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Multi-loop control system design

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Abstract. The approach based on a special case of the Laplace transform, which allows to design multi-loop system is considered. The tuning regulators program on the base of this approach is developed. The numerical example is shown.

Key words: control system, multi-loop system, plant identification, auto tuning controller, real interpolation method.

1. Introduction

One of the main current trends in the control theory is a multi-loop systems design [1]. There are a lot of papers considering this problem [2–4]. However, autotuning of such systems is the more interesting and complicated problem. There is a paper [5], where the adaptive control approach for the second order system with a master model is considered. In the paper [6] the regulator parameters tuning approach on the basis of plural integration is considered. In the paper [7] the new adaptive PI-regulator tuning approach without additional information about plant is suggested. All necessary information is obtained from an impulse response of the plant.

The main disadvantage of these approaches is limitations, which are caused by a regulator or the plant structure. So, the multi-loop systems design problem is a topical issue, but there is no universal solution.

The paper contains the multi-loop systems design approach based on the real Laplace transform.

2. Problem statement

Assume that the multi-loop system is put into consideration. The multi-loop system mathematical model is shown in Fig. 1.

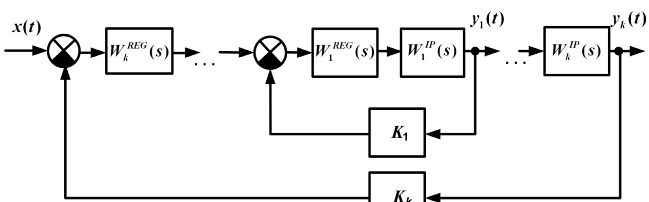

Fig. 1. Multi-loop system structure

Where $W_i^{IP}(s)$ – transfer function of invariable part of i-th loop, $i = \overline{1,k}$, where k – number of control loops; $W_i^{REG}(s)$ – regulator transfer function of i-th loop; K_i – feedback coefficient of i-th loop; $x(t)$ – input signal of a system; $y_i(t)$ – response of i-th loop.

The problem is to tune automatically each loop regulator in order to provide required quality of a main loop step response.

Three-step solution of this problem is proposed:

- The first step: to define plant transfer function $W_P(s)$ and to decompose it into blocks $W_i^{IP}(p)$.
- The second step is the master models $W_i^M(s)$ form, which defines the quality of i -th loop.
- The third step is regulators $W_i^{REG}(s)$ tuning.

3. Real interpolation method

The real interpolation method is a mathematical base for the problem solution. This method includes approaches and algorithms for dynamical systems researching and is based on real integral transform and interpolation procedures.

The real integral transform is defined by formula [8]:

$$F(\delta) = \int_0^\infty f(t)e^{-\delta t}dt, \quad \delta \in [C,\ \infty), \quad C \geq 0, \qquad (1)$$

where original function $f(t)$ corresponds to image function $F(\delta)$.

Formula (1) is called δ-transform.

Practically it is important that δ-transform can be interpreted as a special case of the Laplace transform

$$F(s) = \int_0^\infty f(t)e^{-st}dt, \quad s = \delta + j\omega$$

and be obtained by the substitution of real variable δ instead of complex variable s. This feature is very important for numerical calculations as far as it allows reducing number of calculations to half in comparison with the frequency method because of calculations with the absence of the imaginary part

$$j = \sqrt{-1}.$$

4. Plant identification

To solve an identification problem of the system, all feedbacks are disconnected, and regulators have forms $W_i^{REG}(p)$=1,

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$i = \overline{1,k}$. Then test signal $x(t)$ is given on the system input and output signals $y_i(t)$ are obtained. On the basis of obtained data and Eq. (2) the real transfer function is defined

$$W_{y_i x}(\delta) = \frac{\int\limits_0^{\tau_i} y_i(t) \cdot \exp(-\delta t) dt}{\int\limits_0^{\tau_i} x(t) \cdot \exp(-\delta t) dt}, \tag{2}$$

where τ_i – signal $y_i(t)$ time observation.

So, the plant transfer function may have two forms:

$$W_P(\delta) = W_{y_n x}(\delta), \tag{3}$$

$$W_P(\delta) = \prod_{i=1}^{k} W_i^{IP}(\delta). \tag{4}$$

On the basis of (3) and (4) it is possible to get dependency between $W_i^{IP}(\delta)$ and $W_{y_i x}(\delta)$.

$$W_1^{IP}(\delta) = W_{y_1 x}(\delta),$$

$$W_i^{IP} = \frac{W_{y_i x}(\delta)}{W_{y_{i-1} x}(\delta)}, \tag{5}$$

$$i = \overline{2,k},$$

where $W_{y_i x}(\delta)$ the transfer function from input x to output y_i, in other words the transfer function of i-th loop.

According to (5) real transfer functions of each loop invariant parts are defined.

The following solution of an identification problem defines $W^{IP}(\delta)$ in a fractional rational form

$$W^{IP}(\delta) = \frac{b_m \delta^m + b_{m-1} \delta^{m-1} + ... + b_1 \delta + b_0}{a_n \delta^n + a_{n-1} \delta^{n-1} + ... + a_1 \delta + 1}, \quad n \geq m. \tag{6}$$

The problem of the structure identification that is defining orders of polynomials numerator m and denominator n is solved on the base of the approach described in paper [9].

Let us consider the equation:

$$\lim_{\delta \to \infty} \frac{W^{IP}(\delta)}{W^{IP}(g \cdot \delta)} = g^{n-m}, \tag{7}$$

where $g > 1$ – a real number. From expression (7) the following equation is obtained:

$$\gamma = \widehat{n-m} = \frac{\ln(g^{n-m})}{\ln(g)}. \tag{8}$$

The real number obtained from (8) is rounded up. With the help of the obtained structure parameters estimation γ it is possible to form the transfer function structure identification algorithm. To make this, it is necessary to express a numerator m order through a denominator n order and the estimation value γ

$$m = n - \gamma, \quad n = \begin{cases} 1, 2, ... & \text{if } \gamma = 0, \\ \gamma, \gamma + 1, ... & \text{if } \gamma \neq 0. \end{cases} \tag{9}$$

The value n is a free argument in the last equation. The existence of the fixed parameter γ, allows to get rid of examination of a set of transfer function structures, which do not meet the requirements of Eq. (9). The parameter n value enumeration should be continued until a relative identification error satisfies the specified criterion within the time domain. This is the solution of the structure identification problem.

After the transfer function numerator and denominator defining it is necessary to define coefficients

$$a_1 \div a_n, \qquad b_0 \div b_m.$$

To this effect the Eq. (6) should be transformed into the linear equations system with unknown variables (coefficients

$$a_1 \div a_n, \qquad b_0 \div b_m)$$

and predefined interpolation nodes

$$\delta_j, j = \overline{1, \nu},$$
$$\nu = n + m + 1.$$

Nodes are defined by the equation $\delta_j = j \cdot \delta_1$. The value of δ_1 is taken according to Eq. (8)

$$\delta_1 = \frac{-\ln\left((0.01 \div 0.05)/y(t^{st})\right)}{t^{st}}, \tag{10}$$

where $y(t^{st})$ – output steady-state value, t^{st} – settling time.

Then it is possible to obtain the system

$$\begin{cases} b_m \delta_1^m + b_{m-1} \delta_1^{m-1} + ... + b_1 \delta_1 + b_0 - a_n \delta_1^n W^{IP}(\delta_1) - ... - a_1 \delta_1 W^{IP}(\delta_1) = W^{IP}(\delta_1), \\ b_m \delta_2^m + b_{m-1} \delta_2^{m-1} + ... + b_1 \delta_2 + b_0 - a_n \delta_2^n W^{IP}(\delta_2) - ... - a_1 \delta_2 W^{IP}(\delta_2) = W^{IP}(\delta_2), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ b_m \delta_\nu^m + b_{m-1} \delta_\nu^{m-1} + ... + b_1 \delta_\nu + b_0 - a_n \delta_\nu^n W^{IP}(\delta_\nu) - ... - a_1 \delta_\nu W^{IP}(\delta_\nu) = W^{IP}(\delta_\nu). \end{cases} \tag{11}$$

The result of a system solving is the plant real transfer function. To obtain a transfer function in the Laplace space it is necessary to make a formal substitution $s \to \delta$.

5. Master transfer functions forming

The certain requirements to the step response in a multi-loop system are made only to a main loop, obviously, that causes master transfer functions $W^M(s)$ forming complexity. This as a rule do not work for other loops any requirements. One of the ways to make this problem solution more formal is to consider the maximal time constant of the transfer function $W^{IP}(\delta)$. In the capacity of such a time constant the coefficient a_1 is taken. It is known [1] that the dependency between settling time t^{st} and coefficient a_1 has a view:

$$t^{st} \geq 3 \cdot a_1. \tag{12}$$

On the base of expression (12) and using the Konovalov-Orourke method [10] it is possible to form the master transfer function

$$W^M(p) = \frac{\dfrac{\alpha_1}{2}p + 1}{\alpha_0 p^2 + \alpha_1 p + 1} H,$$

where H – output steady-state value, α_1, α_0 – coefficients defined by equations:

$$\alpha_0 = \frac{[\ln(0.01 \cdot \sigma)]^2}{\dfrac{9}{(t^{st})^2}\left\{[\ln(0.01 \cdot \sigma)]^2 + \pi^2\right\}} H, \quad \alpha_1 = \frac{6 \cdot \alpha_0}{t^{st}},$$

where $\sigma = 100 \cdot \dfrac{H_{\max} - H}{H}$ – required overshoot, $H_{\max}$ – maximum output value.

6. Multi-loop system controllers design

The last step of the multi-loop system control system tuning is a controller design. The controller design procedure consists of one operation set. Controllers tuning are made consequently from an inside loop to outside one. The solution of the design problem is based on solving of the following equation

$$W^{OM}(s) \cong W^{REG}(s) \cdot W^{IP}(s), \tag{13}$$

where $W^{OM}(s)$ is an open-loop master transfer function. It is defined from the equation:

$$W^{OM}(s) = \frac{W^M(s)}{1 - K \cdot W^M(s)}. \tag{14}$$

In order to get Eq. (13) it is necessary to substitute the value of feedback coefficient K to Eq. (14).

The value K can be chosen according to the expression $K = 1/H$, to obtain the first order astaticism of the loop. In this case the solution of Eq. (13) is reduced to defining parameters and a structure of the transfer function $W^{REG}(p)$. As a rule, the structure of a controller is known and the problem is to define the parameters. So, according to the expression (13) the controller transfer function is defined as

$$W^{REG}(s) = \frac{W^{OM}(s)}{W^{IP}(s)}. \tag{15}$$

Also, the controller transfer function has the following form

$$W^{REG}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + .. + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + .. + a_1 s + 1}, \quad n \geq m. \tag{16}$$

From (15) and (16) the following expression is obtained

$$\frac{W^{OM}(s)}{W^{IP}(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + .. + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + .. + a_1 s + 1}. \tag{17}$$

A real transfer function of a controller is obtained after formal substitution$\delta \to s$ in (17).

By taking different values of $\delta = \{\delta_1, \delta_2, ..., \delta_\nu\}$, where $\nu = n + m + 1$, ν linear algebraic equation with ν unknown parameters are received. Nodes $\delta$are spread on the analytical grid $\delta_j = j \cdot \delta_1,\ j = \overline{1, \nu}$. The first node is defined by expression (10).

The design problem solution is the solution of the system (11) in which $W^{IP}(\delta)$ is substituted by Eq. (15).

On the basis of the obtained results the multi-loop system design program was developed that contains blocks: "Identification" and "Master model forming and controller design".

7. Numerical example

Assume that the two-loop control system is considered. The structure of the controlled object is shown in Fig. 2.

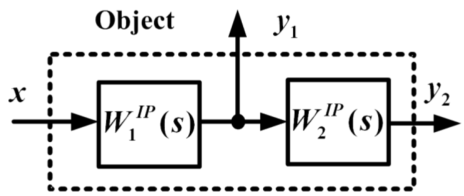

Fig. 2. Structure of the controlled object

To provide identification, the input test signal $x(t)$ is used and output signals $y_1(t)$ and $y_2(t)$ are obtained (Fig. 3).

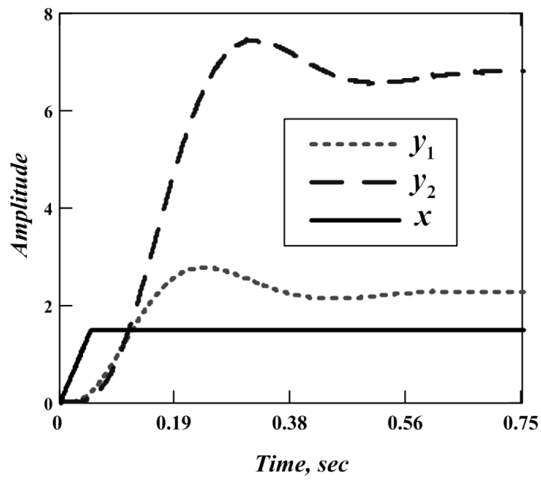

Fig. 3. Input signal and system responses

Figure 3 shows that the time of observation for signals $y_1(t)$ and $y_1(t)$ should be $\tau_1 = \tau_2 = 0.75$ seconds, respectively. According to expression (8) the structure of transfer functions $W_p^(\delta)$, $W_1^{IP}(\delta)$ and $W_2^{IP}(\delta)$ are defined and values $\gamma_p = 3$, $\gamma_1^{IP} = 2$, $\gamma_2^{IP} = 1$ are found. Finally, parameters that describe structures of models are:

– $n_p = 3$, $m_p = 0$ for transfer function $W_p(\delta)$;
– $n_1^{IP} = 2$, $m_1^{IP} = 0$ for transfer function $W_1^{IP}(\delta)$;
– $n_2^{IP} = 1$, $m_2^{IP} = 0$ for transfer function $W_2^{IP}(\delta)$.

For all the calculations associated with the identification interpolation, the node δ_1 is taken according to (10) and has the value $\delta_1 = 6.14$.

Using (2) and (3) to find $W_p^(\delta)$ it is necessary to form the linear equations system (11). As a result of identification the transfer function is obtained:

$$W_P(\delta) = \frac{4.38}{2.32 \cdot 10^{-4}\delta^3 + 6.82 \cdot 10^{-3}\delta^2 + 0.117\delta + 1}.$$

To decompose transfer function $W_p^(\delta)$ into $W_1^{IP}(\delta)$ and $W_2^{IP}(\delta)$ expression (5) is used. After this linear equations system (11) is formed for each transfer function. The result of decomposition is:

$$W_1^{IP}(\delta) = \frac{1.49}{3.47 \cdot 10^{-3}\delta^2 + 0.05\delta + 1},$$

$$W_2^{IP}(\delta) = \frac{2.94}{0.067\delta + 1}.$$

According to (12) performance and other requirements for each control loop are defined.

The requirements of first loop are $t_1^{st} \leq 0.3$ s, $H_1 = 3$, $\sigma_1 \leq 15\%$.

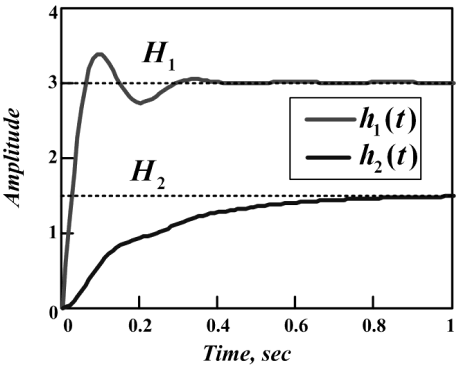

Fig. 4. Step responses

For second loop the requirements are: $t_2^{st} \leq 0.9$ s, $H_2 = 1.5$, $\sigma_2 \leq 1\%$.

The obtained controllers transfer functions in the Laplace space have views:

$$W_1^{REG}(s) = \frac{0.148s^2 + 5.394s + 74.68}{6.99 \cdot 10^{-4}s^2 + s},$$

$$W_2^{REG}(p) = \frac{0.075s + 0.784}{s}.$$

The feedback coefficient for the first loop is $K_1 = 0.333$ and for the second loop is $K_2 = 0.667$.

There are step responses of first $h_1(t)$ and second $h_2(t)$ loops in 8, which are considered separately.

Figure 4 shows that parameters of transients in control loops are:

– First loop: $t_1^{st} = 0.274$ s, $\sigma_1 = 12.9\%$;
– Second loop: $t_2^{st} = 0.869$ s, $\sigma_2 = 0\%$.

Results of numeric example demonstrate that tuned controllers provide the required level of performance and quality in the multi-loop control system.

8. Conclusions

In the paper actuality of the self-tuning of multi-loop system controller is shown. The aim is to develop mathematical apparatus for designing multi-loop control systems. For this purpose the approach based on a real integral transform is proposed. A significant advantage of this approach consists in capability of describing dynamical plants in a form of numeric real values. The approach allows solving the plant identification problem and the regulator designing problem. The numerical example is considered.

REFERENCES

[1] W.S. Levine, *The Control Handbook*, CRC Press, Boca Raton, 1996.

[2] R. Osypiuk, B. Finkemeyer, and F. Whal, "Multi-loop model-based control structure for robot manipulators", *Robotica* 23 (4), 491–499 (2005).

[3] T. Liu, W. Zhang, and D. Gu, "Analytical multiloop PI/PID controller design for two-by-two processes with time delays", *Ind. and Eng. Chem. Res.* 44 (6), 1832–1841 (2005).

[4] D.H. Kim, H. Wang, and E.S. Kim, "Cascade observers for nonlinear systems and application to nonlinear output feedback control", *JSME Int. J. C.* 49 (2), 463–472 (2006).

[5] L. Hsu, "Lyapunov/passivity-based adaptive control of relative degree two MIMO systems with an application to visual servoing", *IEEE Trans. Autom. Contr.* 52 (2), 364–371 (2007).

[6] I. Ganchev, "Auto-tuning of cascade systems with auxiliary corrector", *Proc. SAER*, 46–50 (2004).

[7] N. Petrkov, "A new approach for adaptive tuning of PI controllers. Application in cascade systems", *Proc. Inf. Technol. and Contr.* 6 (1), 19–26 (2008).

[8] V.I. Goncharov, *Real Interpolation Method for the Synthesis of Automatic Control Systems*, TPU Press, Tomsk, 1995.

[9] Y.N. Shalaev, "Modeling of nonstationary dynamical systems by representing the vectors", *Proc. TPU* 309 (7), 44–47 (2006).

[10] A.A. Voronov and I.A. Orourke, *Analysis and Optimal Synthesis of Control Systems Using Computer*, Science Press, Moscow, 1984.