MEASURING INSTRUMENTS AND ELECTRICAL MEASUREMENTS

4.1 INTRODUCTION

So far we have discussed values of voltage, current, resistance, etc, without mentioning the way in which they are masured. In general, the operating principles of instruments are beyond the technical level reached at this point in the book and we are in the 'chicken and the egg' dilemma, insomuch that it is difficult to explain how an instrument works before its principle can be fully comprehended.

However, the way in which measuring instruments operate is so important that they are introduced at this point in the book. Several chapters devoted to electrical principles relevant to the operation of electrical measuring instruments are listed below. You should refer to these chapters for further information:

Electrostatics - Chapter 6
Electromagnetism - Chapter 7
Transformers - Chapter 14
Rectifiers - Chapter 16

4.2 TYPES OF INSTRUMENT

Instruments are classified as either analogue instruments or digital instruments. An **analogue instrument** - see Figure 4.1(a) - is one in which the magnitude of the measured electrical quantity is indicated by the movement of a pointer across the face of a scale. The indication on a **digital instrument** - see Figure 4.1(b) - is in the form of a series of numbers displayed on a screen; the smallest change in the indicated quantity corresponding to a change of ±1 digit in the *least significant digit* (l.s.d.) of the number. That is, if the meter indicates 10.23 V, then the actual voltage lies in the range 10.22 V to 10.24 V.

(a) an analogue multimeter, *(b) a digital multimeter*

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Both types of instrument have their advantages and disadvantages, and the choice of the 'best' instrument depends on the application you have in mind for it. As a rough guide to the features of the instruments, the following points are useful:

1. an analogue instrument does not (usually) need a battery or power supply;
2. a digital instrument needs a power supply (which may be a battery);
3. a digital instrument is generally more accurate than an analogue instrument (this can be a disadvantage in some cases because the displayed value continuously changes as the measured value changes by a very small amount);
4. both types are portable and can be carried round the home or factory.

4.3 EFFECTS UTILISED IN ANALOGUE INSTRUMENTS

An analogue instrument utilises one of the following effects:

1. electromagnetic effect;
2. heating effect;
3. electrostatic effect;
4. electromagnetic induction effect;
5. chemical effect.

The majority of analogue instruments including moving-coil, moving-iron and electrodynamic (dynamometer) instruments utilise the magnetic effect. The effect of the heat produced by a current in a conductor is used in thermocouple instruments. Electrostatic effects are used in electrostatic voltmeters. The electromagnetic induction effect is used, for example, in domestic energy meters. Chemical effects can be used in certain types of ampere–hour meters.

4.4 OPERATING REQUIREMENTS OF ANALOGUE INSTRUMENTS

Any instrument which depends on the movement of a pointer needs three forces to provide proper operation. These are:

1. a deflecting force;
2. a controlling force;
3. a damping force.

The **deflecting force** is the force which results in the movement or deflection of the pointer of the instrument. This could be, for example, the force acting on a current-carrying conductor which is situated in a magnetic field.

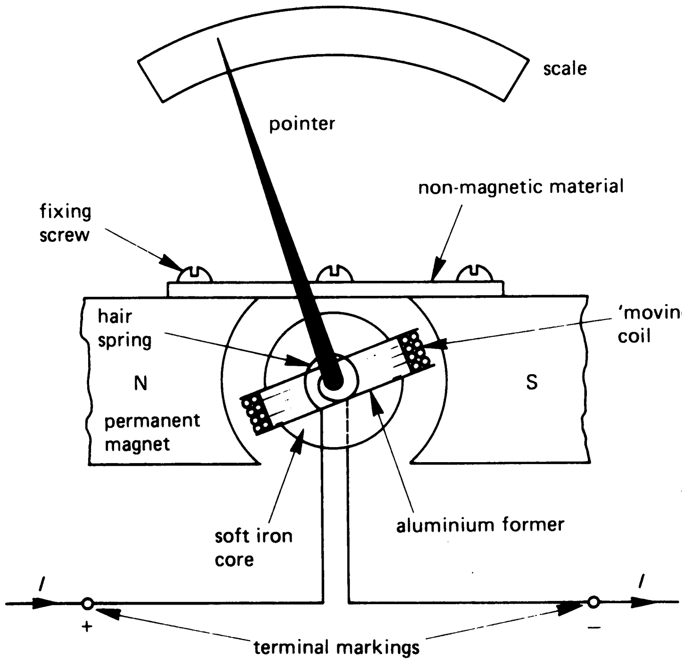
The **controlling force** opposes the deflecting force and ensures that the pointer gives the correct indication on the scale of the instrument. This could be, for example, a hairspring.

The **damping force** ensures that the movement of the pointer is *damped*; that is, the damping force causes the pointer to settle down, that is, be 'damped', to its final value without oscillation.

4.5 A GALVANOMETER OR MOVING-COIL INSTRUMENT

A **galvanometer** or **moving-coil instrument** depends for its operation on the fact that a current-carrying conductor experiences a force when it is in a magnetic field. A basic form of construction is shown in Figure 4.2. The 'moving' part of the meter is a coil wound on an aluminium *former* or frame which is free to rotate around a cylindrical soft-iron core. The moving coil is situated in the magnetic field produced by a permanent magnet; the function of the solf-iron core is to ensure that the magnetic field is uniformly distributed. The soft-iron core is securely fixed between the poles of the permanent magnet by means of a bar of non-magnetic material.

The moving coil can be supported either on a spindle which is pivoted in bearings (often jewel bearings) or on a *taut metal band* (this is the so-called **pivotless suspension**). The current, I, enters the 'moving' coil from the +ve terminal either via a spiral **hairspring** (see Figure 4.2) or via the fig 4.2 *a simple galvanometer or moving-coil meter*



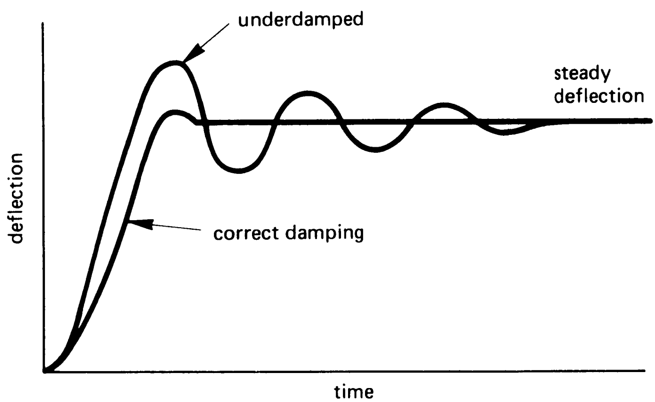
taut band mentioned above. It is this hairspring (or taut band) which provides the *controlling force* of the instrument. The current leaves the moving coil either by another hairspring or by the taut band at the opposite end of the instrument.

When current flows in the coil, the reaction between each current-carrying conductor and the magnetic field produces a mechanical force on the conductor; this is the *deflecting force* of the meter.

This force causes the pointer to be deflected, and as it does so the movement is opposed by the hairspring which is used to carry current into the meter. The more the pointer deflects, the greater the controlling force produced by the hairspring.

Unless the moving system is damped, the pointer will overshoot the correct position; after this it swings back towards the correct position. Without damping, the oscillations about the correct position continue for some time (see Figure 4.3 for the *underdamped response*). However, if the movement is correctly damped (see Figure 4.3) the pointer has an initial overshoot of a few per cent and then very quickly settles to its correct indication. It is the aim of instrument designers to achieve this response.

fig 4.3 *meter damping*



Damping is obtained by extracting energy from the moving system as follows. In the moving-coil meter, the coil is wound on an aluminium former, and when the former moves in the magnetic field of the permanent magnet, a current (known as an **eddy current**) is induced in the aluminium former. This current causes power to be consumed in the resistance of the coil former, and the energy associated with this *damps* the movement of the meter.

4.6 METER SENSITIVITY AND ERRORS

The **sensitivity** of an *ammeter* is given by the value of the current needed to give *full-scale deflection* (FSD). This may be 50 μA or less in a high quality analogue instrument.

The sensitivity of a *voltmeter* is expressed in terms of the *ohms per volt* of the instrument. That is

$$\text{voltmeter sensitivity} = \frac{\text{voltmeter resistance}}{\text{full-scale voltage}}$$

It can be shown that this is the reciprocal of the current needed to give full-scale deflection. A voltmeter having a sensitivity of 20 kΩ/V requires a current of

$$\frac{1}{20\,000} = 50 \times 10^{-6} \text{ A or } 50\ \mu\text{A}$$

to give full-scale deflection. The resistance of the voltmeter is therefore calculated from the equation

meter resistance = 'ohms per volt' × full-scale voltage

The **accuracy** of an instrument depends on a number of factors including the friction of the moving parts, the ambient temperature (which not only affects the resistance of the electrical circuit, but also the length of the hairsprings [where used], and the performance of the magnetic circuit). Another cause of error may be introduced by the user; if the meter is not observed perpendicular to the surface of the meter scale, an error known as *parallax error* may be introduced. This is overcome in some instruments by using a pointer with a knife-edge with an anti-parallax mirror behind it. When the meter is being read, the pointer should be kept in line with its reflection in the mirror (see also Figure 4.1(a)).

Strong magnetic fields can introduce errors in meter readings. These are largely overcome by using instruments which are housed in a magnetic shield which effectively deflects the external magnetic field from the meter.

Further errors can be introduced by using a meter of the 'wrong' resistance. For example, when an ammeter is inserted into a circuit, it increases the total resistance of the circuit by an amount equal to the resistance of the meter; in turn, this reduces the circuit current below its value when the ammeter is not in circuit, giving an error in the reading of current. Hence, *when an ammeter is used in a circuit, the resistance of the ammeter should be very much less (say $\frac{1}{100}$) of the resistance of the remainder of the resistance of the circuit into which it is connected.*

When a voltmeter is used to measure the p.d. across a component, *the resistance of the voltmeter must be much greater (say by a factor of 100) than the resistance of the component to which it is connected.* If this is not the case, the reading of the voltmeter differs from the voltage across the component when the voltmeter is not connected. For example, if you wish to measure the voltage across a 10 kΩ resistor, then the resistance of the voltmeter should be at least 100 × 10 kΩ = 1000 kΩ or 1 MΩ.

4.7 EXTENSION OF THE RANGE OF MOVING-COIL INSTRUMENTS

Moving-coil meters are basically low-current, low-voltage meters, and their circuits must be modified to allow them to measure either high current or high voltage. For example, the current needed to give full-scale deflection may be as little as 50 μA, and the voltage across the meter at FSD may be as little as 0.1 V.

Extending the current range

To extend the current range of the meter, a **low resistance shunt resistor**, S, is connected in parallel or in shunt with the meter as shown in Figure 4.4. The terms used in the diagram are

I = current in the external circuit to give FSD
I_G = current in the moving-coil meter to give FSD
R_G = resistance of the movement of the meter
I_S = resistance of the shunt resistor

The important equations for Figure 4.4 are

total current = meter current + shunt current

or

$$I = I_G + I_S \tag{4.1}$$

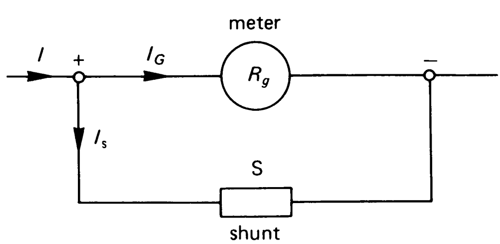
and

p.d. across the meter = p.d. across the shunt

that is

$$I_G R_G = I_S S \tag{4.2}$$

fig 4.4 *extending the current range of a meter using a shunt*



Example

The resistance of a moving-coil meter is 5 Ω and gives FSD for a current of 10 mA. Calculate the resistance of the shunt resistor required to cause it to have a FSD of 20 A.

Solution

Given that I = 20 A; I_G = 10 mA; R_G = 5 Ω from eqn (4.1

From eqn (4.1)

$$I_S = I - I_G = 20\text{A} - 10\text{ mA}$$
$$= 20 - 0.01 = 19.99\text{ A}$$

and from eqn (4.2)

$$\text{shunt resistance}, S = \frac{I_G R_G}{I_S} = (10 \times 10^{-3}) \times \frac{5}{19.99}$$

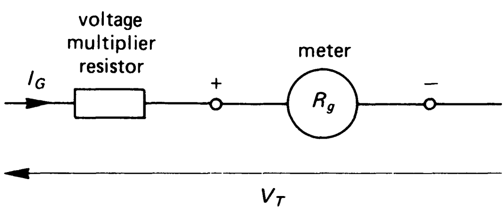
$$= 0.002501\ \Omega$$

Extension of voltage range

The voltage range which can be measured by a moving-coil instrument can be increased by the use of a *voltage multiplier resistor*, R, which is connected in series with the meter as shown in Figure 4.5. Since the meter current, I_G, flows through both R and the meter, the voltage V_T, across the combination is

$$V_T = I_G(R + R_G) \tag{4.3}$$

fig 4.5 *extending the voltage range of a meter using a voltage multiplier resistor*



Example

A moving-coil meter of resistance 5 Ω gives FSD for a current of 10 mA. Calculate the resistance of the voltage multiplier which causes the meter to give a FSD of 100 V.

Solution

$$V_T = 100\text{ V}; I_G = 10\text{ mA}; R = 5\ \Omega$$

From eqn (4.3)

$$V_T = I_G(R + R_G)$$

or

$$R = \frac{V_T}{I_G} - R_G$$

$$= \frac{100}{5 \times 10^{-3}} - 5 = 19\,995\ \Omega$$

4.8 MEASUREMENT OF a.c. QUANTITIES USING A MOVING-COIL METER

A moving-ciol meter or galvanometer is only capable of measuring uni-directional (d.c.) current and voltage. However, it can be used to measure alternating current and voltage when the a.c. quantity has been converted into d.c. by means of a **rectifier** (see Chapter 16 for full details of rectifier circuits).

There are two basic types of rectifier circuit used with moving-coil meters, which are shown in Figure 4.6. In the 'half-wave' circuit in Figure 4.6(a), rectifier diode D1 allows current to flow through the meter when the upper terminal of the meter is positive with respect to the lower terminal; however, this diode prevents current flowing through the meter when the upper terminal is negative. It is the function of diode D2 to bypass the meter when the upper terminal is negative. In this way, the half-wave circuit allows current to flow through the meter **in one direction only** for one half of the a.c. cycle. No current flows through it in the other half-cycle.

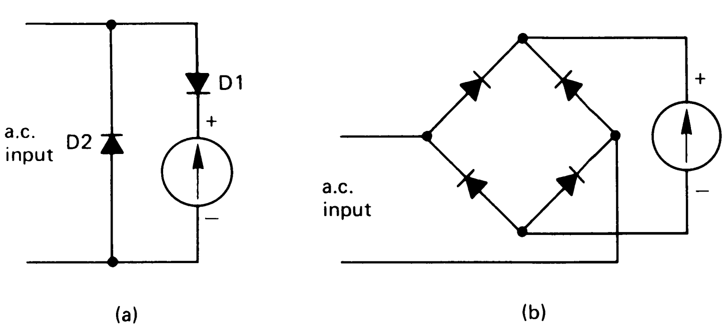
The 'full-wave' rectifier circuit in Figure 4.6(b) allows current to flow through the meter from the terminal marked '+' to the terminal marked '–' whatever the polarity of the a.c. input.

Half-wave rectifiers are generally used in low-cost meters, and full-wave rectifiers are used with more expensive meters.

Extension of the a.c. range of moving-coil meters

The alternating current and voltage range of moving coil meters can, within limits, be extended using shunts and multiplying resistors, respectively, in the manner described earlier.

fig 4.6 *measurement of alternating current using (a) a half-wave circuit and (b) a full-wave circuit*



Very high values of current can be measured by using the meter in conjunction with a **current transformer** to reduce the measured current to, say, 5 A or less. Very high values of voltage can be measured by using the meter in conjunction with a **potential transformer** or **voltage transformer** to reduce the measured voltage level to, say, 110 V or less. The operating principles of the transformer are discussed in Chapter 14.

4.9 MOVING-IRON METERS

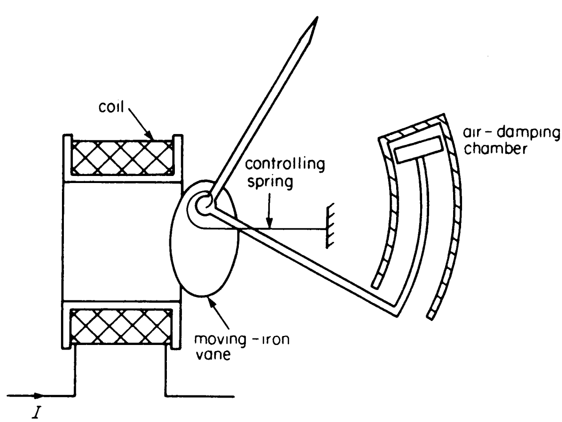
There are two basic types of meter in this category, namely

1. attraction-type meters;
2. repulsion-type meters.

Attraction-type meters depend on the fact that iron is attracted towards a magnetic field. Repulsion-type meters depends on the force of repulsion between two similarly magnetised surfaces (that is to say, there is a force of repulsion between two north poles, and there is a force of repulsion between two south poles).

The general construction of an **attractiont-ype moving-iron meter** is shown in Figure 4.7. The current, I, in the circuit flows through the coil and produces a magnetic field which attracts the 'moving iron' vane into the coil; this is the *deflecting force* of the meter. As the pointer moves it increases the tension in the controlling spring (this provides the *controlling force*). At the same time, it moves a piston inside a cylinder; the piston provides the *damping force* of the meter.

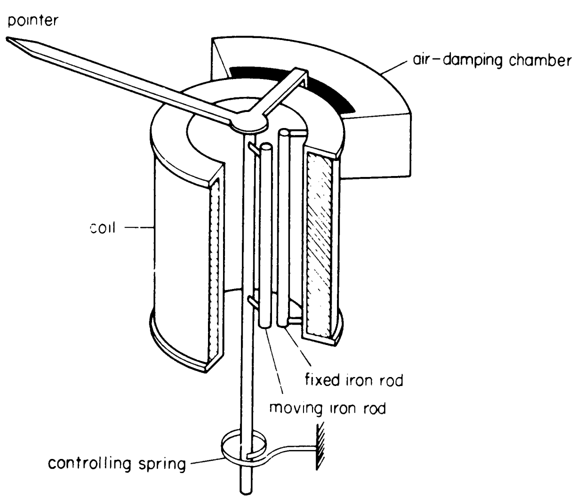
fig 4.7 *attraction-type moving-iron meter*



The basis of a **repulsion-type moving-iron meter** is shown in Figure 4.8. The *deflecting force* is again provided by the current in the coil, the *controlling force* by the controlling spring, and the *damping force* by the air dashpot. The method of operation is as follows. The instrument has two parallel iron rods or vanes inside the coil, one being fixed to the coil whilst the other is secured to the spindle and is free to move. When current flows in the coil, both rods are similarly magnetised, each having N-poles at (say) the upper end in the diagram and S-poles at the lower end. The like magnetic poles mutually repel one another and, since one is fixed to the coil, the rod which is attached to the spindle moves away from the fixed rod, causing the pointer to deflect.

Moving-iron instruments are generally more robust and cheaper than moving-ciol meters, but have a lower accuracy. They are widely used as general-purpose panel meters.

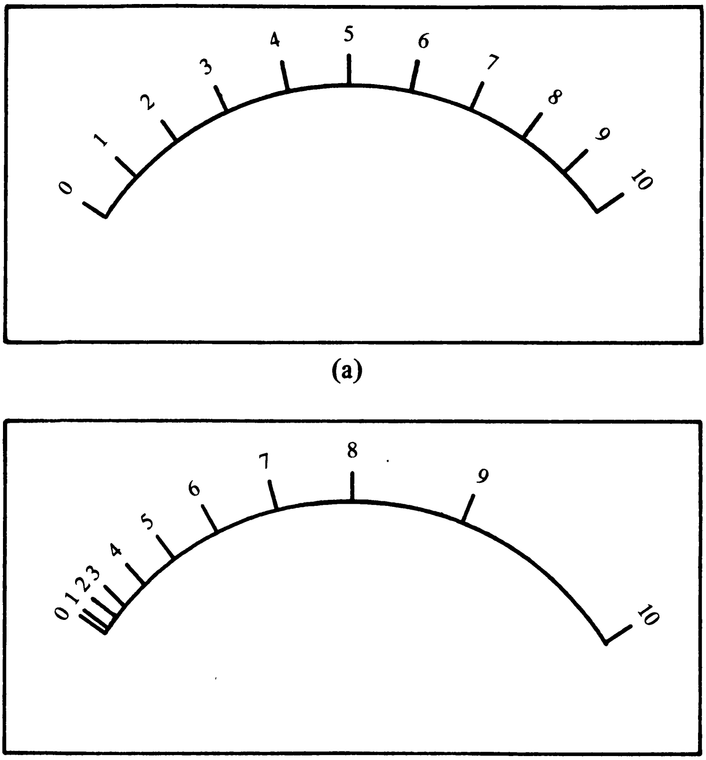
fig 4.8 *repulsion-type moving-iron meter*



4.10 METER SCALES

The length and way in which an instrument scale is calibrated depends on a number of factors and, in general, results in one of two types of scale, namely a linear scale or a non-linear scale. A **linear scale** is one which has an equal angular difference between points on the scale (see Figure 4.9(a)), and a **non-linear scale** has unequal angular differences (Figure 4.9(b)).

A moving-coil meter has a linear scale for both current and voltage scales. A moving-iron meter has a non-linear scale for both voltage and fig 4.9 *instrument scales: (a) linear, (b) non-linear*



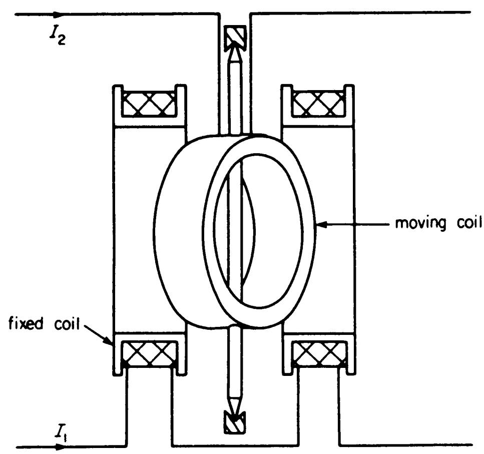
(b)

current; however, it is possible to improve the linearity of the scale of a moving-iron instrument by redesigning the moving vane system.

The total angular deflection of the scale is in the range 90°–250° (the scales in Figure 4.9 have a deflection of about 100°).

4.11 WATTMETERS

As the name of this instrument implies, its primary function is to measure the power consumed in an electrical circuit. The wattmeter described here is called an **electrodynamic wattmeter** or a **dynamometer wattmeter**. It is illustrated in Figure 4.10, and has a pair of coils which are fixed to the frame of the meter (the **fixed coils**) which carry the main current in the circuit (and are referred to as the **current coils**), and a **moving coil** which is pivoted so that it can rotate within the fixed coils. The moving coil generally has a high resistance to which the supply voltage is connected and is called the **voltage coil** or **potential coil**. The pointer is secured to the fig 4.10 *Dynamometer wattmeter*

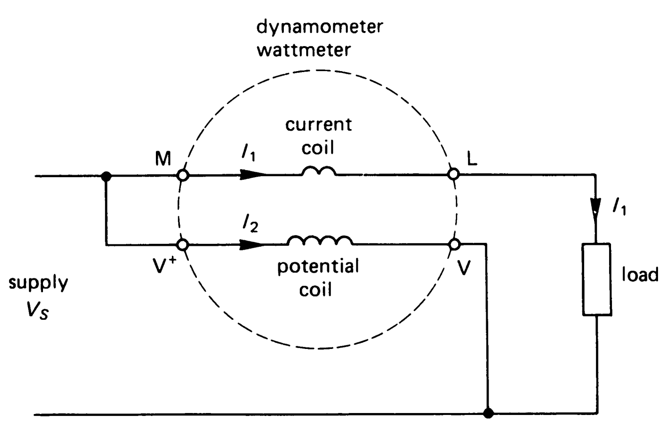


spindle of the moving coil. The magnetic flux produced by the fixed coils is proportional to current I_1 in the figure, and the magnetic flux produced by the moving coil is proportional to I_2 (the former being proportional to the load current and the latter to the supply voltage; see Figure 4.11).

The two magnetic fields react with one another to give a deflecting force proportinal to the product $I_1 \times I_2$. However, since I_2 is proportional to the supply voltage V_S, the deflection of the meter is given by

$$\text{meter deflection} = V_S I_1 = \text{power consumed by the load}$$

fig 4.11 *wattmeter connections*



Dynamometer wattmeters can measure the power consumed in either a d.c. or an a.c. circuit.

Hairsprings are used to provide the *controlling force* in these meters, and air-vane damping is used to *damp* the movement.

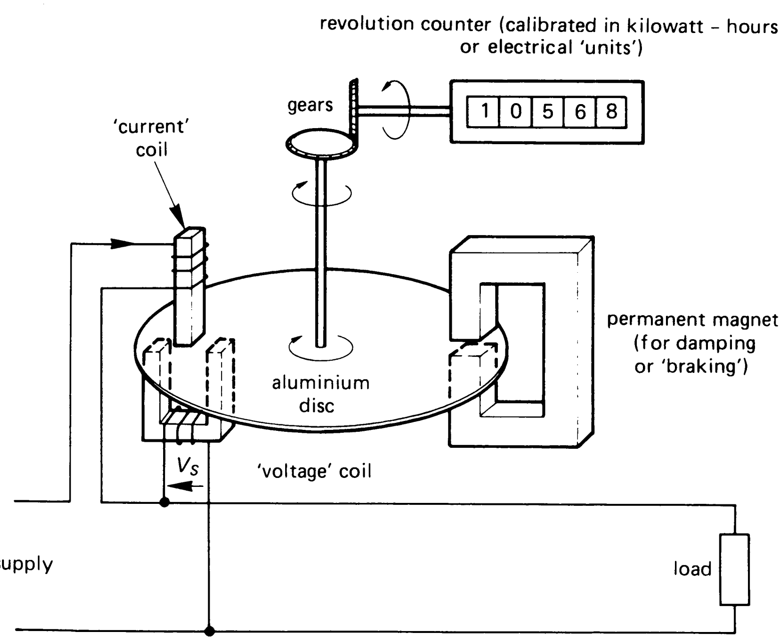
The power consumed by a three-phase circuit (see Chapter 13) is given by the sum of the reading of two wattmeters using what is known as the **two-wattmeter method** of measuring power.

4.12 THE ENERGY METER OR KILOWATT-HOUR METER

The basic construction of an electrical energy meter, known as an **induction meter**, is shown in Figure 4.12. This type of meter is used to measure the energy consumed in houses, schools, factories, etc.

The magnetic field in this instrument is produced by two separate coils. The 'current' coil has a few turns of large section wire and carries the main current in the circuit. The 'voltage' coil has many turns of small section wire, and has the supply voltage connected to it. The 'deflection' system is simply an aluminium disc which is free to rotate continuously (as you will see it do if you watch your domestic energy meter), the disc rotating faster when more electrical energy is consumed.

fig 4.12 *induction-type energy meter*



The effect of the magnetic field produced by the coils is to produce a torque on the aluminium disc, causing it to rotate. The more current the electrical circuit carries, the greater the magnetic flux produced by the 'current' coil and the greater the speed of the disc; the disc stops rotating when the current drawn by the circuit is zero.

The disc spindle is connected through a set of gears to a 'mileometer' type display in the case of a digital read-out meter, or to a set of pointers in some older meters. The display shows the total energy consumed by the circuit.

The rotation of the disc is *damped* by means of a permanent magnet as follows. When the disc rotates between the poles of the permanent magnet, a current is induced in the rotating disc to produce a 'drag' on the disc which damps out rapid variations in disc speed when the load current suddenly changes.

These meters are known as **integrating meters** since they 'add up' or 'integrate' the energy consumed on a continual basis.

4.13 MEASUREMENT OF RESISTANCE BY THE WHEATSTONE BRIDGE

The resistance of a resistor can be determined with high precision using a circuit known as a **Wheatstone bridge** (invented by Sir Charles Wheatstone).

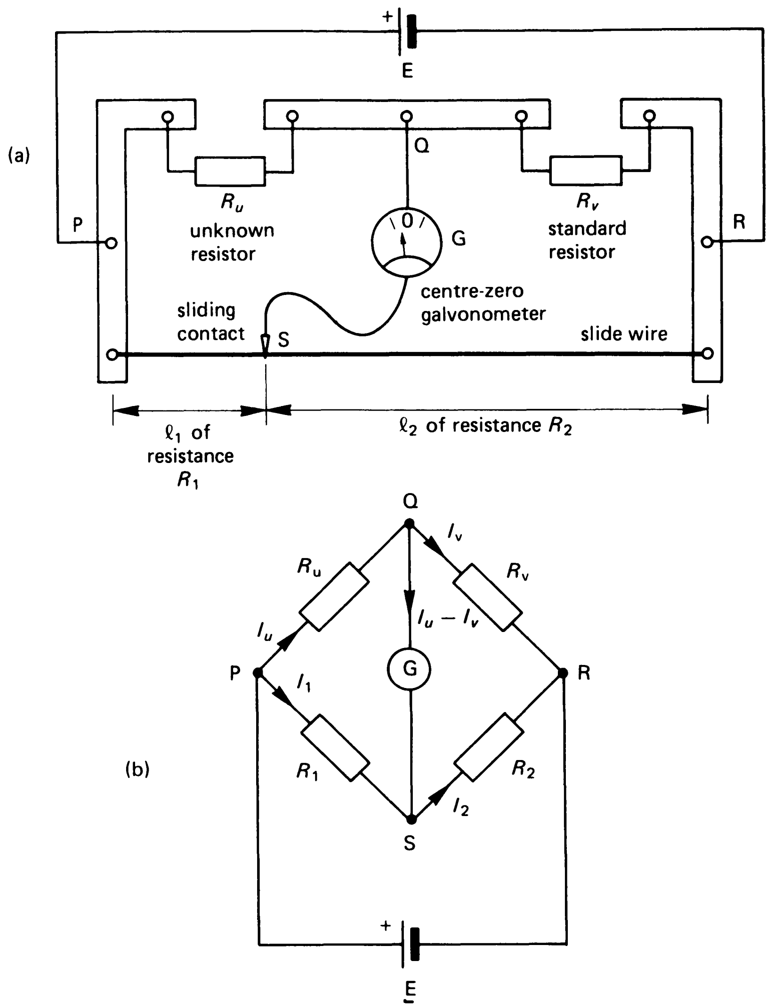
A basic version of the Wheatstone bridge is the familiar 'slide-wire' version shown in Figure 4.13(a). The circuit consists of two parallel branches PQR and PSR supplied by a common source of e.m.f., E. Branch PSR is a wire of uniform cross-sectional area and of length typically 50 or 100 cm; for reasons given later, the two arms l_1 and l_2 are known as the **ratio arms** of the bridge. Branch PQR contains two series-connected resistors R_U and R_V, resistor R_U is the **unknown value of resistance** (to be determined by the measurements), and R_V is a **standard resistor** of precise value and having a high accuracy.

Point Q in the upper branch is connected to the **sliding contact**, S, on the slide-wire via a **centre-zero galvanometer**.

The circuit diagram of the bridge is shown in Figure 4.13(b), in which resistor R_1 replaces the length of slide-wire l_1, and R_2 replaces l_2. The circuit basically comprises two parallel-connected branches (one containing R_U and R_V and the other containing R_1 and R_2) which are linked by the galvanometer G.

When the circuit is initially connected to the battery, the current I_U flowing in R_U does not necessarily equal I_V which flows in R_V; the difference in value between these two currents flows in the galvanometer G.

fig 4.13 *(a) 'slide-wire' version of the Wheatstone bridge, (b) circuit diagram*



Depending on which way the current flows, the needle of the galvanometer will deflect either to the left or to the right of its centre-zero position.

The Wheatstone bridge at balance

If the slider is moved, say, to the left of its initial position the deflection of the valvanometer will (say) give a larger deflection. If the slider is moved to the right, the deflection decreases. By moving the slider along the slide-wire, a position will be reached when the galvanometer gives zero deflection, indicating that the current through it is zero. When this happens, the bridge is said to be **balanced**. Since the current in the galvanometer is zero, then

$$I_U = I_V \tag{4.4}$$

and

$$I_1 = I_2 \tag{4.5}$$

Now, since no current flows through the galvanometer, there is no p.d. across it, hence

potential at Q = potential at S

Furthermore it follows that

p.d. across R_U = p.d. across R_V

or

$$I_U R_U = I_1 R_1 \tag{4.6}$$

and

p.d. across R_V = p.d. across R_2

or

$$I_V R_V = I_2 R_2 \tag{4.7}$$

Dividing eqn (4.6) by eqn (4.7) gives

$$\frac{I_U R_U}{I_V R_V} = \frac{I_1 R_1}{I_2 R_2} \tag{4.8}$$

But, at balance $I_V = I_U$ and $I_1 = I_2$, so that eqn (4.8) becomes

$$\frac{I_U R_U}{I_U R_V} = \frac{I_1 R_1}{I_1 R_2}$$

or

$$\frac{R_U}{R_V} = \frac{R_1}{R_2} \tag{4.9}$$

The **unknown value of resistance** R_U can therefore be calculated as follows

$$R_U = R_V \times \frac{R_1}{R_2} \tag{4.10}$$

In the above equation, R_1 and R_2 correspond to the resistance of l_1 and l_2, respectively, of the slide-wire. Since R_1 and R_2 in eqn (4.10) form a resistance ratio, these two 'arms' of the bridge are known as the **ratio arms**.

Unfortunately we only know the length of l_1 and l_2 and do not know their resistance. However, since the slide-wire has a uniform cross-sectional area and each part has the same resistivity, eqn (4.10) can be rewritten in the form

$$R_U = R_V \times \frac{l_1}{l_2} \tag{4.11}$$

Example

The value of an unknown resistance R_U is measured by means of a Wheatstone bridge circuit of the type in Figure 4.13(a). Balance is obtained when $l_1 = 42$ cm, the total length of the slide-wire being 100 cm. If the value of the standard resistor is 10 Ω, determine the value of R_U.

If the unknown value of resistor is replaced by a 15 Ω resistor, and a standard resistor of 20 Ω is used, what is the value of l_1 which gives balance?

Solution

For the first part of the question $R_V = 10\ \Omega, l_1 = 42$ cm, $l_2 = (100 - 42) = 58$ cm

From eqn (4.11)

$$R_U = \frac{R_V l_1}{l_2} = 10 \times \frac{42}{58} = 7.24\ \Omega \text{ (Ans.)}$$

For the second part of the problem

$$R_U = 15\ \Omega, R_V = 20\ \Omega$$

From eqn (4.11)

$$\frac{l_1}{l_2} = \frac{R_U}{R_V} = \frac{15}{20} = 0.75$$

or

$$l_2 = \frac{l_1}{0.75} = 1.333\, l_1.$$

But $l_1 + l_2 = 100$ cm hence $l_1 + 1.333\, l_1 = 100$ cm therefore

$$l_1 = \frac{100}{(1 + 1.333)} = 42.86 \text{ cm (Ans.)}$$

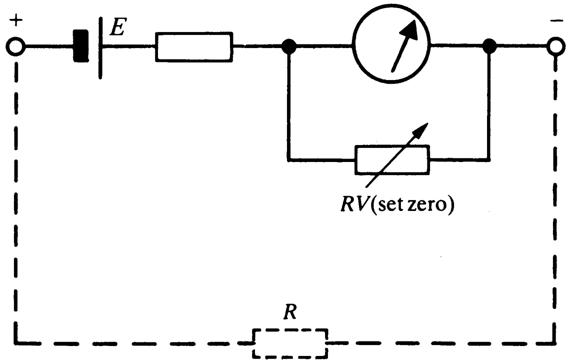
4.14 MEASUREMENT OF RESISTANCE USING AN OHMMETER

An ohmmeter is a moving-coil instrument calibrated directly in ohms. Basically it is a galvanometer fitted with a battery E (see Figure 4.14(a)) to which the unknown resistor R is connected. The current which flows through the meter when the unknown resistance is connected is a measure of the resistance of the resistor; the lower the value of R the larger the current, and vice versa.

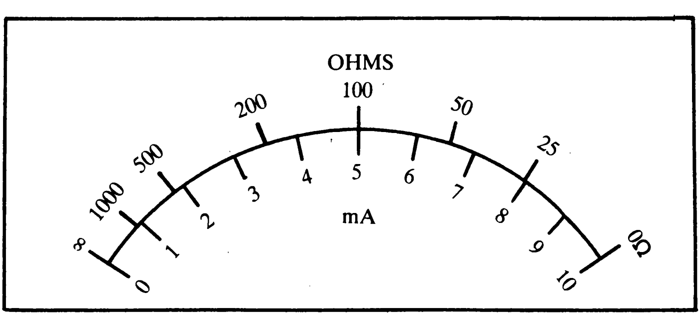
When the unknown resistance is disconnected, the meter current is zero; the ohms scale is therefore scaled to show **infinite (∞) ohms when the current is zero**. The meter is calibrated by applying a short-circuit to its terminals, and then adjusting the SET ZERO control resistor RV until the needle gives full-scale deflection. That is, **maximum current corresponds to zero ohms.**

Since the current in the unknown resistance is proportional to $\frac{1}{R}$, the scale calibration is non-linear (see Figure 4.14(b)).

fig 4.14 *(a) typical ohmmeter circuit and (b) an example of its scale*



(a)



(b)

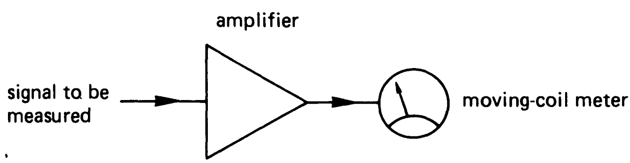
4.15 ELECTRONIC INSTRUMENTS

These are divided into two gategories, namely analogue instruments and digital instruments.

Electronic analogue instruments

These consist of a moving-coil meter which is 'driven' by an amplifier as shown in Figure 4.15. The function of the amplifier is not only to amplify a voltage or a current signal which may have a very low value, but also to give the instrument a high 'input resistance'. The latter means that when the instrument is connected to the circuit, it draws only a minute current from the circuit.

fig 4.15 *the basis of an analogue electronic instrument*

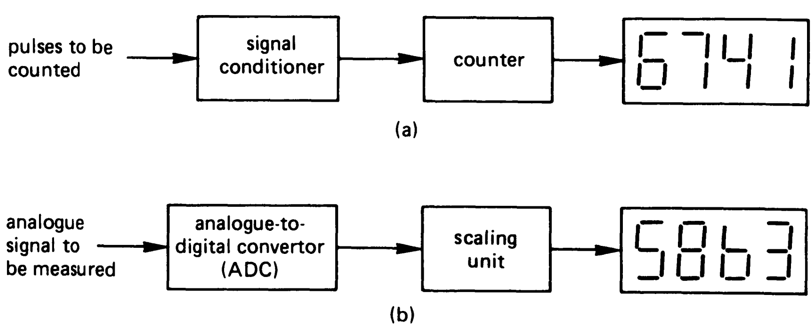


Electronic digital instruments

There are two types of digital instrument, namely those which accept a digital signal, that is, a signal consisting of a series of pulses, and those which accept an analogue signal (similar to a conventional voltmeter or ammeter).

The basis of a digital instrument which directly accepts pulses from a digital system is shown in Figure 4.16(a). The incoming signal may be from an installation which is electronically 'noisy' (an electrical motor or a

fig 4.16 *(a) timer/counter, (b) block diagram of a digital ammeter or voltmeter*



fluorescent lamp are electrically 'noisy' devices); it is first necessary to 'condition' the incoming signal not only to remove the 'noise' but also to bring it to the correct voltage level for the meter. The pulses are then counted and displayed on a suitable read-out device; typical display devices include **light-emitting diodes** (LED), **liquid crystal displays** (LCD), **gas-discharge tubes** and **cathode-ray tubes** (CRT).

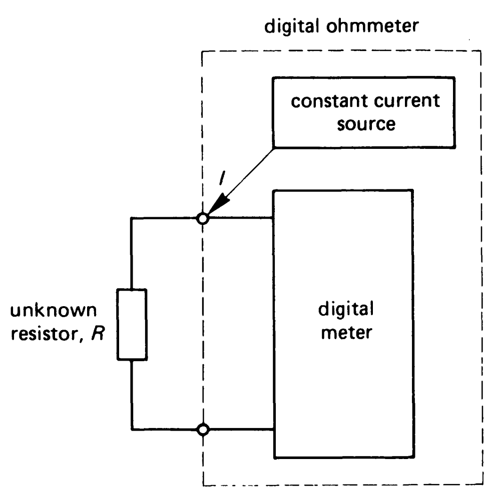
The basis of an instrument which measures an analogue quantity such as voltage or current, but provides a digital display of its value is shown in Figure 4.16(b). The analogue quantity is first converted to its digital form by means of an **analogue-to-digital convertor** (ADC). The output from this part of the circuit is in digital (or on-off) form, the digital signal being related to the analogue value being measured. The digital signal is then displayed on a suitable device. The measuring range of such a meter can be extended using either voltage multiplier resistors or current shunts, in the manner outlined for conventional analogue meters.

4.16 MEASUREMENT OF RESISTANCE USING A DIGITAL METER

The basis of a digital ohmmeter is shown in Figure 4.17. It consists of a digital voltmeter of the type in Figure 4.16(b), together with a constant current source, both being housed in the ohmmeter.

When an unknown resistor R is connected to the terminals of the instrument, the constant value of current (I) from the 'current' source flows through R to give a p.d. of IR at the terminals of the voltmeter. This voltage is measured by the electronic voltmeter and, since the p.d. is pro-

fig 4.17 *principle of a digital ohmmeter*



portional to the resistance of the unknown resistor, the measured voltage is proportional to the resistance of R. The voltage displayed on the meter can therefore be calibrated in 'ohms'.

SELF-TEST QUESTIONS

1. Describe the effects which can be utilised to provide a reliable electrical measuring instrument. Given an example of each kind of effect.
2. What are the basic requirements of an analogue measuring instrument? How are these needs provided in (i) a moving-coil meter, (ii) a moving-iron meter, (iii) an electrodynamic instrument and (iv) an induction instrument?
3. Explain the operation of a moving-coil meter and discuss the way in which the movement is damped.
4. Why can electrodynamic meters be used to measure the power consumed by an electrical circuit?
5. Draw a circuit diagram of a Wheatstone bridge and determine an equation which gives its 'condition of balance'.
6. The value of a resistor of 40-Ω resistance is measured using a Wheatstone bridge. If the value of the standard resistor is 20 Ω and the length of the slide-wire is 100 cm, determine the position of the slider.
7. Draw a circuit diagram of an ohmmeter and explain its operation. Why does the 'zero ohms' position of the scale correspond to the 'maximum current' position?
8. Discuss the operating principles of electronic instruments.
9. Explain how the range of (i) a moving-coil meter and (ii) a moving-iron meter can be extended to measure a large current and a large voltage.

SUMMARY OF IMPORTANT FACTS

An **analogue instrument** indicates the value of the quantity being measured by the smooth movement of a pointer across the scale of the meter; a **digital instrument** gives a display in the form of a set of numbers or digits.

An *analogue meter* needs three forces to ensure proper operation, namely, a **deflecting force**, a **controlling force** and a **damping force**.

Important types of analogue meters are **moving-coil meters (galvanometers)**, **moving-iron meters**, **electrodynamic instruments** and **induction instruments**. The scales used may be either **linear** or **non-linear**. In their basis form, certain meters can only measure direct current (the moving-coil meter for example), others can only measure alternating current (the induction instrument for example), and yet others can measure either d.c. or a.c. (for example moving-iron and electrodynamic meters).

The **range** of a meter can be extended to measure a higher current using a current **shunt**, or to measure a higher voltage using a series-connected **voltage multiplier** resistor.

An *unknown value of resistance* can be measured using a **Wheatstone bridge**; the bridge uses an accurate **standard resistor** together with the two **ratio arms** of the bridge. Resistance can also be measured using an **ohm-meter.**

Electronic instruments are subdivided into **analogue indication** and **digital indication** types. *Analogue indication* types generally consist of a conventional moving-coil meter which is 'driven' by an amplifier. *Digital indication* types are generally more complex than analogue indication types.

CHAPTER 5

ELECTRICAL ENERGY AND ELECTRICAL TARIFFS

5.1 HEATING EFFECT OF CURRENT: FUSES

The amount of heat energy, W joules or watt seconds, produced by a current of I amperes flowing in a resistor R ohms for t seconds is

$$W = I^2Rt \text{ joules (J) or watt seconds}$$

Thus, if 1 A flows for 1 second in a resistance of 1 Ω, the energy dissipated is

$$W = I^2 \times 1 \times 1 = 1 \text{ J}$$

However, if 100 A flows for the same length of time in the same resistor, the energy dissipated is

$$W = 100^2 \times 1 \times 1 = 10\,000 \text{ J}$$

The electrical power rating of an item of electrical plant is related to its ability to dissipate the energy which is created within the apparatus. Since the heat energy is related to I^2, the **rating of an electrical machine** depends on its ability to dissipate the heat generated (I^2R) within it; the rating therefore depends **on the current which the apparatus consumes.**

In order to protect electrical equipment from excessive current, an electrical **fuse** is connected in series with the cable supplying the plant. The function of the fuse is to 'blow' or melt when the load-current exceeds a pre-determined value; needless to say, the current-carrying capacity must be less than that of the cable to which it is connected!

The fuse is simply a piece of wire (which may be in a suitable 'cartridge') which, when carrying a 'normal' value of current remains fairly cool. However, if the current in the circuit rises above the 'rating' of the fuse, the heat generated in the fuse rises to the point where the melting-point of the fuse material is reached. When the fuse melts, the circuit is broken, cutting off the current to the faulty apparatus.

When wiring a fuse, you should always ensure that it is connected in the 'live' wire; this ensures that, when the fuse 'blows', the apparatus is disconnected from the live potential.

In general, the larger the current passing through the fuse, the shorter the 'fusing' time; that is, a fuse has an **inverse-time fusing characteristic**. That is to say, a current which is only slightly greater than the current rating of the fuse may take many minutes to blow the fuse, but a current which is, say, ten times greater than the fuse-rating causes it to blow in a fraction of a second.

Early types of fuses could be rewired, that is, the fuse was simply a piece of wire connected between two terminals in a ceramic fuseholder. The rating of the fuse-wire in a domestic system would be, typically, 5 A, 15 A or 30 A. Modern domestic apparatus is protected by means of a fuse in the plug which terminates the cable. This type of fuse is known as a **cartridge fuse**, which is a cartridge containing the fusible element; the rating of these fuses in the UK is usually one of the values 3 A, 5 A, 10 A or 13 A.

Industrial fuses handle very high values of current (typically many hundreds of amperes) and are frequently special fuses known as **high rupturing-capacity fuses** or **high breaking-capacity fuses**.

Many items of electronic apparatus have fuses in which the fusible element is housed in a glass cartridge, allowing the user to inspect the state of the fuse when it is withdrawn from the fuseholder. Certain types of electronic apparatus draw a sudden rush of current when switched on, but take a much lower current when the apparatus has 'settled down': in this case, a special **anti-surge fuse** is used. This type of fuse tolerates a large rush of current for a few seconds, after which it protects the circuit normally.

5.2 CALCULATION OF ELECTRICAL ENERGY

The basic unit of electrical energy is the joule, and *one joule of energy is consumed when one watt of power is absorbed for a time of one second.* That is

$$\text{energy, } W = \text{power (watts)} \times \text{time (seconds) J}$$

The joule or watt-second is a very small unit of energy, and the unit used to measure energy in domestic and industrial situations is the **kilowatt-hour** (kWh). The type of meter used to measure energy using this unit is the *kilowatt-hour meter*, and is shown in Figure 4.12 of Chapter 4. The energy consumed in kWh is calculated from the equation

$$\text{energy in kilowatt-hours} = \text{power (kW)} \times \text{time (h) kWh}$$

If a 3 kW electric fire is switched on for 4 hours, the energy consumed is

$$\text{energy}, W = 3 \times 4 = 12 \text{ kWh}$$

By comparison, the energy consumed in joules is

$$\text{energy}, W = 3000 \times (4 \times 60 \times 60) = 43\,200\,000 \text{ J}$$

The relationship between joules and kilowatt hours is

$$1 \text{ kWh} = 1 \text{ (kW)} \times 1 \text{ (h)} = 1000 \text{ (W)} \times (60 \times 60) \text{ s}$$
$$= 3\,600\,000 \text{ J} = 3.6 \text{ MJ}$$

The power, P, consumed by a resistor R which carries a current of I ampere is given by

$$\text{power}, P = I^2R = \frac{V^2}{R} = VI \text{ W}$$

where V is the voltage across the resistor. The energy, W, consumed in joules by the resistor in t seconds is

$$\text{energy}, W = I^2Rt = \frac{V^2t}{R} = VIt \text{ J}$$

Also, the power consumed in kW in the resistor is

$$\text{power}, P = \frac{I^2R}{1000} = \frac{V^2}{1000R} = \frac{VI}{1000} \text{ kW}$$

and the energy consumed in kWh in T hours is

$$W = \frac{I^2RT}{1000} = \frac{V^2T}{1000R} = \frac{VIT}{1000} \text{ kWh}$$

Example

Calculate the amount of energy consumed in (i) joules, (ii) kWh by a 10-Ω resistor which is connected to a 9-V battery for 20 seconds.

Solution:

$$R = 10\ \Omega, E = 9 \text{ V}, t = 20 \text{ s}$$

(i) Energy in joules $= E^2tR = 9^2 \times \frac{20}{10} = 162 \text{ J (Ans.)}$

(ii) Energy in kWh $= \dfrac{\text{joules}}{3.6 \times 10^6}$

$= 45 \times 10^{-6}$ kWh or units of electricity.

Example

Calculate the amount of energy consumed (i) in joules, (ii) in kWh by a 100-W electric lamp which is switched on for 5 hours.

Solution

$$P = 100\ \text{W},\ T = 5\ \text{h} = 5 \times 3600\ \text{s} = 18\,000\ \text{s}$$

(i) Energy in joules $=$ power (W) $\times$ time (s)

$$= 100 \times 18\,000 = 1\,800\,000\ \text{J}$$

(ii) Energy in kWh $=$ power (kW) $\times$ time (h)

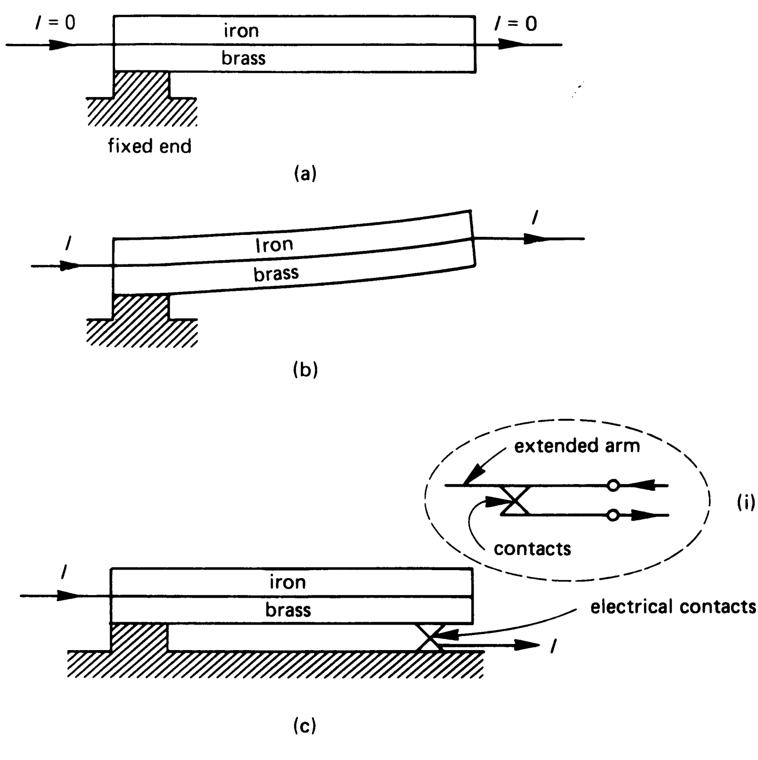
$$= \frac{100}{(1000)} \times 5 = 0.5\ \text{kWh or units of electricity}$$

5.3 APPLICATIONS OF HEATING EFFECTS

Many domestic and industrial applications depend on electro-heat for their operation. The simplest of the devices in this category is the **bimetallic strip** consisting of two metals which are in close contact with one another, but have different coefficients of expansion when heated (see Figure 5.1). When the strip is cold, both metals are the same length (see Figure 5.1(a)). When current passes through the strip both metals heat up but, because of their differing coefficients of expansion, the brass expands more rapidly than the iron (Figure 5.1(b)). Since one end of the strip is secured, the 'free' end bends upwards. This principle has a number of applications.

One application is in the form of a **thermal overcurrent trip** as shown in Figure 5.1(c). If the current drawn by, say, a motor passes through the bimetallic strip (or, more likely, through a heating coil wrapped round the strip), the effect of the load current is to heat up the strip. Under normal conditions of electrical load, the strip does not get hot enough to bend and break the contacts at the free end of the strip. However, when the motor is subject to a heavy overload, the heavy current heats the strip sufficiently to cause the strip to bend upwards and break the contacts; this cuts off the current to the overloaded motor, and prevents it from being damaged. A practical current overload trip should, of course, have a means of 'latching' the contacts open until they are 'reset' (closed) by a qualified engineer.

Alternatively, the bimetallic strip can be used as a temperature-measuring element in a central-heating control system in a house, that is it can be used as a **thermostat**. In this case the bimetallic strip is not heated by an electrical current, but by the room temperature (or boiler temperature). When the room (or boiler) is cool, the bimetallic strip is fairly straight and the contacts shown in inset (i) are closed. The current I_1 flowing through fig 5.1 *bimetallic strip: (a) when cold, (b) when hot, (c) typical applications*



the contacts operates the central heating system and, so long as the contacts are closed, the boiler heats the system up. However, as the room heats up the bimetallic strip begins to bend upwards. When the room reaches the required temperature, the free end of the strip breaks the contacts and cuts off current I_1; this has the effect of turning the central heating off. When the room cools down, the bimetallic strip straightens out and the contacts close again; at this point the central heating system turns on again.

A thermostat has a manual control on it, the function of this control being to alter the separation between the strip and the contacts (a larger gap gives a higher room temperature, and a smaller gap giving a lower temperature).

5.4 ELECTRICITY SUPPLY TARIFFS

The way in which any user of electricity is charged for electricity depends on a number of factors including:

1. the type of consumer they are, for example, whether the electricity is consumed in a house, a factory, a farm, or a place of worship, etc.;
2. the time of the day during which electricity is consumed, for example, all day, or part of the day, or at night, etc.

The simplest form of **electricity tariff** is one where the consumer is charged a fixed amount for each unit of electrical energy (kWh) consumed. Unfortunately, this type of tariff penalises the consumer who uses large amounts of electricity (and is therefore a good customer). The most popular type of tariff for domestic and small industrial consumers is the **two-part tariff**; the two parts to the tariff are

Part 1: this is related to the consumer's contribution to the **standing charge** (see below) of the production of electricity

Part 2: this is related to the consumer's contribution to the **running charges** (see below) of the production of electricity.

Standing charges

These charges are independent of the actual cost of producing electricity, and include the money needed to build the power station and supply lines, together with the rent, rates, telephone bills, etc, and the salaries of the administrative staff. It also includes an amount of money to cover the depreciation of the plant, so that it can be renewed at some later date.

Running charges

These charges are directly related to the electrical output of the power station and include items of money for such things as fuel, stores, repairs, energy loss in the transmission system, operatives wages, etc.

5.5 MAXIMUM DEMAND

The 'maximum demand' that a consumer places on the electricity supply is of interest to the generating authority, since this influences the size of the generating plant he has to install. In general, the maximum demand (MD) is mainly of interest to large industrial consumers, and is the highest value of volt-amperes consumed at the premesis. You should note that whilst the volt-ampere (VA) product in a d.c. circuit is equal to the electrical power consumed in watts, it is not necessarily equal to the power consumed in an a.c. circuit (for details see the chapters on a.c. circuits); for this reason we refer to it as the VA product rather than the power consumed.

Clearly, a consumer who has a high MD consumes a large amount of electricity. It is usually the case that the supply tariff arranges for a large consumer of electricity to pay progressively less for each kilo-volt-ampere (kVA or 1000 VA) or maximum demand he consumes. This is illustrated in section 5.6.

5.6 A TYPICAL SUPPLY TARIFF

The equation of the electricity supply tariff for a domestic or small industrial premises can be written in the form

Cost = £*X* per month (or quarter, or annum) as a standing charge
+ *Y* pence per unit of electricity

For example, a typical domestic tariff may be

a standing charge per quarter of £6.50
a cost per unit of electrical energy of 6.5p

For a large industrial consumer the tariff may take the form

Cost = £*L* per month as a standing charge

+ £*M* per kVA of maximum demand (MD) for the first, say, 100 kVA

+ £*N* per kVA of MD for the next 200 kVA, etc.

A typical industrial tariff may be

a standing charge per month of £20
an additional charge for each kVA of MD
for the first 100 kVA of £1.00
for the next 200 kVA of £0.75
for each additional kVA of £0.50

The industrial consumer may also have to pay an additional 'meter' charge (depending on the voltage at which the energy is metered). He may have to pay a 'fuel cost' charge, which depends on the cost of the fuel purchased by the supply authority.

5.7 ELECTRICITY BILLS

The foregoing indicates that the calculation of the cost of electrical energy can sometimes be complicated. In the following an example involving a domestic installation is calculated.

Example

A family consume 800 units of electrical energy per quarter. If the supply tariff consists of a standing charge of £6.50 and a cost per unit of 6.5p, calculate the cost of electrical energy during the quarter.

Solution

The cost of the electrical energy actually consumed is

$$\frac{\pounds(\text{number of units} \times \text{cost in pence per unit})}{100}$$

$$= \frac{\pounds 800 \times 6.5}{100} = \pounds 52.00$$

The final bill is

$$\begin{aligned}\text{Cost per quarter} &= \pounds(\text{standing charge} + \text{cost of electricity}) \\ &= \pounds(6.50 + 52.00) = \pounds 58.50\end{aligned}$$

SELF-TEST QUESTIONS

1. Calculate the energy consumed in MJ and kWh when a current of 20 A flows in a resistor of 100 Ω for 1 hour.
2. Describe the meaning of (i) a fuse, (ii) a high rupturing-capacity fuse, (iii) an anti-surge fuse.
3. Explain what is meant by a two-part electricity tariff, and state the reason for each 'part' of the tariff.
4. Why do industrial consumers need to pay a 'maximum demand' charge?

SUMMARY OF IMPORTANT FACTS

Electrical equipment can be protected by a **fuse** which, in its simplest form, is a wire which melts when a current in excess of its rated current flows through it for a short length of time.

The unit of **electrical energy** (power × time) is the joule, but a more practical unit is the **kilowatt-hour** (kWh).

An electricity **tariff** is the way in which you are charged for electricity. Tariffs are many and varied, one of the most popular being the **two-part tariff**. One part of this tariff relates to the **standing charges** associated with production of electricity, the second part relates to the **running charges**. Many industrial consumers have to pay a **maximum demand** charge.

CHAPTER 6

ELECTROSTATICS

6.1 FRICTIONAL ELECTRICITY

Many hundreds of years BC it was discovered that when amber was rubbed with fur, the amber acquired the property of being able to attract other objects. The reason for this was not understood until man knew more about the structure of matter. What happens when, for example, amber and fur are rubbed together is that electrons transfer from one to the other, with the result that the charge neutrality of the two substances is upset. The substance which gains electrons acquires a negative charge, and the one which loses electrons acquires a positive charge.

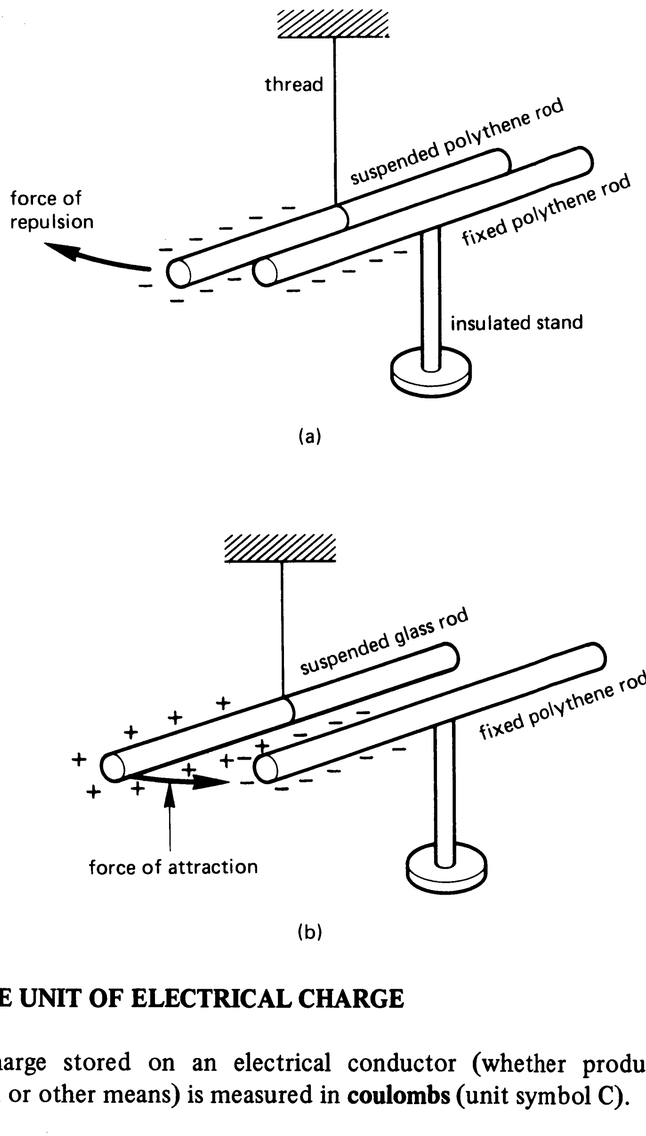
It was also found that the effects of the charge produced by friction can only be observed on a good insulating material, and that the charge itself can only be sustained in dry conditions. These facts are in accord with our knowledge of electricity since, under damp conditions, electrical charge 'leaks' away from charged bodies. This knowledge is used today by aircraft manufacturers; when an aircraft is in flight, it can build up a high frictional charge of electricity, but using tyres containing conducting substances the charge is allowed to leak away when the aircraft lands.

If you rub a polythene rod with a woollen duster, the polythene acquires a negative charge; when a glass rod is rubbed with silk, the rod acquires a positive charge. When you have a charged rod, you can make some interesting observations as illustrated in Figure 6.1.

When two negatively charged polythene rods are brought into close proximity with one another, as in Figure 6.1(a), and if one of the rods is suspended on a thread, there is a force of repulsion between the fixed and suspended rods. However, if a positively charged glass rod is brought close to a negatively charged polythene rode - see Figure 6.1(b) - there is a force of attraction between the rods. This is summarised as follows:

Like electrical charges repel one another.
Unlike electrical charges attract one another.

fig 6.1 *electrostatics: (a) like charges repel, (b) unlike charges attract*



6.2 THE UNIT OF ELECTRICAL CHARGE

The charge stored on an electrical conductor (whether produced by friction or other means) is measured in **coulombs** (unit symbol C).

6.3 ELECTRIC FLUX

A charged electrical body is said to be surrounded by an **electric field**, and a measure of the field is its **electric flux** (symbol Q) having units of the coulomb. *One unit of electrical charge* produces *one unit of electric flux*; that is, 1 coulomb of electric charge produces 1 coulomb of flux. Quite often we think of an electric charge travelling along a *line of flux*; this 'line' is an imaginary concept which scientists have devised to simplify problems when dealing with electrostatics. The *direction of an electric field* at any point is defined as the direction of the force acting on a *unit positive charge* placed at that point. Clearly, since a positive charge experiences a force *away* from a positively charged body and *towards* a negatively charged body, we can say that

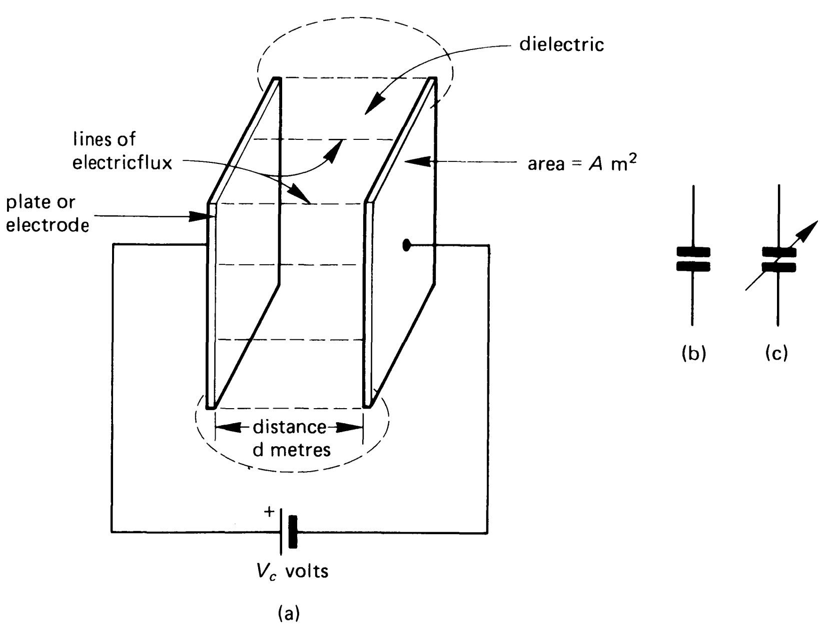
a 'line' of electric force starts on a positive charge and ends on a negative charge.

However, you must remember that a line of electric force is imaginary, and you can only detect it by its effect on other bodies.

6.4 A PARALLEL-PLATE CAPACITOR

A parallel-plate capacitor consists of two parallel metal **plates** or **electrodes** (see Figure 6.2) separated by an insulating material or **dielectric** (in Figure 6.2 it is air). When a potential difference is applied between the plates, an **electric field** is established in the dielectric and lines of electric flux link the plates.

fig 6.2 *(a) parallel-plate capacitor with an air dielectric; (b) symbol for a 'fixed' capacitor and (c) a 'variable' capacitor*



The capacitor in Figure 6.2 is known as a **fixed capacitor** because its geometry is 'fixed', that is, the area of the plates and the distance between the plates cannot be altered. If it is possible to vary either the area of the plates, or the distance between them, or the nature of the dielectric, then the capacitor is known as a **variable capacitor**. Circuit symbols for fixed and variable capacitor are shown in diagrams (b) and (c) of Figure 6.2.

One very useful feature of a capacitor is that it can **store electrical energy**. If, for example, the battery were to be disconnected from the capacitor in Figure 6.2, the capacitor would retain its stored charge for a considerable period of time. The reader should note, however, that *the energy is stored in the dielectric material of the capacitor* and not in the plates.

6.5 POTENTIAL GRADIENT OR ELECTRIC FIELD INTENSITY

The **potential gradient** or **electric field intensity**, symbol E, in volts per metre in the *dielectric* is given by

$$\text{electric field intensity}, E = \frac{\text{thickness of dielectric} \ (m)}{\text{p.d. across dielectric} \ (V)}$$

In the case of Figure 6.2 this is given by the equation

$$E = \frac{V_C}{d} \text{ V/m}$$

Even in low voltage capacitors the electric field intensity can be very high.

Example

Calculate the electric field intensity between the plates of a capacitor whose dielectric is 0.01 mm thick and which has a p.d. of 10 V between its plates.

Solution

$$d = 0.01 \text{ mm} = 0.01 \times 10^{-3} \text{ m}; V_C = 10 \ V$$

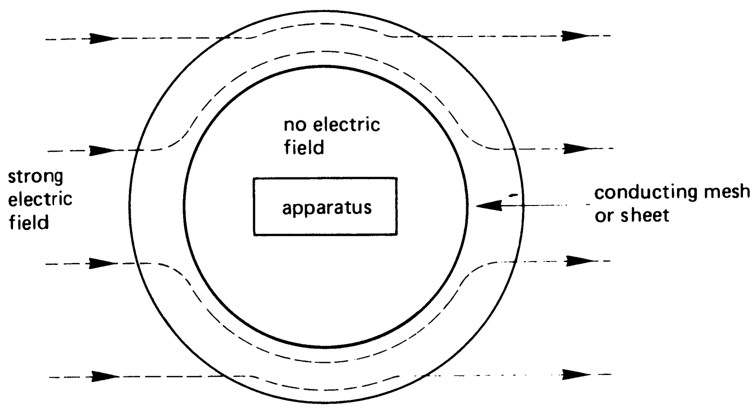
$$\text{Electric field intensity}, E = \frac{V_C}{d} = \frac{10 \ V}{(0.01 \times 10^{-3})}$$

$$= 1\,000\,000 \text{ V/m (Ans.)}$$

6.6 ELECTROSTATIC SCREENING

Many items of electrical plant are sensitive to strong electric fields. To protect the apparatus, it is necessary to enclose it within a conducting mesh or sheath as shown in Figure 6.3. A special cage made from wire mesh designed to protect not only apparatus but also humans from intense electric fields is known as a **Faraday cage**. In effect, the mesh has a very low electrical resistance and prevents the electric field from penetrating it.

fig 6.3 *electrostatic screening*



6.7 UNITS OF CAPACITANCE

The **capacitance**, symbol C, of a capacitor is a measure of the ability of the capacitor to store electric charge; the basic unit of capacitance is the **farad** (unit symbol F). Unfortunately the farad is an impractically large unit, and the following submultiples are commonly used

1 microfarad = 1 μF = 1 x 10^{-6} F = 0.000 001 F
1 nanofarad = 1 nF = 1 x 10^{-9} F = 0.000 000 001 F
1 picofarad = 1 pF = 1 x 10^{-12} F = 0.000 000 000 001 F

6.8 CHARGE STORED BY A CAPACITOR

Experiments with capacitors show that the electric charge, Q coulombs, stored by a capacitor of capacitance C farads is given by

stored charge, $Q = CV_C$ coulombs

where V_C is the voltage across the capacitor.

Example
Calculate the charge stored by a capacitor of capacitance (i) 10 μF, (ii) 100 pF having a voltage of 10 V between its terminals.

Solution

(i) $C = 10\ \mu\text{F} = 10 \times 10^{-6}$ F

$$Q = CV_C = (10 \times 10^{-6}) \times 10 = 100 \times 10^{-6}\ C$$

$$= 100\ \mu C\ \text{(Ans.)}$$

(ii) $C = 100$ pF $= 100 \times 10^{-12}$ F

$$Q = CV_C = (100 \times 10^{-12}) \times 10 = 1000 \times 10^{-12}\ C$$

$$= 1000\ \text{p}C \text{ or } 1\ \text{n}C$$

6.9 ENERGY STORED IN A CAPACITOR

During the time that the electric field is being established in the dielectric, energy is being stored. This energy is available for release at a later time when the electric field is reduced in value. The equation for the energy, W joules, stored in a capacitor of capacitance C farads which is charged to V_C volts is

$$\text{energy stored, } W = \frac{CV_C^2}{2} \text{ joules (J)}$$

Example
Calculate the energy stored by a capacitor of 10 μF capacitance which is charged to 15 V.

Solution

$$C = 10\ \mu\text{F} = 10 \times 10^{-6}\ \text{F};\ V_C = 15\ \text{V}$$

$$\text{Energy stored, } W = \frac{CV_C^2}{2} = 10 \times 10^{-6} \times \frac{15^2}{2}$$

$$= 1.\dot{2}5 \times 10^{-3}\ \text{J or } 1.125\ \text{mJ (Ans.)}$$

6.10 ELECTRIC FLUX DENSITY

Even though electric flux is an imaginary concept, it has some validity in that its effects can be measured. The electric flux emanates from an electrically charged body, and we can determine the **electric flux density**, symbol D, which is the amount of electric flux passing through a unit area, that is, it is the amount of electric flux passing through an area of 1 m^2.

If, for example, Q lines of electric flux pass through a dielectric of area A m^2, the electric flux density in the dielectric is

$$D = \frac{Q}{A} \text{ coulombs per metre}^2 \text{ (C/m}^2\text{)}$$

6.11 PERMITTIVITY OF A DIELECTRIC

The **permittivity**, symbol ϵ, of a dielectric is a measure of the ability of the dielectric to concentrate electric flux. If, for example, a parallel-plate capacitor with air as its dielectric has a flux density in its dielectric of Y C/m^2 then, when the air is replaced by (say) a mica dielectric, the flux density in the dielectric is found to increase to a value in the range $3Y$-$7Y$ C/m^2. That is to say, the mica dielectric produces a higher concentration of electric flux than does the air.

The net result is that a capacitor with a mica dielectric can store more energy than can an air dielectric capacitor. Alternatively, for the same energy storage capacitory, the mica dielectric capacitor is physically smaller than the air dielectric capacitor.

The basic dielectric with which all comparisons are made is a vacuum but, since dry air has similar features, we can regard air as a 'reference' dielectric. Experiments show that the relationship between the electric flux density D and the electric field intensity E (or potential gradient) in a dielectric is given by

$$D = E\epsilon \text{ coulombs per square metre}$$

where ϵ is the **absolute permittivity** of the dielectric. The permittivity of a vacuum (known as the **permittivity of free space**) is given the special symbol ϵ_O and has the dimensions farads per metre (F/m). Its value is

$$\epsilon_O = 8.85 \times 10^{-12} \text{ (F/m)}$$

To all intents and purposes we may regard this as the *permittivity of dry air*.

When air is replaced as the dielectric by some other insulator, the electric flux density is magnified by a factor known as the **relative permittivity**, ϵ_r. The relative permittivity is simply a dimensionless multiplying factor; for most practical dielectrics its value lies in the range 2-7, but may have a value of several hundred in a few cases. Typical values are given in Table 6.1. The absolute permittivity of a material is given by the expression

$$\epsilon = \text{permittivity of free space} \times \text{relative permittivity}$$

$$= \epsilon_O \epsilon_r \text{ F/m}$$

Table 6.1 *Relative permittivity of various materials*

Material	*Relative permittivity*
Air	1.0006
Bakelite	4.5-5.5
Glass	5-10
Mica	3-7
Paper (dry)	2-2.5
Rubber	2-3.5

6.12 CAPACITANCE OF A PARALLEL-PLATE CAPACITOR

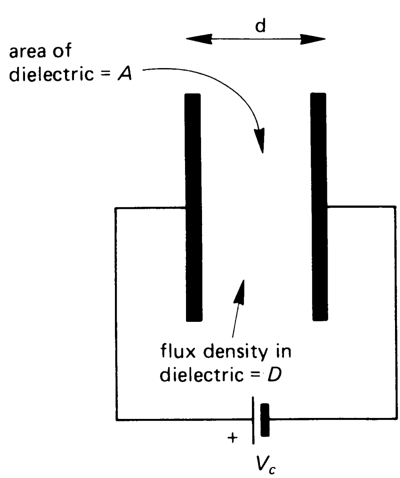
The potential gradient or electric field intensity, E V/m, in the dielectric of the parallel-plate capacitor in Figure 6.4 is given by

$$E = \frac{V_C}{d} \text{ V/m}$$

where V_C is the p.d. between the plates and d is the thickness of the dielectric. If the capacitor stores Q coulombs of electricity, then the electric flux density, D coulombs per square metre, is

$$D = \frac{Q}{A} \text{ C/m}^2$$

fig 6.4 *capacitance of a parallel-plate capacitor*



where A is the area of the dielectric (which is equivalent to the area of the plate). From section 6.11, the relationship between E and D in the dielectric is

$$D = E\epsilon \ \ \mathrm{C/m^2}$$

where ϵ is the absolute permittivity ($\epsilon = \epsilon_O \epsilon_r$) of the dielectric. Hence

$$\frac{Q}{A} = \frac{V_C \epsilon}{d}$$

But the charge stored, Q coulombs, is given by $Q = CV_C$, where C is the **capacitance** of the capacitor (in farads). Therefore

$$\frac{CV_C}{A} = \frac{V_C \epsilon}{d}$$

Cancelling V_C on both sides and multiplying both sides of the equation by A gives the following expression for the capacitance, C, of the parallel-plate capacitor:

$$C = \frac{A\epsilon}{d} = \frac{A\epsilon_O\epsilon_r}{d} \ \text{farads (F)}$$

where A is in m^2, ϵ is in F/m, and d is in m.

Example

A parallel-plate capacitor has plates of area 600 cm^2 which are separated by a dielectric of thickness 1.2 mm whose relative permittivity is 5. The voltage between the plates is 100 V. Determine (a) the capacitance of the capacitor in picofarads, (b) the charge stored in microcoulombs, (c) the electric field intensity in the dielectric in kV/m and (d) the electric flux density in the dielectric in microcoulombs per square metre.

Solution

$A = 600\ \mathrm{cm^2} = 600 \times (10^{-2})^2\ \mathrm{m^2}; d = 1.2\ \mathrm{mm} = 1.2 \times 10^{-3}\ \mathrm{m};$
$\epsilon_r = 5; V_C = 100$

$$\text{(a) Capacitance, } C = \epsilon_O\epsilon_r \frac{A}{d}$$

$$= \frac{(8.85 \times 10^{-12}) \times 5 \times (600 \times 10^{-4})}{1.5 \times 10^{-3}}$$

$$= 1.77 \times 10^{-9}\ \text{F or } 1.77\ \text{pF (Ans.)}$$

(b) Charge stored, $Q = CV_C = (1.77 \times 10^{-9}) \times 100$

$$= 1.77 \times 10^{-7}\ \mathrm{C}$$

$$= 0.177 \times 10^{-6}\ \mathrm{C} \text{ or } 0.177\ \mu\mathrm{C} \text{ (Ans.)}$$

Note

The reader should carefully note the difference here between the use of the symbol C (in italics) to represent the capacitance of the capacitor, and the symbol C (in roman type) to represent the unit of charge (the coulomb).

(c) Electric field intensity $= \dfrac{V_C}{d} = \dfrac{100}{1.2 \times 10^{-3}}$

$$= 83\,333\ \mathrm{V/m} \text{ or } 83.333\ \mathrm{kV/m} \text{ (Ans.)}$$

(d) Electric flux density, $D = \dfrac{Q}{A}$

$$= \frac{1.77 \times 10^{-7}}{(600 \times 10^{-4})}$$

$$= 2.95 \times 10^{-6}\ \mathrm{C/m^2}$$

or $2.95\ \mu\mathrm{C/m^2}$ (Ans.)

6.13 APPLICATIONS OF CAPACITORS

One of the most obvious applications of capacitors is as energy storage elements. However, any capacitor which can store a reasonable amount of energy would be very large indeed and, if only for this reason, it has a very limited use for this type of application.

An application of the capacitor in a number of electrical and electronic circuits is as a 'd.c. blocking' device, which prevents direct current from flowing from one circuit to another circuit. It can be used in this application because the dielectric is an insulator which, once the capacitor is fully 'charged', prevents further direct current from flowing through it.

Capacitors are also widely used in alternating current circuits (see Chapters 11 and 12), and discussion on these applications is dealt with in the specialised chapters.

They are also used in voltage 'multiplying' circuits, in which a voltage much higher than the supply voltage is produced using special circuits using capacitors. An everyday example of this is the electronic flashgun used with a camera; in this case an electronic circuit 'pumps up' the voltage across a 'string' of capacitors, the stored energy being finally discharged into the flash lamp. In industry, capacitors are used to produce voltages of several million volts by this means; the resulting voltage is used to test high voltage apparatus.

Yet another application is the use of a capacitor as a **transducer** or **sensor**. A transducer or sensor is a device which converts energy of one kind to energy of another kind; it can be used, for example, to translate linear movement into a change in voltage. That is, a transducer can be used to *sense a change* in some characteristic. The equation for the capacitance of a parallel-plate capacitor gives us an indication of the way in which the capacitor can be used as a sensor. The equation is

$$C = \frac{\epsilon A}{d}$$

From this equation, the capacitance of the capacitor is

1. proportional to the permittivity ϵ of the dielectric. Any change in the permittivity produces a proportional change in C;
2. proportional to the area A of the dielectric. Any change in this area gives rise to a proportional change in C;
3. inversely proportional to the displacement d between the plates. An increase in d reduces the capacitance, and a reduction in d increases the capacitance.

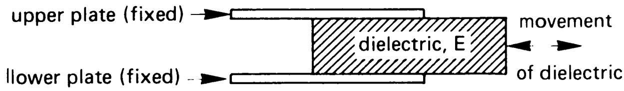
For example, you can detect the movement of, say, the end of a shaft or beam by mechanically linking the shaft or beam so that it changes one of the factors ϵ, A or d. Several basic techniques are shown in Fiure 6.5.

In Figure 6.5(a), the dielectric material is moved either further into or out of the capacitor to change the average permittivity of the capacitor. In Figure 6.5(b), one plate is moved relative to the other plate to alter the total area of the capacitor. In Figure 6.5(c), one plate is moved either towards or away from the other plate to change d. The net effect of any one of these changes is to change the capacitance of the capacitor, the change in capacitance being related to the displacement.

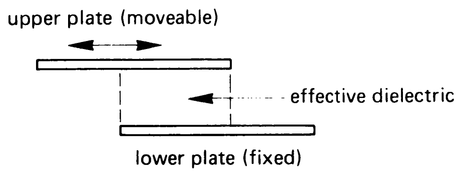
The change in capacitance produced by any of these methods is usually only a few tens of picofarads but, none the less, the change can be measured electrically.

A capacitor can also be used, for example, to measure fluid pressure or flow simply by allowing the pressure produced by a set of bellows to displace a diaphragm. The movement of the diaphragm can be linked to a capacitor to produce one of the changes in the capacitor illustrated in Figure 6.5. The resulting change in capacitance is related in some way to the pressure or flow being measured.

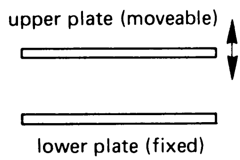
fig 6.5 *the parallel-plate capacitor as a transducer*



(a)



(b)

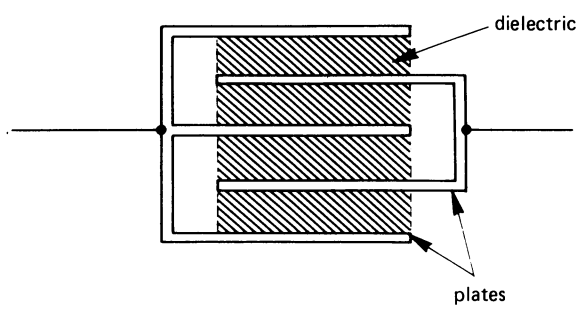


(c)

6.14 MULTI-PLATE CAPACITORS

One method of increasing the capacitance of a capacitor is to increase the area of the dielectric (remember – the energy is stored in the dielectric in a capacitor). A popular method of doing this is to use a multi-plate capacitor of the type shown in Figure 6.6.

fig 6.6 *a multiple-plate capacitor*



A capacitor having n plates has $(n - 1)$ dielectrics; the capacitor in Figure 6.6 has five plates, but only four dielectrics separating the plates. This fact causes the equation for the capacitance of an n-plate capacitor to be

$$C = \frac{(n-1)\epsilon A}{d} \text{ F}$$

6.15 CAPACITORS IN SERIES

When several capacitors are connected in series as shown in Figure 6.7(a), they all carry the same charging current for the same length of time. That is, the (current × time) product is the same for each capacitor; put another way, *each capacitor stores the same amount of charge* (irrespective of its capacitance).

If the three series-connected capacitors in Figure 6.7(a) are replaced by a single **equivalent capacitance**, C_S, then it stores the same amount of charge (say Q coulombs) as the series circuit which to it is equivalent.

If capacitor C_1 stores a charge Q_1, capacitor C_2 stores Q_2, etc, and the equivalent series capacitor C_S stores Q then, since the capacitors are in series and each carries the same charge

$$Q = Q_1 = Q_2 = Q_3$$

Now, for any capacitor

stored charge = capacitance × voltage across the capacitor

then the charge stored by C_1 is $C_1 V_1$, the charge stored by C_2 is $C_2 V_2$, etc, and the charge stored by C_S is $C_S V_S$, hence

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = C_S V_S$$

that is

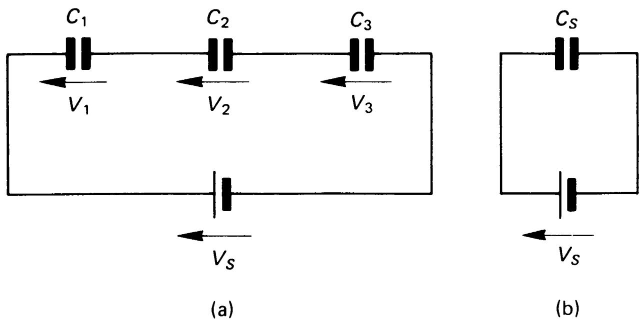
$$V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

$$V_3 = \frac{Q}{C_3}$$

$$V_S = \frac{Q}{C_S}$$

Now, for the series circuit in Figure 6.7(a) fig 6.7 *(a) capacitors in series and (b) their electrical equivalent capacitance*



$$V_S = V_1 + V_2 + V_3$$

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Cancelling Q on both sides of the above equation gives

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

That is, *the reciprocal of the equivalent capacitance of a series connected group of capacitors is the sum of the reciprocals of their respective capacitances.*

It is of interest to note that the *equivalent capacitance, C_S, of the series circuit is less than the smallest individual capacitance in the circuit* (see the example below).

The special case of two capacitors in series

In this case it can be shown that the equation for the equivalent capacitance of the circuit is

$$C_S = \frac{C_1 C_2}{C_1 + C_2}$$

Example

Calculate the equivalent capacitance of three series-connected capacitors having capacitances of 10, 20 and 40 microfarads, respectively.

Solution

$$C_1 = 10\ \mu\text{F}; C_2 = 20\ \mu F; C_3 = 40\ \mu\text{F}$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40}\ \mu F^{-1}$$

$$= 0.1 + 0.05 + 0.025 = 0.175\ \mu F^{-1}$$

hence

$$C_S = \frac{1}{0.175} = 5.714\ \mu F\ (Ans.)$$

Note
The value of C_S is **less than** the smallest capacitance (10 μF) in the circuit.

6.16 CAPACITORS IN PARALLEL

When capacitors are in parallel with one another (Figure 6.8(a)), they have the same voltage across them. The charge stored by the capacitors in the

fig 6.8 *(a) capacitors in parallel and (b) their electrical equivalent capacitance*

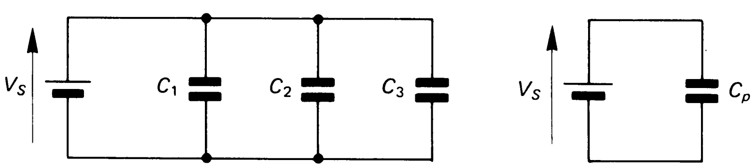


figure is, therefore, as follows

$$Q_1 = C_1 V_S$$

$$Q_2 = C_1 V_S$$

$$Q_3 = C_3 V_S$$

The total charge stored by the circuit is

$$Q_1 + Q_2 + Q_3 = C_1 V_S + C_2 V_S + C_3 V_S = V_S(C_1 + C_2 + C_3)$$

The parallel bank of capacitors in diagram (a) can be replaced by the single equivalent capacitor C_P in diagram (b). Since the supply voltage V_S is applied to C_P, the charge Q_P stored by the equivalent capacitance is

$$Q_P = V_S C_P$$

For the capacitor in diagram (b) to be equivalent to the parallel combination in diagram (a), both circuits must store the same charge when connected to V_S. That is

$$Q_P = Q_1 + Q_2 + Q_3$$

or

$$V_S C_P = V_S(C_1 + C_2 + C_3)$$

when V_S is cancelled on both sides of the equation above, the expression for the capacitance C_P is

$$C_P = C_1 + C_2 + C_3$$

The equivalent capacitance of a parallel connected bank of capacitors is equal to the sum of the capacitances of the individual capacitors. It is of interest to note that *the equivalent capacitance of a parallel connected bank of capacitors is greater than the largest value of capacitance in the parallel circuit.*

Example
Calculate the equivalent capacitance of three parallel-connected capacitors of capacitance 10, 20 and 40 microfarads, respectively.

Solution

$$C_1 = 10\ \mu\text{F};\ C_2 = 20\ \mu\text{F};\ C_3 = 40\ \mu\text{F}$$

equivalent capacitance, $C_P = C_1 + C_2 + C_3 = 10 + 20 + 40$

$$= 70\ \mu\text{F (Ans.)}$$

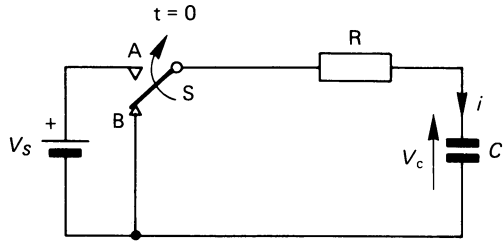
Note
The value of C_P is **greater** than the largest capacitance (40 μF) in the circuit.

6.17 CAPACITOR CHARGING CURRENT

In this section of the book we will investigate what happens to the current in a capacitor which is being charged and what happens to the voltage across the capacitor.

The basic circuit is shown in Figure 6.9. Initially, the blade of switch S is connected to contact B, so that the capacitor is discharged; that is the voltage V_C across the capacitor is zero.

You will observe that we now use the *lower-case* letter v rather than the upper case letter V to describe the voltage across the capacitor. The reason fig 6.9 *charging a capacitor. The blade of switch S is changed from position B to position A at time t = 0*



is as follows. Capital letters are used to describe either d.c. values or 'effective a.c.' values (see chapter 10 for details of the meaning of the latter phrase) in a circuit. Lower case (small) letters are used to describe **instantaneous values**, that is, values which may change with time. In this case the capacitor is initially discharged so that at 'zero' time, that is, $t = 0$, we can say that $v_C = 0$. As you will see below, a little time after switch S is closed the capacitor will be, say, half fully-charged (that is $v_C = \frac{V_S}{2}$). As time progresses, the voltage across C rises further. Thus, v_C changes with time and has a different value at each instant of time. Similarly, we will see that the charging current, i, also varies in value with time. We will now return to the description of the operation of the circuit.

When the contact of switch S is changed to position A at time $t = 0$, current begins to flow into the capacitor. Since the voltage across the capacitor is zero at this point in time, the **initial value** of the **charging current** is

$$i = \frac{\text{supply voltage} - \text{voltage across the capacitor}}{\text{circuit resistance}}$$

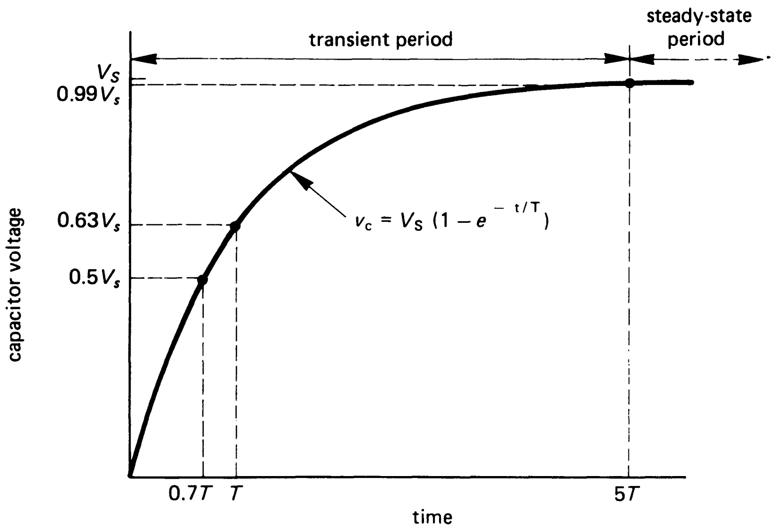
$$= \frac{(V_S - 0)}{R} = \frac{V_S}{R}$$

Let us call this value i_0 since it is the current at 'zero time'. As the current flows into the capacitor, it begins to acquire electric charge and the voltage across it builds up in the manner shown in Figure 6.10(a).

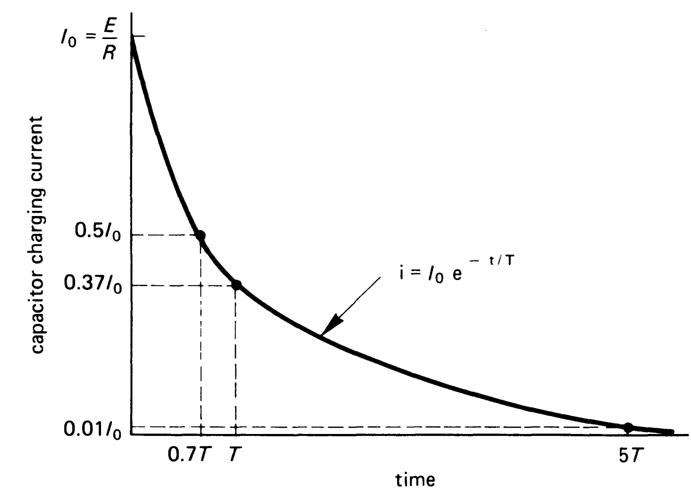
Just after the switch is closed and for a time less than $5T$ (see Figure 6.10), the current through the circuit and the voltage across each element in the circuit change. This period of time is known as the **transient period** of operation of the circuit. During the transient period of time, the voltage v_C across the capacitor is given by the mathematical expression

$$v_C = V_S\,(1 - e^{-t/T}) \text{ volts}$$

fig 6.10 *capacitor charging curves for (a) capacitor voltage, (b) capacitor current*



(a)



(b)

where v_C is the voltage across the capacitor at time t seconds after the switch has been closed, V_S is the supply voltage, T is the *time constant* of the circuit (see section 6.18 for details), and e is the number 2.71828 which is the base of the natural logarithmic series.

For example, if the time constant of an *RC* circuit is 8 seconds, the voltage across the capacitor 10 seconds after the supply of 10 V has been connected is calculated as follows

$$v_C = 10(1 - e^{-10/8}) = 10(1 - e^{-1.125})$$
$$= 10(1 - 0.287) = 7.13 \text{ V}$$

The curve in Figure 6.10(a) is described as an **exponentially rising curve**.

During the transient period, the mathematical expression for the transient current, i, in the circuit is

$$i = I_O e^{-t/T}$$

where I_O is the initial value of the current and has the value

$$I_O = \frac{V_S}{R} \text{ A}$$

The curve in Figure 6.10(b) is known as an **exponentially falling curve.**

After a time equal to $5T$ seconds ($5T = 5 \times 8 = 40$ seconds in the above example) the transients in the circuit 'settle down', and the current and the voltages across the elements in the circuit reach a steady value. The time period beyond the transient time is known as the **steady-state period.**

As mentioned above, T is the *time constant* of the circuit, and it can be shown that after a length of time equal to one time constant, the voltage across the capacitor has risen to 63 per cent of the supply voltage, that is $v_C = 0.63\ V_S$. The charging current at this instant of time is

$$i = \frac{(V_S - \text{voltage across the capacitor})}{R}$$

$$= \frac{(V_S - 0.63\ V_S)}{R} = \frac{0.37\ V_S}{R} = 0.37 I_O$$

This is illustrated in Figure 6.10. That is, as the capacitor is charged, the voltage across it rises and the charging current falls in value.

On completion of the transient period, the voltage across the capacitor has risen practically to V_S, that is the capacitor is 'fully charged' to voltage V_S. At this point in time the current in the circuit has fallen to

$$i = \frac{(V_S - \text{voltage across } C)}{R} \simeq \frac{(V_S - V_S)}{R} = 0$$

Thus, when the capacitor is fully charged, **it no longer draws current from the supply.**

6.18 THE TIME CONSTANT OF AN *RC* CIRCUIT

For a circuit containing a resistor R and a capacitor C, the **time constant**, T, is calculated from

$$T = RC \text{ seconds}$$

where R is in ohms and C is in farads. For example, if $R = 2000\ \Omega$ and $C = 10\ \mu F$, then

$$T = RC = 2000 \times (10 \times 10^{-6}) = 0.02 \text{ s or } 20 \text{ ms}$$

If the supply voltage is 10 V, it takes 0.02 s for the capacitor to charge to 0.63 VS = 0.63 x 10 = 6.3 V.

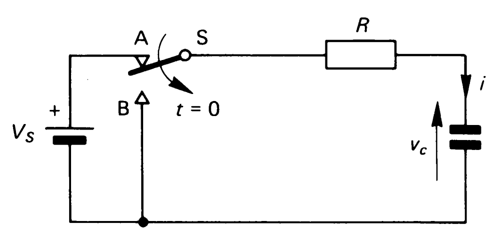
The time taken for the transients in the circuit to vanish and for the circuit to settle to its steady-state condition (see Figure 6.10 is about $5T$ seconds. This length of time is referred to as the **settling time**. In the above case, the settling time is 5 x 0.02 = 0.1 s.

6.19 CAPACITOR DISCHARGE

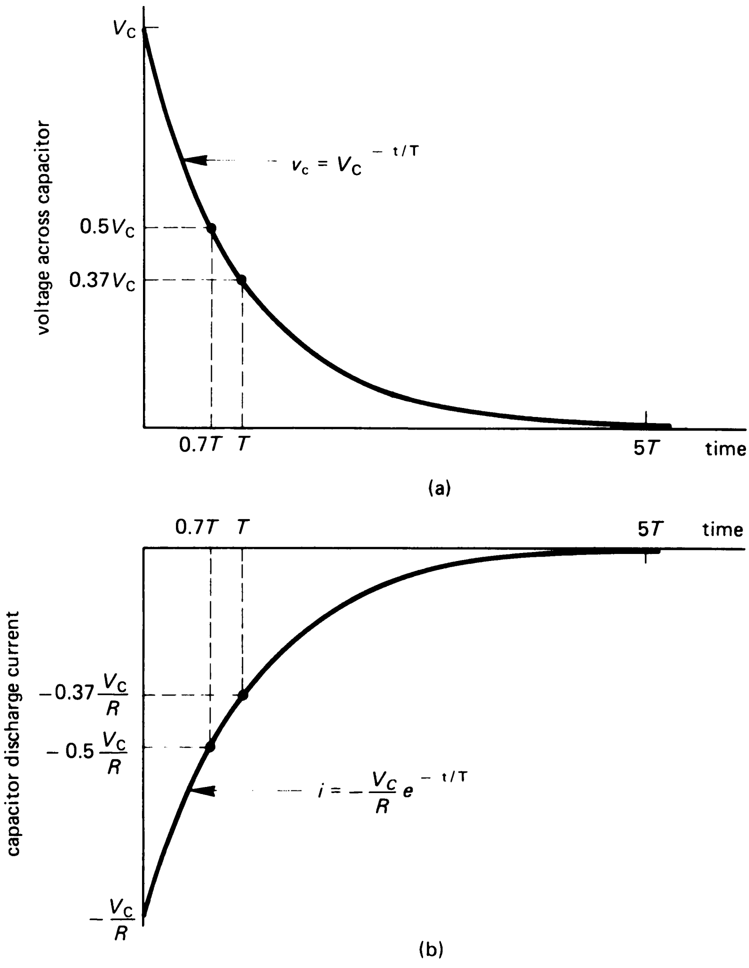
While the contact of switch S in Figure 6.11 is in position A, the capacitor is charged by the cell. When the contact S is changed from A to B, the capacitor is discharged via resistor R.

Whilst the capacitor discharges current through resistor R, energy is extracted from the capacitor so that the voltage v_C across the capacitor gradually decays towards zero value. When discharging energy, current flows out of the positive plate (the upper plate in Figure 6.11); that is, the current in Figure 6.11 flows in the reverse direction when compared with the charging condition (Figure 6.9).

fig 6.11 *capacitor discharge. The blade of switch S is changed from A to B at time t = 0*



The graph in Figure 6.12(a) shows how the capacitor voltage decays with time. The graph in diagram (b) shows how the discharge current rises to a maximum value of $\frac{-VS}{R}$ at the instant that the switch blade is moved fig 6.12 *capacitor discharge curves for (a) capacitor voltage, (b) capacitor current*



to position B (the negative sign implies that the direction of the current is reversed when compared with the charging condition); the current then decays to zero following an exponential curve.

The mathematical expression for the voltage v_C across the capacitor at time *t* ***after*** the switch blade in Figure 6.11 has been changed from A to B is

$$v_C = V_C e^{t/T} \text{ volts}$$

where V_C is the voltage to which the capacitor has been charged just before the instant that the switch blade is changed to position B. The time constant of the circuit is $T = RC$ (T in seconds, R in ohms, C in farads), and $e = 2.71828$. The expression for the discharge current is

$$i = -\frac{V_C}{R} e^{-t/T}$$

Once again, it takes approximately $5T$ seconds for the transient period of the discharge to decay, during which time the current in the circuit and the voltage across the circuit elements change. When the steady-stage period is reached, the current in the circuit and the voltage across R and C reach a steady value (zero in this case, since the capacitor has discharged its energy).

Theoretically, it takes an infinite time for the transient period to disappear but, in practice, it can be thought of as vanishing in a time of $5T$.

6.20 TYPES OF CAPACITOR

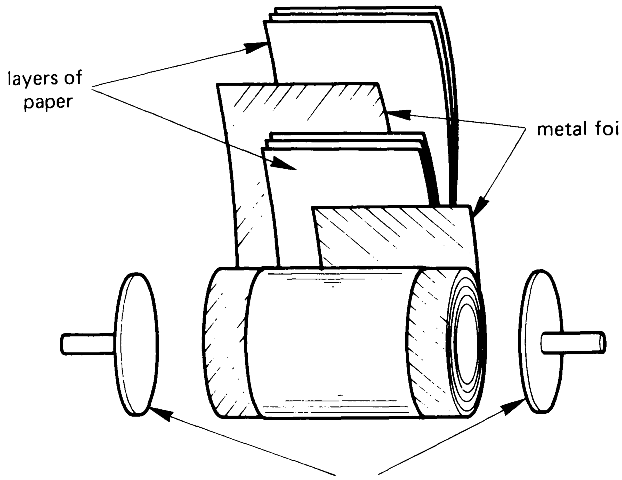
Capacitors are generally classified according to their dielectrics, for example, paper, polystyrene, mica, etc. The capacitance of all practical capacitors varies with age, operating temperature, etc, and the value quoted by the manufacturer usually only applies under specific operating conditions.

Air dielectric capacitors Fixed capacitors with air dielectrics are mainly used as laboratory standards of capacitance. Variable capacitance air capacitors have a set of fixed plates and a set of moveable plates, so that the capacitance of the capacitor is altered as the overlapping area of the plates is altered.

Paper dielectric capacitors In one form of paper capacitor, shown in Figure 6.13, the electrodes are metal foils interleaved with layers of paper which have been impregnated with oil or wax with a plastic (polymerisable) impregnant. In the form of construction shown, contact is made between the capacitor plates and the external circuit via pressure contacts.

In capacitors known as **metallised paper capacitors**, the paper is metallised so that gaps or voids between the plates and the dielectric are avoided. Important characteristics of this type when compared with other 'paper' types are their small size and their 'self-healing' action after electrical breakdown of the dielectric. In the event of the paper being punctured when a transient voltage 'spike' is applied to the terminals of the capacitor (this is a practical hazard for any capacitor), the metallising in the region of the puncture rapidly evaporates and prevents the capacitor from developing a short-circuit.

Plastic film dielectric capacitors These use plastic rather than paper dielectric and are widely used. The production techniques provide low fig 6.13 *construction of one form of tubular paper capacitor*



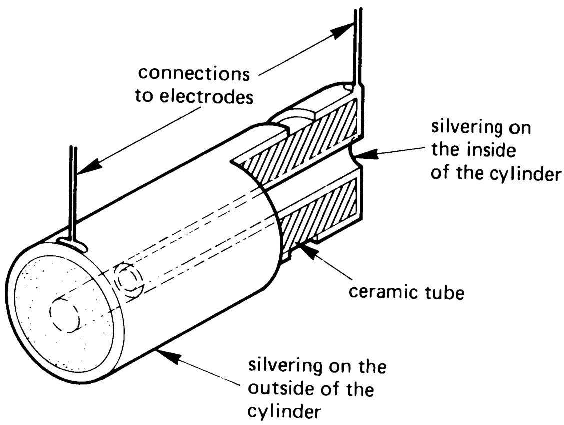
cost, high reliability capacitors. The construction is generally as for paper capacitors, typical dielectrics being polystyrene, polyester, polycarbonate and polypropylene.

Mixed dielectric capacitors Capacitors with dielectrics incorporating plastic films and impregnated paper permit the manufacture of low volume, high voltage capacitors.

Mica dielectric capacitors Mica is a mineral which can readily be split down into thin uniform sheets of thickness in the range 0.025 mm–0.075 mm. In the **stacked construction** (see Figure 6.6) mica and metal foil are interleaved in the form of a multiple plate capacitor, the whole being clamped together to maintain a rigid structure. As with paper capacitors, voids between the foil and the dielectric can be eliminated by metallising one side of the mica (**silvered mica capacitors**).

Ceramic dielectric capacitors These capacitors consist of metallic coatings (usually silver) on opposite faces of discs, cups and tubes of ceramic material. The construction of one form of tubular ceramic capacitor is shown in Figure 6.14, the sectional view through the right-hand end illustrating how the connection to the inner electrode is made. Ceramic capacitors are broadly divided into two classes, namely those with a low value of relative permittivity (**low-ϵ** or **low-k** types), having permittivities in the range 6–100, and those with high values of permittivity (**high-ϵ** or **high-k** types), having permittivities in the range 1500–3000.

Low-ϵ types have good capacitance stability and are used in the tuning circuits of electronic oscillators to maintain the frequency of oscillation fig 6.14 *cut-away section of a tubular ceramic capacitor*



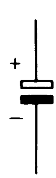
within close limits. High-ϵ types provide a greater capacitance per unit volume than do low-ϵ types, but are subject to a wider range of capacitance variation. They are used in a wide range of electronic applications.

Electrolytic capacitors The dielectric in these capacitors is a thin film of oxide formed either on one plate or on both plates of the capacitor, the thickness of the film being only a few millionths of a centimetre. A consequence is that electrolytic capacitors not only have the highest capacitance per unit volume of all types of capacitor but are the cheapest capacitor per unit capacitance. Offset against these advantages, electrolytic capacitors have a relatively high value of leakage current (particularly so in aluminium electrolytic capacitors) and have a wide variation in capacitance value (from -20 per cent to $+50$ or $+100$ per cent in some types).

The majority of electrolytic capacitors are **polarised**, that is the p.d. between the terminals *must* have the correct polarity, otherwise there may be risk of damage to the capacitor. A typical circuit symbol used to represent an electrolytic capacitor is shown in Figure 6.15; the symbol indicates the correct polarity to which the capacitor must be connected.

Although many metals can be coated with an oxide film, it is found that aluminium and tantalum exhibit the best features for use in electro-

fig 6.15 *a circuit symbol for a polarised electrolytic capacitor*



lytic capacitors. The basis of capacitors using the two materials is described below.

After long periods of inactivity, for instance, if they have been stored for several months, the electrodes of electrolytic capacitors require 're-forming' (this applies particularly to aluminium electrolytics). This is carried out by applying the full rated voltage via a 10 kΩ resistor until the leakage current has fallen to its rated value. Should this not be done, there is a risk when the full voltage is applied that the initial value of the leakage current may be large enough to generate an excessive gas pressure inside the capacitor with consequent hazard of an explosion.

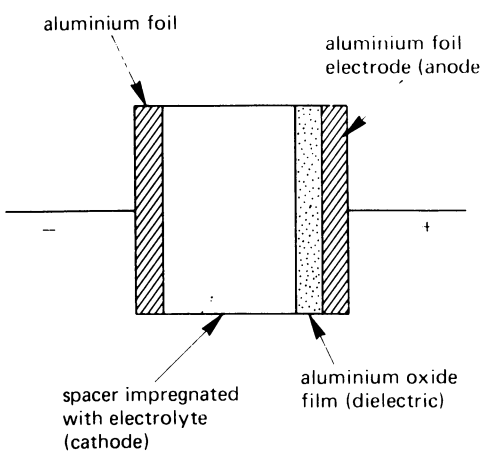
Aluminium electrolytic capacitors The basic construction of a polarised aluminium electrolytic capacitor is shown in Figure 6.16. The anode (positive) foil has an oxide film formed on its surface, the latter being the dielectric with a relative permittivity of about 7–10. The cathode (negative) foil is in contact with the actual cathode electrode, which is a paper film impregnated in an electrolyte such as ammonium borate. The physical construction of tubular capacitors is generally similar to that in Figure 6.13, with the impregnated paper and the aluminium foils rolled in the form of a cylinder.

Non-polar electrolytic capacitors, suitable for use either with d.c. or a.c. supplies, are manufactured by forming oxide layers on both foils.

A feature of electrolytic capacitors is that, at high frequency, they 'appear' to the external circuit as inductors. This effect can be overcome by connecting, say, a polycarbonate capacitor of low value in parallel with the electrolytic capacitor.

Tantalum electrolytic capacitors Two types of tantalum capacitor are available, and are those which use foil electrodes and those which employ

fig 6.16 *basic construction of a polarised electrolytic capacitor*



a tantalum slug as the anode electrode. The construction of tantalum foil types is similar to that of aluminium foil types.

Tantalum capacitors, although more costly per microfarad than aluminium electrolytics, are more reliable and are physically smaller than their aluminium counterparts. The dielectric properties of tantalum oxide are generally superior to those of aluminium oxide, and result in lower leakage current, a longer 'shelf' life, and a flatter temperature–capacitance curve than aluminium capacitors.

SELF-TEST QUESTIONS

1. What is meant by (i) a line of electric flux, (ii) the direction of electric flux and (iii) electric flux density?
2. A dielectric has an electric field intensity in it of 10 MV/m. If the dielectric thickness is 0.1 mm, what p.d. is applied across it?
3. Explain the principle of electrostatic screening. What is a Faraday cage and how is it used?
4. A capacitor stores a charge of 10 μC. Calculate the p.d. between the terminals of the capacitor if its capacitance is (i) 1.0 μF, (ii) 100 pF.
5. What is meant by the permittivity of a dielectric? How does the permittivity of the dielectric affect the capacitance of a parallel-plate capacitor?
6. Calculate the capacitance of a three-plate capacitor if the area of each plate is 500 cm^2, the plates being separated by a dielectric of thickness 1 mm whose relative permittivity is 4.
7. Capacitors of 2, 4 and 6 μF, respectively, are connected (i) in parallel, (ii) in series. Calculate the resultant capacitance in each case.
8. If each of the capacitor combinations in question 6 has a voltage of 10 V across it, calculate in each case the stored charge and the stored energy.
9. An R–C series circuit contains a 1 kΩ resistor and a 10 μF capacitor, the supply voltage being 10 V d.c. Determine (i) the time constant of the circuit, (ii) the initial value of the charging current, (iii) the time taken for the current to have fallen to 0.005 A, (iv) the 'settling' time of the transients in the circuit and (v) the final value of the capacitor voltage and its charging current.

SUMMARY OF IMPORTANT FACTS

Like electrostatic charges **repel** one another and **unlike** charges **attract** one another.

A **parallel-plate capacitor** consists of two parallel **plates** separated by a **dielectric**. A capacitor can **store electrical energy**.

Apparatus can be **screened** from an electrical field by a wire-mesh **Faraday cage**.

The **charge (Q) stored** by a capacitor is given by $Q = CV$ coulombs.

Permittivity is a measure of the property of a dielectric to **concentrate electric flux**. The **absolute permittivity**, ϵ is given by the product $\epsilon_O \times \epsilon_r$ where ϵ_O is the **permittivity free space** and ϵ_r is the **relative permittivity** (a dimensionless number).

For **series-connected capacitors**, the *reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances*. The equivalent capacitance of the series circuit is **less than** the smallest value of capacitance in the circuit.

For **parallel-connected capacitors**, the *equivalent capacitance is the sum of the individual capacitances*. The equivalent capacitance is **greater than** the largest value of capacitance in the circuit.

The **initial value of the charging current** drawn by a *discharged capacitor* is $\frac{V_S}{R}$, where V_S is the supply voltage and R is the circuit resistance. When the capacitor is *fully charged*, the charging current is zero. The **time constant**, T, of an R-C circuit is $\boldsymbol{RC}$ **seconds** (R in ohms, C in farads), and the **settling time** (the time taken for the transients to have 'settled out') is *approximately 5T seconds*.

The **initial value of the discharge current** from a capacitor charged to voltage V_C is $\frac{V_C}{R}$, where R is the resistance of the discharge path. The **settling time** of the discharge transients is *approximately 5T seconds.*

The **energy stored** by capacitor C which is charged to a voltage V_C is $\frac{CV_C^2}{2}$ joules (C in farads, V_C in volts).

CHAPTER 7

ELECTROMAGNETISM

7.1 MAGNETIC EFFECTS

Purely 'electrical' effects manifest themselves as voltage and current in a circuit. It was discovered early in the nineteenth century that electrical effects also produce magnetic effects, and it is to this that we direct studies in this chapter.

A magnetic 'field' or magnetic 'force' cannot be seen by man but (and very importantly) we can detect its existence by its effects on other things. For example, when a current flows in a wire, it produces a magnetic field which can be used to attract a piece of iron (this is the principle of the electromagnet), that is to say, a mechanical force exists between the two (this is also the general basis of the electric motor).

Alternatively, if a permanent magnet is quickly moved across a conductor, that is, the magnetic field 'cuts' the conductor, an e.m.f. is induced in the conductor (this is the basis of the electrical generator).

The presence of a magnetic field is detected by its effects, such as its effect on iron filings. The *direction* in which a magnetic field acts at a particular point in space is said to be the direction in which the force acts on an *isolated north-seeking pole* (a *N-pole*) at that point [we may similarly say that a *S-pole* is a *south-seeking pole*].

Experiments with a pair of permanent magnets show two important features, namely:

1. **like magnetic poles**, that is, two N-poles or two S-poles, **repel one another**;
2. **unlike magnetic poles attract one another**.

Consequently, if an isolated N-pole is placed in a magnetic field, it is repelled by the N-pole which produced the field and towards the S-pole. If the isolated N-pole was free to move, it would *move away from the N-pole of the magnet producing the field and towards the S-pole.* It is for this reason that we say that **lines of magnetic flux leave the N-pole of a magnet and enter the S-pole**. In fact, nothing really 'moves' in a magnetic field, and the 'movement' of magnetic flux is a man-made imaginary concept (none the less, it is sometimes convenient to refer to the 'direction' of the magnetic flux).

However, before studying electromagnetism (that is, magnetism produced by electrical effects) we must understand the basic mechanics of magnetism itself.

7.2 MAGNETISM

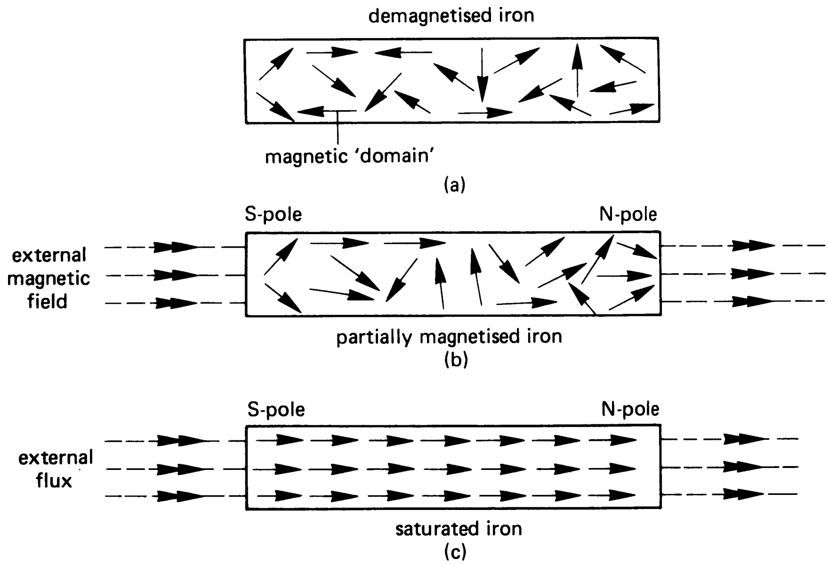
The name **magnetism** is derived from **magnetite**, which is an iron-oxide mineral whose magnetic properties were discovered before man grasped the basic principles of electricity. It was found that a *magnet* had the property of attracting iron and similar materials; the word **ferromagnetism** is frequently used in association with the magnetic properties of iron.

To understand the nature of a magnet we must take a look at the composition of iron. In an atom, the spinning motion of the electrons around the atomic nucleus is equivalent to an electric current in a loop of wire. In turn, this electric current produces a small magnetic effect. There are many electrons associated with each atom, and the magnetic effect of some electrons will cancel others out. In iron there is a slight imbalance in the electron spins, giving rise to an overall magnetic effect. Groups of atoms appear to 'club' together to produce a small permanent magnet, described by scientists and engineers as a **magnetic domain**. Although a magnetic domain contains billions of atoms, it is smaller in size than the point of a needle.

Each domain in a piece of iron has a N-pole and a S-pole but, in a demagnetised piece of iron, the domains point in random directions so that the net magnetic field is zero, as shown in Figure 7.1(a). Each arrow in the figure represents a magnetic domain.

When an external magnetic field is applied to the iron in the direction shown in Figure 7.1(b), some of the magnetic domains turn in the direction of the applied magnetic field and remain in that direction after the field is removed. That is, the external magnetic field causes the piece of iron to be partially magnetised so that it has a N-pole and a S-pole. Repeated application of the external magnetic field results in the iron becoming progressively more magnetised as more of the domains align with the field. A bar of iron can be magnetised in this way simply by stroking it along its length in one direction with one pole of a permanent magnet.

If the magnetic field intensity is increased, all the domains ultimately align with the applied magnetic field as shown in Figure 7.1(a). When this occurs, the iron produces its maximum field strength and the bar is said to fig 7.1 *magnetising a piece of iron*



be magnetically **saturated**. Since no further domains remain to be aligned, any further increase in the external magnetic field does not produce any significant increase in the magnetism of the iron.

7.3 MAGNETIC FIELD PATTERN OF A PERMANENT MAGNET

The magnetic field pattern of a permanent magnet can be traced out by covering the magnet with a piece of paper and sprinkling iron filings on it. When the paper is tapped, the iron filings take up the pattern of the magnetic field (see Figure 7.2). You will note that the flux lines are shown to 'leave' the N-pole and to 'enter' the S-pole.

When unlike magnetic poles are brought close together (see Figure 7.3(a)), the flux leaves the N-pole of one magnet and enters the S-pole of the other magnet, and there is a force of attraction between them. When unlike magnetic poles are brought close together (Figure 7.3(b)) there is a force of repulsion between the poles.

7.4 DIRECTION OF THE MAGNETIC FIELD AROUND A CURRENT-CARRYING CONDUCTOR

Experiments carried out with simple equipment such as a compass needle and a conductor which is carrying current allow us to determine the fig 7.2 *the flux pattern produced by a permanent magnet*

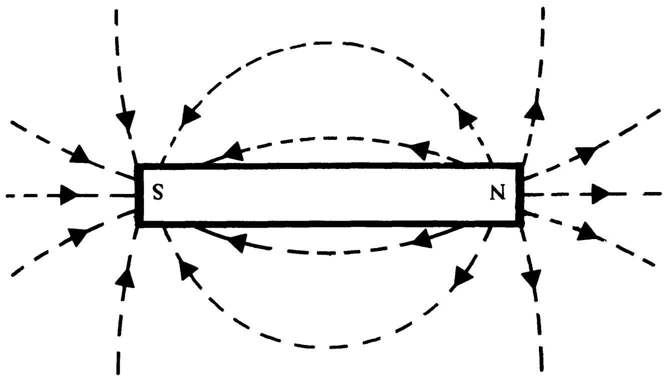
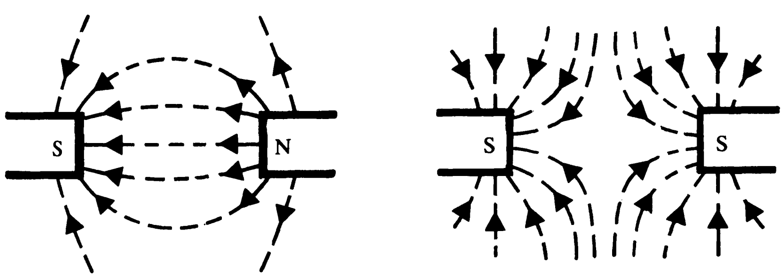


fig 7.3 *the magnetic flux pattern between (a) unlike poles, (b) like poles*



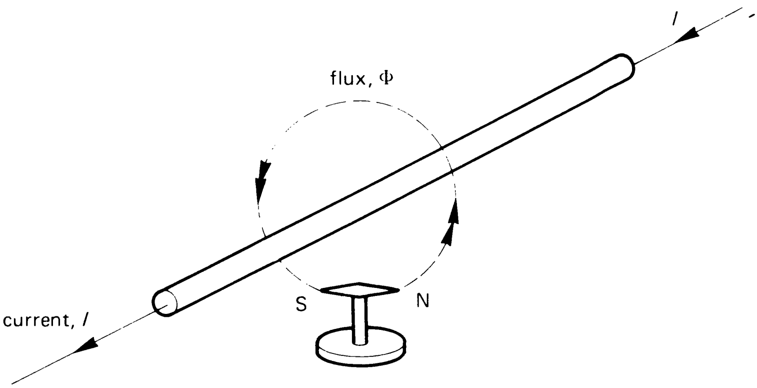
(a) (b)

'direction' of the magnetic flux around the conductor, as shown in Figure 7.4(a). If the direction of the current in the conductor is reversed, the compass needle reverses direction, indicating that the magnetic field has reversed.

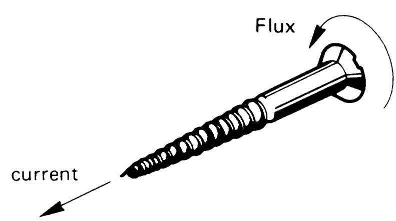
A simple rule known as the **screw rule** allows us to predict the direction of the magnetic field around the conductor as follows:

> **If you imagine a wood screw (Figure 7.4(b)) pointing in the direction of current flow, the direction of the magnetic field around the conductor is given by the direction in which you turn the head of the screw in order to propel it in the direction of the current.**

fig 7.4 *(a) direction of the magnetic flux around a current-carrying conductor, (b) the screw rule for determining the direction of the flux*



(a)



(b)

7.5 SOLENOIDS AND ELECTROMAGNETS

We now turn our attention to the production of magnetism by electrical means. It has been established earlier that whenever current flows in a conductor, a magnetic flux appears around the conductor. If the conductor is wound in the form of a coil, it is known as a **solenoid** if it has an air core and, if it has an iron core, it is known as an **electromagnet**. The current which flows in the coil to produce the magnetic flux is known as the **excitation current**.

The essential difference between an electromagnet and a solenoid is that, for the same value of current, the electromagnet produces a much higher magnetic flux than does the solenoid. This is described in detail in section 7.13.

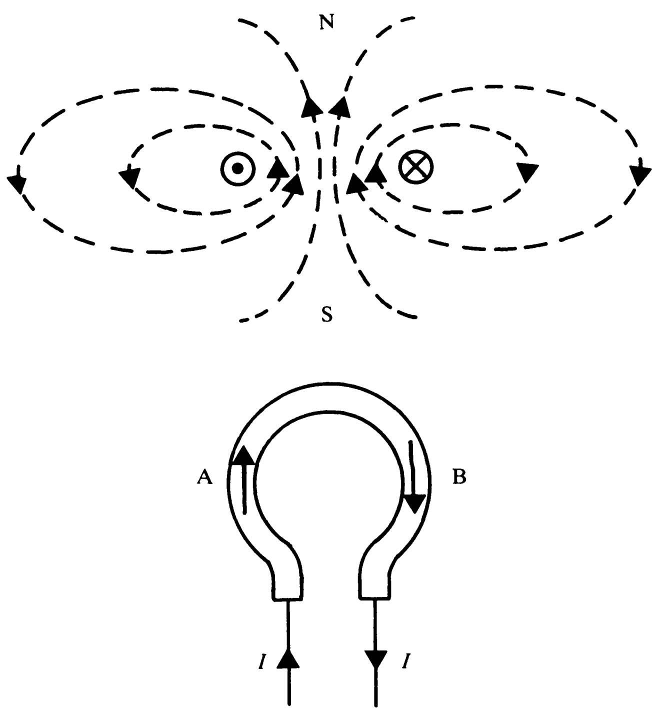
7.6 FLUX DISTRIBUTION AROUND A CURRENT-CARRYING LOOP OF WIRE

Consider the single-turn coil in the lower half of Figure 7.5, which carries current I. We can determine the direction of the flux produced by the coil if we look on the plan view of the coil (which is the section taken through A–B of the coil), illustrated in the upper half of Figure 7.5. The direction of the current is shown in the upper part of the diagram using an 'arrow' notation as follows:

> **If the current approaches you from the paper, you will see a 'dot' which corresponds to the approach of the current 'arrow'.**
> **If the current leaves you (enters the paper), you will see a 'cross' corresponding to the 'crossed feathers' of the arrow.**

In Figure 7.5, the arrow in the plan view approaches you on the left-hand side of the coil and leaves on the right-hand side. Applying the rule

fig 7.5 *magnetic flux pattern produced by current flow in a single-turn coil*



outlined in section 7.4 for the direction of the magnetic flux you will find that, in the plan view, the magnetic flux pattern is as shown.

Referring to the end elevation of the loop (shown in the lower part of Figure 7.5) magnetic flux enters the loop from the side of the page at which you are looking (that is, you are looking at a S-pole) and leaves from the opposite side of the page (that is, the N-pole of the electromagnet is on the opposite side of the loop).

If the direction of the current in the coil is reversed, the magnetic polarity associated with the loop of wire also reverses. You may like to use the information gained so far to verify this.

Simple rules for predicting the magnetic polarity of a current-carrying loop of wire are shown in Figure 7.6.

7.7 MAGNETIC FIELD PRODUCED BY A CURRENT-CARRYING COIL

A coil of wire can be thought of as many single-turn loops connected together. To determine the magnetic polarity of such a coil, we can use the methods outlined in section 7.6.

The first step is to determine the direction of the magnetic flux associated with each wire. This is illustrated in Figure 7.7, and the reader will note that inside the coil the flux produced by the conductors is *additive*. That is the flux produced by one turn of wire reinforces the flux produced by the next turn and so on. In the case shown, the magnetic flux leaves the left-hand side of the coil (the N-pole and enters the right-hand side (the S-pole).

Alternatively, you can deduce the polarity at either end of the coil by 'looking' at the end of the coil and using the method suggested in section 7.6.

If EITHER the direction of the current in the coil is reversed OR the direction in which the coil is wound is reversed (but not both), the magnetic polarity produced by the coil is reversed. The reader is invited to verify this.

fig 7.6 *magnetic polarity of a current-carrying loop of wire*

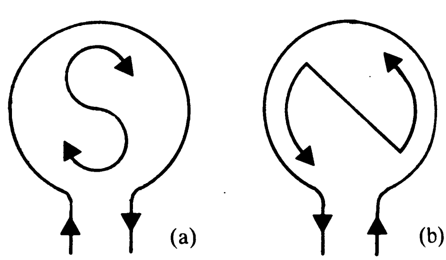
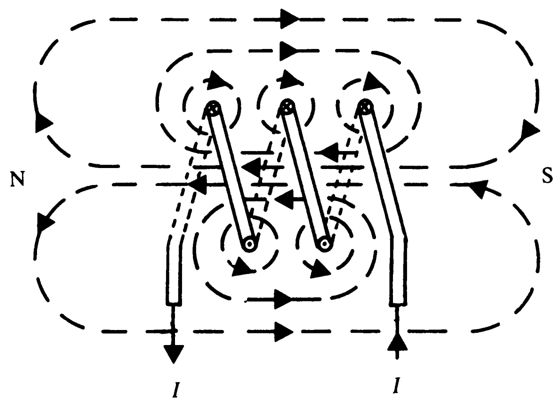


fig 7.7 *magnetic field produced by a coil of wire which carries current*



7.8 MAGNETOMOTIVE FORCE OR m.m.f.

In a coil it is the **magnetomotive force**, F, which causes the magnetic flux to be established inside the coil (its analogy in the electric circuit is the e.m.f. which establishes the current in an electric circuit). The equation for m.m.f. is

$$\text{m.m.f., } F = \text{number of turns} \times \text{current in the coil}$$

$$= NI \text{ ampere turns or amperes}$$

The dimensions of m.m.f. are (amperes × turns) or ampere-turns but, since the number of turns is a dimensionless quantity, the dimensions are, strictly speaking, amperes. However, to avoid confusion with the use of amperes to describe the current in the coil, we will use the ampere-turn unit for m.m.f.

7.9 MAGNETIC FIELD INTENSITY OR MAGNETISING FORCE

The **magnetic field intensity** (also known as the **magnetic field strength** or the **magnetising force**), symbol H, is the m.m.f. per unit length of the magnetic circuit. That is:

$$\text{magnetic field intensity} = \frac{\text{m.m.f.}}{\text{length of the magnetic circuit}}$$

$$= \frac{F}{l} = \frac{NI}{l} \text{ ampere-turns per metre}$$

or amperes per metre

Once again, to avoid confusion between the use of the ampere and the ampere-turn when used in the electromagnetic context, we will use the ampere-turn per metre unit.

Example
A coil has 1000 turns of wire on it and carries a current of 1.2 A. If the length of the magnetic circuit is 12 cm, calculate the value of the m.m.f. produced by the coil and also the magnetic field intensity in the magnetic circuit.

Solution

$N = 1000$ turns; $I = 1.2$ A; $l = 12$ cm $= 12 \times 10^{-2}$ m

m.m.f., $F = NI = 1000 \times 1.2 = 1200$ ampere-turns (Ans.)

$$\text{magnetic field intensity, } H = \frac{F}{l} = \frac{1200}{12 \times 10^{-2}}$$

$= 10\,000$ ampere-turns per metre

7.10 MAGNETIC FLUX

The **magnetic flux**, symbol Φ, produced by a magnet or electromagnet is a measure of the magnetic field, and has the unit of the **weber** (Wb).

7.11 MAGNETIC FLUX DENSITY

The **magnetic flux density**, symbol B is the amount of magnetic flux passing through an area of 1 m^2 which is perpendicular to the direction of the flux; the unit of flux density is the **tesla** (T). That is

$$\text{flux density, } B = \frac{\text{flux, } \Phi}{\text{area, } A} \text{ tesla } (T)$$

where Φ is in Wb and A in m^2. A flux density of 1 T is equivalent to a flux of 1 weber passing through an area of 1 square metre, or

$$1\ T = 1\ Wb/\text{m}^2$$

Example
The magnetic flux in the core of an electromagnet is 10 mWb, the flux density in the core being 1.2 T. If the core has a square cross-section, determine the length of each side of the section.

Solution

$$\Phi = 10 \text{ mWb} = 10 \times 10^{-3} \text{ Wb}; B = 0.75 \text{ T}$$

Since

$$B = \frac{\Phi}{A}$$

then

$$\text{area of core}, A = \frac{\Phi}{B} = \frac{10 \times 10^{-3}}{1.2} = 0.0083333 \text{ m}^2$$

If each side of the section is x metres in length, then

$$x^2 = 0.0083333 \text{ m}^2$$

or

$$x = 0.00913 \text{ m or } 9.13 \text{ cm.}$$

7.12 PERMEABILITY

When an electromagnetic is energised by an electric current, the magnetic field intensity (H ampere turns per metre) produced by the coil establishes a magnetic flux in the magnetic circuit. In turn, this gives rise to a magnetic flux density, B tesla, in the magnetic circuit. The relationship between B and H in the circuit is given by

$$B = \mu H$$

where μ is the **permeability** of the magnetic circuit; the value of the permeability of the magnetic circuit gives an indication of its ability to concentrate magnetic flux within the circuit (the higher its value, the more flux the circuit produces for a particular excitation current).

The **permeability of free space** (or of a vacuum) is given the special symbol μ_O, and is sometimes referred to as the **magnetic space constant.** Its value is

$$\mu_O = 4\pi \times 10^{-7} \text{ henrys per metre (H/m)}$$

where the henry (H) is the unit of inductance and is described in section 7.21. For all practical purposes, *the permeability of air has the same value as the magnetic space constant.*

If a coil has an iron core placed in it, it is found that the magnetic flux produced by the coil increases significantly. The factor by which the flux density increases is given by the **relative permeability**, μ_r, of the material, where

$$\mu_r = \frac{\text{flux density with the iron core}}{\text{flux density with an air (or vacuum) core}}$$

$$= \frac{\mu H}{\mu_O H} = \frac{\mu}{\mu_O}$$

where μ_r is dimensionless. Hence the **absolute permeability**, μ, is given by

$$\mu = \mu_O \mu_r \text{ H/m}$$

Therefore

$$B = \mu H = \mu_O \mu_r H \text{ T}$$

The value of μ_r may have a value in the range 1.0-7000, and depends not only on the type of material but also on the operating flux density and temperature.

Example

An electromagnet produces a magnetic field intensity of 500 ampere-turns per metre in its iron circuit. Calculate the flux density in the core if it (i) has an air core, (ii) has a cast-iron core with $\mu_r = 480$, (iii) has a transformer steel core with $\mu_r = 1590$.

Solution

$H = 500$ ampere-turns per metre.

(i) Air core: $\mu = \mu_O = 4\pi \times 10^{-7}$

$$B = \mu H = 4\pi \times 10^{-7} \times 500$$
$$= 0.628 \times 10^{-3}\ T \text{ or } 0.628 \text{ mT (Ans.)}$$

(ii) Cast-iron core: $\mu = \mu_r \mu_O = 480 \times 4\pi \times 10^{-7}$ H/m

$$B = \mu H = 480 \times 4\pi \times 10^{-7} \times 500 = 0.302 \text{ T (Ans.)}$$

(iii) Transformer steel core: $\mu = \mu_r \mu_O$

$$= 1590 \times 4\pi \times 10^{-7} \text{ H/m}$$

$$B = \mu H = 1590 \times 4\pi \times 10^{-7} \times 500 = 1.0 \text{ T (Ans.)}$$

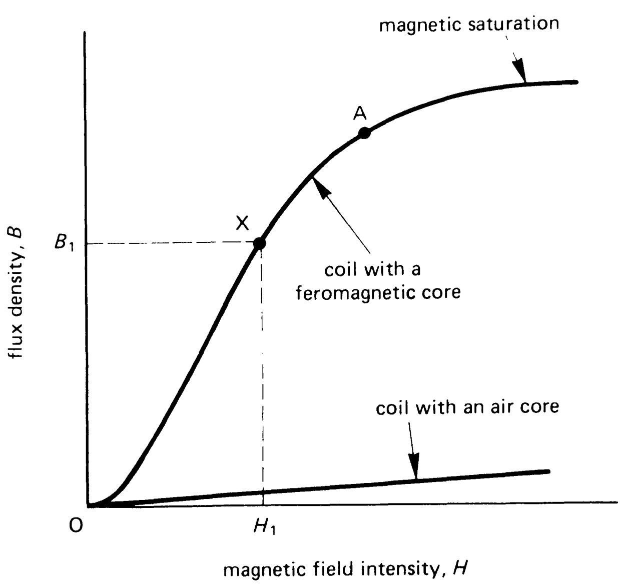
7.13 MAGNETISATION CURVE FOR IRON AND OTHER FERRO-MAGNETIC MATERIAL

If a coil has an air core, and if the current in the coil (and therefore the magnetic field intensity) is gradually increased from zero, the magnetic flux density in the air core increases in a linear manner as shown by the straight line graph at the bottom of Figure 7.8. The curve relating the flux density B to the magnetic field intensity H is known as the **magnetisation curve** of the material. Since in an air core $B = \mu_O H$, the magnetisation curve is a straight line having a slope of μ_O.

If the air is replaced by a **ferromagnetic material** such as iron, we find that the flux density in the core increases very rapidly between O and A (see Figure 7.8), after which the slope of the curve reduces. Finally, at a very high value of H (corresponding to large value of excitation current), the magnetisation curve for the iron becomes parallel to that for the air core. The reason for the shape of the curve is shown in the next paragraph.

As the excitation current in the coil surrounding the iron increases, the magnetic domains in the iron (see section 7.2) begin to align with the magnetic field produced by the current in the coil. That is, *the magnetism of the domains adds to the magnetic field of the excitation current*; as the current increases, more and more of the domains in the iron align with the external magnetic field, resulting in a rapid increase in the flux density in the iron.

fig 7.8 *magnetisation curves for air and iron*



However, when point A in Figure 7.8 is reached, most of the domains have aligned with the field produced by the magnetising current, and beyond this point the rate of increase of the flux density diminishes. This denotes the onset of **magnetic saturation** in the iron.

Any further increase in flux density is caused by the remaining few domains coming into alignment with the applied magnetic field. Finally, the magnetic flux density increases very slowly, and is due only to the increase in excitation current (since all the domains are aligned with the field, no more magnetism can be produced by this means); at this point the magnetisation curve for iron remains parallel to the curve for air.

In a number of cases the magnetisation curve is a useful design tool, allowing calculations on magnetic circuits to be simplified. For example if the value of H is known, the flux density in the iron can be determined directly from the curve. Additionally, the magnetisation curve for a meterial allows us to determine the permeability at any point on the curve; for example, the absolute permeability μ_1 of the iron at point X in Figure 7.8 is

$$\mu_1 = \frac{B_1}{H_1}$$

Example

The following points lie on the magnetisation curve of a specimen of cast steel. Calculate the relative permeability at each point.

Flux density, T	*m.m.f., At/m*
0.04	100
0.6	600
1.4	2000

Solution

Since $B = \mu_O \mu_r H$ then $\mu_r = \dfrac{B}{\mu_O H}$

For $B = 0.04\ T$, $\mu_r = \dfrac{0.04}{(4\pi \times 10^{-7} \times 100)} = 318$ (Ans.)

For $B = 0.6\ T$, $\mu_r = \dfrac{0.6}{(4\pi \times 10^{-7} \times 600)} = 796$ (Ans.)

For $B = 1.4\ T$, $\mu_r = \dfrac{1.4}{(4\pi \times 10^{-7} \times 2000)} = 557$ (Ans.)

7.14 HYSTERESIS LOOP OF A FERROMAGNETIC MATERIAL

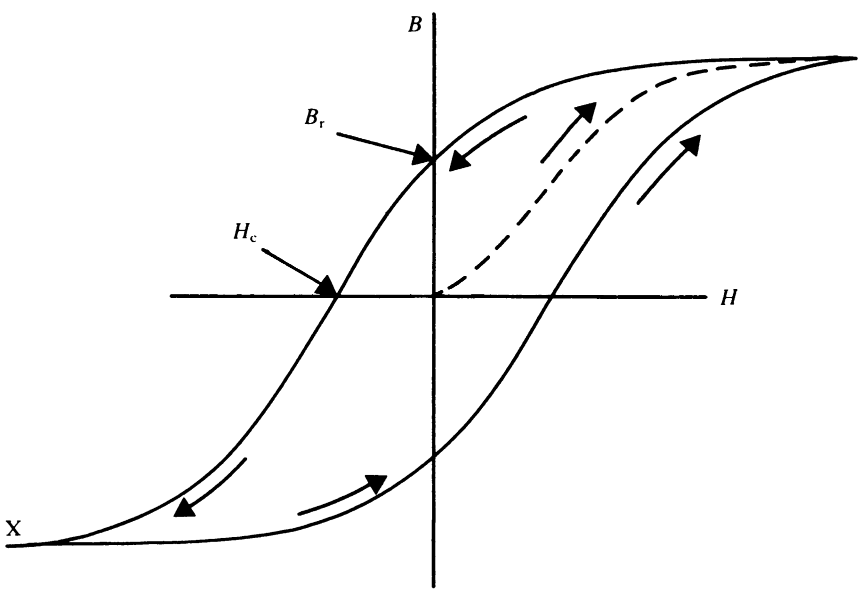
If, after causing the iron core of an electromagnet to be saturated (see the dotted curve in Figure 7.9) the excitation current is reduced, you will find that the *B-H* curve does not return to the zero when the excitation current is reduced to zero. In fact, the iron retains a certain amount of flux density. This is known as the **remanent flux density**, B_r. The reason is that some of the magnetic domains do not return to their original random direction, and still point in the direction of the original magnetising field.

To reduce the flux density to zero, it is necessary to reverse the direction of the excitation current, that is, to apply a negative value of H. At some reverse magnetising force known as the **coercive force**, H_C, the flux density reaches zero.

If the reverse magnetising current is increased in value, the flux density begins to increase in the reverse direction until, finally, the iron becomes saturated in the reverse direction.

Reducing the magnetising current again to zero leaves a remanent flux density once more but in the reverse direction. The flux density is reduced to zero by increasing the magnetising force in the 'forward' direction; if the 'forward' magnetising current is increased to a high value, the iron becomes saturated once more.

fig 7.9 *hysteresis loop of a ferromagnetic material*



The complete loop in Figure 7.9 is known as the **hysteresis loop** or **B-H loop** of the material. The process of repeatedly reversing the magnetic domains as the B-H loop is cycled (as it is when an alternating current [a.c.] is passed through the coil) results in energy being consumed each time the domains are reversed. The energy consumed in this manner appears in the form of heat in the iron and is known as the **hysteresis energy loss**. The total energy consumed depends on a number of factors including the volume (v) of the iron, the frequency (f) of the magnetic flux reversals, and on the maximum flux density (B_m). The hysteresis power loss, P_n is given by

$$P_h = kvfB_m^n \text{ watts}$$

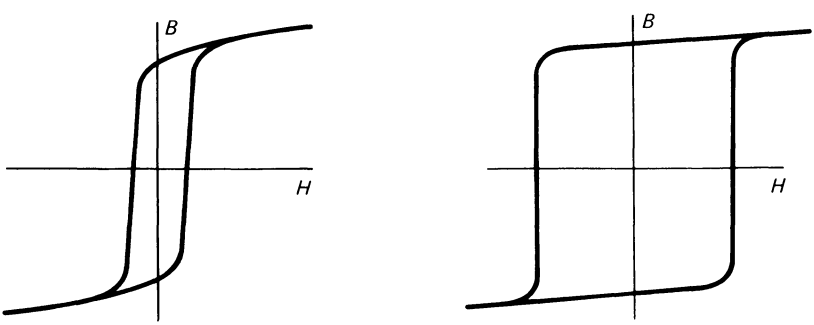
where k is a number known as the **hysteresis coefficient**, and n is a number known as the **Steinmetz index** and has a value in the range 1.6–2.2.

7.15 'SOFT' AND 'HARD' MAGNETIC MATERIALS

Iron used for electromagnets and relays which must lose their magnetism quickly when the current is switched off, are known as **magnetically soft** materials. They are characterised by a narrow B–H loop with a low remanent flux density and low coercive force (see Figure 7.10(a)).

Iron and steel used for permanent magnets must not only retain a high flux density but also be difficult to demagnetise. These are known as **magnetically hard** materials; they have a high value of B_r (about 1 T) and a high value of H_C (about 50 000 At/m). Their B-H loop is 'square', as shown in Figure 7.10(b). Magnetically hard materials consist of iron combined with small amounts of aluminium, nickel, copper and cobalt (for example Alnico, Alcomax III, Triconal G, etc).

fig 7.10 *B-H curve for (a) a 'soft' magnetic material and (b) a 'hard' magnetic material*



(a) (b)

7.16 MAGNETIC CIRCUIT RELUCTANCE

The magnetic circuit is an almost exact analogy of the electric circuit insofar as basic design calculations are concerned. The points where similarities lie are as follows:

Electric circuit	*Magnetic circuit*
Electromotive force, E	Magnetomotive force, F
Current, I	Flux, Φ
Resistance, R	Reluctance, S

The **reluctance** of a magnetic circuit, symbol S, is the effective magnetic 'resistance' to a magnetic flux being established in the magnetic circuit. Electric and magnetic circuits also have a similar 'Ohm's law' to one another as follows:

Electric circuit	*Magnetic circuit*
$E = IR$	$F = \Phi S$

That is, you can consider the m.m.f. in the magnetic circuit to be equivalent to the e.m.f. in the electric circuit, the flux to be equivalent to the current, and the reluctance to be equivalent to the resistance. That is

magnetic circuit 'e.m.f.'	=	magnetic circuit current	×	magnetic circuit 'resistance'

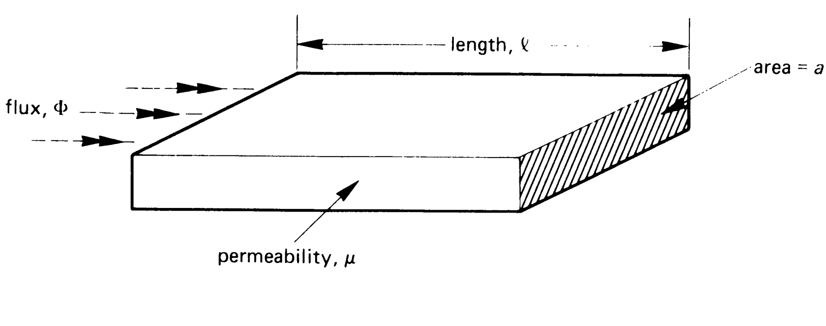
or

m.m.f.	=	flux	×	reluctance

The equation for the reluctance of a piece of magnetic material (see Figure 7.11) is given by

$$\text{reluctance, } S = \frac{1}{\mu a} \text{ ampere-turns per weber}$$

fig 7.11 *Reluctance of a magnetic circuit*



where l is the effective length of the magnetic circuit (in metres), a is the cross-sectional area of the flux path (in m^2), and μ is the absolute permeability of the magnetic material.

7.17 MAGNETIC CIRCUITS

A magnetic circuit is simply a path in which magnetic flux can be established. As in electric circuits, you can have series circuits, parallel circuits, series–parallel circuits, etc; the laws which apply to electrical resistance also apply generally to magnetic circuits.

Consider the simple magnetic circuit in Figure 7.12(a) consisting of an iron path with a coil of N turns wound on it. The following values apply to the circuit

number of turns, $N = 1000$
length of the iron path, $l = 0.25$ m
area of iron path, $a = 0.001\ m^2$
relative permeability of iron path, $\mu_r = 625$
flux in the iron, $\Phi = 1.5 \times 10^{-3}$ Wb

In the following we calculate the value of the current needed in the coil to produce the specified value of flux. The 'equivalent' electrical circuit diagram is shown in Figure 7.12(b) in which S_1 is the reluctance of the magnetic circuit. The value of this reluctance is calculated as follows:

$$\begin{aligned}\text{reluctance, } S_1 &= \frac{l}{\mu_0 \mu_r a}\\ &= \frac{0.25}{(4\pi \times 10^{-7} \times 625 \times 0.01)}\\ &= 0.318 \times 10^6 \text{ At/Wb}\end{aligned}$$

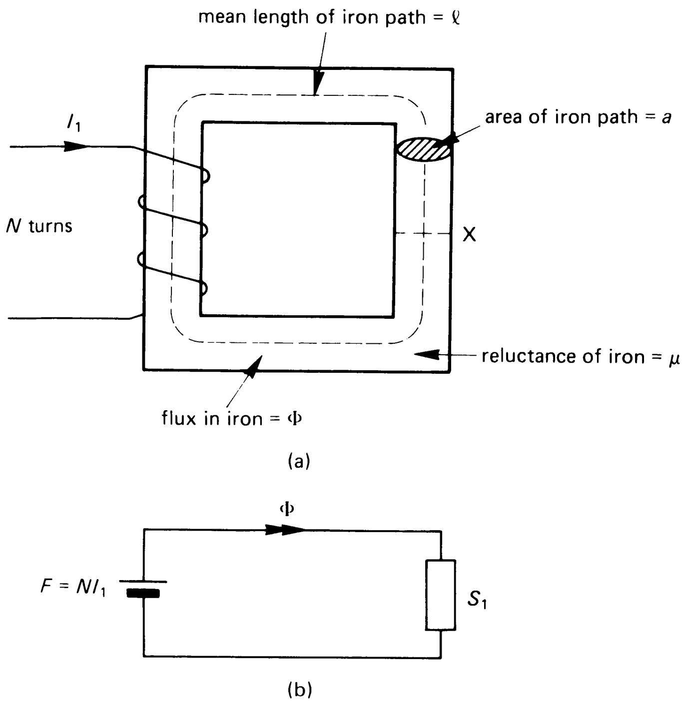
The m.m.f. produced by the coil is calculated as follows

$$\begin{aligned}\text{m.m.f.}, F &= \Phi S_1 = (1.5 \times 10^{-3}) \times (0.318 \times 10^6)\\ &= 477 \text{ ampere-turns}\end{aligned}$$

But $F = NI_1$, where I_1 is the current in the coil. Hence

$$\begin{aligned}I_1 &= \frac{F}{N} = \frac{477}{1000}\\ &= 0.477 \text{ A (Ans.)}\end{aligned}$$

fig 7.12 *(a) a simple magnetic circuit and (b) its electrically 'equivalent circuit*



A series magnetic circuit

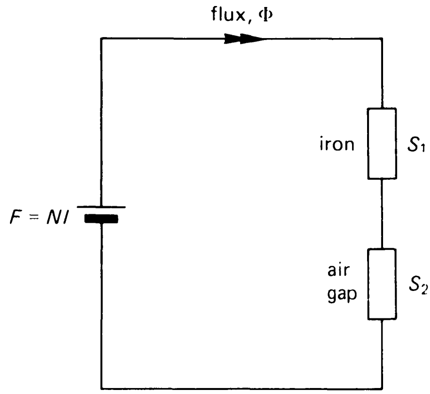
Suppose that the iron path of the magnetic circuit in Figure 7.12(a) is cut at point X so that an air gap, 1 mm-wide, is introduced; this reduces the length of the iron path to 0.249 m. In the following we calculate the current, I_2, which produces the same value of flux (1.5 mWb) in the air gap (see illustrative calculation above). The reluctance of the two parts (the iron path and the air gap) of the magnetic circuit are calculated below (see also Figure 7.13).

$$\text{Reluctance of iron path, } S_1 = \frac{l_{\text{iron}}}{\mu_0 \mu_r a}$$

$$= \frac{0.249}{(4\pi \times 10^{-7} \times 625 \times 0.001)}$$

$$= 0.317 \times 10^6 \ \text{At/Wb}$$

fig 7.13 *the 'equivalent' circuit of Figure 7.12(a) but with a 1 mm air gap at X*



Reluctance of the air gap, $S_2 = \dfrac{l}{\mu_0 a}$ ($\mu_r = 1$ for air)

$$= \frac{0.001}{(4\pi \times 10^{-7} \times 0.001)}$$

$$= 0.796 \times 10^6 \text{ At/Wb}$$

The reader is asked to note the high value of the reluctance of the 1 mm air gap when compared with the reluctance of the 250 mm of iron. The total reluctance of the magnetic circuit is calculated as follows:

Total reluctance, $S = S_1 + S_2 = 1.113 \times 10^6$ At/Wb

The m.m.f. required to produce the flux is

$$\text{m.m.f., } F = \Phi S = (1.5 \times 10^{-3}) \times (1.113 \times 10^6)$$

$$= 1670 \text{ At}$$

The current is calculated from the equation

$$I_2 = \frac{F}{N} = \frac{1670}{1000} = 1.67 \text{ A (Ans.)}$$

The above calculation shows that, in this case, it is necessary to increase the current by a factor of about 3.5 in order to maintain the same magnetic flux in the magnetic circuit when a 1 mm air gap is introduced.

7.18 MAGNETIC SCREENING

Since a strong magnetic field may interfere with the operation of sensitive electrical and electronic apparatus, it may be necessary to screen the apparatus from the field. The method usually adopted is to enclose the apparatus within a screen of low-reluctance material such as iron; this has the effect of placing a magnetic 'short-circuit' around the apparatus.

The screen ensures that the magnetic field cannot penetrate within the shield and reach the apparatus. Magnetic screens are used to protect electrical measuring instruments and cathode-ray tubes from strong fields. In computers, it is possible for a magnetic field produced by, say, a transformer to 'corrupt' the data on a 'floppy disc'; the discs need to be protected against this sort of corruption.

7.19 ELECTROMAGNETIC INDUCTION

When a current flows in a wire, a magnetic flux is established around the wire. If the current changes in value, the magnetic flux also changes in value. The converse is also true, and **if the magnetic flux linking with a wire or coil is changed, then an e.m.f. is induced in the wire or coil**. If the electrical circuit is complete, the induced voltage causes a current to flow in the wire or coil. There are three general methods of inducing an e.m.f. in a circuit, which are

1. self-induction;
2. induction by motion;
3. mutual induction.

The first of the three is fully explained in this chapter, the second and third being only briefly outlined since they are more fully described in Chapters 8 and 14, respectively.

Self induction

If the current in a conductor is, say, increased in value, the magnetic flux produced by the conductor also increases. Since this change of magnetic flux links with the conductor which has produced the fluw, it causes an e.m.f. to be induced in the conductor itself. That is, a change of current in a conductor causes an e.m.f. of **self-induction** to be induced in the conductor.

Induction by motion

An e.m.f. is induced in a conductor when it moves through or 'cuts' a magnetic field. The e.m.f. is due to **induction by motion**, and is the basis of the *electrical generator* (see Chapter 8).

Mutual induction

Suppose that a conductor is situated in the magnetic field of another conductor or coil. If the magnetic flux produced by the other conductor changes, an e.m.f. is induced in the first conductor; in this case the e.m.f. is said to be induced by **mutual induction**. The two conductors or coils are said to be **magnetically coupled** or **magnetically linked**. Mutual inductance is the basic principle of the *transformer* (see Chapter 14).

7.20 THE LAWS OF ELECTROMAGNETIC INDUCTION

The important laws concerned with electromagnetic induction are Faraday's laws and Lenz's law. At this stage in the book we merely state the laws, their interpretation being dealt with in sections appropriate to their practical application.

Faraday's laws

1. **An induced e.m.f. is established in a circuit whenever the magnetic field linking with the circuit changes.**
2. **The magnitude of the induced e.m.f. is proportional to the rate of change of the magnetic flux linking the circuit.**

Lenz's law

> **The induced e.m.f. acts to circulate a current in a direction which opposes the change in flux which causes the e.m.f.**

7.21 SELF-INDUCTANCE OF A CIRCUIT

A circuit is said to have the property of **self-inductance**, symbol L, when a change in the current in its own circuit causes an e.m.f. to be induced in itself. The unit of self-inductance is the **henry**, unit symbol H, which is defined as:

> **A circuit has a self-inductance of one henry if an e.m.f. of one volt is induced in the circuit when the current in that circuit changes at the rate of one ampere per second.**

That is

self-induced e.m.f. = inductance × rate of change of current

or

$$E = \frac{L \times (I_2 - I_1)}{\text{change in time}}$$

or

$$E = L\,\frac{\Delta I}{\Delta t}$$

where the current in the circuit changes from I_1 to I_2 in the length of time Δt seconds; we use the symbol Δ to represent a change in a value such as time or current.

If the change is very small, that is, it is an *incremental change*, we use the mathematical representation

$$E = L\,\frac{di}{dt}$$

where $\frac{di}{dt}$ is the mathematical way of waying 'the change in current during the very small time interval dt seconds'.

According to Lenz's law, the self-induced e.m.f. opposes the applied voltage in the circuit; some textbooks account for this by including a negative sign in the equation as follows

$$E = -L\,\frac{\Delta I}{\Delta t} \text{ and } E = -\,\frac{Ldi}{dt}$$

However, this can sometimes lead to difficulties in calculations, and in this book the positive mathematical sign is used in the equation for the self-induced e.m.f. . The 'direction' of the induced e.m.f. is accounted for by the way in which we use the induced e.m.f. in the calculation.

The self-inductance of a circuit can simply be thought of as an indication of the ability of the circuit to produce magnetic flux. For the same current in the circuit, a circuit with a low self-inductance produces less flux than one with high inductance.

The self-inductance of a coil with an air core may only be a few millihenrys, whereas a coil with an iron core may have an inductance of several henrys.

Example

Calculate the average value of the e.m.f. induced in a coil of wire of 0.5 H inductance when, in a time interval of 200 ms, the current changes from 2.5 A to 5.5 A.

Solution

$L = 0.5$ H; $\Delta t = 200$ ms $= 0.2$ s;

$\Delta I = 5.5 - 2.5 = 3$ A

Induced e.m.f., $E = L\,\dfrac{\Delta I}{\Delta t} = 0.5 \times \dfrac{3}{0.2} = 7.5$ V (Ans.)

7.22 RELATIONSHIP BETWEEN THE SELF-INDUCTANCE OF A COIL AND THE NUMBER OF TURNS ON THE COIL

If the number of turns on a coil is increased, then for the same current in the coil, the magnetic flux is increased. That is to say, increasing the number of turns increases the inductance of the coil. The relationship between the self-inductance, L, the number of turns, N, and the reluctance, S, of the magnetic circuit is given by

$$L = \frac{N^2}{S} \text{ H}$$

that is

L is proportional to N^2

If the number of turns of wire on a coil is doubled, the inductance of the coil is increased by a factor of $2^2 = 4$.

7.23 ENERGY STORED IN A MAGNETIC FIELD

During the time that a magnetic field is being established in a magnetic circuit, energy is being stored. The total stored energy is given by

$$\text{energy stored, } W = \frac{1}{2} LI^2 \text{ joules (J)}$$

where L is the inductance of the circuit in henrys and I is the current in amperes.

Example
Calculate the energy stored in the magnetic field of a 3-H inductor which carries a current of 2 A.

Solution

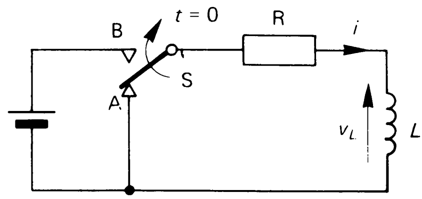
$$L = 3\text{H};\ I = 2\text{A}$$

$$\text{Energy stored, } W = \frac{1}{2} LI^2 = \frac{1}{2} \times 3 \times 2^2 = 6 \text{ J (Ans.)}$$

7.24 GROWTH OF CURRENT IN AN INDUCTIVE CIRCUIT

We now study what happens in a circuit when a practical inductor, that is, one having resistance as well as inductance, is connected to an e.m.f., E.

At the instant of time illustrated in Figure 7.14, the blade of switch S is connected to point A; this connection applies a short circuit to the coil, so that the current through the coil is zero. When the blade S of the switch is fig 7.14 *growth of current in an inductive circuit. The blade of switch S is moving from A to B at the time instant t = 0*



moved to position B (this is described as 'zero' time in this section of the book), the coil (which is an R-L circuit) is connected to the battery; at this point in time, current begins to flow through the coil.

When current flows, the inductor produces a magnetic flux. However, this flux 'cuts' its own conductors and induces a 'back' e.m.f. in them (see Faraday's law in section 7.20). The reader will recall from Lenz's law (section 7.20) that this e.m.f. opposes the change which produces it; that is, the e.m.f. opposes the change in the current (which, in this case, is an increase in current). Consequently, the potential arrow v_L associated with the self-induced e.m.f. is shown to oppose the current flow.

Initially, the self-induced 'back' e.m.f. is equal to the supply voltage, E, so that the current in the circuit at time $t = 0$ is

$$i = \frac{\text{supply voltage} - v_L}{R} = \frac{(E - E)}{R} = 0$$

Since this e.m.f. opposes the current, the current does not rise suddenly to a value of $\frac{E}{R}$ (as it does in a purely resistive circuit which contains no inductance) but begins to rise at a steady rate.

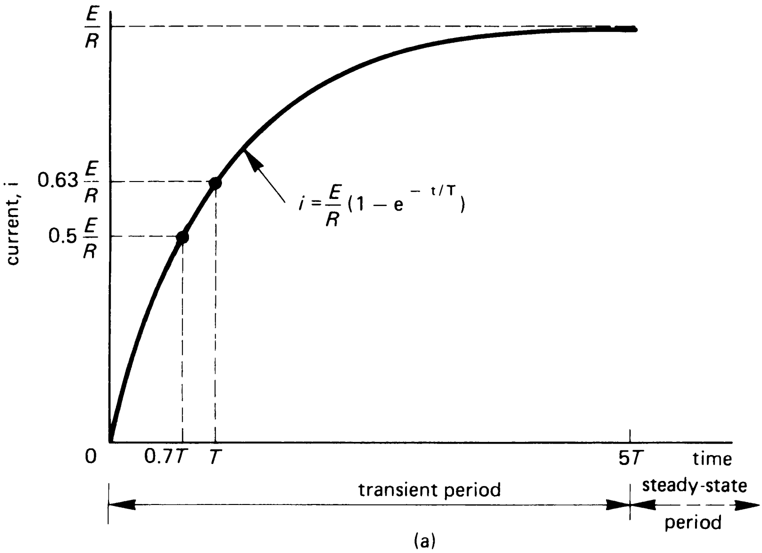
The induced e.m.f. is related to the rate of change of current (remember, induced e.m.f. = $L\ \frac{\Delta I}{\Delta t}$), and as the current increases in value, so its rate of rise diminishes (you can see that the slope of the current graph in Figure 7.15(a) gradually diminishes as the current increases in value). In fact the current builds up according to the relationship

$$i = \frac{E(1 - e^{-t/T})}{R}$$

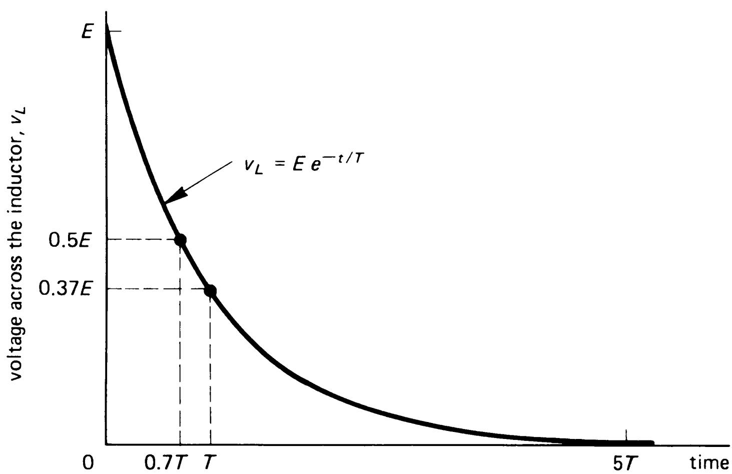
where $e = 2.71828$ (and is the base of the **natural** or **Naperian logarithms**) and T is the **time constant** of the R-L circuit. The time constant of the circuit is calculated from the equation

$$\text{time constant, } T = \frac{L}{R} \text{ seconds}$$

fig 7.15 *(a) rise of current in an inductive circuit and (b) the voltage across the inductor*



(a)



(b)

where L is the inductance in henrys and R is the resistance in ohms. For example, if $L = 150$ mH or 0.15 H and $R = 10\ \Omega$, then

$$T = \frac{150\ (\text{mH})}{10(\Omega)} = \frac{0.15\ (\text{H})}{10\Omega} = 0.015 \text{ s or } 15 \text{ ms}$$

Figure 7.15(a) shows that the current builds up to 50 per cent of its maximum value in a time of 0.7 of a time constant (which is 10.5 ms for the coil above), and reaches 63 per cent of its final value in a time of one time constant (15 ms in the above case) after the switch blade in Figure 7.14 is changed from A to B.

As the current in the circuit builds up in value, so the voltage across the inductance diminishes according to a decaying exponential curve with the formula

$$\text{voltage across } L, v_L = Ee^{-t/T} \text{ volts}$$

where, once again, e - 2.71828, T is the time constant of the circuit in seconds, and E is the supply voltage. The voltage across the inductive element, L, of the coil diminishes from a value equal to E at the instant that the supply is connected, to practically zero after a time of approximately $5T$ seconds. As shown in Figure 7.15(b), it has fallen to $\frac{E}{2}$ after a time of $0.7T$, and to a value of $0.37E$ after a time equal to T.

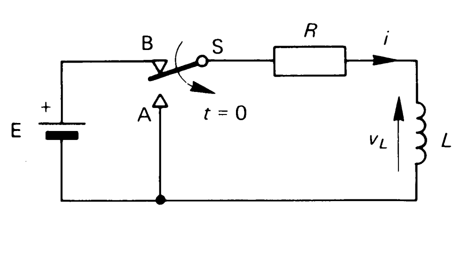
As mentioned above, the transient period of the curve is completed after about $5T$ seconds (or $5 \times 15 = 75$ ms in the case where $T = 15$ ms), by which time the current has risen practically to $\frac{E}{R}$ amperes, and the voltage across the inductive element L has fallen practically to zero.

7.25 DECAY OF CURRENT IN AN INDUCTIVE CIRCUIT

We will now study the effect of cutting off the current in an inductive circuit. The circuit is shown in Figure 7.16 and, at a new 'zero' time, the blade of switch S is changed from B to A; this simultaneously applies a short-circuit to the L-R circuit and disconnects it from the battery.

After this instant of time the magnetic flux in the coil begins to decay; however from Faraday' and Lenz's laws we see that a change in flux associated with the circuit induces an e.m.f. in the circuit. According to Lenz's law, the e.m.f. induced in the circuit opposes the change which produces it; in this case, the 'change' is a reduction in current. Clearly, the 'direction' of the induced e.m.f. is *in the same direction as the current* so

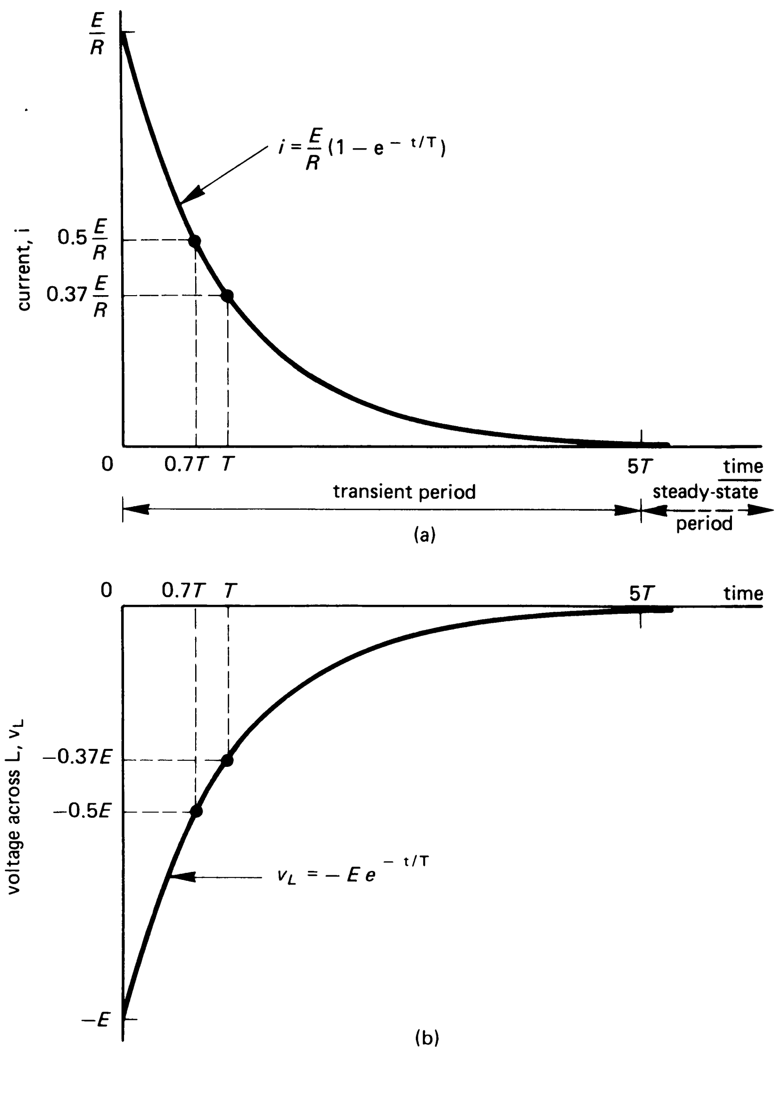
fig 7.16 *decay of current in an inductive circuit*



as to try to maintain its value at its original level. Clearly it cannot do this, but has the effect of decaying the fall in the current. Consequently, the 'direction' of the self induced e.m.f. in the coil in Figure 7.16 reverses when compared with its direction in Figure 7.14.

A curve showing the decay in current is illustrated in Figure 7.17(a),

fig 7.17 *waveforms during the current decay in an inductive circuit: (a) current in the circuit, (b) voltage across the inductive element*



and it follows the exponential law

$$\text{current, } i = \frac{E}{R} e^{-t/T} \text{ A}$$

where $\frac{E}{R}$ is the current in the inductor at the instant that the coil is short-circuited, that is, at $t = 0$; T is the time constant of the circuit $= \frac{L}{R}$ seconds, and $e = 2.71828$. The current decays to half its original value in $0.7T$ seconds and to 37 per cent of its original value in T seconds.

At the instant of time that the coil is short-circuited, that is, $t = 0$, the self-induced e.m.f. in the coil is equal to $-E$ (the negative sign merely implies that its polarity has reversed). This voltage decays to zero along another exponential curve whose equation is

$$\text{voltage across } L,\ v_L = -E\, e^{-t/T} \text{ V}$$

where e, E, t and T have the meanings described earlier. The corresponding curve is given in Figure 7.17(b).

The transient period of both curves in Figure 7.17 is complete after about five time constants ($5T$), and both curves reach zero simultaneously. For all practical purposes we may assume that after this length of time the current in the circuit is zero.

Since a special single-pole, double throw (s.p.d.t.) switch is used in Figure 7.16, it does not represent a practical method of discharging the energy stored in the inductive field. A practical case is considered in section 7.26.

7.26 BREAKING AN INDUCTIVE CIRCUIT

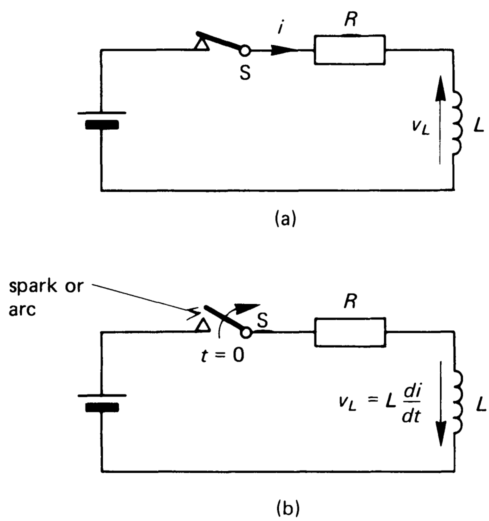
Consider the circuit in Figure 7.18(a) in which the current flows through the single-pole, single-throw (s.p.s.t.) switch S. At the instant of time that the switch contact is opened, that is, at $t = 0$, the switch tries to reduce the current in the circuit from a steady value to practically zero in zero time. According to the laws of electromagnetic induction, the self-induced e.m.f. in the inductor at this instant of time is

$$\text{induced e.m.f.} = \text{inductance, } L \times \text{rate of change of current}$$

Suppose that $L = 3$ H, and a current of 5 A is cut off in 5 ms. The self-induced e.m.f. in the inductor at the instant that the switch is opened is

$$\begin{aligned}\text{induced e.m.f.} &= L \times \text{rate of change of current}\\ &= L \times \frac{\text{current}}{\text{time}}\\ &= \frac{3 \times 5}{(5 \times 10^{-3})} = 3000 \text{ V}\end{aligned}$$

fig 7.18 *breaking the current in an inductive circuit using a simple switch; a spark or arc is produced at the contacts in (b)*

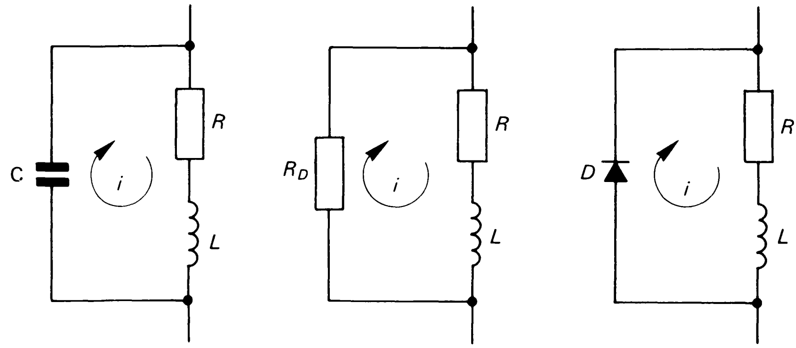


At the instant that the switch is opened, the 'direction' of the induced e.m.f. in the inductor is as shown in Figure 7.18(a) - see also the discussion on Lenz's law in section 7.25. Consequently, **the induced voltage of 3000 V is added to the supply voltage E, and the combined voltage appears between the opening contacts of the switch**. It is for this reason that a spark or an arc may be produced at the contacts of the switch when the current in the inductive circuit is broken. It may be the case that the supply voltage E has a low value, say 10 or 20 V but, none the less, a dangerous voltage is developed between the opening contacts of the switch.

In many applications, switches have been replaced by semiconductor devices such as transistors; these devices are very sinsitive to high voltage, and are easily damaged. Engineers have therefore looked for ways in which the high voltage produced by this means can be limited in value.

The common methods used are shown in Figure 7.19. The principle of all three circuits is the same; that is, when the current through the inductor is suddenly cut off, an alternative path is provided for the flow of the inductive current is produced by the decay of the magnetic field.

The inductor is represented in each diagram by R and L in series. In circuit (a) the inductive circuit is shunted by a capacitor; when the current in the main circuit is cut off, the energy in the magnetic field is converted into current i which flows through capacitor C. In circuit (b), the L-R circuit is shunted by a 'damping' resistor R_D; when the current is cut off in the main circuit, the induced current produced by the collapse of the fig 7.19 *methods of absorbing the inductive energy when the current is broken*



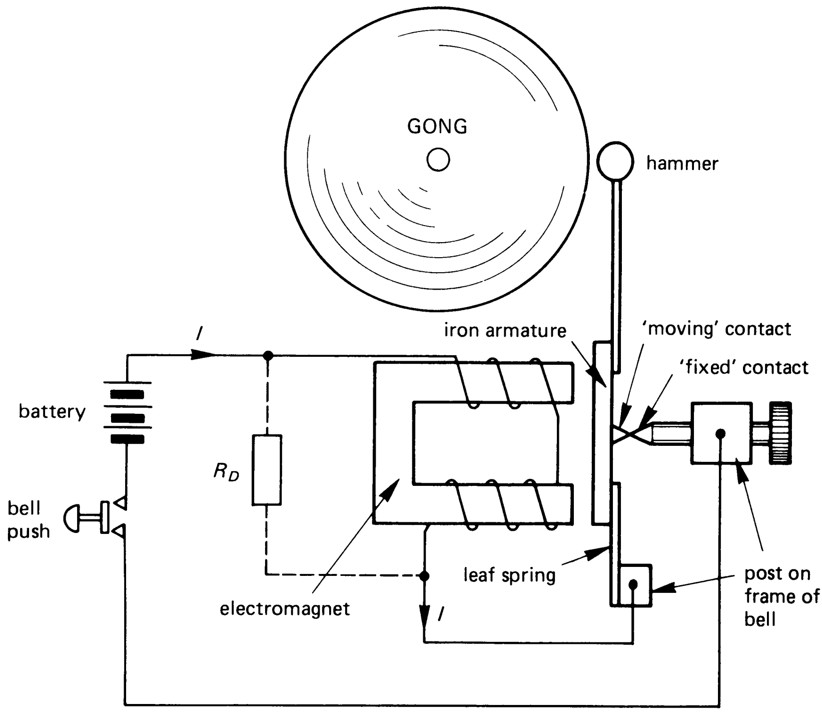
(a) (b) (c)

magnetic field flows through R_D. In circuit (c) the L-R circuit is shunted by a diode; in normal operation, the polarity of the supply is such that it reverse-biasses the diode (the principle of operation of the semiconductor diode is described in detail in Chapter 16), so that it does not conduct. When the current through the L-R circuit is cut off, the direction of the induced e.m.f. in the inductor forward-biasses the diode, causing the inductive energy to be dissipated. When used in this type of application, the diode is known as a **flywheel diode** or as a **spark quench diode**.

7.27 APPLICATIONS OF ELECTROMAGNETIC PRINCIPLES

A basic application with which everyone is familiar is the **electric bell.** A diagram showing the construction of a bell is **electric bell.** A diagram showing the construction of a bell is illustrated in Figure 7.20, its operation being described below. Initially, when the contacts of the bell push are open, the spring on the iron armature of the bell presses the 'moving' contact to the 'fixed' contact. When the bell push is pressed, the electrical circuit is complete and current flows in the bell coils, energising the electromagnet. The magnetic pull of the electromagnet is sufficiently strong to attract the iron armature against the pull of the spring so that the electrical connection between the fixed and moving contacts is broken, breaking the circuit.

However, the armature is attracted with sufficient force to cause the hammer to strike the gong. Now that the circuit is broken, the pull of the electromagnet stops, and the leaf-spring causes the armature to return to its original position. When it does so, the circuit contact between the fixed and moving contacts is 'made' once more, causing the electromagnet to be fig 7.20 *an electric bell*

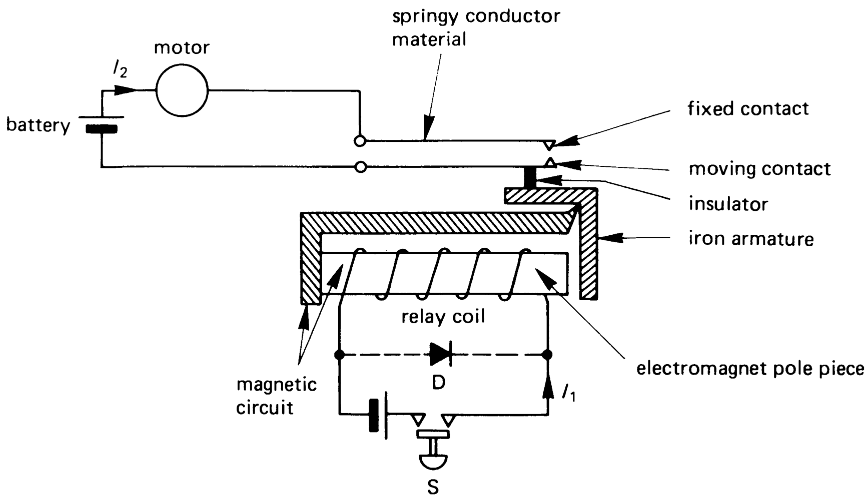


energised and the whole process repeated. Only when the bell push is released is the current cut off and the bell stops ringing.

As described earlier, the release of inductive energy when the fixed and moving contacts separate gives rise to a spark between the two contacts. Any one of the methods outlined in Figure 7.19 can be used to limit the sparking and, in Figure 7.20 a damping resistor R_D is shown in dotted line connected across the coil of the bell.

The **relay** is another popular application of electromagnetism (see Figure 7.21). The relay is a piece of equipment which allows a small value of current, I_1, in the coil of the relay to switch on and off a larger value of current, I_2, which flows through the relay contacts.

The *control circuit* of the relay contains the relay coil and the switch S; when S is open, the relay coil is de-energised and the relay contacts are open (that is, the relay has *normally open* contacts). The contacts of the relay are on a strip of conducting material which has a certain amount of 'springiness' in it; the tension in the moving contact produces a downward fig 7.21 *an electrical relay*



force which, when transferred through the insulating material keeps the iron armature away from the polepiece of the electromagnet.

When switch S is closed, current I_1 flows in the relay coil and energises the relay. The force of the electromagnet overcomes the tension in the moving contact, and forces the moving contact up to the fixed contact. This completes the electrical circuit to the motor, allowing current I_2 to flow in the load.

You might ask why switch S cannot be used to control the motor directly! There are many reasons for using a relay, the following being typical:

1. The current I_1 flowing in the relay coil may be only a few milliamperes, and is insufficient to control the electrical load (in this case a motor which may need a large current to drive it). Incidentally, the switch S may be, in practice, a transistor which can only handle a few milliamperes.
2. The voltage in the control circuit may not be sufficiently large to control the load in the main circuit.
3. There may be a need, from a safety viewpoint, to provide electrical 'isolation' between I_1 and I_2 (this frequently occurs in hospitals and in the mining and petrochemical industries).

Once again, there may be a need to protect the contacts of switch S against damage caused by high induced voltage in the coil when the current

I_1 is broken. The method adopted in Figure 7.21 is to connect a flywheel diode across the relay coil.

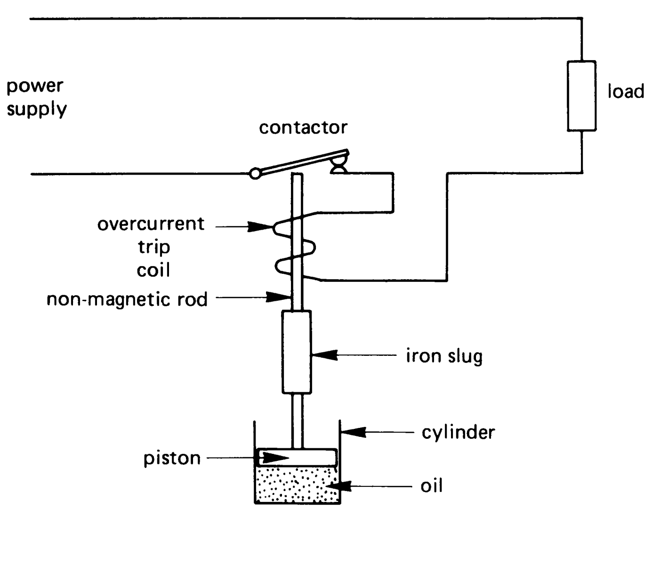
Yet another widely-used application of the electromagnetic principle is to the **overcurrent protection** of electrical equipment. You will be aware of the useof the fuse for electrical protection (see Chapter 5), but, in industry, this can be a relatively expensive method of protecting equipment (the reason is that once a fuse is 'blown' it must be thrown away and replaced by a new one). Industrial fuses tend to be much larger and more expensive than domestic fuses.

In industry, fuses are replaced, where possible, by **electromagnetic overcurrent trips**. A simplified diagram of one form is shown in Figure 7.22. The current from the power supply is transmitted to the load via a 'contactor' (which has been manually closed by an operator) and an overcurrent trip coil. This coil has a non-magnetic rod passing through it which is screwed into an iron slug which just enters the bottom of the overcurrent trip coil; the iron slug is linked to a piston which is an oil-filled cylinder or **dashpot**.

At normal values of load current, the magnetic pull on the iron slug is insufficient to pull the piston away from the drag of the oil, and the contacts of the contactor remain closed.

When an overcurrent occurs (produced by, say, a fault in the load) the current in the circuit rises to a value which causes the magnetic pull produced by the trip-coil to overcome the drag of the oil on the piston. This causes the rod and plunger to shoot suddenly upwards; the top part of the

fig 7.22 *electromagnetic overcurrent trip*



rod hits the contactor and opens the contact to cut off the current to the load. In this way the equipment is protected against overcurrent without the need for a fuse.

The 'value' of the tripping current can be mechanically adjusted by screwing the cylinder and iron slug either up or down to reduce or to increase, respectively, the tripping current.

SELF-TEST QUESTIONS

1. Explain the following terms (i) ferromagnetism, (ii) magnetic domain, (iii) magnetic pole, (iv) magnetic field, (v) direction of the magnetic field, (vi) solenoid, (vii) electromagnet, (viii) magnetic flux.
2. A coil is wound on a non-magnetic ring of mean diameter 15 cm and cross-sectional area 10 cm^2. The coil has 5000 turns of wire and carries a current of 5 A. Calculate the m.m.f. produced by the coil and also the magnetic field intensity in the ring. What is the value of the magnetic flux produced by the coil and the flux density in the ring?
3. Describe the magnetisation curve and the B–H loop of a ferromagnetic material. Explain why a ferromagnetic material 'saturates' when H has a large value. What is meant by 'remanent' flux density and 'coercive force' in connection with the B–H loop?
4. How do magnetically 'soft' and 'hard' materials differ from one another?
5. The relative permeability of a steel ring at a flux density of 1.3 T is 800. Determine the magnetising force required to produce this flux density if the mean length of the ring is 1.0 m.

   What m.m.f. is required to maintain a flux density of 1.3 T if a radial air gap of 1.0 mm is introduced into the ring? You can assume that there is no leakage of flux from the ring.
6. Explain the terms (i) self-induction, (ii) induction by motion and (iii) mutual induction.
7. Outline Faraday's and Lenz's laws and show how they are used to determine the magnitude and direction of an e.m.f. induced in a circuit.
8. Calculate the energy stored in the magnetic field produced by a 2.5 H inductor carrying a current of 10 A.
9. A series R–L circuit containing an inductor of 5 H inductance and a resistance of 10 ohms is suddenly connected to a 10-V d.c. supply. Determine (i) the time constant of the circuit, (ii) the initial value and the final value of the current taken from the supply, (iii) the time taken for the current to reach 0.5 A, (iv) the time taken for the transients in the circuit to have 'settled out', (v) the energy stored in the magnetic field.

SUMMARY OF IMPORTANT FACTS

A **magnetic field** in a **ferromagnetic material** is produced by **magnetic domains**. Lines of magnetic flux are said to **leave a N-pole and enter a S-pole**.

Like magnetic poles attract one another and **unlike magnetic poles repel one another.**

The **magnetomotive force (m.m.f.)** produced by an electromagnet causes a **magnetic flux** to be established in the **magnetic circuit**. The **magnetic field intensity**, H, is the m.m.f. per unit length of the magnetic circuit. The **magnetic flux density**, B, is related to H by the equation

$$B = \mu H$$

where μ is the **permittivity** of the magnetic circuit and

$$\mu = \mu_O \mu_r$$

where μ_O is the permittivity of free space ($= 4\pi \times 10^{-7}$) and u_r is the **relative permittivity** of the material (a dimensionless number).

The **magnetisation curve** for a magnetic material relates B to H for the material. A ferromagnetic material exhibits the property of **magnetic saturation**. The ***B-H* loop** of the material shows how B and H vary when the magnetising force is increased first in one direction and then in the reverse direction. A **soft** magnetic material has a narrow B-H loop, and a **hard** magnetic material has a flat-fopped 'wide' B-H loop.

The effective 'resistance' of a magnetic circuit to magnetic flux is known as its **reluctance**, S. The relationship between the flux (Φ), the reluctance and the m.m.f. (F) is **(Ohm's law for the magnetic circuit)**:

$$F = \Phi S$$

Equipment can be **screened** from a strong magnetic field by surrounding it with a material of low reluctance.

An e.m.f. may be induced in a circuit either by **self-induction**, or **induction by motion in a magnetic field**, or by **mutual induction**. The magnitude and 'direction' of the induced e.m.f. can be predicted using **Faraday's laws** and **Lenz's law.**

The **energy stored**, W, in a magnetic field is given by the equation

$$W = \frac{LI^2}{2}$$

The **time constant**, T, of an L-R circuit is $\frac{L}{R}$ seconds (L in henrys, R in ohms). When an L-R circuit is connected to a d.c. supply, the *final value of the current* in the circuit is $\frac{E}{R}$ amperes, where E is the applied voltage.

The current reaches 63 per cent of its final value after T seconds, and it takes about $5T$ seconds for the transients to 'settle out'. When the voltage applied to the R-L circuit is reduced to zero, it takes T seconds for the current to decay to 37 per cent of its initial value. The transients in the circuit disappear after about $5T$ seconds.

ELECTRICAL GENERATORS AND POWER DISTRIBUTION

8.1 PRINCIPLE OF THE ELECTRICAL GENERATOR

The early activities of scientific pioneers led to the idea that magnetism and electricity were interrelated with one another. It was found that when a permanent magnet was moved towards a coil of wire - see Figure 8.1 - an e.m.f. was induced in the coil. That is, the e.m.f. is induced by the relative movement between the magnetic field and the coil of wire. Lenz's law allows us to predict the polarity of the induced e.m.f.

Lenz's law says that the induced e.m.f. acts to circulate a current in a direction which opposes the change in flux which causes the e.m.f. When the N-pole of the magnet in Figure 8.1 approaches the left-hand end of the coil of wire, the current induced in the coil circulates in a manner to reduce the flux entering the left-hand end of the coil. That is, the current tries to produce a N-pole at the left-hand end of the coil; the effect of the pole produced by the induced current opposes the effect of the approaching magnet. The net result is that the induced current flows in the direction shown in the figure; current therefore *flows out* of terminal Z of the coil (terminal Z is therefore the positive 'pole' of induced e.m.f.).

Consider now the effect of moving the N-pole of the magnet *away* from the coil. When this occurs, the magnetic flux *entering* the left-hand end of the coil is reduced. Lenz's law says that the direction of the induced current is such as to oppose any change. That is, the current induced in the coil must now act to *increase* the amount of flux entering the left-hand end of the coil. The direction of the induced current in the coil must therefore reverse under this condition when compared with the current in Figure 8.1.

If the magnet is repeatedly moved towards the coil and then away from it, the current leaving terminal Z flows alternately away from the coil and then towards it (see Figure 8.2). That is to say, **alternating current** (a.c.) is induced in the coil.

fig 8.1 *the principle of electricity generation*

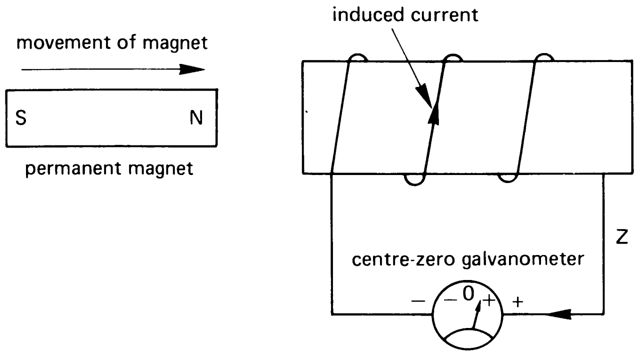
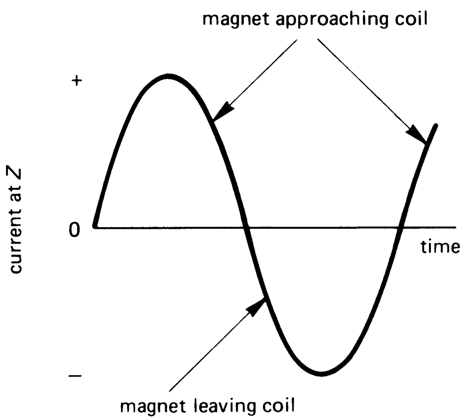


fig 8.2 *a sinusoidal alternating current waveform*

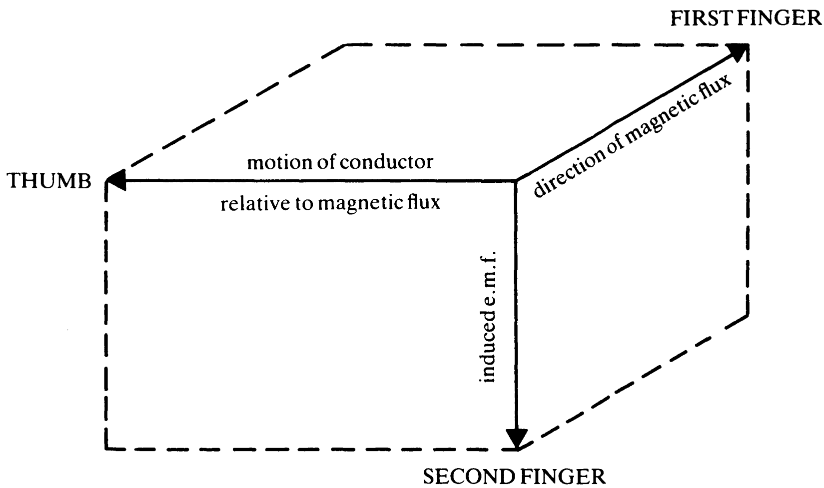


8.2 THE DIRECTION OF INDUCED e.m.f. - FLEMING'S RIGHT-HAND RULE

When studying electrical generators, it is useful to know in which direction the current is urged through a conductor when it moves in a magnetic field. You can predict the direction of the induced e.m.f. by means of **Fleming's right-hand rule** as follows:

> **If the thumb and the first two fingers of the right hand are mutually held at right-angles to one another, and the first finger points in the direction of the magnetic field while the thumb points in the direction of the movement of the conductor relative to the field, then the second finger points in the direction of the induced e.m.f.**

fig 8.3 *Fleming's right-hand rule*

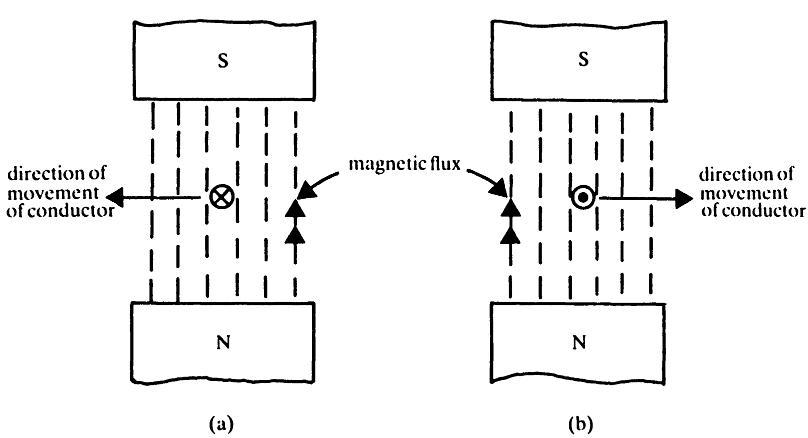


This is illustrated in Figure 8.3 and is summarised below:

*F*irst finger - direction of magnetic *F*lux
s*E*cond finger - direction of induced *E*.m.f.
thu*M*b - *M*otion of the conductor *relative to the flux*.

Examples of the application of Fleming's right-hand rule are given in Figure 8.4.

fig 8.4 *applications of Fleming's right-hand rule*



8.3 ALTERNATORS OR a.c. GENERATORS

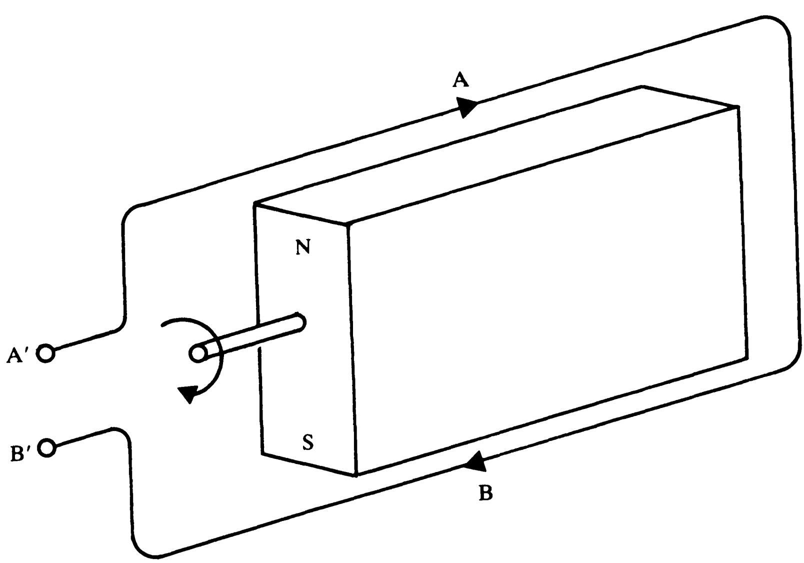
The national electricity supply system of every country is an alternating current supply; in the United Kingdom and in Europe the polarity of the supply changes every $\frac{1}{50}$ s or every 20 ms, and every $\frac{1}{60}$ s or 16.67 ms in the United States of America.

The basis of a simple alternator is shown in Figure 8.5. It comprises a rotating permanent magnet (which is the rotating part or **rotor**) and a single-loop coil which is on the fixed part or the **stator** of the machine. The direction of the induced current in the stator winding at the instant illustrated is as shown in the figure (you can verify the direction of current by using Fleming's right-hand rule). You will see that at this instant of time, current flows into terminal *A*' and out of terminal *B*' (that is, terminal B' is positive with respect to A' so far as the external circuit is concerned).

When the magnet has rotated through 180°, the S-pole of the magnet passes across conductor *A* and the N-pole passes across conductor *B*. The net result at this time is that the induced current in the conductors is reversed when compared with Figure 8.5. That is, terminal B' is negative with respect to A'.

In this way, alternating current is induced in each turn of wire on the stator of the alternator. In practice a single turn of wire can neither have enough voltage induced in it nor carry enough current to supply even one

fig 8.5 *a simple single-loop alternator*



electric light bulb with electricity. A practical alternator has a stator winding with many turns of wire on it, allowing it to deal with high voltage and current. The winding in such a machine is usually *distributed* around the stator in many *slots* in the iron circuit (see Figure 8.6). The designer arranges the coil design so that the alternator generates a voltage which follows a sinewave, that is, the voltage waveform is *sinusoidal* (see Figure 8.7).

The generated e.m.f. rises to a **maximum value** of E_m after 90° rotation. After a further 90° the induced e.m.f. is zero once more and, after a further 90°, the e.m.f. reaches its maximum negative voltage of $-E_m$. A further 90° rotation brings the e.m.f. back to its starting value of zero once more. Figure 8.7 shows one complete **cycle** of the alternating voltage waveform, and the time taken for one complete cycle is known as the **periodic time** of the waveform. The frequency, f, of the wave is the number of complete *cycles per second*, and the frequency of the wave is given in *hertz* (Hz). In the United Kingdom the frequency of the mains power supply is 50 Hz, and the periodic time of the power supply is

$$\text{periodic time} = \frac{1}{\text{frequency}}\ (\text{Hz}) = \frac{1}{f}$$

$$= \frac{1}{50} = 0.02 \text{ s or } 20 \text{ ms}$$

Angular frequency

Engineers use the **radian** for angular measurement rather than the degree. One complete cycle (360°) of the wave is equivalent to 2π radians (abbreviated to rad); that is

$$360^\circ \text{ is equivalent to } 2\pi \text{ rad}$$

so that

$$180^\circ \text{ is equivalent to } \pi \text{ rad}$$

and

$$90^\circ \text{ is equivalent to } \frac{\pi}{2} \text{ rad, etc.}$$

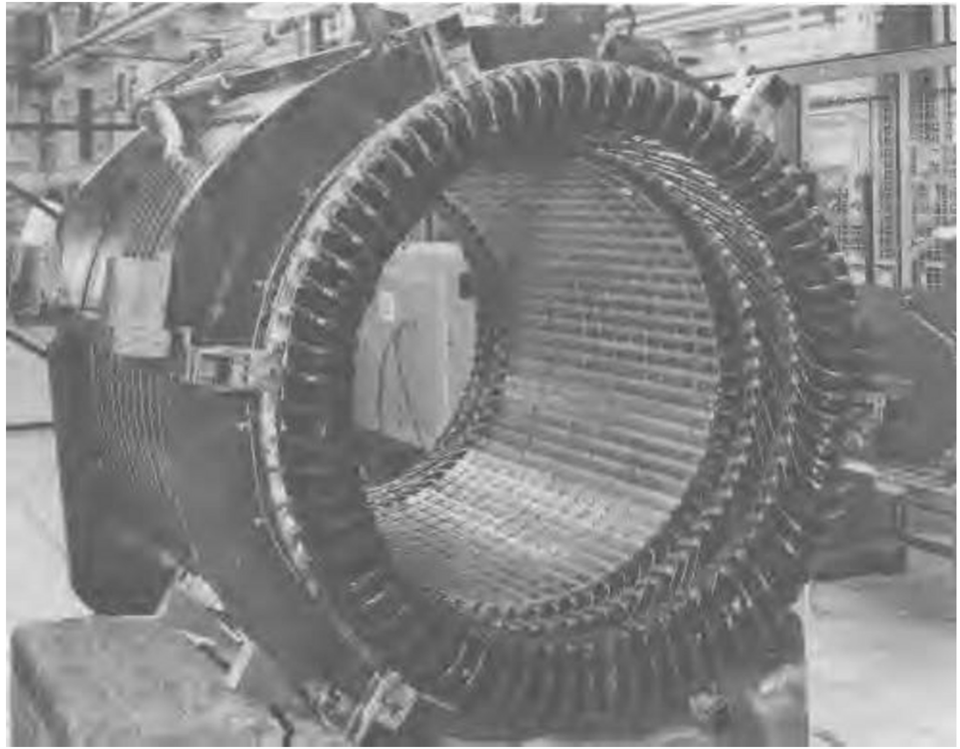
Hence

$$1 \text{ rad} = \frac{360}{2\pi} \text{ degrees} = 57.3^\circ$$

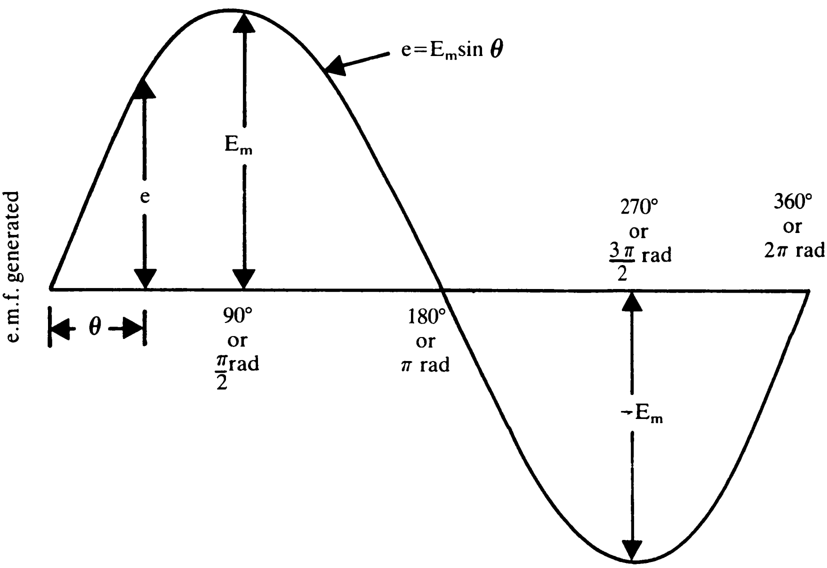
If the frequency of the electricity supply is f Hz or cycles per second, the supply has an **angular frequency**, ω, radians per second of

$$\omega = 2\pi f \text{ rad/s}$$

fig 8.6 *the stator winding of a 3750 k VA alternator*



Reproduced by kind permission of G.E.C. Machines fig 8.7 *sinusoidal voltage waveform*



In the UK the angular frequency of the supply is

$$2\pi \times 50 = 314.2 \text{ rad/s}$$

and in the US it is 377 rad/s.

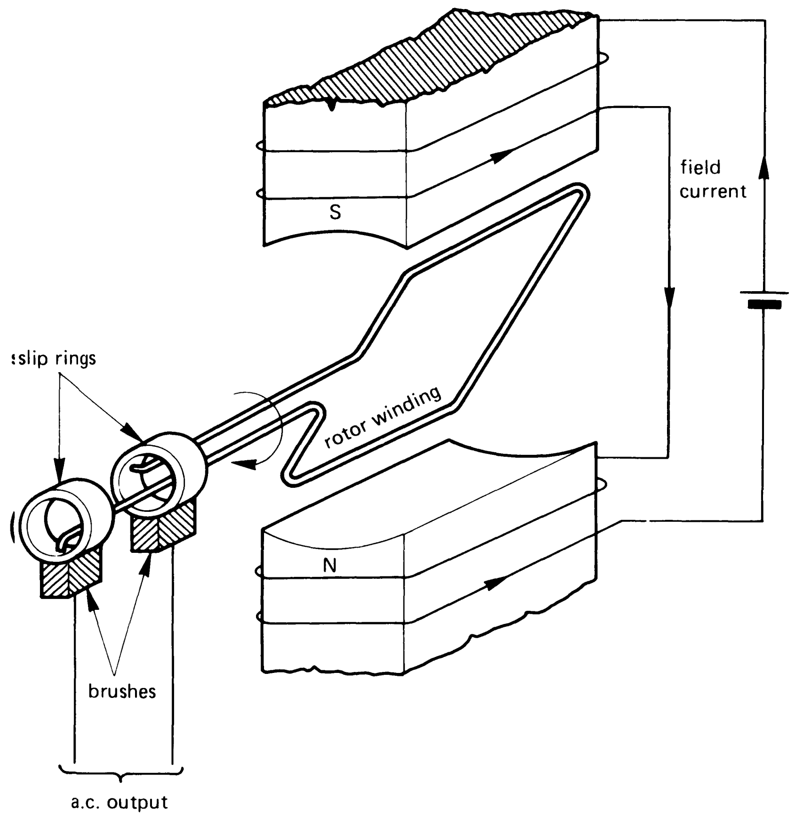
Slip rings

Some alternators use a design which is an 'inversion' of that in Figure 8.5; that is, the magnetic field system is on the stator and the conductors are on the rotor (see Figure 8.8). In this case the field system is an electromagnet excited by a d.c. power supply (in practice it would probably be energised by a d.c. generator known as an **exciter**). The current induced in the rotor winding is connected to a pair of rings, known as **slip rings**, made from either copper, brass or steel; the current is collected from the slip rings by means of carbon brushes which have very little frction and yet have very little voltage drop in them.

In practice, the rotor of the alternator in Figure 8.5 is also an electromagnet which is energised from a d.c. supply via a pair of slip rings.

8.4 SINGLE-PHASE AND POLY-PHASE a.c. SUPPLIES

The alternator described produces a single alternating voltage known as a **single-phase supply**. For a number of reasons which are discussed in fig 8.8 *a simple alternator with slip rings*



Chapter 13, this type of supply has been replaced for the purpose of national and district power supplies by a **poly-phase supply**.

A poly-phase generator can be thought of as one which has a number of windings on it (usually three windings are involved but strictly speaking, there could be almost any number of windings), each producing its own voltage. This type of generator provides a more economic source of power than several individual single-phase systems.

The National Grid System (both in the UK and other countries) is a three-phase system, and provides the best compromise in terms of overall efficiency.

8.5 EDDY CURRENTS

When a conductor moves through or 'cuts' a magnetic field, an e.m.f. is induced in it. The 'conductor' may, quite simply, be the iron part of a magnetic circuit of a machine such as the rotor of a generator, that is, it may not actually form part of the electrical circuit of the generator. The induced voltage causes a current to flow in the iron 'conductor'; such a current is known as an **eddy current**.

In a number of cases, the 'conductor' may not physically 'move' but may yet have an eddy current induced in it. This occurs in the magnetic circuit of a transformer which carries a pulsating magnetic flux. The pulsation of magnetic flux in the iron induce an eddy current in the iron.

The eddy current simply circulates within the iron of the machine and does not contribute towards the useful output of the machine. Since the resistance of the iron is high, the eddy current causes a power loss (an I^2R loss) known as the **eddy-current loss**, which is within the general body of the machine and reduces its overall efficiency; in most cases it is clearly to everyone's advantage to reduce eddy currents within the machine. However, there are applications in which eddy currents are used to heat a metal; for example, iron may be melted in an **induction furnace**, in which eddy currents at a frequency of 500 Hz or higher are induced in order to melt the iron.

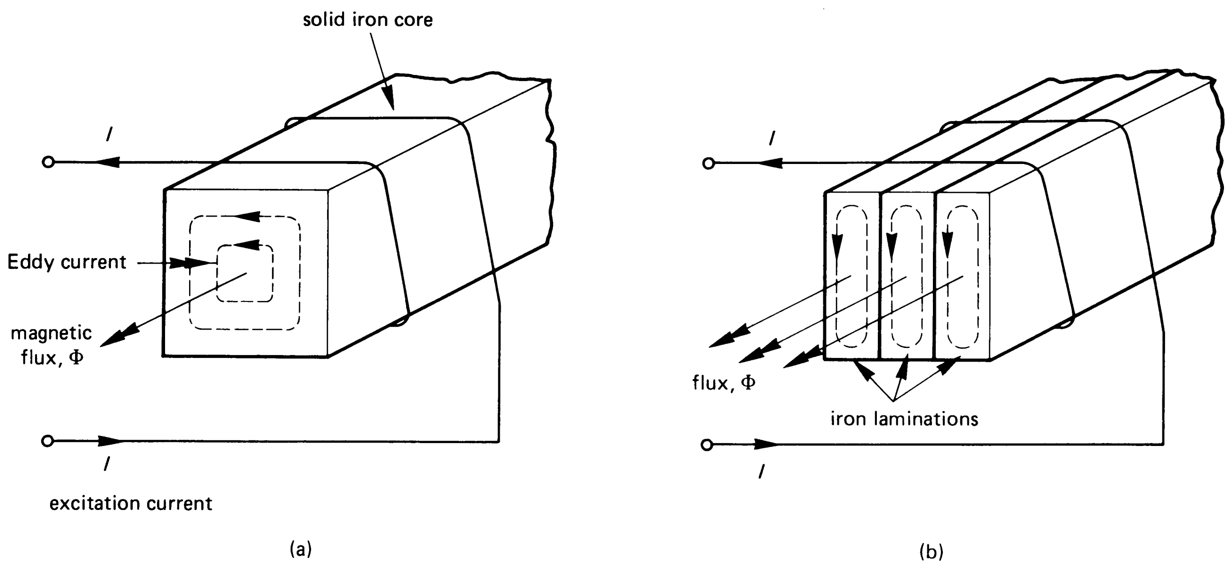
In the following we study the mechanics of the production of eddy currents; this study leads us to the method used in electrical machines to reduce the induced current.

Consider the solid iron core in Figure 8.9(a) which is wound with a coil or wire carrying an alternating current; at a particular instant of time the current flows in the direction shown and is increasing in value. The direction of the magnetic flux in the iron core is predicted by applying the screw rule (see Chapter 7). Since the excitation current, I, is increasing in value, the magnetic flux in the core is also increasing; the increase in flux in the core induces the eddy current in the iron, giving rise to the eddy-current power-loss.

The eddy current can only be reduced by increasing the resistance of the eddy-current path. The usual way of doing this is to divide the solid iron core into a 'stack' of iron **laminations** as shown in Figure 8.9(b). To prevent eddy-current flow from one lamination to another, they are lightly insulated from one another by either a coat of varnish or an oxide coat. Since the thickness of each lamination is much less than that of the solid core, the electrical resistance of the iron circuit has increased so that the eddy current is reduced in value. There is therefore a significant reduction in the eddy-current power-loss when a laminated magnetic circuit is used.

It is for this reason that the iron circuit of many electrical machines (both d.c. and a.c.) is laminated.

fig 8.9 *(a) eddy currents induced in the iron circuit, (b) reduction of eddy currents by laminating the iron circuit*



8.6 DIRECT CURRENT GENERATORS

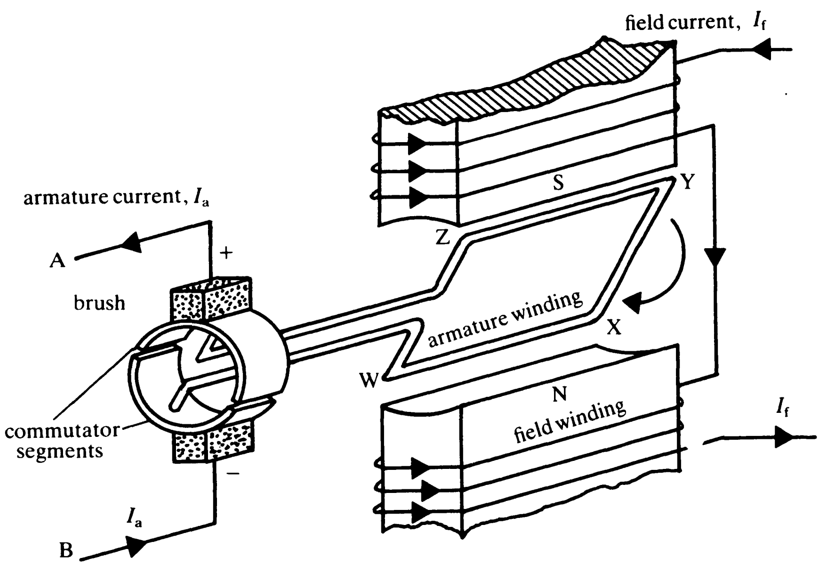
A **direct current** (d.c.) power supply can be obtained by means of a generator which is generally similar to the alternator in Figure 8.8, the difference between the a.c. and d.c. generators being the way in which the current is collected from the rotating conductors.

Basically, a d.c. generator consists of a set of conductors on the rotating part or **armature** of the d.c. machine, which rotate in the magnetic-field system which is on the fixed part or **frame** of the machine. A diagram of a simple d.c. generator with a single-loop armature is shown in Figure 8.10 (the field winding is excited from a separate d.c. supply [not shown]).

You will see from the diagram that each armature conductor alternately passes a N-pole then a S-pole, so that each conductor has an *alternating voltage* induced in it. However, the current is collected from the conductors by means of a **commutator** consisting of a cylinder which is divided axially to give two segments which enable the alternating current in the conductors to be 'commutated' or 'rectified' into direct current in the external circuit. The way the commutator works is described below.

In the diagram, the conductor WX is connected to the lower segment of the commutator, and the conductor YZ is connected to the upper segment. At the instant of time shown, the e.m.f. in the armature causes current to flow from W to X and from Y to Z; that is, current flows out of the upper commutator segment and into the lower commutator segment.

fig 8.10 *a simple single-loop d.c. generator*



When the armature has rotated through 180°, the conductor WX passes the S-pole of the field system and conductor YZ passes the N-pole. This causes the current in the armature conductors to reverse but, at this time, the position of the commutator segments have also reversed so that the current once again flows out of the upper segment and into the lower segment.

The function of the commutator in a d.c. machine is to ensure that the external circuit is connected via the brushgear to a fixed point in space inside the armature, i.e., to a point of fixed polarity.

In Figure 8.10, point A in the external circuit is *always* connected to a conductor in the armature which is moving under the S-pole, and point B is *always* connected to a conductor which is moving under the N-pole.

In practice the armature winding is a coil having many hundreds of turns of wire which is tapped at several dozen points, each tapping point being connected to an individual commutator segment. Each segment is separated from the next one by an insulating material such as mica.

8.7 SIMPLIFIED e.m.f. EQUATION OF THE d.c. GENERATOR

As mentioned earlier, the e.m.f. induced in a conductor in a generator is proportional to the rate at which it cuts the magnetic flux of the machine. That is:

induced e.m.f. α rate of change of flux

The rate at which the conductor cuts the flux is dependent on two factors, namely:

1. the flux produced by each pole, Φ
2. the speed of the armature, n rev/s

Hence

induced e.m.f., $E \;\alpha\; \Phi n$

The above equation is converted into a 'practical' equation by inserting a constant of proportionality, K_E, which relates the induced e.m.f. to the rate of change of flux. That is

induced e.m.f., $E = K_E \Phi n$

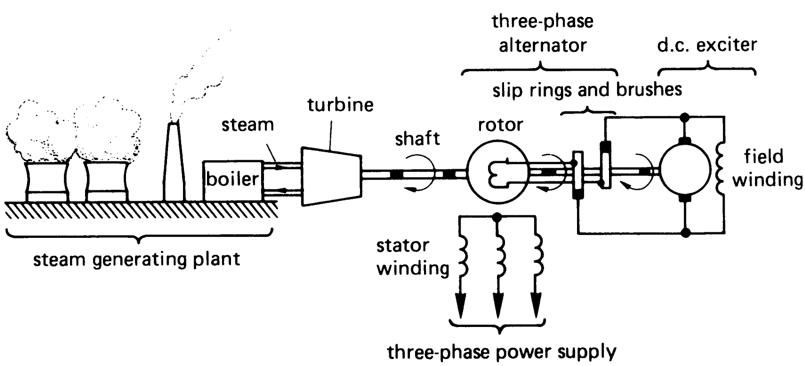
For a given d.c. generator, the value of K_E depends on certain 'fixed' factors in the machine such as the length and diameter of the armature, the dimensions of the magnetic circuit, and the number of conductors on the armature.

8.8 AN ELECTRICITY GENERATING STATION

The basis of an electrical generating plant is shown in Figure 8.11. The power station is supplied with vital items such as water and fuel (coal, oil, nuclear) to produce the steam which drives the turbine round (you should note that other types of turbine such as water power and gas are also used). In turn, the turbine drives the rotor of the alternator round. As shown in the figure, the rotor of the alternator carries the field windings which are excited from a d.c. generator (which is mechanically on the same shaft as the alternator) via a set of slip rings and brushes.

The stator of the alternator has a three-phase winding on it, and provides power to the transmission system. The voltage generated by the alternator could, typically, be 6600 V, or 11000 V, or 33000 V.

fig 8.11 *a simplified diagram of en electrical power station*



8.9 THE a.c. ELECTRICAL POWER DISTRIBUTION SYSTEM

One advantage of an a.c. supply when compared with a d.c. supply is the ease with which the voltage level at any point in the system can be 'transformed' to another voltage level. The principle of operation of the electrical transformer is described in detail in Chapter 14 but, for the moment, the reader is asked to accept that it is a relatively easy task to convert, say, a 6.6 kV supply to 132 kV, and vice versa.

In its simples terms, electrical power is the product of voltage and current and, if the power can be transmitted at a high voltage, the current is correspondingly small. For example, if, in system A, power is transmitted at 11 kV and, in system *B*, it is transmitted at 33 kV then, for the same amount of power transmitted, the current in system A is three times greater than that in system B. However, the story does not finish there because:

1. the voltage drop in the transmission lines is proportional to the current in the lines;
2. the power loss in the resistance of the transmission lines is proportional to (current)^2 [remember, power loss = I^2R].

Since the current in system A is three times greater than that in system B, the voltage drop in the transmission lines in system A is three times greater than that in system B, and the power loss is nine times greater!

This example illustrates the need to transmit electrical power at the highest voltage possible. Also, since alternating voltages can easily be transformed from one level to another, the reason for using an a.c. power system for both national and local power distribution is self-evident.

The basis of the British power distribution system is shown in Figure 8.12 (that of other countries is generally similar, but the voltage levels may differ). Not all the stages shown in the figure may be present in every system. The voltage levels involved may be typically as follows:

Power station: 6.6, 11 or 33 kV, three phase
Grid distribution system: 132, 275 or 400 kV, three phase
Secondary transmission system: 33, 66 or 132 kV, three phase
Primary distribution system: 3.3, 6.6, 11 or 33 kV, three phase
Local distribution system: 415 three-phase or 240 V single phase.

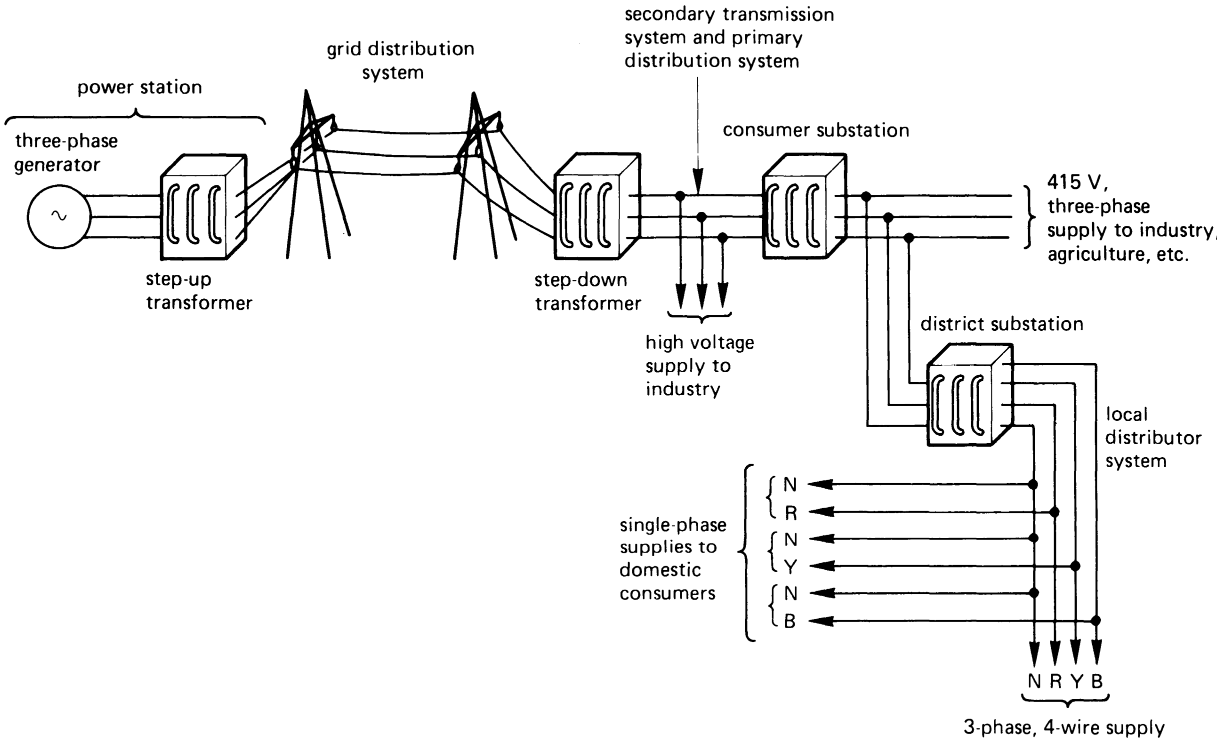
8.10 d.c. POWER DISTRIBUTION

For certain limited applications, power can be transmitted using direct current. The advantages and disadvantages of this when compared with a.c. transmission are listed below.

Advantages

1. A given thickness of insulation on cables can withstand a higher direct voltage than it can withstand alternating voltage, giving a smaller overall cable size for d.c. transmission.
2. A transmission line has a given cable capacitance and, in the case of an a.c. transmission system this is charged continuously. In the case of a d.c. transmission system, the charging current only flows when the line is first energised.
3. The self-inductance of the transmission line causes a voltage drop when a.c. is transmitted; this does not occur when d.c. is transmitted.

fig 8.12 *the national power distribution system*



Disadvantages

1. Special equipment is needed to change the d.c. voltage from one level to another, and the equipment is very expensive.
2. D.c. transmission lends itself more readily to 'point-to-point' transmission, and problems arise if d.c. transmission is used on a system which is 'tapped' at many points (as are both the national grid system and the local power distribution system.

Clearly, d.c. transmission is financially viable on fairly long 'point-to-point' transmission systems which have no 'tapping' points.

Practical examples of this kind of transmission system include the cross-channel link between the UK grid system and the French grid system via a d.c. undersea cable link. A number of islands throughout the world are linked either to the mainland or to a larger island via a d.c. undeasea cable link. In any event, power is both generated and consumed as alternating current, the d.c. link being used merely as a convenient intermediate stage between the generating station and the consumer. The method by which a.c. is converted to d.c. and vice versa is discussed in Chapter 16.

8.11 POWER LOSS AND EFFICIENCY

No machine is perfect, and power loss occurs in every type of electrical device. In machines the power losses can be considered as those which are

1. mechanical in origin
2. electrical in origin.

Power loss which has a **mechanical origin** arises from one of two sources, namely

A. friction power loss
B. windage power loss.

Frictional power loss is due to, say, bearing friction, and is proportional to speed; the faster the machine runs, the greater the friction loss.

Windage power loss or *ventilation* power loss is due to the effort needed to circulate the ventilation (wind) to cool the machine.

Power loss which is **electrical in origin** occurs either in the conductors themselves (the I^2R loss) or in the iron circuit of the machine.

The I^2R loss or **copper loss** (so called because copper is the usual conductor material [in many cases, aluminium is used!]) occurs because of the current flow in the conductors in the system.

The **iron loss,** P_O, is due to the combination of two types of power loss:

1. the **hysteresis power-loss,** P_n, occurs because the state of the magnetisation of certain parts of the magnetic circuit is reversed at regular intervals; work is done in reversing the magnetic 'domains' when this occurs;
2. the **eddy-current loss,** which is due to the heat generated by the flow of eddy currents in the iron circuit.

The iron loss is also known as the **core loss,** P_C. If the supply voltage and frequency are constant, the iron loss is a *constant power loss* and is independent of the load current.

8.12 CALCULATION OF THE EFFICIENCY OF A MACHINE

The efficiency, symbol η, of an electrical machine is the ratio of the electrical power output from the machine to its power input, as follows

$$\text{efficiency}, \eta = \frac{\text{output power}}{\text{input power}}$$

However,

$$\text{output power} = \text{input power} - \text{power loss}$$

and

$$\text{input power} = \text{output power} + \text{power loss}$$

where

$$\begin{aligned}\text{power loss} &= \text{electrical power loss} + \text{mechanical power loss}\\ &= I^2R \text{ loss} + P_O \text{ loss} + \text{friction loss} + \text{windage loss}\end{aligned}$$

Note: not all of the above losses appear in every electrical machine (for instance, the transformer does not have any moving parts and does not, therefore, have either friction or windage losses). The efficiency of an electrical machine can therefore be expressed in one of the following forms:

$$\begin{aligned}\text{efficiency}, \eta &= \frac{\text{output power}}{\text{output power} + \text{losses}}\\ &= \frac{\text{input power} - \text{losses}}{\text{input power}}\end{aligned}$$

In both the above equations, the denominator is greater than the numerator, so that the efficiency of electrical machines can never be 100 per cent (although in many cases, the efficiency is very high).

The result of each of the above equations is a value which is less than unity; for this reason the basic unit of efficiency is expressed as a **per unit** or p.u. value. For example, if the output power from a generator is 90 kW and the mechanical input power is 100 kW, the efficiency is

$$\text{efficiency, } \eta = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{90}{100} = 0.9 \text{ per unit or p.u.}$$

It is sometimes convenient to express the efficiency in a **per cent** form as follows

per cent efficiency = per unit efficiency × 100

so that a 0.9 per unit efficiency corresponds to a 90 pr cent efficiency.

Example
An electrical generator provides an output power of 270 kW. The electrical losses in the machine are found to be 36 kW and the friction and windage loss is 12 kW. Calculate the mechanical input power to the machine and its overall efficiency.

Solution

$$\begin{matrix}\text{Mechanical} \\ \text{input power}\end{matrix} = \begin{matrix}\text{electrical} \\ \text{output power}\end{matrix} + \begin{matrix}\text{electrical} \\ \text{power loss}\end{matrix} + \begin{matrix}\text{mechanical} \\ \text{power loss}\end{matrix}$$

$$= 270 + 26 + 12 = 318 \text{ kW (Ans.)}$$

and

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{270}{318}$$

$$= 0.849 \text{ p.u. or } 84.9 \text{ per cent (Ans.)}$$

SELF-TEST QUESTIONS

1. Draw a diagram of an alternator and explain its operation. If the alternator has 4 poles and its speed of rotation is 6000 rev/min, calculate the frequency of the a.c. supply.

2. Convert (i) 60° and (ii) 120° into radians and convert (iii) 1.6 radians and (iv) 4.6 radians into degrees.
3. Compare the relative merits of single-phase and three-phase supplies.
4. What is meant by an 'eddy current'? How are eddy currents reduced in value in an electrical machine?
5. Explain the principles of operation of a d.c. generator. Compare the functions of 'slip rings' and a 'commutator'.
6. Draw a simplified diagram of an electricity generating station and explain its operation. Draw also a diagram of the power distribution network and describe the purpose of each section of the distribution system.
7. Compare a.c. and d.c. distribution systems.
8. Write down a list of power losses which occur in an electrical machine and explain how it affects the efficiency of the machine.

SUMMARY OF IMPORTANT FACTS

An **alternator** or **a.c. generator** has a rotating part known as a **rotor** and a stationary part or **stator**. The corresponding parts of a d.c. generator are the **armature** and **frame**, respectively. The direction of the current in the conductors can be predicted by **Fleming's right-hand rule.** Current is collected from the **commutator** of a d.c. generator.

The **frequency**, f, in hertz (Hz) of an alternating wave is related to the **periodic time**, T, in seconds by the equation $f = \frac{1}{T}$.

Angles are measured in either **degrees** or **radians**, the two being related as follows:

$$\text{radians} = 2\pi \times \frac{\text{degrees}}{360}$$

Angular frequency, ω in rad/s is given by

$$\omega = 2\pi f \text{ rad/s}$$

where f is the supply frequency in Hz.

Power is distributed through the **National Grid** system using a **three-phase, three-wire** system, and is distributed to domestic consumers using a **three-phase, four-wire** system; each domestic consumer uses power taken from **one phase** of this system. Power is transmitted across short sea passages such as the English Channel using **high voltage direct current** distribution.

An **eddy current** is a current which is induced in the **iron circuit** of an electrical machine (which could be either an a.c. machine or a d.c. machine) whenever the magnetic flux linking with the iron changes. This gives rise to an **eddy-current power-loss** in the iron, causing it to heat up. Eddy currents are minimised by using a **laminated** iron circuit.

Power loss in an electrical machine may either be **mechanical** in origin, that is, due to **friction** or **windage**, or it may be **electrical** in origin, that is, due either to I^2R **loss**, or to **hysteresis loss**, or to **eddy-current loss**. The **efficiency** of an electrical machine is given by the ratio of the output power to the input power and is either expressed in **per unit** or in **per cent**.

CHAPTER 9

DIRECT CURRENT MOTORS

9.1 THE MOTOR EFFECT

The **motor effect** can be regarded as the opposite of the **generator effect** as follows. In a generator, when a conductor is moved through a magnetic field, a current is induced in the conductor (more correctly, an e.m.f. is induced in the conductor, but the outcome is usually a current in the conductor). In a motor, a current-carrying conductor which is situated in a magnetic field experiences a force which results in the conductor moving (strictly speaking, the force is on the *current and not on the conductor*, but the current and the conductor are inseparable).

9.2 THE DIRECTION OF THE FORCE ON THE CURRENT-CARRYING CONDUCTOR - FLEMING'S LEFT-HAND RULE

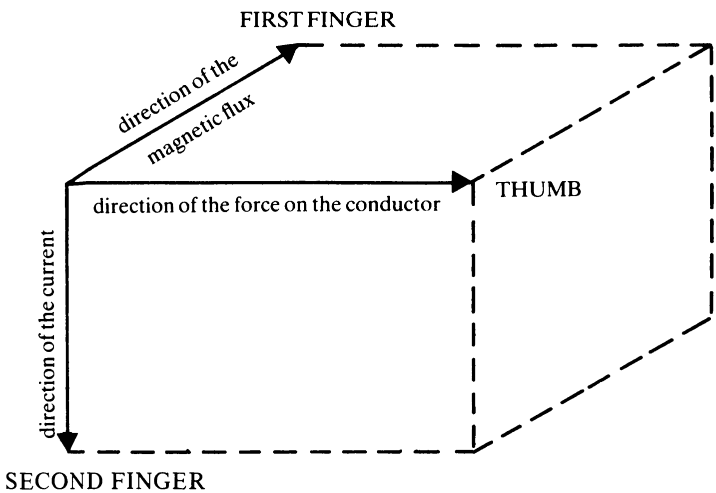
Fleming's left-hand rule allows you to predict the direction of the force acting on a current-carrying conductor, that is, it allows you to study the effect of *motor* action (Fleming's left-hand rule is for **motor action** [you can remember which rule is for motors by the fact that in the UK *motors drive on the left hand side of the road*]). The left-hand rule is as follows:

> **If the thumb and first two fingers of the left hand are mutually held at right-angles to one another, and the first finger points in the direction of the magnetic field whilst the second finger points in the direction of the current, then the thumb indicates the direction of the force on the conductor.**

This is illustrated in Figure 9.1 and can be summarised by:

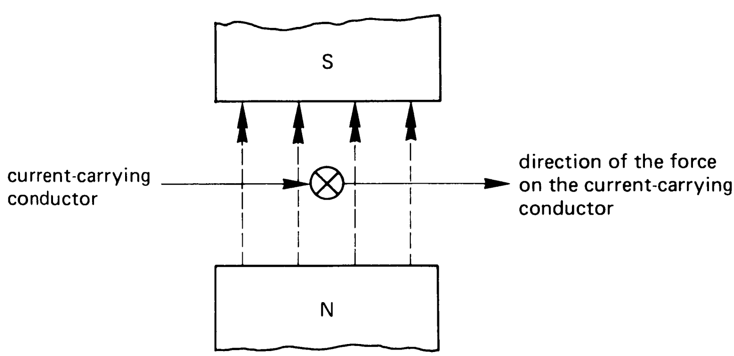
*F*irst finger - direction of the magnetic *F*lux
se*C*ond finger - direction of the *C*urrent
thu*M*b - direction of the *M*otion of (or force on) the conductor.

fig 9.1 *Fleming's left-hand rule*



An application of Fleming's left-hand rule is given in Figure 9.2.

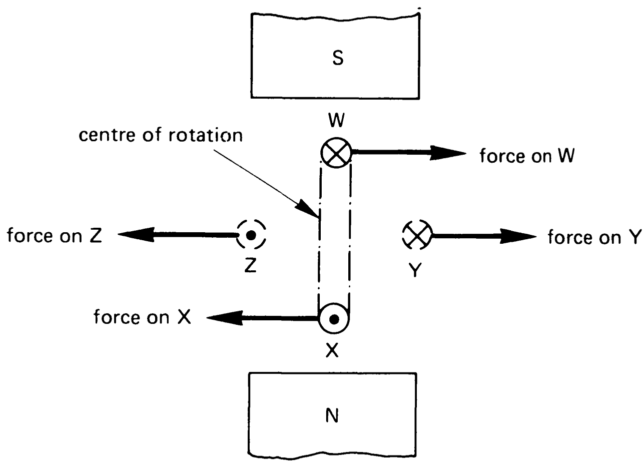
fig 9.2 *an application of Fleming's left-hand rule*



9.3 THE FORCE ON A CURRENT-CARRYING LOOP OF WIRE

Suppose that the loop of wire WX in Figure 9.3 is pivoted about its centre and carries a direct current in the direction shown. Fleming's left-hand rule shows that the force on wire W is in the opposte direction to that on wire X, the combined effect of the two forces causing the loop to rotate in a clockwise direction.

fig 9.3 *force on a current-carrying loop of wire*

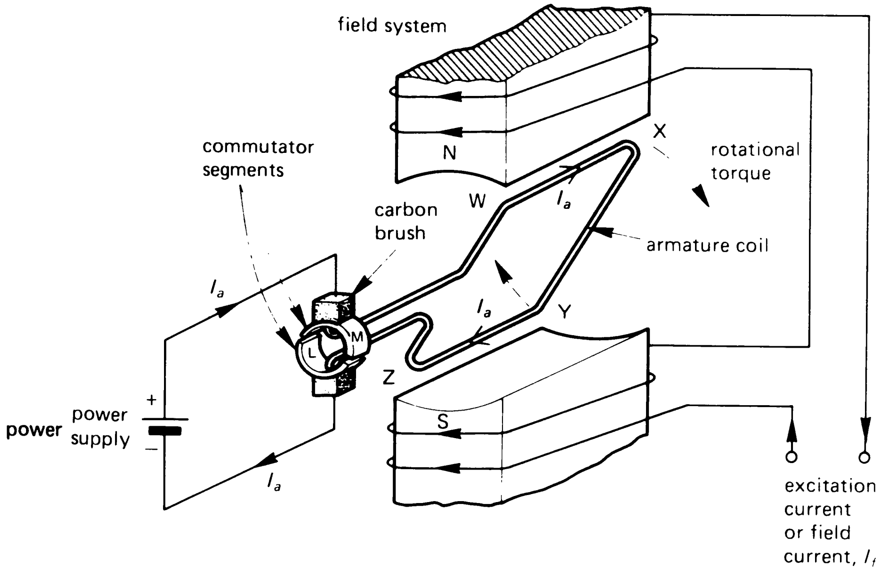


The loop of wire continues to rotate about its centre until it reaches position YZ. At this point, the forces on the two conductors are not only opposite in direction but are in the same plane of action. The net **torque** or mechanical **couple** acting on the coil is zero, and it stops rotating. Clearly, to obtain continuous rotation, the circuit arrangement must be modified in some way; the method adopted is described in section 9.4.

9.4 THE d.c. MOTOR PRINCIPLE

A simple single-loop d.c. motor is shown in Figure 9.4. In the motor shown, the magnetic **field system** is fixed to the frame of the motor, and the rotating part or **armature** supports the current-carrying conductors. The current in the field coils is known as the **excitation current** or **field current**, and the flux which the field system produces reacts with the **armature current** to produce the useful mechanical output power from the motor.

fig 9.4 *the basis of a d.c. motor*



The armature current is conveyed to the armature via carbon brushes and the commutator (the function of the armature having been described in Chapter 8). It is worthwhile at this point to remind ourselves of the functions of the **commutator**. First, it provides an electrical connection between the armature winding and the external circuit and, second, it permits reversal of the armature current whilst allowing the armature to continue to produce a torque in one direction.

At the instant shown in Figure 9.4, current flows from the positive terminal of the power supply into the armature conductor WX, the current returning to the negative pole of the supply via conductor YZ. The force on the armature conductors causes the armature to rotate in the direction shown (the reader might like to verify this using Fleming's rule).

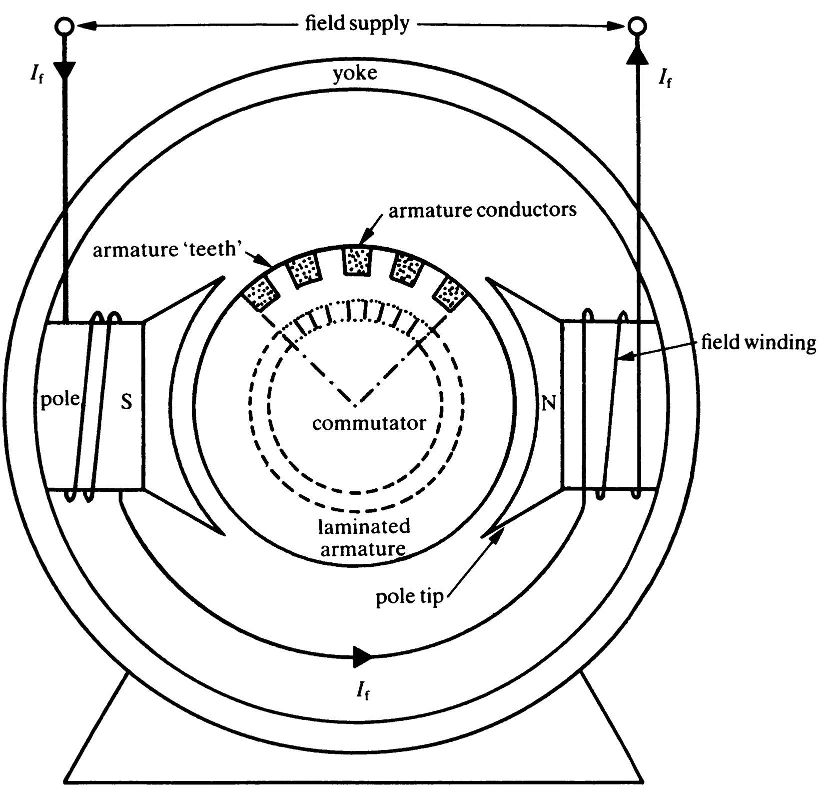
When the armature winding reaches the horizontal position, the gap in the commutator segments passes under the brushes so that the current in the armature begins to reverse. When the armature has rotated a little further, conductor WX passes under the S-pole and YZ passes under the N-pole. However, the current in these conductors has reversed, so that the torque acting on the armature is maintained in the direction shown in Figure 9.4. In this way it is possible to maintain continuous rotation.

9.5 CONSTRUCTION OF A d.c. MOTOR

A cross section through a d.c. motor is shown in Figure 9.5. The **frame** of the machine consists of an iron or a steel ring known as a **yoke** to which is attached the main **pole system** consisting of one or more *pairs of poles*. Each pole has a **pole core** which carries a **field winding**, the winding being secured in position by means of a large soft-iron **pole tip**. Some machines have a smaller pole known as an **interpole** or a **compole** between each pair of main poles; the function of the compoles is to improve the armature current commutation, that is, to reduce the sparking at the brushes under conditions of heavy load.

To reduce the eddy-current power-loss in the armature, the cylindrical **armature** is constructed from a large number of thin iron *laminations*. The armature conductors are carried in, but insulated from, teeth cut in the armature laminations.

fig 9.5 *d.c. machine construction*



9.6 MAGNITUDE OF THE FORCE ON A CURRENT-CARRYING CONDUCTOR

It has been found experimentally that the force, F, acting on a current-carrying conductor placed in a magnetic field is given by

$$\text{force}, F = BIl \text{ newtons (N)}$$

where B is the flux density (tesla) produced by the field system, I is the current (amperes) in the conductor, and l is the 'active' length in metres (m) of the conductor in the magnetic field.

For example, if a conductor of length 0.3 m carries a current of 100 A in a magnetic field of flux density 0.4 T, the force on the conductor is

$$\text{force}, F = BIl = 0.4 \times 100 \times 0.3 = 12 \text{ N}$$

(strictly speaking the force is on the current).

9.7 TORQUE PRODUCED BY THE ARMATURE OF A d.c. MOTOR

Each conductor on the armature of a motor is situated at radius r from the centre of the shaft of the armature. The **torque** or *turning moment* produced by each conductor is shown by

$$\text{torque} = \text{force} \times \text{radius} = Fr \text{ newton metres (N m)}$$

Since the armature has many conductors, each contributing its own torque, the total torque T produced by the armature is

$$\text{total torque}, T = NFr = N \times BI_a l \times r \text{ N m}$$

where N is the number of conductors and I_a is the armature current. If the magnetic flux produced by each pole of the motor is Φ webers and the area of each pole-piece is a square metre, the magnetic flux density, B, in which the armature conductors work is $\frac{\Phi}{a}$ T. That is to say

$$\text{total torque}, T = N \times \frac{\Phi}{a} I_a l \times r \text{ N m}$$

Now, for a given machine, several of the above factors are constant values as follows; the number of conductors (N), the area of the pole-piece (a), the active length of the conductor (l), and the radius (r) of the armature. Suppose that we let

$$\text{constant}, K = \frac{Nlr}{a}$$

Substituting this expression into the above equation for torque we get

$$\text{total torque}, T = K\Phi I_a$$

That is

total torque is proportional to ΦI_a

or

$$T \alpha \Phi I_a$$

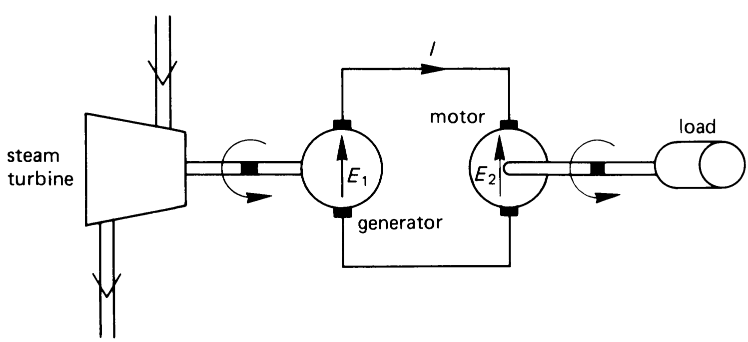
From this it can be seen that the primary factors affecting the torque produced by the motor are the flux per pole (Φ) and the armature current (I_a). Increasing either of these results in an increased torque.

9.8 'BACK' e.m.f. INDUCED IN A d.c. MOTOR ARMATURE

When the armature of a d.c. motor rotates, the conductors cut the magnetic field of the machine. From the earlier work in this book, you will appreciate that when a conductor cuts a magnetic flux, an e.m.f. is induced in the conductor. That is, even though the machine is acting as a motor, there is an e.m.f. induced in the armature conductors. However, in the case of a d.c. motor, the induced e.m.f. opposes the flow of current through the motor (and is another illustration of Lenz's law). For this reason, the e.m.f. induced in the armature of a d.c. motor is known as a **back e.m.f.**

This can be understood by reference to Figure 9.6. The generator armature is driven by a steam turbine, and the generated voltage is E_1; suppose that this voltage is 500 V. When the motor is driving its load, its armature has a 'back' e.m.f., E_2, induced in it. For a current to circulate from the generator to the motor, the value of E_1 must be greater than E_2. Suppose that E_2 is 490 V; the potential difference, $(E_1 - E_2) = 10$ V, is used up in overcoming the resistance of the circuit in order to drive current through it.

fig 9.6 *motor 'back' e.m.f.*



If the value of the armature current I is 10 A, the power generated by the generator armature is

$$E_1 I = 500 \times 10 = 5000 \text{ W}$$

The power consumed by the motor armature is

$$E_2 I = 490 \times 10 = 4900 \text{ W}$$

The difference between the two power values of

$$(5000 - 4900) = 100 \text{ W}$$

is consumed in the resistance of the circuit.

In practice, when a d.c. motor is *running at full speed*, the armature back e.m.f. is slightly less than that of the supply voltage. You should note that this applies at full speed only; if the speed is less than full speed, then the back e.m.f. is also less (this is discussed further in section 9.11).

Since the armature conductors have a 'back' e.m.f. induced in them, you can think of this e.m.f. as being a 'generated' e.m.f. (albeit a 'back' e.m.f.). From this point of view, the relationship between the back e.m.f. and other factors in the machine are the same for the motor as for the generator. We can therefore use the 'generator' equation for the back e.m.f. of the motor as follows:

$$\text{back e.m.f.} = K_E \Phi n$$

where K_E is a constant of the machine, Φ is the magnetic flux per pole of the motor, and n is the armature speed. Later we will use this equation to show how speed control of the motor can be achieved.

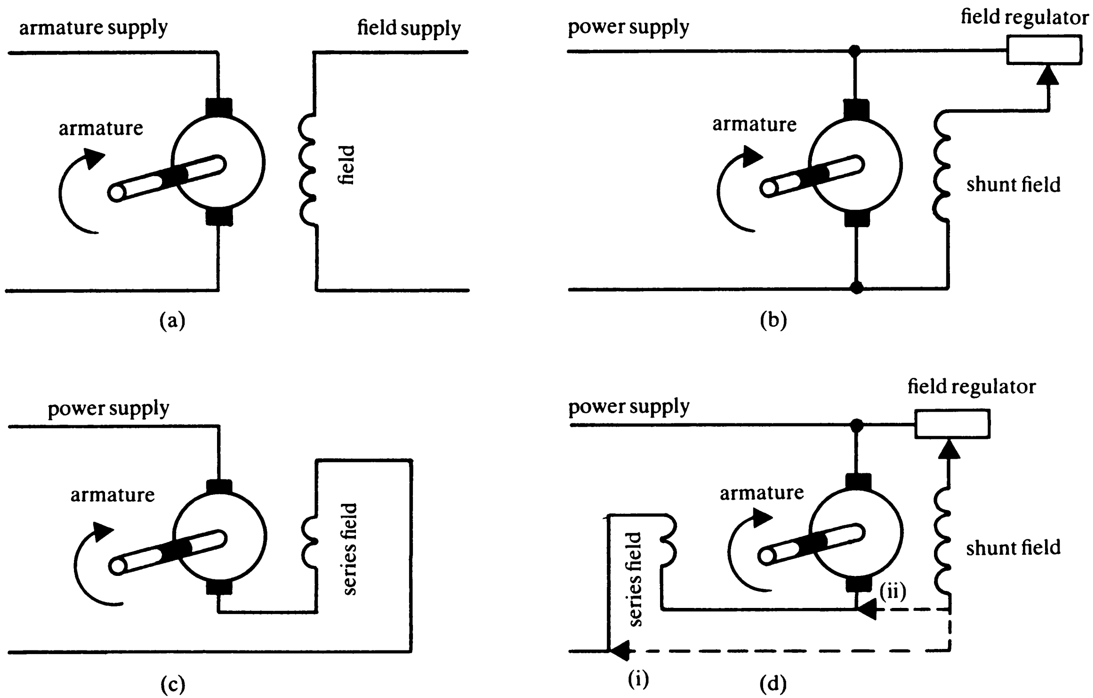
9.9 TYPES OF d.c. MOTOR

The 'name' or 'type' of d.c. motor is given to the way in which the field winding of the motor is connected. The four most popular methods of connection are shown in Figure 9.7.

Separately excited motor (Figure 9.7(a))

This machine has its armature fed from one d.c. source and its field from a separate d.c. source. This connection enables the armature and field voltages to be separately varied and allows the speed of the armature to be smoothly controlled from full speed in one direction to full speed in the reverse direction. Applications of this type of motor include machine-tool drives, steel rolling-mill drives, mine-winder drives, etc.

fig 9.7 *d.c. motor connections*



Shunt-wound motor (Figure 9.7(b))
In this type of motor, the field winding is connected in shunt or in parallel with the armature, both being supplied froma common power supply. Since the current in the shunt-field winding does not produce any useful output power from the motor, its value is kept to a minimum; to achieve this, the shunt field is wound by many turns of relatively fine wire (to give the winding a high resistance). In this type of motor; using the shunt-field **regulator** resistance shown in the figure, the speed of the armature can be regulated over a speed range of about 3:1.

Series-wound motor (Figure 9.7(c))
In this machine the field winding is connected in series with the armature, and must therefore be capable of carrying a heavy current. The series-field windings therefore comprise a few turns of thick wire. For a given armature current, this type of motor produces a much larger torque than the shunt motor. It is used in applications which utilise this large torque including road and rail traction.

Compound-wound motor (Figure 9.7(d))
This type of motor has two field windings, namely a series field which carries the armature current and a shunt field. The magnetic field produced by the two field windings can either assist or oppose one another, allowing a wide variety of machine characteristics to be obtained. Additionally, the shunt winding can be connected either across both the armature and the series winding (giving the *long shunt* version shown by the dotted line (i) in Figure 9.7(d)), or across the armature alone (giving the *short shunt* version shown by the dotted connection (ii)).

Some additional notes on d.c. machines
Although the machines listed here are described as 'd.c.' machines, in some cases it is possible to run them from an a.c. supply.

The essential difference between a d.c. and an a.c. supply is that, in the latter, the current reverses many times each second. Now, provided that the *same a.c. supply* is connected to both armature and field of the 'd.c.' motor, *the direction of the torque on the armature remains the same* (you can check this by simultaneously reversing both the armature current and the magnetic field in Figures 9.2 and 9.3, and applying Fleming's left-hand rule to verify that the direction of the force is unchanged). That is, an a.c. supply is (theoretically) just as good as a d.c. supply so far as certain d.c. motors are concerned. In general, a shunt motor, or a series motor, or a compound-wound motor are equally at home on a d.c. or an a.c. supply.

A problem associated with **large series motors** which does not occur in other d.c. motors is that, if the mechanical load is disconnected from its shaft, the armature 'races'and can reach a dangerously high speed. It is for this reason that large series motors often have an additional shunt-winding which limits the armature speed to a safe maximum value.

Many handtools such as drills and woodworking tools are driven by series motors. In general, the friction and windage power-loss in these machines is a very high proportion of the total load, so that the problem of the armature 'racing' when the load is disconnected is non-existent.

9.10 COMMUTATION PROBLEMS IN d.c. MACHINES

The current which causes the armature to produce its mechanical output power has to flow through the brushes and commutator before reaching the armature. Since the contact between the two is simply a mechanical one, there is a risk of electrical sparking at the point of contact. Several methods are adopted to reduce the sparking to a minimum and include:

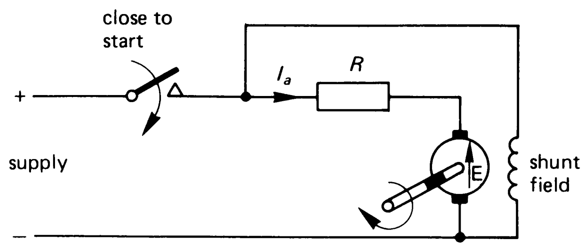
1. the use of brushes whose resistance is not zero (carbon brushes are used since they have about the correct resistance);
2. the use of brushes which span several commutator segments;
3. the use of *interpoles* or *commutating poles* (abbreviated to *compoles*) between the main poles. These poles induce a voltage in the armature which combats the sparking effect (compoles are fairly expensive, and are used only in large machines).

9.11 MOTOR STARTERS

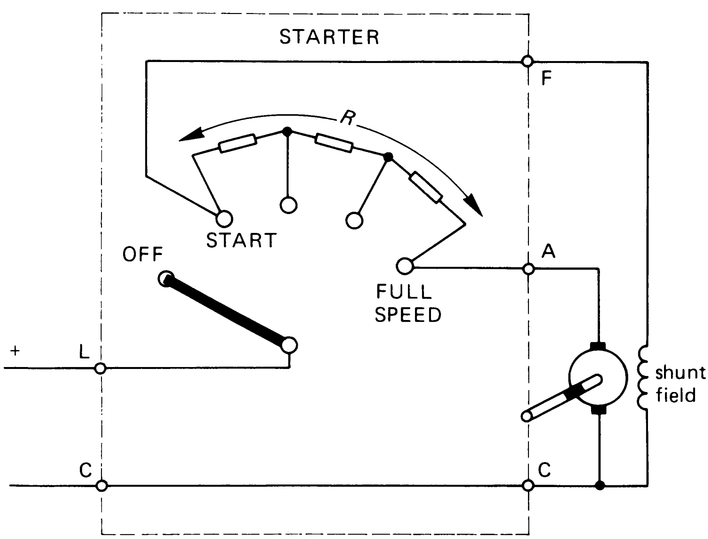
A d.c. motor with a rating above about 100 watts has a very low armature resistance (electrical handtools have a fairly low power-rating and have a fairly high armature-circuit resistance). Consequently, when the d.c. supply is switched on, the current drawn by the armature under starting conditions can be very high. Unless the starting current can be limited to a safe value, there is a considerable dange of the motor being damaged by

1. excessive heating (due to I^2R) in the armature;
2. sparking between the brushes and the commutator at the point of contact.

The simplest method of reducing the starting current is to insert resistor R (see Figure 9.8(a)) in series with the armature. Under starting conditions, the armature is stationary and the 'back' e.m.f. in the armature is zero. The p.d. across resistor R under starting conditions is therefore practically the whole of the supply voltage. If, for example, the supply voltage is fig 9.8 *simplified starter for a d.c. shunt motor*



(a)



(b)

240 V and the starting current must be limited to 10 A, the value of R is calculated from

$$R = \frac{V_S}{I} = \frac{240}{10} = 24\ \Omega$$

This current causes the armature to produce a torque, so that it accelerates from standstill. As it does, a back e.m.f. E is induced in the armature. Since this e.m.f. opposes the supply voltage, the p.d. across R (= $V_S - E$)

decreases. For example, when $E = 100$ V, the p.d. across R is $(240 - 100) = 140$ V; the armature current is then reduced to

$$\frac{(V_S - E)}{R} = \frac{140}{24} = 5.83 \text{ A}$$

Since the armature can safely withstand a surge of current of 10 A, it is possible to reduce the value of R to allow more current to flow, and to enable the armature to accelerate further.

One way in which this is done in a manually-operated starter is shown in Figure 9.8(b). The starting resistance R is divided into several steps; the arm of the starter is moved from the OFF position to the START position, and is then slowly moved from one stud to the next as the starting resistance is cut out of the armature circuit. When full speed is reached, the starting resistance is completely cut out of the circuit, and the supply voltage is connected directly to the armature. At this point in time, the back e.m.f. E in the armature has a value which is only slightly less than that of the supply voltage.

Although not shown in the starter circuit in Figure 9.8(b), a practical starter would incorporate circuits which protect the motor not only against *overcurrent* but also against *undervoltage* during normal operation.

Automatic starters are available which automatically reduce the starting resistance either on a timing basis or on a current value basis.

SELF-TEST QUESTIONS

1. Explain how Fleming's left-hand rule can be used to determine the direction of the current induced in a conductor.
2. Explain what is meant by the following terms in association with a d.c. machine: armature, commutator, brushes, field system, frame, interpole.
3. An armature conductor carries a current of 86 A. If the length of the conductor is 0.25 m and the force on the conductor is 8 N, calculate the flux density in the machine.
4. The torque produced by a motor is 1000 N m. If the magnetic flux is reduced by 20 per cent and the armature current is increased by 20 per cent, what torque is produced?
5. Name four types of d.c. machine and discuss applications for each of them.
6. Discuss the need for a d.c. motor starter. Draw a circuit diagram of a motor starter and explain its operation.

SUMMARY OF IMPORTANT FACTS

Motor action is caused by the force acting on a current-carrying conductor in a magnetic field. The direction of the force can be predicted by **Fleming's left-hand rule**.

A **d.c. motor** consists of a *rotating part* (the **armature**) and a *fixed part* (the **frame**). Electrical connection to the armature is made via **carbon brushes** and the **commutator**. The **torque** produced by the armature is proportional to the *product of the field flux and the armature current*. When the armature rotates, a **back e.m.f.** is induced in the armature conductors (this is by *generator action*) which opposes the applied voltage.

The four main types of d.c. motor are the **separately excited**, the **shunt wound**, the **series wound** and **compound wound** machines.

d.c. machines experience **commutation problems**; that is, sparking occurs between the brushes and the commutator. These problems can be overcome, in the main, by using brushes which have a finite resistance and which span several commutator segments (wide *carbon brushes*) together with the use of *interpoles* or *compoles*.

d.c. motors larger than about 100 W rating need a **starter** in order to limit the current drawn by the motor under starting conditions to a safe value.