ISSN 0015-4628, Fluid Dynamics, 2014, Vol. 49, No. 1, pp. 108–111. © Pleiades Publishing, Ltd., 2014.
Original Russian Text © A.N. Golovanov, F.M. Pakhomov, 2014, published in Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Zhidkosti i Gaza, 2014, Vol. 49, No. 1, pp. 126–130.

Supersonic Axisymmetric Flow past a Blunt Cone Executing Longitudinal Low-Frequency Oscillations

A. N. Golovanov[†] **and F. M. Pakhomov**

Received September 27, 2012

Abstract—The aerodynamic parameters of an oscillating cone in unsteady axisymmetric supersonic flow are investigated on the basis of the inviscid perfect gas model both in the absence and in the presence of strong air injection from its flat end into the shock layer.

Keywords: supersonic axisymmetric flow, inviscid flow model, flat end, longitudinal low-frequency oscillations, injection from a surface.

DOI: 10.1134/S001546281401013X

Longitudinal oscillations of a body in motion at a supersonic velocity may either occur as a result of natural vibrations or be forced [1]. Strong gas injection from the leading part of the body in the oncoming counter stream can be used for controlling its aerodynamic characteristics [2].

The purpose of this study is to investigate the effect of longitudinal low-frequency oscillations on the drag coefficient of a slender flat-ended cone both in the absence and in the presence of strong localized gas injection from the surface.

1. FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

The numerical modeling of supersonic flow past a body is based on the inviscid perfect gas model. In the cylindrical coordinate system $(x,\ r,\ \varphi)$, under the assumption of the axial symmetry the integral laws of conservation of the mass, the momentum components, and the energy take the form [3]:

$$\frac{\partial}{\partial t}\iint\limits_{s}\rho\,dr\,dx=\oint\limits_{L}\rho(\mathbf{V}-\omega,\,\mathbf{n})\,dL-\iint\limits_{s}\frac{1}{r}\rho v\,dx\,dr,$$

$$\frac{\partial}{\partial t}\iint\limits_{s}\rho u\,dr\,dx=\oint\limits_{L}\left[pn_x+\rho u(\mathbf{V}-\omega,\,\mathbf{n})\right]dL-\iint\limits_{s}\frac{1}{r}\rho uv\,dx\,dr,$$

$$\frac{\partial}{\partial t}\iint\limits_{s}\rho v\,dr\,dx=\oint\limits_{L}\left[pn_r+\rho v(\mathbf{V}-\omega,\,\mathbf{n})\right]dL-\iint\limits_{s}\frac{1}{r}\rho v^2\,dx\,dr,$$

$$\frac{\partial}{\partial t}\iint\limits_{s}\rho e\,dr\,dx=\oint\limits_{L}\left[p(V,\,n)+\rho e(\mathbf{V}-\omega,\,\mathbf{n})\right]dL-\iint\limits_{s}\frac{1}{r}(\rho e+p)v\,dx\,dr.$$

Here, t is time, ρ is the density, p is the pressure, u and v are the components of the velocity vector $\mathbf{V}$, n_x and n_r are the components of the vector $\mathbf{n}$ of the inward normal to the boundary L, ω is the boundary L displacement velocity vector, and e is the total specific energy of the gas.

[†] Deceased.

The governing system of equations is closed by the equation of state for a perfect gas with a constant adiabatic exponent γ

$$e = \xi + \frac{u^2 + v^2}{2}, \qquad \xi = \frac{1}{\gamma - 1}\frac{p}{\rho}.$$

The boundaries of the integration domain S are the bow shock, the body surface, the axis of symmetry of the flow, and the closing normal to the body surface at the cone periphery.

For the boundary conditions on the shock the Rankine–Hugoniot relations were taken. The symmetry conditions were preassigned at the axis of symmetry. At the exit boundary, which was located in the supersonic region of the shock layer to ensure the well-posedness of the problem, soft boundary conditions, when the flow parameters were downstream transferred, were preassigned. On the impermeable regions of the body surface the impermeability condition

$$(\rho v_n)_w = 0$$

was imposed (here, the subscript w refers to the body surface, while the subscript n means the projection on the normal).

As in [2], in the subsonic injection regions on the body surface the gasdynamic parameters were determined from the solution of the following nonlinear system of equations

$$(\rho v_n)_w = \text{const},$$

$$\frac{\gamma}{\gamma - 1}\frac{p_w}{\rho_w} + \frac{v_{n,w}^2}{2} \equiv H_{0,w} = \text{const},$$

$$v_{n,1} - v_{n,w} + \frac{2}{\gamma - 1}\sqrt{\gamma\frac{p_1}{\rho_1}}\left[1 - \left(\frac{p_w}{p_1}\right)^{(\gamma-1)/2\gamma}\right] = 0.$$

The first and second equations determine the specific flow rate of the injected gas and its total enthalpy, while the third equation, in which $v_{n,1}$, p_1, and ρ_1 are the gas parameters at the body surface, ensures the agreement between the injection parameters and the external conditions and is responsible for the continuity of the Riemann invariants on the left expansion wave relative to the inward normal to the body surface.

The longitudinal oscillations were modeled by the following law of variation of the body surface coordinates [1]

$$x^n = x^0 + A\sin(2\pi f t^n), \qquad y^n = y^0,$$

where f is the oscillation frequency, A is the amplitude, t is time, x^0 and y^0 are the coordinates of the initial position of the body in a flow, and x^n and y^n are the body surface coordinates at the moment $t = t^n$.

The gasdynamic parameters determined in the process of solution are dimensionless: the velocity components are divided by the maximum freestream velocity $V_{\max,\infty}$, the density to the freestream density ρ_∞, the pressure to the quantity $\rho_\infty V_{\max,\infty}^2$, and the linear dimensions to the base radius.

The governing system of gasdynamic equations with the corresponding initial and boundary conditions was solved using the explicit difference Godunov method of the first order in the independent variables [3]. Introducing a movable computation grid made it possible to realize the bow shock fitting. The contact surface, which existed in the case of gas injection from the body surface, was not explicitly fitted in the process of the numerical calculation of the problem. The results presented below were calculated on a computation grid containing 5000 cells (100×50).

2. SUPERSONIC FLOW PAST THE OSCILLATING BODY WITH AN IMPERMEABLE SURFACE

An analysis of the effect of small longitudinal low-frequency oscillations of a body on its aerodynamics was made with reference to the example of the supersonic ($M_\infty = 2$, $\gamma = 1.4$) axisymmetric flow past a

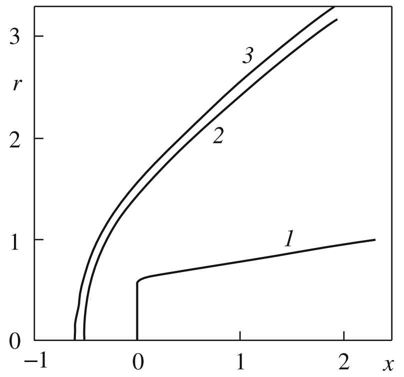

Fig. 1. Patterns of the flow past the model; (*1*) cone generator contour; (*2*) bow shock in the case of the impermeable body surface; and (*3*) bow shock in the case of injection from the flat end of the body.

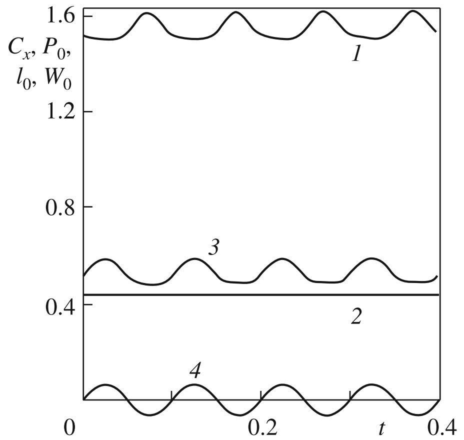

Fig. 2. Time dependence of the flow parameters in the absence of injection; (*1*) drag coefficient C_x; (*2*) pressure at the stagnation point P_0; (*3*) distance between the bow shock and the origin at the axis of symmetry l_0; and (*4*) distance between the body surface and the origin at the axis of symmetry W_0.

slender flat-ended cone. The contour of the cone generator is given by curve *1* in Fig. 1. The geometric parameters of the model are as follows: the flat end radius is 9.5 mm, the base section radius is 15.5 mm, and the model length is 36 mm. The corner point of the intersection between the flat end and the lateral generator of the cone is rounded by a circle, 1.5 mm in radius.

The initial data before the onset of the body oscillations corresponded to the parameters of steady flow past the model at the given freestream Mach number $M_\infty = 2$. The corresponding bow shock position is given by curve *2* in Fig. 1.

Figure 2 presents the time dependences of the main aerodynamic parameters in the case of the longitudinal oscillations of the impermeable body at a frequency of 10 Hz and an amplitude of 1 mm. Curve *1* corresponds to the variation of the drag coefficient C_x calculated without account for the base pressure and curve *2* to the dimensionless pressure P_0 at the stagnation point. Curves *3* and *4* are the dimensionless coordinates of the points of intersection of the axis of symmetry with the bow shock l_0 and the flat end W_0, respectively. The positive regions of curves *3* and *4* relate to the bow shock motion counter the freestream.

3. SUPERSONIC FLOW PAST THE OSCILLATING BODY IN THE CASE OF STRONG LOCALIZED INJECTION FROM THE SURFACE

Apart from the investigation of the supersonic aerodynamics of the impermeable oscillating body, we considered the case in which strong subsonic air injection into the shock layer was realized from the flat end of the body at the flow rate of 112 kg/(m^{2}s) and the stagnation temperature of 300 K. The injection is

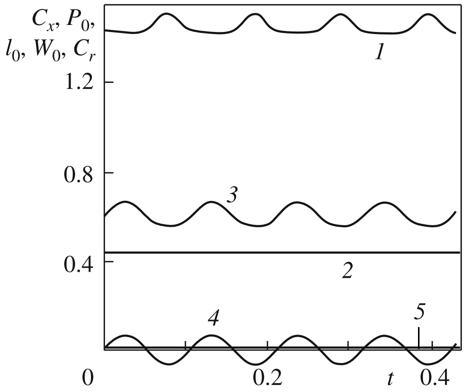

Fig. 3. Time dependence of the flow parameters in the presence of injection; (*1*–*4*) same as in Fig. 3 and (*5*) the reactive drag coefficient C_r.

so strong that it pushes the bow shock from the body surface, compared with the flow past the impermeable surface (curve *3* in Fig. 1).

In Fig. 3 the same characteristics, as in Fig. 2, are presented in the case of the injection from the flat end of the surface and the longitudinal oscillations at a frequency of 9.46 Hz and the same amplitude. The only difference is that we have one more curve *5* corresponding to the reactive drag coefficient C_r.

To check whether the hysteresis of the flow parameters exists in the case of gas injection from the model oscillating in a supersonic flow we carried out a calculation with an opposite initial phase of the body motion. In the process under consideration the hysteresis was not observed.

4. ANALYSIS OF THE RESULTS

The results presented in Sections 2 and 3 show that small longitudinal low-frequency oscillations of the body lead to low-frequency oscillations of the drag coefficient at a frequency coinciding with the body oscillation frequency and an amplitude not greater (in the cases considered) than 7% of its value under standard flow conditions. This is true both in the case of the impermeable surface and in the case of strong injection from the bluntness (curves *1* in Figs. 2 and 3). In both cases the bow shock stand-off distance at the axis of symmetry (the difference between curves *3* and *4* in Figs. 2 and 3) and stagnation pressure (curves *2*) remain constant in the process of the supersonic flow past the oscillating body. In the case of injection the reactive drag coefficient is also constant (curve *5* in Fig. 3); its sum with the wave drag coefficient is less than that in the absence of injection, which is in agreement with [2].

Summary. Small longitudinal low-frequency oscillations of a body in a supersonic flow do not lead to the expected reduction of its drag but, contrariwise, deteriorate its aerodynamic qualities.

Strong subsonic localized injection from the body bluntness has no effect on the nature of the interaction between the oscillating body and the supersonic flow.

The study was carried out with the support of the Analytical Departmental Program “Development of the Scientific Potential of the Higher School” under grant No. 2269.

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