



TOMSK POLYTECHNIC UNIVERSITY

**O.M. Demidova, V.M. Zamyatin**

**ENGINEERING MECHANICS.  
ENGLISH FOR SPECIFIC PURPOSES**

**Teacher's book**

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МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РОССИЙСКОЙ ФЕДЕРАЦИИ  
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**«НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ  
ТОМСКИЙ ПОЛИТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ»**

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**О.М. Демидова, В.М. Замятин**

**ТЕОРЕТИЧЕСКАЯ МЕХАНИКА.  
ПРОФЕССИОНАЛЬНЫЙ АНГЛИЙСКИЙ ЯЗЫК**

**Книга для учителя**

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Книга для учителя содержит ключи к заданиям, посвященным профессионально-ориентированным темам по курсу «Теоретическая механика», а также тексты на аудирование. Ключи к заданиям расположены в порядке следования глав пособия, что значительно облегчит работу преподавателей-лингвистов и преподавателей профилирующих кафедр.

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**ББК Ш143.21-923**

*Рецензенты*

Кандидат технических наук, доцент ТГАСУ

*В.М. Педиков*

Кандидат филологических наук, доцент НИ ТГУ

*Д.А. Олицкая*

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**Chapter 1.**  
**Basic Concepts and Principles.**  
**1.1. The Subject of Statics**

**Exercise 1**

- 1) The branches of mechanics are statics, dynamics, hydrostatics, aerostatics and general mechanics.
- 2) Dynamics deals with the accelerated motion of bodies. Statics deals with the equilibrium of bodies that are either at rest or move with a constant velocity.
- 3) Aerostatics studies the equilibrium of gases. Hydrostatics studies the equilibrium of liquids.
- 4) The problems of statics may be solved either by geometrical constructions (the graphical method) or by mathematical calculations (the analytical method).

**Exercise 3**

Отрасль механики, сложение сил, законы сложения сил, условия равновесия, состояние покоя, система координат, данное тело, абсолютное равновесие, относительное равновесие, абсолютно твердое тело, удовлетворять условиям равновесия, определять условия равновесия, находить (определять) условия равновесия, ускоренное движение, равновесие твердых тел, до определённой степени, степень деформации, размеры (измерения) тела, постоянное расстояние, пара частиц, система сил, основная задача статики, различные системы сил, принцип сложения, геометрическое построение, математическое вычисление.

**Exercise 4**

Laws of composition of forces, under the action of the force system, state of rest, rigidly connected (bound), concept of equilibrium, to be subjected to the action of a force system, principle of composition (addition), principle of replacing, principle of reduction, relatively, to solve the problems of statics, it should be borne in mind, a branch of mechanics, a material body, material system, constant velocity, equilibrium of solids, degree of deformation, dimensions of the body, distance between particles, to satisfy the conditions of equilibrium, in order to, geometrical constructions, mathematical calculations.

**Exercise 5**

1. Compose (складывать) (v.) – composed (сложенный) (p.2) – composing (складывая) (p.1) – composer (композитор) (n.) – composition (композиция, состав) (n.)
2. Act (действовать) (v.) – active (активный, действующий) (adj.) – acting (действуя) (p.1) – action (действие) (n.)

3. Relating (относящийся) (p.1) – relation (отношение) (n.) – relate (относиться) (v.) – related (связанный) (p.2) – relational (относительный) (a.) – relative (относительный) (a.) – relatively (относительно) (adj.).
4. Justify (подтверждать) (v.) – justifiable (может быть подтверждено) (a.) – justification (подтверждение) (n.)
5. Respect (отношение) – respectable (приемлемый) (a.) – respector (почтительный человек) (n.) – respectful (почтительный) (a.) – respectfully (почтительно) (adj.) – respective (соответственный) (adj.) – respectively (соответственно) (adv.)
6. Form (форма) (n.) – formation (образование)(n.) – formal (официальный)(a.) – to form (составлять) (v.) – deform (деформировать)(v.) – deformable (деформирующийся)(a.) – undeformable (недеформируемый) (a.) – deformation (деформация) (n.)
7. Place (место, расположение) (n.) – to place (помещать, располагать) (v.) – placed (расположенный) (p.2) – replace (перемещать) (v) – replacing (перемещая) (p.1).
8. Reduce (уменьшать, редуцировать) (v.) – reduced (уменьшенный, редуцированный) (p.2) – reducible (допускающий уменьшение, редукцию) (adj.) – reducing (уменьшая, редуцируя) (p.1) – reduction (уменьшение, сокращение) (n.).
9. Absolute (абсолютный) (a.) – absolutely (абсолютно)(adv.)
10. Rigid (жесткий, твердый) (a.) – rigidly (жестко, твердо) (adv.).
11. Strict (точный, строгий)(a.) – strictly(точно, строго)(adv.)

### Exercise 6

**Match the subjects and the main verbs in the following sentences.**

1. In studying the general conditions of equilibrium, we will treat solid bodies as undeformable or absolutely rigid, ignoring the small deformations that actually occur under specific loads.
2. A perfectly rigid body, or absolutely rigid body, is said to be the one in which the distance between any pair of particles is always constant.
3. For a rigid body to be in equilibrium (at rest) under the action of a system of forces, the system must satisfy certain conditions of equilibrium.

### Exercise 7

**In the text all the words in bold refer to something mentioned before or after in the text. Read the passage carefully and complete the table underneath.**

lines	words in bold	refers to something before	refers to something after	what it refers to
1	which	+		branch
6	that	+		bodies
14	this		+	deformation
17	that	+		deformations
19	which	+		rigid bodies
24	one		+	problems

### Exercise 8

**In the following text all punctuation has been removed. Can you put it back? Start a new paragraph when you think it is necessary. Capitalize the words when necessary.**

Mechanics can be defined as that branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject is subdivided into three branches: rigid body mechanics, deformable – body mechanics, and fluid mechanics. This book treats only rigid – body mechanics, since it forms a suitable basis for the design and analyses of many types of structural, mechanical, or electrical devices encountered in engineering. Also, rigid body mechanics provides part of the necessary background for the study of the mechanics of deformable bodies and the mechanics of fluids.

### Exercise 9

Statics is a branch of mechanics which studies the laws of composition of forces and the conditions of equilibrium of material bodies under the action of forces. Dynamics deals with the accelerated motion of bodies. The equilibrium of liquids and gases is studied in the courses of hydrostatics and aerostatics respectively. General mechanics deals with the equilibrium of solids or perfectly rigid bodies. For a rigid body to be in equilibrium under the action of a system of forces, the system must satisfy certain conditions of equilibrium. In order to determine the conditions of equilibrium one should know the principles of statics. The problems of statics may be solved either by geometrical constructions or by mathematical calculations.

### Exercise 10

1. Statics is a branch of mechanics which studies the laws of composition of forces and the conditions of equilibrium of material bodies under the action of forces.
2. The other branches of mechanics are dynamics, aerostatics, hydrostatics, and general mechanics.
3. Statics of rigid bodies treats of two basic problems:

- 1) the composition of forces and reduction of force systems acting on rigid bodies to as simple form as possible
- 2) the determination of conditions of equilibrium of force systems acting on rigid bodies.
4. The main methods of solution of these problems are geometrical constructions (the graphical method) or by mathematical calculations (the analytical method).

### **Exercise 11**

- 1) statics
- 2) equilibrium
- 3) absolute equilibrium
- 4) gaseous
- 5) liquids
- 6) a perfectly rigid body
- 7) a particle
- 8) the principles of composition of forces; the principles of replacing one force system by another, the principles of reduction of a given force system.
- 9) Graphical; analytical

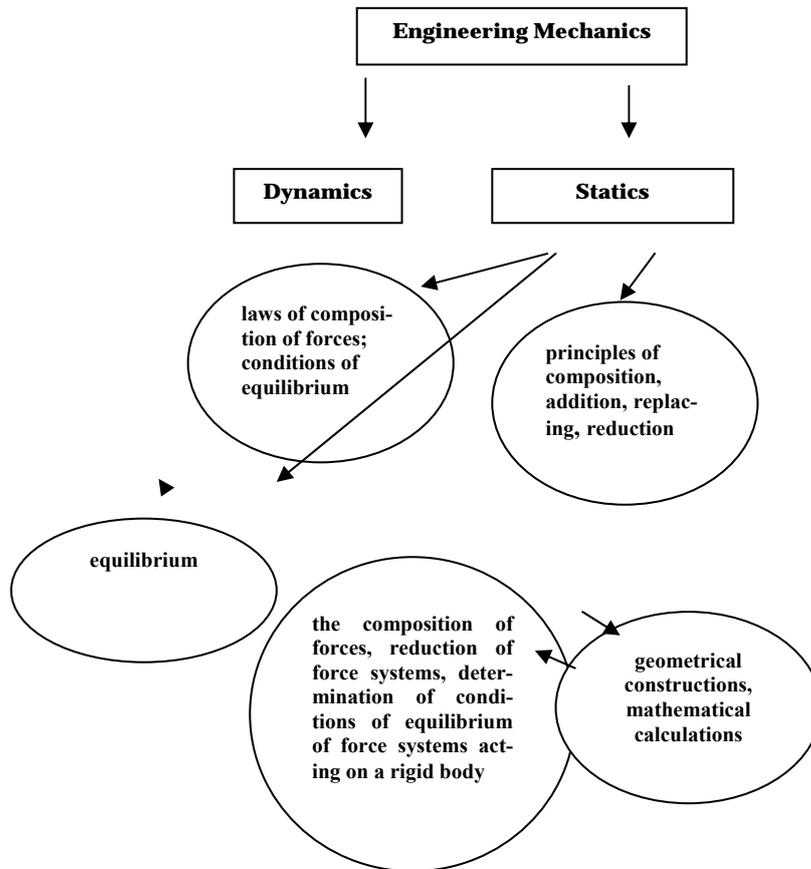
### **Exercise 12**

- 1) Equilibrium is the state of rest of a body relative to other material bodies.
- 2) Conditions of equilibrium depend on whether a given body is solid, liquid or gaseous.
- 3) General mechanics deals essentially with the equilibrium of solids.
- 4) All solid bodies change their shape to a certain extent when subjected to the external forces.
- 5) A perfectly rigid body is one in which the distance between any pair of particles is always constant.
- 6) A particle is a point which has a mass but a size that can be neglected.
- 7) The problems of statics may be solved either by geometrical constructions or by mathematical calculations.

### **Exercise 13**

**Example:** The article tells about the main branches of statics. It describes such main notions as: a perfectly rigid body, a particle, conditions of equilibrium, the problems of statics, and methods of solution of these problems.

## Exercise 14



## Chapter 1 Basic Concepts and Principles

### 1.2. Force

#### Exercise 1

1. A force is a quantitative measure of the mechanical interaction of material bodies.
2. A force is characterized by its magnitude, direction, point of application.
3. There may be an equilibrant force, a resultant, an external force, an internal force, a concentrated force, a distributed force.
4. A scalar is a quantity characterized by a positive or negative number. A vector is a quantity that has both a magnitude and a direction.

#### Exercise 3

А) частица; чисто теоретическое понятие; соответствующая система сил; состояние равновесия; механические взаимодействия; количественная мера; скаляр; вектор; объем; точка приложения; линия действия силы; не нарушая; первоначальное условие покоя; оставаться в покое; противоположны по направлению; условия движения; графически.

#### Exercise 4

State of rest; to depend on mechanical interaction; quantities employed in mechanics; the force is directed along the line E; the line of action of a force; to be connected with other bodies; to be displaced in any direction in space; force system acting on a rigid body; to remain at rest; a force system equivalent to a single force; a force collinear with the resultant; a force equal in magnitude; forces can be divided into two groups; to apply a force to a single point; forces treated in mechanics.

#### Exercise 5

<b>noun</b>	<b>adjective</b>	<b>Verb</b>	<b>adverb</b>
	given	to give	
mechanics mechanic mechanician mechanism mechanist	mechanical mechanistic mechanization	to mechanize	
action	acting	to act	active
direction	direct	to direct	directly
place	placed	to place	

### Exercise 6

- interaction
- external
- concentrated
- corresponding
- quantitative
- frequently
- equilibrant

### Exercise 7

1. h
2. c
3. g
4. i
5. d
6. e
7. f
8. k
9. a
10. j
11. b

### Exercise 8

1. Quantities employed in mechanics are either scalars or vectors.
2. In statics the vector quantities frequently encountered are position, force and moment.
3. Its action on a body is characterized by its (1) magnitude, (2) direction, and (3) point of application.
4. The line DE along which the force is directed is called the line of action of a force.

### Exercise 9

lines	Words in bold	Refers to something before	Refers to something after	What it refers to
2	<b>its</b>	+		body
3	<b>it</b>	+		body
9	<b>that</b>	+		quantity
11	<b>its</b>	+		force
14	<b>which</b>	+		line
21	<b>which</b>	+		body
30	<b>each other</b>	+		force systems

### Exercise 10

- a. 2
- b. 3
- c. 4
- d. 1

### Exercise 11

It is an exercise with a free students' answer

### Exercise 12

State of equilibrium  
Given body  
Equivalent force/system  
Line of action  
Vector quantity  
Mechanical interaction  
Positive number  
Equilibrant force/system  
Qualitative measure

### Exercise 13

**A vector** – is a quantity that has both a magnitude and a direction.

**A free body** – a body not connected with other bodies and which from any given position can be displaced in any direction in space

**A scalar** – is a quantity characterized by a positive or negative number.

**Equivalent system of forces** – force systems acting on a free rigid body which can replace each other without disturbing the body's initial condition of rest or motion.

**A line of action of a force** – the line along which the force is directed.

**A balanced force system** – a force system under the action of which a free rigid body remains at rest.

**An equilibrant force** – a force equal in magnitude, collinear with, and opposite in direction to the resultant

**A concentrated force** – a force applied to one point of a body.

**External forces** – represent the action of other material bodies on the particles of a given body.

**Distributed forces** – forces acting on all the points of a given volume or given area of a body.

**Internal forces** – are those with which the particles of a given body act on each other.

**A particle** – is a point which has a mass but a size that can be neglected.

## Chapter 1 Basic Concepts and Principles

### 1.3. Fundamental Principles

#### Exercise 1

1. These principles are known as axioms of statics.
2. Internal forces are those with which the particles of a given body act on each other.
3. In order to define the equilibrium of a chain, we should consider it as a rigid rod.
4. The other name of the consequence is a corollary.

#### Exercise 3

##### A)

1. deduct(вычитать) (v) – deductive (вычитание)(n) – deduction (вычитание)
2. formula (формула) (n) – formulae (формулы)(n) – formulate (формулировать) (v) – formulation (формулировка) (n) – formulism (формулизм) (n)
3. observe (наблюдать) (v) – observer (наблюдатель) (n) – observed (наблюдаемый) (p.2) – observing (наблюдая) (p.1) – observable (поддающийся наблюдению) (a) – observance (соблюдение) (n) – observant (наблюдательный) (a) – observation (наблюдение) (n) – observational (наблюдательный) (a)
4. motion (движение) (n) – motional (двигательный) (a) – motionless (неподвижный) (a)
5. simple (простой) (a) – simpler (проще) (a) – simplest (самый простой) (a)
6. act (действовать) (v) – active (действующий) (a) – action (действие) (n) – interaction (взаимодействие) (n) – interactive (взаимодействующий) (n)
7. subtract (вычитать) (v) – subtracting (вычитая) (p.1) – subtracted (вычтенный) (p.2)
8. oppose (противопоставлять) (v) – opposed (противопоставленный(p.1), противоположный) (a) – opposite(противоположный) (a) – opposition (оппозиция) (n) – oppositely (противоположно) (adv)
9. direct (направлять) (v) – director (руководитель) (n) – direction (направление) (n) – directed (направленный) (p.2) – directing (направляя) (p.1)

10. apply (прикладывать) (v) – applied (приложенный) (p.2) – applying (прикладывая) (p.1) – application (приложение) (n)
11. form (образовывать) (v) – deform (деформировать) (v) – deformable (деформируемый) (a) – formate (формировать) (v) – deformate (деформировать) (v) – deformation (деформация) (n)
12. solid (твёрдый) (a) – solidify (затвердевать) (v) – solidity (твёрдость) (n) – solidification (затвердевание) (n)

## **B)**

1. exchange – change – interchangeable – unchangeable
2. equal – equality – equation – unequal
3. length – long – lengthen
4. act – action – active – interaction
5. direct – director – directed – direction
6. deformation – deformable – deform – form
7. oppose – opposite – opposition – oppositely
8. observe – observed – observing – observer – observation

## **Exercise 4**

Основные принципы; математическое доказательство; аксиомы статики; общие формулировки; большое количество наблюдений; следствие основных законов; равные по величине; противоположные по направлению; уравновешенная (сбалансированная) система сил; точка приложения; твёрдое тело; произвольная точка; две равные силы; противоположно направленные силы; образовывать сбалансированную систему сил; рассматриваться; аннулировать друг друга; быть перенесённым в точку; скользящий вектор; прикладные задачи; равновесие структуры; внутреннее напряжение; прикладывать в ту же самую точку; диагональ параллелограмма; геометрическая сумма; результирующая двух сил; материальное тело, противоположно направленная реакция, основные законы механики, противоположны по направлению, общие условия равновесия, в соответствии с первым принципом, состояние равновесия, принцип солидификации (затвердевания), скреплённые звенья, гибкая нить, изогнутый жесткий стержень, покоящееся тело, растяжимые силы.

## **Exercise 5**

Some fundamental principles; general formulations; vast number of experiments; motion of a body; laws of mechanics; the course of dynamics; a free body; equal; collinear; oppositely directed forces; to remain unchanged; orig-

inal force system; to transfer along the line of action of a force; let's consider a rigid body; an arbitrary point; to influence the action of a force; to conceal each other; equal in magnitude and in sense; along a line; a sliding vector; to use the law; to take into account; to construct a parallelogram; to construct a parallelogram; geometrical sum; to formulate a principle; to act simultaneously; the law of action and reaction; it follows from the law; according to the principles of statics; studying the conditions of equilibrium; state of equilibrium; links of a chain; flexible string, a bend rigid rod, a reposing body; the same force system; to satisfy the conditions of equilibrium: sufficient and necessary conditions of equilibrium.

### Exercise 6

Fundamental principles(laws); mathematical proof; axioms of statics; a vast number of experiments (observations); motion of a body; corollaries of principles; laws of mechanics; course of statics; a free rigid body; equal in magnitude; the simplest balanced force system; to be added to a force; the point of application; to transfer to a point; along the line of action; an arbitrary point; forces conceal each other; a sliding vector; a perfectly rigid body; to take into account; the parallelogram law; the diagonal of a parallelogram; the geometrical sum; the law of action and reaction; two particles act on each other; general conditions of equilibrium; tensile forces; a reposing body; to stretch a body.

### Exercise 7

1. Now take an arbitrary point B on the line of action of the force and apply to that point two equal and oppositely directed forces  $F_1$  and  $F_2$  such that  $F_1 = F_2$  and  $F_2 = -F_1$ .
2. From the 1st principle it follows that forces  $F_1$  and  $F_2$  also form a balanced system and thus cancel each other.
3. Thus, the vector denoting force  $F$  can be regarded as applied at any point along the line of action.
4. To any action of one material body on another there is always an equal and oppositely directed reaction.
5. The term "force" will henceforth be used in the sense of "external force".

### Exercise 8

Such that = so that.

Thus = so; in this way.

Hence = consequently, therefore.

However = but, though.

Since = as.

Though = however.

### **Exercise 9**

- Cause: since
- Consequence: hence, thus, such that
- Time sequence: –
- Concession: though, however
- Opposition: –

### **Exercise 10**

1. All theorems and equations in statics are deduced from a few fundamental principles which are known as axioms of statics.
2. A free rigid body subjected to the action of two forces can be in equilibrium if, and only if, the two forces are equal in magnitude, collinear, and opposite in direction.
3. The action of a given force system on a rigid body remains unchanged if another balanced force system, is added to, or subtracted from the original system.
4. Two forces applied at one point of a body have as their resultant a force applied at the same point and represented by the diagonal of a parallelogram constructed with the two given forces as its sides.
5. To any action of one material body on another there is always an equal and oppositely directed reaction.
6. If a deformable body subjected to the action of a force system is in equilibrium, the state of equilibrium will not be disturbed if the body solidifies (becomes rigid).

### **Exercise 11**

1. c
2. e
3. d
4. a
5. b

## Exercise 12.

1. f
2. n
3. i
4. a
5. c
6. d
7. b
8. e
9. g
10. h
11. k
12. j
13. l
14. m

**Chapter 1 Basic Concepts and Principles**  
**1.4. Constraints and Their Reactions,**  
**1.5. Axiom of Constraints**

**Exercise 1**

1. There may be several types of constraints. They are a smooth plane (surface) or support, a string, a cylindrical pin (bearing), a ball-and-socket joint, a step bearing, a rod.
2. The axiom of constraints: any constrained body can be treated as a free body relieved from its constraints, provided the latter are represented by their reactions.
3. If in the structure a rod AB is used as a constraint the reaction N will be directed along its axis.
4. The reaction T of the string is thus directed along the string towards the point of suspension.
5. The reaction N of a smooth surface or support is directed normal to both contacting surfaces and is applied at the point of the contact.
6. The reaction R of a pin can have any direction in the plane perpendicular to the axis of the joint.

**Exercise 3**

Реакция (противодействие), направление, связь (опора), ограниченное тело, ограничивающий, примерное округление, нерастяжимая нить, цилиндрическая опора (болт), свободно вращаться, плоскость диаграммы, перпендикулярный, подшипниковое соединение, шаровая пятка, тренога, сравнивать, равновесие, практическое значение, сопротивление.

**Exercise 4**

To be joined, displacement, to solve, the force of the reaction of the constraint, corresponding problems of statics, contacting surfaces, the point of the suspension of a string, axial line, axis of the joint, the plane of the diagram, neither...nor, ball-and-socket joint, ball pivot, tripod, collinear, tension, compression, equilibrium, the following axiom, determination of the reactions, to calculate the strength of structural elements.

**Exercise 6**

1. As it has been defined above, a body not connected with other bodies and capable of displacement in any direction is called a *free body* (e.g., a balloon floating in the air).

2. The constraints in these cases are the surface of the table, which prevents the weight from falling, and the hinges, which prevent the door from sagging from its jamb.
3. The force with which a constraint acts on a body, thereby restricting its displacement, is called the force of reaction of the constraint (force of constraint), or simply the reaction of the constraint.
4. When two bodies are joined by means of a pin passing through holes in them, the connection is called a pin joint or hinge.
5. The axial line of the pin is called the axis of the joint.
6. If a constraint prevents the displacement of a body in several directions, the resultant of reactions is not immediately apparent and has to be found by solving the problem at hand.
7. Let a rod  $AB$  secured by hinges at its ends be the constraint of a certain structure
8. Consequently, a rod subjected to forces applied at its tips, where the weight of the rod as compared with the magnitude of the forces can be neglected, can be only under tension, or under compression.

### Exercise 7

**In the text all the words in bold refer to something mentioned before or after in the text. Read the passage carefully and complete the table underneath.**

lines	words in bold	refers to something before	refers to something after	what it refers to
7	which	+		Surface of the table
14	its	+		Body
24	one	+		Surface
24	whose	+		Surface
51	such	+		Constraint
54	its	+		Reaction
65	its	+		Rod

### Exercise 8

1. as in the case
2. before
3. however
4. and
5. when

6. as in the two dimensional case
7. whereas
8. for example
9. therefore

### **Exercise 9**

As it has been defined above, a body not connected with other bodies and capable of displacement in any direction is called a free body (e.g., a balloon floating in the air). A body whose displacement in space is restricted by other bodies is called a constrained body. A constraint is anything that restricts the displacement of a given body in space.

Examples of constrained bodies are a weight lying on a table or a door swinging on its hinges. The constraints in these cases are the surface of the table, which prevents the weight from falling, and the hinges, which prevent the door from sagging from its jamb.

A body whose displacement is restricted by a constraint acts on that constraint with a force, which is called the load. At the same time, according to the 4th principle, the constraint reacts with a force of the same magnitude and opposite in the sense. The force with which a constraint acts on a body, thereby restricting its displacement, is called the force of reaction of the constraint (force of constraint), or simply the reaction of the constraint.

### **Exercise 11**

d) improper constraints

### **Exercise 12**

1. d
2. f
3. a
4. g
5. e
6. h
7. b
8. i
9. c
10. k
11. j

### Exercise 13

1. f
2. a
3. g
4. c
5. b
6. d
7. e
8. h
9. i
10. j

### Exercise 14

1. with; of; in
2. of; in
3. of; on; on
4. in; of; from; from; from
5. by; on; with.
6. with; on; of; of.
7. of; in; of; by; at.
8. of; or; is; to; is; at; of.
9. of; along; to; of.
10. of; in; to; of.
11. of; in.
12. of, with.
13. of, in; on; of
14. of; of; of
15. of; on; of.

**Chapter 2**  
**Composition of Forces.**  
**Concurrent Force Systems**  
**2.1. Geometrical Method of Composition of Forces. Resultant**  
**of Concurrent Forces**

**Exercise 1**

1. Forces whose lines of action intersect at one point are called concurrent.
2. The geometric sum  $\mathbf{R}$  of two concurrent forces is determined either by the parallelogram rule or by constructing a force triangle.
3. The geometrical sum of three non coplanar forces is represented by the diagonal of a parallelepiped with the given forces as its edges (*the parallelepiped rule*)
4. The geometric sum is the principal vector of a set of forces. It is a vector. The algebraic sum it is the sum of numbers. It is a number.
5. (An open answer of students)
6. The principal vector of the system of forces is the geometric sum of all forces of a given system.

**Exercise 2**

1.  $R = F_1 + F_2 = F_2 = F_1.$
2.  $R = \sqrt{F_1^2 + F_2^2} = 2F_1F_2 \cos \alpha .$
3.  $R = F_1 + F_2 + \dots + F_n$  or  $R = \sum F_k .$

**Exercise 4**

Concurrent forces, the resultant of concurrent forces, vector quantity, problems of mechanics, laws of vector algebra, studying of statics, method of composition of forces, a principle vector, composition of two forces, two concurrent forces, a parallelogram rule, a force triangle, in fact, corresponding parallelogram, diagonal of a parallelogram, connecting the tail of the vector to the head of the vector, corresponding vectors, in a similar way, component forces, law of sines, non coplanar forces, diagonal of the parallelepiped, parallelepiped rule, consecutive application, successive composition of forces, to verify a rule, a force polygon, to lay off a vector, an arbitrary point, penultimate vector, magnitude and the direction of a vector, triangle rule, set of forces, in one sense, in the opposite sense, to come to the conclusion.

### Exercise 5

1. Operate (работать) (v) – operated (обрабатываемый) (p.2) – operating (обрабатывая) (p.1) – operation (действие, операция) (n) – operational (действующий) (a) – operative (действующий) (a).
2. add (прибавлять) (v) – adding (прибавляя) (p.1) – addition (прибавление) (n) – additional (добавочный) (a) – additive (присадка к маслу) (n)
3. quantum (количество, сумма) (n) – quantify (определять) (v) – quantity (количество) (n) – quantitative (количественный) (a)
4. principle (принцип, правило) (n) – principled (принципиальный) (p.2) – principal (глава, начальник) (n) – principality (княжество) (n) – principally (главным образом) (adv)
5. resolve (решать) (v) – resolving (решая) (p.1) – resolved (решенный) (p.2)
6. sector (сектор) (n) – section (сечение, секция) (n) – sectional (секционный) (a) – intersection (пересечение) (n)
7. angle (угол) (n) – triangle (треугольник) (n) – triangular (треугольный)(a) – rectangular (прямоугольный)(a)
8. similar (подобный) (a) – similarity (подобие) (n) – similarly (подобным образом) (adv) .
9. determine (определять) (v) – determined (определённый) (p.2) – determinate (определённый) (a) – determinative (определяющий) (a) – determination (определение) (n) – determinant (определитель) (n).
10. succeed (преуспевать, следовать) (v) – success (успех) (n) – successful (успешный) (a) – succession (последовательность) (n) – successive (последовательный) (a) – successor (последователь) (n).
11. represent (представлять) (v)- representing (представляя) (p.1) – represented (представленный) (p.2) – representation (представление) (n) – representative (представляющий) (a).
12. equal (равный) (a) – equality (равенство) (n) – equalization (уравнение, уравнение) (n) – equalize (уравнивать) (v) – equalizer (балансир, уравнитель) (n) -equally (равно) (adv).
13. conclude (заключать) (v) – conclusion (заключение) (n) – conclusive (заключительный) (a)
14. obtain(получать) (v) – obtaining (получая) (p.1) – obtained (полученный) (p.2) – obtainable (доступный) (a)

### Exercise 6

1. apply – applied – application
2. act – acting – active – action – interaction – counteraction
3. know – known – unknown – knowledge
4. angle – angular – rectangular – triangular

5. mechanic – mechanics – mechanical – mechanism – mechanize – mechanization
6. geometry – geometrical – geometrically – geometer
7. construct – construction – constructive – constructional – constructor – constructing – constructed
8. system – systematical – systematically – systematize – systematizing – systematized
9. resolve – resolving – resolved – resolution
10. determine – determination – determined – determinative – determination – determinant
11. compose – composing – composed – composite – compositor – composition – composition – composition of forces
12. equal – equality – equation – to equalize – equalizer – equally

### Exercise 7

Geometric method of composition, geometric sum of all forces, should not be confused, as we shall see later on, lines intersect at a point, “head-to-tail”, in a similar manner, the side of the triangle (parallelogram, polygon), the law of sines, composition of a system of forces, to lay off a vector, from the tail to the head, a set of forces, resultant of forces, resultant is applied at the same point.

### Exercise 8

1. The geometric sum  $\mathbf{R}$  of two concurrent forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  is determined either by the parallelogram rule (Fig. 2.1a) or by constructing a force triangle (Fig. 2.1b), which is in fact one-half of the corresponding parallelogram.  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are joined at their tails.
2. We can also add  $\mathbf{F}_1$  to  $\mathbf{F}_2$  using a triangle construction, which is a special case of the parallelogram law, whereby vector  $\mathbf{F}_1$  is added to vector  $\mathbf{F}_2$  in a “head to tail” fashion, i.e., by connecting the head of  $\mathbf{F}_1$  to the tail of  $\mathbf{F}_2$ .
3. Consecutively applying the parallelogram rule, we come to the conclusion that the resultant of a system of concurrent forces is equal to the geometrical sum (principal vector) of these forces and that it is applied at the point of their intersection.
4. The magnitude of the vector is indicated by the length of the arrow, the direction is defined by the angle between the reference axis and the arrow’s line of action, and the sense is indicated by the arrowhead.
5. As a special case, if the two vectors  $\mathbf{A}$  and  $\mathbf{B}$  are collinear, i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition*  $\mathbf{R}=\mathbf{A}+\mathbf{B}$ .

6. Using the parallelogram law to add more than two forces, as shown here, often requires extensive geometric and trigonometric calculation to determine the numerical values for the magnitude and direction of the resultant.

### Exercise 9

lines	Words in bold	Refers to something before	Refers to something after	What it refers to
3	Which	+		quantity
8	That	+		concept
17	Which	+		force triangle
21	Which	+		diagonal
23	Which	+		triangle construction
31	Which	+		the angles
40	The latter	+		the force polygon (method)
53	Its	+		force polygon

### Exercise 10

As force is a vector quantity, many problems of mechanics are solved according to the laws of vector algebra. We shall commence our study of statics with the geometric method of composition of forces. The quantity which is the geometric sum of all the forces of a given system is called the *principal vector* of the system. As noted in 1.3, the concept of the geometric sum of forces should not be confused with that of the resultant. As we shall see later on, many force systems have no resultant at all, but the geometric sum (principal vector) can be calculated for any force system.

**Composition of Two Forces.** The geometric sum  $\mathbf{R}$  of two concurrent forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  is determined either by the parallelogram rule (Fig. 2.1a) or by constructing a force triangle (Fig. 2.1b), which is in fact one-half of the corresponding parallelogram.  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are joined at their tails. Parallel lines drawn from the head of each vector intersect at a common point, thereby forming the sides of a parallelogram. As shown, the resultant  $\mathbf{R}$  is the diagonal of the parallelogram, which extends from the tails of  $\mathbf{A}$  and  $\mathbf{B}$  to the intersection of the lines.

We can also add  $\mathbf{F}_1$  to  $\mathbf{F}_2$  using a **triangle construction**, which is a special case of the parallelogram law, whereby vector  $\mathbf{F}_1$  is added to vector  $\mathbf{F}_2$  in a “head to tail” fashion, i.e., by connecting the head of  $\mathbf{F}_1$  to the tail of  $\mathbf{F}_2$ . The resultant  $\mathbf{R}$  extends from the tail of  $\mathbf{F}_1$  to the head of  $\mathbf{F}_2$ . In a similar manner,

$\mathbf{R}$  can also be obtained by adding  $\mathbf{F}_2$  to  $\mathbf{F}_1$ . In other words, the vectors can be added in either order, i.e.,  $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{F}_2 + \mathbf{F}_1$ .

The magnitude of  $\mathbf{R}$  is the side  $A_1C_1$  of the triangle  $A_1B_1C_1$ , i.e.,

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos(180^\circ - \alpha) ,$$

where  $\alpha$  is the angle between the two forces. Hence,  

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

The angles  $\beta$  and  $\gamma$  which the force  $\mathbf{R}$  makes with the component forces can be determined by the law of sines. Since  $\sin (180^\circ - \alpha) = \sin \alpha$ , we have:

$$\frac{F_1}{\sin \gamma} = \frac{F_2}{\sin \beta} = \frac{R}{\sin \alpha}$$

**2. Composition of Three Non-Coplanar Forces.** The geometrical sum  $\mathbf{R}$  of three non-coplanar forces  $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$  is represented by the diagonal of a parallelepiped with the given forces as its edges (the *parallelepiped rule*). This rule can be verified by successive applying the parallelogram rule (Fig. 2.2).

**3. Composition of a System of Forces.** The geometrical sum (principal vector) of any force system can be determined either by successive composition of the forces of the system by the parallelogram rule, or by constructing the **force polygon**. The latter method is simpler and more convenient. In order to find the sum of the forces  $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \dots, \mathbf{F}_N$  (Fig. 2.3a) we lay off a vector from an arbitrary point  $O$  (Fig. 2.3b) a vector  $\overline{Oa}$  equal to the force  $\mathbf{F}_1$ . Now from point  $a$  we lay off a vector  $\overline{ab}$  equal to the force  $\mathbf{F}_2$ , from point  $b$  we lay off a vector  $\overline{bc}$  equal to the force  $\mathbf{F}_3$ , and so on. From the head  $m$  of the penultimate vector we lay off the vector  $\overline{mn}$  equal to the force  $\mathbf{F}_n$ . Vector  $\overline{On} = \mathbf{R}$ , laid off from the tail of the first vector to the head of the last vector, is the geometrical sum, or the principal vector, of the component forces:  $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n$  or  $\mathbf{R} = \sum \mathbf{F}_k$

The magnitude and direction of  $\mathbf{R}$  do not depend on the order in which the vectors are laid off. It will be noted that the construction of the force polygon is a consecutive application of the triangle rule.

Thus, *the geometrical sum, or the principal vector, of a set of forces is represented by the closing side of a force polygon constructed with the given forces as its sides (the polygon rule)*. In constructing a vector polygon, we should arrange all the component vectors in one sense along the periphery of the polygon, with vector  $\mathbf{R}$  being drawn in the opposite sense.

**Resultant of Concurrent Forces.** *Forces whose lines of action intersect at one point are called concurrent* (see Fig. 2.3a). It follows from the first two principles of statics that a system of concurrent forces acting on a rigid body is equivalent to a system of forces applied at one point (point  $A$  in Fig. 2.3a). Consecutively applying the parallelogram rule, we come to the conclusion that the *resultant of a system of concurrent forces is equal to the geometrical sum (principal vector) of these forces and that it is applied at the point of their intersection*. Hence, if the forces  $F_1, F_2, \dots, F_n$  intersect at point  $A$  (Fig. 2.3a), the resultant of the system is a force equal to the principal vector  $R$ , obtained by constructing a force polygon, and applied at point  $A$ .

### Exercise 11

(An open answer of the students)

### Exercise 12

Concurrent forces

Geometrical method

Force systems

Principal vector

Vector addition law

Parallelogram rule

Common point

Triangle construction

### Exercise 15

1. 5
2. 2
3. 6
4. 9
5. 7
6. 3
7. 1
8. 8
9. 4
10. 10

### Exercise 16

1. of, with, of, of.
2. of, of
3. at

4. from, of, at, of.
5. to, in, to, by, of, with.
6. by.
7. of, by, of, with.
8. in, of, off, from.
9. of, on, in, off.
10. by, in, along, of, with, in.
11. of, at.
12. from, of, of, on, to, of, at.

**Chapter 2**  
**Composition of Forces.**  
**Concurrent Force Systems**  
**2.2. Resolution of Forces;**  
**2.3. Equilibrium of a System of Concurrent Forces.**

**Exercise 1**

**2.2. Resolution of forces**

To resolve a force into two or more components means to replace it by a force system whose resultant is equal to the original force. Two cases are of particular interest:

1. Resolution of a force into two components of given direction. The problem can be solved by applying the parallelogram rule and the triangle rule. That is  $P+Q=F$ .
2. Resolution of a force into three components of given direction. If the given directions are not coplanar, the problem is specified and it reduces to the construction of a parallelepiped having the given force  $F$  as its diagonal and the sides parallel to the given directions.

The method of resolution of forces is useful in determining the pressure on constraints induced by the applied forces. This method can be applied only if the directions of their reactions are quite evident.

**2.3. Equilibrium of a System of Concurrent Forces.**

For a body subjected to the action of a system of concurrent to be in equilibrium it is necessary for the resultant of the forces to be zero.

1. **Graphical Conditions of Equilibrium.** A system of concurrent forces is in equilibrium if and only if the force polygon drawn by these forces is closed.
2. **Analytical Conditions of Equilibrium.** The necessary and sufficient condition for the equilibrium of a three-dimensional system of concurrent forces is that the sums of the projections of all these forces on each of three coordinate axes must be zero.
3. **The Theorem of Three Forces.** The following theorem will often be used in solving problems of statics: if a free rigid body remains in equilibrium under the action of three nonparallel coplanar forces, then the lines of action of these forces intersect at one point. It should be noted that the reverse is not true, i.e., if the action lines of three forces intersect at one point, the body on which they are acting is not necessarily in equilibrium. Thus, the theorem expresses a necessary, but not sufficient, condition for the equilibrium of a free rigid body acted upon by three forces.

### Exercise 3

Resolution of forces; original force; additional conditions; given direction (sense); respectively; forces parallel correspondingly; to carry out the resolution of a force; to apply a triangle rule; to draw the lines to the point of their intersection; to replace a force; along the line of action of a force; three components of a given direction; parallel sides; method of resolution of forces; to determine pressure; pressure on a constraint; reactions are evident; the system of concurrent forces; to express the conditions; graphical conditions of equilibrium; the side of a force polygon; if and only if; a force polygon is closed; magnitude of the resultant; the square root of (under the square root); sum of the components; necessary and sufficient conditions; three-dimensional force system; sum of the projections; to lay in one plane; coplanar system of concurrent forces; to express the conditions of equilibrium; necessary equations; system of three forces; to remain in equilibrium; under the action of forces; three non-parallel coplanar forces; according to the first principle; a force passes through A; the proved theorem; to determine the unknown directions of the reactions.

### Exercise 4

Разложение сил, заменять (замещать) системой сил, дополнительные условия, быть параллельным линиям, построить параллелограмм, провести линии через начало и конец, соответствующие компоненты, выполнять разложение, вдоль линии действия, уточнять задачу, направления не копланарны, давление производимое силой, графические условия равновесия, закрывающая сторона многоугольника, аналитические условия равновесия, величина результирующей, выражение под корнем, проекции на координатные оси, проекции сил, лежать в одной плоскости, два уравнения, выражать условия равновесия, теорема трех сил, использовать теорему, оставаться в равновесии, три непараллельные копланарные силы, линии действия, пересекаться в точке А, доказывать теорему, та же самая плоскость, перенести силы к точке, необходимые но не достаточные условия, замещать опоры(связи), свободное тело в равновесии.

### Exercise 5

1. to resolve (раскладывать) (v) – resolving (раскладывая) (p.1) – resolved (разложенный) (p.2) – resolution (разложение) (n)
2. place (место) (n) – to place (помещать) (v) – placed (размещенный) (p.2) – to replace (перемещать) (v) – replaced (перемещённый) (p.p)

3. to determine (определять) (v) – determined (определённый) (p.p) – determinate (определённый) (a) – determinant (определитель) (n) – determination (определение) (n) – determinative (определяющий) (a) – indeterminate (неопределённый) (a).
4. to solve (решать) (v) – solving (решая) (p.1) – solved (решенный) (p.p) – solution (решение) (n) .
5. direct (прямой) (a) – direction (направление) (n) – directional (направленный) (a) – directive (направляющий) (a)- directly (прямо) (adv) – director (управляющий) (n).
6. dimension (измерение) (n) – dimensional (пространственный) (a) – three-dimensional (трех-пространственный) (a)
7. section (сечение) (n) – sectional (секционный) (a)- to intersect(пересекаться) (v) – intersecting (пересекая) (p.1)- intersection (пересечение) (n).
8. necessary (необходимый) (a) – necessarily (необходимо) (adv) – necessity (необходимость) (n)
9. constrain (ограничивать) (v) – constrained (ограниченный) (p.p) – constrainedly (по принуждению) (a) – constraint (связь, ограничение, принуждение) (n).
10. know (знать ) (v) – knowing (зная) (p.1) – unknown(неизвестный) (a) – well-known (известный) (a) – knowledge (знание) (n).

### Exercise 6

- a. to subject – subjecting – subjected – subject – subjective – subjectivity – subjection
- b. condition – conditional – conditioned – conditioning
- c. to express – expressible – expression – expressive – expressively
- d. analyze – analyzing – analyzed – analysis – analyst – analytical – analytics
- e. necessarily – necessary – necessitate – necessity
- f. project – to project – projecting – projected – projection – projector
- g. coordinate – coordination – coordinates
- h. follow – followed – following – followers
- i. parallel – parallelepiped – parallelism – parallelogram
- j. to apply – to apply – applying – applied – application – applicable – application (form) – applicant
- k. consequence – consequent – consequential – consequently
- l. to constrain – constrained – constrainedly – constraint – constraint
- m. to act – action – active – counteraction – interaction

### Exercise 7

To resolve a force, to lay off a vector, to construct a parallelogram, to apply the force, lines intersect at one point, along the line of action, three components of a force, a system of forces, graphical or analytical conditions of equilibrium, a closing side of a polygon, the expression under the radical, necessary and sufficient, three- dimensional system of concurrent forces, three coordinate axes, to lie in one plane, to intersect at one point, to prove the theorem, to replace a force system by a resultant, it should be noted, under the action of forces, to pass through a point, to determine the direction of forces.

### Exercise 9

1. the tail of the vector
2. the head of the vector
3. magnitude
4. load
5. rigid
6. apparent
7. equilibrium
8. concurrent forces
9. zero
10. simultaneously
11. plane
12. theorem

### Exercise 10

1. To resolve a force into two or more components means to replace it by a force system whose resultant is equal to the original force.
2. If the given directions are not coplanar, the problem is specified and it reduces to the construction of a parallelepiped having the given force  $F$  as its diagonal and the sides parallel to the given directions.
3. Loads acting on rigid constraints are determined by resolving the given forces along the directions of the reactions of the constraints, force acting on a constraint and its reaction have the same line of action.
4. As the resultant  $R$  of a system of concurrent forces is defined as the closing side of a force polygon constructed by the given forces, then  $R$  can be zero only if the head of the last force vector of the polygon coincides with the tail of the first force vector, that is, if the polygon is closed.
5. If the body is to be in equilibrium, then, according to the 1st principle, forces  $R$  and  $F_3$  must be directed along the same line, i.e., along  $AB$ .

6. By replacing the constraints with their reactions we can treat the beam as a free body in equilibrium under the action of three forces  $\mathbf{P}$ ,  $\mathbf{N}_D$  and  $\mathbf{R}_A$ , whose lines of action, according to the theorem just proved, must intersect at one point.

### Exercise 11

lines	words in bold	refers to something before	refers to something after	what it refers to
3	It	+		Forces
3	Whose	+		Force system
12	Which	+		Components
21	Their	+		Lines
25	It	+		Problem
26	Its	+		Parallelepiped
29	Their	+		Components
34	Its	+		Constraint
59	They	+		Concurrent forces
70	Them	+		Forces
70	Their	+		Forces
76	They	+		Forces
84	They	+		Lines of action

### Exercise 12

**1. Graphical Condition of Equilibrium.** As the resultant  $\mathbf{R}$  of a system of concurrent forces is defined as the closing side of a force polygon constructed by the given forces, then  $\mathbf{R}$  can be zero only if the head of the last force vector of the polygon coincides with the tail of the first force vector, that is, if the polygon is closed.

Thus, a system of concurrent forces is in equilibrium if and only if the force polygon drawn by these forces is closed.

**2. Analytical Conditions of Equilibrium.** The magnitude of the resultant of a system of concurrent forces is given by the formula  $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

As the expression under the radical is a sum of positive components,  $R$  can be zero only if simultaneously  $R_x = 0$ ,  $R_y = 0$ ,  $R_z = 0$ , i.e.,  $\sum F_{kx} = 0$ ,  $\sum F_{ky} = 0$ ,  $\sum F_{kz} = 0$ .

Eqs. (2.5) provide the conditions of equilibrium in analytical form: *The necessary and sufficient condition for the equilibrium of a three-dimensional system of concurrent forces is that the sums of projections of all these forces on each of three coordinate axes must be zero.*

If all the concurrent forces acting on a body lie in one plane, they are said to form a *coplanar system of concurrent forces*. Obviously, for such a force system only two equations are required to express the conditions of equilibrium:  $\sum F_{kx} = 0$ ,  $\sum F_{ky} = 0$ .

### Exercise 13

c) Vector multiplication

### Exercise 14

The part about the theorem of three forces.

### Exercise 15

1. 5
2. 4
3. 7
4. 1
5. 6
6. 3
7. 2

### Exercise 16

1. to the original force
2. triangle rule
3. forces
4. three-dimensional force, coordinate
5. three forces
6. forces, one
7. equilibrium
8. reactions

### Exercise 17

1. We can resolve a force into two components by constructing a parallelogram.
2. we can resolve a force into three components by constructing a parallelepiped
3. Conditions of equilibrium can be expressed either in graphical or in analytical form.

4. The system of concurrent forces is in equilibrium if the force polygon is closed.
5. If all the concurrent forces lie in one plane they form a coplanar system of concurrent forces.
6. The theorem of three forces is used in solving the problems of statics.

**Exercise 18**

1. T
2. F
3. F
4. F
5. T
6. T
7. F
8. F
9. F
10. F

**Chapter 2**  
**Composition of Forces.**  
**Concurrent Force Systems**  
**2.4, 2.5. Problems Statically Determinate and Statically Indeterminate.**  
**Solution of Problems of Statics.**

**Exercise 1**

A problem of statics can be solved only if the number of unknown reactions is not greater than the number of equilibrium equations which can be used to determine these reactive forces. Such problems are called statically determinate. When a body has more supports than are necessary to hold it in equilibrium, it becomes statically indeterminate.

**Solution of problems of Statics**

The problems solved by the methods of statics fall into two types:

1. Problems in which some or all the forces acting on the body are known and we have to determine under what conditions the body will be in equilibrium.
2. Problems in which the body is known to be in equilibrium and we have to determine all or some of the forces acting on it.

The solution of the problems of statics consists of the following steps:

1. Choose the body whose equilibrium should be examined.
2. Isolate the body from its constraints and draw the given forces and the reactions of the removed constraints.
3. State the conditions of equilibrium.
4. Determine the unknown quantities, verify the answer and analyze the results.

Equilibrium problems involving concurrent forces can be solved by the graphical or the analytical method. The graphical method is suitable when the total number of given and required forces acting on a body is three. The analytical method can be applied for any number of forces.

**Exercise 2**

- 1) A problem of statics can be solved only if the number of unknown reactions is not greater than the number of equilibrium equations which can be used to determine these reactive forces.
- 2) Statically indeterminate problems are studied in the courses of strength of materials and statics of structures.

- 3) The problems of statics fall into two types: **A.** Problems in which some or all the forces acting on the body are known and we have to determine under what conditions the body will be in equilibrium. **B.** Problems in which the body is known to be in equilibrium and we have to determine all or some of the forces acting on it.
- 4) In practical applications, problems of statics are used to determine the conditions of equilibrium of structures (when they are not properly held or constrained) and the loads on the supports in different parts of a structure.
- 5) A structure is an association of several connected bodies.
- 6) The solution of the problems of statics consists of the following steps:
  1. Choose the body whose equilibrium should be examined.
  2. Isolate the body from its constraints and draw the given forces and the reactions of the removed constraints.
  3. State the conditions of equilibrium.
  4. Determine the unknown quantities, verify the answer and analyze the results.
- 7) To solve the problems involving concurrent forces we use the graphical and analytical methods. (See the description of these methods in the text)
- 8) An open answer of the students.
- 9) An open answer of the students.

#### **Exercise 4**

1. G
2. S
3. F
4. T
5. B
6. E
7. H
8. U
9. C
10. W
11. V
12. D
13. I
14. A
15. Q
16. L
17. J
18. P
19. Y

- 20. X
- 21. N
- 22. M
- 23. R
- 24. O
- 25. K

**Exercise 5**

nouns	adjectives	Verbs	adverbs
equilibrium	equilibrant	Solve	statically
constraint	constrained	Determine	properly
solution	reactive	Hold	carefully
quantity	corresponding	Move	
motion	necessary	Construct	
structure	different	Examine	
exam	respective	Isolate	
reaction	total	Verify	
statement	graphical	Simplify	
computation	coplanar		
	concurrent		
	arbitrary		

**Exercise 6**

The reactions of the constraint, the number and the type of the constraint, the number of unknown reactions, statically determinate problems, to hold the body in equilibrium, solutions of problems of statics, to fall into two types, in practical application, conditions of equilibrium, to draw the given forces, the reactions of the removed constraints, graphical method of solution, to determine the unknown quantities, to carry out the computations, to be written out in a general form, to analyze the results, to substitute the numerical values, to obtain the unknown quantities, coplanar system of concurrent forces.

**Exercise 7**

- 1. of, of
- 2. of, of, of,
- 3. in, of,
- 4. i.e; in, of, as, of.
- 5. for, of, of, into.
- 6. by, of, into
- 7. of, in, of
- 8. on, into

**Exercise 8**

- 1. c
- 2. e
- 3. a
- 4. b

- 5. d
- 6. f
- 7. a

### Exercise 9

- For the solution of statically indeterminate problems the assumption of the rigidity of the bodies under consideration must be given up and their deformations taken into account.
- The problems solved by the methods of statics fall into one of two types: (1) Problems in which some or all of the forces acting on the body are known and where we have to determine under what conditions the body will be in equilibrium. (2) Problems in which the body is known to be in equilibrium (or in inertial motion) and we have to determine all or some of the forces acting on it.
- For the problem to be solved, the given and required forces, or their equivalents, should all be applied to the body whose equilibrium is examined (for instance, if the problem is to determine a load acting on a support, we can examine the equilibrium of the body experiencing the reaction of the support, which is equal in magnitude to the required load).
- In solving a problem it is important to have a carefully drawn diagram, which helps to choose the correct method of solution and prevents errors in stating the conditions of equilibrium.
- Before writing the conditions of equilibrium which for a coplanar system of concurrent forces will be in the form of the two eqs. (2.6), and for a three-dimensional system the three Eqs. (2.5) the coordinate axes must be chosen.
- Identify the unknowns, and in general show all the unknown components having a positive sense if the sense cannot be determined.
- Instead, it is recommended that one choose the direction of an axis for moment summation such that it intersects the lines of action of as many unknown forces as possible.
- By the proper choice of axes, it may be possible to solve directly for an unknown quantity, or at least reduce the need for solving a large number of simultaneous equations for the unknowns.
- To ensure the equilibrium of a rigid body, it is not only necessary to satisfy the equations of equilibrium, but the body must also be properly held, or constrained by its supports.
- Some bodies may have more supports than are necessary for equilibrium, whereas others may not have enough or the supports may be arranged in a particular manner that could cause the body to collapse.

### Exercise 10

In the text all the words in bold refer to something mentioned before or after in the text. Read the text carefully and complete the table underneath.

lines	words in bold	refers to something before	refers to something after	what it refers to
2	their	+		Reactions of the constraints
5	which	+		Equilibrium equation
13	them	+		Number of reactions
19	where	+		Problems
22	it	+		Body
29	whose	+		Body
46	this	+		Should be written
50	they	+		Equilibrium problems

### Exercise 11

1. however
2. because of
3. therefore
4. hence
5. furthermore
6. since
7. as a result

### Exercise 12

Construct a free – body diagram for the body. Be sure to include all the forces and couple moments that act on the body. These interactions are commonly caused by the externally applied loadings, contact forces exerted by adjacent bodies, support reactions, and the weight of the body if it is significant compared to the magnitudes of the other applied forces. Establish the origin of the  $x, y, z$  axes at a convenient point, and orient the axes so that they are parallel to as many of the external forces and moments as possible. Identify the unknowns, and in general show all the unknown components having a positive sense if the sense cannot be determined. Dimensions of the body, necessary for calculating the moments of forces, are also included on the free-body diagram.

### Exercise 13

4) equilibrium of a rigid body.

### Exercise 14

1. 4
2. 2
3. 1
4. 7
5. 5
6. 6
7. 8
8. 3

### Exercise 15

1. analytical method
2. statically indeterminate problems
3. a problem
4. a constraint
5. graphical method
6. a constrained body
7. equilibrium
8. quantity
9. statically determinate problems
10. force
11. the reaction of the constraint
12. structure

### Exercise 16

- 1) e
- 2) g
- 3) j
- 4) i
- 5) h
- 6) b
- 7) d
- 8) c
- 9) f
- 10) a

**Chapter 3**  
**Parallel Forces and Force Couples in a Plane**  
**3.1. Composition and Resolution of Parallel Forces**

**Exercise 1**

1. A force couple is a pair of parallel forces acting on a rigid body.
2. A plane of action of a couple is a plane in which two concurrent forces lie.
3. Concurrent forces pass through a common point.
4. The magnitude of the resultant of the concurrent forces of the same sense is equal to the sum of their magnitudes, parallel to them and is of the same sense.

The magnitude of the resultant of two concurrent forces of the opposite directions acting on a body is equal in magnitude to the difference between their magnitudes.

**Exercise 3**

Давайте..., твердое тело, такого же направления, противоположного направления, сложение 2 сил, две параллельные силы, равная система сходящихся сил, две сбалансированные силы, правило параллелограмма, перемещать результирующую, раскладывать на первоначальные компоненты(составляющие), замещать результирующей, результирующая параллельных сил, рассматривать треугольники, соответствующие треугольники, по свойству пропорций, из подобия (сходства) треугольников, если какие-нибудь, первоначальные компоненты, принимать в расчет, быть равным сумме их величин, разделять отрезок прямой (линии) , точки приложения сил, обратно пропорциональный, рассматривать конкретный случай, точка на продолжении АВ, расстояние С удовлетворяет уравнениям, складывая силы ...мы находим..., быть равным по величине, быть приложенным в точке В, силы могут быть отброшены, замещать результирующей R, силы противоположные по направлению, составляющие силы, несколько параллельных сил, последовательное приложение, правило сложения сил.

**Exercise 4**

Let's find the resultant, two cases are possible, forces of the same sense, forces of the opposite sense, parallel forces, applying the first and the second principles of statics, equivalent system of concurrent forces, according to the parallelogram rule, to transfer the resultant to the point O, to resolve into the components, two balanced forces, to determine the location of C, let's consider the triangles OAC and OCB, equality of the corresponding triangles, by the property of the proportions, to take into account, the resultant of two

concurrent forces, sum of their magnitudes, the line of action of the resultant, to divide the line segment into the parts, inversely proportional to the forces' magnitudes, let's consider a specific case, to satisfy the equations, forces can be discarded, is equal to the difference between the magnitudes, consecutive application of the rule.

### Exercise 5

Of the same sense, of the opposite sense, the resultant of two forces, system of concurrent forces, by the parallelogram rule, initial components, similarity of the respective triangles, property of the proportions, taking into account, line of action, inversely proportional, a specific case, to satisfy the equations, the point of application, extension of the line, consecutive application, the rule of the composition of forces.

### Exercise 6

Inversely proportional

Parallel forces

The same sense

Rigid components

Given body/system/forces/rule etc.

Concurrent forces

Parallelogram rule

Initial components

Property of proportions

Respective triangles

Line segment

### Exercise 7

a)

1. Compose (складывать, составлять) (v) – composed (составленный) (p.p) – composer (композитор) (n) – composing (составляя) (p.1) – composite (составной) (a) – composition (состав) (n)
2. Act (действовать) (v – action (действие) (n) – active (действующий) (a) – activity (деятельность) (n) – interaction (взаимодействие) (n)
3. Possible (возможный) (a) – possibly (возможно) (adv) – possibility (возможность) (n)
4. Balance (равновесие) (n) – balancer (эквilibрист) (n) – balancing (балансируя) (p.1) – balanced (сбалансированный) (p.p)
5. Direct (прямой) (a) – direction (направление) (n) – directional (направленный) (a) – directive (направляющий) (a) – directly (прямо) (adv) – director (руководитель)(n).

6. Transfer (перемещать) (v) – transferable (перемещаемый) (a) – transferee (человек, которому что-то передается) (n) – transference (передача) (n) – transferor (лицо, передающее что-либо) (n).
7. Angle (угол) (n) – angular (угловой) (a) – angularity (угловатость) (n) – rectangular (прямоугольный) (a) – triangle (треугольник) (n)
8. Proportion (пропорция) (n) – proportional (пропорциональный) (a) – proportionality (пропорциональность) (n) – proportionate (делать пропорциональным) (v).
9. Count (счет) (n) – countable (исчисляемый) (a) – uncountable (неисчисляемый) (a)
10. Apply (прилагать) (v) – applied (прилагаемый) (p.p) – applying (прилагая) (p.1) – applicable (применимый) (a)
11. Connect (соединять) (v) – connected (соединенный) (p.p) – connecting (соединяя) (p.1) – connective (соединительный) (a) – connection (соединение) (n).
12. Magnify(увеличивать) (v) – magnifier (увеличитель) (n) – magnificent (величественный) (a) – magnification (увеличение) (n) – magnitude(величина) (n)
13. Equal (равный) (a) – equality (равенство) (n) – equalization (уравнение) (n) – equalize (уравнивать) (v) – equalizer (уравнитель) (n) – equally (равно) (adv).
14. Differ (различаться) (v) – difference (различие)(n) – different (различный) (a) – differently(по – разному) (adv)

**b)**

1. rigid – rigidity – rigidly
2. possible – possibility – possibly
3. force – enforce – enforcement – reinforce
4. angle – angular – rectangular – rectangle – triangle – triangular
5. proper – properly – property
6. proportion – proportional – proportionality – proportionalize
7. oppose – opposed – opposite – opposition – oppositionist – oppositely
8. location – locate – locating – located – dislocated
9. specific – specification – specify – specifying – specified

**Exercise 8**

Composition, location, resolution, equation, case, difference, sense, sufficient, equivalent, concurrent.

**Exercise 9**

1. Thus, the resultant of two parallel forces of the same sense acting on a rigid body is equal to the sum of their magnitudes, parallel to them, and is of same sense.

2. The line of action of the resultant divides the line segment between the points of the forces application into the parts which are inversely proportional to the forces' magnitudes.
3. Composing forces  $F_2$  and  $R'$ , we find from Eqs. (3.1) and (3.2) that their resultant  $Q$  is equal in magnitude to  $F_2 + R'$ , that is, it is equal to force  $F_1$  and is applied at point  $A$ .
4. Thus, the resultant of two parallel forces of opposite sense acting on a rigid body is equal in magnitude to the difference between their magnitudes, parallel to them, and has the same sense as the greater force; the line of action of the resultant divides the extension of the line segment connecting the points of application of the component forces into the parts which are inversely proportional to the forces' magnitudes.
5. If a system of forces is either concurrent, coplanar or parallel, it can always be reduced, as in the above case, to a single resultant force  $F$  acting through a unique point  $P$ .
6. Parallel force systems, which can include couple moments that are perpendicular to the forces, can be reduced to a single resultant force, because when each force is moved to any point  $O$  in the  $x$ - $y$  plane, it produces a couple moment that has components only about the  $x$  and  $y$  axes.
7. Most often a scalar analysis can be used to apply these equations, since the force components and the moment arms are easily determined for either coplanar or parallel force systems.
8. The technique used to reduce a coplanar or parallel force system to a single resultant force follows the general procedure outlined in the previous section.

### Exercise 10

lines	Words in bold	Refers to something before	Refers to something after	What it refers to
12	Them	+		Forces
15	Their	+		Resultants
17	Them	+		Forces
22	Their	+		Forces
31	Their	+		Forces
33	Which	+		Parts
41	Their	+		Forces
47	Them	+		Forces
50	Which	+		Parts
52	Their	+		Forces

### Exercise 11

An open answer of the students

### Exercise 12

The moment of a force will be formulated using Cartesian vectors in the next section. Before doing this, however, it is first necessary to expand our

knowledge of vector algebra and introduce the cross-product method of vector multiplication.

The cross product of two vectors **A** and **B** yields the vector **C**, which is written  $\mathbf{C} = \mathbf{A} * \mathbf{B}$  and is read “**C** equals **A** cross **B**”.

### Exercise 13

1. resultant, forces, rigid
2. forces, same, forces, opposite
3. point, forces
4. determine, triangles
5. parallel, sense, rigid, equal
6. force, segment, inversely
7. forces, successive, composition

### Exercise 14

1. body
2. triangle
3. parallel lines
4. parallelogram
5. point
6. statics
7. resultant
8. segment
9. line
10. system
11. rigid
12. proportion
13. concurrent
14. equation

### Exercise 15

1. g
2. j
3. i
4. h
5. a
6. b
7. c
8. e
9. f
10. d

## Chapter 3.

### 3.2. Moment of a force about a point.

#### Exercise 1

1. A moment of a force about a center  $O$  is a measure of the tendency of that force to turn a body.
2. It is characterized by the following elements: 1) magnitude of the moment, which is equal to the product of the force and the moment arm, 2) the plane of rotation  $OAB$  passing through the line of action of the force  $F$  and the centre  $O$ , 3) the sense of rotation in that plane.
3. When all the given forces are coplanar the moment of a force can be defined as a scalar algebraic quantity equal to  $\pm Fh$ , where the sign indicates the sense of rotation. If the forces are not coplanar, the planes of rotation have different aspects for different forces and have to be specified additionally.
4. A **unit vector** in a normed vector space is a vector whose length is 1.  $i, j$  and  $k$  are the unit vectors on the coordinate vectors.

#### Exercise 3

Нет необходимости, в этом случае, данные силы, плоскости разные для разных сил, направление вектора, момент силы около точки, векторное выражение момента, предположим, мера (величина) стремления, характеризуется тремя элементами, величина момента, произведение момента на плечо момента, плоскость вращения, координаты точки приложения, направление вращения, линия действия силы, скалярная алгебраическая величина, разные виды (аспекты), положение плоскости в пространстве, модуль этого вектора, направление вращения, перпендикулярный к плоскости, рассматриваемый с конца вектора, произведение векторов, двойная площадь треугольников, через меньший угол, векторное произведение радиус вектора и силы, вычислять момент  $M_o$  аналитически (алгебраически), через начало  $O$ , единичные векторы на координатных осях, правая часть уравнения, расширять первым рядом, то есть, проходить через.

#### Exercise 4

Product of the force and the moment arm, plane of rotation, the line of action of a force, magnitude of the moment, to rotate the body, round the axis, sign, to specify additionally, different aspects, position of the plane in space, scalar algebraic quantity, modulus of the vector, with respect to the centre  $O$ , normal to the plane  $OAB$ , moment of a force about a point, to direct a vector, to view the rotation, from the arrowhead, the position of the moment centre,

vector product, equal in magnitude, double area of the triangle, to carry through the smaller angle between them, to lay off from the same point, thus, consequently, radius vector, the point of application of the force, to us equations, to compute the moment, the known formula in vector algebra, to be characterized by the three elements, coplanar forces, to define the moment of a force, to determine the moment of a force, vector applied to O, in the same sense(direction), the point of application of a force, determinant.

### **Exercise 5**

Vector expression of a moment; forces, acting on a rigid body; forces are directed along the plane ABC; to transfer forces to the point O; resolve the forces into the initial components; from the similarity of the respective triangles; by the property of the proportions; to take into account; the resultant is equal to the sum; the resultant is of the same sense; to divide into the universally proportional parts; we find from the equations; as a result; the difference between the magnitudes; moment of a force; measure of the tendency; plane of rotation; the sense of rotation;

In that plane; the quantity equal to  $Fh$ ; the position of a plane in space; line vector is normal to the plane; a moment of a force; with respect to a given centre; the rotation viewed from the arrowhead; expression of the moment in terms of cross product; rotation would carry OA into F through the smaller angle; two forces are laid off from the same point; to be expanded by the first row.

### **Exercise 6**

A rigid body

The center of rotation

In the same sense

Plane of rotation

Equal in magnitude

Arm of a moment

Respective triangles

Composition of forces

Right-hand part of the equation

Coplanar forces

Point of application

Moment of a force

Parallel lines

Resolution of forces

Coordinate axes  
Analytical methods

**Exercise 7**

1. of, about, of, of, to
2. of, for
3. of, in, to, of, of
4. of, to,
5. of, about, at
6. in, to, of, through
7. in, to, of
8. to
9. in, to, between, in, as
10. at, of
11. of, in, by, to
12. in, to, of, to
13. of, about, to, of, from, of.
14. through, of, on, of, of
15. of, by, of, to, of, on

**Exercise 8**

Multiplication = product  
Imaginary surface = plane  
Direction = sense  
Perpendicular = normal  
Anticlockwise = counterclockwise  
Place = location  
The smallest vector = unit vector  
Determining element = determinant  
Right sided = right-hand sided

**Exercise 9**

1. point
2. tendency
3. to rotate
4. consider
5. distance
6. that
7. to turn
8. length
9. rotation
10. moment
11. perpendicular
12. intersects

### Exercise 10

1. It is characterized by the following three elements: (1) magnitude of the moment, which is equal to the product of the force and the moment arm, that is,  $Fh$ ; (2) the plane of rotation  $OAB$  passing through the line of action of the force  $F$  and the centre  $O$ ; and (3) the sense of rotation in that plane.
2. The modulus of this vector is equal to the magnitude of the force moment; the direction of the vector denotes the sense of rotation.
3. This vector is equal in magnitude to the product of the force  $F$  and the moment arm  $h$ , and normal to the plane  $OAB$  through  $O$  and  $F$ .
4. Vector  $(\overline{OA} \times \underline{F})$  is normal to plane  $OAB$ . In the direction from which a counterclockwise rotation would carry  $\overline{OA}$  into  $F$  through the smaller angle between them (when the two forces are laid off from the same point), i.e., it is in the same direction as vector  $M_O$ .
5. If the determinant in the right-hand part of the equation is expanded by the first row, the factors of  $\bar{i}$ ,  $\bar{j}$  and  $\bar{k}$  will be equal to the projections  $M_x$ ,  $M_y$ ,  $M_z$  of vector  $M_O$  on the coordinate axes (that is  $M_O = M_x \bar{i} + M_y \bar{j} + M_z \bar{k}$ ).

### Exercise 10

lines	words in bold	refers to something before	refers to something after	what it refers to
2	It	+		moment of a force
4	Which	+		magnitude
16	It	+		plane
40	Them	+		vectors OA and F
41	It	+		vector OA
51	Its	+		force F

### Exercise 12

An open answer of the students.

### Exercise 13

The moment of a force  $F$  about a centre  $O$ , is a measure of the tendency of that force to turn a body. It is characterized by the following three elements: (1) magnitude of the moment, which is equal to the product of the force and the moment arm, that is,  $Fh$ ; (2) the plane of rotation  $OAB$  passing through the line of action of the force  $F$  and the centre  $O$ ; and (3) the sense of rotation in that plane. When all the given forces at the centre  $O$  are coplanar there is no need to specify the plane of rotation  $OAB$ . In this case, the moment of a force can be defined as a scalar algebraic quantity equal to  $\pm Fh$ , where the sign indicates the sense of rotation.

**Exercise 14**

Points 1, 2, 4 are mentioned in the text.

**Exercise 16**

1. g
2. j
3. a
4. i
5. b
6. c
7. d
8. e
9. f
10. h

**Exercise 17**

1. triangle
2. the point of application
3. equation
4. the moment of a force
5. vector
6. a unit vector
7. coplanar vectors
8. radius vector

**Exercise 19**

1. F
2. T
3. T
4. T
5. T
6. F
7. T
8. F

**Exercise 20**

An open answer of students.

## Chapter 3. Parallel Forces and Force Couples in a Plane.

### 3.3. A Force Couple. Moment of a Couple.

#### 3.4. Vector Expression of the Moment of a Couple

##### Exercise 1.

##### A force couple. Moment of a couple.

*A force couple is a system of two parallel forces that have the same magnitude, opposite directions. A couple has no resultant. The plane passing through the lines of action of both forces of a couple is called the plane of action of the couple. The perpendicular distance  $d$  between the lines of action of the forces is called the arm of the couple. The moment of a couple is defined as the product of the magnitude of one of the couple's forces and the perpendicular distance between the forces (or a moment arm).*

##### 3.4. Vector expression of the Moment of a couple.

The action of a couple on a body is characterized by:

- 1) the magnitude of the moment of the couple,
- 2) the aspect of the plane of action, and
- 3) the sense of rotation in that plane.

*The moment of a couple is denoted by a vector  $\mathbf{m}$  or  $\mathbf{M}$  whose modulus is equal to the product of one of its forces and the moment arm. The vector is normal to the plane of action of the couple in the direction from which the rotation induced by the couple would be observed as counterclockwise. The moment of a couple is said to be positive if the action of the couple tends to turn a body counterclockwise, and negative if clockwise.*

##### Exercise 3

Момент пары	Направление плоскости
Две параллельные силы	При рассмотрении пар в пространстве
Противоположные направления	для того чтобы определить
Особый вид взаимодействия	Обозначен вектором
Проходя через линии взаимодействия	По масштабу
Плоскость действия пары	Обозначая символами
Перпендикулярные расстояния	Вычислять момент $M_0$ аналитически
Расположение плоскости	Координатные оси
Направление вращения	Из известной формулы в векторной алгебре

#### Exercise 4

Subject of special studies	Magnitudes of the forces of the couple
Plane of rotation of a couple	Location of the plane
Location of a couple	The product of the forces and a moment arm
Passing through the lines of action of a moment	Perpendicular distance
Denoting the moment by symbol M	Moment arm
In considering couples in a plane	Tendency to turn a couple
In order to define a couple	Let's prove the theorem
To some scale	Algebraic sum of couple moments
The vector is normal to the plane of the couple	Counterclockwise rotation (motion)
Rotation induced by the couple	Vector is applied to any point
Location of the point	Free vector
Equal to the moment of the couple	To calculate the moment of a couple
Considering an arbitrary point O	Moment of a couple about a centre
Adding these equations	Computation of couple moments about a centre
Noting that $F^2 = F$	

#### Exercise 6

A force couple  
Parallel forces  
Plane of action of a couple  
To satisfy the equations of equilibrium  
Algebraic sum  
Rigid body  
Moment of a force  
To prove the theorem  
The couple moment  
The sense of rotation  
Opposite directions  
Mechanical interactions  
The arm of the couple  
It should be borne I mind  
The sense of rotation  
A free body  
An arbitrary point

### Exercise 7

1. The moment of a couple is defined as the product of the magnitude of one of the couple's forces and the perpendicular distance between the forces (or a moment arm).
2. The moment of a couple is denoted by a vector **m** or **M** whose modulus (to some scale) is equal to the product of one of its forces and the moment arm.
3. The vector is normal to the plane of action of the couple in the direction from which the rotation induced by the couple would be observed as counter-clockwise.
4. Since a couple may be located anywhere in its plane of action or in a parallel plane, it follows that vector m can be attached to any point of the body (such a vector is known as a free vector).
5. Let us prove the following theorem of the moments of the forces of a couple: The algebraic sum of the moments of the forces of a couple about any point in its plane of action is independent of the location of that point and is equal to the moment of the couple.

### Exercise 8

lines	words in bold	refers to something before	refers to something after	what it refers to
2	That	+		Two parallel forces
15	It	+		Body
15	It	+		The action of a couple
31	Whose	+		Vector
33	Its	+		Couple
36	Which	+		Direction
48	Its	+		Forces of a couple

### Exercise 9

A *force couple* is a system of two parallel forces **that** have the same magnitude, opposite directions. A force system constituting a couple is not in equilibrium (see the 1<sup>st</sup> principle). Furthermore, a couple has no resultant. Thus, a couple cannot be replaced or balanced by a single force. For this reason the properties of the couple as a special mode of mechanical interaction between bodies are the subject of a special study. The plane passing through the lines

of action of both forces of a couple is called the *plane of action of the couple*. The perpendicular distance  $d$  between the lines of action of the forces is called the *arm of the couple*. The action of a couple on a rigid body is tendency to turn **it**. **It** depends on:

- (1) the magnitude  $F$  of the forces of the couple and the perpendicular distance  $d$  between them;
- (2) the location of the plane of action of the couple; and
- (3) the sense of rotation in that plane.

A couple is characterized by its *moment*.

*The moment of a couple is defined as the product of the magnitude of one of the couple's forces and the perpendicular distance between the forces (or a moment arm).*

### Exercise 13

1. a couple
2. equilibrium
3. plane of action of a couple
4. the arm of the couple
5. tendency
6. moment
7. sliding vector
8. counterclockwise
9. the moment of couples about any centre
10. a single force
11. a parallel plane

### Exercise 14

1. g
2. f
3. h
4. a
5. i
6. j
7. e
8. d
9. b
10. k
11. l
12. c

### **Exercise 15**

A force couple – is a system of two parallel forces that have the same magnitude, opposite directions

Moment of a couple – the product of the magnitude of one of the couple's forces and the perpendicular distance between the forces (or a moment arm)

Resultant – it is a geometrical sum (principal vector) of the forces

Plane of action of the couple – the plane passing through the lines of action of both forces of a couple.

The arm of the couple – the perpendicular distance between the lines of action of the forces.

A moment arm – is the tendency of a force to rotate an object about an axis.

A free vector – a vector that can be attached to any point of the body.

An arbitrary point – any chosen point of the plane.

Force – the quantitative measure of the mechanical interaction of material bodies.

### **Exercise 17**

1. F
2. F
3. F
4. F
5. F
6. T
7. T
8. F
9. F
10. T

### Chapter 3.

#### 3.5. Moment of a Force with Respect to an Axis.

##### Exercise 1

1. The moment of a force about an axis is the measure of the tendency of the force to cause a body to rotate about that axis.
2. The moment of a force about an axis is a scalar, it is an algebraic quantity.
3. The magnitude of the moment of a force about an axis is equal to the moment of the projection of that force on a plane normal to the axis with respect to the point of intersection of the axis and the plane.
4. A moment of a force is positive if the rotation induced by a force is seen as counterclockwise when viewed from the positive end of the axis, and negative if it is seen as clockwise.
5. If a force is parallel to an axis, its moment about that axis is zero.

##### Exercise 3

По отношению к оси, величина стремления, заставляя тело вращаться, проводить плоскость через точку, проектировать силу на плоскость, перемещать тело вдоль оси, таким образом, мы определяем что, мы делаем вывод что, ось пронзает плоскость, выводим определение, пересечение оси и плоскости, Декартовская система координат, эти уравнения, точка приложения, изображение (цифра), алгебраический, необходимый, производить.

##### Exercise 4

Moment of a force about an axis, tendency of a force to rotate a body, let's consider a rigid body, let a force  $A$  act on a body, force being parallel to the axis  $Z$ , to translate the body along the axis, the total tendency of a force, we conclude that, the rotational effect of the force  $F_{xy}$ , to deduce the definition, with respect to the point of intersection, an algebraic quantity equal to the moment of a force, viewed from the positive end of the axis, in order to determine, , determining a force, determine the sense of the moment, it should be borne in mind that, perpendicular distance from the force to the axis, analytical expression of the moment of a force, rectangular coordinate system, to resolve into rectangular components, components equal in magnitude, from Varignon's theorem, we obtain the moments in the same way, the axes of the Cartesian coordinate system, using these equations.

#### Exercise 4

1. Place (место) (n) – to place (помещать) (v) – to replace (перемещать) (v).
2. Consider (рассматривать) (v) – consideration (рассмотрение) (n) – considerable (значительный) (a) – inconsiderable (незначительный) (a) – considerably (значительно) (adv)
3. Reduce (уменьшать) (v) – redactor (редуктор) (n) – reduction (уменьшение) (n).
4. Rotate (вращаться) (v) – rotation (вращение) (n) – rotational (вращательный) (a) – rotor (ротор) (n).
5. Change (изменение) (n) – to change (менять) (v) – changing (меняя) (p.1) – changeable (меняющийся) (a) – unchangeable (неизменяемый) (a)
6. Center (центр) (n) – central (центральный) (a).
7. Origin (происхождение) (n) – original (первоначальный) (a) – originate (происходить) (v).
8. Necessary (необходимый) (a) – unnecessary (ненужный) (a) – necessity (необходимость) (n)
9. Algebra (алгебра) (n) – algebraic (алгебраический) (a).
10. Define (определять) (v) – definite (определённый) (a) – indefinite (неопределённый) (a) – definition (определение) (n)
11. Axis (ось) (n) – axial (осевой) (a)
12. Add (складывать) (v) – addition (сложение) (n) – additional (дополнительный) (a) – additionally (дополнительно) (adv);
13. Geometry (геометрия) (n) – geometrical (геометрический) (a);
14. Clock (часы) (n) – clockwise (по часовой стрелке) (adv) – anticlockwise (против часовой стрелки) (adv) – counterclockwise (против часовой стрелки) (adv)
15. Direct (прямой) (a) – direction (направление) (n) – director (руководитель) (n) – directly (прямо) (adv)
16. Compute (вычислять) (v) – computer (компьютер) (n) – computation (вычисление) (n)
17. Examine (изучать, экзаменовать) (v) – examination (экзамен) (n) – examiner (экзаменатор) (n) – examinee (экзаменуемый) (n) – exam (экзамен) (n);

#### Exercise 6

1. e
2. j
3. k
4. g
5. i

- 6. l
- 7. b
- 8. a
- 9. h
- 10. f
- 11. c
- 12. d

**Exercise 7**

- 1. d
- 2. f
- 3. e
- 4. a
- 5. b
- 6. g
- 7. c

**Exercise 8**

Moment, components, measure, counterclockwise, coplanar, Varignon's theorem, vector.

**Exercise 9**

1. Let us now pass a plane  $xy$  through point  $A$  normal to the axis  $z$  and let us resolve the forces into rectangular components  $F_z$  parallel to the  $z$ -axis and  $F_{xy}$  in the plane  $xy$  ( $F_{xy}$  is in fact the projection of force  $F$  on the plane  $xy$ ).
2. But the rotational effect of force  $F_{xy}$ , which lies in a plane perpendicular to axis  $z$ , is the product of the magnitude of this force and its distance  $h$  from the axis.
3. The moment of a force about an axis is an algebraic quantity equal to the moment of the projection of that force on a plane normal to the axis with respect to the point of intersection of the axis and the plane.
4. Thus, in order to determine the moment of a force about axis  $z$  in Fig. 3.8, we have to: (1) pass an arbitrary plane  $xy$  normal to the axis; (2) project force  $F$  on the plane and compute the magnitude of  $F_{xy}$ ; (3) erect a perpendicular from point  $O$ , where the plane and axis intersect, to the action line of  $F_{xy}$  and determine its length  $h$ ; (4) compute the product  $F_{xy}h$ ; (5) determine the sense of the moment.
5. Consider a rectangular coordinate system with an arbitrary beginning  $O$  (Fig. 3.9) and a force  $F$  applied at a point  $A$  whose coordinates are  $x, y, z$ .
6. Using these equations, we can determine the moments if we know the projections of a force on the coordinate axes and the coordinates of the point of its application.

### Exercise 10

It – a force

It – axis z

That – the total tendency

Its – a force

Its – a force

### Exercise 12

Recall that when a moment of a force is computed about a point, the moment and its axis are always perpendicular to the plane containing the moment arm and the force. In some problems it is important to find the component of this moment long a specified axis that passes through the point. To solve this problem either a scalar or vector analysis can be used.

As a numerical example of this problem, consider the pipe assembly, which lies in the horizontal plane and is subjected to the vertical force of  $F = 20 \text{ N}$  applied at point A. The moment of this force about point O has a magnitude of  $M_o = (20 \text{ N})(0.5\text{m}) = 10 \text{ N}\cdot\text{m}$  and a direction defined by the right-hand rule.

### Exercise 13

Gravitation as always fascinated human beings since its immaterial nature is somewhat magic. According to Aristotle's philosophy, nature was made of four basic elements referred to as air, fire, water and earth. The very principles that presided to the motion of each of these elements were supposed to be embedded in them. So the Aristotelian universe is a geocentric one. Newtonian and classical mechanics have shed a new rational light on gravitation, including it in the great edifice of the laws of nature. Using a very simple formula, human beings were able to compute the trajectories of the planets as well as artificial satellites thereby penetrating the so-called god's wills. Today, three centuries after the publication of the *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) by Isaac Newton, it is worth asking ourselves about the intimate nature of gravitation.

### Exercise 15

1. is the measure of the tendency of the force to cause a body to rotate about that axis.
2. can not turn the body about that axis
3. quantity equal to the moment of the projection of that force on a plane normal to the axis with respect to the point of intersection of the axis and the plane.

4. if the rotation induced by a force  $F_{xy}$  is seen as counterclockwise from the positive end of the axis.
5. if the rotation induced by a force  $F_{xy}$  is seen as clockwise.
6. its moment about that axis is zero.
7. then its moment with respect to that axis is zero.
8. its moment about that axis is equal to the product of the force magnitude and the perpendicular distance from the force to the axis.
9. the moment of a force  $F$  with respect to the axis  $Z$ .

### **Exercise 16**

1. F
2. F
3. T
4. F
5. T
6. F
7. F
8. T
9. F
10. F

### **Exercise 18**

1. rotating
2. through, normal
3. the axis, round, axis
4. respect, pierces
5. special, borne
6. perpendicular, equal, product, perpendicular
7. rectangular
8. axis
9. determine, projections, coordinate, its application

### **Exercise 19**

1. 3
2. 2
3. 5
4. 4
5. 1

## Chapter 3.

### 3.6. Theorem of Translation of a Force.

### 3.7. Reduction of a Force System in Space to a Given Centre.

### 3.8. Conditions of Equilibrium of an Arbitrary Force System in Space.

#### Exercise 1

1. For the given system of forces to be in equilibrium it is necessary and sufficient that  $R=0$  and  $M_o=0$ . But vectors  $R$  and  $M_o$  can be zero only if all their projections on the coordinate axes are zero.

2. The necessary and sufficient condition for the equilibrium of any force system in space are that the sums of the projections of all the forces on each of the three coordinate axes and the sums of the moments of all the forces about those axes must separately vanish.

3. Any system of forces acting on a rigid body can be reduced to an arbitrary centre  $O$  and replaced by a single force  $R$ , equal to the principal vector of the system applied at the centre of reduction, and a couple with a moment  $M_o$ , equal to the principal moment of the system with respect to  $O$ .

4. A force, acting on a rigid body can be moved parallel to its line of action to any point of the body, if we add a couple of a moment equal to the moment of the force about the point to which it is translated.

#### Exercise 2

Теорема перемещения силы, система сходящихся сил, для случая двух параллельных сил, очевидно, пара момента, сила  $F$  приложенная в точке  $A$ , результирующая система трех сил, это уравнение следует из уравнения (14), центр редукции, складывать соответствующие пары, складывать пары методом геометрического сложения, главный вектор системы, главный момент системы по отношению к центру  $O$ , произвольный центр  $O$ , необходимо и достаточно, проекции на координатные оси, проекции на три координатные оси.

#### Exercise 3

Theorem of translation of a force, equivalent system of concurrent forces, to move parallel to the line of action, let's consider force  $F$ , the resulting three-force system, the theorem is proved, reduction of a force system to a given centre, centre of reduction, to compose corresponding couples, geometrical addition, magnitude  $R$ , principle vector of the system, an arbitrary centre  $O$ , with respect to the centre  $O$ , necessary and sufficient conditions, sum of the projections of the forces, to vanish separately, projections on the coordinate axes, to satisfy the conditions, to translate to an arbitrary centre, to replace by a principle vector, with respect to  $O$ .

#### Exercise 4

1. Direct (прямой) (a) – director (управляющий) (n) – directed (направленный) (p.2) – direction (направление) (n) – directly (направленно) (adv).
2. Follow (следовать) (v) – follower (последователь) (n) – followed (преследуемый) (p.2) – following (следуя) (p.1).
3. Act (действовать) (v) – action (действие) (n) – reaction (противодействие) (n) – acting (действуя) (p.1) – interaction (взаимодействие) (n).
4. Equal (равный) (a) – equality (равенство) (n) – equalization (уравнивание) (n) – equalize (уравнивать) (v) – equalizer (эквалайзер) (n) – equally (одинаково) (adv) – equation (уравнение) (n).
5. Reduce (уменьшать) (v) – reduced (уменьшенный) (p.2) – reducible (допускающий уменьшение) (a) – reducing (уменьшая) (p.1) – reduction (уменьшение) (n).
6. Transfer (перемещать) (v) – transferable (допускающий перемещение) (a) – transferal (перемещение) (n) – transference (передача) (n).
7. Geometer (геометр) (n) – geometrical (геометрический) (a) – geometrically (геометрически) (adv) – geometry (геометрия) (n).
8. Condition (условие) (n) – conditional (условный) (a) – conditioned (обусловленный) (p.2) – conditioning (обуславливая) (p.1).
9. Project (проект) (n) – projection (проекция) (n) – projector (проектор) (n).
10. Relate (относиться) (v) – related (связанный) (p.2) – relation (отношение) (n) – relational (относительный) (a) – relationship (взаимоотношение) (n).
11. Suffice (быть достаточным) (a) – sufficiently (достаточно) (adv) – sufficiency (достаточность) (n) – sufficient (достаточный) (a).
12. Separability (отделимость) (n) – separable (отделимый) (a) – to separate (отделять) (v) – separate (отдельный) (a) – separation (отделение) (n).

#### Exercise 7

1. found directly with the help of the parallelogram rule.
2. its line of action to any point of the body
3. if two balanced forces are applied at any point B of the body.
4. of the system to it and add the corresponding couples.
5. a single force R applied at the same point
6. geometric addition the vectors of their moments
7. the principal vector of the system
8. the principal moment of the system with respect to O
9. can be reduced to an arbitrary centre O and replaced by a single force R
10. that the sums of the projections of all the forces on each of the three coordinate axes and the sums of the moments of all the forces about those axes must separately vanish.

### Exercise 8

1. We solved the problem for the case of two parallel forces by replacing them with an equivalent system of concurrent forces.
2. A force acting on a rigid body can be moved parallel to its line of action to any point of the body, if we add a couple of a moment equal to the moment of the force about the point to which it is translated.
3. Take any point  $O$  as the centre of reduction, transfer all the forces of the system to it and add the corresponding couples.
4. As in the case of a coplanar system, the quantity  $\mathbf{R}$  (the geometric sum of all the forces) is called the principal vector of the system; vector  $\mathbf{M}_O$  (the geometric sum of moments of all forces with respect to  $O$ ) is called the principal moment of the system with respect to  $O$ .
5. Any system of forces acting on a rigid body can be reduced to an arbitrary centre  $O$  and replaced by a single force  $\mathbf{R}$ , equal to the principal vector of the system applied at the centre of reduction, and a couple with a moment  $\mathbf{M}_O$ , equal to the principal moment of the system with respect to  $O$ .
6. Thus, the necessary and sufficient conditions for the equilibrium of any force system in space are that the sums of the projections of all the forces on each of the three coordinate axes and the sums of the moments of all the forces about those axes must separately vanish.

### Exercise 9

lines	words in bold	refers to something before	refers to something after	what it refers to
3	Them	+		Parallel forces
4	It	+		Force system
6	Its	+		Force
9	Which	+		Point
9	It	+		Force
18	It	+		Point
24	Their	+		Couples
39	Their	+		Vectors

### Exercise 10

Cause:

Consequence: thus, and, i.e.

Time sequence:

Concession: obviously, if, only if

Opposition: but

Comparison: as

**Exercise 11**

The direction and sense of  $\mathbf{M}_O$  are determined by the right-hand rule as it applies to the cross product thus extending  $\mathbf{r}$  to the dashed position and curling the right-hand fingers from  $\mathbf{r}$  toward  $\mathbf{F}$   $\mathbf{r}$  cross  $\mathbf{F}$  the thumb is directed upward or perpendicular to the plane containing  $\mathbf{r}$  and  $\mathbf{F}$  and this is in the same direction as  $\mathbf{M}_O$  the moment of the force about point O note that the curl of the fingers like the curl around the moment vector indicates the sense of rotation caused by the force since the cross product is not commutative it is important that the proper order of  $\mathbf{r}$  and  $\mathbf{F}$  be maintained

**Exercise 13**

c) Cartesian vector notation

**Exercise 14**

1. the theorem of translation of forces
2. reduction of a force system in space to a given centre
3. the magnitude of the resultant force R of a system of forces
4. the moment of the couple reducing the system of couples

**Exercise 15**

1. F
2. F
3. T
4. F
5. F
6. T
7. F
8. F
9. F
10. T

### Chapter 3.

#### 3.9. Varignon's Theorem of the Moment of a Resultant with Respect to an Axis.

#### 3.10 Conditions of Equilibrium of a Coplanar Force System. The Case of Parallel Forces

##### Exercise 1

Thus, it states that the moment of a force about a point is equal to the sum of the moments of the forces components about the point.

For the given system of forces to be in equilibrium it is necessary and sufficient that:  $R=0$ ,  $M_o=0$ .

Where O is any point in a given plane, as for  $R=0$  the magnitude of  $M_o$  does not depend on the location of O.

Any coplanar force system is in equilibrium if and only if the sums of the projections of all the forces on each of the two coordinate axes and the sum of the moments of all the forces about any point in the plane are zero.

The necessary and sufficient conditions of the equilibrium of any coplanar force system are that the sums of the moments of all the forces about any two points A and B and the sums of the projections of all the forces on axis Ox not perpendicular to AB must separately vanish.

The necessary and sufficient conditions of the equilibrium of any coplanar force system are that the sums of the moments of all the forces about any three non-collinear points A, B, C must separately vanish.

##### Exercise 3

Varignon's theorem; let a system of forces; to pass through any point; moment of the resultant force; to satisfy the conditions; sum of the moments of the forces components; horizontal shaft; bearing; pulley; driving portion of the belt; problem solution; to apply the equations of equilibrium; determine the reactions of the bearing; to draw the coordinate axes; to denote the forces; perpendicular to the axes; rectangular components; substituting the values; coplanar force system; in case of the parallel forces; to depend on the location of O; to reduce the system of forces to the resultant; analytical conditions of equilibrium; necessary and sufficient conditions of equilibrium; sum of the projections of forces; free rigid body; to verify the equations; identity.

##### Exercise 5

Balance = equilibrium

A part = portion

The same = identity

Total = sum  
 Weight = load  
 Response = reaction  
 Perpendicular = normal  
 Place = location  
 Pair = couple

### Exercise 6

1. b
2. a
3. c
4. c
5. b
6. c
7. a
8. c
9. b
10. a
11. a
12. c

### Exercise 7

1. A horizontal shaft is supported by bearings  $A$  and  $B$  (Fig. 3.13). A pulley of radius  $r_1 = 20$  cm and a drum of radius  $r_2 = 15$  cm are attached to the shaft.
2. Neglecting the weight of the construction, determine the reactions of the bearings and the tension  $T_1$  in the driving portion of the belt, if it is known that  $T_1 = 2 T_2$  and if  $a = 40$  cm,  $b = 60$  cm, and  $\alpha = 30^\circ$ .
3. Drawing the coordinate axes as shown and regarding the shaft as a free body, denote the forces acting on it: the tension  $F$  of the cable, which is equal to  $P$  in magnitude, the tensions  $T_1$  and  $T_2$  in the belt, and the reactions  $Y_A, Z_A, Y_B,$  and  $Z_B$  of the bearings (each of the reactions  $R_A, R_B$  can have any direction in planes normal to axis  $x$  and they are therefore denoted by their rectangular components).
4. For the given system of forces to be in equilibrium it is necessary and sufficient that:  $R = 0, M_O = 0$  where  $O$  is any point in a given plane, as for  $R = 0$  the magnitude of  $M_O$  does not depend on the location of  $O$ .
5. The conditions (3.20) are necessary, for if one of them is not satisfied the force system acting on a body is reduced either to a resultant (when  $R \neq 0$ ) or to a couple (when  $M_O \neq 0$ ) and consequently is not balanced.
6. Any coplanar force system is in equilibrium if and only if the sums of the projections of all the forces on each of the two coordinate axes and the sum of the moments of all the forces about any point in the plane are zero.
7. The necessary and sufficient conditions of the equilibrium of any coplanar force system are that the sums of the moments of all the forces about any two

points A and B and the sum of the projections of all the forces on axis Ox not perpendicular to AB must separately vanish.

### Exercise 8

lines	words in bold	refers to something before	refers to something after	what it refers to
3	Whose	+		Resultant
10	It	-		(subject)
27	It	+		Shaft
30	It	+		Shaft
33	They	+		Reactions
76	Each		+	Coordinate axes
88	These	+		Conditions
90	It	+		A force system

### Exercise 10

- two – force members

### Exercise 11

- $\sum m_x(\mathbf{F}_k) + m_x(\mathbf{R}') = 0; m_x(\mathbf{R}) = \sum m_x(\mathbf{F}_k)$
- $R = \sqrt{R_x^2 + R_y^2}, M_O = \sum m_O(\mathbf{F}_k)$
- $\sum m_A(\mathbf{F}_k) = 0, \sum m_B(\mathbf{F}_k) = 0, \sum m_C = 0.$
- $\sum F_{ky} = 0, \sum m_O(\mathbf{F}_k) = 0$

### Exercise 13

- d
- h
- j
- l
- g
- b
- c
- i
- a
- e
- f
- k

### Exercise 14

- Projection – the act of projecting (all senses), sth. That projects or has been projected.
- A force – the quantitative measure of the mechanical interaction of material bodies.
- Reaction – action or state resulting from, in response to.

4. Force system – several forces acting on a body.
5. Location – place.
6. A rigid body – (in the text) any material object of our physical world.
7. Equilibrium – state of being balanced.
8. A Point – mark or position, real or imagined, in space or time
9. A plane – (in the text) imaginary level surface such that the straight line joining any points on it is touching it at all points.
10. A vector – a segment of a straight line with the tail and the head in the form of an arrow.
11. Parallel – lines continuing at the same distance from one another.

### Exercise 15

1. be acting on a rigid body
2. to the sums of the moments of the forces components about the point
3. it is necessary and sufficient that  $R=0$ ,  $M_o=0$
4. of all the forces on each of the two coordinate axes and the sums of the moments of all the forces about any point in the plane are zero.
5. of the moments of all the forces about any two points A and B and the sums of the projections of all the forces on axis Ox not perpendicular to AB must separately vanish.
6. the forces will not be in equilibrium
7. non – collinear points A, B, C must separately vanish.
8. we can take axis  $x$  perpendicular to them and the axis  $y$  parallel to them
9. the forces acting on it are in equilibrium and the equations of equilibrium can be applied
10.  $\sum F_{kx} = 0$ ,  $\sum F_{ky} = 0$ ,  $\sum m_o(\mathbf{F}_k) = 0$ .  
 $\sum m_A(\mathbf{F}_k) = 0$ ,  $\sum m_B(\mathbf{F}_k) = 0$ ,  $\sum F_{kx} = 0$ .  
 $\sum m_A(\mathbf{F}_k) = 0$ ,  $\sum m_B(\mathbf{F}_k) = 0$ ,  $\sum m_C = 0$

### Exercise 16

1. shaft, bearings
2. forces; resultant; line
3. construction; bearings; driving; belt
4. uniformly; equilibrium; applied
5. projections; axes; moments; axes
6. equilibrium; coordinate; forces; plane
7. moments; points; projections; axis
8. obvious; satisfied; equilibrium

### Exercise 18

1. the basic equation of equilibrium
2. the third form of the equations of equilibrium
3. the third form of the equations of equilibrium
4. equilibrium of a coplanar system of parallel forces

## Chapter 3.

### 3.11 Equilibrium of Systems of Bodies.

#### Exercise 1

An engineering structure may not necessarily remain rigid when the external constraints are removed. According to the principle of solidification, for a system of forces acting on such a structure to be in equilibrium it must satisfy the conditions of equilibrium for a rigid body. In order to solve such a problem it is necessary to examine additionally the equilibrium of one or several parts on the given structure. Another method of solving such problems is to divide a structure into separate bodies and write the equilibrium equations for each body as for a free body.

#### Exercise 2

Строительная конструкция, оставаться жестким, внешние связи, арка с тремя креплениями, опора, в соответствии с, принцип затвердения, левосторонние элементы, система шести уравнений, внутренние связи правосторонний элемент, равный по величине, противоположный по направлению, статически неопределимый, наклоненный элемент, быть прикрепленным к точке D, условия равновесия, отбрасывать связи, находить неизвестные, разделять на отдельные составляющие, составлять пары сил.

#### Exercise 3

Engineering structure, a three-pin arc, turn about a pin, according to the principle of solidification, to satisfy the conditions of equilibrium, in order to solve the problem, three equations with four unknowns, right-hand side member of the arc, to solve the system of six equations, to divide a structure into separate parts, reactions of the internal constraints, forces equal in magnitude and opposite in sense, to be subjected to the action of the system of forces, a bracket hinged to the wall, rejecting the external constraints, as a whole, on the other hand, to contain four unknowns, shown in the diagram, projections of forces.

#### Exercise 4

Construction = structure

Outside = external

Supports = constraints

Hard = rigid

Balance = equilibrium

Inside = internal

Pair = couple

Direction = sense  
 Size = magnitude  
 A part = an element

**Exercise 5**

Solidification  
Equilibrium  
Subjected  
Indeterminate  
Reaction

**Exercise 6**

1. An engineering structure may not necessarily remain rigid when the external constraints are removed. An example of such a structure is the three-pin arch in Fig. 3.16
2. According to the principle of solidification, for a system of forces acting on such a structure to be in equilibrium it must satisfy the conditions of equilibrium for a rigid body.
3. For a structure of  $n$  bodies, each of which is subjected to the action of a coplanar force system, we thus have  $3n$  equations from which we may determine  $3n$  unknowns (for other force systems the number of equations may be, of course, different).
4. On the other hand, by the principle of solidification, if it is in equilibrium the forces acting on it must satisfy the conditions of static equilibrium.

**Exercise 7**

lines	words in bold	refers to something before	refers to something after	what it refers to
4	Its	+		arch
8	It	+		System of forces
22	Each of which	+		Bodies
38	It	+		A bracket
40	Its		+	A bracket
44	It	+		A bracket
59	That	+		Sense

**Exercise 8**

Cause: for  
 Consequence: thus

Sequence: therefore  
Concession:  
Opposition: but, on the other hand  
Condition: if  
Purpose: in order to

### **Exercise 9**

An engineering structure may not necessarily remain rigid when the external constraints are removed. An example of such a structure is the three-pin arch. If supports A and B are removed the arch is no longer rigid, for its parts can turn about pin C.

According to the principle of solidification, for a system of forces acting on such a structure to be in equilibrium it must satisfy the conditions of equilibrium for a rigid body. In order to solve such a problem it is necessary to examine additionally the equilibrium of one or several parts on the give structure.

For example, for the forces acting on the three-pin arch we have three equations with four unknowns,  $X_A$ ,  $Y_A$ ,  $X_B$ ,  $Y_B$ . By investigating the conditions for the equilibrium, the left- or right-hand side members of the arch we obtain three more equations with two additional unknowns,  $X_C$  and  $Y_C$ . By solving the system of six equations we can find six unknowns

### **Exercise 10**

a) frames and machines as system of bodies

### **Exercise 11**

1. A three – pin arc is an example of an engineering structure.
2. If the supports A and B are removed the arc is no longer rigid.
3. By solving the system of six equations we can find six unknowns.
4. The reactions of the internal constraints will constitute pairs of forces equal in magnitude and opposite in sense.
5. A bracket consists of a horizontal member and an inclined member.
6. When the constraints are removed the bracket is no longer a rigid body.
7. If the number of unknowns is greater then the number of equations the problem is statically indeterminate.
8. Another method of solving such problems is to divide a structure into separate parts.

### **Exercise 12**

1. f
2. a
3. n
4. L; I

5. c
6. k
7. b
8. o
9. L; I
10. d
11. e
12. j
13. h
14. m
15. g

### Exercise 13

- 1.-1) arc
- 2.-2) system of forces
- 3.-3) bodies
4. 2) a bracket
5. 2) bracket
- 6.1) sense of forces

### Exercise 14

1. when the external constraints are removed
2. a three- pin arc
3. we have three equations with three unknowns
4. to divide a structure into separate bodies and write the equilibrium equations for each body as for a free body.
5. pairs of forces equal in magnitude and opposite in sense.
6. the number of equations, the problem is statically indeterminate.
7. hinged to the wall and an inclined member  $CB$
8. a rigid body, because the members can turn about pin  $B$

### Exercise 15

1. Rigid – stiff, unbending, that cannot be bent.
2. External – situated on the outside.
3. Constraints – something that restricts the movement.
4. A problem – a question to be solved or decided, esp. something difficult.
5. Equilibrium – the state of being balanced.
6. To examine – to look at carefully in order to learn about or from something.

7. An unknown – there is no information about this object or concept.
8. A system – group of things or parts working together in a regular relation.
9. An equation – a formula expressing equality between two quantities by the sign =
10. Indeterminate – that which can not be determined.
11. To solve a problem – to find an answer to the problem or question.

### Chapter 3.

#### 3.12. Distributed forces.

##### Exercise 1.

1. The intensity  $q$  of such a system is a constant. In solving problems of statics such a force system can be replaced by its resultant  $Q$  of the magnitude applied to the middle of AB.
2. The resultant  $Q$  of the forces distributed along a straight line according to a linear law is determined in the same manner as the resultant of the gravity forces acting on a homogeneous triangular lamina ABC.
3. The magnitude of the resultant  $Q$  of the forces uniformly distributed along the arc of a circle is determined by the formula  $Q = Q_x = \sum (q\Delta l_k) \cos \varphi_k$

##### Exercise 3

1. e
2. l
3. a
4. f
5. b
6. c
7. m
8. d
9. g
10. k
11. h
12. j
13. i

##### Exercise 4

Question = problem

To be concerned with = to deal with

To spread = to distribute

Flat = plane  
Unchangeable = constant  
Uniform = homogenous  
Weight = load  
Summation mark = plus

### **Exercise 5**

Distributor  
Characteristics  
Long  
Applied  
To determine  
Intersection  
Proportion  
Cylinder  
Sum

### **Exercise 6. Say what comes next**

Engineering problem, loads distributed over some area, mathematical law, coplanar forces, load per unit length, Newtons per metre, uniformly distributed, linear law, in the same sense (direction), gravity forces, homogenous triangular lamina, to be drawn to some scale, the centre of gravity, hydrostatic pressure, common multiplier, to take outside the summation mark

### **Exercise 7**

1. A plane system of distributed forces is characterized by the load per unit length of the line of application, which is called the intensity  $q$ .
2. An example of such a load is the pressure of water against a dam, which drops from a maximum at the bottom to zero at the surface.
3. The resultant  $Q$  is determined in the same manner as the resultant of the gravity forces acting on a homogeneous triangular lamina  $ABC$ .
4. The magnitude of the resultant  $Q$  of such forces is, by analogy with the force of gravity, equal to the area of the figure  $ABDE$  drawn to scale, and  $Q$  passes through the centre of gravity of that area.
5. An example of such forces is the hydrostatic pressure on the sides of a cylindrical vessel.
6. It is apparent from the laws of symmetry that the sum of the projections on the  $Oy$ -axis, which is perpendicular to the axis of symmetry  $Ox$ , is zero and, consequently, their resultant  $Q$  is directed along  $Ox$ .

### Exercise 9

lines	words in bold	refers to something before	refers to something after	what it refers to
4	Which	+		Load
8	Its	+		A force system
14	Which	+		Pressure
18	Its	+		Lamina
28	Which	+		Oy – axis
29	Their	+		Projections
29	Its	+		Resultant

### Exercise 10

A flat rectangular plate of constant width is submerged in a liquid having a specific weight  $\gamma$ . The plane of the plate makes an angle with the horizontal, such that its top edge is located at a depth  $z_1$  from the liquid surface and its bottom edge is located at a depth  $z_2$ . Since pressure varies linearly with depth, the distribution of pressure over the plate's surface is represented by a trapezoidal volume having an intensity of  $p_1 = \gamma z_1$  at depth  $z_1$  and  $p_2 = \gamma z_2$  at depth  $z_2$ . As noted earlier, the magnitude of the resultant force  $\mathbf{F}_r$  is equal to the volume of this loading diagram and  $\mathbf{F}_r$  has a line of action that passes through the volume's centroid C. Hence  $\mathbf{F}_r$  does not act at the centroid of the plate; rather, it acts at point P, called the centre of pressure.

### Exercise 11

In engineering problems we often have to deal with loads distributed over some area according to the known mathematical law. Let us examine some simple cases of distributed coplanar forces. A plane system of distributed forces is characterized by the load per unit length of the line of application, **which** is called the *intensity*  $q$ . The dimension of intensity is Newton's per metre (N/m).

**1. Forces Uniformly Distributed Along a Straight Line** (Fig. 3.18a). The intensity  $q$  of such a system is a constant. In solving problems of statics such a force system can be replaced by **its** resultant  $\mathbf{Q}$  of the magnitude  $Q = aq$  applied to the middle of  $AB$ .

**2. Forces Distributed Along a Straight Line According to a Linear Law**

The resultant  $\mathbf{Q}$  is determined in the same manner as the resultant of the gravity forces acting on a homogeneous triangular lamina  $ABC$ . As the weight of a homogeneous lamina is proportional to **its** area, the magnitude of  $\mathbf{Q}$  is  $Q = \frac{1}{2} aq_m$

**3. Forces Distributed Along a Straight Line According to an Arbitrary Law** (Fig. 3.18c). The magnitude of the resultant  $\mathbf{Q}$  of such forces is, by analogy with the force of gravity, equal to the area of the figure  $ABDE$  drawn to scale, and  $\mathbf{Q}$  passes through the centre of gravity of that area.

**4. Forces Uniformly Distributed Along the Arc of a Circle their resultant  $Q$  is directed along  $Ox$ . Its magnitude is  $Q = Q_x = \sum (q\Delta l_k) \cos \varphi_k$  where  $q\Delta l_k$  is the pressure on an arc element of length  $\Delta l_k$  and  $\varphi_k$  is the angle between the force and the axis  $x$ . But from the diagram it is apparent that  $\Delta l_k \cos \varphi_k = \Delta y_k$ . Taking the common multiplier  $q$  outside the summation sign, we obtain  $Q = \sum q\Delta y_k = q\Delta y_k = q \cdot AB$ , whence  $Q = qh$  where  $h$  is the length of the chord intersecting the arc  $AB$ .**

**Exercise 13**

1. problems, with, over
2. us, coplanar
3. forces, per, intensity
4. intensity, Newton
5. dam, maximum, zero
6. manner, forces, triangle
7. straight, equal, scale, centre
8. from, axis, along
9. distributed, hydrostatic, vessel
10. multiplier, mark

**Exercise 14**

1. d
2. f
3. e
4. b
5. c
6. a

**Exercise 15**

1. h
2. d
3. a
4. g
5. i
6. c
7. j
8. f
9. e
10. b

**Exercise 16**

1. Problem – a question to be solved.
2. Mathematical law – a law explaining the relation of things, objects of reality in the form of numbers and equations.

3. Coplanar – geometric objects lying in a common plane are said to be coplanar.
4. Intensity – state or quality of being intense
5. Resultant – it is the vector sum of two or more vectors. It is *the result* of adding two or more vectors together.
6. Triangular – in the shape of a triangle.
7. Arbitrary law – a law which is based on opinion or impulse only, not on reason.
8. Arc – a part of the circumference of a circle or other curved line.
9. Scale – series of marks at regular intervals for the purpose of measuring (as on a ruler or a thermometer)
10. Projection – the act of projecting, something that projects or has been projected.
11. Magnitude – the size of the object
12. Common multiplier – the least quantity that contains two or more given quantities exactly.
13. Summation sign – a plus.
14. Element – substance which has not so far been split up into a simpler one by ordinary chemical methods.

### **Exercise 17**

1. F
- 2.
3. F
4. F
5. T
6. F
7. F
8. F
9. F
10. F

## **Тексты на аудирование**

### **Text 1**

#### **Historical Development**

The subject of Statics developed very early in history, because the principles involved could be formulated simply from measurements of geometry and force. For example, the writings of Archimedes (287-212 B.C.) deal with the principle of the lever. Studies of the pulley, inclined plane, and wrench are also recorded in ancient writings – at times when the requirements of engineering were limited primarily to building construction.

Since the principles of dynamics depend on an accurate measurement of time, this subject developed much later. Galileo Galilei (1564-1642) was one of the

first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642-1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

Before beginning our study of rigid-body mechanics, it is important to understand the meaning of certain fundamental concepts and principles.

## Text 2

### Basic Quantities

The following four quantities are used throughout rigid-body mechanics.

**Length.** *Length* is needed to locate the position of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then quantitatively define distances and geometric properties of a body as multiples of the unit length.

**Time.** *Time* is conceived as a succession of events. Although the principles of statics are time independent, this quantity does play an important role in the study of dynamics.

**Mass.** *Mass* is a property of matter by which we can compare the action of one body with that of another. This property manifests itself as a gravitational attraction between two bodies and provides a quantitative measure of the resistance of matter to a change in velocity.

**Force.** In general, *force* is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application.

## Text 3

### Idealizations.

Models or idealizations are used in mechanics in order to simplify application of the theory. A few of the more important idealizations will now be defined. Others that are noteworthy will be discussed at points where they are needed.

**Particle.** A *particle* has a mass but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital mo-

tion. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form, since the geometry of the body will not be involved in the analysis of the problem.

**Rigid Body.** A *rigid body* can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another both before and after applying a load. As a result, the material properties of any body that is assumed to be rigid will not have to be considered when analyzing the forces acting on the body. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.

**Concentrated Force.** A *concentrated force* represents the effect of a loading which is assumed to act at a point on a body. We can represent this effect by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.

#### Text 4

##### **Newton's Three Laws of Motion.**

The entire subject of rigid-body mechanics is formulated on the basis of Newton's three laws of motion, the validity of which is based on experimental observation. They apply to the motion of a particle as measured from a non accelerating reference frame and may be briefly stated as follows.

**First Law.** A particle originally at rest, or moving in a straight line with constant velocity, will remain in this state provided the particle is *not* subjected to an unbalanced force.

**Second Law.** A particle acted upon by an *unbalanced force* experiences acceleration that has the same direction as the force and a magnitude that is directly proportional to the force.

**Third Law.** The mutual forces of action and reaction between two particles equal, opposite, and collinear.

#### Text 5

##### **External and Internal Forces**

Since a rigid body is a composition of particles, both *external* and *internal* loadings may act on it. It is important to realize, however, that if the free-body diagram for the body is drawn, the forces that are *internal* to the body are *not represented* on the free-body diagram. These forces always occur in equal but opposite collinear pairs, and therefore their *net effect* on the body is zero. In some problems, a free-body diagram for a "system" of connected bodies may be used for an analysis. An example would be the free-body diagram of an entire automobile (system) composed of its many parts. Obvi-

ously, the connecting forces between its parts would represent *internal forces* which would *not* be included on the free-body diagram of the automobile. To summarize, then, internal forces act between particles which are located *within* a specified system which is contained within the boundary of the free-body diagram. Particles or bodies outside this boundary exert external forces on the system, and these alone must be shown on the free-body diagram.

### Text 6

#### **Weight and the Center of Gravity.**

When a body is subjected to a gravitational field, each of its particles has a specified weight as defined by Newton's law of gravitation. If we assume the size of the body to be "small" in relation to the size of the earth, then it is appropriate to consider these gravitational forces to be represented as a *system of parallel forces* acting on the particles contained within the boundary of the body. Such a system can be reduced to a single resultant force acting through a specified point. We refer to this force resultant as the *weight* of the body, and to the location of its point of application as the *center of gravity*.

If the weight of the body is important for the analysis, this force will then be reported in the problem statement. Also, when the body is *uniform* or made of homogeneous material, the center of gravity will be located at the body's *geometric center* or *centroid*: however, if the body is non-homogeneous or has an unusual shape, then its center of gravity will be given.

### Text 7

#### **Redundant Constraints**

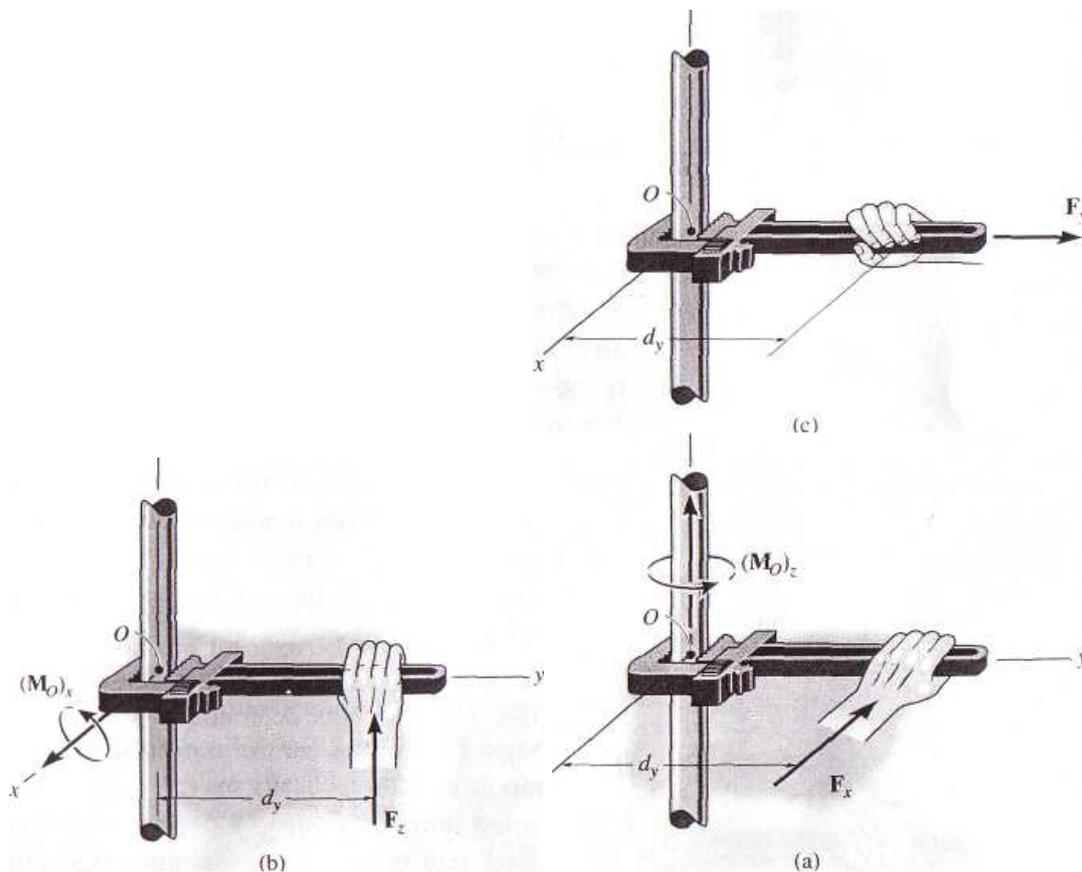
When a body has redundant supports, that is, more supports than are necessary to hold it in equilibrium, it becomes statically indeterminate. *Statically indeterminate* means that there will be more unknown loadings on the body than equations of equilibrium available for their solution. For example, the two-dimensional problem and the three-dimensional problem, shown in the picture together with their free body diagrams, are both statically indeterminate because of additional support reactions. In the two-dimensional case, there are five unknowns, for which only three equilibrium equations can be written. The three-dimensional problem has eight unknowns, for which only six equilibrium equations can be written. The additional equations needed to solve indeterminate problems generally obtained from the deformation conditions at the points of support. These equations involve the physical properties of the body which are studied in subjects dealing with the mechanics of deformation, such as "mechanics of materials."

## Дополнительные тексты для аудирования

### Text 1

#### Moment of a force – Scalar formulation

The *moment* of a force about a point or axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis. For example, consider the horizontal force  $F_x$ , which acts perpendicular to the handle of the wrench and is located a distance  $d_y$  from point  $O$ . It is seen that this force tends to cause the pipe to turn about the  $z$  axis. The larger the force or the length  $d_y$ , the greater the turning effect. This tendency for the rotation caused by  $F_x$  is sometimes called a *torque*, but most often it is called the *moment of a force* or simply the *moment* ( $M_O$ ). In particular, note that the *moment axis* ( $z$ ) is perpendicular to the shaded plane ( $x$ - $y$ ) which contains both  $F_x$  and  $d_y$  and that this axis intersects the plane at point  $O$ .



### Text 2

#### A principle of Moment

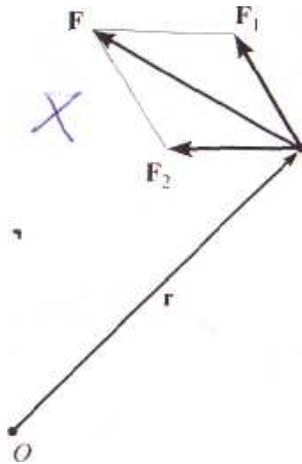
A concept often used in mechanics is the *principle of moments*, which is sometimes referred to as *Varignon's theorem* since it was originally developed

by the French mathematician Varignon (1654-1722). It states that *the moment of a force about a point is equal to the sum of the moments of the force's components about the point*

. The proof follows directly from the distributive law of the vector cross product. To show this, consider the force  $F$  and two of its components, where  $F = F_1 + F_2$ . We have

$$M_o = r \times F + r \times F_2 = r \times (F_1 + F_2) = r \times F$$

This concept has important applications to the solution of problems and proofs of theorems that follow, since it is often easier to determine the moments of a force's components rather than the moment of the force itself.



### Text 3

#### General Procedure for Analysis

The most effective way of learning the principles of engineering mechanics is to *solve problems*. To be successful at this, it is important always to present the work in a *logical and orderly manner*, as suggested by the following sequence of steps:

1. Read the problem carefully and try to correlate the actual physical situation with the theory studied.
2. Draw any necessary diagrams and tabulate the problem data.
3. Apply the relevant principles, generally in mathematical form.
4. Solve the necessary equations algebraically as far as practical, then, making sure they are dimensionally homogeneous, use a consistent set of units and complete the solution numerically. Report the answer with no more significant figures than the accuracy of the given data.
5. Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.
6. Once the solution has been completed, review the problem. Try to think of other ways of obtaining the same solution.

Учебное издание

ДЕМИДОВА Ольга Михайловна  
ЗАМЯТИН Владимир Маркович

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Научный редактор *кандидат филологических наук,  
доцент Е.В. Швагрукова*

Компьютерная верстка *О.М. Демидова*

Дизайн обложки *А.И. Сидоренко*

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ИЗДАТЕЛЬСТВО  ТПУ. 634050, г. Томск, пр. Ленина, 30  
Тел./факс: 8(3822)56-35-35, www.tpu.ru