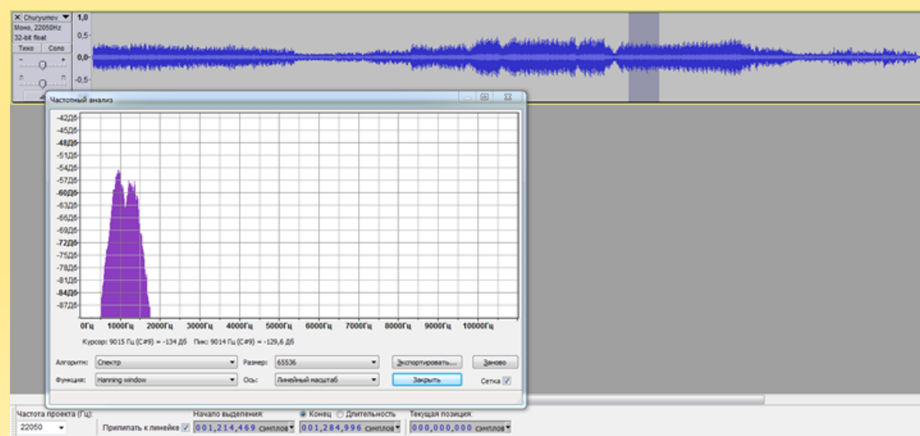
$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{i2\pi kn/N}, \quad n \in \mathbb{Z}$$

Part 3. DPA: Fourier Transform: DTFT

- The Discrete-Time Fourier Transform (DTFT) is a form of Fourier analysis that is applicable to the uniformly-spaced samples of a continuous function. The term discrete-time refers to the fact that the transform operates on discrete data (samples) whose interval often has units of time.

$$X_{1/T}(f) = \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} x[n]\cdot\delta(t-nT)\right\}$$



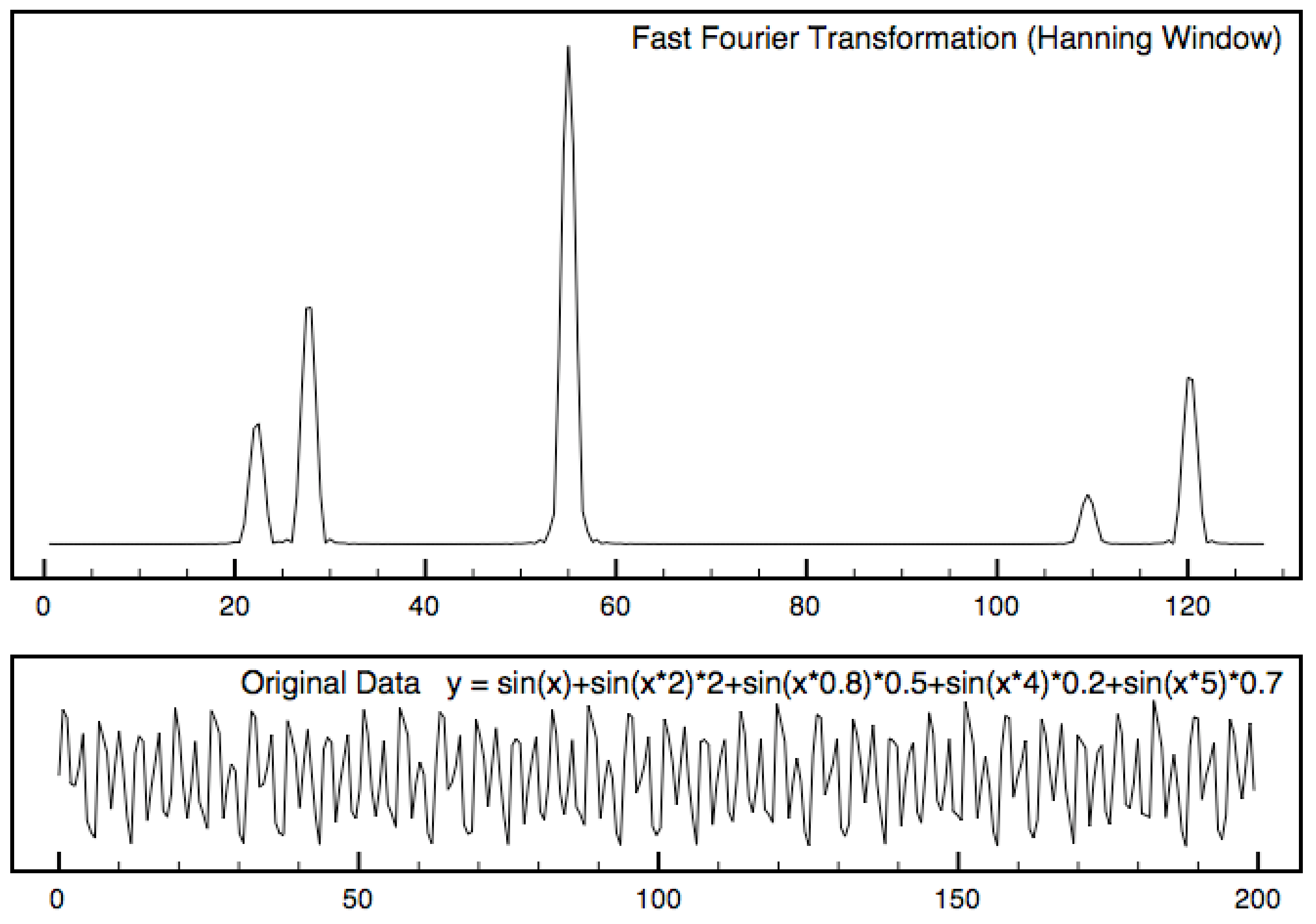


Part 3. DPA: Fourier Transform: FFT

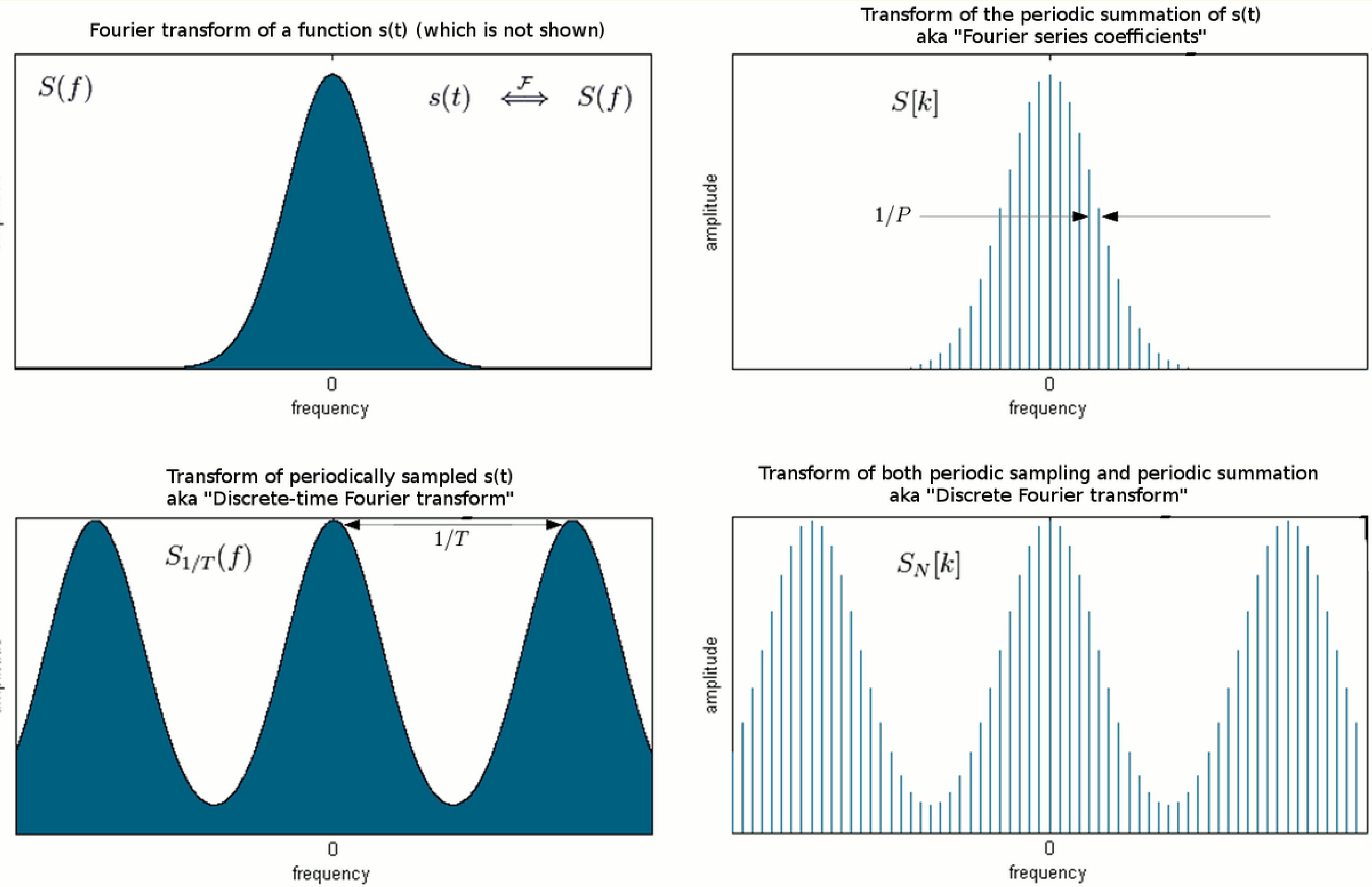
- A fast Fourier Transform (FFT) is an algorithm to compute the discrete Fourier transform (DFT) and its inverse.
- Fourier analysis converts time (or space) to frequency and vice versa; an FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors.
- As a result, Fast Fourier Transforms are widely used for many applications in engineering, science, medicine, and mathematics.
- The most known algorithms: Cooley–Tukey FFT algorithm, Prime-factor FFT algorithm, Bruun's FFT algorithm, Rader's FFT algorithm, Bluestein's FFT algorithm, etc.

Part 3. DPA: Fourier Transform: FFT

Fast Fourier Transforms have been described as “the most important numerical algorithm of our lifetime”.



Part 3. DPA: Fourier Spectrum



Part 3. DPA: Wavelet Transform

- Continuous Wavelet Transform (CWT) of research signal $S(t)$ is:

$$W(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty}S(t)\cdot\Psi\left(\frac{t-b}{a}\right)dt$$

- The continuous wavelet transform of a discrete sequence $S(t_i)$ is defined as the convolution of $S(ti)$ with a scaled and translated version of mother wavelet Ψ.

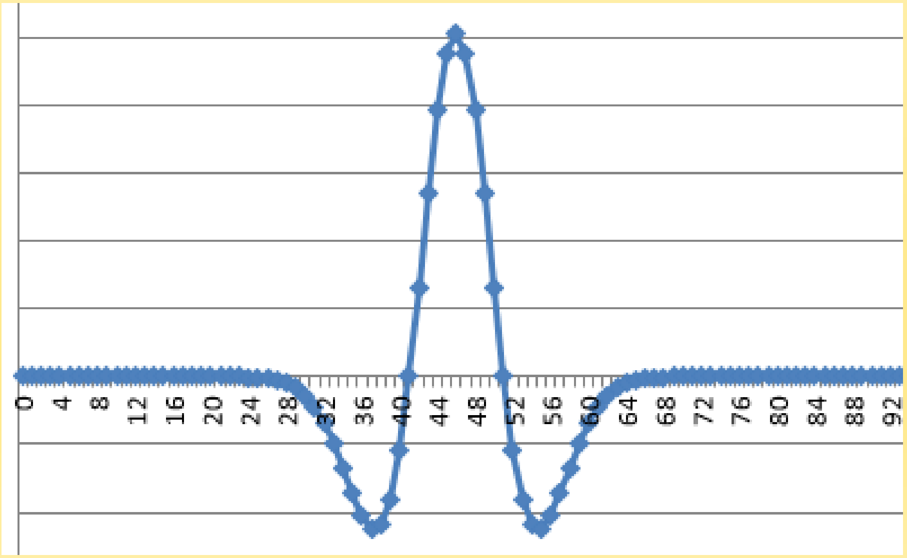
$$W(a_j,b_k)=\frac{1}{\sqrt{a_j}}\sum_{i=0}^{N}S(t_i)\cdot\Psi\left(\frac{t_i-b_k}{a_j}\right)\Delta t$$

where *i*, *j*, *k* are indexes of time *t*, scale *a*, and translation *b*, respectively; *N* is the number of time steps; *M* is the number of scales; Δt_i is the sampling interval.

Part 3. DPA: Wavelet Transform

- Mother wavelets is very time consuming to calculate, e.g., wavelet known as the “Mexican Hat” (MHAT) is the second derivative of a Gaussian function:

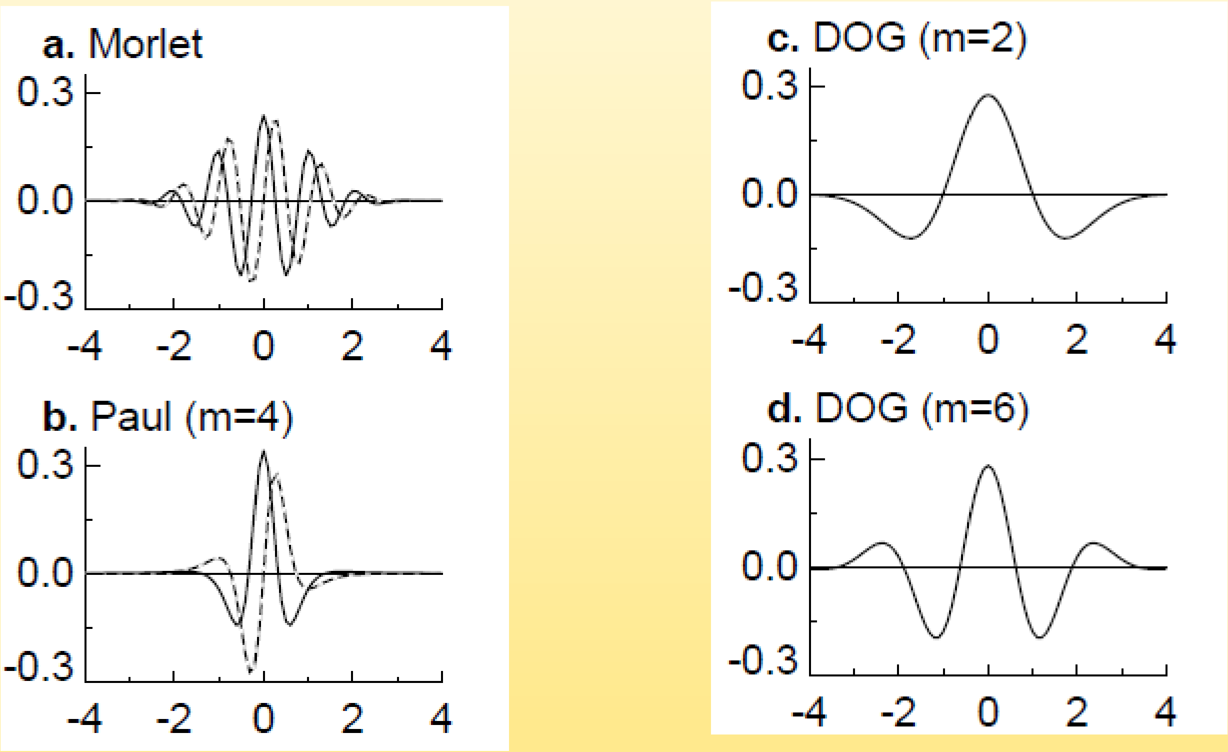
$$\Psi(x) = \frac{1}{\sqrt{2\pi\sigma^3}}\left[e^{\frac{-x^2}{2\sigma^2}} \cdot \left(1 - \frac{x^2}{\sigma^2}\right)\right],$$



where Sigma (the standard deviation, sometimes called the Gaussian RMS width) controls the width of the “Hat".

Part 3. DPA: Wavelet Transform

- There are many other functions that may be used as mother wavelet, e.g.:



- The choice of wavelet is dictated by the signal or image characteristics and the application's area. You can choose a mother wavelet that will be optimised for your task.

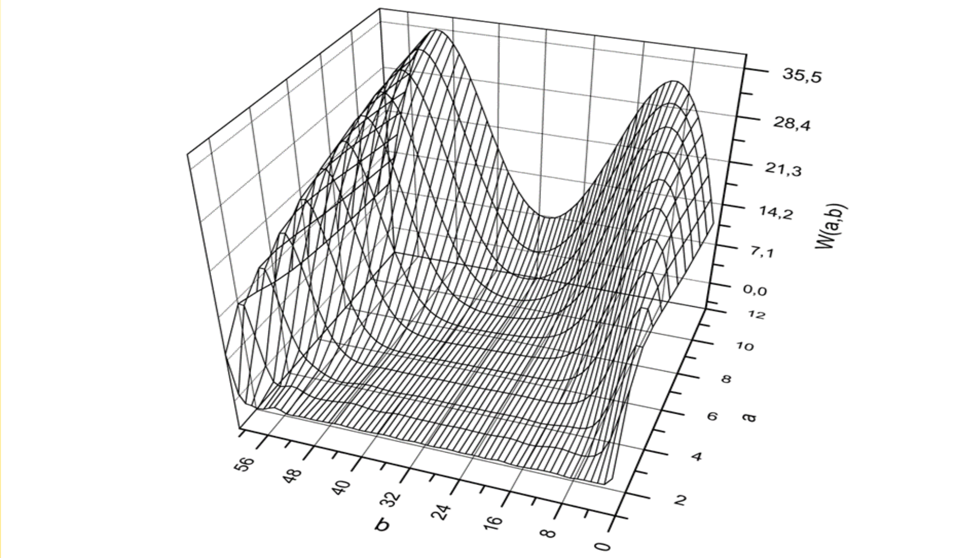
Part 3. DPA: Wavelet Transform

Wavelet families vary in terms of several important properties. Examples include:

- Support of the wavelet in time and frequency and rate of decay.
- Symmetry or antisymmetry of the wavelet. The accompanying perfect reconstruction filters have linear phase.
- Number of vanishing moments. Wavelets with increasing numbers of vanishing moments result in sparse representations for a large class of signals and images.
- Regularity of the wavelet. Smoother wavelets provide sharper frequency resolution. Additionally, iterative algorithms for wavelet construction converge faster.

Part 3. DPA: Wavelet Transform

- **Scalogram** is a visual representation of a Wavelet Transform, having axes for time, scale, and coefficient value, analogous to a spectrogram.



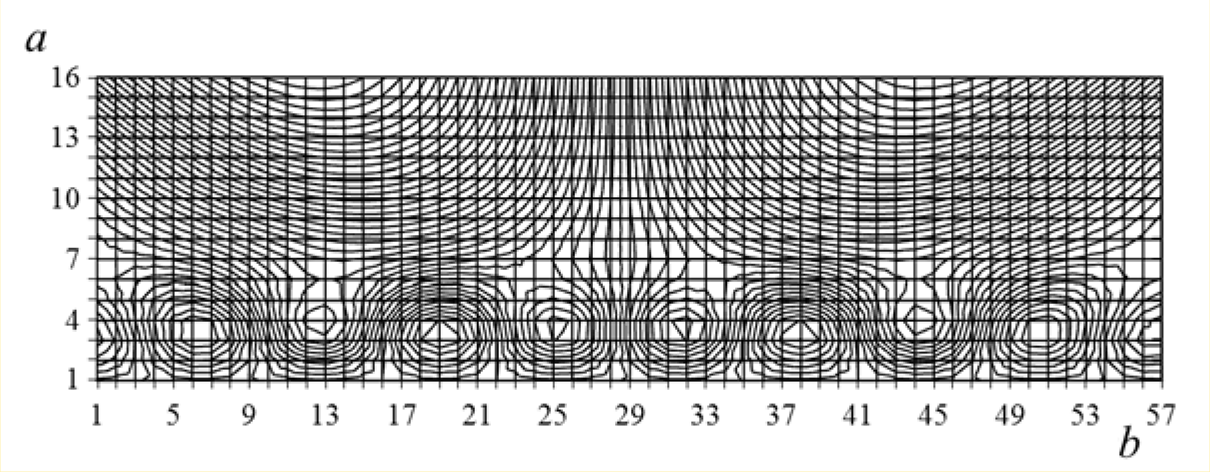
- **Integral Wavelet Spectrum** is a visual representation of a Wavelet Transform, having axes for scale only:

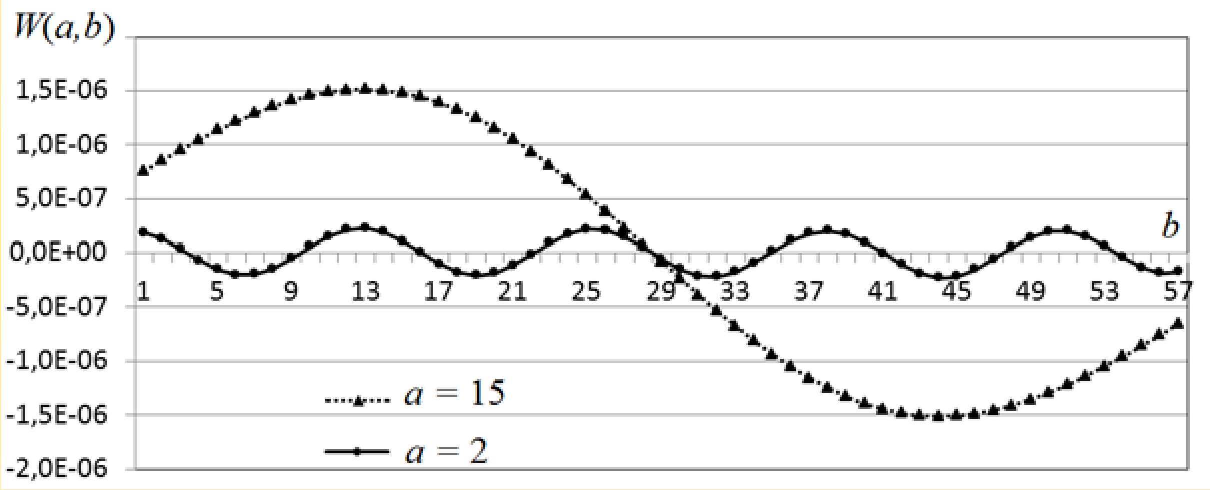
$$W^*(a) = \frac{1}{B}\int_0^B \left|W(a,b)\right|^2 db$$

Part 3. DPA: Wavelet transform

- Test example of CWT implementation:

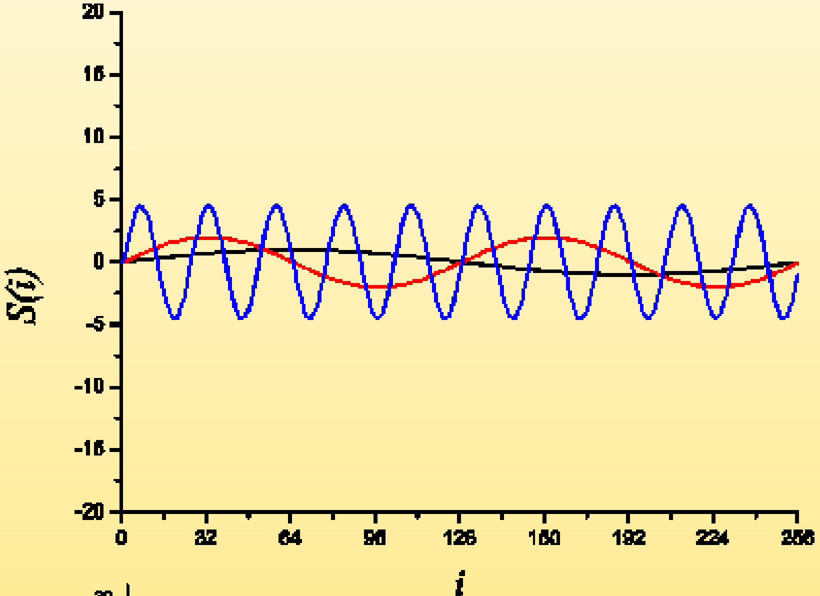
$$S(t) = A_1 \sin(\omega t) + A_2 \sin(5\omega t),$$

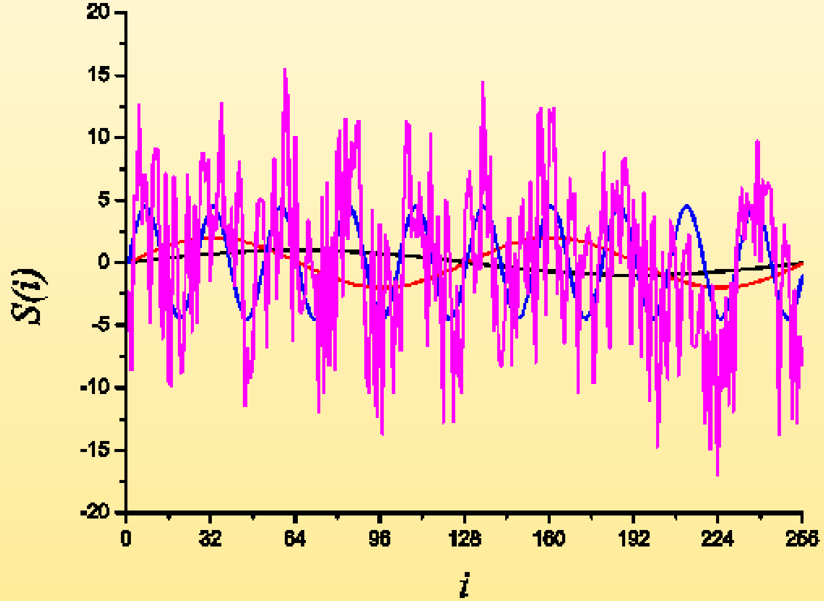


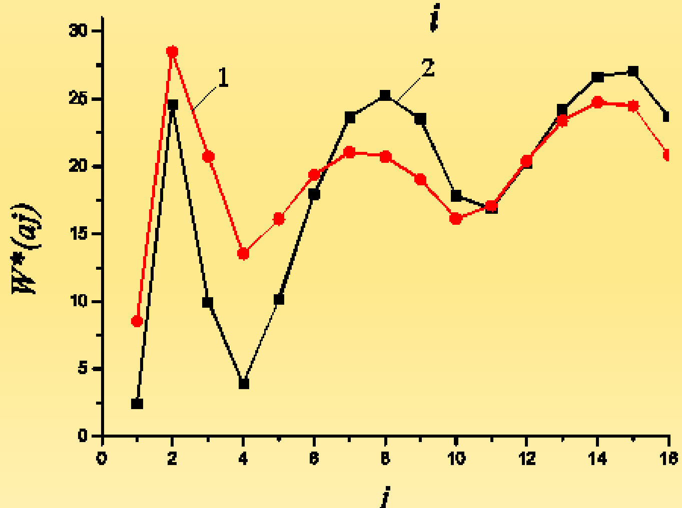


Part 3. DPA: Wavelet Transform

- Example of CWT implementation for a weak signal detection in a strong noise:



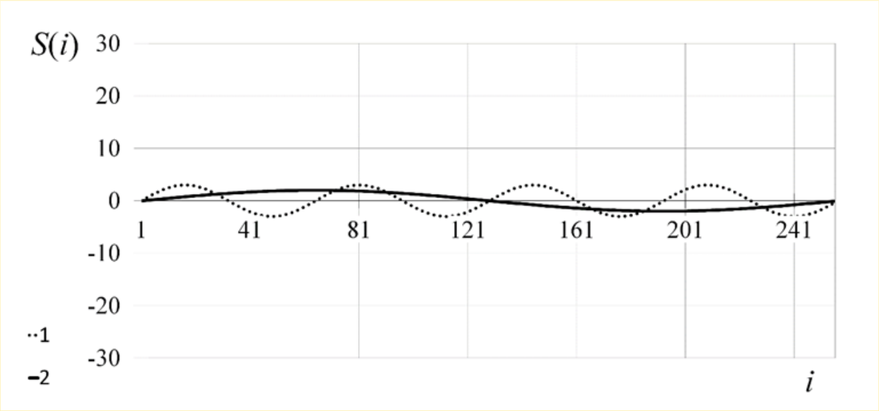


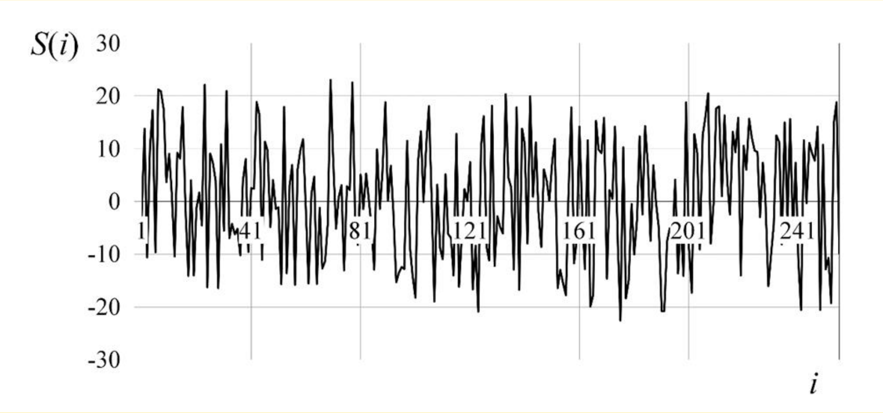


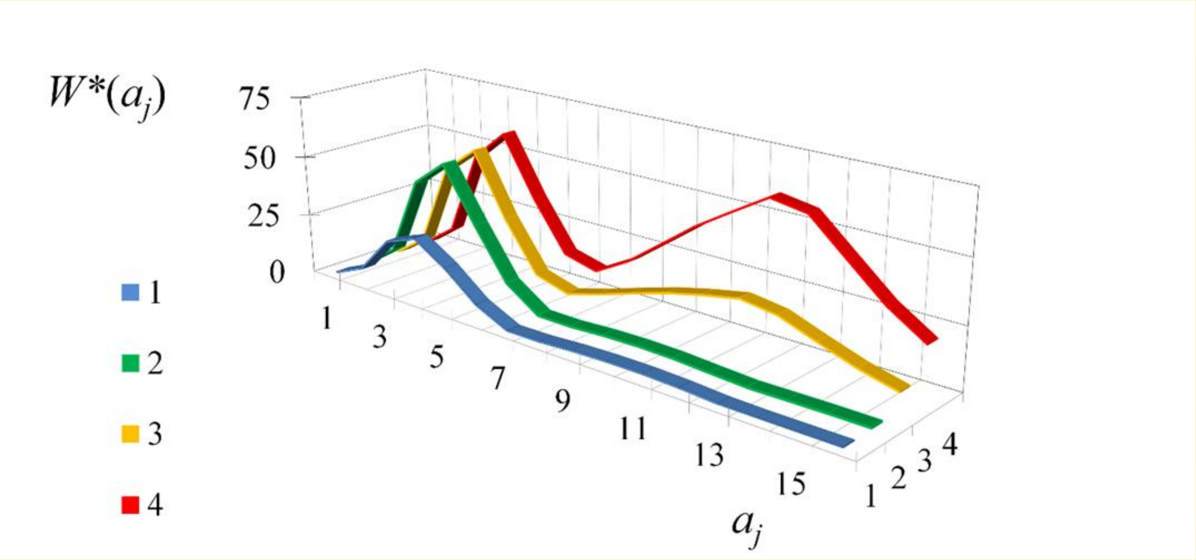
$$S(i) = 2\sin\left(\frac{2\pi}{N} i\right) + 3\sin\left(\frac{8\pi}{N} i\right) + A\left[2Rnd(1) - 1\right]$$

Part 3. DPA: Wavelet Transform

Continuous Wavelet Transform allows to view CWT-imaging sequence (1, 2, 3, and 4) from strong noise with negative SNR







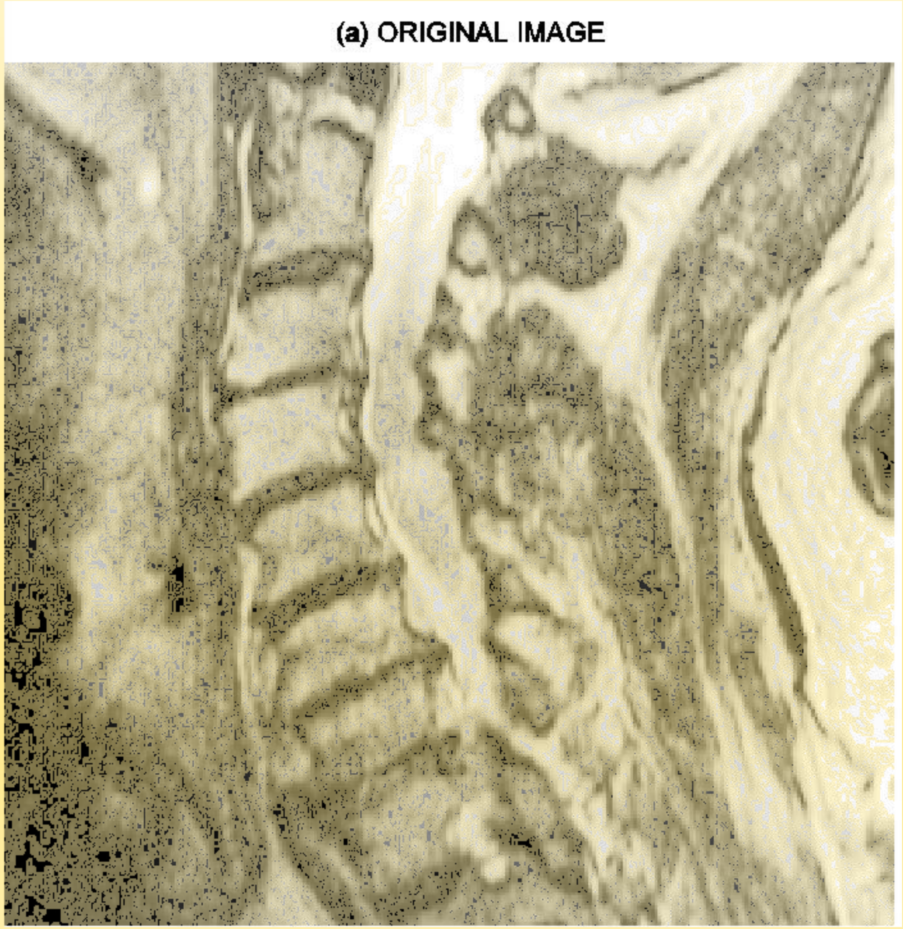
Part 3. DPA: Image Interpolation

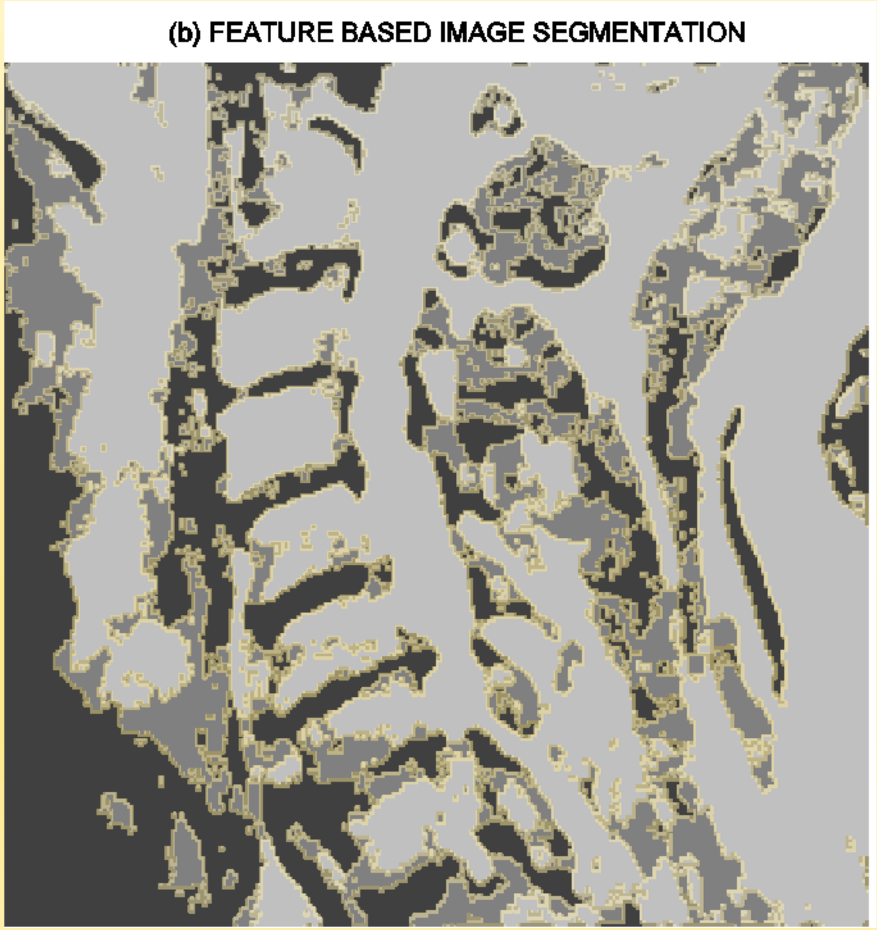
50% Missing Pixels

Interpolation by learning the basis from the corrupted image

Part 3. DPA: Image Segmentation

Each root pixel of the original image is associated with its feature derived from its neighborhood.   Pixels are individually classified into selected number of levels

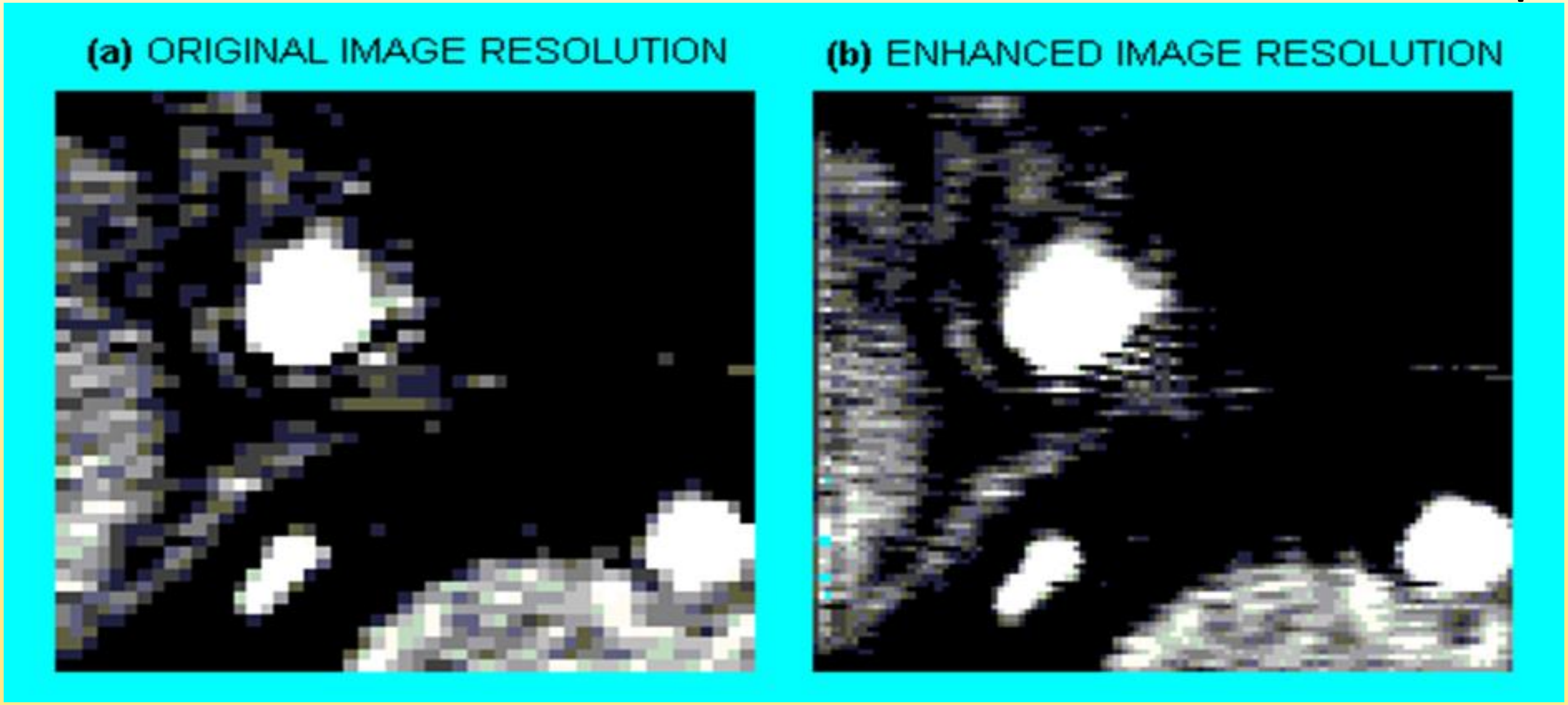




Part 3. DPA: Wavelet Transform in image resolution enhancement

Original resolution: 512 x 512

Resolution enhancement: 1024 x 1024 pixels



DWT: sharper edges obtained

Part 4. Digital Signal and Image Processing Tools

Implementation of the methods within the Computer Software in English: Mathcad, Matlab, Ansys, etc. (optional).

The data for the estimation of the resonance frequencies is presented graphically in figures 1, 2, and 3. In Table II are summarized the values of the resonance frequencies.

This description will be detailed in the next tutorial:

Laboratory Practice

Digital Signal and Image Processing

Thank you for your attention

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